# New Keynesian Perspectives on Monetary Policy and the Business Cycle in Closed Economies and Monetary Unions

INAUGURAL-DISSERTATION

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# Vorwort

Die vorliegende Arbeit entstand während meiner Tätigkeit als wissenschaftlicher Mitarbeiter am Lehrstuhl für Geld und Währung an der Bayerischen Julius-Maximilians-Universität Würzburg. Im November 2005 wurde sie von der wirtschaftswissenschaftlichen Fakultät der Universität Würzburg als Dissertation angenommen.

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Würzburg, im Oktober 2006 Eric Mayer

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# Definitions of Variables

## General Issues

$U_x$: First derivative of utility with respect to x

$U_{xx}$: Second derivative of utility with respect to x

$U_{xy}$: Partial derivative with respect to x and y

$\overline{X}$: Natural level of a variable X

$x_{-i}$: All countries except country i

$x^e$: Expected value of x

$\Delta x$: $LnX - LnX_{-1}$

$x^b$: Backward-looking expectations on x

$x^f$: Flex-price equilibrium

$x^{NT}$: Non-Tradables

$x^{rat}$: Rational expectations formed on variable x

$x^T$: Tradables

$\hat{x}$: Logarithmic deviation of a variable from its steady state level $\overline{X}$

$\overline{x}$: Four quarter moving average of x

## Variables

A: Technology shock

B: Nominal bonds

C: Consumption

g: Nominal fiscal balance

$\overline{g}_1$: Structural fiscal balance

$i$: Nominal interest rate administered by the central bank

J: Value function

L: Loss function of the central bank

M: Nominal money balances

MC: Marginal cost

N: Number of hours worked

P: Price level

$P_j$: Price charged by firm j in the intermediate good sector for product $Y_j$

Q: Real exchange rate

$\tilde{Q}$: Optimal reset price

S: Nominal exchange rate

Y: Aggregate income

$Y_j$: Production of firm j in the intermediate good sector

W: Nominal wages

Z: Optimal reset price

$c_i$: Responsiveness of the output gap to TOT-effects

$v_t$: Index of real wages

## Greeks

$p$ : Inflation rate

$\pi_0$: Inflation target of the central bank

$\Pi_j$: Nominal profits of firm j in the intermediate good sector

$\Delta$ : Stochastic discount factor, pricing kernel

# 1 INTRODUCTION AND OVERVIEW

By the end of the 1990's Real Business Cycle Theory had become the dominant macroeconomic doctrine (Plosser (1989)). According to that theory economic cycles are driven by technological innovations reflecting the natural volatility grounded out of the dynamic Walrasian general equilibrium system. The old Schumpeterian idea (Schumpeter (1912)) that the business cycle is nothing but a manifestation of the dynamic process of capitalism itself had found a first rudimentary mathematical foundation. This theory leaves no meaningful role for a benevolent central bank as recessions are no application for stabilization policy but on the contrary have cleansing effects driving inefficient producers out of markets.

The return of Keynesian theory in the early 1990's was driven by two major streams coming from academia and institutional changes. On the one hand empirical evidence seemed to suggest that major central banks implement their policy by setting a nominal short term interest rate as their operating target. This empirical finding reemphasized the role of interest rates for monetary transmission and its implications for the business cycle. In a seminal paper Blinder and Bernankee (1992) find evidence using Vector Autoregressive Analysis (VAR) for a decline in bank loans and real output roughly contemporaneously after a monetary tightening in the form of higher interest rates. Several years later first seminal papers were published that implemented interest rate policy into a dynamic stochastic general equilibrium framework (DSGE) (Clarida, Gali and Gertler (1999)). In another influential study Taylor (1993) reviewed earlier work by Bryant, Hooper and Mann (1993). He reports evidence that leading state of the art models had three major common conclusions concerning the conduct of monetary policy rules. First, it prevailed that monetary targeting rules were outperformed by interest rate rules in terms of the loss inflicted on society. Second, interest rate rules that react on inflation and on output performed better than rules that just focused on only one of these variables. Third, instrument rules that directly react on movements of the exchange rate lead to worse results for society than rules that neglect the exchange rate. In a quest to condense these key insights into a simple and transparent rule Taylor proposed the by now well known Taylor rule.

This major academic stream was accompanied by institutional changes that took place in the strategic framework of leading central banks. At the beginning of the 1990's many central bank's stood in front of a pile of broken classes as monetary targeting or exchange rate targeting had failed. The Bank of England and other leading central banks implemented a full fledged regime of ‘Inflation Targeting’. Due to the clear cut theoretical concept and the obvious success in terms of keeping the inflation rate close to the inflation target ‘Inflation Targeters’ have become prime examples for central banks that implement a successful and transparent strategy that combines new theoretical insights and its practical implementation. From an academic perspective key for the return of New Keynesian macroeconomics was the reinvention of a non-vertical Phillips curve derived from solid microeconomic relationships which made Keynesian economics presentable at a theoretical level as it provided a micro-founded justification for stabilization policy. Due to the existence of nominal inertia the central bank had a meaningful role to protect society from aggregate shocks. Nominal inertia enables the central bank to manage aggregate demand via steering the real interest rate according to its preferences. New Keynesian macroeconomics created a new apparatus of thought. It is now possible to think Keynesian but micro-founded at the same time.

As an important contribution to literature we present within this study a simple but at the same time powerful static version of a New Keynesian macromodel. In spite of its simplicity it can carry the main insights of New Keynesian macroeconomis (see Clarida, Gali and Gertler 1999) to an intermediate level and deal with issues such as inflation targeting, monetary policy rules, and central bank credibility.

Within this study we build on this new apparatus of thought to find convincing answers to questions surrounding the conduct of monetary policy in a currency area. The unique feature of a currency area is that different macroeconomic agents, the ECB, national governments and labor unions focus on different levels of macroeconomic aggregates. The ECB whose policy we assume to be conducted according to the notion of Inflation Targeting focuses on union wide averages, whereas national governments focus on national aggregates. Surprisingly the effects of diverging real interest rates and its impact on economic activity is not yet well understood. In a monetary union idiosyncratic supply shocks might be destabilizing for individual member states even if the common central bank implements the Taylor principle.

The study is structured as follows: In the first chapter we will derive and review the theoretical and mathematical foundations of New Keynesian economics. In the focus of our analysis stands the interaction between a representative household and a representative firm. We will analyze in depth the habitat of a representative agent. We will see that the advancement of New Keynesian economics was driven by two factors. On the one hand by the quest to derive macroeconomic equations from solid microeconomic relationships and on the other hand from the desire to be able to explain the data. We will identify the key parameters of New Keynesian macromodels governing the disequilibrium dynamics. In particular we will analyze to what extend different mechanisms of expectation formation impact on the correlation structure of the model. As a contribution to literature we will show how to simplify the New Keynesian model into a simple but powerful framework.

Equipped with this apparatus we will then analyze the neuralgic points of a currency union. Quite surprisingly this stream of literature seems heavily under researched. The main focus of research (e.g., Dixit and Lambertini (2003)) is still on potential target conflicts which may arise if the common central bank and the national governments have diverging preferences on the inflation target or trend growth in output. The question how monetary and fiscal policy should react to asymmetric shocks originating in some parts of the currency area is not addressed at all. Our analysis will show that the creation of a currency area calls for a renaissance of fiscal policy from a stabilization perspective. In particular we will show that the current macroeconomic design as enshrined in the Stability and Growth Pact (SGP) is too one dimensional as it neglects the interplay between monetary and fiscal policy in a currency area. We will make some propositions along which we think the SGP should be reformed.

# 2 THE STRUCTURE AND MECHANICS OF NEW KEYNESIAN MACROECONOMICS

Over the last decade a new consensus model has emerged in monetary macroeconomics, labeled New Keynesian macroeconomics (Clarida, Gali and Gertler (1999), Woodford (2003)). It consists of three simple building blocs: a forward-looking IS-equation that is derived from the constraint optimization problem of a representative household, a forward-looking Phillips curve that maps the optimal pricing decision of monopolistically competitive firms facing restrictions on their ability to adjust prices in a flexible manner, and a relationship that describes how monetary policy is conducted

This introductory study serves as a map that comprises the key elements of New Keynesian macroeconomics. On the one hand we will supply in depth descriptions of state of the art New Keynesian macromodels derived from utility functions of representative households and the intertemporal optimization calculus of monopolistically competitive firms (Woodford 2003). We will see how it is possible to legitimize macroeconomic stabilization policy from a microeconomic perspective (Woodford (2001)). This roundtrip through New Keynesian macroeconomics will show the virtues and shortcomings of this macroeconomic doctrine. In principle we will see that it is possible to reduce the New Keynesian macromodel to a system of three equations.

As an important contribution to literature we will present a simplified framework that condenses the key insights of New Keynesian macroeconomics into a static model. In spite of its simplicity it can carry the main insights of New Keynesian macroeconomis (see Clarida, Gali and Gertler 1999) to an intermediate level and deal with issues such as inflation targeting, monetary policy rules, and central bank credibility. Compared to existing literature we propose a more general approach to model expectations in a static framework (Walsh, 2002). Additionally we present the linkages between our static approach and a dynamic macromodel by means of impulse response analysis.

After having identified these key relationships we will systematically analyze the implicit dynamics nested within a New Keynesian macromodel. In particular we will show that the cognitive abilities of forming expectations are key to understand the abilities of central banks to smooth out macroeconomic fluctuations. If economic agents have a high degree of awareness on the functioning of the economy they will react stronger on changes in the real interest rate. Economic agents that are backward looking will ignore changes in the monetary stance and just be guided by actual macroeconomic outcomes.

## 2.1 The Structure of New Keynesian Macromodels: A Review

This introductory chapter serves as a roadmap for the chapters to come. Within this chapter we describe the "social habitat" of our representative agent. This seems necessary as in depth descriptions combined with complete and accurate mathematical derivations are rarely found. Walsh (2003) and Woodford (2003) are notable exceptions although concrete derivations are often left to the interested reader.

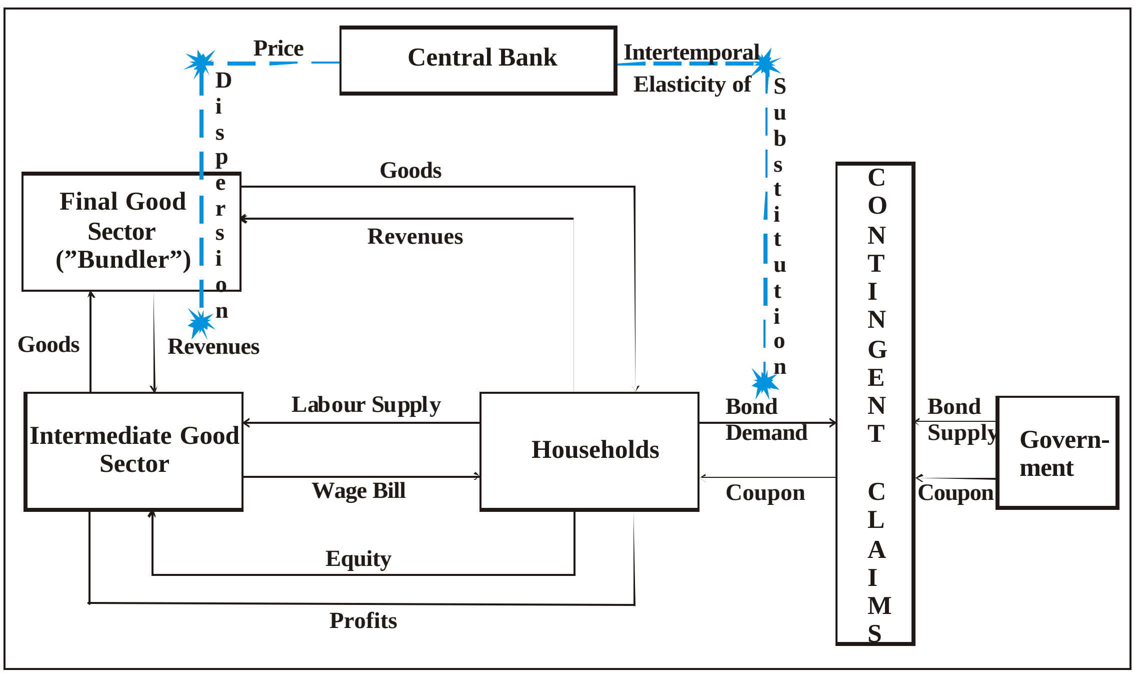



Figure 1: The Interaction between Sectors in the New Keynesian Model

Own Source

Before going into details let us take a preliminary look at the individual sectors of the economy. The representative household supplies work $N_t$, consumes the final good $C_t$ and receives profits $\Pi_t$. Households work in the intermediate good sector from which they receive their wage bill $W_tN_t$ for the work effort they supply. In the simplest version of the New Keynesian macromodel labor markets are assumed to be perfectly flexible. Households will spent a part of their wage bill on consumption $P_tC_t$; the rest will be saved either in terms of money $M_t$ or bond $B_t$ holdings. The existence of complete contingent claims markets enables households to spread consumption over time. One unit of government bonds today will be redeemed tomorrow including the interest rate payments $(1+i_t)B_t$. Money holdings do not earn an interest. Additionally our representative household is a shareholder of the intermediate good sector. As these firms operate in an environment of monopolistic competition they earn profits $\Pi_t$ on their fixed capital stock $\bar{K}$. Profits are transferred to shareholders. Therefore the flow budget constraint, which traces the different types of activities households unfold, can be stated as follows:

$$P_{t+j}C_{t+j} + M_{t+j} + B_{t+j} \leq W_{t+j}N_{t+j} + \left(1+i_{t-1+j}\right)B_{t+j-1} + M_{t+j-1} + \Pi_{t+j} \quad \text{for } j=0,1,2,\ldots \quad (2.1)$$

The second single most important sector that gives a Keynesian flavor to our economy is the intermediate good sector which operates in an environment of monopolistic competition and sticky prices (Dixit and Stiglitz (1977)). The monopolistically competitive firm sells its products to the final good sector subject to a standard isoelastic demand function (Dixit and Stiglitz (1977))

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-e} Y_t, \quad (2.2)$$

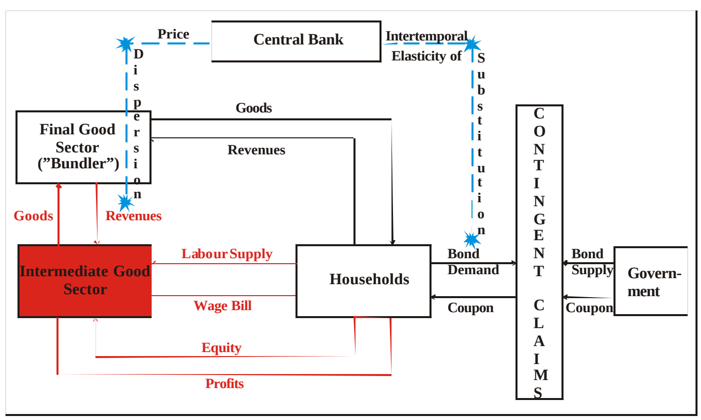
where $p_{jt}$ denotes the price of its own product in relation to the overall price index $P_t$, and the elasticity of demand is denoted by $\varepsilon$.

The final good sector which bundles the output of the intermediate good sector into the aggregate commodity $Y_t$ is assumed to operate in an environment of perfect competition. This means in particular that the final good is sold at marginal costs. This assumption implies that the revenues of the final good sector are simply the weighted average of the input prices charged by the individual firms operating in the intermediate good sector. This statement can be written more compactly in mathematical terms as follows:

$$P_t Y_t - \int p_t(i) y_t(i) di = 0 . \quad (2.3)$$

The very fact that prices are not adjusted in a synchronized way throughout the economy creates welfare distortions. The existence of sticky prices calls for a benevolent policymaker that limits the detrimental impact on consumer welfare (Woodford (2003), 382 pp.). We assume this to be the central bank which controls the nominal interest rate $i_t$ in the economy. As prices are predetermined some economic agents will not be able to adapt to a changing economic environment. Due to the assumption of sticky prices monetary policy has a leverage on real short term interest rates. Thereby the central bank can manage aggregate demand as a change in the interest rate changes the slope of the intertemporal budget constraint which induces households to reallocate their consumption patterns through time. Interest rate policy will be conducted in such a way that the inflation rate will be close to the inflation target while equally having a concern for economic activity. This is of course nothing but a short cut for the well known strategy of inflation-forecast-targeting (Svennsson (1999)). We will address in depth the question how monetary policy will deal with demand and supply shocks under this central bank strategy. We will see that in the case of a demand shock monetary policy will maneuver interest rates to a level where those firms that are allowed to reset prices will charge the same price as those firms that are not allowed to change prices. In effect we will see ex post neither a quantity nor a price reaction in the data. In other words the impact of demand shocks on economic activity can be completely undone. In the case of a supply shock the central bank will spread the macroeconomic loss inflicted on society equally across the two target variables according to its preferences.

### 2.1.1 The New Keynesian Phillips Curve: Optimizing Firms




The Phillips curve was always at the heart of macroeconomic debate and to a certain extent the dominating Phillips curve of its time always mirrored the dominating macroeconomic paradigm. Since the mid 1990's a new paradigm has come into reign in monetary macroeconomics named "New Keynesian Macroeconomics". Just as in earlier periods of macroeconomic history the dominating paradigm namely "Real Business Cycle Theory" was redeemed by New Keynesian Macroeconomics by the invention of a new Phillips curve (e.g. Sbordone (2002)). The same happened about 35 years earlier when

Friedman (1969) in his presidential address to the American Economic Association casted his doubt on the existence of a stable trade-off between unemployment and inflation.

Within this chapter we will highlight the NKPC in its different versions by focusing on mathematical derivations as well as on the economic intuition. The NKPC is a behavioral relationship that tells us why we observe inflation in an economy and which forces reduce the purchasing power of money. In a market economy firms are those agents that set prices. To that extend the NKPC curve tells the story of a representative firm that sets its price. This implies market power as a price taker has no room to maneuver the price for the product it sells.

In the following section we will discuss the various alternative derivations of the NKPC. We will start with Taylor's (1979) model and end the chapter with state of the art hybrid versions of the NKPC as proposed by Gali, Gertler and Salido-Lopez (2001) or by Christiano, Eichenbaum and Evans (2005). The advancement of the NKPC was driven throughout the last two decades by two factors. On the one hand the quest to derive macroeconomic relationships from microeconomic optimization calculus and on the other hand from the desire to be able to explain stylized facts as embedded in the data.

#### 2.1.1.1 Taylor Contracts

In the early 1980's the proposition of a non-vertical Phillips curve by Taylor (1979) virtually reinvented New Keynesian Macroeconomics. Sticky prices imply a meaningful role for macroeconomic stabilization policy in the short to medium run as the central bank has the power to smooth out macroeconomic fluctuations triggered by exogenous shocks. In the focus of Taylor's model are monopolistically competitive firms that negotiate wage contracts. In a simplified theoretical framework (Bofinger (2001), pp.102-103) one can tell Taylor's story as follows. In each period a mass of 50% of workers renegotiate wages. According to standard microeconomic theory a monopolistically competitive firm will price its output at a constant margin over marginal costs. As the capital stock is fixed, wages $w_t$ are the only source of variations in variable costs as the log price level $P_t$ is equal to the log wage $w_t$ plus the mark-up $\mu$:

$$p_t = w_t + m \ . \qquad (2.4)$$

The log-price level is simply the weighted average of the level of log-wage contracts negotiated over the current period and over the last period:

$$p_t = \frac{1}{2}\left(w_t + w_{t-1}\right). \tag{2.5}$$

Taylor offers the following theory for wage negotiations. He assumes that workers care on two components while negotiating wages. On the one hand they want to participate in economic activity as measured by the output gap. On the other hand they bargain for a weighted average of those contracts fixed over the lifetime of the contract:

$$w_t = \frac{1}{2}\left(w_t + E_t w_{t+1}\right) + g\,y_t\,. \tag{2.6}$$

Inserting the contracting equation (2.6) into equation (2.5) yields

$$p_t = \frac{1}{2}\left[\left(\frac{1}{2}\left(p_t + E_t p_{t+1}\right) + g\,y_t\right) + \left(\frac{1}{2}\left(p_{t-1} + E_{t-1} p_t\right) + g\,y_{t-1}\right)\right], \tag{2.7}$$

where we have set $m = 0$. After some algebraic manipulation this expression can be rewritten as:

$$p_t = \frac{1}{2} p_{t-1} + \frac{1}{2} p_{t+1} + \frac{g}{2}\left(y_t - y_{t-1}\right) + \frac{1}{2} e_t\,. \tag{2.8}$$

where $e_t = E_{t-1} p_t - p_t$

Hence Taylor succeeds in explaining why the price level might be sticky as a consequence of staggered wage setting. As each period only a fraction of workers resets wages, decisions taken in the past still influence the presence. Although workers are assumed to built rational expectations some economic agents are not able to process new information as their hands are tied due to settled contracts. Therefore macroeconomic shocks $\varepsilon_t$ need time to be incorporated into pricing decisions. Nevertheless those workers that renegotiate wages in the current period look into the future so that expected events also have an impact on the current price level $P_t$.

Equation (2.8) can equally be expressed in terms of inflation rates $\pi_t$. Subtracting $p_{t-1}$ from both sides of the equation yields:

$$p_t = E_t p_{t+1} + 2g\left(y_t + y_{t-1}\right) + h_t \,. \tag{2.9}$$

Equation (2.9) shows that the price level $P_t$ is inertial but not the inflation process $p_t$ itself. This result reflects the assumption that workers negotiate on wage levels and not on wage changes. This assumption has important implications for the conduct of monetary policy. In particular monetary policy can design a credible cold turkey disinflation at zero costs in terms of output. Therefore credible disinflations can go hand in hand with a constant real interest rate $\left(i - p^e\right)$, whereby stabilization recessions can be avoided. Unfortunately this implication of Taylor's specification stands in sharp contrast to work by Ball (1994), who presented sound empirical evidence that disinflation's can only be designed if society is willing to temporarily sacrifice output. Ball concluded that policy should legislate regulations against long labor market contracts as they impair the ability of economic agents to react to a changing economic environment.

Additionally Taylor's version of the NKPC conflicts with stylized facts according to which inflation is an inertial process as a shock to the inflation rate just produces a single jump in the inflation rate (see Christiano, Eichenbaum and Evans (2005)). Not surprisingly given the poor empirical predictions of Taylor's model follow-up models tried to remedy these deficiencies. But before going into a detailed description of the so called Fuhrer and Moore (1995) approach, we will highlight Calvo's version of a NKPC as it has become the main engine of today's macroeconomic state of the art models in monetary macroeconomics. To summarize: Taylor succeeded in giving a meaningful role to stabilization policy, but he failed to present a convincing empirical specification.

### 2.1.1.2 Fuhrer and Moore

The Fuhrer and Moore (1995) approach to the NKPC can be seen as a direct extension of Taylor's version of staggered wage contracts. Taylor's version implied that inflation is a non-inertial process and that periods of disinflation can be designed at zero output costs. This implication is not backed up by the data. Therefore Fuhrer and Moore (1995) intended to design a version that was closer to the data but in the spirit of Taylor (Bofinger (2001), p. 103, Walsh (2003), p 228). In a simplified framework one can sketch Fuhrer and Moore's (1995) story as follows. Assume that each period a mass of 50 percent of all workers renegotiate wages. Then the index of real wages $v_t$ is defined as the weighted average of this periods contracts $\left(w_t - p_t\right)$ and the real value of last periods contracts $\left(w_{t-1} - p_{t-1}\right)$:

$$v_t = \frac{1}{2}\left((w_t - p_t) + (w_{t-1} - p_{t-1})\right). \qquad (2.10)$$

Fuhrer and Moore (1995) propose that workers care on two components while negotiating wage contracts. On the one hand they want to be compensated for the state of the economic cycle $y_t$. On the other hand workers bargain for a weighted average of the real wage index over the lifetime of the settled contract. In a nutshell the real contract $\left(w_t - p_t\right)$ can then be stated as:

$$w_t - p_t = \frac{1}{2}\left(v_t + E_t v_{t+1}\right) + k y_t \,. \qquad (2.11)$$

Inserting the relevant expressions yields the following real wage contracting equation:

$$\mathrm{w}_t - \mathrm{p}_t = \frac{1}{2}\left(\mathrm{w}_{t-1} - \mathrm{p}_{t-1} + \mathrm{E}\left(\mathrm{w}_{t+1} - \mathrm{p}_{t+1}\right)\right) + 2\mathrm{k}\mathrm{y}_t \,. \qquad (2.12)$$

As prices are set by monopolistically competitive firms at a constant mark-up over marginal costs prices evolve as a weighted average of the wage level. Subtracting $w_{t-1}$ from both sides of the equation and collecting terms yields the following expression:

$$\Delta w_t = \frac{1}{2}\left(p_t - E_t p_{t+1}\right) + 2k y_t \,. \qquad (2.13)$$

As the inflation rate is defined by

$$\pi_t = \Delta \mathrm{w}_t - \Delta \mathrm{w}_{t-1}, \qquad (2.14)$$

we can state the Phillips curve in terms of inflation by subtracting $\Delta w_{t-1}$ from both sides of the equation:

$$\pi_t = \frac{1}{2}\pi_{t-1} + \frac{1}{2}E_t\pi_{t+1} + \gamma\bar{y}_t + \eta_t\,. \qquad (2.15)$$

where: $\bar{y}_t = (y_t + y_{t-1})$

$\eta_t = -(\pi_{t-1} - E_{t-1}\pi_t)$

Equation (2.15) nicely depicts that Fuhrer and Moore (1995) succeed in deriving an inertial Phillips curve (Roberts (1997)). If inflation is high in the current period it will remain above average in the following periods. Thereby they reconcile the NKPC with stylized facts from VAR-analysis according to which inflation is a persistent process.

#### 2.1.1.3 Calvo Pricing

Calvo's (1983) proposition of a NKPC is the workhorse of today's state of the art models in monetary macroeconomics (see Smets and Wouters (2005), Christiano, Eichenbaum and Evans (2005)). According to Calvo (1983) the process of price adjustment follows a time dependent rule. Each period only a fraction of $(1-q)$ firms in the economy receive a signal to reset prices optimally. The rest keeps its old price. Those firms that reoptimize will take in particular into account the probability of being stuck with the new reset price for j periods to come. The adjustment price is determined by the projected path of marginal cost over the expected time horizon that elapses until the next price adjustment signal arrives. Obviously the approach to model changing prices based on a time dependent black-box approach is taken for reasons of mathematical convenience rather than for sound empirical evidence. From microeconomic price data we know that prices are fixed mainly due to menu costs, implicit customer relationships and pressure of competition (Blinder (1994)).

There are two common ways to derive a NKPC a la Calvo. The "quick and dirty way", which is for instance used by Gali, Gertler and Salido-Lopez (2001) and the more sophisticated approach taken by Christiano, Eichenbaum and Evans (2005). Due to the unchallenged dominance of Calvo-pricing in monetary macroeconomics we will present both approaches.

**Way I: Deriving it the quick and dirty way**

The “quick and dirty way” to derive the New Keynesian Phillips curve can be sketched as follows (Whelan (2005), Walsh (2003)). Assume that only a fraction of $(1-q)$ percent of firms are allowed to reoptimize its price in the current period, while the remaining part of all firms has to keep its old price. Given this assumption one can show that the average duration of a reset price is equal to:

$$D_q = \frac{1}{1-q}. \tag{2.16}$$

Hence, if a mass of $q = 0.75$ have to keep their old prices every quarter, the average price duration will be equal to four quarters. Assume as auxiliary assumption that those firms that are allowed to reset prices are guided by the following quadratic loss function:

$$L_t(z_t) = \sum_{k=0}^{\infty} (qb)^k \left(z_t^* - p_{t+k}^*\right)^2, \tag{2.17}$$

where β denotes the discount factor and $z_t^*$ is the optimal reset price. Naturally, it is the objective of the representative firm to minimize $L_t$. Therefore at each point in time the bliss point is given by $z_t^* = p_{t+i}^* \ \forall i \in N_+$. The term $\left(z_t^* - p_{t+k}^*\right)$ measures the distance between the optimal price $p_{t+k}^*$ and the new reset price $z_t^*$. If a firm would be stuck for k periods with the new reset price $z_t^*$ then the expected quadratically measured distance can be evaluated as follows:

$$\left(z_t - p_t^*\right)^2 + (qb)\left(z_t - p_{t+1}^*\right)^2 + (qb)^2\left(z_t - p_{t+2}^*\right)^2 + (qb)^3\left(z_t - p_{t+3}^*\right)^2 + \ldots + (qb)^k\left(z_t - p_{t+k}^*\right)^2. \tag{2.18}$$

An optimizing firm will choose $z_t^*$ in such a way that this weighted sum is minimized. Taking the derivative with respect to $z_t^*$ it has to hold that:

$$
\begin{aligned}
\frac{\partial L_t}{\partial z_t} = {} & 2\left(bq\right)^0\left(z_t - p_t^*\right) \\
& + 2\left(bq\right)^1\left(z_t - p_{t+1}^*\right) \\
& + 2\left(bq\right)^2\left(z_t - p_{t+2}^*\right) \\
& + 2\left(bq\right)^3\left(z_t - p_{t+3}^*\right) \\
& + \ldots \\
= {} & 0,
\end{aligned}
\tag{2.19}
$$

which can equally be rewritten more compactly as:

$$z_t^* \sum_{k=0}^{\infty} (qb)^k = \sum_{k=0}^{\infty} (qb)^k E_t p_{t+k}^* . \tag{2.20}$$

Equation (2.20) nicely shows that firms choose their prices in such a way that on average the geometrically weighted reset prices $z_t^*$ is equal to the expected cumulated sum of optimal prices $p_{t+k}^*$. As $|qb| < 1$ the left hand geometric sum can be simplified to:

$$z_t^* \sum_{k=o}^{\infty} (qb)^k = \frac{1}{1-qb} z_t^* . \tag{2.21}$$

Inserting (2.21) into (2.20) the optimal reset price is given by:

$$z_t^* = (1-qb) \sum_{k=0}^{\infty} (qb)^k E_t p_{t+k}^* . \tag{2.22}$$

Equation (2.22) can be further simplified by noticing that there are two ways of representing first-order difference equations. A first-order inhomogeneous difference equation can be stated as follows:

$$y_t = ax_t + by_{t+1} . \tag{2.23}$$

Iterating this relationship forward yields under the assumption that $|b| < 1$ the following equivalent representation (see Hamilton (1994), p. 28):

$$y_t = a\sum_{n=0}^{\infty} b^k E_t x_{t+k} \ . \tag{2.24}$$

Accordingly equation (2.23) can be rewritten as:

$$z_t^* = qb\, E_t z_{t+1} + (1-qb)\, p_t^* . \tag{2.25}$$

where: $a = (1-qb)$

$b = qb$

So far we have analyzed the behavior of those firms that are called upon to reoptimize. All other firms keep the price level $p_{t-1}$ of the previous period. Accordingly the aggregate price level evolves according to the following weighted average

$$p_t = q\, p_{t-1} + (1-q)\, z_t^* , \tag{2.26}$$

which can equally be written when solved for the reset price $z_t^*$ as follows:

$$z_t^* = \frac{1}{1-q}\left( p_t - q\, p_{t-1}\right) . \tag{2.27}$$

Substituting out the optimal reset price $z_t^*$ in equation (2.25) and multiplying the equation by $(1-q)$ yields the following equation

$$p_t - q\, p_{t-1} = qb E_t \left( p_{t+1} - q\, p_t \right) + (1-qb)(1-q)(m_t + mc_t) , \tag{2.28}$$

which can be transformed into:

$$q\left( p_t - p_{t-1}\right) = qb\, E_t \left( p_{t+1} - q\, p_t \right) + (1-qb)(1-q)(m_t + mc_t) - p_t + q\, p_t . \tag{2.29}$$

Collecting terms yields:

$$q\left(p_t - p_{t-1}\right) = qb\,E_t\left(p_{t+1} - q\,p_t\right) + \left(1-qb\right)\left(1-q\right)\left(m_t + mc_t\right) - p_t + q\,p_t . \qquad (2.30)$$

Equation (2.30) can equally be expressed in terms of the inflation rate as follows:

$$p_t = b\,E_t p_{t+1} + \frac{\left(qb - bq^2 + q - 1\right)}{q}\,p_t + \frac{\left(1-qb\right)\left(1-q\right)}{q}\left(m_t + mc_t\right). \qquad (2.31)$$

Simplifying and collecting terms we arrive at the NKPC:

$$p_t = b\,E_t p_{t+1} + \frac{\left(1-q\right)\left(1-qb\right)}{q}\left(m_t + mc_t - p_t\right). \qquad (2.32)$$

Note that the term $m + mc_t - p_t$ is nothing but the deviation of prices from their distorted steady state level. Therefore let us define the following variable, which measures the degree of disequilibrium:

$$mc_t^r = m + mc_t - p_t . \qquad (2.33)$$

Let us assume that there is a proportional relationship between the output gap and deviations of marginal costs from its steady state level:

$$mc_t^r = l\,y_t . \qquad (2.34)$$

By this assumption we can substitute out $mc_t^r$ in equation (2.32) with the help of equation (2.34):

$$p_t = bp_{t+1} + g\,y_t , \qquad (2.35)$$

where it holds that: $g = l\left(\left(1-q\right)\left(1-qb\right)/q\right)$. Note that equation (2.29) is not yet a stochastic relationship as we did not introduce an error term. Therefore let us assume that there are stochastic shocks to the degree of monopolistic power in the good markets $m_t = m + e_t$:

$$p_t = bE_t p_{t+1} + \frac{(1-q)(1-qb)}{q}(m + mc_t - p_t) + \frac{(1-q)(1-qb)}{q} e_t . \qquad (2.36)$$

Compared to Taylor's version of a Phillips curve two major differences stand out. First, in the focus of Calvos model is an optimizing firm that sets prices and not workers that renegotiate wages. The question which process drives wages is left open. For direct comparisons additional assumptions on the functioning of labor markets would have to be made. Second, as the reset signal follows a Poisson process firms automatically have a larger time horizon while reoptimizing. Additionally $q$, the share of Calvo price setters might be considered as a deep parameter of the economy, which is invariant to monetary policy. Thereby one might argue that equation (2.36) is immune to the Lucas-critique (Lucas (1976)).

**Way II: The sophisticated approach**

The previous section showed how to derive the NKPC under the auxiliary assumption that firms are confronted with quadratic adjustment costs (Rotemberg (1987)). Quite naturally one might ask the question why not to evaluate the implications of price stickiness directly in terms of their implications for profit maximization. The answer to this question is the second approach to evaluate the implications of price stickiness on the firm level. Following the seminal work of Dixit and Stiglitz (1977) let us assume that there exists the following market structure on goods and labor markets (see Figure 2). A continuum of intermediate firms $(I_1,...I_n)$ produces differentiated goods. These firms operate in an environment of monopolistic competition. Firms hire their labor input $N_t$ from a perfectly competitive labor market. The final good sector which bundles the output of the intermediate good sector into a homogeneous aggregate commodity $Y_t$ operates under conditions of perfect competition.

For reasons of mathematical convenience we assume that firms have the following simple linear production technology:

$$y_{jt} = A_t N_{jt} , \qquad (2.37)$$

where $y_{jt}$ denotes the output of firm j at time t and $N_{jt}$ refers to the labor input of firm j at time t. The term $A_t$ depicts a productivity shock which is assumed to be iid with $E(A_t) = 1$ and variance $s_a^2$. In the simplest scenario one may assume perfectly flexible labor markets where just a homogeneous type of labor is supplied. As shown by Dixit and Stiglitz (1977) each firm in the intermediate good sector faces an isoelastic demand function for its output (see equation (2.2)) which depends on its own price $p_{jt}$ in relation to the overall price index $P_t$, and the elasticity of demand $\varepsilon$. In the absence of shocks the demand for $y_{jt}$ is synchronized with aggregate output $Y_t$. Let us make the standard assumption that the prime target of a monopolistically competitive firm is to maximize profits. The intertemporal profit function of the monopolistically competitive firm operating in the intermediate good sector can be constructed as follows.

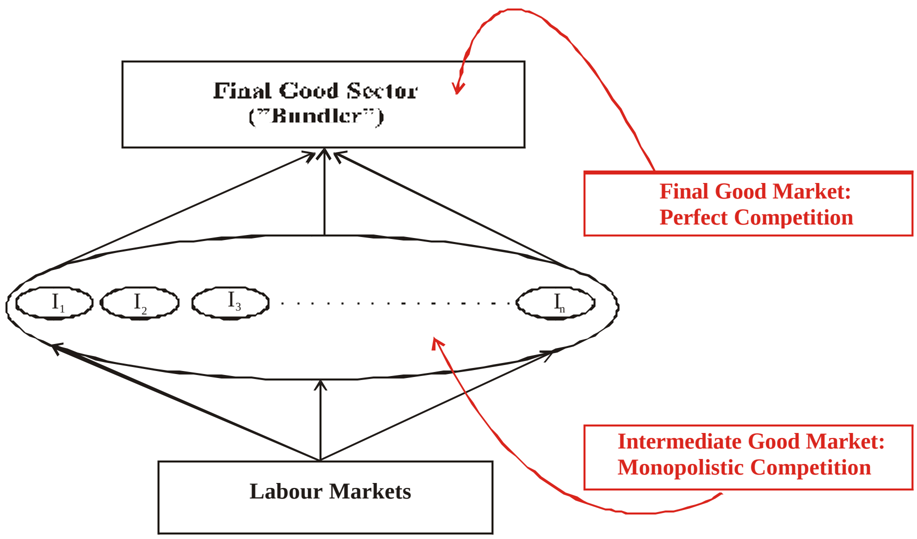



**Figure 2** : The Hypothesized Market Structure in New Keynesian Models

Each firm will receive a marginal profit of $\left(p_{jt}/P_t\right) - mc_t$ per unit it sells. The period profit flows for the expected time horizon over which the firm is not allowed to reoptimize can be stated as:

$$\begin{aligned}
&q^0\Delta_{t+0}\left[\left(\frac{p_{jt}}{P_t}\right)-mc_t\right]y_{jt}\\
&+q^1\Delta_{t+1}\left[\left(\frac{p_{jt}}{P_{t+1}}\right)-mc_{t+1}\right]y_{jt+1}\\
&+q^2\Delta_{t+2}\left[\left(\frac{p_{jt}}{P_{t+2}}\right)-mc_{t+2}\right]y_{jt+2}\\
&+\dots
\end{aligned}, \tag{2.38}$$

where $q$ denotes the probability of receiving no reset signal and $\Delta_{t+i}$ depicts the stochastic discount factor of households for risky assets. Making use of the isoelastic demand function (2.2) one can substitute out $y_{jt}$ in terms of the aggregate commodity $Y_t$. Collecting terms the intertemporal profit function can be written more compactly as follows (see Walsh (2003) p. 235):

$$\Pi^J = E_0 \sum_{k=0}^{\infty} q^k \Delta_{t+k} \left[ \left( \frac{p_{jt}}{P_{t+k}} \right)^{1-e} - mc_{t+k} \left( \frac{p_{jt}}{P_{t+k}} \right)^{-e} \right] Y_{t+k} \,. \tag{2.39}$$

The firms' action parameter is to choose the optimal reset price $p_t^*$ such that expected profits are maximized. Taking the derivative with respect to the optimal reset price $p_t^* = p_{jt}$ yields:

$$\begin{aligned}
\frac{\partial \Pi^j}{\partial p_t^*} = & \left[ q^0 \Delta_t \left( (1-e) \left( \frac{p_t^*}{P_t} \right)^{-e} \frac{1}{P_t} + e\, mc_t \left( \frac{p_t^*}{P_t} \right)^{-e-1} \frac{1}{P_t} \right) \right] Y_t \\
& + \left[ q^1 \Delta_{t+1} \left( (1-e) \left( \frac{p_t^*}{P_{t+1}} \right)^{-e} \frac{1}{P_{t+1}} + e\, mc_{t+1} \left( \frac{p_t^*}{P_{t+1}} \right)^{-e-1} \frac{1}{P_{t+1}} \right) \right] Y_{t+1} \\
& + \left[ q^2 \Delta_{t+2} \left( (1-e) \left( \frac{p_t^*}{P_{t+2}} \right)^{-e} \frac{1}{P_{t+2}} + e\, mc_{t+2} \left( \frac{p_t^*}{P_{t+2}} \right)^{-e-1} \frac{1}{P_{t+2}} \right) \right] Y_{t+2} \\
& + \ldots \\
& = 0.
\end{aligned} \tag{2.40}$$

Which can be rewritten as:

$$\sum_{k=0}^{\infty} q^k \Delta_{t,\, t+k} \left[ (1-e) \left( \frac{p_t^*}{P_{t+k}} \right) + e\, mc_{t+k} \right] \frac{1}{p_t^*} \left[ \frac{p_t^*}{P_{t+k}} \right]^{-e} Y_{t+k} = 0 \,. \tag{2.41}$$

Note as profits are redeemed to shareholders they are discounted with the stochastic discount factor of households, which own the portfolio shares of the firms operating in the intermediate good sector. Given the isoelastic utility function (2.138) households trade off consumption today versus tomorrow by the following stochastic discount factor:

$$\Delta_{t+k}^{k} = b^{\,t+k}\left(\frac{Y_{t+k}}{Y_t}\right)^{-s}. \qquad (2.42)$$

Inserting this so-called pricing kernel of shareholders into equation (2.41) and rearranging the equation yields:

$$(1-e)E_t\sum_{k=0}^{\infty}(qb)^k Y_{t+k}^{1-s}\left(\frac{p_t^*}{P_{t+k}}\right)^{1-e} Y_{t+k}^{1-s}\,\frac{1}{p_t^*} - e\,E_t\sum_{k=0}^{\infty}(qb)^k\,Y_{t+k}^{1-s}\,mc_{t+k}\left(\frac{p_t^*}{P_{t+k}}\right)^{-e}\frac{1}{p_t^*} = 0 \qquad (2.43)$$

which can be rewritten as:

$$p_t^* = \frac{e}{e-1}\,\frac{p_t^{*e}}{p_t^{*(1+e)}}\,\frac{\sum_{k=0}^{\infty}E_t\,(qb)^k\,Y_{t+k}^{1-s}\,mc_{t+k}\left(\frac{1}{P_{t+k}}\right)^{-e}}{\sum_{k=0}^{\infty}E_t\,(qb)^k\,Y_{t+k}^{1-s}\left(\frac{1}{P_{t+k}}\right)^{1-e}}. \qquad (2.44)$$

By dividing both sides of the equation by the aggregate price level $P_t$, equation (2.44) can be restated as:

$$\frac{p_t^*}{P_t} = \frac{e}{e-1}\,\frac{\sum_{k=0}^{\infty}E_t\,(qb)^k\,Y_{t+k}^{1-s}\,mc_{t+k}\left(\frac{P_{t+k}}{P_t}\right)^{e}}{\sum_{k=0}^{\infty}E_t\,(qb)^k\,Y_{t+k}^{1-s}\left(\frac{P_{t+k}}{P_t}\right)^{e-1}}. \qquad (2.45)$$

In order to give some economic intuition to this non-linear first order condition let us assume that prices are fully flexible. This means that each period all firms receive a signal to change prices. Inserting $(q=0)$ into equation (2.45) yields (King, Robert and Wolman (1996)):

$$p_t^* = \left(\frac{e}{e-1}\right)mc_t. \qquad (2.46)$$

In the flex-price equilibrium monopolistically competitive firms price their output at a constant mark-up over marginal costs. The size of the mark-up depends on the pricing power of the monopolistically competitive firm which can be directly related to the inverse of the elasticity of demand $e$. The more elastic demand is, the lower will be the mark-up in equilibrium. The first order condition nicely shows that intertemporal optimality implies in principle that firms have to make forecasts on future demand as well as on future prices unless both variables are driven by common factors of the business cycle.

The first order condition for $p_t^*$ is a highly non-linear expression. Under the assumption that we are only interested in small perturbations around steady state we can derive linear approximations by the technique of log-linearization. Assume that $X_t$ is a strictly positive variable and $\overline{X}_t$ is its natural level. Then the variable $\hat{x}_t$ denotes the logarithmic deviation of the variable from its steady state level (see e.g. Edmond (2004)):

$$\begin{aligned} X_t = \overline{X}_t\left(\frac{X_t}{\overline{X}_t}\right) &= \overline{X}_t e^{\ln\left(X_t/\overline{X}_t\right)} = \overline{X}_t e^{\hat{x}_t} \\ &\cong \overline{X}_t e^0 + \overline{X}_t e^0\left(\hat{x}_t - 0\right) \\ &= \overline{X}_t\left(1+\hat{x}_t\right), \end{aligned} \tag{2.47}$$

where we have taken a first-order Taylor approximation around steady state. By applying the same log-linearization technique to two stochastic variables one can show that:

$$X_t Y_t \cong \overline{X}_t \overline{Y}_t\left(1+\hat{x}_t+\hat{y}_t+\hat{x}_t\hat{y}_t\right). \tag{2.48}$$

If the cross terms $\hat{x}_t$ and $\hat{y}_t$ are sufficiently small it is legitimate to drop them out. But let us turn to a more general case where we have the following continuous and differentiable functions f() and g():

$$f\left(X_t, Y_t\right) = g\left(Z_t\right). \tag{2.49}$$

We assume that $X_t$, $Y_t$ and $Z_t$ are strictly positive variables. By applying the ln function to both sides of equation (2.49) it holds that:

$$\ln\left[f\left(e^{\ln X_t}, e^{\ln Y_t}\right)\right] = \ln\left(g\left(Z_t\right)\right). \tag{2.50}$$

Taking a Taylor approximation of the left hand side of the equation it has to hold that:

$$\ln\left[ f\left(e^{\ln X_t}, e^{\ln Y_t}\right)\right] \cong \ln\left[ f\left(\bar{X}_t, \bar{Y}_t\right) + \frac{1}{f\left(\bar{X}_t, \bar{Y}_t\right)} \left[ \begin{array}{l} f_{\bar{X}_t}\left(\bar{X}_t, \bar{Y}_t\right)\bar{X}_t\left(\ln\left(X_t\right) - \ln\left(\bar{X}_t\right)\right) \\ + f_{\bar{Y}_t}\left(\bar{X}_t, \bar{Y}_t\right)\bar{Y}_t\left(\ln\left(Y_t\right) - \ln\left(\bar{Y}_t\right)\right) \end{array} \right]\right] \tag{2.51}$$

The right hand side can be approximated by applying the same Taylor approximation by noting that:

$$\ln\left[ g\left(e^{\ln Z_t}\right)\right] \cong \ln\left[ g\left(\bar{z}_t\right) + \frac{1}{g\left(\bar{z}_t\right)}\left[ g_{z_t}(\bar{z}_t)\bar{z}_t\left(\ln\left(z_t\right) - \ln\left(\bar{z}_t\right)\right)\right]\right]. \tag{2.52}$$

Equating equation (2.51) and equation (2.52) results in:

$$\begin{array}{l} f_{\bar{X}_t}\left(\bar{Y}_t, \bar{X}_t\right)\bar{X}_t\left(\ln\left(X_t\right) - \ln\left(\bar{X}_t\right)\right) \\ + f_{\bar{Y}_t}\left(\bar{Y}_t, \bar{X}_t\right)\bar{X}_t\left(\ln\left(Y_t\right) - \ln\left(\bar{Y}_t\right)\right) \cong g_z\left(\bar{z}_t\right)\bar{Z}_t\left(\ln\left(Z_t\right) - \ln\left(\bar{Z}_t\right)\right) \end{array}, \tag{2.53}$$

For the most general case

$$f\left(X_t^1, X_t^2, ..., X_t^n\right) = g\left(Y_t^1, Y_t^2, ..., Y_t^m\right), \tag{2.54}$$

the log-linearized approximation can be written as:

$$\sum\nolimits_{i=1}^{n} f_{\bar{x}_t^i}\left(\bar{X}_t^1, \bar{X}_t^2, ..., \bar{X}_t^n\right)\bar{X}_t^i\hat{x}_t^i \cong \sum\nolimits_{i=1}^{m} f_{\bar{Y}_t^i}\left(\bar{Y}_t^1, \bar{Y}_t^2, ..., \bar{Y}_t^m\right)\bar{Y}_t^i\hat{y}_t^i. \tag{2.55}$$

To get some intuitive understanding let us assume that $f\left(x\right) = x^2$. A simple linear Taylor-series expansion around the point $\bar{x}$ =2 can be written as:

$$\begin{aligned} f(x) &\cong 2 + f_x(x-2) \\ &= 2 + 2\Delta x \end{aligned}. \tag{2.56}$$

The log-linear approximation of $f(x) = x^2$ at the point x=2 is:

$$\hat{y}_t = 2\hat{x}_t . \tag{2.57}$$

Hence in log-linear approximation a deviation of the variable $X_t$ from its steady state value $\overline{X}$ causes a deviation of the variable $Y_t$ from its steady state by approximately 2 percent.
Let us now return to our economic application. So far we have only analyzed the behavior of those firms that receive the signal to optimize. Of course, by the very assumption of Calvo pricing there remains a mass of $q$ percent of firms in the intermediate good sector which have to keep the old price. Assume for reasons of mathematical convenience that the inflation target of the central bank is equal to zero (Sbordone (2002), p. 270). Note that the aggregate price level evolves according to the following formula, where we have made use of the fact that all firms that are called upon to reset prices face the same optimization problem:

$$P_t^{1-q} = \left[ (1-a)\, p_t^{*1-q} + a\, P_{t-1}^{1-q} \right]. \tag{2.58}$$

Log-linearizing equation (2.58) around its steady state can be done by applying the following formula:

$$F_{\overline{P}_t}\left(\overline{P}_t, \overline{p}_t^*, \overline{P}_{t-1}\right)\overline{P}_t \hat{p}_t = F_{\overline{p}_t^*}\left(\overline{P}_t, \overline{p}_t^*, \overline{P}_{t-1}\right)\overline{p}_t^* \hat{p}_t^* + F_{\overline{P}_t}\left(\overline{P}_t, \overline{p}_t^*, \overline{P}_{t-1}\right)\overline{P}_{t-1}\hat{p}_{t-1}, \tag{2.59}$$

where:

$$F_{P_t}\left(\overline{P}_t, \overline{p}_t^*, \overline{P}_{t-1}\right) = (1-q)\,\overline{P}_t^{-q} \tag{2.60}$$

$$F_{P_t}\left(\overline{P}_t, \overline{p}_t^*, \overline{P}_{t-1}\right) = (1-q)(1-a)\,\overline{P}_t^{-q} \tag{2.61}$$

$$F_{P_{t-1}}\left(\overline{P}_t, \overline{p}_t^*, \overline{P}_{t-1}\right) = q\,(1-a)\,\overline{P}_{t-1}^{-q} \tag{2.62}$$

Substituting out the partial derivatives and collecting terms yields:

$$0 = (1-a)\left(\hat{p}_t^* / \hat{p}_t\right) + a\left(\hat{p}_{t-1} / \hat{p}_t\right), \tag{2.63}$$

which can be simplified to:

$$0 = (1-a)\,\tilde{\tilde{q}}_t - a\hat{p}_t\,, \tag{2.64}$$

where $\tilde{\tilde{q}}_t = \left(p_t^* / \hat{p}_t\right)$ is the ratio of the optimal reset price in relation to the aggregate price level $P_t$ expressed in terms of percentage deviations around a non-inflationary steady state and $\hat{p}_t$ denotes the percentage deviation of the inflation rate from the inflation target. Equation (2.64) shows that deviations of the inflation rate from the inf lation target depend in particular on the ratio of firms that are allowed to optimize versus those that do not optimize:

$$\hat{p}_t = \frac{a}{1-a}\,\tilde{\tilde{q}}_t\,. \tag{2.65}$$

As we have already computed the optimal relative reset price $\tilde{Q}_t$ (equation (2.45)) it remains from a mathematical perspective the tedious task to log-linearize the expression derived for $\tilde{Q}_t$ in order to substitute out $\tilde{\tilde{q}}_t$ in equation (2.65). The first-order condition of the optimizing firm can be stated as:

$$E_t \sum_{k=0}^{\infty} (qb)^k Y_{t+k}^{1-s} \left(\frac{P_{t+k}}{P_t}\right)^{e-1} \tilde{Q}_t = mE_t \sum_{k=0}^{\infty} (qb)^k Y_{t+k}^{1-s}\, \hat{m}c_{t+i} \left(\frac{P_{t+k}}{P_t}\right)^{e}. \tag{2.66}$$

As indicated by equation (2.66) we can separately log-linearize the left-hand side and right hand-side functions. So let us in a first step log-linearize the left hand side function.

The steady state is defined as:

$$\bar{F}\left(\bar{Y}_{t+k}, \bar{\tilde{Q}}_t, \bar{P}_{t+k}, \bar{P}_t\right) = E_t \sum_{k=0}^{\infty} (qb)^k\, \bar{Y}_{t+k}^{1-s} \left(\frac{\bar{P}_{t+k}}{\bar{P}_t}\right)^{e-1} \bar{\tilde{Q}}_t\,. \tag{2.67}$$

The partial derivatives we needed to log-linearize are given by:

$$f_{\bar{Y}_{t+k}}\left(\bar{Y}_t,\tilde{\tilde{Q}}_t,\bar{P}_t,\bar{P}_{t+i}\right)\bar{Y}_{t+k} = E_t\sum_{k=0}^{\infty}(qb)^k\,\bar{Y}_{t+k}^{-s}\,\tilde{\tilde{Q}}_t\left(\frac{\bar{P}_{t+k}}{\bar{P}_t}\right)^{e-1}(1-s) \quad (2.68)$$

$$f_{\bar{P}_t}\left(\bar{Y}_t,\tilde{\tilde{Q}}_t,\bar{P}_t,\bar{P}_{t+k}\right)\bar{P}_t = -E_t\sum_{k=0}^{\infty}(qb)^k\,\bar{Y}_{t+k}^{-s}\,\tilde{\tilde{Q}}_t\left(\frac{\bar{P}_{t+k}}{\bar{P}_t}\right)^{e-1}(e-1) \quad (2.69)$$

$$f_{\bar{P}_{t+1}}\left(\bar{Y}_t,\tilde{\tilde{Q}}_t,\bar{P}_t,\bar{P}_{t+k}\right)\bar{P}_{t+k} = E_t\sum_{i=0}^{\infty}(qb)^k\,\bar{Y}_{t+k}^{-s}\,\tilde{\tilde{Q}}_t\left(\frac{\bar{P}_{t+k}}{\bar{P}_t}\right)^{e-1}(1-e) \quad (2.70)$$

$$f_{\tilde{\tilde{Q}}_t}\left(\bar{Y}_t,\tilde{\tilde{Q}}_t,\bar{P}_t,\bar{P}_{t+k}\right)\tilde{\tilde{Q}}_t = E_t\sum_{i=0}^{\infty}(qb)^k\,\bar{Y}_{t+k}^{1-s}\left(\frac{\bar{P}_{t+k}}{\bar{P}_t}\right)\tilde{\tilde{Q}}_t\,. \quad (2.71)$$

Applying the formulae (2.55) it will have to hold that:

$$\begin{aligned} &f_{Y_t}\left(\bar{Y}_t,\bar{P}_t,\bar{P}_{t+k},\tilde{\tilde{Q}}_t\right)\bar{Y}\hat{y}_{t+1} + f_{P_t}\left(\bar{Y}_t,\bar{P}_t,\bar{P}_{t+k},\tilde{\tilde{Q}}_t\right)\bar{P}\hat{p}_t + f_{P_{t+1}}\left(\bar{Y}_t,\bar{P}_t,\bar{P}_{t+k},\tilde{\tilde{Q}}_t\right)\bar{P}\hat{p}_{t+1} \\ &\qquad + f_{Q_t}\left(\bar{Y}_t,\bar{P}_t,\bar{P}_{t+k},\tilde{\tilde{Q}}_t\right)\tilde{\tilde{Q}}_t\hat{q}_t \cong 0 \end{aligned} \quad (2.72)$$

Substituting out the partial derivatives $f_x(\ldots)$ in (2.72) yields:

$$F\left(\bar{Y}_t,\tilde{\tilde{Q}}_t,\bar{P}_t,\bar{P}_{t+k}\right) = \frac{1}{1-qb}\bar{Y}^{1-s} + \frac{1}{1-qb}\bar{Y}^{1-s}\,\hat{\tilde{q}}_t + \bar{Y}^{1-s}\left[\sum_{k=1}^{\infty}(qb)^k\left((1-s)\,\hat{y}_{t+k} + (e-1)\left(\hat{p}_{t+k}-\hat{p}_t\right)\right)\right] \quad (2.73)$$

The right hand side function has the following steady state:

$$F\left(\bar{Y}_t,\bar{mc}_t,\bar{P}_t,\bar{P}_{t+k}\right) = m\,E_t\sum_{k=0}^{\infty}(qb)^k\,\bar{Y}_{t+k}^{1-s}\,\bar{mc}_{t+k}\left(\frac{\bar{P}_{t+k}}{\bar{P}_t}\right)^{e}. \quad (2.74)$$

The necessary partial derivatives to log-linearize are given by:

$$f_{\bar{y}_t}\left(\bar{Y}_t,\bar{mc}_t,\bar{P}_t,\bar{P}_{t+k}\right)\bar{Y}_t = m\;mc\sum_{i=0}^{\infty}(qb)^k\bar{Y}_t^{1-s}\left(\frac{\bar{P}_{t+1}}{\bar{P}_t}\right)^{e}(1-s) \quad (2.75)$$

$$f_{\overline{P}_t}\left(\overline{Y}_t,\overline{mc}_t,\overline{P}_t,\overline{P}_{t+k}\right)\overline{P}_t = m\ \mathrm{mc}E_t\sum_{k=0}^{\infty}(qb)^k\overline{Y}^{1-s}\left(\frac{\overline{P}_{t+1}}{\overline{P}_t}\right)^e(-e) \qquad (2.76)$$

$$f_{\overline{P}_{t+k}}\left(\overline{Y}_t,\overline{mc}_t,\overline{P}_t,\overline{P}_{t+k}\right)\overline{P}_{t+k} = m\ \mathrm{mc}E_t\sum_{k=0}^{\infty}(qb)^k\overline{Y}^{1-s}\left(\frac{\overline{P}_{t+1}}{\overline{P}_t}\right)^e e \qquad (2.77)$$

$$f_{\overline{mc}_{t+k}}\left(\overline{Y}_t,\overline{mc}_t,\overline{P}_t,\overline{P}_{t+k}\right)\overline{mc}_{t+k} = mE_t\sum_{i=0}^{\infty}(qb)^k\overline{Y}^{1-s}\left(\frac{\overline{P}_{t+1}}{\overline{P}_t}\right)^e\overline{mc}_{t+k}. \qquad (2.78)$$

Log-linearizing we have to apply the following formulae to a non-inflationary steady state:

$$f_{Y_t}\left(\overline{Y}_t,\overline{mc}_t,\overline{P}_t,\overline{P}_{t+k}\right)\overline{Y}_t\hat{y}_{t+1}+f_{P_t}\left(\overline{Y}_t,\overline{mc}_t,\overline{P}_t,\overline{P}_{t+k}\right)\overline{P}_t\hat{p}_t+f_{P_{t+1}}\left(\overline{Y}_t,\overline{mc}_t,\overline{P}_t,\overline{P}_{t+k}\right)\overline{P}_{t+1}\hat{p}_{t+1}$$
$$+f_{mc_t}\left(\overline{Y}_t,\overline{mc}_t,\overline{P}_t,\overline{P}_{t+k}\right)\tilde{\overline{Q}}_t\hat{q}_t\cong 0. \qquad (2.79)$$

Substituting out the partial derivatives and collecting terms results in the following expression:

$$m\ \overline{\mathrm{mc}}\frac{\overline{Y}^{1-s}}{1-qb}+m\ \overline{\mathrm{mc}}\overline{Y}_t^{1-s}\sum_{k=0}^{\infty}(qb)^kE_t\left(\hat{m}c_{t+k}+(q-1)\left(\hat{p}_{t+k}-\hat{p}_t\right)+(1-s)\hat{y}_{t+k}\right). \qquad (2.80)$$

Equating the left hand side approximation (2.73) and the right hand side approximation (2.80) it holds true that:

$$\tilde{\bar{q}}_t+\hat{p}_t=(1-qb)\sum_{k=0}^{\infty}(qb)^k\left(E_t\hat{m}c_{t+k}+E_t\hat{p}_{t+k}\right). \qquad (2.81)$$

Note that we have used the fact that $m\ \overline{\mathrm{mc}}=1$. Under the assumption that $|b|<1$ it holds that the following equation

$$y_t=a\sum_{i=0}^{\infty}b^kE_tx_{t+k}, \qquad (2.82)$$

can be rewritten as:

$$y_t = a x_t + b E_t y_{t+1} . \tag{2.83}$$

Therefore equation (2.81) can be restated as:

$$\hat{\tilde{q}}_t = (1 - qb)\hat{m}c_t + qb\left(E_t \hat{\tilde{q}}_{t+1} + E_t \hat{p}_{t+1} - \hat{p}_t\right) . \tag{2.84}$$

where it holds that:

$$a = 1 - wb$$

$$b = wb$$

$$y_t = \hat{q}_t$$

$$\hat{x}_t = \left(\hat{m}c_{t+i} + \left(\hat{p}_{t+i} - \hat{p}_t\right)\right)$$

Remember for small perturbations of the inflation rate around its non-inflationary steady state we have derived the following expression:

$$\hat{p}_t = \frac{q}{1 - q}\hat{q}_t . \tag{2.85}$$

This equation can be used to substitute out $\hat{q}_t$ in terms of $\hat{p}_t$.

$$\frac{q}{1 - q}\hat{p}_t = (1 - qb)\hat{m}c_t + qb\left(\frac{q}{1 - q}E_t\hat{p}_{t+1} + E_t\hat{p}_{t+1}\right), \tag{2.86}$$

which can be transformed into:

$$\hat{p}_t = b E_t \hat{p}_{t+1} + l\,\hat{m}c_t + e_t . \tag{2.87}$$

where: $l = \left[\frac{(1 - q)(1 - qb)}{q}\right]$

Equation (2.87) relates the inflation rate to the projected path of marginal costs. There is a long tradition in macroeconomic theory that relates the inflation rate directly to some output measure. This has the advantage that the inflation rate can be directly explained in terms of the economic cycle. Nevertheless substituting out marginal costs by the output gap has its downside risk. As we will see below, we have to make assumptions on technology and more importantly on the functioning of labor markets. In other words, replacing marginal costs by the output gap in the NKPC is more than "just mathematics". Assume that firms produce according to a linear production technology $Y_t = A_t N_t$. Log-linearizing the production function yields (Walsh (2003), p. 238):

$$\bar{Y}_t \hat{y}_t = \bar{N}_t \bar{A}_t \hat{a}_t + \bar{A}_t \bar{N}_t \hat{n}_t . \tag{2.88}$$

Which can be simplified to:

$$\hat{y}_t = \hat{n}_t + \hat{a}_t . \tag{2.89}$$

Hence deviations of production from its trend path are driven by technology and labor. But how is labor input determined? Let us assume for the baseline scenario that labor markets are perfectly competitive. From the households' equilibrium condition we know that the real wage is equal to the disutility of labor and the marginal utility of consumption. Given our concrete isoelastic utility function it holds that:

$$MC_t = \frac{W_t}{P_t} - \frac{c\, N_t^h}{C_t^{-s}} . \tag{2.90}$$

Log-linearizing this expression yields:

$$\hat{m}c_t = \left( \frac{\bar{W}_t}{\bar{P}_t} \right) [\hat{w}_t - \hat{p}_t] - \left( \frac{c\, \bar{N}_t^h}{\bar{C}_t^{-s}} \right) [h \hat{n}_t + s\, \hat{y}_t] . \tag{2.91}$$

The steady state level of the real wage and the ratio of marginal disutility of work in relation to the marginal utility of consumption can be written as:

$$\hat{m}c_t = h\hat{n}_t + s\hat{y}_t - (\hat{y}_t - \hat{n}_t). \qquad (2.92)$$

Substituting out labor $\hat{n}_t$ by the production function we can rewrite marginal costs as follows:

$$\hat{m}c_t = (1\text{-}h)\hat{a}_t + (h + s)\hat{y}_t. \qquad (2.93)$$

Which can be further simplified to:

$$\hat{\mathrm{m}}\mathrm{c}_t = (h + s)\left[\hat{y}_t - \left(\frac{1+h}{h+s}\right)\hat{a}_t\right]. \qquad (2.94)$$

So far we succeeded to express marginal costs in terms of output $\hat{y}_t$ and technology $\hat{a}_t$. By analyzing the flex-price equilibrium we will show that we can further simplify equation (2.94). If prices are flexible it will hold that

$$\frac{W_t}{P_t^f} = \frac{A_t}{m} = \frac{c N_t^h}{C_t^{-s}}, \qquad (2.95)$$

where the superscript f denotes the flex-price equilibrium. Applying the same log-linearization techniques we can express the equilibrium value as follows:

$$h\hat{n}_t^f + s\hat{c}_t^f = \hat{a}_t. \qquad (2.96)$$

From the production function we know that:

$$\hat{y}_t^f = \hat{n}_t^f + \hat{a}_t^f. \qquad (2.97)$$

Combining equation (2.91) and (2.92) by substituting out $\hat{n}_t^f$ and replacing $\hat{c}_t^f$ by $\hat{y}_t^f$ we see that the flex-price output gap can be stated as:

$$\hat{y}_t^f = \left( \frac{1+h}{s+h} \right) \hat{a}_t \,. \qquad (2.98)$$

Substituting the flex-price equilibrium $\hat{y}_t^f$ in equation (2.94) we can make the following prediction on the relationship between marginal costs $\hat{m}c_t$ and the output gap $\hat{y}_t$:

$$\hat{m}c_t = (s+h)\left[ \hat{y}_t - \hat{y}_t^f \right]. \qquad (2.99)$$

Substituting out $\hat{m}c_t$ in the Phillips curve (2.87) we arrive at the following well known NKPC which relates inflation rates to the output gap $\hat{x}_t = \left[ \hat{y}_t - \hat{y}_t^f \right]$:

$$\hat{p}_t = b E_t \hat{p}_{t+1} + g \frac{(1-q)(1-bq)}{q} \hat{x}_t \,. \qquad (2.100)$$

This expression nicely shows that the output gap is the driving variable in the inflation process. Iterating the NKPC forward it can be rewritten as follows under the assumption that $|b| < 1$:

$$p_t = E_t \sum\nolimits_{k=0}^{\infty} \left( b\bar{k} \right)^i y_{t+k} \,. \qquad (2.101)$$

Without any doubt the NKPC a la Calvo has its merits as it is directly derived from solid microeconomic foundations, which many macroeconomists regard as a virtue of its own. Nevertheless it suffers intrinsically from the same drawbacks as Taylor's (1979) version. As optimizing firms decide on price levels and not on price changes, the NKPC implies the same serious empirical defects. In particular it is unable to explain the persistent response of inflation to macroeconomic shocks. Not surprisingly this was sufficient impetus for some economists to propose alternatives which are closer to the data. In the following two sections we will highlight three different alternatives. One is a direct extension of Taylor's version, namely Fuhrer and Moore (1995) and one is a direct extension of Calvo-pricing (Christiano, Eichenbaum and Evans (2005)). Another string in monetary macroeconomics was initiated by Mankiw and Rice (2001) and Ball, Mankiw and Reis (2001), who categorically reject the notion of sticky prices. They argue instead that the degree of information updating in the economy is sticky.

#### 2.1.1.4 The Sticky Information Phillips Curve

The NKPC has been criticized on a number of grounds which led to fruitful extensions (McCallum (1997)). Besides these extensions which we will discuss in this section an alternative proposition to the NCPC which has been labeled as "Sticky Information Phillips Curve" (SIPC) which was originally proposed by Mankiw and Reis (2001). Mankiw and Reis criticize in particular that a sticky price Phillips curve implies disinflationary booms, if a real balance effect is present. Typically they assume that aggregate demand can be described by a quantity relationship of the following form (see Mankiw and Reis (2001), p. (1301):

$$m_t - p_t = y\, y_t . \qquad (2.102)$$

Therefore, announced disinflations boost aggregate demand as the real quantity of money increases.

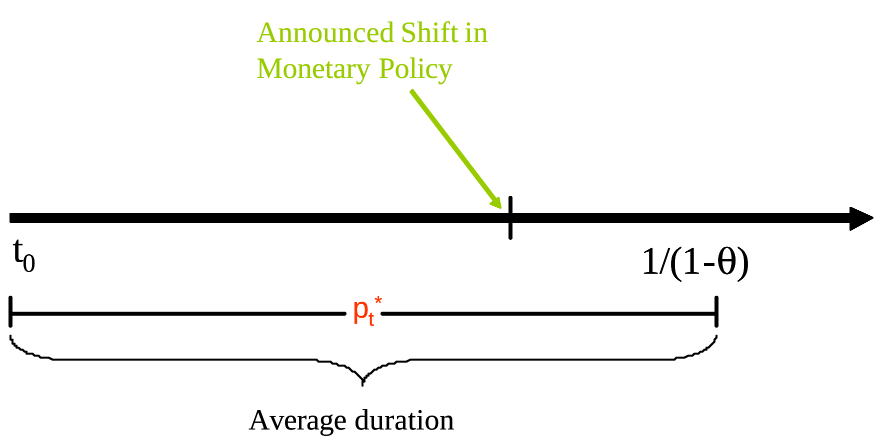



**Figure 3:** The "Dilemma" of a Calvo Price Setter

The detailed story goes as follows. A Calvo price setter that is allowed to reset its price with a probability of (1-θ) is likely to be stuck with the reset price on average for (1/(1-θ)) periods to come. If a shift in the inflation target is announced in three quarters from period $t_0$ on a Calvo price setter chooses its prices to be optimal on average. This implies that the Calvo price setter anticipates that he will not be able to adjust prices in the period the inflation target is changed as his price is still fixed with a certain probability. This implies that prior to the announced disinflation prices will be cut which boosts economic activity. To remedy this deficiency Mankiw and Reis propose an alternative Phillips curve.

Assume that at each point in time only a fraction of firms engages into the process of acquiring costly new information in order to update prices. So just as in the Calvo price model prices are adjusted infrequently at random intervals. But in contrast to the Calvo case prices are not sticky, but firms just price their output at an outdated information set. Mankiw and Reis postulate that the optimal price $p_t^*$ for those firms that are allowed to reoptimise is given by (Mankiw and Reis (2001), p. 1299):

$$p_t^* = p_t + \alpha y_t . \tag{2.103}$$

Hence the price depends on the aggregate log price level $P_t$ and the state of the cycle $y_t$. Those firms that do not update their information set in period j are stuck with their old state of knowledge. Accordingly they set the price for period t guided by an outdated information set they acquired j periods ago:

$$z_t^k = E_{t-k} p_t^* . \tag{2.104}$$

As we assume to have an infinite amount of firms j in the economy, the aggregate price level is just the weighted average of all prices currently charged

$$p_t = \theta \sum_{k=0}^{\infty} (1-\theta)^k z_t^k , \tag{2.105}$$

where θ is the rate of arrival of price adjustment. Higher values of (1-θ) indicate that the economy on average updates information more promptly. Substituting out $z_t^j$ in equation (2.104) the aggregate price level $p_t$ can be stated as follows:

$$p_t = \theta \sum_{k=0}^{\infty} (1-\theta)^k E_{t-k} ( p_t + \alpha y_t ) . \tag{2.106}$$

Obviously this formulation nests a Phillips curve as it relates the price level to some output measure. In order to transform equation (2.99) in terms of the inflation rate some further substitutions are necessary. Mankiw and Reis (2001) propose to take the first term out of the sum.

$$p_t = q\,(p_t + a\,y_t) + q\sum_{j=0}^{\infty}(1-q)^{j+1}E_{t-1-j}(p_t + a\,y_t)\,. \tag{2.107}$$

Iterating this equation one period backwards yields:

$$p_{t-1} = q\sum_{k=0}^{\infty}(1-q)^{k}E_{t-1-k}(p_{t-1} + a\,y_{t-1})\,. \tag{2.108}$$

Subtracting $p_{t-1}$ from $p_t$ one can retrieve the following expression for the inflation rate

$$p_t = q\,(p_t - a\,y_t) + q\sum_{k=0}^{\infty}(1-q)^{k}E_{t-1-k}(p_t + \Delta y_t) - q^2\sum_{k=0}^{\infty}(1-q)^{k}E_{t-1-k}(p_t + a\,y_t)\,. \tag{2.109}$$

Note from equation (2.101) we know that:

$$p_t - q\,(p_t - a\,y_t) = q\sum_{k=0}^{\infty}(1-q)^{k}E_{t-1-k}(p_t + a\,y_t) \tag{2.110}$$

Substituting out this expression in equation (2.107) we can restate the (SIPC) as follows:

$$p_t = [a\,q/(1-q)]\,y_t + q\sum_{k=0}^{\infty}(1-q)^{k}E_{t-1-k}(p_t + a\Delta y_t) \tag{2.111}$$

As we will show below this Phillips curve is immune against disinflationary booms. Assume that monetary policy announces a credible cold turkey disinflation in three quarters to come. Optimizing agents that update their information set over this period will choose $E_{t_0+1}p^*_{t+3}$, $E_{t_0+2}p^*_{t+3}$ and $E_{t_0+3}p^*_{t+3}$ in such a way that it will incorporate the new stance of monetary policy. As a consequence the price level will not change due to the announced shift in monetary policy until the shift actually occurs. Hence those agents that update the information set j periods in advance will choose $z^k_t = E_{t-k}p^*_t$ in such a way that it incorporates the shift in monetary policy. As prices are not intrinsically sticky there is no incentive to lower the price level in advance. In each period all prices are reset but some prices are based on outdated information.

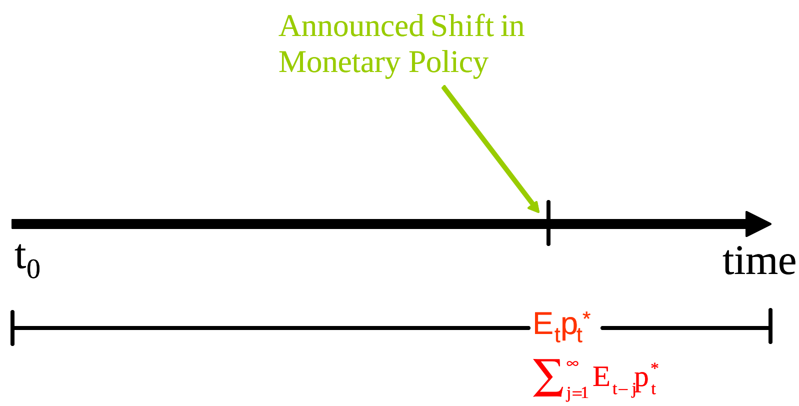



**Figure 4:** Price Setting under Sticky Information Phillips Curve

Of course one might critically ask whether this conclusion applies to a scenario where the central bank conducts inflation targeting by means of manipulating the real interest rate. In a regime of interest rate targeting lower inflationary expectations might give a restrictive monetary impulse as expected real interest rates increase. Typically in a purely New Keynesian IS-framework a real balance effect is not present unless one deviates from the assumption that money and consumption enter the utility function separately (see Woodford, ch. 4, pp. 301). The fact that the SIPC does not have more adherents can be traced back to extensions of the basic Calvo model. Within the next section we will highlight these extensions.

#### 2.1.1.5 Hybrid New Keynesian Phillips Curve

The NKPC combines elements of the "Real Business Cycle" framework with Keynesian elements of monopolistic competition and sticky prices. Nevertheless it has its problems when it meets the data. Therefore extensions have been developed to reconcile the virtues of micro foundation with stylized facts. These Phillips curves are labeled as "Hybrid New Keynesian Phillips Curves" (HNKPC) as they combine forward looking elements with backward looking behavior. Gali, Gertler and Salido Lopez (2001) propose the following derivation for a

HNKPC. The log price index $p_t$ can be defined as a weighted average of last period's prices $p_{t-1}$ and those prices that are reset in the current period:

$$p_t = \theta p_{t-1} + (1-\theta) p_t^* . \tag{2.112}$$

Those prices $p_t^*$ that are reset in the current period can be decomposed into $p_t^{rat}$, where the index (rat) denotes forward looking and $p_t^b$, where the index (b) denotes backward looking. Clarida, Gali and Lopez-Salido (2001) propose the following updating scheme for backward looking price setters:

$$p_t^b = p_{t-1}^* + \pi_{t-1} . \tag{2.113}$$

Accordingly backward looking firms update $p_{t-1}^*$ by last periods inflation rate. Of course alternative rules of thumb are thinkable and actually implemented (see Christiano, Eichenbaum and Evans 2005). So for instance one might assume that backward looking agents update their prices by steady state inflation.

$$p_t^b = p_{t-1}^* + p \ . \tag{2.114}$$

Generally rule-of-thumb behavior has become a very common theme in monetary macroeconomics as it is a straightforward way to introduce inertia in macroeconomic models. Rule-of-thumb behavior can be rationalized by a broad list of arguments (Amato and Laubach (2003)). It does not produce any computational costs as the information needed to update prices is assumed to be publicly available. The fraction of firms that updates by rule-of-thumb implicitly learns as yesterdays inflation rate incorporates the pricing decisions of those agents that optimize. In steady state rule-of-thumb setters will set prices equal to those who do Calvo pricing. Under the assumption of Calvo pricing forward looking firms set prices according to the following rule:

$$p_t^{rat} = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left( \hat{mc}_t + p_{t+k} \right) . \tag{2.115}$$

Equation (2.112) can be rewritten as follows:

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)\left(p_t^* - p_t\right). \tag{2.116}$$

Equation (2.116) can be simplified by substituting out the reset price $p_t^*$ by the weighted average of those price setters that follow a rule of thumb and those that optimize.

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)\left[(1-\omega)\left(p_t^{rat} - p_t\right) + \omega\left(p_t^b - p_t\right)\right] \tag{2.117}$$

Based on the seminal work of Sbordone (2002) one can derive the following expression for average marginal costs if firms implement a CES technology

$$\hat{m}c_t = \hat{m}c_{t+k} - \frac{\tilde{a}e}{1-\tilde{a}}\left(p_t^f - p_{t+k}\right), \tag{2.118}$$

where $\tilde{a}$ denotes the labor share and $\varepsilon$ the elasticity of demand. Accordingly marginal costs are given by $MC_t = \left(\left(W_t/P_t\right)/\left(1-\tilde{a}\right)\left(Y_t/N_t\right)\right)$ in levels. In order to obtain a Phillips curve in terms of inflation and deviations of marginal costs from their flex-price values we need to substitute out $\left(p_t^{rat} - p_t\right)$ and $\left(p_t^b - p_t\right)$ in (2.117). The distance between the price set by forward looking agents and the log price level can be stated as follows by substituting out (2.18) in (2.115):

$$p_t^{rat} - p_t = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\hat{m}c_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^{rat} - p_t\right) + \beta\theta\left(1+\frac{\varepsilon\alpha}{1-\alpha}\right)\pi_{t,t+k+1}\right]. \tag{2.119}$$

With this expression at hand it will have to hold that:

$$p_t^{rat} = (1-\beta\theta)\xi\sum_{k=0}^{\infty}(\beta\theta)^k E_t\hat{m}c_{t+k} + \sum_{k=1}^{\infty}E_t\pi_{t+k}. \tag{2.120}$$

where $\xi = \dfrac{1-\alpha}{1+\alpha(\varepsilon-1)}$

Combining these expressions yields:

$$p^b - p_t = \frac{1}{1-\theta}\pi_{t-1} - \pi_t . \tag{2.121}$$

Next insert (2.119) and (2.121) into (2.117) in order to obtain:

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)\left[\omega\left(\frac{1}{1-\theta}\right)\pi_{t-1} - \pi_t\right] + (1-\omega)\left[(1-\beta\theta)\xi\sum_{k=0}^{\infty}(\beta\theta)^k E_t\hat{m}c_{t+k} + \sum_{k=1}^{\infty}(\beta\theta)^k E_t\pi_{t+k}\right] \tag{2.122}$$

Collecting variables and multiplying the equation by the forward operator $(1-bqF)$ yields:

$$p_t = \frac{bq\left(q+(1-q)v\right)}{vbq+bq+(1-q)v}p_{t+1} + \frac{v}{vbq+bq+(1-q)v}p_{t-1} + \frac{(1-q)(1-v)}{vbq+bq+(1-q)v}(1-bqF)[...] \tag{2.123}$$

By applying the operator $(1-bqF)$ to the bracket the equation can be rewritten as:

$$p_t = \frac{bq\left(q+(1-q)v\right)}{vbq+bq+(1-q)v}p_{t+1} + g_b p_{t-1} + \tilde{l}\,mc_t + \frac{bq(1-q)(1-v)}{vbq+bq+(1-q)v}p_{t+1} . \tag{2.124}$$

This can ultimately be written in the standard form of a HNKPC:

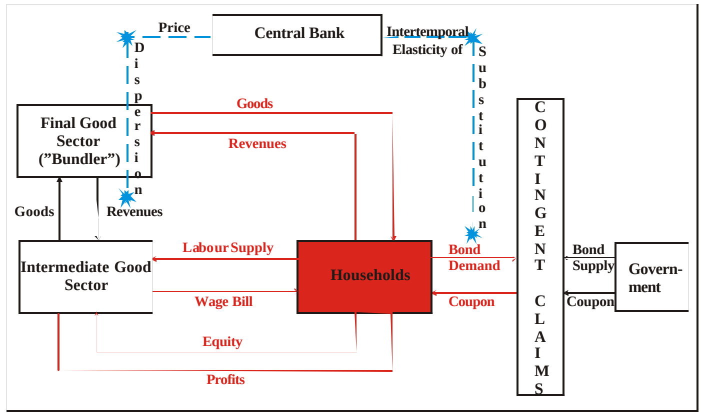
$$\pi_t = \gamma_f E_t\pi_{t+1} + \gamma_b\pi_{t-1} + \tilde{\lambda}mc_t + \varepsilon_t . \tag{2.125}$$

$$\text{where: } \lambda = (1-\theta)(1-\beta\theta)(1-\omega)\xi\phi^{-1}$$
$$\gamma_f = \beta\theta\phi^{-1},\ \gamma_b = \omega\phi^{-1},\ \phi = \theta + \omega\left[1-\theta(1-\beta)\right]$$

The hybrid specification nests the purely backward looking NKPC as well as the purely forward looking one. Therefore it bridges the gap between the old style accelerationist type of Phillips curve and the New Keynesian one. Although the HNKPC is intrinsically inertial it is common practice in applied work (Smets and Wouters ((2005)); Rabanal and Rubio-Ramirez (2003)) to augment equation (2.125) by a serially correlated error term. This indicates that the HNKPC is still not able to generate enough inertia out of their structural relationships. Based on this notion of the Phillips curve we will explore the true degree of forward-lookingness $\gamma_f$ and backward-lookingness $\gamma_b$ nested in the data. Equation (2.125) nests the case of a purely backward looking Phillips curve ($g_b = 1$) as well as the standard NKPC ($g_b = 0$). The dynamics enshrined in the NKPC crucially depend on two relations. On the one hand on the relative magnitude of $g_b$ in relation to $g_f$. On the other hand on $k_p^{'}$ which depicts the responsiveness of inflation to deviations of marginal cost from its steady state level. The relative size of $g_b$ in relation to $g_f$ critically determines the persistence of the inflation process. The higher the degree of backward-lookingness the higher will be the persistence of the inflation process as embedded in the autocorrelation functions. The degree of backward lookingness depends in particular on the percentage of price setters that update by rule of thumb and the share of Calvo-price setters in the economy. The second crucial parameter $k_p^{'}$ denotes the sensitivity of inflation with respect to marginal cost and indirectly over the production function to output. Therefore the parameter $k_p^{'}$ can be interpreted as the slope of the Phillips Curve. Note in particular that the parameter $k_p^{'}$ depends negatively on the degree of Calvo-price setters. Hence the more economic agents are able to adjust prices to changing economic conditions the looser becomes the link between changes in the economic cycle and the inflation process itself. Given the absolute magnitudes of $g_b, g_f$ and $k_p^{'}$ it is easy to see that by far the most important variable in explaining the inflation process is the inflation rate itself and not the deviation of marginal costs from its flex-price equilibrium. In section (4.3) we will systematically evaluate the implications of variations in the degree of forward and backward lookingness and its implication for the model dynamics. To summarize. This section analyzed the price setting behavior of firms. We saw that firms are only called at random intervals to reset prices. This type of price stickiness leads to price dispersion in the economy, which has detrimental effects on consumer welfare. Therefore the non vertical (NKPC) curve leaves a meaningful role to a central bank that smoothes out the impact of macroeconomic shocks on welfare.

### 2.1.2 Optimizing Households

New Keynesian macroeconomics centers around a representative household that cares on utility. It is the dominant doctrine to assume that households are guided by optimization behavior. In other words households behave as if they are maximizing an intertemporal utility function (Woodford, ch. 2, (2003)).




$$E_0 \sum_{i=0}^{\infty} b^i U_{t+i} \,, \qquad (2.126)$$

where $U_t$ denotes the instant utility. We will assume throughout the section that the utility function is separable in its arguments, which means nothing but that the individual variables enter the utility function additively. So if we assume for instance that the period utility depends in particular on consumption $C_t$ and the work load $N_t$ an additive utility function will have the following generalized form:

$$U(C_t, N_t) = U(C_t) + U(N_t)\,. \qquad (2.127)$$

This assumption simplifies the mathematical exposition as the derivative with respect to e.g. consumption $C_t$ only depends on consumption itself:

$$U_C(C_t) = h(C_t)\,. \qquad (2.128)$$

So let us turn our attention to a representative household that designs its optimal intertemporal consumption path. The standard assumption that marginal utility is increasing ($U_C$>0) at a decreasing rate ($U_{CC}$<0) holds. This implies in particular that a representative household is risk averse and would be willing to give up a fraction of his overall consumption spending in order to be isolated from stochastic shocks beyond its control. This property of the utility function is key to understand why the representative household appreciates stabilization policy as an effective tool to smooth out fluctuations in macroeconomic income (Campbell, Lo, McKinlay (1997)).

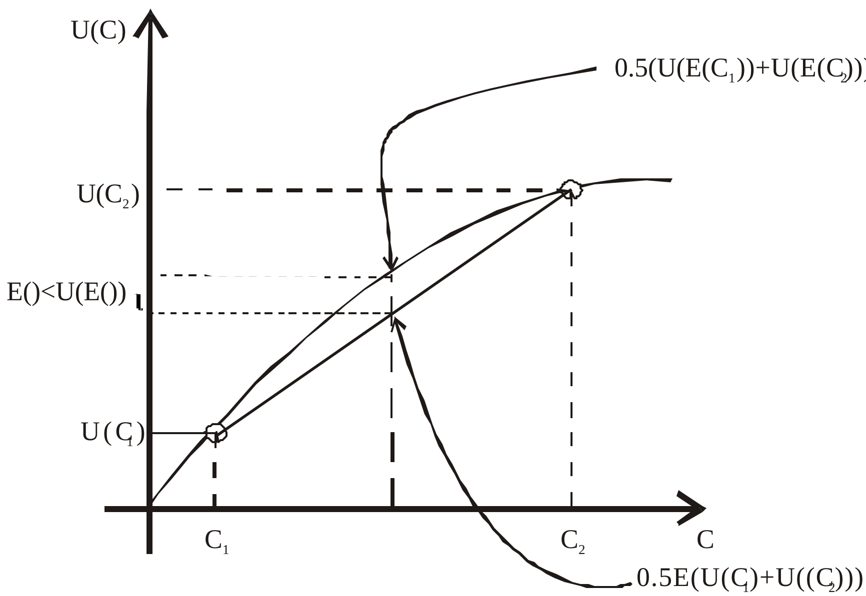



**Figure 5:** A Risk Averse Household

We will build on this property later on when we rationalize monetary policy from a utility based welfare criterion (Woodford (2001)). The figure shows that a risk averse economic agent will always prefer the certain outcome $0.5\left(U\left(E\left(C_1\right)\right)+U\left(E\left(C_2\right)\right)\right)$ to a gamble where he receives $U(C_1)$ and $U(C_2)$ with a 50% probability, although the expected value of consumption is the same.

### 2.1.2.1 The Purely Forward Looking IS-Equation

Given these preliminaries let us take a somewhat deeper look at the habitat of our representative agent. It is common practice to assume the existence of complete contingent claims markets. Assets are traded that offer complete insurance against any risk that stems from (firm-) specific income, (firm-) specific price or taste shocks. This implies that the wealth is equal across households in equilibrium, so that all households face the same flow budget constraint. In particular all representative agents own a proportionate share of the market portfolio of firms supplying in the intermediate good sector. The presence of complete contingent claims markets imply that a risk free bond exists. As it is well known from finance literature a general class of asset pricing models can be written as follows (Campbell Lo Mckinelay (1997))

$$B_t = E_t\left[\Delta_{t,t+1} A_{t+1}\right], \tag{2.129}$$

where $B_t$ stands for the nominal price of a bond today, and $A_{t+1}$ is the stochastic price of the bond tomorrow, $Q_{t+1}$ is the so called stochastic discount factor, which is often equally labeled as the pricing kernel. Generally the pricing kernel is defined as (Cochrane, 2004, p. 8.):

$$\Delta_{t,k} = b^k \frac{U_C(C_{t+1})}{U_C\left(C_t\right)}. \tag{2.130}$$

Equation (2.130) tells us that risky assets are typically traded at a discount as $U(C_{t+1}) < U\left(C_t\right)$. Different models of asset pricing, e.g. the Capital Asset Pricing Model (CAPM) are nothing but alternative theories for $\Delta_{t,k}$. Let us assume that a risk free asset which has a face value of $B_t$=€1 exists and pays of a certain nominal return of $A_{t+1}=1+i_t$ tomorrow. According to the fundamental pricing relationship it will have to hold that:

$$1€ = E_t\left[\Delta_{t,t+1}\left(1+i_t\right)\right]. \tag{2.131}$$

As $A_{t+1}$ is not a random variable, households do not have to build expectations on $A_{t+1}$:

$$1 = E_t\left[\Delta_{t,t+1}\right]\left(1+i_t\right). \tag{2.132}$$

Under these assumptions $E_t\Delta_{t,t+1}$ can be defined as:

$$\Delta_{t,t+1} = \frac{1}{1+i_t}. \tag{2.133}$$

Hence with complete contingent claims markets there exists a stochastic discount factor $\Delta_{t,t+1}$ that is equal to $\left(1+i_t\right)^{-1}$ for risk less assets. If one assumes that government bonds $B_t$ are risk free it is legitimate to discount $B_{t+j-1}$ in the flow budget constraint of households with the discount factor $\Delta_{t,t+1}$:

$$P_{t+j}C_{t+j} + M_{t+j} + B_{t+j} \leq W_{t+j}N_{t+j} + \Delta_{t,t+j+1}B_{t+j-1} + \Pi_{t+j}. \quad \text{for j=1,2,3,..} \quad (2.134)$$

Substituting out the pricing kernel of households (2.134) can be rewritten as:

$$P_{t+j}C_{t+j} + M_{t+j} + B_{t+j} \leq W_{t+j}N_{t+j} + \left(1+i_{t-1+j}\right)B_{t+j-1} + \Pi_{t+j}, \text{ for j=1,2,3,.. } (2.135)$$

Accordingly our representative household allocates his recourses on consumption spending $P_{t+j}C_{t+j}$ and his money holdings $M_{t+j}$ and bond holdings $B_{t+j}$. His spending patterns are financed by labor income $W_{t+j}N_{t+j}$, dividend payments and profits $\Pi_{t+j}$ from the intermediate good sector. In equilibrium it will have to hold that

$$\frac{U_c\left(C_t;x_t\right)}{U_c\left(E_tC_{t+1};x_{t+1}\right)} = \frac{b}{\Delta_{t,t+1}}\frac{P_t}{P_{t+1}}, \quad (2.136)$$

where $x_t$ denotes stochastic shocks to preferences. Substituting out the stochastic discount factor $\Delta_{t,t+1}$ this can equally be written as:

$$U_C\left(C_t;x_t\right) = b\,E_t\left[U_C\left(C_{t+1};x_t\right)\frac{P_t}{P_{t+1}}\right]\left(1+i_t\right). \quad (2.137)$$

This equation is nothing but the well known intertemporal-Euler equation, which is also often labeled as the intertemporal IS-equation. The equation states that the marginal utility of consumption today has to be equal to the discounted marginal utility of consumption tomorrow corrected for changes in the price level. In equilibrium it holds that households cannot improve their welfare by reallocating their spending patterns. To animate the above said let us assume that the representative household has the following additive period utility function in consumption

$$U\left(C_t;x_t\right)=\frac{C_t^{1-s}}{1-s}e^{x_t}\ , \qquad (2.138)$$

where $s$ measures the intertemporal elasticity of substitution of log consumption with respect to changes in the real interest rate. This so called power utility function is the single most used function in monetary macroeconomics. The marginal utility of consumption $U_C$ can be written as follows:

$$U_C\left(C_t;x_t\right)=C_t^{-s}e^{x_t}\ . \qquad (2.139)$$

Substituting out $U_c$ in equation (2.137) one obtains the following non-linear equation:

$$1=b\left(1+i_t\right)E_t\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_{t+1}^{-s}}{C_t^{-s}}\right)e^{x_{t+1}-x_t}\ . \qquad (2.140)$$

Log-linearizing the intertemporal Euler-equation the following approximation holds:

$$\begin{aligned}1\cong\ & f_{\bar{i}}(\bar{i}_t,\bar{P}_{t+1},\bar{P}_t,\bar{C}_{t+1},\bar{C}_t)\bar{i}_t\hat{i}_t+f_{\bar{P}_{t+1}}(\bar{i}_t,\bar{P}_{t+1},\bar{P}_t,\bar{C}_{t+1},\bar{C}_t)\bar{P}_{t+1}\hat{p}_{t+1}+f_{\bar{P}_t}(\bar{i}_t,\bar{P}_{t+1},\bar{P}_t,\bar{C}_{t+1},\bar{C}_t)\bar{P}_t\hat{p}_t\\ &+f_{\bar{C}_{t+1}}(\bar{i}_t,\bar{P}_{t+1},\bar{P}_t,\bar{C}_{t+1},\bar{C}_t)\bar{C}_{t+1}\hat{c}_{t+1}+f_{\bar{C}_t}(\bar{i}_t,\bar{P}_{t+1},\bar{P}_t,\bar{C}_{t+1},\bar{C}_t)\bar{C}_t\hat{c}_t\end{aligned}\ .$$

(2.141)

Substituting out the partial derivatives and simplifying terms yields

$$1\cong\left[b\,E\left(\frac{\bar{P}_t}{\bar{P}_{t+1}}\right)\left(\frac{\bar{C}_{t+1}^{-s}}{\bar{C}_t^{-s}}\right)\right]\left[\left(\hat{i}-\hat{p}_t\right)+s\,\hat{c}_t-s\,\hat{c}_{t+1}+x_{t+1}-x_t\right]. \qquad (2.142)$$

This can be approximated by:

$$\hat{c}_t=E_t\hat{c}_{t+1}-\frac{1}{s}\left(i_t+p_{t+1}\right)+\frac{1}{s}\left(x_{t+1}-x_t\right). \qquad (2.143)$$

Hence the Euler equation predicts that today's consumption depends negatively on the real interest rate $(i_t - p_{t+1})$ and positively on the expected consumption level $C_{t+1}$. If we assume that the capital stock is constant we can substitute out consumption by output $(\hat{c}_t = \hat{y}_t)$:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{s}(i_t + \hat{p}_{t+1}) + \frac{1}{s}(x_{t+1} - x_t). \tag{2.144}$$

This equation is the well known intertemporal IS-equation. It has many supporters as it is derived from solid microeconomic foundations, namely from the constraint optimization problem of a representative household. The New Keynesian IS-equation nicely depicts that monetary policy can manage aggregate demand by temporarily changing the slope of the intertemporal budget constraint. Increasing real interest rates imply that households have an incentive to postpone consumption into the future. This means nothing but that monetary policy can steer real interest rate according to its ultimate objectives. Unfortunately the purely New Keynesian IS-equation is unable to explain stylized facts (Fuhrer (1997)). In particular it implies that consumption growth $(\ln C_{t+1} - \ln C_t)$ and real interest rates should be positively correlated:

$$(E_t \hat{y}_{t+1} - \hat{y}_t) = \frac{1}{s}(i_t - \hat{p}_t). \tag{2.145}$$

This is at odds with the data. High real interest rates today foreshadow an economic depression and not a period of prosperity as predicted by the intertemporal IS-equation. Moreover the New Keynesian IS-equation predicts that a shock to aggregate demand will generate only a single jump in output which stands in sharp contrast to the hump shaped behavior documented in VAR studies (Christiano, Eichenbaum and Evans (2005)). In order to address these issues extensions of the New Keynesian IS-equation have been designed which we will highlight in the next sections.

#### 2.1.2.2 Hybrid Specifications of the New Keynesian IS-Equation

Extensions of the purely forward looking IS-equation have been designed to scope with issues like hump shaped responses of output to macroeconomic shocks and negative correlation's of the real interest rate with and output. Abel (1990) has proposed the following period utility function which was for instance implemented by Smets and Wouters (2003) in a widely appreciated New Keynesian model

$$U_t = e^{x_t} \frac{1}{1-s} \left( C_t - H_t \right)^{1-s} , \tag{2.146}$$

where $H_t$ denotes a variable that measures the degree of habit formation in consumption which they assume to be given by $H_t = hC_{t-1}$. By this assumption Smets and Wouters (2003) succeed to introduce inertia in consumption decisions as period utility depends on last periods consumption level $C_{t-1}$. The marginal utility of consumption can now be stated as:

$$U_C \left( C_t ; x_t \right) = e^{x_t} \left( C_t - hC_{t-1} \right)^{-s} . \tag{2.147}$$

Inserting these expressions into the Euler equation (2.137) yields:

$$e^{x_t^b} \left( C_t - hC_{t-1} \right)^{-s} = b \left( C_{t+1} - hC_t \right)^{-s} \frac{P_t}{P_{t+1}} \left( 1 + i_t \right) e^{x_{t+1}^b} \tag{2.148}$$

By applying the same linearization apparatus as we did beforehand we need to compute the following expressions:

$$f_{\overline{C}_{t-1}} (...) \overline{C}_{t-1} \hat{c}_{t-1} = hs\, b\, \hat{c}_{t-1} \overline{C}_{t-1} \left[ \left( \frac{\left( \overline{C}_{t+1} - h\overline{C}_t \right)}{\left( \overline{C}_t - h\overline{C}_{t-1} \right)} \right)^{-s} \left( 1 + i_t \right) \frac{\overline{P}_t}{\overline{P}_{t+1}} \frac{1}{\left( h\overline{C}_{t-1} - \overline{C}_t \right)} \right] e^{\left( x_{t+1} - x_t \right)} \tag{2.149}$$

$$f_{\overline{C}_t} (...) \overline{C}_t \hat{c}_t = s\, \hat{c}_t b \overline{C}_t \left[ \left( \frac{\left( \overline{C}_{t+1} - h\overline{C}_t \right)}{\left( \overline{C}_t - h\overline{C}_{t-1} \right)} \right)^{-s} \left( 1 + i_t \right) \frac{\overline{P}_t}{\overline{P}_{t+1}} \frac{\left( h^2 \overline{C}_{t-1} - \overline{C}_{t+1} \right)}{\left( h\overline{C}_{t-1} - \overline{C}_t \right) \left( h\overline{C}_{t-1} - \overline{C}_t \right)} \right] e^{\left( x_{t+1} - x_t \right)} \tag{2.150}$$

$$f_{\overline{C}_{t+1}} (...) \overline{C}_{t+1} \hat{c}_{t+1} = s\, \hat{c}_{t+1} b\, \overline{C}_{t+1} \left[ \left( \frac{\left( \overline{C}_{t+1} - h\overline{C}_t \right)}{\left( \overline{C}_t - h\overline{C}_{t-1} \right)} \right)^{-s} \left( 1 + i_t \right) \frac{\overline{P}_t}{\overline{P}_{t+1}} \frac{1}{\left( h\overline{C}_t - \overline{C}_{t+1} \right)} \right] e^{\left( x_{t+1} - x_t \right)} \tag{2.151}$$

$$f_{\bar{P}_{t+1}}(\ldots)\,\bar{P}_{t+1}\hat{p}_{t+1} = -\hat{p}_{t+1} b\left[\left(\frac{\left(\bar{C}_{t+1} - h\bar{C}_t\right)}{\left(\bar{C}_t - h\bar{C}_{t-1}\right)}\right)^{-s}\left(1+i_t\right)\frac{\bar{P}_t}{\bar{P}_{t+1}}\right] e^{\left(x_{t+1}-x_t\right)} \qquad (2.152)$$

$$f_{\bar{P}_t}(\ldots)\,\bar{P}_t\hat{p}_t = \hat{p}_t b\left[\left(\frac{\left(\bar{C}_{t+1} - h\bar{C}_t\right)}{\left(\bar{C}_t - h\bar{C}_{t-1}\right)}\right)^{-s}\left(1+i_t\right)\frac{\bar{P}_t}{\bar{P}_{t+1}}\right] e^{\left(x_{t+1}-x_t\right)} \qquad (2.153)$$

$$f_{\bar{i}_t}(\ldots)\,\bar{i}_t\hat{i}_t = \hat{i}_t b\left[\left(\frac{\left(\bar{C}_{t+1} - h\bar{C}_t\right)}{\left(\bar{C}_t - h\bar{C}_{t-1}\right)}\right)^{-s}\left(1+i_t\right)\frac{\bar{P}_t}{\bar{P}_{t+1}} e^{\left(x_{t+1}-x_t\right)}\right] \qquad (2.154)$$

$$f_{\left(\bar{x}_{t+1}-\bar{x}_t\right)}(\ldots) e^{\left(\bar{x}_{t+1}-\bar{x}_t\right)}\left(\hat{x}_{t+1} - \hat{x}_t\right) = b\left[\left(\frac{\left(\bar{C}_{t+1} - h\bar{C}_t\right)}{\left(\bar{C}_t - h\bar{C}_{t-1}\right)}\right)^{-s}\left(1+i_t\right)\frac{\bar{P}_t}{\bar{P}_{t+1}}\right] e^{\left(x_{t+1}-x_t\right)}\left(\hat{x}_{t+1} - \hat{x}_t\right)$$

(2.155)

Using the partial derivatives and assuming that the steady state of consumption is equal to $\bar{C}_t = \bar{P}_t = 1$ we arrive at the following approximation:

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}\hat{c}_{t+1} - \frac{1-h}{(1+h)s}\left(i_t - \hat{p}_{t+1}\right) + \frac{1-h}{(1+h)s}\left(x_{t+1} - x_t\right). \qquad (2.156)$$

Let us assume that the capital stock in the economy is fixed, so that the only source of short run variation from steady state is consumption: $\hat{y}_t = \hat{c}_t$, then it holds that:

$$\hat{y}_t = \frac{h}{1+h}\hat{y}_{t-1} + \frac{1}{1+h}\hat{y}_{t+1} - \frac{1-h}{(1+h)s}\left(i_t - \hat{p}_{t+1}\right) + \frac{1-h}{(1+h)s}\left(x_{t+1} - x_t\right) \qquad (2.157)$$

This is the so called hybrid version of a New Keynesian IS-equation. It collapses to the purely forward looking IS-equation if we set the degree of habit formation h equal to zero. The higher the degree of habit formation the more economic agents center their optimal consumption choices around last periods consumption level $C_{t-1}$. Just as in the case of a HNKPC it proofed to be necessary to augment the intertemporal IS-equation by highly auto correlated shocks (Smets and Wouters (2005)) to display a gradual and hump shaped response to diverse categories of macroeconomic shocks (Fuhrer (2000)).

To summarize: This section proposed two possible specifications that govern the intertemporal allocation schemes of households. Basically we saw that "anything goes" in the sense that plausible variations in the utility function lead to alternative specifications of the intertemporal IS-equation.

### 2.1.2.3 The Optimal Labor Supply Decision

The last section has derived the optimal allocation scheme for consumption through time. Besides his consumption decision a representative household has to decide how much labor to supply. Before analyzing the labor supply decision let us take a somewhat closer look at the interaction between firms and households in the New Keynesian framework. So far we have only analyzed the monopolistically competitive firm taking its decisions on production in the light of expectations on the future path of marginal costs. Additionally we have analyzed a representative household that allocates consumption optimally through time. We did not analyze labor markets where the labor demand of firms meets the labor supply of households. Nevertheless as the basic New Keynesian framework assumes labor markets to be perfectly flexible they are uninteresting from a stabilization perspective as wages are able to adjust to a changing economic environment. Let us clarify this statement by a thought experiment. For the implications of sticky wages see Erceg, Henderson and Levin (2000).

Monopolistically competitive firms price their output above marginal costs. Therefore it will be profitable for these firms in the intermediate good sector to accommodate demand shocks as long as prices are equal or above marginal cost ($p_{jt}^{fix} \geq mc_{jt}$) as profits will increase. Monopolistic competition is a necessary condition for monetary policy to be able to manage aggregate demand. Firms can only induce workers to work more by increasing nominal wages. As prices are sticky for a fraction of $q$ percent of all firms this implies that some firms will operate in an environment of decreasing marginal unit profits. Note if product markets would be perfectly competitive firms would have no incentive to accommodate demand shocks as $p_{jt}^{fix} \leq mc_{jt}$.

Due to price stickiness those firms that are fixed with their prices can only react on the production side whereas those that are allowed to reoptimize will change prices and quantities. With these preliminaries in mind let us now analyze in some depth labor markets.

From the perspective of an individual household the well known relationship that the marginal utility of consumption divided by the marginal disutility of labor has to be equal to the real wage has to hold in equilibrium:

$$\frac{U_N}{U_C} = \frac{W_t}{P_t}. \tag{2.158}$$

So let us for instance assume that the following additive separable intertemporal utility function is able to explain the behavior of our household (Walsh (2003), p.232):

$$E_0 \sum_{i=0}^{\infty} b^i \left[ \frac{C_{t+i}^{1-s}}{1-s} - c\,\frac{N_{t+i}^{1+h}}{1+h} \right]. \tag{2.159}$$

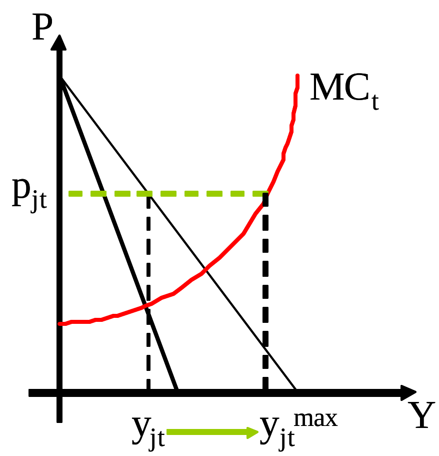



**Figure 6:** Demand Shock Accommodation by a Monopolistically Competitive Firm

If firms operate with a linear technology of the form

$$y_{jt} = A_t N_{jt}, \tag{2.160}$$

it will have to hold in equilibrium that:

$$\frac{c\,N_t^h}{xC_t^{-s}} = \frac{W_t}{P_t} = \frac{A_t}{m} \tag{2.161}$$

Log linearly approximated the flex-price equilibrium can be written as

$$h\hat{n}_t^f + s\,\hat{c}_t^f = \hat{a}_t^f , \tag{2.162}$$

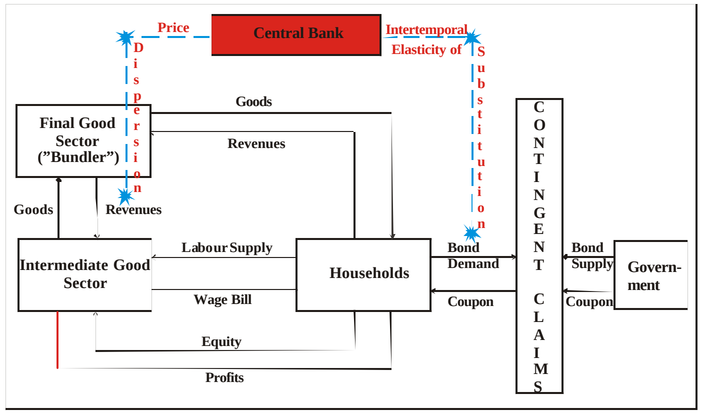
where the superscript f denotes flexible prices. Substituting out the steady-state level of employment $\hat{n}_t^f$ by means of the production function $y_t^f = \hat{n}_t^f + \hat{a}_t^f$ the flex-price equilibrium can be written as a function of technology shocks as follows:

$$\hat{y}_t^f = \left(\frac{1+h}{s+h}\right)\hat{a}_t^f . \tag{2.163}$$

Note as long as we assume that labor markets are flex-price markets they are not a source of welfare losses. Therefore the simple New Keynesian macromodel does not take labor markets into account. Of course this assumption has downsides. Obviously labor markets are not perfectly competitive; therefore a model that neglects to put inertial reactions in labor markets will underestimate the global need for stabilization policy in terms of household's willingness to give up consumption in order to be isolated from stochastic shocks beyond its control.

The degree of natural output volatility $\hat{y}_t^f$ depends besides the size of technology shocks themselves on the intertemporal elasticity of substitution $s$ and the Frisch elasticity $c$. If economic agents are more risk averse fluctuations from technology to output will be somewhat dampened, as economic agents have a strong incentive to smooth consumption (Canzoneri, Cumby. and Diba (2004)).

## 2.1.3 The Role of Monetary Policy




So far we have only analyzed the private sector in the New Keynesian economy. We studied the interaction between households and monopolistically competitive firms.

Now we turn our attention to a benevolent central planer in the form of a central bank. The overall goal of monetary policy is to promote welfare. This is usually interpreted in terms of keeping the inflation rate close to the inflation target while equally having a concern for economic activity. The implementation of monetary policy is based on a so-called monetary strategy. The strategy facilities the internal decision-making process as well as the transparency and accountability in relation to the public. The strategy of inflation-forecast targeting has become more and more popular throughout the last decade. Countries like New Zealand, Canada, the UK, Sweden, Finland, Australia and Brazil have introduced a full-fledged inflation-targeting regime. Other central banks most notably the FED and the ECB implicitly implemented such an approach. Following Bofinger (2001), Svensson (2002) and Woodford (2003) inflation forecast targeting can be defined by the following main characteristics:

- There is a numerical value for the inflation target. Achieving this inflation rate is the dominant goal of monetary policy although some space for other goals like stabilizing output around its trend is left.
- Interest rates are set in such a way that the inflation forecast will return to the inflation target in the periods to come. Therefore the inflation forecast plays a prominent role in the decision-making process. The speed of dis- and reinflation is determined by preferences.
- The decision-making process is characterized by a high degree of transparency and accountability.

### 2.1.3.1 The Quadratic Loss Function

It is common practice to characterize the preferences of central banks by quadratic loss functions. The goal variables are modeled in terms of the output gap and the inflation rate. The central bank's intertemporal optimization problem can be stated within the linear quadratic framework as follows:

$$L_t = E_0 \sum_{k=0}^{\infty} b^k \left\{ l_p \left(p_{t+k} - p_0\right)^2 + l_y y_{t+k}^2 \right\} \qquad (2.164)$$

The parameter $b$ denotes the discount factor and $\lambda$ measures the relative weight attached to the goal of output stabilization. Note under the assumption that $\beta$ converges to one the intertemporal loss function converges to the sum of unconditional variances:

$$L_t = Var\left[p_t\right] + l_y Var\left[y_t\right]. \qquad (2.165)$$

The popularity of the quadratic stems from the fact that it is able to map the popular strategy of 'inflation-forecast targeting'. The nested regimes can be stated as follows:

- Strict-inflation targeting: $\lambda_y = 0$
- Flexible-inflation targeting: $\lambda_y > 0$

The intuition behind the quadratic loss function is quite simple. Policymakers stabilize squared deviations of the inflation rate around the inflation target while equally holding squared deviations of the output gap near null. The quadratic implies that positive and negative deviations of target values impose an identical loss on economic agents. Additionally large deviations from target values generate a more than proportional loss. If $\lambda_y$ is equal to null policymakers only care about inflation. This type of central bank is called inflation nutter. If $\lambda_y$ goes to infinity, policymakers only care on output. This preference type will be called output junkie. Although the approach to use a quadratic function is very plausible it has been criticized as being ad hoc. In particular Woodford (2001) has argued that there is a need to find a microeconomic rationale for the quadratic loss function.

### 2.1.3.2 A Welfare Based Approach to a Quadratic Loss Function

Quite arguably the overall goal of monetary policy is to promote economic welfare. This means in particular that consistent with the structural equations of the model the central bank sets a path for its instrument $\{i\}_{\{0,\infty\}}$ consistent with its targets in such a way that the expected utility of a representative household is maximized. Nevertheless given no general consensus on relevant variables there is quite some ambiguity how to specify a concrete loss function. Woodford (2001) has shown that it is possible under certain assumptions to derive a "natural welfare criterion" based on a representative household's utility function (Woodford(2003), p. 382 ch. 6)):

"An important advantage of using a model founded upon private-sector optimization to analyze the consequences of alternative policy rules is that there is a natural welfare criterion in the context of such a model, provided by the preferences of private agents, which are displayed in the structural relations that determine the effects of alternative policies."

The following pages reproduce the mathematical derivation of a micro-founded approach to the quadratic loss function. This section draws on Grimm and Ried (2005), Walsh (2003) and Woodford (2001/ 2003). The starting point of our analysis is the following expected utility:

$$E_0\left\{\sum_{t=0}^{\infty} b^t U_t\right\}. \tag{2.166}$$

Let us assume for reasons of mathematical convenience that the period utility contribution to the expected utility of a representative household can be stated as follows:

$$U_t = u\left(C_t; z_t\right) - \int_0^1 \tilde{v}\left(n_t(i); z_t\right)di\,. \tag{2.167}$$

Just as in the previous sections we assume that households draw utility from consumption whereas the production of commodity $y_t$ imposes disutility. The vector $z_t$ is composed of macroeconomic shocks. Using the national income identity $Y_t = C_t$ and the production technology $y_t(i) = A_t f(n_t(i))$ of firm i which links labor input to output the utility function can be restated as follows:

$$U_t = u\left(Y_t; z_t\right) - \int_0^1 v\left(y_t(i); z_t\right) di \,, \tag{2.168}$$

where the aggregate commodity is composed of the weighted average of the individual intermediate goods:

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{q-1}{q}}\right]^{\frac{q}{q-1}}. \tag{2.169}$$

For notational purposes let us restate the following conventions. Let $\bar{Y}$ be the steady state of a variable, then the deviation around the steady state is defined as: $\tilde{Y}_t = Y_t - \bar{Y}$, where the log deviation is given by $\hat{Y}_t = \log\left(Y_t / \bar{Y}\right)$. Given these notational conventions the variables $Y_t$ and $Y_t^2$ can be approximated up to second order as follows:

$$\frac{Y_t - \bar{Y}}{\bar{Y}} \approx \hat{y}_t + \frac{1}{2}\hat{y}^2 \tag{2.170}$$

$$\hat{y}_t^2 \approx \left(\frac{Y_t - \bar{Y}}{\bar{Y}}\right)^2. \tag{2.171}$$

Let us now consider the following second-order approximation to output of a households' utility function around the steady state level $\left(\bar{Y}, 0\right)$ (Woodford(2001), p. 16):

$$U\left(Y_t; z_t\right) \approx U\left(\bar{Y}, 0\right) + U_C \tilde{Y}_t + U_z z_t + \frac{1}{2} U_{CC} \tilde{Y}_t^2 + U_{C,Z}\, z_t \tilde{Y}_t + \frac{1}{2} z_t^{'} U_{z,z} z_t + O\left(\|z\|^3\right). \tag{2.172}$$

This can be rewritten as

$$U\left(Y_t, z_t\right) \approx U\left(\bar{Y}, 0\right) + U_C \bar{Y}\left(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right) + U_z z_t + \frac{1}{2} U_{CC} \bar{Y}^2 \hat{Y}_t^2 + U_{C,Z}\, z_t^{'} \bar{Y}_t \hat{Y}_t + \frac{1}{2} z_t^{'} U_{z,z} z_t + O\left(\|z\|^3\right), \tag{2.173}$$

by substituting out $\tilde{Y}_t$ and $\tilde{Y}_t^2$ by using equations (2.170) and (2.171) and dropping out cross terms such as $z_t\hat{Y}_t^2$. Equation (2.173) can be further simplified by defining the following coefficients:

$$\tilde{s} = -\frac{\bar{Y}U_{CC}}{U_C},\ \tilde{f}_t = -\frac{U_{C,Z}}{\bar{Y}U_{CC}} z_t .$$

So that:

$$U\left(Y_t, z_t\right) \approx \bar{Y}U_C\left\{\hat{Y}_t + \frac{1}{2}\left(1-\tilde{s}^{-1}\right)\hat{Y}_t^2 + \tilde{s}^{-1}\tilde{f}_t\hat{Y}_t\right\} + t.i.p. \tag{2.174}$$

where t.i.p. denotes terms independent of policy. Equation (2.174) gives a second order approximation to output. Let us now take a second order approximation of the household's disutility of producing commodity i around the steady state $\left(\bar{Y};0\right)$:

$$v\left(y_t(i), z_t\right) = v\left(\bar{y},0\right) + n_y\tilde{y}\left(i\right) + n_z z_t + \frac{1}{2}n_{yy}\tilde{y}_t\left(i\right)^2 + n_{yz}z_t\tilde{y}_t\left(i\right) + \frac{1}{2}z_t n_{zz}z_{tt} + O\left(\|\ \|^3\right).$$

(2.175)

Substituting out $\tilde{y}_t(i)$ and $\tilde{y}_t^2(i)$ by the analogue to equation (2.170) and (2.171) we can rewrite (2.175) as follows:

$$n\left(y_t(i); z_t\right) \approx n_y\,\bar{y}\left[\hat{y}_t(i) + \frac{1}{2}\left(1+\tilde{h}\right)\hat{y}_t(i)^2 - \tilde{h}\tilde{q}_t\hat{y}_t(i)\right] + t.i.p., \tag{2.176}$$

where we have defined the following coefficients:

$$\tilde{h} = \frac{n_{yy}\bar{y}}{n_y}; \tilde{q}_t = -\frac{n_{y,z}z_t}{n_{yy}\bar{y}}.$$

So far we have succeeded in taking a second-order approximation of a household's utility function in terms of output. This is a step in the right direction, as the variable output can be rewritten in terms of the output gap and the inflation rate via the Phillips curve. Accordingly it is possible in principle to restate the approximations (2.173) and (2.176) in terms of a quadratic loss function.

To proceed we need to define the distorted steady state level that prevails if prices would be flexible. In equilibrium it holds that households equate the marginal disutility of labor divided by the marginal utility of consumption to the real wage. Given the assumptions made on the production technology (see Woodford, 2001 p. 13) it holds that the efficient steady state level is given by $s\left(Y^{*};0\right)=1$, whereas the distorted steady state under monopolistic competition is defined as:

$$\left(n_{y}/U_{C}\right)=\frac{q-1}{q}=1-\Phi. \tag{2.177}$$

with: $\Phi=\left(1/q\right)$.

Accordingly the parameter $\Phi$ summarizes the degree of macroeconomic distortion. If one assumes for reasons of mathematical convenience that the overall distortions are sufficiently small in size then the terms $\Phi\hat{y}^{2}$ and $\Phi q_{t}\hat{y}$, can be dropped out so that equation (2.176) simplifies to:

$$n\left(y_{t}(i),z_{t}\right)\approx U_{C}\bar{Y}_{t}\left[\left(1-\Phi\right)\hat{y}_{t}(i)+\frac{1}{2}\left(1+\tilde{h}\right)\hat{y}_{t}(i)^{2}-\tilde{h}q_{t}\,\hat{y}_{t}(i)\right]+t.i.p. \tag{2.178}$$

where: $q_{t}=\left(v_{yz}z_{t}\right)/\left(\bar{Y}y_{yy}\right)$.

If percentage deviations of aggregate output from steady state are equal to those across individual outputs in the intermediate good sector the following second-order Taylor approximation holds:

$$\hat{Y}\approx E_{i}\hat{y}(i)+\frac{1}{2}\left(1-q^{-1}\right)\operatorname{var}_{i}\hat{y}(i), \tag{2.179}$$

So far we have only taken a second-order approximation for a single household working for firm i. To make inference on the aggregate level of the economy we have to integrate over all commodities produced in the economy. It approximately holds that:

$$\int_0^1 n\left(y_t(i);z_t\right)=\bar{Y}u_C\left[\left(1-\Phi\right)E\hat{y}_t(i)+\frac{1}{2}\left(1+\tilde{h}\right)\left[\left(E\hat{y}_t(i)\right)^2+\operatorname{var}_i\hat{y}_t(i)\right]-\tilde{h}\tilde{q}_tE\hat{y}_t(i)\right]+t.i.p$$
$$\approx U_c\bar{Y}_t\left[\left(1-\tilde{f}-\tilde{h}q_t\right)\hat{Y}_t+\frac{1}{2}\left(1+\tilde{h}\right)\hat{Y}_t^2+\frac{1}{2}\left(q^{-1}+h\right)\operatorname{var}_i\hat{y}_t(i)\right]+t.i.p. \tag{2.180}$$

Now we can combine the utility of consumption and the disutility of labor. The aggregate approximation can be stated as follows:

$$V\approx\bar{Y}U_C\left\{\left[\Phi+\tilde{s}f_t+\tilde{h}q_t\right]\hat{Y}_t-\frac{1}{2}\left(\tilde{s}+\tilde{h}\right)\hat{Y}_t^2-\frac{1}{2}\left(q^{-1}+\tilde{h}\right)\operatorname{var}_i\hat{y}_t(i)\right\}+t.i.p. \tag{2.181}$$

As it is our ultimate goal to show that the period utility function can equally be stated in terms of a standard loss function which is commonly used in monetary macroeconomics we need a measure for the flex-price output $\hat{y}_t^f$. Given that we have made the assumption that the aggregate price index is normalized to one it will have to hold in equilibrium that:

$$\left(\frac{q}{q-1}\right)n_y=U_C. \tag{2.182}$$

To get a linear approximation of this non-linear equation one can take a first-order approximation at the flexible-price output level $\hat{Y}_t^f$, which can be stated as (see Walsh p. 552):

$$\left(\frac{q}{q-1}\right)\left[u_y\left(\bar{y},0\right)+u_{yy}\bar{Y}Y_t^f+u_{y,z}\hat{z}_t\right]=U_C\left(\bar{Y},0\right)+U_{cc}\left(\bar{Y},0\right)\bar{Y}\hat{Y}_t^f+U_{c,z}\hat{z}_t. \tag{2.183}$$

In order to obtain a closed form solution for the flex-price output gap $\hat{y}_t^f$ divide both sides of the equation by $U_C(\bar{Y},0) = qu_y(\bar{y},0)/(q-1)$ to obtain:

$$\tilde{h}\hat{Y}_t^f - \tilde{h}q_t = -\tilde{s}\hat{Y}_t^f + \tilde{s}f_t, \tag{2.184}$$

so that the flex-price equilibrium can be written as:

$$\hat{Y}_t^f = \left(\frac{\tilde{s}f_t + \tilde{h}q_t}{\tilde{s}+\tilde{h}}\right). \tag{2.185}$$

The utility function can now be written more compactly as follows:

$$\begin{aligned} V &\approx -\left(\frac{1}{2}\right)(\tilde{s}+\tilde{h})\bar{Y}U_C\left\{\hat{Y}_t^2 - \frac{[\Phi + \tilde{s}f_t + \tilde{h}q_t]}{\tilde{s}+\tilde{h}}\hat{Y}_t + \left(\frac{q^{-1}+\tilde{h}}{\tilde{s}+\tilde{h}}\right)\mathrm{var}_i\,\hat{y}_t(i)\right\} + t.i.p \\ &\approx -\left(\frac{1}{2}\right)(\tilde{s}+\tilde{h})\bar{Y}U_C\left\{(x_t - x^*)^2 + \left(\frac{q^{-1}+\tilde{h}}{\tilde{s}+\tilde{h}}\right)\mathrm{var}_i\,\hat{y}_t(i)\right\} + t.i.p \end{aligned},$$

$$\tag{2.186}$$

where the output gap is defined as:

$$x_t \equiv \hat{Y}_t - \hat{Y}_t^f; x^* = \frac{\Phi}{s+h}. \tag{2.187}$$

The final step is now to substitute out $\hat{y}_t(i)$ by the inflation rate. Under the assumption of an isoelastic demand function

$$\log \hat{y}_t(i) = \log Y_t - q(\log p_t(i) - \log P_t), \tag{2.188}$$

the following holds true for the degree of price dispersion:

$$\mathrm{var}_i \log \hat{y}_t(i) = q^2\,\mathrm{var}_i \log \hat{p}_t(i). \tag{2.189}$$

Substituting out $\mathrm{var}_i\ \hat{p}_t(i)$ in (2.186) the utility function can be stated as follows:

$$U_t \approx -\left(\frac{1}{2}\right)\bar{Y}U_C\left\{\left(x_t - x^*\right)^2 + \left(q^{-1} + \bar{h}\right)q^2\ \mathrm{var}_i\ \hat{p}_t(i)\right\} + t.i.p. \qquad (2.190)$$

As noted by Woodford (2001) equation (2.190) is derived under very general assumptions where no specific assumptions on the nature of price dispersion in the economy have been made. Nevertheless if we want to simplify the approximation further we need to specify the relationship which governs inflation, namely the NKPC:

$$p_t = kx_t + bE_t p_{t+1}. \qquad (2.191)$$

Following Woodford (2001, p. 396) let us define the following measures:

$$P_t \equiv E_i \log p_t(i) \text{ and } \Delta_t = \mathrm{var}_i \log p_t(i).$$

The very fact that prices are not adjusted in a synchronized fashion throughout the economy puts welfare burdens on consumers. If price changes are not adjusted in a synchronized fashion it has distortionary effects on the equilibrium allocation which are comparable to the dead weight loss generated by taxes as prices are out of equilibrium. Therefore a policy that limits price dispersion in the economy fosters economic welfare. Let us define the following measure:

$$\bar{P}_t - \bar{P}_{t-1} = E_i\left[\log p_t(i) - \bar{P}_{t-1}\right]. \qquad (2.192)$$

Maintaining the standard assumption of Calvo-pricing the aggregate price level can be rewritten as:

$$\begin{aligned} &= qE_i\left[\log p_{t-1}(i) - \bar{P}_{t-1}\right] + (1-q)\left(\log p_t^* - \bar{P}_{t-1}\right) \\ &= (1-q)\left(\log p_t^* - \bar{P}_{t-1}\right) \end{aligned}, \qquad (2.193)$$

where $a E_i\left[\log p_{t-1}(i)-\overline{P}_{t-1}\right]$ is equal to zero by definition. Note that we can equally define the dispersion measure $\Delta_t$ as follows:

$$\Delta_t = \operatorname{var}_i\left[\log p - \overline{P}_{t-1}\right]. \tag{2.194}$$

Making use of the fact that $Var(x) = E(x)^2 - \left[E(x)\right]^2$ we can rewrite (2.194) as follows:

$$\Delta_t = E_t\left[\left(\log p_t(i) - \overline{P}_{t-1}\right)^2\right] - \left[E_t\left(\log p_t(i)\right) - \overline{P}_{t-1}\right]^2. \tag{2.195}$$

Inserting equation (2.193) into (2.195) it will hold have to hold that:

$$\Delta_t = q E_t\left[\log p_{t-1}(i) - \overline{P}_{t-1}\right]^2 + (1-q)\left(\log p_t^* - \overline{P}_{t-1}\right)^2 - \left(\overline{P}_t - \overline{P}_{t-1}\right)^2. \tag{2.196}$$

This can be rewritten as:

$$\Delta_t = (1-q)\left(\log p_t^* - \overline{P}_{t-1}\right)\left(\log p_t^* - \overline{P}_{t-1}\right) - \left(\overline{P}_t - \overline{P}_{t-1}\right)^2, \tag{2.197}$$

as $\left(\overline{P}_t - \overline{P}_{t-1}\right) = (1-q)\left(\overline{P}_t - \overline{P}_{t-1}\right)^2 - (\overline{P}_t - \overline{P}_{t-1})^2$ it holds that we can restate this relationship as follows:

$$\Delta_t = \frac{1}{1-q}\left(\overline{P}_t - \overline{P}_{t-1}\right)^2 - \left(\overline{P}_t - \overline{P}_{t-1}\right)^2. \tag{2.198}$$

Which canbe simplified to:

$$\Delta_t = \frac{q}{1-q}\left(\overline{P}_t - \overline{P}_{t-1}\right)^2. \tag{2.199}$$

Accordingly it will hold that:

$$\Delta_t = q\Delta_{t-1} + \frac{q}{1-q}\left(\bar{P}_t - \bar{P}_{t-1}\right)^2. \qquad (2.200)$$

Up to a first-order approximation that it holds that: $\bar{P}_t = \log \bar{P}_t + O\left(\| \|\right)^2$. Equation (2.200) can be rewritten as:

$$\Delta_t = q\Delta_{t-1} + \frac{q}{1-q} p_t^2. \qquad (2.201)$$

Integrating forward from any initial level this relationship can be rewritten as:

$$\Delta_t = q^{t+1}\Delta_{t-1} + \sum q^{t-s}\left(\frac{q}{1-q}\right) p_s^2 + O\left(\| \|\right)^3. \qquad (2.202)$$

Note that it is our aim to approximate expected utility, therefore we have to iterate the dispersion measure forward in order to substitute it out of equation (2.190):

$$\sum_{t=0}^{\infty} b^t \Delta_t = \sum_{t=0}^{\infty} b^t \left[ t.i.p + \sum_{s=0}^{t} q^{t-s} \frac{q}{1-q} p_s^2 + o\left(\|a\|^3\right)\right]. \qquad (2.203)$$

Equation (2.203) can be simplified by the following transformations:

$$\begin{aligned}
E_t \sum_{i=0}^{\infty} b^i \Delta_{t+i} &= \left[\frac{q}{(1-q)(1-qb)}\right] E_t \sum_{i=0}^{\infty} b^i \left[p_{t+i}^2\right] + t.i.p \\
&= \frac{q}{1\text{-}q} \sum_{t=0}^{\infty} b^t \sum_{s=0}^{t} q^{t-s} p_s^2 + t.i.p. + o\left(\|a\|^3\right) \\
&= \frac{q}{1\text{-}q} \sum_{t=0}^{\infty} \sum_{s=0}^{t} b^{t-s} q^{t-s} b^s p_s^2 + t.i.p. + o\left(\|a\|^3\right), \\
&= \frac{q}{1-q} \frac{1}{1-qb} \sum_{t=0}^{\infty} b^t p_s^2 + t.i.p. + o\left(\|a\|^3\right) \\
&= \frac{1}{l_y} \sum_{t=0}^{\infty} b^t p_t^2 + t.i.p. + o\left(\|a\|^3\right)
\end{aligned} \qquad (2.204)$$

where it holds that $l_y = \frac{(1-q)(1-qb)}{q}$. Finally one can substitute out the degree of price dispersion in the utility function to obtain:

$$E_t \sum_{i=0}^{\infty} b^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} b^i \left[ p_{t+i}^2 + l_y y_{t+i}^2 \right] + t.i.p.. \tag{2.205}$$

where

$$\Omega = \frac{1}{2} \bar{Y} U_C \left[ \frac{q}{(1-q)(1-qb)} \right] \left( q^{-1} + h \right) q^2$$

$$l_y = \left[ \frac{(1-q)(1-qb)}{q} \right] \frac{(\tilde{s} + \tilde{h})}{(1+\tilde{h}q)q}$$

Hence based on Woodford's exposition we have shown up to a second-order approximation it holds that the utility function of a representative household can be approximated by a scaled version of a standard loss function. To restate the case: The main advantage of this procedure is that it might be interpreted as a natural welfare measure in a general equilibrium framework. In particular it gives some indication which relative weight to assign to which target variable from a micro-founded perspective. Nevertheless there is some downside risk. As we saw, a substantial amount of assumptions is necessary to approximate a utility function by a loss function. In particular we had to make assumptions on the functioning of labor markets and on technology.

Equation (2.205) and the implied weight $l_y$ have quite substantial implications for the conduct of monetary policy. Assume that a central bank aims at keeping fluctuations of the inflation rate around the inflation target within a certain range denotes $\Delta p$. If the central bank faces a steep Phillips curve, the output gap will fluctuate by $\Delta y_{steep}$, whereas in the case of a flat Phillips curve the output gap will be set by the central bank within the interval $\Delta y_{flat}$. Accordingly Figure 7 shows that a central bank that has its prime focus on stabilising the inflation rate cannot simultaneously put a high weight on output stabilisation if the Phillips curve is flat. This result can be explained by the transmission mechanism nested in New Keynesian macromodels. As monetary policy uses the interest rate to manipulate the output gap a flattening Phillips curve implies that monetary policy needs to use its nominal interest rate more rigorously to move the output gap in order to have the desired impact on the inflation rate. In the case of a steep Phillips curve modest movements in the output gap can already have a large impact on the inflation rate.

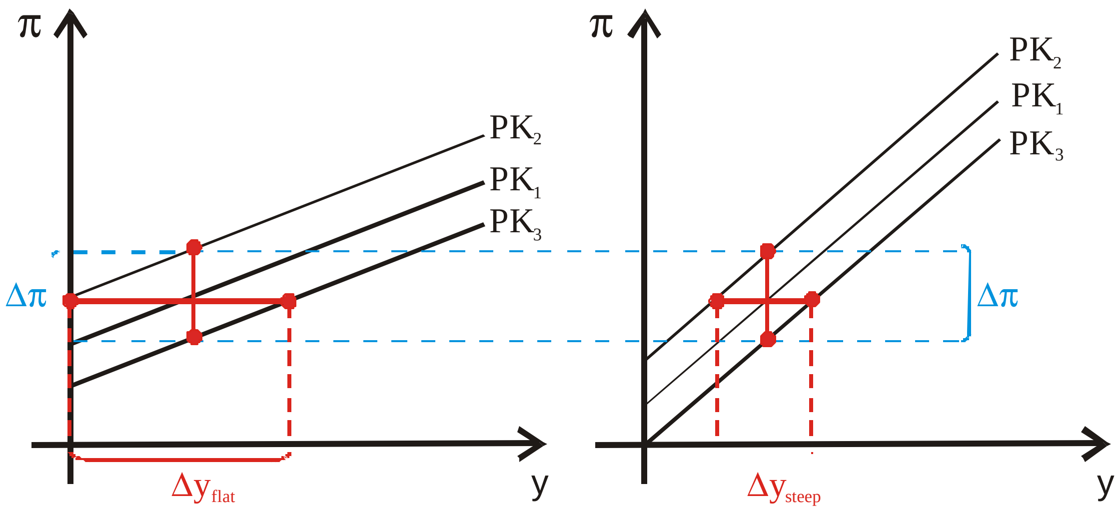



**Figure 7:** On the Relation between the Phillips Curve and Output Stabilization

## 2.2 New Keynesian Macroeconomics Made Simple

In the last section we have derived in depth the framework of New Keynesian macroeconomics. We have seen that the individual equations mirror the equilibrium behaviour of interacting sectors in a highly stylized economy. Each sector can be described by a single equation. As a consequence the New Keynesian macromodel can be reduced to three main building blocs (see Bofinger, Mayer and Wollmershäuser (2005)).

- A HNKPC depicting the pricing decisions of monopolistically competitive firms in the intermediate good sector.
- An intertemporal IS-equation depicting the optimal allocation schemes of households allocating consumption and bond holdings over time.
- And the policy rule that tell us how monetary policy is conducted.

In the following section we will show the model in action. In particular we demonstrate that it is possible to simplify the New Keynesian framework to an intermediate level while equally preserving its main insights. Quite arguably this endeavour has many advantages. The least of them is not that it is a powerful alternative to the IS/LM-AS/AD model which is still the central tool of macroeconomic teaching in most textbooks. In its basic version it is at the same time more simple and more powerful than the IS/LM-AS/AD model. In its more complex version it can analyze important concepts such as inflation targeting and monetary policy rules that have become standard tools in New Keynesian macroeconomics. Compared with other approaches such as Walsh (2002) or Taylor (2001) it deals explicitly with the central bank's reaction to demand shocks and focuses on the concept of central bank credibility, which plays a pivotal role in the concept of inflation targeting.

### 2.2.1 A Simplified Framework for Monetary Economics

This section will present a model that develops the Romer (2000) approach into a simple, but at the same time comprehensive macroeconomic model. In spite of its simplicity it can carry the main insights of the New Keynesian macroeconomics (see Clarida, Gali and Gertler (1999)) to an intermediate level and deal with issues such as inflation targeting, monetary policy rules, and central bank credibility. Compared to existing approaches (Walsh (2002) and Taylor (2001)) we offer an important contributions. Our approach has the advantage of modeling expectations in a more general way so that issues of credibility that play a pivotal role in inflation targeting can be addressed. In this respect, a main innovation of the model is that we integrate the Barro and Gordon (1983) time inconsistency problem into our analysis. Finally, our model explicitly treats the reaction of monetary policy to demand shocks. This issue is crucial to an understanding of central banking but has been neglected, above all in the graphical analysis by Walsh (2002).

#### 2.2.1.1 Its Main Building Blocs

The model consists of three building blocs:

- An IS-equation
- A Phillips curve, and
- A monetary policy rule

Let us postulate that the output gap $y$ depends on autonomous demand components $a$, the real interest rate $r$ and $a$ demand shock $\varepsilon_1$:

$$y = a - br + \varepsilon_1 . \qquad (2.206)$$

The output gap $y$ is defined as the deviation of (the logarithm) of aggregate output from its potential, or full capacity level. This approach is very much in line with Romer (2000). It is clear from this equation that in the absence of shocks the output gap (which is zero then) depends on the real interest rate which is given by $a/b$. In accordance with Blinder (1998, p.31) this rate is called the neutral real short-term interest rate ($r_0$). From a New Keynesian perspective the IS-curve depicts the optimal allocation of consumption of households in equilibrium over time.

Second, let us postulate that the second building block can be simplified to the following expectations-augmented Phillips curve:

$$\pi = \pi^e + d\, y + \varepsilon_2 . \qquad (2.207)$$

The inflation rate is determined by expectations about inflation $\pi^e$, the output gap $y$, and a supply shock $\varepsilon_2$. The parameter $d$ is nonzero and positive. For reasons of simplicity we assume in the most basic model that the central bank is credible, that is that private sector inflation expectations are identical with the central bank's inflation target $\pi_0$. This automatically translates into $\pi^e = \pi_0$ so that the Phillips curve can be rewritten as:

$$\pi = \pi_0 + d\, y + \varepsilon_2 . \qquad (2.208)$$

In a later section we discuss inflation expectations in a more general way. In particular we show that our approach of modeling the Phillips curve can be regarded as a special case of the New Keynesian perspective, in which expectations are formed rationally and the current inflation rate is related to the expected future inflation rate. Walsh (2002), by contrast, assumed that the private sector has adaptive expectations and he did not show under which condition inflation fluctuates on average around the central bank's inflation target. As a third building bloc we specify the way according to which monetary policy is conducted. The strategy of inflation targeting can be derived as follows:

$$L = \left(\pi - \pi_0\right)^2 + \lambda_y y^2 \text{ with } l_y \geq 0. \qquad (2.209)$$

Accordingly, the central bank aims at stabilizing squared deviations of the inflation rate from the inflation target while also being concerned with economic activity.

Given the transmission structure of a change in the monetary policy stance, which runs from the real interest rate, optimal monetary policy can be derived logically best by applying the following two-step procedure. First insert the Phillips-curve (2.208) into the loss function (2.209) and second, we minimize the modified loss function with respect to y. The solution gives an optimal value for the output gap:

$$y = -\frac{d}{d^2 + l_y} e_2 . \qquad (2.210)$$

The result nicely shows the impact of varying degrees of preferences on output stabilization. If the central bank only cares on output economic activity will be totally stabilized in the limit:

$$\lim_{l_y \to \infty} y = 0 . \qquad (2.211)$$

In the case of an inflation nutter supply shocks that hit the economy will be amplified by a factor of $(1/d)$ .

$$\lim_{l_y \to 0} y = -\frac{1}{d} e_2 . \qquad d<1, \qquad (2.210)$$

If we insert equation (2.210) into the Phillips curve (2.208) we can derive the following reduced form expression for the deviation of the inflation rate from the inflation target:

$$p - p_0 = \frac{l_y}{d^2 + l_y} e_2 . \qquad (2.212)$$

Just as beforehand we can analyze equation (2.212) in the limit:

$$\lim_{l_y \to \infty} p = p_0 + e_2 . \qquad (2.213)$$

It prevails that in the case of an output junkie supply shocks hit the economy undampend, whereas in the case of an inflation nutter ($\lambda$=0) the effects of a supply shock on inflation will be totally undone by suitable monetary policy action:

$$\lim_{l_y \to 0} p = p_0 . \tag{2.214}$$

Under a strategy of inflation targeting one way to conduct monetary policy is to follow an instrument rule (Svensson and Woodford (2003)). Such a rule makes the reaction of the instrument of monetary policy depend on all the information available at the time the instrument is set, that is the exogenous variables $\varepsilon_1$ and $\varepsilon_2$ and the structure of the economy. In our framework, the instrument rule can be derived by inserting equation (2.210) into (2.206) and by solving the resulting expression for $r$:

$$r^{opt} = \frac{a}{b} + \frac{1}{b}\varepsilon_1 + \frac{d}{b\left(d^2 + \lambda_y\right)}\varepsilon_2 . \tag{2.215}$$

The rule shows the following characteristics: The optimal response to demand shocks $\varepsilon_1$ does not depend on the central bank's preferences $\lambda_y$. As the interest rate changes according to $(1/b)\varepsilon_1$ the output gap remains zero, irrespective of the preference type. Thus, as long as demand shocks are part of the information set of the central bank they do not inflict any costs on society. The reaction of the central bank to supply shocks depends on preferences $\lambda_y$. A central bank that only cares about inflation ($\lambda_y$=0), requires a strong real rate response and, accordingly, a large output gap (see point A in Figure 8). With an increasing $\lambda_y$ the real interest rate response declines (see point B in Figure 8). In equilibrium $(\varepsilon_1 = \varepsilon_2 = 0)$ the real interest rate will be given by the neutral real short-term interest rate $r_0 = a/b$.

### 2.2.1.2 The Model in Action

The strategy of inflation targeting can also be presented with our graphical analysis (see Figure 8). The instrument rule enters as a horizontal line in the $y$-$r$ space (marked by $r\left(\varepsilon_1, \varepsilon_2\right)$ to highlight the shift parameters of the monetary policy line). As before, the AD-curve that could be derived from inserting the policy rule (2.215) into the IS curve (2.206) would be a vertical line. The loss function of the central bank can be illustrated by circles around a bliss point in the $y$-$\pi$ space. The bliss point that represents the first best outcome with a loss of zero is defined by an inflation rate $\pi$ equal to the inflation target $\pi_0$ and an output gap of zero. We can derive the geometric form of the circle by transforming the loss function (2.209) into:

$$1 = \left. \frac{\left(\pi - \pi_0\right)^2}{\left(\sqrt{L}\right)^2} + \frac{\left(y - 0\right)^2}{\left(\sqrt{L}\right)^2} \right|_{\lambda_y = 1}, \qquad (2.216)$$

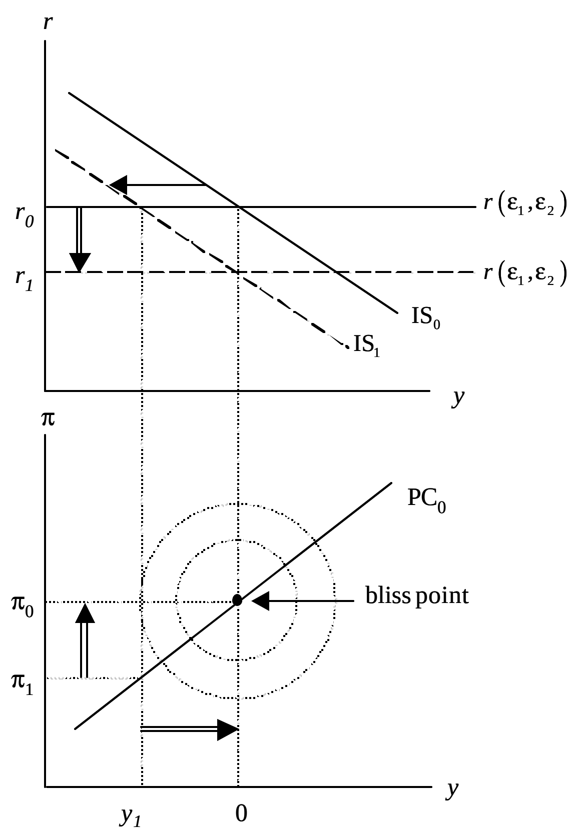



**Figure 8:** Demand Shock under Inflation Targeting

where (0; $\pi_0$) is the center of the circle and the radius is given by $\sqrt{L}$ . In the case of a demand shock we can see from Figure 8 that monetary policy is always able to maintain the bliss combination. In the case of a supply shock the loss function helps us to identify the optimum combination of $\pi$ and $y$. Here the shifted Phillips curve serves as a constraint under which the radius of the circle has to be minimized. The optimum combination is graphically given by the locus on the Phillips curve $(y_1;\pi_1)$ that is tangent to an isoquant of the loss function (see Figure 9). In order to attain this point the central bank will adjust its instrument so as to realize the optimum output gap $y_1$. An alternative view of inflation targeting is given by the so-called "targeting rule" of the central bank (Svensson and Woodford (2003)). Such a rule gives a high level specification of monetary policy that can be directly derived from the central bank's strategy. Targeting rules are an important device to describe actual central banks as the institutional changes that took place during the last two decades aimed at committing central banks at the target level, that is specifying a concrete inflation target.

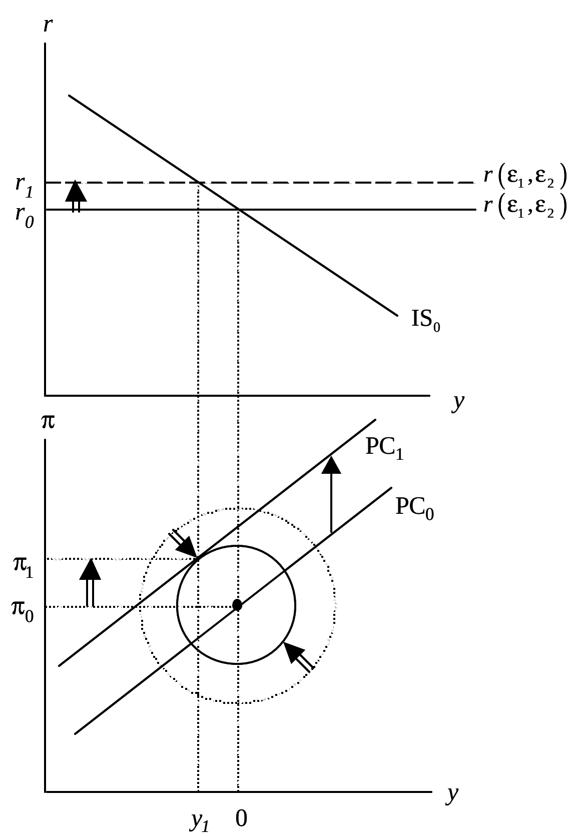



**Figure 9:** Supply Shock under Inflation Targeting

By eliminating the supply shock $\varepsilon_2$ from equations (2.210) and (2.212) we arrive at the following consolidated first-order condition:

$$\pi = \pi_0 - \frac{\lambda_y}{d} y. \tag{2.217}$$

In contrast to instrument rules, targeting rules are a linear relationship between endogenous target variables that will have to hold with equality if monetay policy is conducted optimally. By the very definition of a first-order condition this ensures that, for a given value of private sector expectations and thus for any given location of the Phillips curve, the loss function (2.207) is minimized.

Graphically, the optimal outcome is thus described by the intersection of the Phillips curve $PC_1$ with the targeting rule of the central bank (see Figure 10). Equation (2.217) shows that an increasing $\lambda_y$ (i.e., an increasing weight on output stabilization) leads to a steepening of the reaction function $RF(\lambda_y)$.

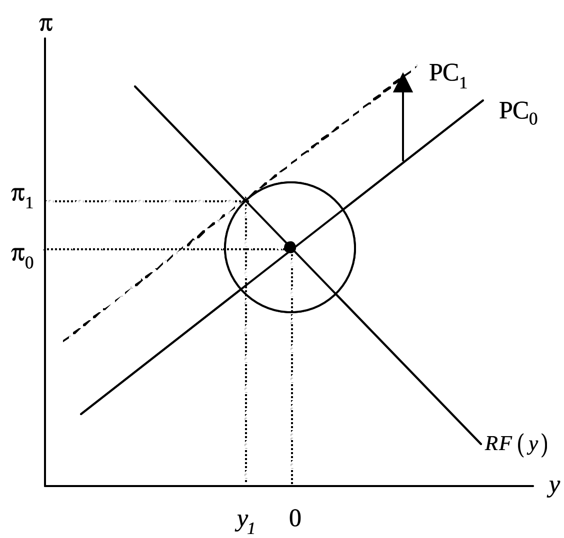



**Figure 10:** Supply Shock and the Targeting Rule

## 2.2.2 Three Approaches for the Specification of Inflation Expectations

So far we modeled inflation expectations $\pi^e$ in a very simple way. For a more general analysis we have to specify in detail how expectations may be formed. Thereby we can distinguish three cases:

- adaptive expectations: $\pi^e = \pi^e\left(\pi_{-1}\right)$;
- rational expectations: $\pi^e = E\left(\pi | I\right)$, where $I$ is defined as the private sector's information set;
- rational expectations and a credible central bank: $\pi^e = \pi_0$.

Adaptive expectations are at the core of the model developed by Walsh (2002). He implicitly assumed that initially expectations are exogenously given. This can be seen in his graphical analysis (see Walsh (2002), Figure 1. p. 335) where he started with inflation expectations that are higher than the inflation target of the central bank. Although Walsh did not explain how these initial expectations are formed, his case can be translated into our framework as an unexpected shift of the Phillips curve to the left.

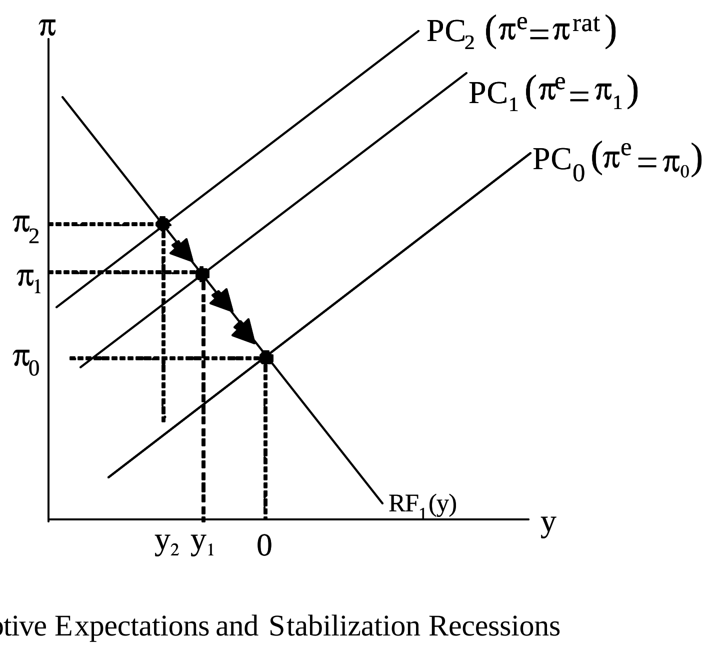



**Figure 11:** Adaptive Expectations and Stabilization Recessions

The new equilibrium is the intersection of the reaction function $RF_{rat}$ with the unchanged Phillips curve. However with a negative output gap the Phillips curve which is the inflation determining relationship tells us that the inflation will start to fall, so that in the next period $p^e = p_1$. If expectations are purely autoregressive this process will continue until some periods later $p^e = p_0$. During the transition period the economy will go through a stabilization recession.

In our view, it seems useful to endogenize expectations. In order to map the standard New Keynesian Phillips curve in which current inflation is determined by rational expectations about future economic conditions (Calvo (1983)) into a static framework, we have to impose that the disturbance term $\varepsilon_2$ is purely white noise. Under this assumption the private sector expects inflation to return immediately to equilibrium in the period following a shock. And this equilibrium is exactly defined by the central bank's inflation target, provided that the inflation target is credible. This proposition can be verified as follows: We assume that the central bank is guided by the loss function (2.209) and the structure of the economy is given by equations (2.206) and (2.208). Substituting the Phillips curve into the loss function and deriving the optimal output gap yields:

$$y = -\frac{d}{d^2 + \lambda_y}\left(\pi^e - \pi_0\right) - \frac{d}{d^2 + \lambda_y}\varepsilon_2 . \tag{2.218}$$

Inserting equation (2.218) into the Phillips curve (2.208) we get the following optimal inflation rate for the central bank as a function of private sector expectations:

$$\pi\left(\pi^e\right) = \frac{\lambda_y}{d^2 + \lambda_y}\pi^e + \frac{d^2}{d^2 + \lambda_y}\pi_0 + \frac{\lambda_y}{d^2 + \lambda_y}\varepsilon_2 . \tag{2.219}$$

At the beginning of a period private agents settle goods and labor market contracts. Therefore they have to build expectations on the inflation rate. We assume that the private sector is guided by the following loss function:

$$L = \left\{\pi\left(\pi^e\right) - \pi^e\right\}^2 . \tag{2.220}$$

The first-order condition of the private sector is given by:

$$\pi^{opt}\left(\pi^e\right) = \pi^e . \tag{2.221}$$

While forming its expectations, the private sector takes the optimal inflation rate of the central bank, equation (2.219), into account. Accordingly it has to hold that:

$$\pi^e = \frac{\lambda_y}{d^2+\lambda_y}\pi^e + \frac{d^2}{d^2+\lambda_y}\pi_0 + \frac{\lambda_y}{d^2+\lambda_y}\varepsilon_2 . \tag{2.222}$$

Solving for the private sector expectations yields:

$$\pi^e = \pi_0 , \tag{2.223}$$

because the current supply shock is not an element of the information set of the private sector. Thus, assuming that the private sector's inflation expectations are identical with the central bank's medium term inflation target is simply a special case of rational expectations.

### 2.2.3 A Central Bank with an Inflation Bias

The rational expectations solution also allows a discussion of a central bank with an inflation bias, which correspondingly suffers from low credibility (see Barro and Gordon 1983). For this purpose we have to modify the central bank's loss function as follows:

$$L = \left(\pi - \pi_0\right)^2 + \lambda_y \left(y - k\right)^2 \text{ with } k > 0. \tag{2.224}$$

By introducing the parameter $k$, the central bank targets an output gap that is above zero. This could be rationalized by monopolistic distortions in goods and labor markets that keep potential output below an efficient level. Compared with the loss function that we have used so far, the bliss point (k; $\pi_0$) has moved to the right (see Figure 12).

In line with Barro and Gordon (1983) the game between the private sector and the central bank can be modelled as follows. The private sector forms its inflation expectations, which enter in the contracts settled on the goods and labor market. Using these private sector expectations the central bank chooses an inflation rate that minimizes its loss function so that we arrive at the following targeting rule:

$$\pi = \pi^e + \frac{\lambda_y}{d} k - \frac{\lambda_y}{d} y. \tag{2.225}$$

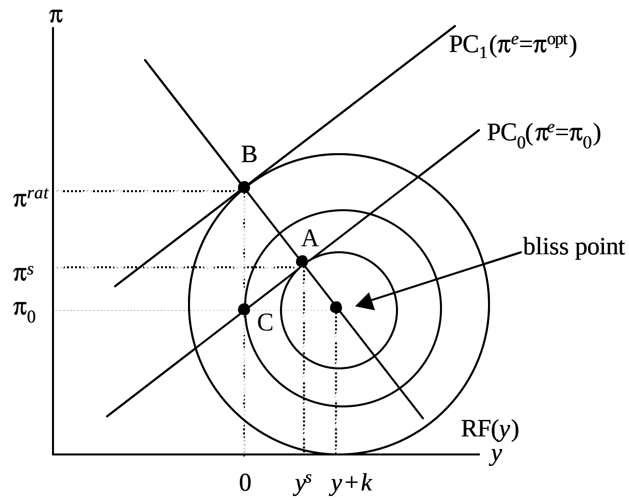



**Figure 12:** Different Monetary Policy Outcomes

In comparison to equation (2.217) the new reaction function of the central bank has shifted to the right because of the inflationary bias. If the private sector forms expectations rationally, it minimizes the following loss function:

$$L = \left\{\pi\left(\pi^e\right) - \pi^e\right\}^2. \tag{2.226}$$

Accordingly, the private sector will take the first order condition of the central bank into account while building its expectations. Thus, our framework comes to the result that monetary cheating does not pay off as the economy will end up with no gains in output ($y^{rat} = 0$) but higher rates of inflation ($\pi^{rat} > \pi_0$) (see point B in Figure 12). Compared to surprise inflation (point A with $\pi = \pi^s$ and $y = y^s$, which is based on a Phillips curve with $\pi^e = \pi_0$) this solution is clearly inferior because it leads to a higher inflation rate without a positive gain in output. The loss circle lies outside the circle attached to the solution with surprise inflation. Even if the central bank announces an inflation target, rational agents will realize that it has a strong incentive to renege on its announcement. In order to avoid the high social loss under discretion, a mechanism is required that credibly commits the central bank to the inflation target $\pi_0$.

Thus, our framework can be easily extended for an analysis of the issues that are related to the Barro and Gordon (1983) model. Although these results are well-known in literature, the model has the advantage that it provides a coherent framework for a discussion of monetary policy, which includes both traditional stabilization issues and the topics related to time inconsistency.

### 2.2.4 The Dynamics of the Models: A Comparison

In the previous section we have simply postulated a proximity between New Keynesian macroeconomics and the presented static three equation model. In this section we will justify this claim (Bofinger, Mayer, Wollmershäuser (2004)). The proximity of this simple reduced form New Keynesian macromodel and the standard New Keynesian macromodel can be explained by analyzing the impulse responses.

The objective function of the central bank is an intertemporal loss function, summing up the expectations about discounted current and future deviations of inflation from target and output from potential:

$$L_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \left( \pi_{t+\tau} - \pi_0 \right)^2 + \lambda_y y_{t+\tau}^2 \right\}. \tag{2.227}$$

For the solution of the central bank's dynamic optimization problem we adopted an approach which basically draws on Clarida et al. (1999) and Svensson (2003). The intertemporal loss function (2.226) is minimized subject to the Phillips curve equation. This leads to the following Lagrangian:

$$H_t = \sum_{t=0}^{\infty} b^t \left\{ \left[ \left( p_{t+t,t} - p_0 \right)^2 + l_y y_{t+t,t}^2 \right] + \tilde{x}_{t+t,t} \left( p_{t+t,t} - bp_{t+t+1,t} - dy_{t+t,t} \right) \right\} \tag{2.228}$$

where $x_{t+\tau,t}$ denotes the $\tau$-period-ahead expectations of variable x, conditional on the central bank's information in period t on the state of the economy and the transmission mechanism of monetary policy (which is equal to $E_t x_{t+\tau}$). The term in parentheses following the dynamic Lagrange multiplier $\tilde{x}_{t+t,t}$ represents the central bank's $\tau$-period-ahead forecast of the NKPC in period t. Differentiating with respect to $\pi_{t+\tau,t}$ and $y_{t+\tau,t}$ gives the two first-order conditions:

$$\tilde{x}_{t+t,t} = -2\left(p_{t+t,t} - p_0\right) \tag{2.229}$$

and

$$\tilde{x}_t = \frac{2\, l\, y_{t+t,t}}{d}. \tag{2.230}$$

A basic assumption underlying the first foc is that the central bank takes private sector expectations about next period inflation rate $\pi_{t+\tau+1,t}$ as given. The literature typically refers to this kind of procedure as discretionary optimization, in contrast to optimization under commitment. If a central bank credibly commits to a once-and-for-all policy rule, it internalizes the effects of its own interest rate decision on the expectations of the private sector. For $\tau \geq 1$ the first foc would then be $2\beta^\tau\left(\pi_{t+\tau,t} - \pi_0\right) - \beta^\tau \xi_{t+\tau,t} - \beta^{\tau-1}\beta\xi_{t+\tau-1,t} = 0$. Setting $\tau = 0$ and eliminating the Lagrange multiplier leads to the consolidated first-order condition:

$$y_t = \frac{d}{l_y}\left(p_t - p_0\right) \tag{2.231}$$

Obviously the targeting rule of the central bank is identical to relationship (2.217). Henceforth optimal monetary policy is conducted in an identical fashion. Inserting (2.231) into NKPC yields the following forward-looking first-order difference equation

$$\frac{l_y + d^2}{b\, l_y} p_t = E_t p_{t+1} + \frac{d^2}{b\, l_y} p_0 + \frac{1}{b} e_{2,t}, \tag{2.232}$$

which can be solved using the MSV (minimum state variables) approach of McCallum (1983). The MSV-approach can be applied as follows. Note let us assume that the shock terms in the Phillips curve and the IS-equation can be specified as follows:

$$e_{1,t} = r_1 e_{1,t-1} + \hat{e}_{1,t} \tag{2.233}$$

and

$$e_{2,t} = r_2 e_{2,t-1} + \hat{e}_{2,t} \tag{2.234}$$

In this system the minimal set of state variables includes only $\varepsilon_{2,t}$, so the solution will be of the form

$$p_t = \tilde{\tilde{a}} + \tilde{b} e_{2,t} \tag{2.235}$$

Taking expectations of (2.235),

$$E_t p_{t+1} = \tilde{\tilde{a}} + \tilde{b}\, r_2 e_{2,t}, \tag{2.236}$$

and inserting (2.236) into (2.232), and solving the resulting expression for $\pi_t$ yields

$$p_t = \left( \frac{b\, l_y \tilde{\tilde{a}} + d^2 p_0}{l_y + d^2} \right) + \left( \frac{l_y + \tilde{b} b\, l_y r_2}{l_y + d^2} \right) e_{2,t}. \tag{2.237}$$

Setting the first term in paranthesis equal to $\alpha$ and the second term in paranthesis equal to $\beta$, and solving the resulting equations for $\alpha$ and $\beta$, respectively, finally gives

$$a = \frac{d^2}{l_y + d^2 - \tilde{b} l_y} p_0, \tag{2.238}$$

and

$$\tilde{b} = \frac{l_y}{l_y + d^2 - b\, l_y r_2}. \tag{2.239}$$

The solution of (2.232) then is:

$$p_t = \frac{d^2}{l_y + d^2 - b\,l_y} p_0 + \frac{l_y}{l_y + d^2 - b r_2} e_{2,t} . \tag{2.240}$$

For d=1 the dynamics of the inflation rate can be simplified to:

$$p_t = p_0 + \frac{l_y}{d^2 + l_y - l_y r_2} e_{2,t} . \tag{2.241}$$

Taking one-period-ahead expectations of (2.240) (and considering equation 2.234) gives:

$$E_t p_{t+1} = \frac{d^2}{d^2 + l_y - b\,l_y} + \frac{l_y r_2}{d^2 + l_y - b\,l_y r_2} e_{2,t} . \tag{2.242}$$

Inserting (2.240) into the consolidated foc (2.231) yields the dynamic law of motion of the output gap:

$$y_t = d\left(\frac{1-b}{d^2 + l_y - b\,l_y}\right) p_0 + \frac{d^2}{d^2 + l_y - b r_2} e_{2,t} . \tag{2.243}$$

Taking again one-period-ahead expectations of (2.243) (and considering equation (2.234)) gives:

$$E_t y_{t+1} = d\left(\frac{1-b}{d^2 + l_y - b\,l_y}\right) p_0 - \frac{d\,r_2}{d^2 + l_y - b\,l_y r_2} e_{2,t} . \tag{2.244}$$

With the dynamics of inflation and output at hand we can finally derive the optimal interest rate rule. Inserting (2.242), (2.243) and (2.244) into the IS-curve and solving the resulting expression for the monetary policy instrument $i_t$ yields the following instrument rule:

$$i_t = r_0 + \frac{d^2}{d^2 + l_y - b\,l_y} p_0 + \frac{1}{b} e_{1,t} + \frac{b\,l_y r_2 - d\,r_2 + d}{b\left(d^2 + l_y - b\,l_y r_2\right)} e_{2,t} . \tag{2.245}$$

If a central bank follows this rule:

- it perfectly offsets demand shocks $\varepsilon_{1,t}$ as the interest rate impacts on the output gap with a factor b;
- it faces a trade-off in the case of supply shocks $\varepsilon_{2,t}$ which crucially depends on the preferences of the central bank $\lambda_y$;
- it keeps the nominal interest rate constant in the absence of shocks.

A basic requirement for ensuring the long-run neutrality of money is that $\delta$ approaches unity. From a theoretical point of view setting $\beta$ equal to unity is somewhat problematic as $\delta$ depicts the discount factor of a representative household that maximizes its utility. It can be shown that the neutral real interest rate $r_0$ is defined as $-\log(\beta)$. Thus, in order to avoid a value of $r_0$ equal to zero, $\beta$ must be below 1.[1] The discount factor $\beta$ also appears in the Phillips curve as profits of firms are assumed to be transferred to households so that they are discounted with $\beta$. From an empirical perspective the postulation that $\delta$ should be one is less problematic as estimated discount factors are typically not statistically different from one (Rotemberg and Woodford (1999)). In the case of $\beta = 1$, the long-run inflation rate and the long-run inflation expectations converge to the level of the inflation target ($\pi_t = E_t \pi_{t+1} = \pi_0$), the long-run output gap is zero ($y_t = 0$), and the long-run nominal interest rate equals the sum of the equilibrium real interest rate and the inflation target ($i_t = r_0 + \pi_0$). Otherwise there will be a long-run trade-off between the level of the inflation target (which can be freely chosen by the central bank) and the level of the output gap. To see this assume that $b = 1$, meaning that the costs resulting from the anticipation of deviations of inflation from its target level and of output from potential are weighted more strongly as they occur earlier in time. Inflation will then be biased downwards ($\pi_t < \pi_0$) at the expense of a positive output gap which crucially depends on the central bank's choice of $\pi_0$:

$$y = d\left(\frac{1-b}{d^2 + l_y - b\, l_y}\right) p_0 > 0 . \qquad (2.246)$$

The point that the long-run Phillips curve is steep and not vertical was also made – among others – by Woodford (1999, p. 32). The dynamics of the New-Keynesian model can be

[1] Quarterly models often assume $\delta = 0.99$ (0.995), so that $r_0 = 4.0$ % (2.0 %).

simplified substantially, if we specify two of the model’s parameters appropriately. First, we set equal to one. This has the convenient effect that in the limit, after scaling the intertemporal loss function (2.227) by a factor of (1-β), the intertemporal loss approaches the weighted sum of the unconditional variances of inflation and the output gap (Svensson, 2003):

$$\lim_{b \to 1}(1-b)L_t = Var[p_t] + l_y Var[y_t]. \tag{2.247}$$

By interpreting the intertemporal loss in terms of the variances of the goal variables, the optimality of an interest rate rule (such as (2.245)) can then be illustrated by the so-called efficiency frontier which depicts the second-order trade-off between the variances of inflation and output (Taylor, 1979). Hence although there is no trade off at the level of the variables, there is a trade-off in the second moments that is compatible with the same steady state solution. Second, we will gradually lower the autocorrelation of the supply shock to zero. This exercise is most crucial for the purpose of the present Section as it turns out to be the exclusive source of dynamic movements in a simple New-Keynesian macromodel as originally proposed by Clarida, Gali and Gertler (1999). For $b = 1$ the dynamics of the inflation rate as expressed in equation (2.240) reduces to

$$p_t = p_0 + \frac{l_y}{d^2 + l_y - l_y r_2} e_{2,t}. \tag{2.248}$$

According to (2.244) deviations of the inflation rate from its target only occur in the event of supply shocks. The extent of the deviation crucially depends on the preference parameter of the central bank, and hence on the extent to which the central bank accommodates supply shocks. By additionally setting $\rho_2 = 0$ equation (2.244) further reduces to

$$p_t = p_0 + \frac{l_y}{d^2 + l_y} e_{2,t}, \tag{2.249}$$

which is identical to equation (2.240) of the BMW model. The expected inflation rate for the next period which was given by equation (2.242) can also be substantially simplified after inserting $b = 1$ and $\rho_2 = 0$:

$$E_t p_{t+1} = p_0 \,. \tag{2.250}$$

Equation (2.249) implies that in the long-run inflation is expected to be anchored by the central bank's inflation target. Recall that this was a basic simplification for the formulation of the Phillips curve in the BMW model. In Section (2.2.2) we justified equation (2.250) by the assumption that the central bank's monetary policy is credible and that the private sector therefore believes in the central bank's commitment to the inflation target. Now we provide the analytical proof of this simplification which is valid in a macroeconomic environment in which the duration of shocks is limited to one period. If we set $b = 1$, the non-neutrality of money in equation (2.246) disappears and the dynamics of the output gap evolve according to:

$$y_t = -\frac{d}{d^2 + l_y - l_y r_2} e_{2,t} \,. \tag{2.251}$$

As was the case with the inflation rate, deviations of output from potential only occur in response to supply shocks which are only partially compensated by the central bank. By setting ß=1 and $\rho_2 = 0$ equation (2.251) can be further simplified to

$$y_t = -\frac{d}{d^2 + l_y} e_{2,t} \,, \tag{2.252}$$

which is then identical to the reduced form solution in the simplified framework.

For $b = 1$ and $\rho_2 = 0$ the optimal interest rate rule of the dynamic New-Keynesian model simplifies to

$$i_t = r_0 + p_0 + \frac{1}{b} e_{1,t} + \frac{d}{b\left(d^2 + l_y\right)} e_{2,t} . \tag{2.253}$$

With the nominal interest rate being defined as $i_t = r_t + E_t \pi_{t+1}$ and with equation (2.242), the policy rule can be expressed in terms of the real interest rate

$$r_t = r_0 + \frac{1}{b} e_{1,t} + \frac{d}{b\left(d^2 + l_y\right)} e_{2,t} \,. \tag{2.254}$$

which is identical to the optimal policy rule (2.215) of the BMW model if the neutral real short-term interest rate $r_0$ equals $a/b$.

The dependence of the dynamic behavior of the New-Keynesian model on the autocorrelation coefficient of the supply shock $\rho_2$ and its identity with the BMW model for $b=1$ and $\rho_2=0$ can be illustrated by calculating and depicting the *impulse response functions* of the New-Keynesian model. Figure 1 shows the responses of the nominal interest rate, the output gap and the inflation rate to a one standard deviation supply shock which hits the economy in period 1. For this simulation the model was calibrated as follows: $b=0.4$, $d=0.34$, $\lambda=1$, ß=1, $\mathrm{Var}\left[\varepsilon_{2,t}\right]=1$, $\pi_0=2$, and $r_0=2$. The basic message of Figure 13 is that the lower $\rho_2$, the lower the persistence of the deviation of $i_t$, $y_t$, and $\pi_t$ from their equilibrium levels 4 ($=r_0+\pi_0$), 0, and 2 ($=\pi_0$), respectively. For $\rho_2=0$, the dynamics are reduced to a single peak in period 1 which is typical for a comparative static model since in the period directly following the shock (period 2) the model's variables immediately return to their equilibrium values.

While the comparative statics appear to be plausible at first sight, the high initial jump and the gradual return of the variables that follows the jump for $\rho_2>0$ require a somewhat deeper look at the dynamics of the New-Keynesian model. To explain this we take the Phillips curve as an example. The NKPC not only produces a positive correlation between the level of inflation and real output, it also defines a negative correlation between the expected change in inflation and real output (for ß=1). The dynamic implication of these opposite-signed correlations is that, in response to, say, a positive shock to inflation, the level of inflation will rise, while the change in inflation will always be negative. This can only occur if inflation jumps up immediately in response to the shock, and subsequently falls back to its equilibrium.[2]

The fact that the presented simplified framework represents a special case of the New-Keynesian model can also be demonstrated by computing the efficiency frontier. On the basis of equations (2.249) and (2.252) the variances of inflation and output can be calculated as:

[2] While the New Keynesian models is derived from sound economic principles, this dynamic implication is seriously at odds with the data. There is a host of empirical evidence suggesting that both inflation and output exhibit gradual and 'humpshaped' responses to real and monetary shocks, instead of the 'jump' behavior resulting from purely forward-looking model specifications (see e.g. Estrella and Fuhrer, 2002).

$$Var[\pi_t] = \left( \frac{\lambda_y}{\delta + \lambda_y - \lambda_y \rho_2} \right)^2 Var\left[\varepsilon_{2,t}\right] \tag{2.255}$$

$$Var\left[y_t\right] = \left( \frac{\delta}{\delta^2 + \lambda_y - \lambda_y \rho_2} \right)^2 Var\left[\varepsilon_{2,t}\right]. \tag{2.256}$$

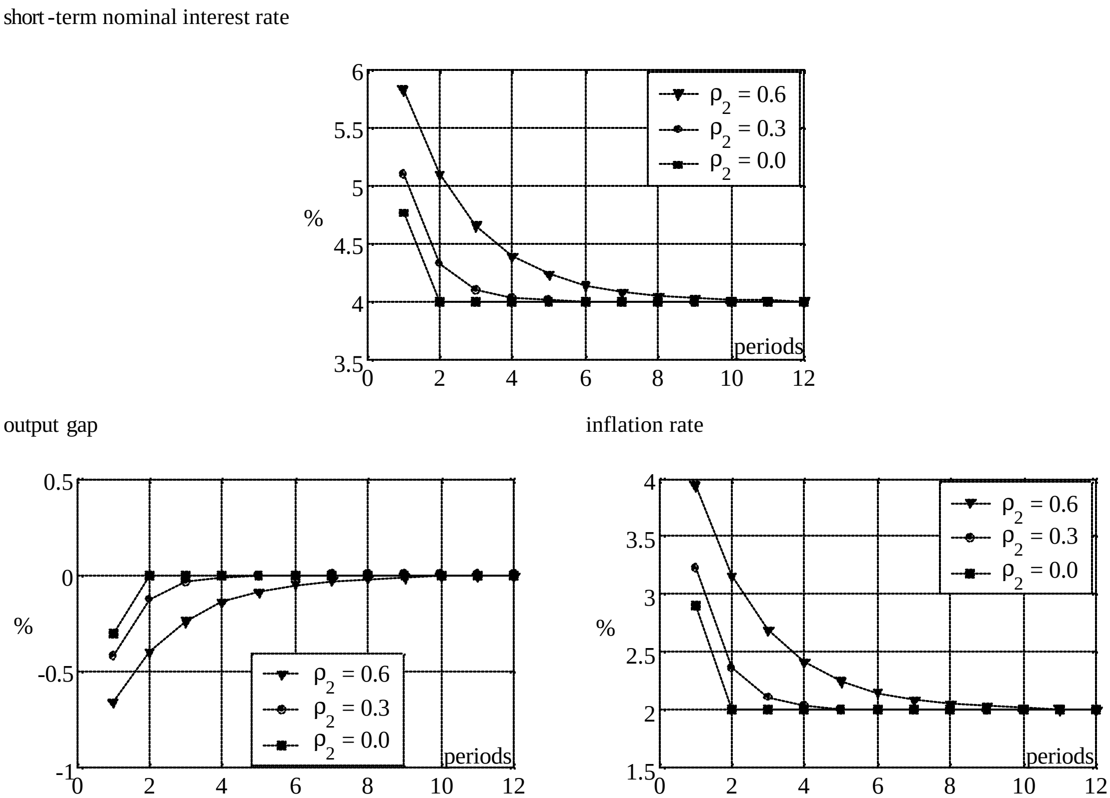



**Figure 13:** Responses to a Supply Shock

Since $\varepsilon_{2,t}$ follows a first-order autoregressive process (see equation (2.234)), its variance can be expressed as:

$$Var\left[\varepsilon_{2,t}\right] = \frac{Var\left[\hat{\varepsilon}_{2,t}\right]}{1 - \rho_2^2}. \tag{2.257}$$

The values of $\mathrm{Var}\left[y_t\right]$ and $\mathrm{Var}\left[\pi_t\right]$ that are associated with different values of $\lambda$ are the plotted as the convex efficiency frontiers in Figure 2. At points on the frontiers, it is impossible for the policymakers to reduce the variance of inflation without increasing the variance of the output gap, given that the central bank sets interest rates according to the optimum policy rule (2.254). Policymakers can, however, choose alternative points along the frontier by varying the relative weight $\lambda$ that they put on output versus inflation stabilization. For the construction of the curves we increased the preference parameter $\lambda$ from 0.01 (high preference for inflation stabilization; the lower right end of the frontier) to 10 (a high preference for output stabilization; the upper left end of the frontier) in steps of 0.01. With a falling $\rho_2$, both, $\mathrm{Var}\left[\varepsilon_{2,t}\right]$ and the squared term in brackets in equations (2.255) and (2.256) will become smaller so that the efficiency frontier shifts towards the origin of the $\mathrm{Var}[y_t]-\mathrm{Var}[\pi_t]$ space. For $\rho_2 = 0$ the efficiency frontier is identical across the models which underlines that the presented framework is appropriate.

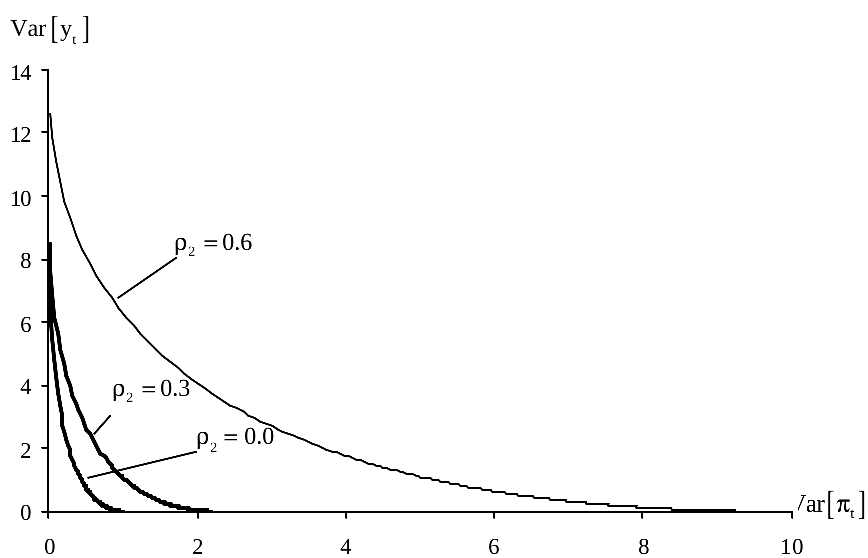



**Figure 14**: The Efficiency Frontier

## 2.3 Concluding Remarks

Within the next chapters we will apply a standard reduced form New Keynesian model to address a variety of questions linked to the interaction between varies economic agents. The purpose of this chapter was to underline that the popularity of the New Keynesian framework stems from its micro foundations. The individual reduced form equations mirror the equilibrium behavior of interacting sectors in a highly stylized economy. We have shown that each sector can be described in a highly reduced form by a single equation. As a consequence the New Keynesian macromodel can be reduced to three main building blocs. Accordingly in the following chapters we will base our models on three equations grounded out of the dynamic general equilibrium framework of New Keynesian macroeconomics.

Additionally the purpose of this paper was to demonstrate the proximity of the presented comparative-static framework to a standard dynamic New Keynesian macro model à la Clarida, Gali and Gertler (1999). The key to understand this proximity is to see that under discretion the first-order conditions of both models are identical. Therefore, we showed that when supply shocks converge from an first-order autoregressive process to a white noise process the 'dynamics' of the two models (as encapsulated in the consolidated first-order condition) become the same. To illustrate this point, we showed the convergence of the impulse response functions and the efficiency frontiers.

# 3 MONETARY AND FISCAL POLICY INTERACTION IN THE EURO AREA WITH DIFFERENT ASSUMPTIONS ON THE PHILLIPS CURVE ♣

In this chapter we apply a static version of a New Keynesian macromodel which we have developed in section 2.2 to a monetary union potentially describing EMU. With the launch of the third stage EMU its member countries have delegated monetary policy to an independent central bank setting monetary conditions in line with the average macroeconomic environment in the union. The unique feature of a currency area is given by the fact that the different macroeconomic agents, the ECB, national governments and labour unions focus on different levels of target variables. The common central bank whose policy we assume to be conducted according to the notion of inflation targeting (Svensson (1999)) focuses on union wide aggregates. It sets its nominal interest rate for the currency area consistent with its inflation target while equally having a concern for economic activity. This means in particular that the interest rate policy of the ECB will be indifferent against mean preserving distributions of macroeconomic outcomes across member countries. In contrast national governments basically focus on national aggregates. This constellation calls for rules which balance the chances and perils that are nested in monetary and fiscal policy interaction with decentralised fiscal authorities (Aarle, Bartolomeo, Engwerda and Plasmans (2002)). On the one hand unsustainable national policies, e.g. non anticipated fiscal expansions that are not consistent with the inflation target of the ECB, lead to prolonged business cycles with a boom in the home country and negative spill over effects for the rest of the union. On the other hand fiscal policy serves as a buffer to prevent idiosyncratic shocks from spreading to other member countries. Therefore a monetary union calls for a renaissance of fiscal stabilisation policy and stringent rules at the same time. As consequence the Maastricht Treaty which led to the Stability and Growth Pact (SGP) superimposed some broad guidelines on fiscal policy such as the 3% deficit criterion (Bofinger (2003)). Among the rich universe of aspects we analyse in particular whether fiscal policy should actively engage in stabilising business cycles or whether the fiscal stance should be state independently neutral. Our analysis will in

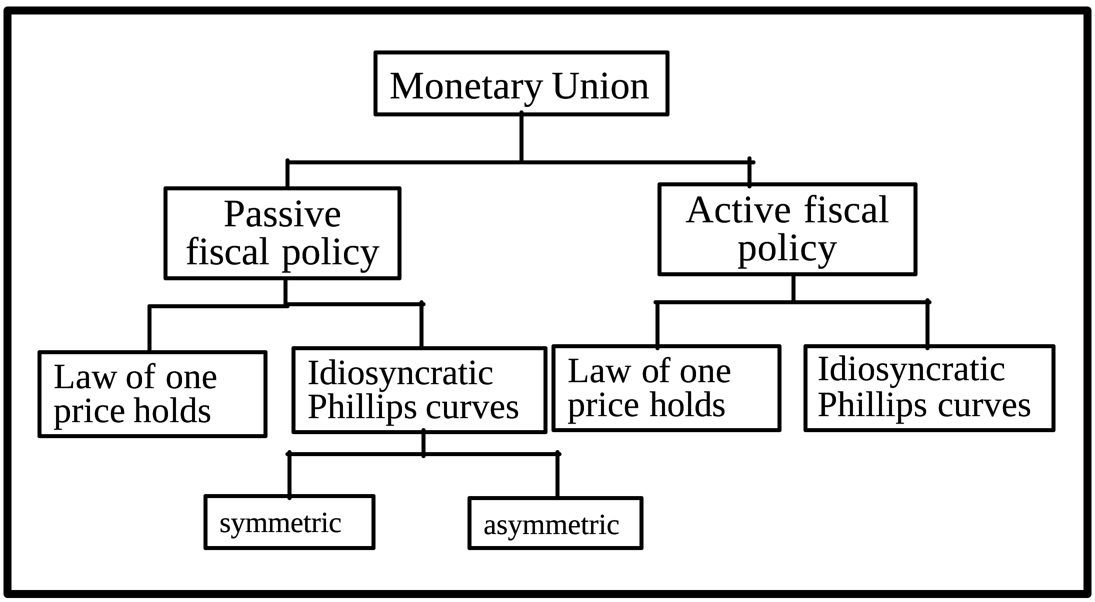
♣ This chapter benefited from presentation in Dresden (Annual Meeting of the German Economic Association (2004) and Göttingen (6th Göttingen Workshop on International Economic Relations). For valuable comments the author would like to thank in particular Michael Carlberg (Helmut Schmidt Universität Hamburg) and Timo Wollmershäuser (ifo - Institute for Economic Research) .

particular focus on the sustainability of fiscal policy and provide a rationale for the 3% deficit criterion as well as for its suspension.

There is a large body of related literature that analysis welfare under various assumptions on the conduct of monetary policy. The literature related to Barro and Gordon (1983) shows that monetary policy should be committed or delegated to a conservative central banker to limit the detrimental effects associated to the time consistency issue. As noted by Dixit and Lambertini (2003) this literature neglects to analyze these topics in the face of a second stabilization agent in the form of fiscal policy. Dixit and Lambertini ((2001), (2003)) analyze fiscal and monetary policy interaction in a closed economy and a monetary union, under the most general scenario when monetary and fiscal policy has different target values for output and inflation. They show that the Nash equilibrium is dominated by fiscal or monetary leadership. Otherwise prices will be inefficiently high and output inefficiently low. A joint commitment is the best solution whereas fiscal discretion has the potential to evaporate the advantages of monetary commitment. Unfortunately Dixit and Lambertini entirely focus on the issue of time consistency. Thereby they neglect to analyze the beneficial impact of stabilization policy if the union is hit by symmetric or asymmetric shocks. To that extend we extend Dixits and Lambertinis joint commitment solution to the case where the common monetary union is hit by symmetric or asymmetric supply and demand shocks. Throughout the paper we will focus in particular on two aspects. First we will show that life in a monetary union is easier if the law of one price holds. If product markets are highly integrated the currency area as a whole shares one common real interest rate $(i-p)$ which prevents that a wedge can be driven between macroeconomic outcomes in the vague of demand shocks. Second, we will analyse a scenario when all countries only produce non-tradables. Such a setting implies the existence of national inflation rates $p_i$ which translate into national real interest rates $(i-p_i)$ that amplify shocks.

In line with Dornbusch (1997), we can show that restrictions on the fiscal instrument might be harmful under such a setting (see also Chari and Kehoe (1998)), (Beetsma, Favero and Missale (2004)). In total we analyze sixteen different scenarios within this chapter conditioned on an active versus passive fiscal stance and on synchronized versus non-synchronized supply and demand shocks (See Figure 15). In order to crosscheck the robustness of our results we have additionally computed the model under different assumptions on the way expectations are formed and under different assumptions on the way fiscal policy is conducted (Appendix 3. A and 3. B ).

**Figure 15:** Analysed Scenarios



## 3.1 Monetary Policy with a Passive Fiscal Policy

In this section we assume that monetary policy is the only macroeconomic player in a monetary union, i.e. national fiscal policies remain completely passive. This means in particular that only the central bank will respond with its instrument –the nominal interest rate- to shocks in order to stabilize economic activity.

We assume that monetary policy is guided by a loss function that we have derived and introduced in section 2.1.3.2 The objective function of the central bank is given by:

$$L_{ECB} = (p - p_0)^2 + l_y y^2 . \tag{3.1}$$

The ECB tries to stabilise squared deviations of the inflation rate and the output gap from their target values respectively. The preference parameter $l_y$ depicts the weight monetary policy attaches to stabilise the output gap versus stabilising the inflation rate.

Hence it is the task of the common central bank to set the interest rate in response to exogenous disturbances and consistent with the structural equations of the model so that the loss function $L_{ECB}$ is minimised. Note that the ECB only targets at euro wide averages, whereas it does not take care of the dispersion of goal variables across countries. In other words the ECB does not consider the spread as a problem as long as it is mean preserving. This means for example that the ECB is indifferent between the following two macroeconomic outcomes as depicted in Figure 16. This convention established in literature (linear quadratic loss function in inflation and output) is to our understanding somewhat inconvenient. Nevertheless throughout the exposition we take it as granted that conventional wisdom says that the ECB should only take care of euro wide averages of the inflation rate and the output gap.

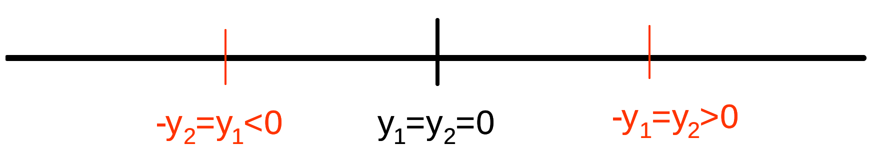



**Figure 16:** Mean Preserving Distribution of Macroeconomic Outcomes

### 3.1.1 The Law of One Price Holds

Let us assume that in a monetary union only tradables are produced. Since we abstract from trade barriers or any other country specific features such as inhomogeneous preferences the law of one price will hold. Technically speaking this assumption means in particular that the currency area is only hit by a common supply shock. Additionally we reintroduce a common real interest rate $(i - p)$ for the whole area. Relying on these assumptions one can derive the common Phillips curve as follows. Each period all firms negotiate new wage contracts for one period. Workers are assumed to care about the current state of economic activity y as well as on the expected inflation rate $\pi^e$ over the life of the contract. For the sake of simplicity we assume that monetary policy is credible ( $\pi^e = \pi_0$). The nominal change in wages is then given by:

$$\Delta w = p_0 + dy. \tag{3.2}$$

As firms are assumed to be monopolistic competitors which price their output at a constant markup over marginal costs, markup pricing translates wage inflation into price inflation:

$$p = \Delta w + m. \tag{3.3}$$

Let us assume that the mark-up factor is equal to zero ($m$ =0). By inserting equation (3.2) into equation (3.3) we get a static version of a Phillips curve:

$$p = p_0 + dy + e_2. \tag{3.4}$$

Obviously as monetary conditions measured in real terms $r = (i - p)$ are identical for all member countries i, we can specify the aggregate demand relationship for country i as follows:

$$y_i = a - b(i - p) + e_{i,1}. \tag{3.5}$$

Given this description of the economy the ECB solves the following optimisation problem.

$$L_{ECB} = (p - p_0)^2 + l_y y^2 \tag{3.6}$$

s.t.: $y = a - b(i - p) + e_1$ [4]

$$p = p_0 + dy + e_2 .$$

Inserting the Phillips curve into the loss function and solving the optimisation problem the output gap on average will be given by:

$$y = -\frac{d}{d^2 + l_y} e_2 . \tag{3.7}$$

Inserting (3.7) into the Phillips curve we see that the euro wide inflation rate will only depend on supply shocks:

$$p = p_0 + \frac{l_y}{d^2 + l_y} e_2 . \tag{3.8}$$

The ECB can protect the union on average from demand shocks. Nevertheless across countries as we will see there can be a great dispersion in output, even if the law of one price holds. Inserting the reduced form expressions for the inflation rate and the output gap into the aggregate demand relationship yields the following reduced form for the interest rate:

$$i = \frac{a}{b} + p_0 + \frac{1}{b} e_1 + \frac{(d + b l_y)}{b(d^2 + l_y)} e_2 . \tag{3.9}$$

Equation (3.9) nicely depicts that the reaction to demand shocks is not preference dependent whereas the reaction to supply shocks depends on preferences. With an increasing concern for output stabilisation (increasing λ) the coefficient $\left((d + b l_y) \big/ \left(b\left(d^2 + l_y\right)\right)\right)$ will converge to one, which reflects that the Taylor Principle also holds for the "output junkie". Inserting the

[4] Note that the ECB solves its optimization problem subject to the average IS-curve and the average Phillips curve. Assuming that the different member states share an identical economic structure you can easily retrieve the average structural relationship by computing $y = (1/n)\sum_{i=1}^{n} y_i$ and $p = a + b\left(1/n\sum_{i=1}^{n} y_i\right)$ respectively.

inflation rate and the interest rate rule (3.9) into the national aggregate demand equation (3.5) one can easily determine the output gap for country i as follows:

$$y_i = \left(\varepsilon_{i,1} - \varepsilon_1\right) - \frac{\delta}{\delta^2 + \lambda_y}\varepsilon_2 . \tag{3.10}$$

Equation (3.10) displays the key difference between a closed economy like the US and a monetary union like EMU. Even if the average output gap is equal to zero, this can go hand in hand with dispersion in national aggregates. Obviously non-synchronised demand shocks $corr\left(\varepsilon_{i,1};\varepsilon_1\right) \neq 1$ can drive a wedge between country specific output gaps. This can in the long run undermine the very existence of a union as each country would need notably different monetary conditions which is of course impossible by the very definition of a monetary union itself (see also Uhlig (2002)). To clarify this statement let us make the assumption of uncorrelated shocks $corr\left(\varepsilon_{i,1};\varepsilon_1\right) = 0$ and equally sized countries. What happens if only country i is hit by a shock at time t? To illustrate this case let us assume that the GDP share of country i is α and $\varepsilon_2 = 0$. Then we can rewrite the aggregate demand shock as the following weighted average:

$$\varepsilon_1 = \alpha\varepsilon_{i,1} + \left(1 - \alpha\right)\varepsilon_{-i,1} . \tag{3.11}$$

Since shocks are uncorrelated $corr\left(\varepsilon_{i,1},\varepsilon_{-i,1}\right) = 0$ by assumption it holds that:

$$\varepsilon_1 = \alpha\varepsilon_{i,1} . \tag{3.12}$$

Inserting equation (3.12) into (3.10), we see that output in country i will be given by

$$y_{i,1} = \left(1 - \alpha\right)\varepsilon_{i,1} , \tag{3.13}$$

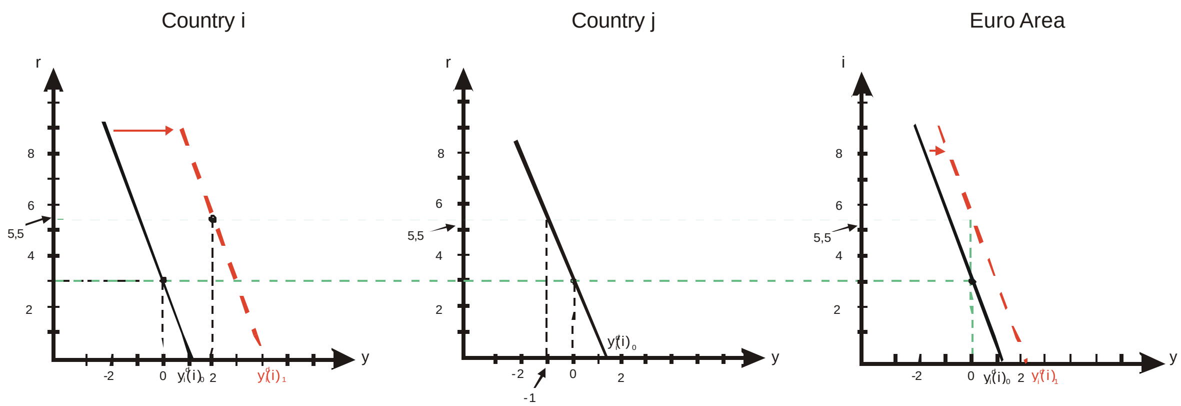
whereas output in the rest of the union is equal to:

$$y_{-i,1} = -\alpha\varepsilon_{i,1} . \tag{3.14}$$

Equations (3.13) and (3.14) depict potential conflicts which might prevail in a monetary union. As a consequence of the shock originating in country i, output will be above its potential whereas the rest of the union suffers from a somewhat depressed economic activity. Obviously equation (3.13) shows that asymmetric shocks are a major problem for small countries participating in a union as the real interest rate set by the ECB is not coined for a country with a low GDP weight unless $corr(e_i;e_{-i})=1$. In the limit, when the GDP share of an individual member country is almost zero, the shock will be passed through completely on the output gap if fiscal policy remains passive. Therefore as we will see in section (3.2) fiscal policy is in particular needed in small countries to squeeze the impact of shocks on the output gap and the inflation rate.

We can equally retrieve these results with the help of a graphical analysis (see Figure 16 ). Country i is hit by a demand shock and accordingly the aggregate demand curve shifts from $y_0^d(r)$ to $y_1^d(r)$. As we assume that fiscal policy remains completely passive over the cycle only the ECB reacts to the extend that the shock influences the global output gap. The demand shock in country i translates into a shift of the European demand curve from $y_0^d(r)$ to $y_1^d(r)$ of size $(1/n)e_{1,i}$. The ECB will tighten real interest rate conditions from $r_0$ to $r_1$ in order to stabilise economic activity on average. Nevertheless, as Figure 26 shows this stabilisation on the aggregate goes hand in hand with a dispersion of output across member states. Real interest rates for country i will be too loose giving a boost to economic activity, output will be above its potential ($y_i$>0) whereas real interest rates for the rest will be too high resulting in a somewhat depressed economic environment$\left(y_{-i}<0\right)$.

For symmetric shocks one can make use of the graphs developed in Bofinger, Mayer and Wollmershäuser (2005). Table 1 summarizes the net reaction of all variables under consideration to a positive shock.




Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 17:** Uncorrelated Demand Shock in Country i: $e_{1,i} = 3$

| | $e_1^* > 0$ | $e_{i,1} > 0$ | $e_2 > 0$ |
|---|---|---|---|
| **Output gap ($y_i$)** | ↓* | ↑ | ↓ |
| **Inflation Rate ($p_i$)** | / | / | ↑ |
| **Interest Rate (i)** | ↑ | ↑ | ↑ |
| **Aggregate Output (y)** | / | / | ↓ |
| **Aggregate Inflation (p )** | / | / | ↑ |

* Throughout the expostion we assumed that the correlation of shocks is $r\left(e_{1,i};e_1\right)=0$

**Table 1:** Net Change of a Variable to a Positive Shock

### 3.1.2 Idiosyncratic Phillips Curves

Let us now assume that the country specific output is not tradable. Accordingly the law of one price can be violated and each member state will be characterized by an idiosyncratic Phillips curve. Nevertheless as we take idiosyncratic supply shocks to be iid distributed with mean zero and a constant variance the conditional as well as the unconditional expectations of the inflation rate of the individual member states are identical. Given this assumption our set of equations can be stated as follows:

$$p_i = p_0 + dy_i + e_{i,2} \tag{3.15}$$

$$y_i = a - b(i - p_i) + e_{i,1} \, . \tag{3.16}$$

Assuming that the ECB only targets at averages its optimization problem remains unaltered. In other words the aggregate values for the output gap and the inflation gap are identical to the previous scenario on average. Following this line of argumentation we can state in particular that the nominal euro wide interest rate is still given by:

$$i = \frac{a}{b} + p_0 + \frac{1}{b} e_1 + \frac{\left(d + b\, l_y\right)}{b\left(d^2 + l_y\right)} e_2 \, . \tag{3.17}$$

The output gap of country i is now given by (3.18):

$$y_i = \frac{1}{1 - db}\left(e_{i,1} - e_1\right) + \frac{1}{(1 - bd)}\left[ be_{2,i} - \frac{d + b\, l_y}{\left(d^2 + l_y\right)} e_2 \right]. \tag{3.18}$$

Equation (3.18) shows that an uncorrelated demand shock $corr\left(e_{i,1}; e_1\right) \neq 1$ can drive a wedge between national cycles. Additionally the dispersion across national outputs is amplified by a factor of $\left(1/(1 - bd)\right)$ compared to a scenario where the law of one price holds (see (3.16)). As we will see below this can be explained by diverging real interest rate conditions $\left(i - p_i\right)$ across member states. Perhaps somewhat surprisingly equation (3.16) shows that supply shocks originating in country i can give a boost to domestic economic activity whereas union wide supply shocks depress economic activity. The argument goes as follows: A supply shock in country i (e.g., excessive wage demands) gives a push to inflation $\pi_i$ that lowers its real interest rate $\left(i - p_i\right)$. This calls the ECB upon to act only insofar as the European inflation rate raises. Therefore, the expansionary impact of declining real interest rates in country i is not totally undone by subsequent raising nominal interest rates so that output will increase. Thus the ECB can not punish individual member states by rising average real rates which clearly shows that stringent rules for labour unions as well as for national governments are a prerequisite for a well functioning monetary union, to prevent free rider behaviour and negative spill over effects for other member states. The inflation rate of country i is given by the following equation:

$$\pi_i = \pi_0 + \frac{\delta}{1-\beta\delta}\left(\varepsilon_{1,i} - \varepsilon_1\right) + \frac{1}{1-\delta\beta}\left[\varepsilon_{2,i} - \frac{\delta\left(\delta + \beta\lambda_y\right)}{\delta^2 + \lambda_y}\varepsilon_2\right]. \tag{3.19}$$

The individual inflation rates in a monetary union- in sharp contrast to a closed economy- depend on demand shocks. Although the ECB will meet its inflation target on average this can go hand in hand with a significant dispersion in inflation rates across countries. In the case of symmetric supply shocks $\varepsilon_{2,i} = \varepsilon_2$ the inflation rate will again be described by equation (3.8). To further illustrate the results let us analyze again the case of uncorrelated demand shocks. The real interest rate is given by: $r = i - \pi_i$. Making use of the reduced form of the inflation rate and the nominal interest rate in country i we can compute real interest rate conditions for country i as follows:

$$r_i = \left(i - \pi_i\right) = \frac{\alpha}{\beta} + \frac{\alpha - \beta\delta}{\beta\left(1-\delta\beta\right)}\varepsilon_{i,1}. \tag{3.20}$$

Monetary conditions for the rest of the union are given by

$$r_{-i} = \frac{\alpha}{\beta} + \frac{1}{\beta\left(1-\beta\delta\right)}\varepsilon_{i,1}. \tag{3.21}$$

which translates into the following inflation rates:

$$\pi_i = \pi_0 + \frac{\delta}{1-\delta\beta}\left(1-\alpha\right)\varepsilon_{1,i} \tag{3.22}$$

$$\pi_{-i} = \pi_0 + \frac{\delta}{1-\delta\beta}\left(-\alpha\varepsilon_{1,i}\right). \tag{3.23}$$

With equations (3.22) and (3.23) at hand we can easily compute the corresponding output gaps:

$$y_i = \frac{1}{1-\beta\delta}\left(1-\alpha\right)\varepsilon_{i,1} \tag{3.24}$$

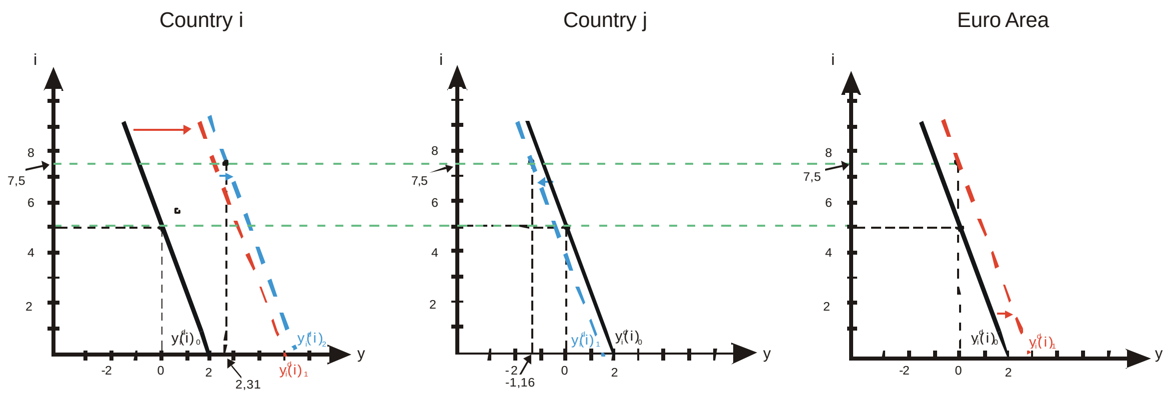
$$y_{-i} = \frac{-a}{1-bd} e_{i,1} . \qquad (3.25)$$

This set of equations depicts that if country i is hit by an uncorrelated shock and the ECB only cares on averages, then national outcomes may greatly diverge. Additionally compared to a scenario where the law of one price holds the degree of dispersion in output is amplified by a factor of $(1/1-bd)$ as a consequence of diverging real interest rates across countries. Hence the previous two sections underline that from the perspective of monetary policy a higher degree of integration in product markets is favourable as the central bank can influence more directly the real interest rate in each country.

In a scenario without fiscal policy it essentially depends on the size of the individual member country whether idiosyncratic shocks will be stabilizing or destabilizing. According to the Taylor Principle uncorrelated demand shocks will be destabilizing if real interest rates $(i-p_i)$ will not be raised. This will only be the case if (see (3.20)):

$$a - bd < 0 \Rightarrow a < bd . \qquad (3.26)$$

Given our baseline calibration (b=0.4 and d=0.34) equation (3.26) indicates that idiosyncratic shocks will be destabilizing if the GDP share of the individual country under consideration is smaller than approximately 14%. An intuition for this result is easy to find. As the ECB is the only macroeconomic agent that stabilises shocks, it only reacts to euro wide averages. The smaller the individual country in size the smaller the impact of an idiosyncratic shock on the currency area and hence the smaller the reaction of the ECB to this idiosyncratic shock. This underlines that by far most countries in EMU need fiscal policy as an independent institution in order to deal with asymmetric shocks (APPENDIX 3. C.). Some further intuition to these results can be given by taking a look at Figure 18 and Figure 19. Figure 18 depicts a scenario where country i is hit by a demand shock of size $e_{1,i} = 3$. This translates into a shift of the aggregate demand curve from $y_0^d(i)$ to $y_1^d(i)$. In response to the boom in economic activity the ECB raises real interest rates from $i_0$ to $i_1$ inducing a change in economic activity that exactly compensates the impact of the initial demand shock on the euro wide economic activity. Hence we arrive at the result that demand shocks can be totally stabilised for the currency are on average. Nevertheless this goes hand in hand with a dispersion at the national level. The increase in nominal rates leads to a decreased economic activity in the rest of the union. As the inflation rate is a shift parameter in the (i;y)-space the aggregate demand curve is shifted inwards in the rest of the union. In country i the boom in economic activity leads to an additional outward shift in aggregate demand. As we already indicated the size of shifts critically depends on the GDP share of country i.

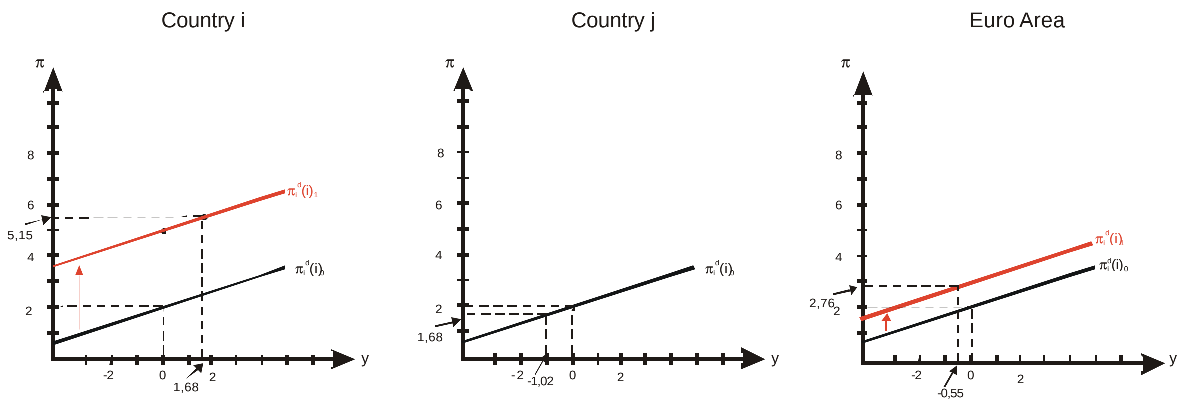



* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 18:** Idiosyncratic Demand Shock in Country i: $e_{1,i} = 3$

Figure 19 depicts a currency area where country i is hit by a supply shock of size $e_{2,i} = 3$. This translates into a shift of the aggregate inflation rate by a factor of $\left(e_2 = ae_{2,i}\right)$. Depending on preferences the ECB chooses its preferred stabilisation mix on the aggregate level by setting nominal rates in line with its preferences. This increase in euro wide nominal rates partially stabilises the inflation rate in country i. The rest of the union suffers from a deflationary environment. Table 13 underlines that national real interest rates – if existent- can drive a massive wedge between national outcomes and call for stringent rules that prevent unsustainable policies in individual member states which inflict negative spill over effects for the rest of the union. Additionally the figures display that we need fiscal policy as an additional macroeconomic agent in order to squeeze idiosyncratic shocks. The impact of the negative spill-over effect depends again on the GDP share of country i.

Table 2 summarises the reaction of the variables under consideration to positive shocks.




* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 19:** Idiosyncratic Supply Shock in Country i: $e_{2,i} = 3$

| | $e_{-i,1} > 0$ | $e_{i,1} > 0$ | $e_2 > 0$ | $e_{2,i} \geq 0$ |
|---|---|---|---|---|
| **Output gap ($y_i$)** | ↓* | ↑* | ↓* | ↑ |
| **Inflation Rate ($p_i$)** | ↓* | ↑* | ↑ | ↑* |
| **Interest Rate (i)** | ↑ | / | ↑ | / |
| **Aggregate Output (y)** | / | ↑* | ↓ | / |
| **Aggregate Inflation ($p$)** | / | ↓* | ↑ | / |

*Note that we implicitly assume that idiosyncratic demand and supply shocks are uncorrelated.

**Table 2:** Net Change of a Variable to a Positive Shock

### 3.1.3 Idiosyncratic Phillips Curves and TOT Effects in the IS-Equation

As in section 2.2 we assume that the PPP does not hold in the short run. Accordingly given the definition of the real exchange rate

$$\Delta q = \Delta s + p_{-i} - p_i, \qquad (3.27)$$

it holds that the change in intra-European competitiveness is equal to the difference in the national inflation rates. The inflation rate of country i is governed by the following Phillips curve:

$$p_i = p_0 + dy_i + e_{i,2}\,. \tag{3.28}$$

Taking care of these terms of trade effects (TOT) the IS-equation of country i can be written as follows:

$$y_i = a - b\left(i - p_i\right) + c_i\left(p_{-i} - p_i\right) + e_{i,1}\,, \tag{3.29}$$

where $c_i$ denotes the real exchange rate elasticity of aggregate demand. Accordingly the impact of domestic inflation on output is somewhat different compared to the previous scenario as an increase in the inflation rate triggers two effects which are simultaneously at work. On the one hand an increase in the inflation rate lowers ceteris paribus in a first round effect the real interest rate $\left(i - p_i\right)$ which gives a boost to economic activity. On the other hand an increase in the inflation rate decreases the competitiveness so that foreign demand for domestically produced goods is somewhat depressed. As the euro-area is modeled as closed economy, it has to hold for reasons of model consistency that:

$$a\, y_i + \left(1 - a\right) y_{-i} = y\,. \tag{3.30}$$

It can be shown that a necessary and sufficient condition for this equation to be valid is:

$$c_i = \left((1-a)/a\right) c_{-i}\,. \tag{3.31}$$

As the resulting reduced forms are somewhat lengthy we only present numerical results in this section. Throughout this section we calibrate $c_{-i}$ equal to $c_{-i} = 0.02$ and α equal to $a = \left(1/3\right)$. Therefore the small bloc of the union can be labeled as Germany as its GDP-weight is approximately one third of the currency area and $c_i = 0.04$ is in line with estimates as provided by Angeloni and Ehrmann (2004) for large open economies.

As the euro-area in total is modeled as a closed economy the aggregate demand side can be stated as:

$$y = a - b(i - p) + e_1. \tag{3.32}$$

The ECB still solves the same optimization problem which is given by:

$$L_{ECB} = \left(p - p_0\right)^2 + l_y y^2 \tag{3.33}$$

$$\text{s.t.:} \quad \begin{array}{l} y = a - b(i - p) + e_1 \\ p = p_0 + dy + e_2 \end{array},$$

so that the instrument is set according to the following reaction function:

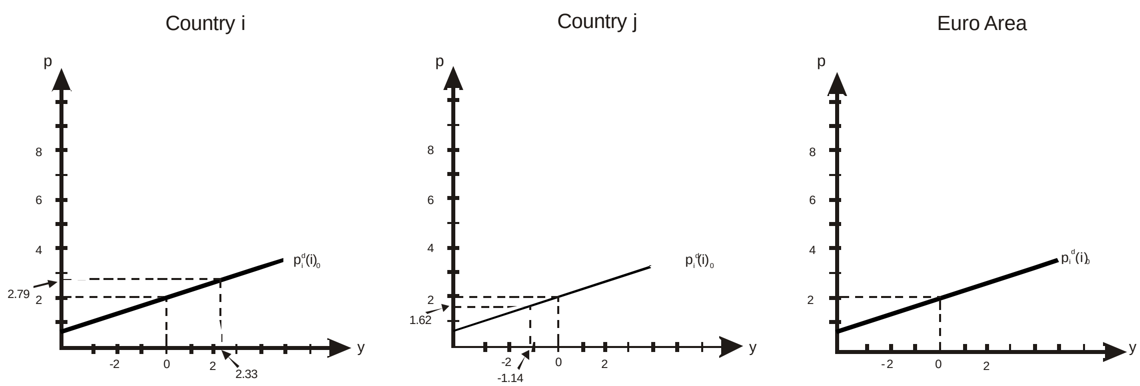
$$i = \frac{a}{b} + p_0 + \frac{1}{b} e_1 + \frac{\left(d + b\, l_y\right)}{b\left(d^2 + l_y\right)} e_2 . \tag{3.34}$$

Given this set up we can easily retrieve the reduced forms as follows. Inserting the idiosyncratic Phillips curves and the interest rate equation into the IS-equation we can compute the reduced form for the output gaps. Having these at hand and inserting them in the Phillips curves we can then compute the idiosyncratic inflation rates. In the following we will analyze what happens if country i is hit by an idiosyncratic demand shock $corr(e_{i,1}, e_{-i,1}) = 0$. To illustrate this scenario we assume that the GDP share $\alpha$ of country i is one third of the currency area and that the currency area is not hit by a supply shocks ($e_2 = 0$). Accordingly it holds that:

$$e_1 = a e_{i,1} . \tag{3.35}$$

Table 3.C.1 depicts the scenario when country i is hit by a demand shock of size $e_{i,1} = 3$. As we have attached a weight of $a = 0.33$ to country i, this translates into an initial increase of 1% of the European inflation rate. Nevertheless, as we have seen in the scenarios beforehand the ECB is able to set real interest rate conditions for the currency area in such a way that it

hits its targets on average. This goes hand in hand with a dispersion on the national level. As we have seen before, real interest rates are too expansionary to stop the economic boom in country i and too restrictive for the rest of the union which translates into the observed dispersion in economic conditions. Given the weights we have chosen the results depict the macroeconomic outcomes for an economy like Germany. Note in particular that the effects triggered by decreasing competitiveness are much too weak to undo the effects triggered by real interest rates.




* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 20:** Idiosyncratic Demand Shock in Country i

To generalize the derived results we will evaluate the baseline to an arbitrary country size α. Note that the graph depicts the output gaps $y_i$ and $y_{-i}$ and the inflation rates $p_i$ and $p_{-i}$ of the two blocs of the currency area under consideration. The graph nicely depicts the following results: For the case of a supply shock the ECB is always able to maintain its bliss point for the currency area on average. Nevertheless the dispersion of macroeconomic outcomes across the currency area is strongly governed by the size of the country which was hit by the asymmetric shock. The ECB will increase interest rates by

$$i = \frac{a}{b} + p_0 + \frac{1}{ab} e_{i,1}, \tag{3.36}$$

in response to the demand shock in country i which has a GDP-weight of α. Obviously real interest rates will be too loose for that economy and too restrictive for the rest of the union.

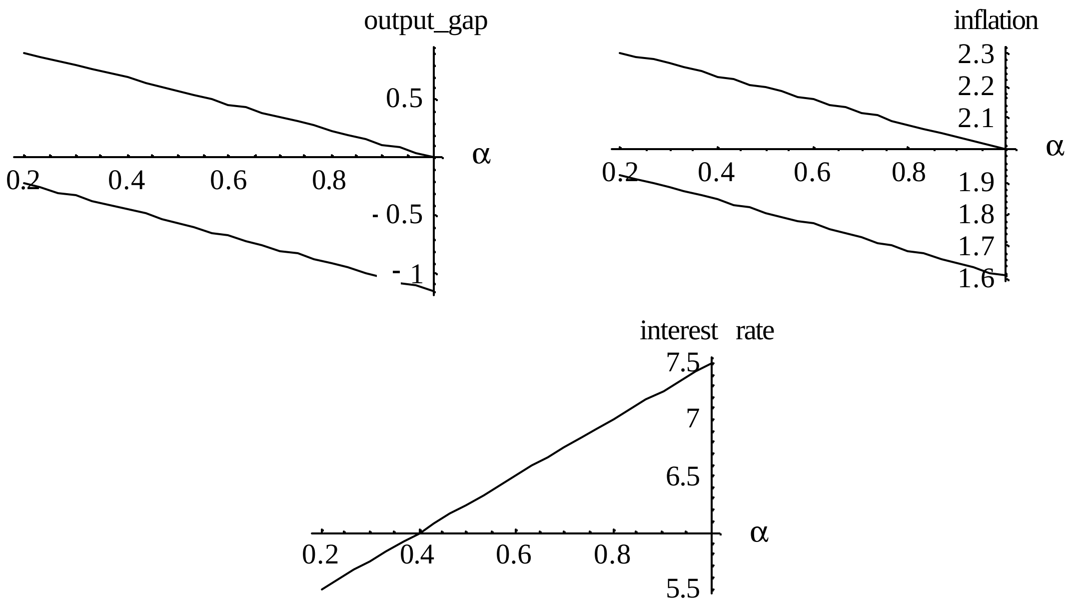
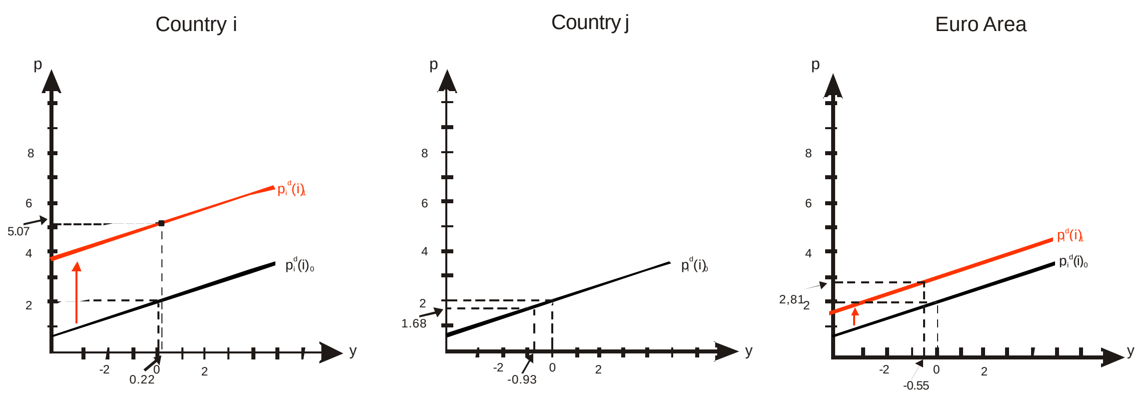



**Figure 21**: Reaction of the Output Gap and Inflation Rate as a Function of the Country Size: Demand Shock

If the economy that was hit by the shock becomes larger, the ECB reacts stronger which creates more turbulence in the rest of the union. The created mismatch in monetary conditions triggers a deeper recession in the rest of the union. The nominal interest rate reaction to a demand shock monotonically increases with respect to the size of country α. Figure 22 shows a scenario when the currency area is hit by a supply shock of size $e_{i,2} = 3$.




* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 22**: Idiosyncratic Supply Shock in Country i: $e_{i,2} = 3$

Depending on its preferences the ECB will choose its preferred stabilization mix on the aggregate level by setting the nominal interest rate conditions in line with its preferences. This induces a macroeconomic dispersion which only partially stabilizes the economic boom in country i whereas the rest of the union suffers from a mild depression. Compared to a scenario where the TOT effects are not operating the dispersion is somewhat dampened as the country that booms looses competitiveness so that the increase in economic activity is less pronounced. Nevertheless for a country of the size of Germany the real interest rate effect clearly has the potential to dominate the TOT effect (see Figure 22).

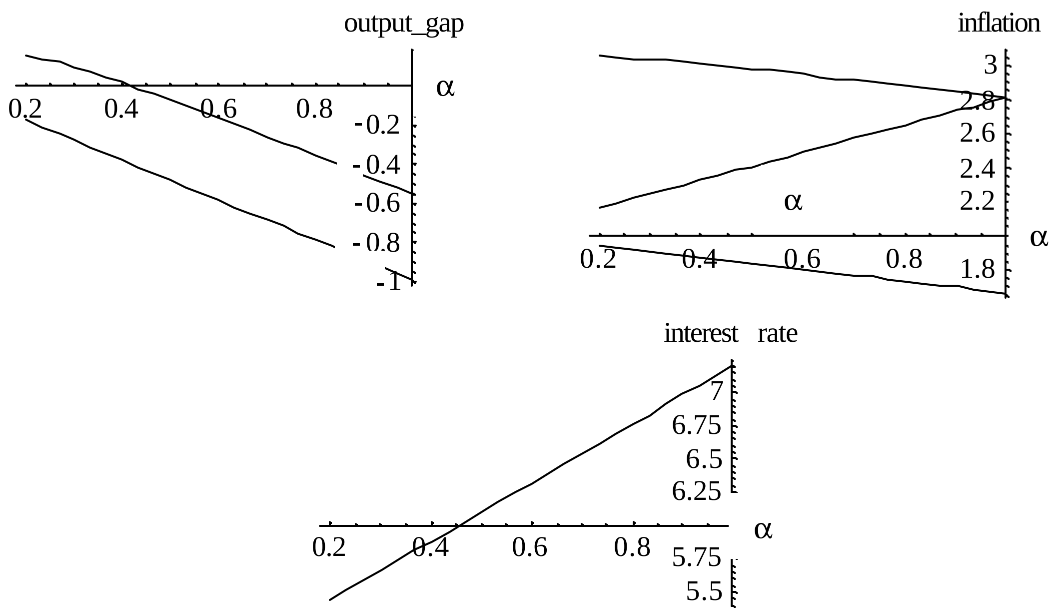



**Figure 23**: Sensitivity of the Output Gap and Inflation Rate as a Function of the Country Size: Supply Shock

Hence our analysis gives support to the result that an idiosyncratic supply shock in country i may give a boost to economic activity. We claim that this is an important contribution to literature that seems largely neglected in related studies.

## 3.2 Monetary and Fiscal Policy Interaction

In the previous section we modeled a monetary union when monetary policy is the only macro economic agent that actively stabilizes shocks. We basically saw for two possible specifications of a Phillips curve that life in a monetary union is easier if shocks are correlated and product markets are integrated. In this section we introduce a fiscal authority in each member state that is guided by a loss function and which has g, the fiscal stance parameter as its only instrument. The stance of fiscal policy is defined as expenditures minus revenues. Hence if g>0 the fiscal stance is expansionary if g<0 the fiscal stance is contractionary.

### 3.2.1 The Loss Function of Fiscal Authorities

We assume that national fiscal authorities are guided by a loss function.

$$L_{G,i} = y_i^2 + \varphi\, g_i^2 .\text{[5]} \tag{3.37}$$

Each government is interested in stabilising output around its potential. The second term in the loss function captures the notion that governments behaviour might be motivated for instance by the Treaty of Maastricht that penalises excessive (downward) movements in the fiscal stance parameter g. Additionally if g would be permanently larger than null the solution would exhibit an unpleasant debt arithmetic's as the fiscal balance exhibits a structural deficit[6]. The parameter φ scales the costs of using the fiscal policy instrument.

As a specific characteristic of a monetary union the common central bank targets at union wide aggregates whereas the individual governments focus on national aggregates. This set-up nests possible conflicts as the ECB can only on average meet its targets which is likely to go hand in hand, depending on the correlation of country specific shocks, with a dispersion in the individual target variables under consideration in each member state. The question we will answer now is to what extend fiscal policy can prevent national outcomes from diverging across the currency area[7]. Hence we will look to what extend national fiscal policies can mitigate asymmetric shocks.

[5] Note that we implicitly assume that both macroeconomic agents have an identical output target. For diverging targets see (Dixit and Lambertini (2001)).

[6] For a paper that focuses more strongly on the political interaction between the national governments and a common central bank see (Demertzis (1999)).

[7] For a focus on automatic stabilizers see Gali and Perotti 2003.

### 3.2.2 The Law of one Price Holds

Let us assume that the law of one price holds. Then the Phillips curve for all countries is given by:

$$p = p_0 + dy + e_2 . \tag{3.38}$$

Hence the commodity bundles produced in each country are perfect substitutes with a common inflation rate π. The currency union has only one common real interest rate $r = i - p$ . Additionally the union is hit only by a common supply shock. The second building bloc of the model is the IS-equation:

$$y_i = a - b\left(i - p\right) + k\, g_i + e_{i,1} . \tag{3.39}$$

Aggregate demand now also depends on the fiscal stance parameter. We assume that $g = g^{opt}$ . Hence g is set in order to minimise the loss function of fiscal policy. Given the structure of the economy the ECB solves the following optimisation problem:

$$L_{CB} = \left(p - p_0\right)^2 + l_y y^2 \tag{3.40}$$

s.t.

$$y = a - b\left(i - p\right) + k\, g + e_1 \tag{3.41}$$

$$p = p_0 + dy + e_2 . \tag{3.42}$$

Depending on the structural parameters of the economy and its preferences the ECB chooses the following stabilisation mix:

$$p = p_0 + \frac{l_y}{d^2 + l_y} e_2 \tag{3.43}$$

$$y = -\frac{d}{d^2 + l_y} e_2 . \tag{3.44}$$

Equations (3.43) and (3.44) underline that the ECB is the dominating actor of the game as it can push its preferred bliss point through. In other words it can always completely offset the effects of fiscal policy on average. The inflation and output gap are identical to those we already saw for the scenario without fiscal policies. The reaction function of the central bank is given by:

$$i = \frac{a}{b} + p_0 + \frac{1}{b} e_1 + \frac{b\, l_y + d}{b\left(d^2 + l_y\right)} e_2 + \frac{k}{b} g \; . \tag{3.45}$$

The reaction function specifies the optimal nominal interest rate if governments of the individual member states play $\left(\frac{1}{n}\sum_{i=1}^{n} g_i = g\right)$ on average. It depicts the optimal response of the central bank to the average current stance of fiscal policy across the currency area. Equation (3.45) is characterised by the following features: In the absence of macroeconomic shocks $\varepsilon_1 = \varepsilon_2 = 0$ the ECB will set interest rates equal to their long run equilibrium value $\mathrm{i} = (\mathrm{a}/\mathrm{b}) + \pi_0$ which corresponds to a union wide output gap of null and an inflation rate that is equal to the inflation target. The global response to demand shocks in a union compared to a scenario of a closed economy is on average unaltered and given by: $\Delta \mathrm{i} = (1/\mathrm{b})\varepsilon_1$. Again the response to supply shocks depends on preferences.

Fiscal authorities in each member state solve the following optimisation problem[8]:

$$L_{G,i} = y_i^2 + j\, g_i^2 \tag{3.46}$$

s.t.:

$$\text{s.t.: } y_i = a - b\left(i - p\right) + k\, g_i + e_{i,1} \; .^9 \tag{3.47}$$

Solving this optimisation problem we arrive at the following relationship depicting the way according to which fiscal policy is conducted:

$$g_i = \frac{-ak}{k^2 + j} + \frac{bk}{k^2 + j}\left(i - p\right) - \frac{k}{k^2 + j} e_{i,1} \, . \tag{3.48}$$

---

[8] Note that we do not intend to model alliances between individual member states (see Aarle, Bas van, Bartolomeo Giovanni. Di, Engwerda Jacob, Plasmans, Joseph (2005) ).

[9] For an analysis that includes the real exchange rate in the strategic analysis between the central bank and the government see Leitemo (2003).

It depicts the optimal reaction of the government to the current stance of monetary policy. The equation is characterised by the following features: The partial derivative of g with respect to r is $(\partial g/\partial r) = \left(\kappa b / \left(\kappa^2 + \varphi\right)\right) > 0$. Hence if monetary policy gets more restrictive the government will switch to a more expansionary stance. The higher the weight on stabilising its instrument $\varphi$, the lower will be the strategic interaction between the two macroeconomic agents. Following e.g. a negative demand shock $\varepsilon_1$ fiscal policy will become more expansionary. Note that in contrast to monetary policy the government does not face a lower bound. Hence g can become negative (g<0). The strategic interaction between fiscal and monetary authorities results from the fact that the ECB responds to union-wide averages:

$$e_1 = a e_{i,1} + (1-a) e_{-i,1}. \tag{3.49}$$

Hence if only country i is hit by a demand shock, this triggers a feedback mechanism as all member countries have to share the adjustment burden of higher interest rates. The extend of the strategic feedback depends on the GDP share $a$ of country i. Nevertheless to simplify the exposition we will assume symmetry in the following.

Given the reaction function of n fiscal authorities and the ECB we can easily compute the reduced form solution as we have n+1 unknowns $(g_1;...;g_n;i)$ and n+1 reaction functions. Inserting (3.43) in (3.48), averaging and plugging the resulting expression into (3.45) we get the following reduced form equation for the interest rate:

$$i = p_0 + \frac{a}{b} + \frac{1}{b} e_1 + \frac{b l_y j + d\left(k^2 + j\right)}{b\left(d^2 + l_y\right) j} e_2. \tag{3.50}$$

In the absence of macroeconomic shocks ($\varepsilon_1 = \varepsilon_2 = 0$) the ECB will set interest rates equal to their long run equilibrium value $i = (a/b) + \pi_0$ which corresponds to a union wide output gap of null and an inflation rate that is equal to the inflation target. The global response to monetary shocks in a union compared to a scenario of a closed economy is on average unaltered an given by: $\Delta i = (1/b)\varepsilon_1$.

The reduced form for the fiscal stance parameter can be computed by inserting the inflation rate and the interest rate into the reaction function of the central bank.

$$g_i = \frac{k}{k^2 + j}\left(e_1 - e_{1,i}\right) + \frac{kd}{j\left(d^2 + l_y\right)} e_2 . \tag{3.51}$$

Equation (3.51) displays the difference between a closed and open economy setup of a static version of a New Keynesian macromodel. First we see that fiscal authorities have a stabilisation task in response to demand shocks as long as these exhibit a degree an asymmetry. Most importantly as individual shocks are assumed to be iid there is some positive probability $\left(\frac{k^2 + j}{k}\int_{-\infty}^{-0,03} f\left(e_i\right) de_i\right)$ that the 3% deficit criterion cannot be met. In other words if the size of the shocks is large (3.51) clearly demonstrates that even under an optimal and sustainable fiscal stance (defined as g=0 in the absence of shocks) the Maastricht deficit criterion is likely to be violated with some positive probability. Nevertheless as long as the violation stems from the size of exogenous shocks and not from a fiscal policy that is conducted in an unsustainable fashion (g>0), (see chapter 4.4) the violation of the Maastricht criterion is a necessary precondition to restore the overall optimal outcome. Exactly for that reason the 3% deficit criterion can be suspended if a country is hit by a large shock. The same holds of course true for large demand and supply shocks.

Inserting (3.50) and (3.51) into the aggregate demand equation we arrive at the following expression for the country specific output gap:

$$y_i = -\frac{d}{d^2 + l_y} e_2 + \frac{j}{k^2 + j}\left(e_{i,1} - e_1\right). \tag{3.52}$$

Note given standard parameterisation $\left(k = j = 0,5\right)$ uncorrelated demand shocks are likely to have a smaller impact on the overall economic activity compared to a scenario where fiscal policy remains passive. So indeed we can state that a Keynesian stabilization policy is able to dampen economic cycles compared to a policy that sets g=0[10]. Nevertheless the stabilisation of shocks will not be perfect. The argument goes as follows. Assume that only one country is hit by a negative demand shock. Obviously, given the Nash equilibrium, real interest rates $\left(i - p\right)$ will be too tight for that country, too weak to restore an output in line with potential

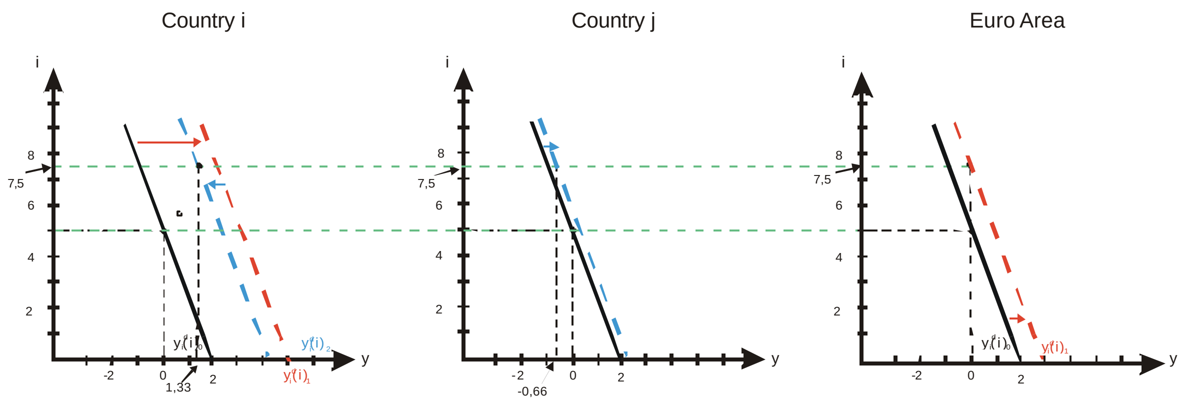
[10] For a critical view that stresses that fiscal shocks itself might be a source of dispersion in output see for instance Canova and Pappa (2003).

$(y_i < 0)$. In contrast the real interest rate for the rest of the union will be too loose giving a boost to economic activity ($y_{-i} > 0$). At first glance this result might seem at odds with intuition. One might ask why fiscal authorities do not use their instrument more rigorously in response to demand shocks in the equilibrium. The key to this answer lies in the strategic interaction between the agents. A more expansionary fiscal policy triggers higher interest rates for the currency area so that the marginal costs of an expansionary fiscal policy outweigh the marginal benefits.

The degree of conflict potential can be summarised by the correlation between the idiosyncratic demand shocks versus the euro wide average $corr(e_i;e_{-i})$. Equation (3.280) depicts that in a union where demand shocks are perfectly correlated $corr(e_i;e_{-i}) = 1$ the output gaps of individual member states $y_i$ are identical at each point in time. Obviously a maximum dispersion in output will be given if $corr(e_i;e_{-i}) = -1$. Then the individual output gaps $y_i$ would exhibit a maximum dispersion which could potentially undermine the existence of the union in the long run as at each point in time country i finds it beneficial-evaluated in terms of $L_{G,i}$- to leave the union as it requires significantly different real interest rates. Therefore our simple static analysis clearly makes the prediction that if the law of one price holds life within a monetary union is easier if demand shocks are highly correlated and fiscal policy actively engages into stabilising shocks. Additionally the exposition provided a rationale for the suspension of the 3% deficit criterion in the vague of large shocks as a necessary condition for fiscal policy to be conducted optimally.

It is important to note that if we set $j = 0$ shocks can be completely stabilised. In other words if fiscal policy does not put any weight on smoothing its instrument it is possible to completely offset uncorrelated demand shocks. Nevertheless the smoothing objective is a common theme in literature.

We can present the same results with the help of a graphical analysis. Let us assume that country i is hit by an uncorrelated demand shock. The shock shifts the aggregate demand curve from $y_0^d(i)$ to $y_1^d(i)$. As a result the aggregate European demand curve shifts from $y_0^d(i)$ to $y_1^d(i)$. As the ECB can stabilize shocks on average, it will raise real interest rates from $i_0$ to $i_1$ which brings output back to its potential and the inflation rate to the inflation target. The new nominal rate depresses economic activity in the rest of the union so that fiscal policy becomes expansionary which leads to an outward shift of the aggregate demand curve.




* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 24:** Idiosyncratic Demand Shock in Country i: $e_{i,1} = 3$

In country i the increase in nominal rates is too small so that fiscal policy will become contractionary leading to an inward shift of the $y_0^d(i)$ curve. Figure 3 summarizes the net reaction of the variables to a positive shock respectively.

| | $e_{-i,1} > 0$ | $e_{i,1} > 0$ | $e_2$ |
|---|---|---|---|
| **Fiscal stance (g)** | ↑ | ↓ | ↑ |
| **Output gap (y)** | ↓ | ↑ | ↓ |
| **Inflation rate (p)** | / | / | ↑ |
| **Interest rate (i)** | ↑ | ↑ | ↑ |

**Table 3:** Net Change of a Variable to a Positive Shock

## 3.2.3 Idiosyncratic Phillips Curves

In this section we analyse the strategic interaction between fiscal and monetary authorities in a union if the law of one price does not hold. We will again focus on uncorrelated idiosyncratic demand and supply shocks. As already shown in section (3.1.2) the existence of country specific real interest rates drives a further wedge between macroeconomic outcomes compared to a scenario where the law of one price holds. Nevertheless fiscal policy has stabilizing effects on the performance of member countries.

Like in section (3.1.2) the Phillips curve can be specified as:

$$p_i = p_0 + dy_i + e_{i,2} . \tag{3.53}$$

This means in particular that each country only produces non-tradable commodities. Note that this assumption does not mean that the country specific inflation rates can diverge arbitrarily over time, as we take non auto correlated shocks to be the workhorse throughout our exposition. The inflation rate in country i is driven by the country specific output gap $(y_i)$ and the idiosyncratic supply shock $e_{i,2}$, e.g. non-sustainable wage policies. With equation (3.281) we effectively reintroduce country specific real interest rates. The government in the individual member state (i) has to solve the following optimization problem:

$$L_{G,i} = y_i^2 + j\, g_i^2 \tag{3.54}$$

s.t.

$$y_i = a - b\left(i - p_i\right) + kg_i + e_{i,1} . \tag{3.55}$$

The reaction function of fiscal policy can than be stated as follows:

$$g_1 = -\frac{k\,a}{k^2 + j} + \frac{k\,b}{k^2 + j}\left(i - p_i\right) - \frac{k}{k^2 + j} e_1 . \tag{3.56}$$

In order to solve the game we impose symmetry, hence we assume that not only the coefficients in the country specific Phillips curves and the IS curves are identical but that additionally the countries are of equal size. Consequently averaging over the fiscal stance parameter results in:

$$\bar{g} = \frac{1}{n}\left[g_1 + g_2 + ... + g_n\right] = g = -\frac{k\left(a + b\left(i - p\right) + e_1\right)}{k^2 + j} . \tag{3.57}$$

Inserting (3.47) into the following equation:

$$i = \frac{a}{b} + p_0 + \frac{1}{b} e_1 + \frac{b\,l_y + d}{b\left(d^2 + l_y\right)} e_2 + \frac{k}{b} g . \tag{3.58}$$

and solving for (3.58) yields (3.59). Most notably equation (3.59) is identical to the reduced form we already saw under scenario 1. This cannot come as a surprise as the averages of the variables under consideration (output gap, fiscal stance parameter,...) from the perspective of the ECB are identical under both scenarios. Hence from the viewpoint of monetary policy it does not matter whether the supply side of the economy is characterised by only one or many Phillips curves as long as the ECB only cares on shocks and is indifferent between mean preserving spreads:

$$i = \frac{a}{b} + p_0 + \frac{1}{b} e_1 + \frac{b l_y j + d\left(k^2 + j\right)}{b\left(d^2 + l_y\right) j} e_2 . \tag{3.59}$$

The fiscal stance parameter is given by:

$$g_i = q_1\left(e_1 - e_{1,i}\right) + q_2 e_{i,2} + q_3 e_2 . \tag{3.60}$$

where:

$$q_1 = \frac{\kappa}{\kappa^2 + (1 - \mathrm{bd})\varphi} < 0$$

$$q_2 = -\frac{\kappa \mathrm{b}}{\kappa^2 + (1 - \mathrm{bd})\varphi} < 0$$

$$q_3 = \frac{\kappa\left(\mathrm{b}\lambda_y\varphi + \mathrm{d}\left(\kappa^2 + \varphi\right)\right)}{\left(\mathrm{d}^2 + \lambda_y\right)\varphi\left(\kappa^2 + (1 - \mathrm{db})\varphi\right)} > 0$$

Fiscal policy exhibits a higher level of activity compared to a scenario where the law of one price holds as $q_1$ is larger than the corresponding coefficient in equation (3.51). This shows that fiscal policy needs to become more countercyclical as country specific real rates ($i-\pi_i$) amplify shocks that hit the individual economies. A negative demand shock originating in the own country leads to a fiscal expansion as a negative output shock in the other member states leads to a contraction in the own fiscal stance parameter which nicely depicts that the ECB will relax monetary conditions which would give a boost to output in country j if fiscal policy would not contract. This result clearly shows the macroeconomic assignment which is nested in the Nash equilibrium. Demand shocks are mainly stabilised by the ECB and not- as one might expect- by the individual member states. As expected a foreign inflation shock leads to a more expansionary fiscal stance since the government is only concerned about output and not about inflation. Therefore as a response to tighter monetary conditions for the whole area the fiscal stance becomes more expansionary. These results are qualitatively identical to those we already saw in section 3.2.2.

The output gap equation is given by:

$$\mathrm{y_i} = \mathrm{q_5}\left(\varepsilon_{i,1} - \varepsilon_1\right) + \mathrm{q_6}\varepsilon_2 + \mathrm{q_7}\varepsilon_{i,2}\,. \quad (3.61)$$

[11]

where:

$$\mathrm{q_5} = \frac{\varphi}{\left(\kappa^2 + (1-\mathrm{bd})\varphi\right)} > 0$$

$$\mathrm{q_6} = -\frac{\mathrm{b}\lambda_y\varphi + \mathrm{d}\left(\kappa^2 + \varphi\right)}{\left(\mathrm{d}^2 + \lambda_y\right)\left(\kappa^2 + (1-\mathrm{bd})\varphi\right)} < 0$$

$$\mathrm{q_7} = \frac{\mathrm{b}\varphi}{\left(\kappa^2 + (1-\mathrm{bd})\varphi\right)} > 0$$

Note in particular given our standard calibration $\left(k = j = 0{,}5; d = 0{,}34, b = 0{,}4\right)$ the stabilisation of idiosyncratic demand shocks is only partial compared to a scenario where the law of one price holds. This underlines that diverging real interest rates ($i$-$\pi_i$) amplify shocks. Accordingly by the very definition of a (stable) Nash equilibrium fiscal policy has no incentive to deviate from the final outcome of the game as otherwise monetary policy would have an incentive to raise real interest rates. Hence, we come to the result that a country specific supply shock, e.g. wage demands that are not consistent with the inflation target of the ECB ($\Delta w > p_0$) lead to an increase in domestic inflation and to a drop in national real interest rates. Thus the ECB cannot punish individual member states which calls for a wage policy that is consistent with the inflation target of the ECB. For a foreign and an aggregate supply shock we come to the same conclusions as in section 3.1.2. But again the analysis shows that fiscal policy as an independent agent is able to stabilise the impact of supply

[11] Note if we set $e_{i,1} = e_1$ and $e_{i,2} = e_2$, hence if the currency area is hit by symmetric shocks then equation (289) simplifies to (236).

shocks. So indeed as in the case of demand shocks equation (3.57) clearly demonstrates the advantageous of a Keynesian stabilisation policy as the impact of supply and demand shocks on the macroeconomic goal variables is significantly reduced. To complete the reduced form description of the economy we compute the inflation rate. The reduced form expression for the inflation rate is characterised by the following expression:

$$\pi_i = \pi_0 + q_8\left(\varepsilon_{i,1} - \varepsilon_1\right) + q_9\varepsilon_2 + q_{10}\varepsilon_{i,2}.\text{[12]} \qquad (3.62)$$

where:

$$q_8 = \frac{dj}{\left(k^2 + (1-bd)j\right)} > 0$$

$$q_9 = -\frac{d\left(bl_y j + d\left(k^2 + j\right)\right)}{\left(d^2 + l_y\right)\left(k^2 + (1-bd)j\right)} < 0$$

$$q_{10} = \frac{\left(k^2 + j\right)}{\left(k^2 + (1-bd)j\right)} > 0$$

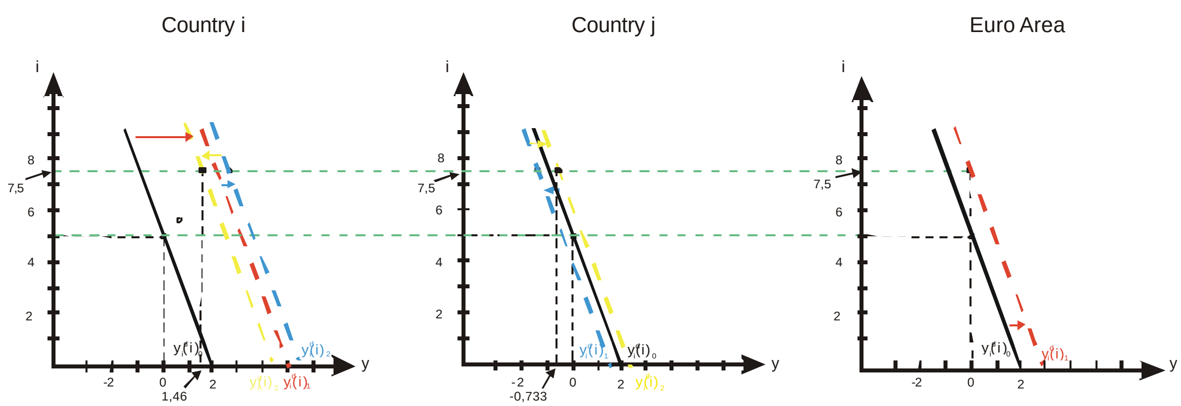
The reduced form inflation rate is characterised by the following features: In the absence of macroeconomic shocks that hit the euro area the individual inflation rate will be equal to the inflation target. Demand shocks will only have an impact on the idiosyncratic inflation rate to the extend that they are uncorrelated. Compared to a scenario where only monetary policy takes care of shocks the introduction of a Keynesian stabilization policy $g = g^{opt}$ reduces the impact of demand shocks on the national inflation rate and the output gap. The same dramatic decrease (given our standard calibration) can be recorded following idiosyncratic supply shocks.

Let us illustrate the results of this section. Country i is hit by a positive demand shock of size $e_{i,1} = 3$ which gives a massive boost to economic activity in that country given unchanged real interest rates ($\pi$ serves as a shift parameter) (see Figure 25). The aggregate demand curve in country i is shifted from $y_0^d(i)$ to $y_1^d(i)$. Nevertheless the idiosyncratic shock in country i

[12] Note if we set $e_{i,1} = e_1$ and $e_{i,2} = e_2$, hence if the currency area is hit by symmetric shocks then equation (290) simplifies to (235).

translates into an average euro-wide shock of size $(1/n)e_1$. This calls the ECB upon to act. As we already saw, in the case of demand shocks, the ECB can always maintain its bliss point. Accordingly it will tighten monetary conditions and raise real interest rates from $i_0$ to $i_1$ which induces a change in economic activity for the whole currency area that exactly compensates the initial demand shock. As output on average will be back to potential for the currency area, the inflation rate will equally return to the inflation target. Nevertheless the policy stance in country i will be too loose. On contrary for the rest of the union real interest rates will be too tight resulting in a somewhat depressed economic activity. Accordingly the inflation rate in the country that was hit by the initial demand shock will be above the inflation target of the ECB whereas inflation in the rest of the union will be below the ECB's inflation target. But remember for the union as a whole inflation will be back to target. This result nicely depicts that the common central bank is indifferent when it comes to mean preserving macroeconomic outcomes. Given this global picture we still need to look at the behaviour of the individual member states in equilibrium. Obviously the government in country i initiates a fiscal contraction as output is above its potential shifting the aggregate demand curve inward. In the rest of the union the governments relax the fiscal stance in order to stabilize economic activity shifting the aggregate demand curve outward. The degree of strategic interaction critically depends on the size of country i. Compared to a scenario where monetary policy is the only stabilizing actor fiscal authorities succeed in partially stabilizing output as depicted in Figure 25. Given this battery of shifts and back shifts we arrive at a final policy outcome in response to the idiosyncratic demand shock that is described by the following features. In country i output will be above potential and the inflation rate will be higher than the inflation target. In the rest of the union the economic environment is characterized by the opposite picture: output will be below potential and inflation will be below its target level. As in the case of a closed economy the shock will be stabilized on average.

Figure 26 and Figure 27 depict what happens if country i is hit by an idiosyncratic supply shock. Assume that country i is hit by a supply shock of size $e_{i,2} = 3$. As in the case of a closed economy the ECB determines the overall outcome of the game depending on preferences $\lambda$ by setting the nominal interest rate accordingly. Equations (3. 61) and (3. 62) depict the union wide outcomes that will prevail given an aggregate supply shock of size $(1/n)e_{i,2} = e_{i,2}$. For $\lambda_y$ equal to 0.5 we can see that the inflation rate will increase to 2.81% and the output gap will drop to a level of –0.55%. Now the interesting question is how this global outcome translates into national macroeconomic performances. Obviously the rest of the union will suffer from a recession as it will face higher real interest rates which translates into a negative output gap. Therefore we will move along the Phillips curve to a point that is characterized by a lower output and a lower inflation rate. In the rest of the union the fiscal stance is expansionary to (partially) unwind the effects of the contractionary monetary stance. For country i itself the massive increase in inflation by 3% leads to almost unchanged real rates so that fiscal policy is somewhat contractionary to prevent real interest rats from decreasing.



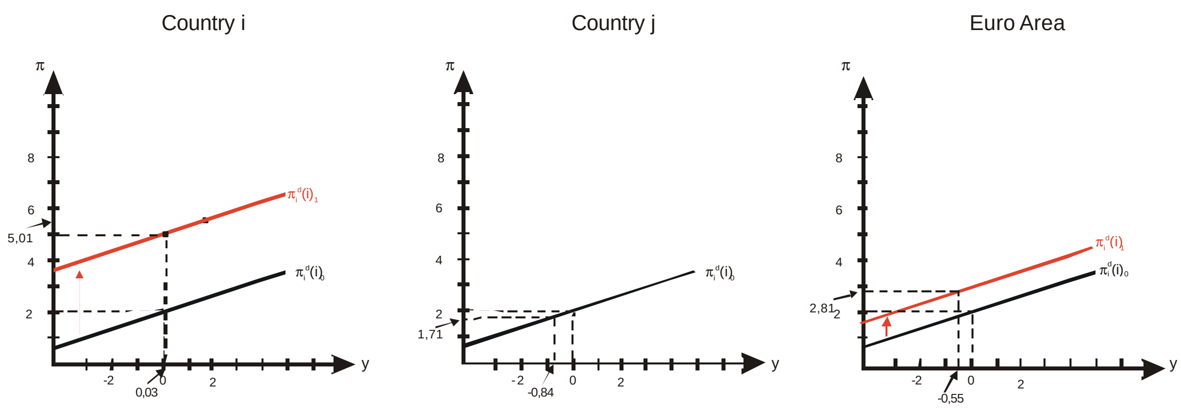

* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 25:** Idiosyncratic Demand: Shock in Country i: $e_{i,1} = 3$[13]

Figure 26 nicely maps the 'dynamics' captured in a static version of a New Keynesian macromodel: Supply shocks are only contractionary in sum to the extend that monetary policy reacts to them. As the massive inflationary shock only translates by (1/n) on the aggregate the reaction of the ECB for that individual country will be far too weak to contract economic activity. Within a monetary union labour unions can potentially hide behind the (1/n)-effect as the ECB cannot 'punish' a particular country for a wage policy that is not in line with its inflation target. Of course we can equally look at the effects of a supply shock by mapping the strategic interaction between the agents in the (i,y)-space. Given that the policy of the ECB is conducted optimally we have to take into account that the inflation rate as well as the fiscal stance parameter serves as a shift factor in the (i,y)-space. Given the initial supply shock in country i the aggregate demand curve will shift due to the increase in economic activity by

[13] For an analysis within the classical AS/AD framework see Hagen and Mundschenk 2002.

$b\Delta p$ . This shift in economic activity is translated into a shift of the aggregate demand curve by a factor of $(1/n)\,b\Delta p$ .



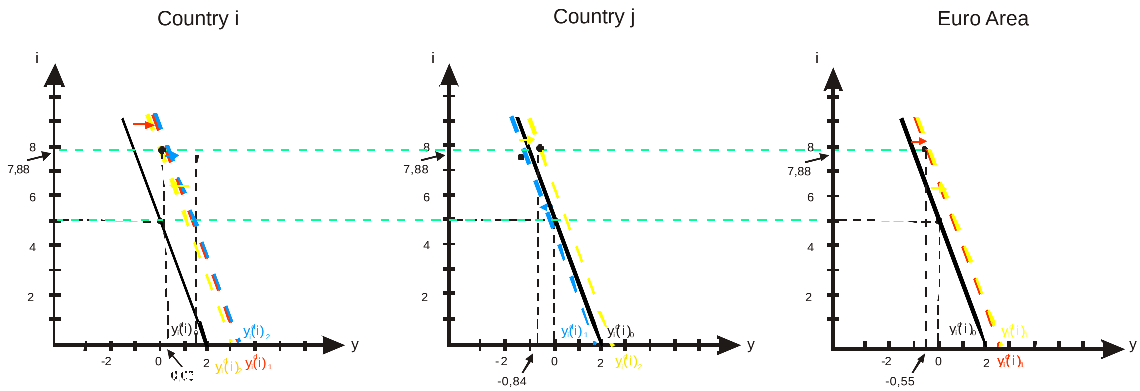

* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 26:** Idiosyncratic Supply Shocks in Country i: $e_{i,1} = 3$

Now the ECB steps in and chooses its preferred stabilisation mix taking the reaction of fiscal authorities appropriately into account. Given the ECB's preferences it will raise nominal interest rates and induce a stabilisation recession in order to minimize its loss function. This move by the ECB triggers an expansionary fiscal stance in the rest of the monetary union and a somewhat contractionary stance in country i. The overall policy outcome is depicted in Figure 27.

Finally to demonstrate the advantageous of a Keynesian stabilisation policy we can compute real interest rates for individual member states in the vague of asymmetric demand shocks. Making use of the reduced form the real interest rate for country i that was hit by the shock can be written as:

$$(i - p_i) = \frac{a}{b} + \frac{k^2 + (a - bd)\,j\,e_{i,1}}{b(k^2 + (1 - db)\,j\,)} \,. \qquad (3.63)$$




* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 27:** Idiosyncratic Supply Shocks in Country i: $e_{i,1} = 3$

With the help of equation (3.291) we can see that shocks will not be destabilising unless:

$$a \leq \frac{j\,bd - k^2}{j}. \tag{3.64}$$

Given our standard parameterisation this scenario can be virtually ruled out. Accordingly the analysis clearly demonstrates the advantageous of a Keynesian stabilisation policy that dramatically reduces the risk that shocks will be amplified. Table 4 shows the reaction of all variables under consideration to a positive shock.

| **National aggregates** | $e_1 > 0$ | $e_{i,1} > 0$ | $e_2 > 0$ | $e_{i,2} > 0$ |
|---|---|---|---|---|
| **Fiscal stance (g)** | ↑ | ↓ | ↓ | ↓ |
| **Output gap (y)** | ↓ | ↑ | ↓ | ↑ |
| **Inflation rate** (p) | ↑ | ↑ | ↑ | ↑ |
| **Interest rate (i)** | ↓ | ↑ | ↑ | ↑ |

**Table 4:** Net Change of a Variable to a Positive Shock

### 3.2.4 Idiosyncratic Phillips curves and TOT Effects in the IS-Equation

As in the previous section we assume that the PPP does not hold in the short term. Therefore we include TOT effects in the IS equation as in section 2.3. Accordingly the IS-equation can be stated as follows:

$$y_i = a - b(i - p_i) + c_i \Delta q + e_{i,1}\,. \tag{3.65}$$

As in the previous sections we now assume that the government in country i is guided by the following loss function:

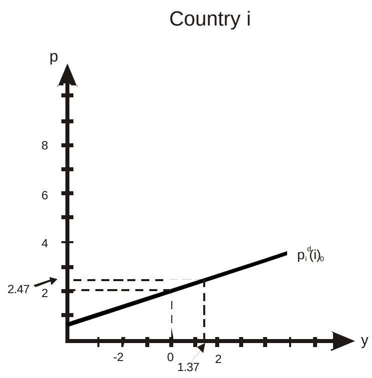
$$L_{G,i} = y_i^2 + j\, g_i^2 \tag{3.66}$$

s.t.

$$y_i = a - b\left(i - p_i\right) + c\Delta q + k\, g_i + e_{i,1}\,.$$

As the analytical results are somewhat too lengthy we only present numerical results for this scenario. We adopt the following solving strategy to compute the Nash equilibrium. In a first step we compute the reaction functions of fiscal authorities where we have substituted out the inflation rates by making use of the national Phillips curves. Then we substitute these reaction functions into the IS-equation and get a reduced form expression for the individual output gaps. With the output gaps at hand we can compute the inflation rates for the individual countries and all other variables of interest. The following section presents the results.

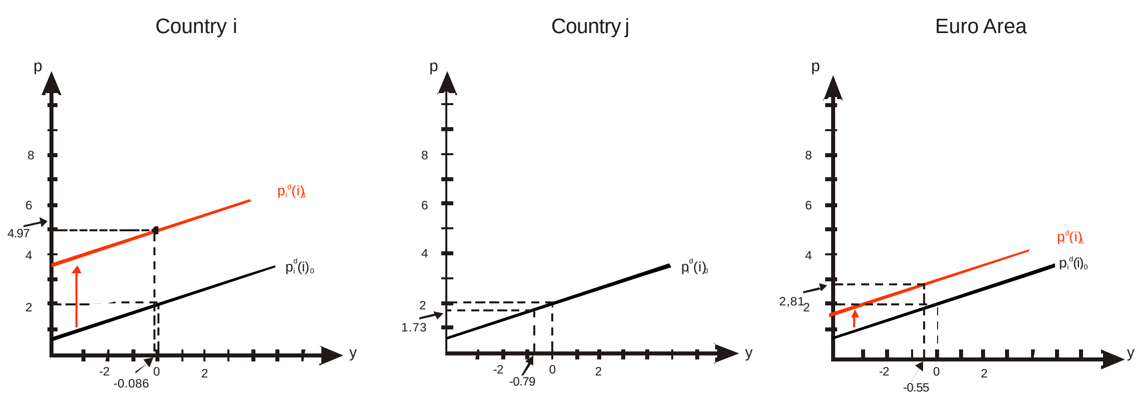
Figure 28 depicts the scenario if country i is hit by a demand shock of size three. Note in particular that compared to the previous scenario the TOT effects are partially stabilizing as the amplitudes of macroeconomic dispersion across the currency area are not as pronounced as in the previous scenarios. Nevertheless even including the TOT effects the economic forces triggered by diverging real interest rates are still dominant as Germany would boom whereas the rest of the union suffers from a mild depression. As we have seen throughout the chapter many times before the economic situation is characterized by a mismatch of real interest rate conditions as in non of the described countries real interest rates are in line with the economic environment. Nevertheless as in the scenarios beforehand the ECB is able to adjust the monetary environment on average so that it will attain its targets.




* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 28:** Idiosyncratic Demand Shocks in Country i: $e_{i,1} = 3$

Figure 29 depicts the ideal outcome from a stabilisation perspective. It shows that the ECB is able to realise its favourite macroeconomic outcome. Nevertheless TOT effects in combination with national fiscal policies that aim at stabilising the output gap almost completely succeed in stabilising the output gap in the country that was the source of macroeconomic turbulence. The macroeconomic outcome is very comparable to the case where TOT effects were not present. Hence in sum we conclude that the introduction of TOT effects does neither qualitatively nor qualitatively alter the results if we analyse the macroeconomic interaction between two large blocs of a monetary union.




* Note that the figure maps the situation in which the monetary union consists of three countries of equal size. For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 and d=0.34.

**Figure 29** Idiosyncratic Supply Shock in Country i: $e_{i,2} = 3$

## 3.3 CONCLUSIONS

In this chapter we applied a static version of a New Keynesian macromodel a la Clarida, Gali and Gertler (1999) to a currency union. We focussed in particular on the impact of asymmetric shocks and the integration of product markets and its implication for the functioning of a currency union. Our results are very easy to state: Life within a monetary union is much easier if shocks are highly correlated and product markets are integrated. Under such a scenario shocks are unlikely to be amplified across individual member states as the ECB can within an inflation targeting regime easily deal with them. Additionally we find that in particular small countries are in a vulnerable position as the ECB almost neglects their idiosyncratic situations unless shocks are correlated. This is of course a strong argument for a Keynesian stabilisation policy that actively fights shocks to stabilise economic activity. We showed that by this very argument one can provide a strong rationale for the suspension of the 3% deficit criterion in the vague of strong asymmetric demand and supply shocks that hit individual countries as a necessary precondition to restore optimal outcomes. Our analysis showed that in order to avoid negative spill-over effects stringent rules are necessary in order to prevent national governments as well as national labour unions to conduct a beggar-my-neighbour policy. Therefore the fathers of the Stability and Growth Pact (SGP) were right to implement rules that endorse a sustainable fiscal stance in each member state. We have shown numerically that these results are not qualitatively and quantitatively altered if we include TOT effects for a large open economy like Germany. From a theoretical perspective we have extended Dixits and Lambertinis (2003). They entirely focus on the issue of time consistency. Thereby they neglect to analyze the beneficial impact of stabilization policy if the union is hit by symmetric or asymmetric shocks. To that extend we extend Dixits and Lambertinis joint commitment solution to the case where the common monetary union is hit by symmetric or asymmetric supply and demand shocks.

# Appendix: Some Alternative Scenarios

## 3. A: PHILLIPS CURVE WITH TRADABLE AND NON- TRADABLE SECTOR

Let us discuss a third scenario which nests the two previously derived solutions as corner cases. We assume that each country has a tradable and a non-tradable sector. Therefore the consumer price inflation is given by a weighted average of the two product bundles:

$$p_i^{CPI} = a p^T + (1 - a) p_i^{NT} . \tag{3.A.1}$$

In each sector- tradables and nontradables- the inflation rate is determined by the difference between increases in nominal wages minus productivity:

$$p_i = w_i - prod_i + e_i \; . \tag{3.A.2}$$

It is generally assumed that the productivity growth $q_i$ in those sectors that face international competition is larger than in those sectors that only produce for domestic markets, hence $q_i > v_i$. To simplify the exposition we assume that in each sector wages are negotiated separately. Very much in line with a static version of Fuhrer and Moore (1995) we assume that the nominal wage is determined as:

$$w_i^T - q_i = p_0 + dy \tag{3.A.3}$$

$$w_i^{NT} - v_i = p_0 + dy_i \, . \tag{3.A.4}$$

Hence the union in each sector negotiates wages above productivity that are consistent with the inflation target of the ECB. Additionally workers wages depend on the state of the cycle. It seems plausible to assume that wage changes depend on overall activity as the sector specific characteristics are already taken into account by $q_i$ and $v_i$. Sectors that face international competition are assumed to depend on the overall cycle in the union whereas wage demands for non-tradables are orientated on domestic markets.

$$p^{T} = p_0 + dy + e_2^{T} \tag{3.A.5}$$

$$p_i^{NT} = p_0 + dy_i + e_{i,2}^{NT} \; . \tag{3.A.6}$$

Inserting leads to the following expression for the consumer inflation rate:

$$p_i^{CPI} = p_0 + a\,dy + \left(1 - a\right) dy_i + e_{i,2} \tag{3.A.7}$$

$$\text{with: } e_{i,2} = a e_2^{T} + \left(1 - a\right) e_{i,2}^{NT}$$

Note that this specification nests the two corner solutions discussed in section 3.1 and section 3.2. If the law of one price holds (α=1), the Phillips curve is given by:

$$p_i^{CPI} = p_0 + dy + e_2^{T} \; . \tag{3.A.8}$$

If each country only produces a non-tradable commodity bundle (α=0), the Phillips curve can be depicted as:

$$p_i^{CPI} = p_0 + dy_i + e_{i,2}^{NT} \; . \tag{3.A.9}$$

Now we turn to the specification of the aggregate demand side. The static version of the usual IS-equation can be specified as in the previous sections:

$$y_i = a - b\left(i - p_i^{CPI}\right) + k\,g_i + e_1 \; . \tag{3.A.10}$$

In each member state the political party in power solves the following optimisation problem:

$$L_{G,i} = y_i^2 + j\,g_i^2 \tag{3.A.11}$$

s.t.:

$$y_i = a - b\left(i - p_i^{CPI}\right) + k\,g_i + e_{i,1} \, . \tag{3.A.12}$$

Solving gives the following reaction function:

$$g_i = -\frac{k}{k^2 + j}\left(-a + b(i - p_i^{CPI}) - e_{1,i}\right). \tag{3.A.13}$$

The union wide output gap is given by:

$$y = -\frac{d\left(ae^T - (a-1)e^{NT}\right)}{d^2 + l}. \tag{3.A.14}$$

The union wide inflation rate is given by:

$$p = p_0 - \frac{d^2 ae^T + (a-1)\, le^{NT}}{d^2 + l} + ae^T. \tag{3.A.15}$$

The reaction function of the interest rate is given by:

$$i = p_0 + \frac{a}{b} + \frac{k}{b}g + \frac{\left(d - bd^2\right)a + ba\left(d^2 + l\right)}{b\left(d^2 + l\right)}e_T + \frac{(1-a)(d + bl)}{b\left(d^2 + l\right)}e_{NT} + \frac{1}{b}e_1. \tag{3.A.16}$$

which underlines that the interest rate setting behaviour is equal under the two scenarios previously considered. This result cannot come as a surprise as the ECB only reacts to euro-wide averages, which are identical under the two scenarios as the shocks are iid. This underlines that the behaviour of the ECB remains unaltered.

$$i = p_0 + \frac{a}{b} + \frac{d\left(k^2 + j - bdj\right)a + ba\left(d^2 + l\right)j}{b\left(d^2 + l\right)j}e_T + \frac{(1-a)dk^2 + (1-a)dj + (1-a)blj}{b\left(d^2 + l\right)j}e_{NT} + \frac{1}{b}e_1 \tag{3.A.17}$$

Applying the usual solving strategy we get the following reduced form equations:

$$p_i^{CPi} = p_0 + \frac{1}{\left(d^2+l\right)\left(k^2+\left(1+bd\left(a-1\right)\right)j\right)} \left[ \begin{array}{l} \left(-bd^3\left(a-1\right)j + l\left(k^2+j\right)\right)ae_T \\ +\left(a-1\right)\left( \begin{array}{l} -d\left(d^2+l\right)je_1 + d\left(d^2+l\right)je_1 \\ d^2k^2e_{NT} + d^2je_{NT} + bdlje_{NT} - bdalje_{NT} \\ -d^2k^2e_{NTi} - k^2le_{NTi} - k^2le_{NTi} - d^2je_{NTi} \\ -lje_{NTi} + bda\left(d^2+l\right)je_T \end{array} \right) \end{array} \right]$$

(3.A.18)

Output gap:

$$y_i = \frac{1}{\left(d^2+l\right)\left(k^2+\left(1+bd\left(a-1\right)\right)j\right)} \left[ \begin{array}{l} \left(bd^2\left(a-2\right)j - blj + d\left(k^2+j\right)\right)ae_T \\ \left(d^2+l\right)j\left(e_{i,1}-e_1\right) + \left(dk^2 - dak^2\right)e_{NT} \\ +dje_{NT} - daje_{NT} + bd^2aje_{NT} - bd^2a^2je_{NT} \\ +blje_{NT} - balje_{NT} - bd^2je_{NTi} + bd^2aje_{NTi} \\ -blje_{NTi} - blje_{NTi} + ba\left(d^2+l\right)je_T \end{array} \right]. \quad (3.A.19)$$

Fiscal stance parameter, which nests the two corner solutions:

$$g_i = \frac{1}{\left(d^2+l\right)j\left(k^2+\left(1+bd(a-1)\right)j\right)} \left[ \begin{array}{l} k\left(\left(bd^2\left(a-2\right)j - blj + d\left(k^2+j\right)\right)ae_T\right) \\ -\left(d^2+l\right)je_1 + \left(d^2+l\right)je_3 + dk^2e_{NT} - dak^2e_{NT} \\ +dje_{NT} - daje_{NT} + bd^2aje_{NT} - bd^2a^2je_{NT} \\ +blje_{NT} - balje_{NT} + ba\left(d^2+l\right)je_T \end{array} \right] (A.1).$$

(3.A.20)

# 3. B: ALTERNATIVE ASSUMPTIOMS ON THE CONDUCT OF FISCAL POLICY

Of course each theoretical model critically depends on the assumptions one makes about the functioning of the economy. In order to check the robustness of our results we have derived throughout the main part of the text we want to alter our set of assumptions along two dimensions. First of all, we illustrate the effects of introducing the Fisher equation in the IS-curve instead of the real interest rate. Second of all, we analyze the impact if each government in country i internalizes its impact on the euro-wide inflation rate. To shorten the appendix we just calculate for each alternative assumption the most complicated case with monetary and fiscal policy interaction when the law of one price does not hold.

***<u>Introducing the Fisher Equation</u>***

Following other strands of literature (e.g. Uhlig (2002)) we introduce the Fisher equation into the IS-curve. The Fisher equation states:

$$i-\pi^{e}=r\,. \tag{3.B.1}$$

Making use of the Fisher equation we can restate the IS-curve as follows:

$$y=\alpha-\beta\left(i-\pi^{e}\right)+\kappa g_{i}+\varepsilon_{i,1} \tag{3.B.2}$$

In order to simplify the exposition we assume-without loss of generality- that the inflation target of the central bank is equal to zero $\left(\pi_{0}=0\right)$. Accordingly we can state the Phillips curve as follows:

$$\pi=\delta y+\varepsilon_{2} \tag{3.B.3}$$

Let us assume that the private sector builds rational expectations according to the following loss function:

$$\mathrm{L}=\left(\pi\left(\pi^{\mathrm{e}}\right)-\pi^{\mathrm{e}}\right)^{2}. \tag{3.B.4}$$

Hence the private sector is happy if it anticipates at the outset of the game the inflation rate correctly, which boils down to the following equation:

$$\pi^{e}=\pi_{0}\,. \tag{3.B.5}$$

Given this somewhat altered structure of the economy the ECB solves the following optimization problem subject to the aggregate Phillips curve:

$$L_{ECB}=\pi^{2}+\lambda_{y}y^{2} \tag{3.B.6}$$

which translates into the following average area wide output gap:

$$y = -\frac{d}{d^2 + l_y} e_2 \,. \tag{3.B.7}$$

Inserting the output gap into the Phillips curve yields the following expression for the inflation rate:

$$p = \frac{l}{d^2 + l_y} e_2 \,. \tag{3.B.8}$$

Making use of this assumption as well as on the timing of the game we arrive at the following interest rate equation:

$$i = \frac{a}{b} + \frac{1}{b} e_1 + \frac{d}{b\left(d^2 + l_y\right)} e_2 + \frac{k}{b} g \,. \tag{3.B.9}$$

Note that this equation is exactly equal to the one we derived in PART I of the book. This cannot come as a surprise as a nominal instrument rule that targets zero inflation should be identical to a monetary policy that targets the real interest rate. But let us now turn more importantly to the optimisation problem of fiscal authorities. Now the government faces the following optimisation problem:

$$L_{G,i} = y_i^2 + j\, g_i^2 \tag{3.B.10}$$

s.t.:

$$y_i = a - b\left(i - p_i^e\right) + k g_i + e_{i,1} \,. \tag{3.B.11}$$

Given the assumptions we have made on the private sector and the way according to which expectations are formed it holds that in each member state $p_i^e = 0$. Making use of this result the reaction function of fiscal policy can be stated as follows:

$$g_i = -\frac{ak}{k^2+j} + \frac{bk}{k^2+j} i - \frac{k}{k^2+j} e_{i,1}. \tag{3.B.12}$$

Taking expectations over the average fiscal stance parameter $g_i$ and inserting it into the reaction function of monetary policy we arrive at the following reduced form expression for the interest rate:

$$i = \frac{a}{b} + \frac{1}{b} e_1 + \frac{d\left(k^2+j\right)}{b\left(d^2+l_y\right)j} e_2. \tag{3.B.13}$$

This can of course be used to solve for the fiscal stance parameter,

$$g_i = \frac{k}{k^2+j}\left(e_1 - e_{1,i}\right) + \frac{kd}{j\left(d^2+l_y\right)} e_2 \tag{3.B.14}$$

the output gap in the individual member country i,

$$y_i = \frac{j}{\left(k^2+j\right)}\left(e_{i,1} - e_1\right) - \frac{d}{\left(d^2+l_y\right)} e_2 \tag{3.B.15}$$

and the corresponding inflation rate in member country i:

$$p_i = \frac{dj}{k^2+j}\left(e_{i,1} - e_1\right) + \frac{1}{d^2+l_y}\left[\left(d^2+l_y\right)e_{2,i} - d^2 e_2\right]. \tag{3.B.16}$$

In order to shortly evaluate the plausibility of the results one can see that if shocks are symmetrical $r\left(e_{i,1};e_1\right)=1$ and $r\left(e_{i,2};e_2\right)=1$ than the equations simplify to:

$$y_i = -\frac{d}{\left(d^2+l_y\right)} e_2 \tag{3.B.17}$$

$$p_i = \frac{l_y}{d^2+l_y} e_2. \tag{3.B.18}$$

As this setup may be a natural alternative to the structure of the economy as assumed throughout the main Part of the text let us give some comments on the results:

- Demand shocks only have an impact on the average macroeconomic outcomes even if they are not synchronized.
- In the absence of shocks the output gap will be equal to zero and the inflation rate will be equal to the inflation target.
- The model setup is internally consistent as in the case of synchronized supply and demand shocks the country specific equations boil down to the euro area equations.

Nevertheless one result is dramatically altered. As we assume that not the actual real interest rate matters but the expected real interest rate, real interest rates are de facto equal across countries. Hence we do have no longer the phenomenon that country specific real interest rates can drive a wedge between country specific macroeconomic outcomes. In the main part of the text we saw that a dispersion across national outcomes could be amplified by diverging real interest rate conditions. By assumption this scenario is ruled out if we replace the real interest rate by the Fisher equation as $p_i^e$ is always zero and additionally impose that the shocks are white noise. (Uhlig 2002) additionally makes an interesting point which can equally be retrieved within our model. The model presented in the paper and Uhlig's version of the model share the characteristic, that real interest rate volatility is larger if fiscal policy is conducted according to a discretionary policy compared to a scenario where it remains passive. This point can easily be seen by comparing the interest rate reaction as described by the equations (3.B.9) and (3.B.13). To state the case we compute a numerical example. Let us assume that the currency area is hit by a symmetric of size $e_2 = 1$.

The table shows that discretionary policy has the side effect of an increasing interest rate volatility in the case of an aggregate supply shock. Note that this result is rooted in the Nash-equilibrium as fiscal policy does not endogenize that monetary policy can implement its preferred stabilization mix by compensating the more expansionary fiscal stance by higher interest rates. This result might call for monetary leadership which could be implemented by means of a Stackelberg equilibrium.

**Table 3.B.1:** The Impact of Fiscal Policy on the Interest Rate Volatility

| | **FISCAL POLICY REMAINS PASSIVE** | | **DISCRETION** | |
|---|---|---|---|---|
| Shock | +1 | -1 | +1 | -1 |
| Output Gap | -0.55 | 0.55 | -0.55 | 0.55 |
| Inflation | 2.81 | 1.19 | 2.81 | 1.19 |
| Nominal Interest Rate | 7.19 | 2.81 | 7.88 | 2.12 |
| Real Interest Rate | 4.38 | 1.62 | 5.07 | 0.93 |
| Interest rate volatility | $3 \pm 1.38$ | | $3 \pm 2.07$ | |

***Alternative assumptions on the optimization problem of fiscal authorities***

In this part of the appendix we want to illustrate that the results derived in the main text are qualitatively the same, irrespectively whether we assume that the government in country i internalizes the Phillips curve. Internalizing the Phillips curve means that the government takes account for the effects its own actions have on the euro wide inflation rate. As in the previous sections we assume that the ECB solves the identical optimization problem:

$$L_{CB} = p^{2} + l\, y^{2} \tag{3.B.19}$$

s.t.

$$p = dy + e_{2}. \tag{3.B.20}$$

Using this setup we arrive at the following results:

$$p = p_{0} + \frac{l_{y}}{d^{2} + l_{y}} e_{2}, \tag{3.B.21}$$

this translates into the following output gap equation

$$y = -\frac{d}{d^{2} + l_{y}} e_{2}. \tag{3.B.22}$$

Which still translates into the following reaction function for monetary policy:

$$i=\frac{a}{b}+p_0+\frac{1}{b}e_1+\frac{b\,l_y+d}{b\left(d^2+l_y\right)}e_2+\frac{k}{b}g \qquad (3.B.23)$$

Now let us turn to fiscal policy: As a novelty compared to the main Part of the text we assume that the government in country i internalizes the effects of its individual actions on the euro-area wide inflation rates:

$$L_{G,i}=y_i^2+j\,g_i^2 \qquad (3.B.24)$$

s.t.:

$$y_i=a-b(i-p_i)+k\,g_i+e_{i,1} \qquad (3.B.25)$$

$$p_i=p_0+dy_i+e_{i,2}\,. \qquad (3.B.26)$$

Consolidating the constraint we can equally state the constraint of the optimization problem as follows:

$$y_i=\frac{a}{1-bd}+\frac{bp_0}{1-bd}-\frac{b}{1-bd}i+\frac{k}{1-bd}g_i+\frac{1}{1-bd}e_{1,i}+\frac{b}{1-bd}e_{2,i}\,. \qquad (3.B.27)$$

Given this somewhat altered optimization problem we arrive at the following reduced forms: For the interest rate

$$i=\frac{a}{b}+p_0+\frac{1}{b}e_1+\frac{-dk^2+(bd-1)\left(d+b\,l_y\right)j}{b\left(d^2+l_y\right)(bd-1)j}e_2\,, \qquad (3.B.28)$$

the fiscal stance parameter:

$$g_i=\frac{k}{k^2+(bd-1)^2j}\left(e_1-e_i\right)-\frac{kb}{k^2(bd-1)^2j}e_{i,2}+\frac{k\left(-dk^2+(bd-1)\left(d+b\,l_y\right)j\right)}{(bd-1)\left(d^2+l_y\right)j\left(k^2+(bd-1)^2j\right)}e_2\,, \qquad (3.B.29)$$

the reduced form output gap parameter:

$$y_i = -\frac{(bd-1)j}{k^2+(bd-1)^2 j}\left(e_{i,1}-e_1\right)+\frac{-dk^2+(bd-1)\left(d+bl_y\right)j}{\left(d^2+l_y\right)\left(k^2+(bd-1)^2 j\right)}e_2+\frac{(1-bd)b}{k^2+(bd-1)^2}e_{i,2}, \tag{3.B.30}$$

and the inflation rate:

$$p_i = p_0+\frac{d(1-bd)j}{k^2+(bd-1)^2 j}\left(e_1-e_{1,i}\right)+\left(\frac{db(1-bd)j}{\left(k^2+(bd-1)^2 j\right)}+1\right)e_{i,2}+\frac{d\left(-dk^2+(bd-1)\left(d+bl_y\right)j\right)}{\left(d^2+l_y\right)\left(k^2+(bd-1)^2 j\right)}e_2 \tag{3.B.31}$$

In order to shortly evaluate the plausibility of the results one can see that if shocks are symmetrical $r\left(e_{i,1};e_1\right)=1$ and $r\left(e_{i,2};e_2\right)=1$ the equations simplify to:

$$y_i = -\frac{d}{\left(d^2+l_y\right)}e_2 \tag{3.B.32}$$

$$p_i = \frac{l_y}{d^2+l_y}e_2 \tag{3.B.33}$$

<u>The following results stand out:</u>

- Demand shocks only have an impact on the overall results if demand shocks are not perfectly synchronized.
- In the absence of macroeconomic shocks the inflation rate is equal to the inflation target and the output gap is equal to zero.
- The results are qualitatively unaltered to the results derived in the main part of the text.

In order to compare the results somewhat deeper we compute the value for the reduced form coefficients given our standard calibrations in comparison to those derived in the main text. Without going into detail the tables demonstrate that the internalization of the aggregate inflation rate does not alter the quantitative results significantly.

**Table 3.B.2**: Fiscal Stance

$$g_i = q_1 (e_1 - e_i) + q_2 e_{i,2} + q_3 e_2$$

| | MAIN PART | APPENDIX |
|---|---|---|
| **$q_1$** | 0.733 | 0.802 |
| **$q_2$** | -0.293 | -2.143 |
| **$q_3$** | 0.846 | 1.4 |

Calibration: $b = 0.4; d = 0.34; l = 0.5; j = k = 0.5$

**Table 3.B.3**: Output Gap

$$y_i = q_1 (e_1 - e_i) + q_2 e_2 + q_3 e_{i,2}$$

| | MAIN PART | APPENDIX |
|---|---|---|
| **$q_1$** | 0.733 | 0.69314 |
| **$q_2$** | -0.846 | -0.319 |
| **$q_3$** | 0.293 | 0.867 |

Calibration: $b = 0.4; d = 0.34; l = 0.5; j = k = 0.5$

**Table 3.B.4:** Inflation Rate

$$p_i = p_0 + q_1 (e_1 - e_i) + q_2 e_{i,2} + q_3 e_2$$

| | MAIN PART | APPENDIX |
|---|---|---|
| **$q_1$** | 0.250 | 0.2357 |
| **$q_2$** | -0.287 | -0.282; |
| **$q_3$** | 1.10 | 1.094 |

Calibration: $b = 0.4; d = 0.34; l = 0.5; j = k = 0.5$

## 3. C: TABLES

**Table 3.C.1** GDP-Weights[14]:

| Country | EU11 |
|---|---|
| **Belgium** | 3.3 |
| **Germany** | 29.9 |
| **Greece** | 2.6 |
| **Spain** | 10.9 |
| **France** | 20.5 |
| **Ireland** | 1.3 |
| **Italy** | 19.2 |
| **Luxemburg** | 0.3 |
| **Netherlands** | 5.4 |
| **Austria** | 3.2 |
| **Portugal** | 2.1 |
| **Finland** | 1.6 |

Data were taken from (ECB 2003)

**Table 3.C.2 :** Figure 17

| | Country one | REST OF THE UNION | Euro Area | Initial Levels |
|---|---|---|---|---|
| **Interest Rate** | 7.5 | 7.5 | 7.5 | 5 |
| **Output Gap** | 2 | -1 | -1 | 0 |
| **Fiscal stance** | / | / | / | 0 |
| **Inflation Rate** | 2 | 2 | 2 | 2 |
| **Real Interest Rate** | 5.5 | 5.5 | 5.5 | 3 |

[14] For the sake of illustration we have used concrete numerical values. As baseline calibration we have set b=0.4 d=0.34 and $j = k = 0.5$.

**Table 3.C.3 :** Figure 18

| | Country one | REST OF THE UNION | Euro Area | Initial Levels |
|---|---|---|---|---|
| **Interest Rate** | 7.5 | 7.5 | 7.5 | 5 |
| **Output Gap** | 2.31 | -1.16 | -1.16 | 0 |
| **Fiscal stance** | 0 | 0 | 0 | 0 |
| **Inflation Rate** | 2.79 | 1.61 | 1.61 | 2 |
| **Real Interest Rate** | 4.71 | 5.89 | 5.89 | 3 |

**Table 3.C.4:** Figure 19

| | Country one | REST OF THE UNION | Euro Area | Initial Levels |
|---|---|---|---|---|
| **Interest Rate** | 7.19 | 7.19 | 7.19 | 5 |
| **Output Gap** | 1.68 | -1.02 | -1.02 | 0 |
| **Fiscal stance** | 0 | 0 | 0 | 0 |
| **Inflation Rate** | 5.15 | 1.68 | 1.66 | 2 |
| **Real Interest Rate** | 2.07 | 5.53 | 5.53 | 3 |

**Table 3.C.5 :** Figure 20

| | COUNTRY ONE | REST OF THE UNION | EURO -AREA | INITIAL LEVELS |
|---|---|---|---|---|
| **INTEREST RATE** | 7.5 | 7.5 | 7.5 | 5 |
| **OUTPUT GAP** | 2.33 | -1.14 | 0 | 0 |
| **FISCAL STANCE** | 0 | 0 | 0 | 0 |
| **INFLATION RATE** | 2.79 | 1.62 | 2 | 2 |
| **REAL INTEREST RATE** | 4.71 | 5.88 | 5.5 | 3 |

**Table 3.C.6 :** Figure 22

| | Country one | Rest of the union | Euro-Area | Initial Levels |
|---|---|---|---|---|
| **Interest Rate** | 7.19 | 7.19 | 7.19 | 5 |
| **Output gap** | 0.22 | -0.93 | -0.55 | 0 |
| **Fiscal Stance** | 0 | 0 | 0 | 0 |
| **Inflation rate** | 5.07 | 1.71 | 2.81 | 2 |
| **Real Interest Rate** | 2.12 | 5.51 | 4.36 | 3 |

**Table 3.C.7 :** Figure 24

| | Country one | Rest of the union | Euro-Area | Initial Levels |
|---|---|---|---|---|
| **Interest Rate** | 7.5 | 7.5 | 7.5 | 5 |
| **Output Gap** | 1.33 | -0.66 | -0.66 | 0 |
| **Fiscal stance** | -1.33 | 0.66 | 0.66 | 0 |
| **Inflation Rate** | 2 | 2 | 2 | 2 |
| **Real Interest Rate** | 5.5 | 5.5 | 5.5 | 3 |

**Table 3.C.8 :** Figure 25

| | Country one | Rest of the union | Euro-Area | Initial Levels |
|---|---|---|---|---|
| **Interest Rate** | 7.5 | 7.5 | 7.5 | 5 |
| **Output Gap** | 1.46 | -0.73 | -0.73 | 0 |
| **Fiscal stance** | -1.46 | 0.73 | 0.73 | 0 |
| **Inflation Rate** | 2.50 | 1.75 | 1.75 | 2 |
| **Real Interest Rate** | 5.00 | 5.75 | 5.75 | 3 |

**Table 3.C.9:** Figure 26/ Figure 27

| | Country one | REST OF THE UNION | Euro-Area | Initial Levels |
|---|---|---|---|---|
| **Interest Rate** | 7.88 | 7.88 | 7.88 | 5 |
| **Output Gap** | 0.03 | -0.84 | 0.85 | 0 |
| **Fiscal stance** | -0.46 | 0.42 | 0.42 | 0 |
| **Inflation Rate** | 5.01 | 1.71 | 1.71 | 2 |
| **Real Interest Rate** | 2.87 | 6.17 | 6.17 | 3 |

**Table 3.C.10:** Figure 28

| | COUNTRY ONE | REST OF THE UNION | EURO-AREA | INITIAL LEVELS |
|---|---|---|---|---|
| INTEREST RATE | 7.5 | 7.5 | 7.5 | 5 |
| OUTPUT GAP | 1.37 | -0.69 | 0 | 0 |
| FISCAL STANCE | -1.57 | 0.79 | 0 | 0 |
| INFLATION RATE | 2.47 | 1.77 | 2 | 2 |
| REAL INTEREST RATE | 5.03 | 5.73 | 5.5 | 3 |

**Table 3.C.11:** Figure 29

| | COUNTRY ONE | REST OF THE UNION | EURO-AREA | INITIAL LEVELS |
|---|---|---|---|---|
| INTEREST RATE | 7.88 | 7.88 | 7.88 | 5 |
| OUTPUT GAP | -0.086 | -0.79 | -0.55 | 0 |
| FISCAL STANCE | 0.07 | 0.83 | 0.58 | 0 |
| INFLATION RATE | 4.99 | 1.75 | 2.81 | 2 |
| REAL INTEREST RATE | 2.89 | 6.13 | 5.07 | 3 |

**Table 3.C.12:** Comparison of Impact Coefficients in the Vague of Idiosyncratic Demand Shocks: $e_{i,1}$

| | | Only Monetary Policy | | Monetary and Fiscal Policy | |
|---|---|---|---|---|---|
| | | **Law of one Price** | **Many Phillips Curves** | **Law of One Price** | **Many Phillips Curves** |
| **$y_i$** | **General** | 1 | $\frac{1}{1-db}$ | $\frac{j}{k^2+j}$ | $\frac{j}{k^2+(1-bd)j}$ |
| | **Calibrated** | 1 | 1.16 | 0.67 | 0.73 |
| **$g_i$** | **General** | / | / | $\frac{k}{k^2+j}$ | $\frac{k}{k^2+(1-bd)j}$ |
| | **Calibrated** | / | / | 0.67 | 0.73 |
| $p_i$ | **General** | / | $\frac{d}{1-bd}$ | / | $\frac{dj}{k^2+(1-bd)j}$ |
| | **Calibrated** | | 0.40 | | 0.25 |

**Table 3.C.13:** Comparison of Impact Coefficients in the Vague of Idiosyncratic Supply Shock: $e_{i,2}$

| | | Only Monetary Policy | | Monetary and Fiscal Policy | |
|---|---|---|---|---|---|
| | | **Law of one Price** | **Many Phillips Curves** | **Law of One Price** | **Many Phillips Curves** |
| **$y_i$** | **General** | / | $\frac{b}{(1-bd)}$ | / | $\frac{bj}{\left(k^2+(1-bd)j\right)}$ |
| | **Calibrated** | / | 0.46 | / | 0.29 |
| **$g_i$** | **General** | / | / | / | $-\frac{kb}{k^2+(1-bd)j}$ |
| | **Calibrated** | / | / | / | -0.29 |
| $p_i$ | **General** | / | $\frac{1}{1-bd}$ | / | $\frac{k^2+j}{k^2+(1-bd)j}$ |
| | **Calibrated** | | 1.16 | | 0.25 |

**Table 3.C.14**: Comparison of Impact Coefficients in the Vague of Global Supply Shocks: $e_2$

| | | **Only Monetary Policy** | | **Monetary and Fiscal Policy** | |
|---|---|---|---|---|---|
| | | **Law of one Price** | **Many Phillips Curves** | **Law of One Price** | **Many Phillips Curves** |
| $\mathbf{y_i}$ | **General** | $-\frac{d}{d^2+l_y}$ | $-\frac{d+bl_y}{(1-bd)\left(d^2+l_y\right)}$ | $-\frac{d}{d^2+l_y}$ | $-\frac{d}{d^2+l_y}$ |
| | **Calibrated** | -0.55 | -1.02 | -0.55 | -0.55 |
| $\mathbf{g_i}$ | **General** | / | / | $\frac{kd}{j\left(d^2+l_y\right)}$ | $\frac{kd}{j\left(d^2+l_y\right)}$ |
| | **Calibrated** | / | / | 0,55 | 0.55 |
| $p_i$ | **General** | $\frac{l_y}{d^2+l_y}$ | $\frac{l_y}{d^2+l_y}$ | $\frac{l_y}{d^2+l_y}$ | $\frac{l_y}{d^2+l_y}$ |
| | **Calibrated** | -0.81 | -0.81 | -0.81 | -0.81 |

# 4 MONETARY POLICY AND THE BUSINESS CYCLE IN CLOSED ECONOMIES AND MONETARY UNIONS: TWO APPLICATIONS

In chapter 2 and chapter 3 we have reviewed and extended the framework of New Keynesian macroeconomics. We have seen that the existence of nominal inertia has a fundamental impact on the functioning of the economy and the role macroeconomic stabilization agent's play. In particular it prevailed that nominal inertia leaves leverage for the central bank on the real interest rate by which it can steer the economy according to its preferences. In a monetary union we have shown as a contribution to literature that diverging real interest rates have the potential to destabilize the very stability of the currency area itself. This calls for a renaissance of fiscal policy from a stabilization perspective.

In this chapter we will apply the New Keynesian reduced form three equation apparatus to the data. We will discuss two separate topics. On the one hand we will estimate key parameters of a New Keynesian macromodel for closed economies, namely the USA and evaluate the implied mechanics of the model. This means that we analyze the implied autocorrelations and cross-correlations with respect to changes in key parameters like preference vectors of monetary policy and the degree of forward-lookingness of economic agents. By this analysis we gain insights into an economy with rational agents and nominal inertia. The second application focuses on the European monetary union. We will analyze the sense and nonsense of the SGP. This chapter extends the basic framework of chapter 3 to a dynamic setting. As we hold it to be unrealistic that real world fiscal policy is conducted by optimal control we replace it by a simple rule. After having analyzed the SGP we will make some broad guidelines along which we think it proofs necessary to rebuild the SGP.

This chapter is structured as follows. In the next section we will extend the basic New Keynesian framework to a quarterly setting. This means in particular that we will introduce a richer lag structure in the Phillips-curve and the IS-equation. Having specified the economic model we will show how to rewrite them in state space notation and explain in some depth the econometric estimation technique. Following the technical issues we will address the two above mentioned topics.

## 4.1 A New Keynesian Macro Model as a Vehicle to Model a Large Closed Economy and the European Monetary Union

In this section we will extend the basic New Keynesian three equation apparatus to a quarterly setting to realistically describe the data. This means in particular that we will augment the intertemporal Euler-equation and the Phillips curve by a richer lag structure. This seems necessary to generate enough persistence in order to be able to explain stylized facts like hump shaped responses of impulse responses following a demand or supply shock (see Walsh, (2003) ch. 1). Additionally we will amend the basic equations by open economy characteristics. Concerning the IS-equation we take care of possible international linkages of a country associated with the real exchange rate and the real interest rate channel. Additionally we will pay attention to direct spill over effects. With respect to the Phillips curve we will add imported inflation to model the effects of diverging inflation rates in a monetary union. In the following we will introduce two quarterly models, namely a closed economy and a monetary union model.

### 4.1.1 The Empirics of the New Keynesian Phillips Curve

As shown in section (2.1.1.5) a cornerstone of New Keynesian macromodels is the HNKPC (e.g., Jondeau and Bihan (2001); Roberts (1997); Sbordone (2002)). In its most sophisticated version (see section (2.1.1.5)) it can be stated as follows:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \tilde{\lambda} mc_t + \varepsilon_t . \tag{4.1}$$

$$\text{where: } \tilde{\lambda} = (1-\theta)(1-\beta\theta)(1-\omega)\xi\phi^{-1}$$
$$\gamma_f = \beta\theta\phi^{-1},\ \gamma_b = \omega\phi^{-1},\ \phi = \theta + \omega\left[1-\theta(1-\beta)\right]$$

As we will fit the Phillips curve to a quarterly data set we allow for a more generalized lag-structure of the following form:

$$p_t = g_f \sum_{k=0}^{K} E_{t-1} p_{t+k} + g_b \sum_{j=1}^{s} b_{pj} p_{t-j} + \sum_{i=1}^{n} b_{yi} y_{t-i} + e_t . \tag{4.2}$$

The current rate of inflation is explained by a weighted average of past and future inflation rates as well as the current and lagged value of the output gap. Equation (4.2) nests all possible specifications of the Phillips curve as outlined in section (2.1.1). If we set $g_f$ equal to null equation (4.2) is equal to a purely backward looking specification as proposed by Svensson and Rudebusch (1999).

| **Study** | **Phillips-Curve** | **Period Region** | **Estimation Method** |
|---|---|---|---|
| Castelnuovo (2003) | $p_t = 0.1 \cdot E_{t-1} p_{t+3} + 0.141 \cdot y_{t-1} + 0.9 \cdot [0.282 \cdot p_{t-1} - 0.025 \cdot p_{t-2} + 0.292 \cdot p_{t-3} + 0.385 \cdot p_{t-4}]$ | 1987Q3-2001 Q1 USA | Minimum Distance Estimation |
| Linde (2002) | $p_t = 0.463 E_t p_{t+1} + 0.72 p_{t-1} + 0.032 y_t + e_p$ | 1960Q1-1997Q4 USA | FIML |
| Söderlind et al. (2005) | $p_t = 0.1 \cdot E_{t-1} p_{t+3} + 0.13 \cdot y_{t-1} + 0.9 \cdot [0.67 \cdot p_{t-1} - 0.14 \cdot p_{t-2} + 0.4 \cdot p_{t-3} + 0.07 \cdot p_{t-4}]$ | 1987Q4-1999Q4 USA | Matching Moments |
| Domenech et al. (2001) | $p_t = 0.537 E_t p_{t+1} + 0.463 p_{t-1} + 0.063 y_{t-1}$ | 1986Q1-2000Q4 USA | GMM |
| Gali et al. (2001) | $p_t = 0.364 E_{t+1} p_{t+1} + 0.599 p_{t-1} + 0.02 mc_t$ | 1960:1-1994:4 Euro-Area | GMM |
| Jondeau et al (2001) | $p_t = 0.747 E_{t+1} p_{t+1} + 0.462 p_{t-1} + 0.037 mc_t$ | USA | GMM |
| Rudd et al (2001) | $p_t = 0.605 E_{t+1} p_{t+1} + 0.393 p_{t-1} - 0.000 y_t$ | 1960:Q1-1997 Q4 Euro-Area | GMM |
| Rudebusch (2000) | $p_t = 0.29 E_t p_{t+1} + 0.71 p_{t-1} + 0.13 y_t$ | 1968:3-1996:Q4 Euro-Area | OLS |
| Gali et al. (1999) | $p_t = 0.682 E_{t+1} p_{t+1} + 0.252 p_{t-1} + 0.037 mc_t$ | 1960:1-1994:4 Euro-Area | GMM |

*Note as most authors present a battery of estimates we have taken the ones which we considered as the most relevant ones.

**Table 5:** Hybrid Phillips Curves

Setting $g_b = g_f = 0.5$ yields the Fuhrer and Moore (1995) specification. If we set $g_f = 1$ we have a purely forward-looking NKPC where inflation only depends on expected future inflation like in Taylor's (1979), Calvo's (1983), and Clarida, Gali and Gertler's (1999) specification. No consensus has yet emerged up to which degree the price setting behaviour of economic agents is governed by forward-looking behaviour. Table 5 presents some evidence from estimated and calibrated 'baseline' versions for the USA and the Euro-Area. The presented baseline estimates of the degree of forward-lookingness vary from 0.1 to 0.75. This dispersion in estimates is somewhat inconvenient as the dynamics of the reduced form system depend critically on the true degree of forward and backward-lookingness embedded in the Phillips curve and the IS -equation.

In the second part of this chapter when we analyze the SGP we need to modify the closed economy NKPC. In order to describe the inflation dynamics in a monetary union we augment the HNKPC by the inflation rate that prevails in the rest of the union $\bar{p}_{-i,t}$ .

| Study | Identified Loss Function | Period | Estimation Method |
|---|---|---|---|
| Di Bartolomeo et al. (2003) | $p_t = g_b p_{t-1} + g_f E_t p_{t+1} + g_y \hat{y}_t + h q_t + e_t$ | 1994-2002 | GMM-Estmination |
| Leitemo et al. (2001) | $p_t = (1-g_q) p_t^d + g_q p_t^m$<br>$p_t^m = p_t + (q_t - q_{t-1})$<br>$p_t = g_b p_{t-1} + g_f E_{t-1} p_{t+1} + g_y E_{t-1} y_t + g_{qq} E_{t-1} q_t + e_t^p$ | Calibration | |
| Batini et al (2001) | $p_t = g_b p_{t-1} + g_f E_t p_{t+1} + g_y y_{t-1} + g_q \Delta\left( \frac{1}{4} \sum_{j=1}^{4} q_{t-j} \right) + e_t^p$ | Calibration | |
| Batini et al. (1999) | $p_t = g_b p_{t-1} + g_f p_{t+1} + g_y (y_t - y_{t-1}) + g_q \left[ (1-i_b) \Delta q_t + i_f E_t \Delta q_{t+1} \right] + e_t$ | - | Calibrated |

**Table 6:** Open Economy Phillips Curve

The basic idea for this open-economy version of a Phillips curve is as follows. When foreign inflation rates start to pick up, than domestic inflation rates will equally accelerate, as parts of the products that domestic agents purchase come from abroad.

$$p_{i,t} = g_f E_{i,t-1}\bar{p}_{i,t+k} + g_b \sum_{j=1}^{s} b_{pj} p_{i,t-j} + \sum_{i=1}^{n} b_{yi} y_{i,t-i} + x\bar{p}_{-i,t} + e_{i,t}\ . \tag{4.3}$$

Note, compared to related studies (see Table 6) we have directly implemented the foreign inflation rate in the Phillips curve as nominal exchange rate movements can be excluded as an independent source of real exchange rate movements in a monetary union, where the nominal exchange rate is fixed once and for all.

### 4.1.2 The Empirics of the New Keynesian IS-Curve

The second building bloc of New Keynesian macromodels is the intertemporal IS-equation. It gives a description of the demand side of the economy. The New Keynesian IS-curve is a relationship that relates the output gap negatively to the expected real interest rate and to tomorrow's output gap. As we have shown in section (2.1.1.5) state of the art hybrid IS-equations can be stated as follows

$$y_t = \frac{h}{1+h} y_{t-1} + \frac{1}{1+h} y_{t+1} - \frac{1-h}{(1+h)s}\left(i_t - p_{t+1}\right) + \frac{1-h}{(1+h)s}\left(x_t - x_{t+1}\right), \tag{4.4}$$

where h depicts the degree of habit formation in consumption. The stronger the representative household centres its consumption decisions on last period's consumption level the more inertial becomes the output gap. As we will apply the equation to a set of quarterly data we augment the equation by a richer lag structure[15]:

$$y_t = v_y E_{t-1} \sum_{i=1}^{n} b_{yi} y_{t+i} + \left(1 - v_y\right) \sum_{j=1}^{m} b_{yj} y_{t-j} - b_r v_r \left[\bar{i}_{t-1} - E_{t-1}\bar{p}_{t+3}\right] + h_t\ . \tag{4.5}$$

As in the case of the Phillips curve this very general specification nests the different types of Euler equations we have highlighted in section (2.1.2) as corner solution. Setting $v_y$ equal to zero equation (4.5) collapses to the case of a purely forward-looking intertemporal Euler

[15] Note the convention that $x_t = E_{t-1}x_t + h_t$ is an easy way to introduce an error term in equation (4.2) as the recent realization is defined as the expected realization $E_{t-1}x_t$ plus the expectational error term $h_t$.

equation, whereas in the case of $v_y$ equal to one we are left with a purely backward looking specification. Table 7 presents some baseline estimates for the IS-curve. Reviewing these studies there seems to crystallise a consensus that a substantial degree of backward lookingness is needed to fit the actual data. At least half of the economic agents are assumed to be backward-looking according to the reviewed studies.

| Study | Phillips-Curve | Period | Method |
|---|---|---|---|
| Castelnuovo (2003) | $y_t = 0.2 \cdot E_{t-1} y_{t+1}$ $+0.8 \cdot \left[1.229 \cdot y_{t-1} - 0.244 y_{t-2} - 0.073\left(i_{t-1} - E_{t-1}\bar{p}_{t+3}\right)\right]$ $+h_t$ | 1987:Q3-2001:Q1 | Minimum distance Estimation |
| Smets et al (2003) | $y_t = 0.41 E_t y_{t-1} + 0.588 y_{t-1} - 0.88\left(r_t - p_{t+1}\right) + h_t$ | | Bayesian econometrics |
| Söderlind et al. (2005) | $y_t = 0.5 \cdot E_{t-1} y_{t+1}$ $+0.5 \cdot \left[1.15 \cdot y_{t-1} - 0.27 y_{t-2} - 0.09\left(i_{t-1} - E_{t-1}\bar{p}_{t+3}\right)\right]$ $+h_t$ | 1987Q4-1999Q4 | Matching moments |
| Domenech et al. (2001) | $y_t = 0.499 E_t y_{t+1} + 0.488 y_{t-1} + 0.047 y_{t-2} - 1.09 y_{t-3}$ $+0.161 y_{t-4} - 0.08181 r_{t-2} - 0.00819 r_{t-3} + h_t$ | 1986Q1-2000Q4 | GMM |
| Smets et al (2003) | $y_t = 0.41 E_t y_{t-1} + 0.588 y_{t-1} - 0.88\left(r_t - p_{t+1}\right) + h_t$ | | Bayesian econometrics |

**Table 7:** Hybrid IS-Equations

In the second part of this chapter when we analyze the SGP we need an open economy specification of the closed economy IS-equation. In order to capture the international linkages we augment the IS-equation by the following features:

$$\Delta q_t = \left(\bar{p}_{-i,t} - \bar{p}_{i,t}\right) \tag{4.6}$$

$$ex_t^d = V y_{-i,t} \tag{4.7}$$

$$g_{1,t} = \bar{g}_1 - c\, y_{1,t} + l\, g_{1,t-1} + e_{1,t}^f . \tag{4.8}$$

Relationship (4.6) is the change in the real exchange rate as a measure for intra-European competitiveness. If foreign inflation rates are higher than domestic ones domestic products become more attractive and hence the output gap will be pushed above its potential until the new equilibrium is reached. With ex$^d$ (see equation (4.7)) we measure the excess demand that results if a foreign country has a boom in output, so that exports and hence economic activity start to accelerate. Equation (4.8) is the fiscal policy rule. Combining these equations we arrive at the following open economy IS-relationship:

$$y_{1,t} = v_f E_{1,t-1} \sum_{s=1}^{n} b_{iy s} y_{1,t+s} + v_b \sum_{j=1}^{m} b_{yj} y_{1,t-j} - b_r v_r \left[ \bar{i}_{t-1} - E_{1,t-1} \bar{p}_{1,t+3} \right] \\ + f g_{1,t} + i \Delta \bar{q}_{1,t-1} + j\, y_{2,t} + h_{1,t} . \qquad (4.9)$$

| Study | Identified Loss Function | Period | Estimation Method |
|---|---|---|---|
| Di Bartolomeo et al. (2003) | $y_t = v_b y_{t-1} + v_f E_t y_{t+1} - b_i \left( i_t - E_t p_{t+1} \right) + i q + h_t$ | 1994-2002 | GMM-Estmination |
| Leitemo et al. (2001)<br>Söderström (2001) | $y_t = b_y \left[ v_b y_{t-1} + v_f E_{t-1} y_{t+1} \right] - b \left( i_{t-1} - 4 E_{t-1} p_t^d \right) + i q_{t-1} + b_{yf} y_t^f + h_t$ | - | Calibration |
| Batini and Nelson (2000) | $y_t = v_f E_t y_{t+1} - b \left( i_t - E_t p_{t+1} \right) + b_q \left( \frac{1}{4} \sum_{j=1}^{4} q_{t-j} \right) + h_t$ | - | Calibration |

**Table 8:** Open economy IS-Equations

Compared to related studies (see Table 8) we have directly implemented the change in the real exchange rate in the IS-equation which is equal to the difference in national inflation rates as the nominal exchange rate is fixed in a monetary union.

### 4.1.3 The Empirics of Quadratic Loss Functions

The third building bloc of a New Keynesian model is a relationship depicting the way according to which monetary policy is conducted. In section (2.1.3) we have proposed the following intertemporal loss function:

$$L_t = E_0 \sum_{i=0}^{\infty} b^i \left\{ l_p p_{t+i}^2 + l_y y_{t+i}^2 \right\} . \qquad (4.10)$$

In empirical work it has proven to be necessary to augment the standard loss function by an interest smoothing term in order to be able to explain the data. Therefore we introduce interest rate smoothing as an independent goal of monetary policy. It is an observable fact that monetary policy is implemented gradually. Typically short-term rates are not changed by more but 25 or 50 basis points (see e.g., Martin and Salmon (1999)). In other words monetary authorities do not implement their desired interest rate target cold turkey but perform a gradual adjustment to the desired target level. This observable interest rate setting behaviour can be rationalised among others by the following argument: Policymaker's are confronted with three major types of uncertainties. Model uncertainty, parameter uncertainty and data uncertainty. It is well documented that each of these uncertainties tends to reduce the aggressiveness with which policymakers react with their instrument to the set of predetermined variables. In other words the coefficients in the optimal monetary policy rules are smaller in absolute values.

This automatically translates into a smoother interest rate setting behaviour. One straightforward way to introduce interest rate smoothing in the model is to introduce an additional term $\Delta i_t$ in the loss function that penalizes excessive movements in the interest rate. Given these goals of monetary policy we can state the loss function as follows (e.g., Svensson (2003)):

$$L_t = E_0 \sum_{i=0}^{\infty} b^k \left\{ l_p p_{t+i}^2 + l_y y_{t+i}^2 + l_{\Delta i} \Delta i_{t+i}^2 \right\}. \tag{4.11}$$

There are only a few studies available that try to pin down the true preferences $\left( l_p ; l_y ; l_{\Delta i} \right)$ of monetary policy makers for the US and the Euro-area. Reviewing these studies (see Table 9) there seems to emerge the following consensus: Central banks seem to put a higher weight on stabilising the inflation rate around the inflation target than stabilising output at its full capacity level. Additionally a high weight is put on interest rate smoothing. Output stabilisation only seems to play a minor role for the conduct of monetary policy. Note that we already gave an analytical explanation for this finding in section (2.1.3.2).

| Study | Identified Loss Function | Period/ Region | Estimation Method |
|---|---|---|---|
| Castelnuovo (2003) | $L_t = p_t^2 + 0.5y_t^2 + 0.5\Delta i_t^2$ | 1987-2001 USA | Minimum Distance |
| Woodford (2003) | $L_t = p_t^2 + 0.048y_t^2 + 0.077\Delta i_t^2$ | / USA | Estimation Second Order Approximation |
| Söderlind et al. (2002) | $L_t = p_t^2 + 0.1y_t^2 + 1.5\Delta i_t^2$ | 1987-1999 USA | Matching Moments |
| Dennis (2001) | $L_t = p_t^2 + 0.23y_t^2 + 12.3\Delta i_t^2$ | 1979-2000 FED | FIML |
| Bavero et. al. (2002) | $L_t = p_t^2 + 0.00125y_t^2 + 0.0085\Delta i_t^2$ | 1980-1998 FED | GMM, Euler Equation |
| Cecchetti et. al. (1999)) | $L_t = p_t^2 + 0.25y_t^2$ | 1987-1999 Germany | Slope of the Aggregate Supply Relationship |

**Table 9:** Loss Functions

As a summary statistic the following box collects the closed economy equations in a New Keynesian setting.

*Central bank is guided by the following period loss function*

$$L_t = l_p p_t^2 + l_y y_t^2 + l_{\Delta i}\Delta i_t^2$$

*Quarterly New Keynesian Phillips curve*

$$p_{1,t} = g_f E_{1,t-1}\bar{p}_{1,t+k} + g_b \sum_{j=1}^{s} b_{pj} p_{1,t-j} + \sum_{i=1}^{n} b_{yi} y_{1,t-i} + x\bar{p}_{2,t} + e_{1,t}$$

*Quarterly New Keynesian IS-curve*

$$\begin{aligned} y_{1,t} = {} & v_y E_{1,t-1} \sum_{s=1}^{n} b_{iys} y_{1,t+s} + (1 - v_y) \sum_{j=1}^{m} b_{yj} y_{1,t-j} - b_r v_r \left[ \bar{i}_{t-1} - E_{1,t-1}\bar{p}_{1,t+3} \right] \\ & + f g_{1,t} + i\bar{q}_{1,t-1} + j\,\bar{y}_{2,t} + h_{1,t} \end{aligned}$$

**Box 1:** New Keynesian Macromodel for a Closed Economy

### 4.1.4 The Current Setting of Fiscal Policy

With respect to the open economy monetary union part of this chapter we need to augment the model by fiscal policy in order to address the SGP. The institutional design of the European monetary union was heavily shaped by the „Delors Report“ that called for stringent rules for national fiscal policies as a prerequisite for an efficient functioning of monetary policy (see Bofinger (2003)). In particular the German side was anxious that individual member states could conduct an unsustainable fiscal policy that would trigger a chain reaction of higher average inflation and nominal interest rates for the rest of the union. Therefore the fathers of the SGP intended to design fiscal rules for national policymakers that prevented fiscal authorities itself from being a major source of economic disturbance (Canova and Pappa (2003); Canzoneri, Cumby and Diba (2002)). This was laid down in particular by the following two interrelated rules which are intended to serve as a firewall against myopic fiscal policymakers:

- The budget should be balanced over the cycle. If the economy is hit by a large shock the ratio of the current nominal balance to GDP should not exceed the 3%-line unless the economy is hit by a large shock.
- The debt to GDP ratio should be in the medium run close to or below 60%.

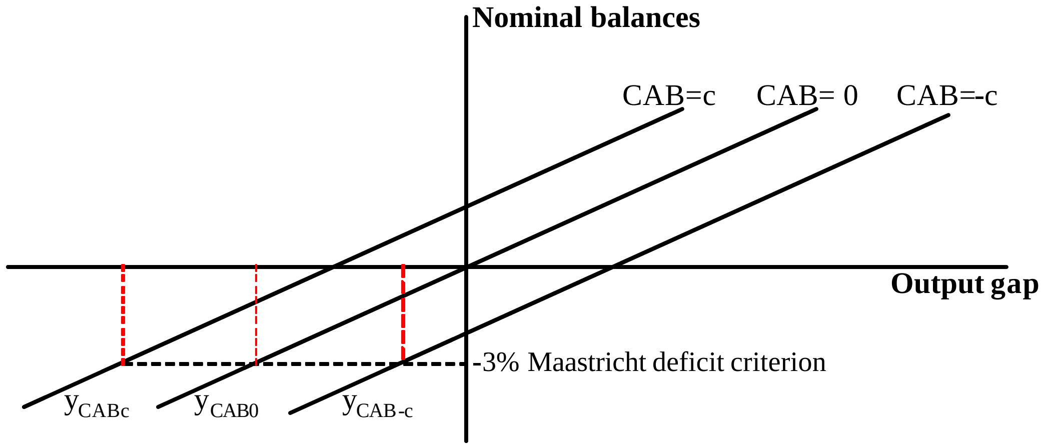
According to the SGP the cyclically adjusted balance should be balanced over the cycle. Nevertheless this does of course not rule out the possibility that the cyclically adjusted balance is used in a discretionary manner. As automatic stabilisers and discretionary fiscal policy are freely allowed to operate the definition of a sustainable fiscal policy combines at least from the perspective of the Commission long run sustainability with short run flexibility (EEAG, (2003))[16].

If the cyclically adjusted balance is zero on average, automatic stabilisers can freely operate and the likelihood that the 3%-deficit criterion will be broken is low (see Figure 30). Only if the economy is hit by a large shock so that $y<y_{CAB}$ the monitoring procedure will be triggered. Nevertheless if the cyclically adjusted balance is on average –c% small shocks are likely to run fiscal policy into troubled waters as normal output fluctuations already trigger the monitoring procedure[17]. Obviously a fiscal policy stance that exhibits a negative nominal

[16] Note that the European Council has recently agreed to modified the SGP (March 2005). In particular the conditions under which the 3% deficit criterion can be broken have been relaxed. Additionaly the period by which deficit violations have to be reversed have been relaxed substantially by the introduction of additional factors such as negative output gaps or the quality of public finances (see Bundesbank 2005).

[17] For an overview on the deficit procedure see ECB (2003) Monthly Bulletin Box 7, p. 58.

balance even if the output gap is zero increases the likelihood to break the 3%-deficit criterion in the vague of shocks.




Based on EEAG (2003), p.54.

**Figure 30:** The Actual Budget Balance as a Function of the Output Gap

In order to incorporate fiscal policy into our small scale macromodel we follow Taylor (2000) who has proposed for reasons of plausibility that US fiscal policy can be described by the following simple rule:

$$g_{1,t} = \bar{g}_1 - c\, y_{1,t} + e^{g}_{1,t}\,, \tag{4.12}$$

where $g_{1,t}$ denotes the nominal balance in percent of GDP. With $-c_1 y_{1,t}$ measuring the reaction of fiscal policy to the state of the cycle. The constant $\bar{g}_1$ depicts the structural fiscal balance over the sample period; $e^{g}_{1,t}$ denotes a fiscal spending shock. For the sample period 1983-1999 Taylor has estimated $c$ to be -0.37 and the constant was estimated to be 0.31. Hence Taylor makes the prediction that a decline in the output gap by 1% induces an increase in government financial deficit by 0.37 percent. Additionally Ballabriga and Martinez-Mongay (2002) proposed to introduce inertia in fiscal spending decisions:

$$g_{1,t} = \bar{g}_1 - c\, y_{1,t} + \breve{l}\, g_{1,t-1} + e^{f}_{1,t}\,. \tag{4.13}$$

*Central Bank is Guided by the Following Period Loss Function*

$$L_t = l_p \hat{p}_t^2 + l_y \hat{y}_t^2 + l_{\Delta i} n \Delta i_t^2$$

*The Area Wide Aggregates*

$$\hat{p}_t = v_1 p_{1,t} + v_2 p_{2,t} \quad \sum_{i=1}^{2} v_i = 1$$

$$\hat{y}_t = v_1 y_{1,t} + v_2 y_{2,t} \quad \sum_{i=1}^{2} v_i = 1$$

*Augmented Hybrid New Keynesian Phillips Curve*

$$p_{1,t} = g_f E_{1,t-1} \bar{p}_{1,t+k} + g_b \sum_{j=1}^{s} b_{pj} p_{1,t-j} + \sum_{i=1}^{n} b_{yi} y_{1,t-i} + x \bar{p}_{2,t} + e_{1,t}$$

$$p_{2,t} = g_f E_{2,t-1} \bar{p}_{2,t+k} + g_b \sum_{j=1}^{s} b_{pj} p_{2,t-j} + \sum_{i=1}^{n} b_{yi} y_{2,t-i} + x \bar{p}_{1,t} + e_{2,t}$$

*Augmented Hybrid New Keynesian IS-Curve*

$$\begin{aligned} y_{1,t} = {} & v_y E_{1,t-1} \sum_{s=1}^{n} b_{i,ys} y_{1,t+s} + \left(1 - v_y\right) \sum_{j=1}^{m} b_{yj} y_{1,t-j} - b_r v_r \left[ \bar{i}_{t-1} - E_{1,t-1} \bar{p}_{1,t+3} \right] \\ & + f g_{1,t} + i \bar{q}_{1,t-1} + j \bar{y}_{2,t} + h_{1,t} \end{aligned}$$

$$\begin{aligned} y_{2,t} = {} & v_y E_{2,t-1} \sum_{s=1}^{n} b_{2,ys} y_{2,t+s} + \left(1 - v_y\right) \sum_{j=1}^{m} b_{yj} y_{2,t-j} - b_r v_r \left[ \bar{i}_{t-1} - E_{t-1} \bar{p}_{2,t+3} \right] \\ & + f g_{2,t} + i \bar{q}_{2,t-1} + j \bar{y}_{1,t} + h_{2,t} \end{aligned}$$

*Fiscal Policy Rule*

$$g_{1,t} = \bar{g}_1 + f\left(-c \bar{y}_{1,t-1}\right) + \breve{l} g_{1,t-1} + e_{1,t}^{g}$$

$$g_{2,t} = \bar{g}_2 + f\left(-c \bar{y}_{2,t-1}\right) + \breve{l} g_{2,t-1} + e_{2,t}^{g}$$

*Change of the Real Exchange Rate*

$$\Delta \bar{q}_{1,t} = \left(\bar{p}_{2,t} - \bar{p}_{1,t}\right)$$

$$\Delta \bar{q}_{2,t} = \left(\bar{p}_{1,t} - \bar{p}_{2,t}\right)$$

**Box 2:** Open Economy New Keynesian Macromodel for a Monetary Union

Obviously this simple specification of fiscal policy does not disentangle whether the cyclical stance is automatic (automatic stabilizers) or intentional (discretionary policy). But as it is our aim to measure the overall impact of fiscal policy on the cycle this cannot come as a drawback. Throughout the paper we will not take debt smoothing as an independent goal of fiscal policy into account. Ballabriga and Martinez-Mongay (2002) have shown that the output gap is equally influenced by the level of debt. Auerbach (2002) comes to a similar finding for the USA as he reports that fiscal policy seems to respond systematically to both: cyclical factors and the fiscal balance during recent decades. As a summary box 2 collects the equations characterising the monetary union model.

## 4.2 Econometric Methodology

In this section we will show in some depth how to take the two models to the data. In a first step we will rewrite the two models in state space notation. The state space notation allows by standard software routines (see Söderlind (1999)) to solve for the rational expectations equilibrium of the model, to generate the impulse response functions and define relevant concepts of interest, such as variance-covariance matrices. Having rewritten the models in state space notation we will then estimate them by matching moments. Additionally we will shortly highlight the alghorithms applied to estimate the structural parameters of the model.

### 4.2.1 Rewriting the Model in State Space Notation[18]

Within this section we will set-up the general state space representation of the models. Let us assume that we can rewrite the model in the following generalized form (Söderlind 2003):

$$A_0 x_{t+1} = A_1 x_t + B_0 i_t + n_{t+1}. \tag{4.14}$$

Equation (4.14) can be rewritten as follows:

$$A_0 \begin{bmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{bmatrix} = A_1 \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + B_0 i_t + \begin{bmatrix} e_{1,t,t+1} \\ 0_{n_2 \times 1} \end{bmatrix}, \tag{4.15}$$

[18] All codes for basic computations were taken from Paul Söderlind homepage: http://www.hhs.se/personal/PSoderlind/Research/MonEEAMatLab.zip.

where $x_t$ is a $(n_1+n_2)\times 1$ vector of $n_1$ predetermined variables and $n_2$ "forward-looking" or "non-predetermined" variables. The shocks that drive the system are stacked into $e_{1t,t+1}$. Due to the specific model set up the variance-covariance matrix $\Sigma$ is diagonal. Therefore we interpret the individual shocks as structural shocks. The nominal short term interest rate $i_t$ is the instrument of the central bank. The matrices $A_0$, $A_1$ and $B_0$ denote the parameters of the model. Premultiplying equation (4.15) by $A_0$ it can equally be written as follows (see e.g.: Söderlind (1999); Svensson (1999)):

$$\begin{bmatrix} x_{1,t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + Bi_t + n_{t+1}. \tag{4.16}$$

Since $A_0$ is block diagonal with an identity matrix as its upper left block ($1{:}n_1;1{:}n_1$) and the lower block of ($n_1+1$, $n_1+n_2$) is zero it has to holds that:

$$A_0^{-1} \begin{bmatrix} e_{1,t+1} \\ 0_{n_2\times 1} \end{bmatrix} = \begin{bmatrix} e_{1,t+1} \\ 0_{n_2\times 1} \end{bmatrix}. \tag{4.17}$$

For details how to rewrite the two models (see Box 1 and Box 2) see appendix 4.A. Concerning the sequence of events the following holds true. At the start of period t $X_{1t}$, driven by the shock terms $e_t$ is realised. Then the central bank, conditional on the available information set $(e_t, X_{1t}, e_{t-1}, X_{t-1}, i_{t-1}, ...)$ chooses $i_t$. At the end of period t $X_{2t}$ results. Finally rational expectations on $E_t x_{2t+1}$ are formed on the available information at the end of period t. Note that one can solve for the rational expectations equilibrium as outlined by Backus and Driffil (1986) and Oudiz and Sachs (1985). Following Söderlind (2003) the rational expectations equilibrium shares the following characteristics (see Söderlind (2003), p.26).

- Although the policy maker reoptimizes each period it is possible to find each period stable time invariant stationary policy rule if one lets the algorithm iterate to infinity.
- Once the rational expectations equilibrium has been determined the rational expectations variables as well as the monetary policy rule can be expressed as a linear function of the state variables $x_{1t}$.

In order to initialize the state space routines we need to specify a measurement vector that defines the goal variables of monetary policy

$$z_t = C_x x_t + C_i i_t , \qquad (4.18)$$

with $x_t = \begin{bmatrix} x_{1,t} & x_{2,t} \end{bmatrix}$ and $C_x$ and $C_i$ are defined appropriately. Given this vector of target variables we can then define the following period loss function as stated in equation (4.18). By applying standard matrix algebra the loss function can be rewritten as:

$$\begin{aligned} L_t &= z_t' K z_t \\ &= \begin{bmatrix} x_t' & i_t' \end{bmatrix} \begin{bmatrix} C_x' \\ C_i' \end{bmatrix} K \begin{bmatrix} C_x & C_I \end{bmatrix} \begin{bmatrix} x_t \\ i_t \end{bmatrix}. \end{aligned} \qquad (4.19)$$

$$\text{with: } K = \begin{bmatrix} l_p & 0 & 0 \\ 0 & l_y & 0 \\ 0 & 0 & l_{\Delta i} \end{bmatrix}$$

Multiplying out equation (4.19) it holds that

$$\begin{aligned} L_t &= x_t' C_x' K C_x x_t + x_t' C_x' K C_i i_t + i_t' C_i' K C_x x_t + i_t' C_i' K C_i i_t \\ &= x_t' Q x_t + x_t' U i_t + i_t' U' x_t + i_t' R i_t \end{aligned} , \qquad (4.20)$$

where it holds that:

$$\begin{aligned} Q &= C_x' K C_x \\ U &= C_x' K C_i \; . \\ R &= C_i' K C_i \end{aligned}$$

Additionally we need to specify the unconditional variance-covariance matrix $\Sigma$ of the disturbance vector $e_{1,t+1}$.

Under the assumption that a rational expectations equilibrium exists it holds that the instrument of monetary policy can be expressed as a linear function of the predetermined state variables:

$$i_t = -F x_{1,t} . \qquad (4.21)$$

The closed loop dynamics of the model (the economy in conjunction with the policy rule) which serves as a starting point to generate diverse measures which we are interested in can be stated as follows:

$$x_{1,t} = \left(A_{11} + A_{12}C\right) x_{1,t} + e_{1,t} \tag{4.22}$$

$$x_{2,t} = Cx_{1,t}, \tag{4.23}$$

where $A_{11}$ and $A_{12}$ are the respective sub-matrices of $A = A_0^{-1}A_1$, which have been partitioned conformably with $x_{1,t}$ and $x_{2,t}$. Using the algorithms as described in Söderlind (1999), the matrix C which maps the predetermined into the non-predetermined variables is determined numerically. Equipped with equations (4.22) and (4.23) we can compute the variance-covariance matrix of the predetermined variables $x_{1,t}$ and the goal variables $z_t$:

$$vec(\Sigma_{x_{1t}x_{1t}}) = \left[I - M \otimes M\right]^{-1} vec(\Sigma) \tag{4.24}$$

$$\text{with: } M = \left(A_{11} + A_{12}C\right)$$

and

$$\Sigma_{zz} = E[z_t z_t^{'}] = \tilde{C}X_{1t}(\tilde{C}X_{1t})' = \tilde{C}\Sigma_{XX}\tilde{C}'. \tag{4.25}$$

$$\text{with } \tilde{C} = C_{X1} + C_{X2}C + C_iF$$

Note that the equations (4.24) and (4.25) are very useful as they allow us to compute the variances, covariances, autocorrelations and cross-correlation of the theoretical New Keynesian model implied by a particular parameter constellation.

### 4.2.2 Econometric Methodology

In this section we will present the estimation technique. The estimation is based on the following state space representation:

$$X_{t+1} = MX_t + u_{t+1}, \tag{4.26}$$

which is a short hand notation for equation (4.22) and (4.23). The closed loop dynamics of the model serves as a starting point to generate the variances, covariances and cross-correlations.

For matching the theoretical New Keynesian model to the data we have to estimate a set of parameters. For the closed economy model this set of parameters is given by:

$$V^{us} = \left( g_f \quad v_y \quad l_p \quad l_y \quad l_{\Delta i} \right). \tag{4.27}$$

We fit the closed economy US model to the term of Alan Greenspan (1987:4 – 2002:1). For the open economy part of this chapter we estimate an extended set of parameters to capture the international linkages:

$$V^{euro} = \left( g_f \quad v_y \quad l_p \quad l_y \quad l_{\Delta i} \quad i \quad c \quad d \quad x \quad j \right). \tag{4.28}$$

The euro-area model covers the time period starting from the soft European monetary system in 1983 to the second quarter of 2003. The synthetic European data set was provided by the ECB. The applied estimator $V$ minimises a distance measure $J(V)$ (see e.g. Christiano, Eichenbaum and Evans (2005))

$$J = \min_{V} \left( \hat{\Psi} - \Psi(V) \right)' V^{-1} \left( \hat{\Psi} - \Psi(V) \right), \tag{4.29}$$

where $\hat{\Psi}$ denotes the empirical sample moments and $\Psi(V)$ describes the mapping from $V$ to the theoretical sample moments of the New Keynesian model implied by that particular parameter constellation. Note that any positive semi definite matrix assures consistent estimates (see Verbeek, p. 135). The matrix V denotes the weighting matrix which we have set equal to the identity matrix. Hence we estimate both models by minimizing a quadratic norm between the theoretical moments (variances, sample correlations, cross correlations) of the New Keynesian model and the empirical sample moments which characterize the specific data sets. For a detailed definition of the individual criteria for the US-data (1987:4-2002:2) We additionally impose the restriction that the individual standard deviations of the goal variables should not display a greater percentage deviation but c from historical counterparts. We set c=0.5.

The open economy model was fitted to the sample autocorrelation function, where we opted for a lag length of twenty. Note that we can assume that following a macroeconomic shock it basically takes around 20 quarters before the initial shock is completely undone. To model the disequilibrium dynamics it seems sufficient to model a lag length of 20 quarters.

Note that the choice which parameters to estimate and which to calibrate is rarely to nowhere discussed in literature. Nevertheless common wisdom applies. Generally one should not try to estimate parameters which make no difference. In other words only if a variation in an element of $V$ is likely to have a significant impact on the value function $J$ we can expect to retrieve meaningful estimates. In other words it has to hold that:

$$\left|\frac{\Delta J}{\Delta V_{i,-i}}\right| >> 0\,. \tag{4.30}$$

If on contrary the value function is very flat with respect to large variations of a specific parameter it does not make sense to try to estimate that parameter as the concrete parameterization does not make a difference for the value function over a large interval. Therefore one says that a parameter is locally non identifiable. Dividing the set of parameters in those to be estimated and those to be calibrated we have relied on related studies by Christiano, Eichenbaum and Evans (2005). Concerning the group of calibrated parameters we proceed as follows. The backward looking inflation polynomial in the Phillips curve $a_{pi}$, the impact of economic activity on inflation $a_y$, the interest rate sensitivity of economic activity in the IS-curve $b_r$, and the autoregressive part in the output gap equation $b_{yi}$ were specified by estimates as reported by Rudebusch (2000) which are displayed in Table 10. Rudebusch (2000) used the following specifications: $p_t$ was specified as the quarterly inflation rate in the GDP chain-weighted price index $p_t$ seasonally adjusted and calculated at an annual rate $4\left(\ln P_t - \ln P_{t-1}\right)$; $\bar{p}_t$ is the four quarter moving average constructed as $\left(1/4\right)\sum_{i=0}^{3} p_{t-j}$ ; $\bar{i}_t$ is the four quarter average federal funds rate, hence $\frac{1}{4}\sum_{i=0}^{3} i_{t-j}$ ; $y_t$ is the output gap constructed as the percentage deviation of the output $Y_t$ from trend output $Y_t^*$, where $Y_t^*$ was taken from the Congressional Budget Office. All variables were demeaned prior to estimation. Note in particular that the specification as proposed by Rudebusch (2000) implies that the sum over the inflation polynomial ($\sum_{i=1}^{4} b_{pi} = 1$) is equal to one, so that the long run neutrality of money holds. This means in steady state ($p_t = p_{t-1} = p_{t-2} = p_{t-3} = \ldots = \bar{p}$ .) it holds that:

$$\bar{y} = \left(1 - \left[b_{p1} + b_{p2} + b_{p3} + b_{p4}\right]\right) a^{-1} p_0 . \qquad (4.31)$$

Obviously the property of long run neutrality is violated as long as $\left(b_{p1} + b_{p2} + b_{p3} + b_{p4}\right) \neq 1$. Higher inflation targets $p_0$ could boost output permanently, which would violate the long run neutrality of money. Thus, it is desirable to set the slope coefficient equal to one β=1, which translates into $\left(1 - b\right) a^{-1} = 0$. This is from an economic point of view somewhat problematic as $b$ should be interpreted as a discount factor.

| PARAMETER | SYMBOL | ESTIMATE |
|---|---|---|
| **Phillips Curve** | | |
| Inflation Polynom | $a_{p_1}$ | 0.67 |
| | $a_{p_2}$ | -0.14 |
| | $a_{p_3}$ | 0.4 |
| | $a_{p_4}$ | 0.07 |
| Output Coefficient | $a_y$ | 0.13 |
| **IS-Curve** | | |
| Output Polynom | $b_{y_1}$ | 1.15 |
| | $b_{y_2}$ | -0.27 |
| Interest Rate Elasticity | $b_r$ | -0.09 |
| **Fiscal Policy** | | |
| Structural fiscal balance | $\bar{g}_1$ | -1.8 |

**Table 10:** Parameter Calibration

Based on this partitioning Table 11 summarizes the estimated set of parameters $V$ that minimizes the distance measure (4.29).

For the closed economy US-model the estimates can be characterized as follows: The weight $\lambda_y$ on stabilising squared deviations of the output gap around zero is rather small compared to the weight put on the other two goal variables of monetary policy. It is well known that this does not mean that monetary policy does not care on the output gap. This is quickly confirmed if one takes a look at the optimal monetary policy rule which is given by:

$$i_t = 0.2947p_t + 0.1140p_{t-1} + 0.1169p_{t-2} + 0.0166p_{t-3} + 0.2348y_t + 0.0701y_{t-1} + 0.6391i_{t-1} \quad (4.32)$$

Hence monetary policy reacts on impact with an increase of 0.2348 to current changes in the output gap and with a coefficient of 0.0701 to changes in last period's output gap. This can be explained as follows: Even a central bank that only puts a modest weight on output stabilization opts to react on movements in economic activity in order not to loose control over the inflation rate as the output gap is the driving variable of the inflation process (e.g., Svensson (2003)).

| PARAMETER | SYMBOL | ESTIMATE | |
|---|---|---|---|
| **Degrees of forward lookingness** | | USA | Euro-Area |
| Phillips Curve | $g_f$ | 0.6 | 0.35 |
| IS Curve | $v_y$ | 0.4 | 0.24 |
| **Monetary Policy** | | | |
| Weight on inflation | $l_p$ | 1 | 1 |
| Weight on output | $l_y$ | 0.15 | 0.61 |
| Weight on interest rate smoothing | $l_{\Delta i}$ | 1.85 | 0.2836 |
| **Fiscal Policy** | | | |
| Automatic Stabilization | $c$ | / | -0.53 |
| **Other parameters** | | | |
| Demand Externalities | $j$ | / | 0.2586 |
| TOT effect in IS-equation | $i$ | / | 0.3144 |
| Imported Inflation | $x$ | / | 0.3090 |
| Fiscal Policy multiplier | $f$ | / | 0.3144 |

**Table 11:** Parameter Estimates

The finding that output gap stabilization only seems to be of minor importance as an independent goal of monetary policy is well in line with related studies that coherently come to the same result. The relatively high weight on financial market stability as an independent goal of monetary policy confirms earlier results by Dennis (2001) and Söderlind, Söderström and Vredin (2005). The high weight on interest rate smoothing is reflected in the optimal discretionary monetary policy rule as the coefficient on $i_{t-1}$ is equal to 0.6391. The degree of forward-lookingness in the Phillips curve is identified to be equal to 0.4. Hence 40% of economic agents seem to build rational expectations on the inflation rate whereas 60% set their prices based on rule of thumbs. This result lies in the midst of the estimates presented by related studies. Accordingly the estimation results give further evidence that purely forward-looking Phillips curves do not fit the facts. The degree of forward-lookingness in the IS-equation is estimated to be equal to 0.4. Hence only a modest degree of forward-lookingness seems to be present in the data, which confirms earlier results by Fuhrer (2000). In other words a purely forward-looking IS-equation is not able to describe the optimal consumption plan of households. Consumption decisions seem to be mainly driven by rule-of-thumb behaviour and habit formation. Households centre their current and future spending decisions on yesterday's consumption level or alternatively around some targeted level of consumption.

| RANK $\psi_i$ | LEVELS | | | | ONE-QUATER-CHANGES | | | |
|---|---|---|---|---|---|---|---|---|
| | STANDARD DEVIATION | AC(1) | AC(2) | AC(3) | STANDARD DEVIATION | AC(1) | AC(2) | AC(3) |
| | | | | **Inflation** | | | | |
| **Data** | **0.9794** | **0.649** | **0.514** | **0.585** | **0.8105** | **-0.32** | **-0.283** | **0.101** |
| Fitted | 1.4668 | 0.6733 | 0.4970 | 0.4938 | 1.1857 | -0.2303 | -0.2649 | |
| | | | | **Output Gap** | | | | |
| **Data** | **1.6953** | **0.945** | **0.865** | **0.755** | **0.5462** | **0.28** | **0.278** | **0.049** |
| Fitted | 1.4906 | 0.7823 | | | 0.9837 | | | |
| | | | | **Federal Funds Rate** | | | | |
| **Data** | **1.9326** | **0.930** | **0.814** | **0.671** | **0.5365** | **0.58** | **0.303** | **0.191** |
| Fitted | 1.5113 | 0.58 | 0.303 | 0.191 | 0.4626 | 0.5873 | 0.2810 | |

**Table 12:** Time Series Properties: Simulated and Actual Data: (1987:4-2002:1)

As Table 12 indicates, the estimated vector $V^{US} = [1 \quad 0.15 \quad 1.85 \quad 0.4 \quad 0.4 \quad 1]$ captures the correct signs of the autocorrelation functions over all relevant variables. Nevertheless the model has some problems in displaying the low variance in the inflation rate and the low variance in the first difference of the output gap.

Qualitatively the estimation results retrieved for the euro-area model are very comparable. In particular it prevails that the number of backward-looking agents is more important than the number of forward-looking households and firms. Additionally monetary policy puts a higher weight on stabilizing the inflation rate than stabilizing the output gap. Nevertheless the weight on output stabilization seems to be more important for the euro-area sample than for the term of Alan Greenspan. Of course one should keep in mind that we rely for the euro-area on a synthetic data set starting in 1983. As Italy had average inflation rates of 9.6%, Spain of 9.3%

and Greece of 19% in the 1980's, there is of course a bias towards persistent inflation cycles in the data sample. Nevertheless it is common practice to estimate Euro-area parameters on long data samples. Related studies like Smets and Wouters (2005) use data samples ranging back to the first quarter of 1974. Note that the model fits the inflation and interest dynamics quite well whereas it had some problems in capturing the output gap dynamics (see Figure 31). The open economy parameters are in line with those reported in literature. The estimated parameter for automatic stabilization is with a point estimate of -0.53 close to the value proposed by Aarle, Bartolomeo, Engwerda and Plasmans (2002) who have calibrated $c$ equal to $c = 0.5$.

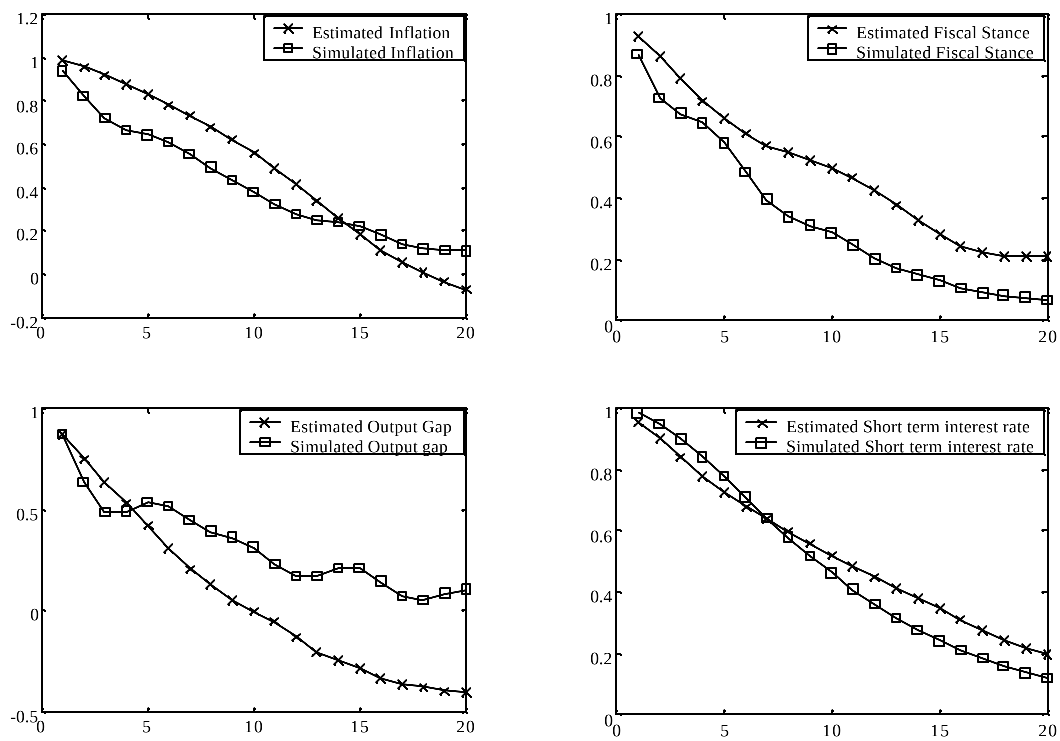



**Figure 31:** Minimum Distance Estimation by Matching Theoretical to the Empirical SACF

## 4.2.3 Excursus: A Note on the Applied Algorithm and Determinacy

In this section we will shortly highlight the implemented algorithms. Like in many applications there is a trade-off between robustness and speed. Depending on the overall computational task we have chosen the appropriate algorithm. For the closed economy part the computational burden is manageable so that we have opted to implement a tight grid-search method. The overall computational task can be handled within a few hours. In the case of the open economy model grid search algorithms are not feasible due to the excessive computational burden. Therefore we applied more sophisticated techniques like Nelder-Mead search algorithms. Additionally we will show that both estimates are stable and determinate which assures that we have found two unique rational expectations equilibria.

#### 4.2.3.1 Closed Economy Model

For the closed economy model we have applied a simple but robust method, namely a grid search algorithm. This method is robust but time consuming. Nevertheless as we only try to estimate in total five parameters it is still feasible. We iterate the individual parameters over the following ranges:

- $g_f$ In the interval from 0 to 1 with step size 0.1.
- $v_y$ From 0 to 1 with step size 0.1.
- $v_r$ Was set alternatively equal to null or one.
- $l_y$ In the interval from 0 to 4 with step size 0.05.
- $l_{\Delta i}$ In the interval from 0 to 6 with step size 0.1.

This procedure generates a total of 960,000 constellations for the value function. We have chosen the one that produces the global minimum $J^{ot}$ within the grid. Note that this procedure is explicitly based on the assumption that the underlying value function is well behaved. This assumes of course that the grid is reasonably dense so that we hit the global optimum sufficiently close. Nevertheless under the assumption that the quadratic value function $J(V)$ is sufficiently well behaved we have a priori no reason to believe that we miss the optimum by a large scale.

#### 4.2.3.2 Euro-Area Model

In the open economy part of the chapter we estimate in total 10 coefficients simultaneously. Therefore a grid based procedure as applied in the closed economy part of the chapter is no longer feasible. Applying a sufficiently accurate grid would mean that we would have in total

96,000,000,000 parameter constellations to compute. This would hardly be feasible in terms of time consumption. Therefore we have opted to estimate the open economy part of the chapter by faster algorithms that do not scan the whole value function within a prespecified space but rely on more sophisticated techniques. To initialize the minimum distance estimator we apply the following two step procedure:

$$1. \quad \begin{aligned} &\min J\left(V_i \,|\, V_{-i}\right) \\ &\forall i : V_i \in \left[V_{i,\min}; V_{i,\max}\right] \end{aligned} \tag{4.33}$$

$$2. \quad \min_{V_i^{prior}} J\left(V_i^{prior}\right). \tag{4.34}$$

In the first step we try to obtain good starting values for the algorithm. Therefore we fix all elements in the vector $V_i$ except one. We minimize the function $J\left(V_i \,|\, V_{-i}\right)$ on the bounded interval $\left[V_{\min}; V_{\max}\right]$. The formulated priors for the individual parameters as well as the specific bounds are formulated on the basis of plausibility. By this procedure we retrieve a vector $V^{prior}$. In a second step we use these optimized priors to estimate the global optimum letting all parameters in $V_{opt}^{prior}$ variable. The concrete optimization was performed by a multidimensional unconstraint non-linear minimization procedure. The applied algorithm is a so called derivative free Nelder-Mead algorithm. The Nelder Mead algorithm is based on simplex transformations. For details see Mirand and Fackler (2002), pp.62-64.

### 4.2.3.3 Technical Equilibrium Analysis[19?]

Following Blanchard and Khan (1980) we test for uniqueness and stability by computing the eigenvalues. It has to hold that the number of unstable eigenvalues is equal to the number of forward-looking variables. A look at the partitioned state vector tells us that the number of predetermined variables is equal to nine. The number of forward-looking variables is equal to four:

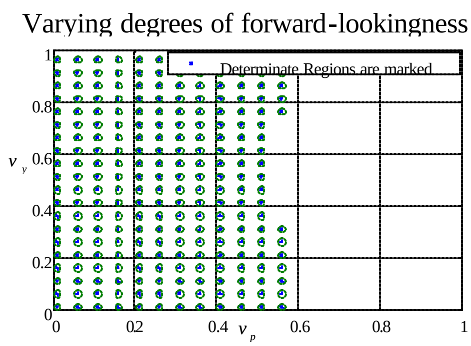
- $X_{1t} = \{p_t, p_{t-1}, p_{t-2}, p_{t-3}, y_t, y_{t-1}, i_{t-1}, i_{t-2}, i_{t-3}\}'$

[19] All codes for basic computations were taken from Paul Söderlind homepage: http://www.hhs.se/personal/PSoderlind/Research/MonEEAMatLab.zip

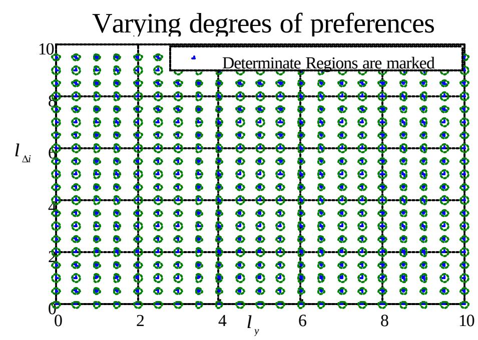
- $X_{2t} = \{E_t p_{t+3}, E_t p_{t+2}, E_t p_{t+1}, E_t y_{t+1}\}'$.

Figure 32 confirms that for $V^{US} = [1 \quad 0.15 \quad 1.85 \quad 0.4 \quad 0.4 \quad 1]$ the number of forward-looking variables satisfies the proposition as stated by Blanchard and Khan (1980). Hence we conclude that the identified baseline configuration $V^{US}$ generates a stable and unique solution.

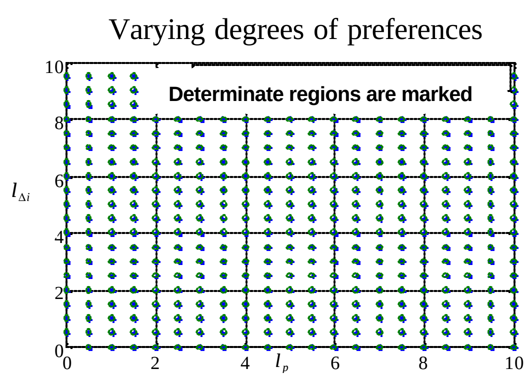
(a)




(b)




(c)

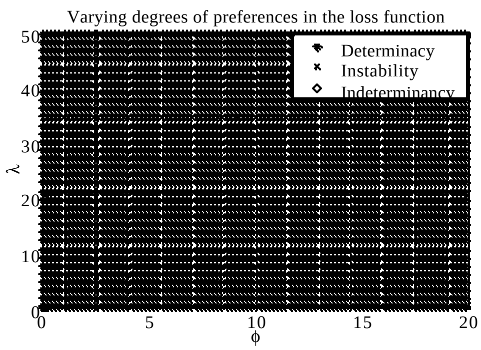



The simulations were performed under the baseline estimate as reported in Table 11.

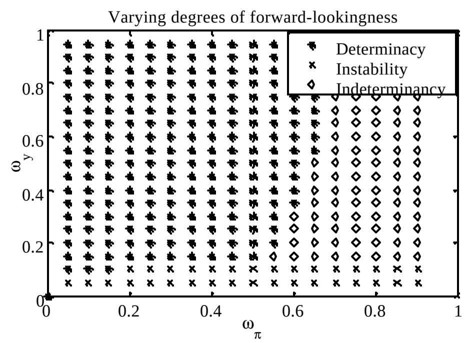
**Figure 32:** Regions of Determinacy: US-Model

For the open economy model one can apply the same procedure as beforehand. As can be seen the model is remarkably robust against changes in the preference vector of monetary policy. For a vast range of parameter constellations $[l_y \quad l_{\Delta i}] \in [0 \quad 50]$ uniqueness and stability holds. On contrary the model seems to display indeterminacy for combinations of high degrees of forward-lookingness in the Phillips curve and the IS-equation. This scenario occurs if we have low degrees of habit persistence in the IS-equation and a high share of Calvo price setters in the Phillips curve. Additionally indeterminacy and instability seems to be an issue for combinations of very active fiscal (high degree of automatic stabilization) paired with varies degrees of activism on the side of monetary policy (varying degrees of output stabilization). Figure 33 shows how the determinacy property reacts to changes in the baseline calibration as monetary policy reacts more strongly to changes in inflation and output. It impressively illustrates that as soon as monetary policy puts a weight on price stability in its main focus determinacy is assured in the quarterly setting. Nevertheless in the case where monetary policy neglects its legal mandate to safeguard stable prices ( $l_p = 0$ ) the model becomes indeterminate. Additionally combinations of high degrees of forward-lookingness in price setting and high degrees of forward-lookingnes in consumption decisions induce indeterminacy, whereas a higher degree of price stickiness builds in 'path-dependency' that generates determinacy.

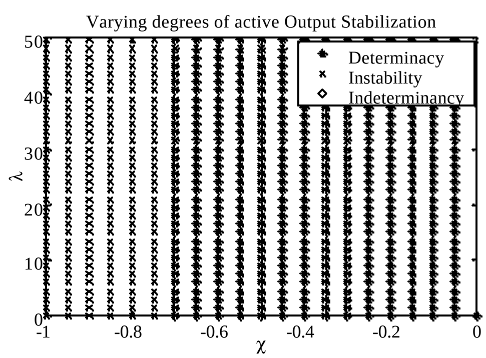
(a)




(b)




(c)




The simulations were performed under the baseline estimate as reported in Table 11.

**Figure 33:** Regions of Determinacy: Euro-Area Model

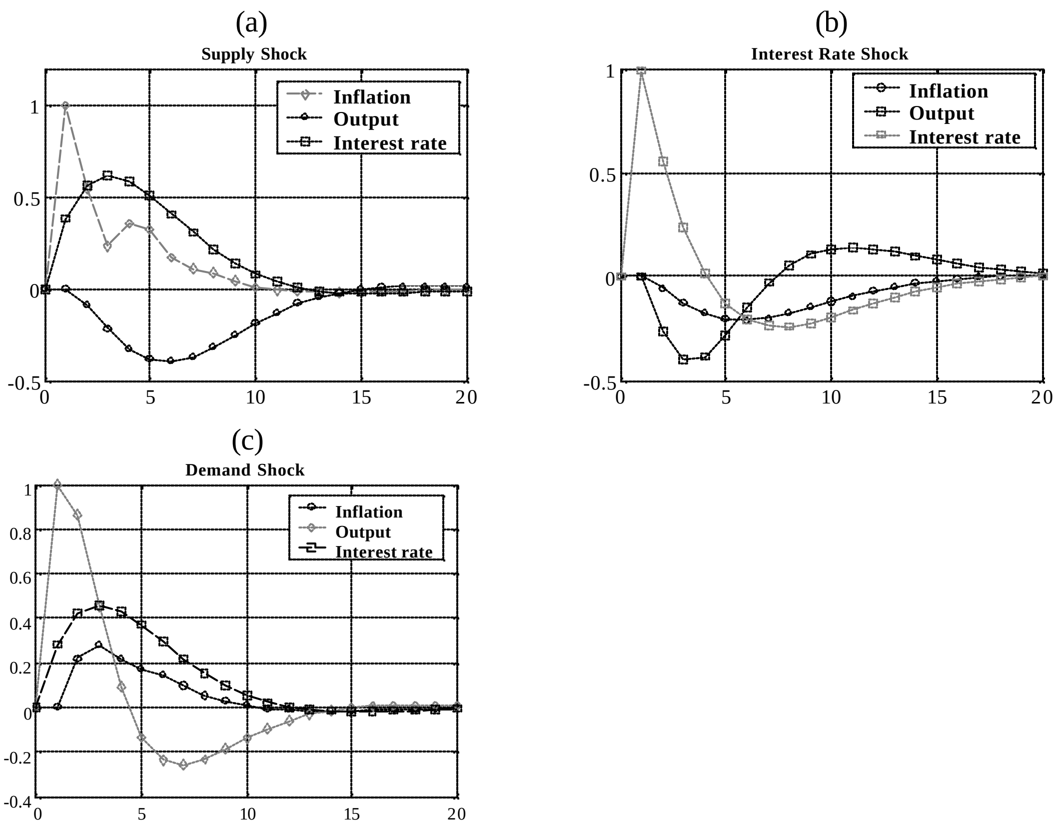
## 4.3 Evaluating the Closed Economy Model: The Mechanics of an Estimated New Keynesian Macromodel

In the following we will discuss in detail the mechanics of a New Keynesian macromodel as embedded in the data. The following section will proceed as follows. In a first step we will analyze the disequilibrium behaviour of the model by means of impulse response analysis. Then we will systematically analyze the correlation structure embedded in the model with respect to changes in key parameters like the degree of forward-lookingness and the preference vector of monetary policy.

### 4.3.1 Impulse Response Functions

The main characteristics of the estimated baseline configuration for the closed economy US-model are depicted in Figure 34. The high degree of interest rate smoothing and the lags in the hybrid Phillips-curve and the IS-equation translate into hump shaped impulse response functions that can be considered in line with conventional New Keynesian macromodels (e.g., Walsh (2003), ch. 11). We will shortly discuss each impulse response function in term.

Quite remarkably the impulse response function of the inflation rate with respect to an interest rate shock does not exhibit a prize puzzle (see Figure 34(b)). Following an interest rate shock the impulse response function of the interest rate starts to decline and reaches its peak response after three quarters. Due to the drop in economic activity the inflation rate equally starts to decline and reaches its peak response with a lag of six quarters. After approximately 20 periods all series are back at their baseline values. Hence long run neutrality holds. The impulse response functions nicely depict the transmission structure encapsulated within this particular specification of a New Keynesian macromodel. The peak response in the output gap leads the peak response in the inflation rate which can be explained by the backward-looking inflation dynamics in the HNKPC. This reflects that the output gap is the driving variable of the inflation process within a hybrid specification and that monetary policy can only disinflate by deeds. Monetary policy seems largely to accommodate supply shocks (see Figure 34(a)). The initial unit supply shock leads to a pronounced but modest increase in the interest rate, which goes hand in hand with a drop in the output gap induced by a tighter stance in monetary policy (peak response after 3 quarters). Consequently the inflation rate starts to decline and returns to its baseline after 13 quarters. The output gap exhibits a pronounced reaction, which reaches its peak response after 6 quarters. Following a positive unit demand shock (see Figure 34(b)), monetary policy reacts by raising real interest rates (peak response after 3 quarters). Due to the stronger economic activity the inflation rate equally starts to rise. It reaches its peak response after 3 quarters. All depicted time series return to their baseline values after 13 quarters. This somewhat pronounced response compared to a supply shock reflects that monetary policy only puts a modest weight on output gap stabilisation ( $l = 0.15$ ).



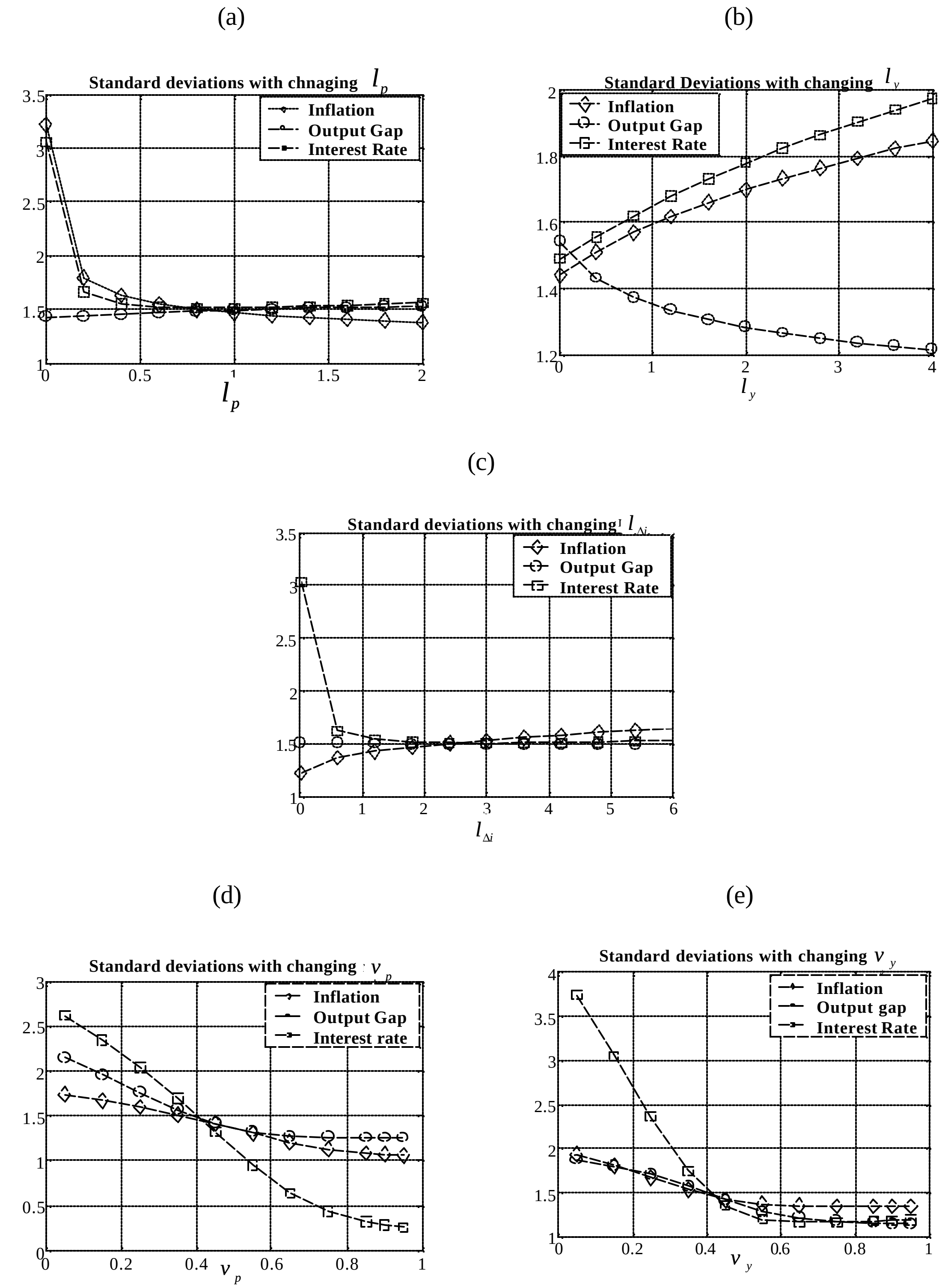

Impulse Response Function for the baseline configuration: $V = [1 \quad 0.15 \quad 1.85 \quad 0.4 \quad 0.4 \quad 1]$.

**Figure 34:** Impulse Response Function

## 4.3.2 Baseline Evaluation and the Implied Model Dynamics

In the following section we will perform a battery of baseline evaluations to get a deeper understanding of the mechanics of the model. In particular we will take a look at the sensitivity of the variances, covariance's and the implied autocorrelations and cross correlations with respect to changes in the individual elements of the identified vector $V^{US} = [\, l_p \quad l_y \quad l_{\Delta i} \quad g_f \quad v_y \,]$. Figure 35 shows how the variances of the goal variables respond ceteris paribus to a change in the individual elements of $V$. The results are largely in line with expectations. An increasing weight on the individual goal variables, hence the inflation rate, the output gap and the change in interest rates respectively lead to a drop in the variances of each of these variables. E.g. if monetary policy puts an increasing weight on interest rate smoothing (increasing $l_{\Delta i}$) the variance of the interest rate starts to decline. The same holds true for the other target variables of monetary policy. Nevertheless reducing the variance of one goal variable is no free lunch. Let us assume that monetary policy puts a higher weight on stabilizing the inflation rate (increasing $l_p$). As side effect the variance of the interest rate increases. In other words the central bank needs to make a more rigorous use of its instrument in response to supply or demand shocks.

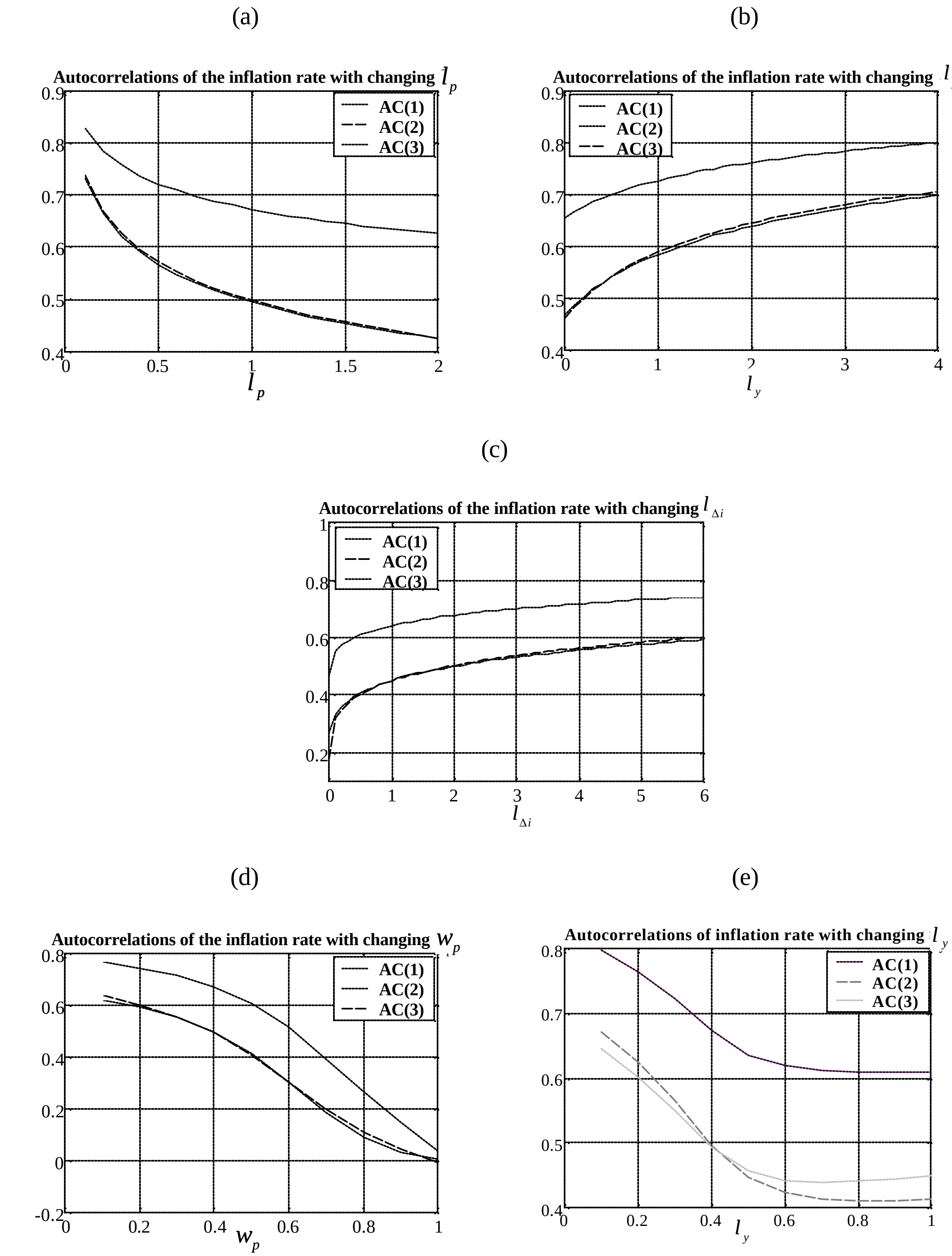
This is in particular obvious if we take a look at Figure 35(b). Figure 35(b) depicts the implications if monetary policy puts a greater concern for economic activity. As we see the variance of the output gap drops with an increasing $l_y$. Nevertheless this can only be realized at the cost of an increase in the variance of the inflation rate. This means in particular that central banks take a less vigorous stance on supply shocks thereby increasing the fluctuations in inflation. With respect to the degree of forward-lookingness the following seems to hold true. An increasing degree of forward lookingness in the hybrid Phillips curve $g_f$ and in the intertemporal IS-curve $v_y$ implies a sharp drop of the variance of the interest rate. Hence if we keep the preference vector corresponding to the period loss function $L_t = p_t^2 + 0.15 y_t^2 + 1.85 \Delta i_t^2$ fixed an increasing degree of forward-lookingness serves as a substitute for a more aggressive monetary policy stance. Therefore one might say that an increasing degree of forward-lookingness implies that monetary policy does not need to “lean against strong persistence” in the data. Hence the results presented by purely backward looking models stating that estimated response coefficients in monetary policy rules are smaller than optimal coefficients retrieved by control methods may be spurious. In the light of the presented results these studies might simply neglect to capture the degree of forward-lookingness $v_y = g_f = 0.4$ present in the data. Influential backward looking models are for instance Ball (1997) or Rudebusch et al (1999). Figure 36 evaluates the impact of changes in the estimated vector $V = [\, l_p \quad l_y \quad l_{\Delta i} \quad m_p \quad m_y \,]$ on the autocorrelation patterns of the inflation rate. As one would expect, an increasing weight on stabilizing the inflation rate around the inflation target leads to a drop in the persistence of the inflation process (see Figure 36 (a)).




*setting $v_r = 1$ dominated setting $v_r = 0$ in terms of the chosen criterion, therefore we kept $v_r = 1$ for all possible specifications.

**Figure 35:** Variances with Changing $V^{US} = [\, l_p \quad l_y \quad l_{\Delta i} \quad g_f \quad v_y \,]$*

In other words if monetary policy uses its instrument more rigorously to keep the inflation rate close to the inflation target the degree of persistence in the inflation process declines. This underlines that the degree of persistence is endogenous to the monetary policy regime. Nevertheless the 'beneficial' reduction of persistence in one variable comes at a cost. E.g., an increasing weight on stabilizing the output gap leads to an increase in persistence of the inflation process. One likely explanation can be given as follows: As monetary policy tends to react stronger to movements in the output gap it will tend to 'overlook' supply shocks leading to a higher degree of persistence in the inflation rate. An increasing weight on stabilizing the change in interest rates leads to an increase in the inflation persistence, which can be quite naturally explained by the fact that monetary policy uses its instrument less vigorously to keep the inflation rate on track. As expected an increasing degree of forward lookingness leads to a drop in the degree of persistence of the inflation rate. Forward looking price setters anticipate that the central bank will raise real interest rates in order to keep inflation under control. Therefore price increases tend to be more modest and deviations from the inflation target are less pronounced. Note that in the limit with $v_y$ converging to one, when we approximate the NKPC the inflation process converges towards white noise. Figure 37 shows the sensitivity of the autocorrelations of the interest rate with respect to the individual elements of the estimated vector. Increasing weights on interest rate stabilization raises the persistence in interest rates as monetary policy uses its instrument more cautious and gradual (Figure 37 (c)). Hence the interest rate reaction in response to shocks will be more sustained. This automatically leads to a higher degree of persistence. Varying weights on stabilizing the output gap do not have a significant impact on the autocorrelation structure (Figure 37 (b)). Figure 37 depicts some cross-correlations inherently nested in the chosen baseline calibration $V$. Evaluating the cross-correlations is of key interest as the purely forward looking New Keynesian macromodel makes two strong predictions. On the one hand it states that there should be a positive correlation between changes in the output gap and the inflation rate. Hence higher real interest rates today foreshadow an economic boom tomorrow. Secondly the New Keynesian macromodel predicts that increasing inflation rates are negatively correlated with economic activity. Both predictions are at odds with the data. Therefore one needs to introduce backward-looking behaviour in order to change the signs of the relevant cross-correlations. The following results stand out: Figure 37 depicts the cross correlation of the inflation rate $p_t$ with the lagged differences of the interest rate $\Delta i_t, \Delta i_{t-1}$ and $\Delta i_{t-2}$.

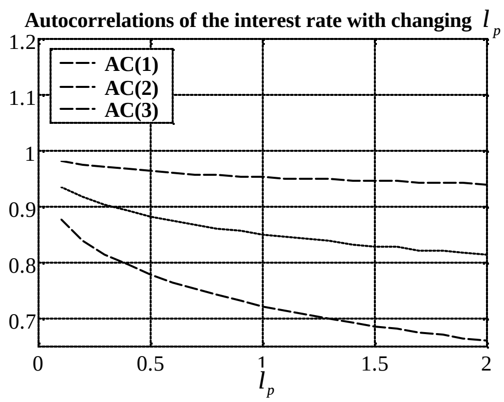



*setting $v_r = 1$ dominated setting $v_r = 0$ in terms of the chosen criterion, therefore we kept $v_r = 1$ for all possible specifications.

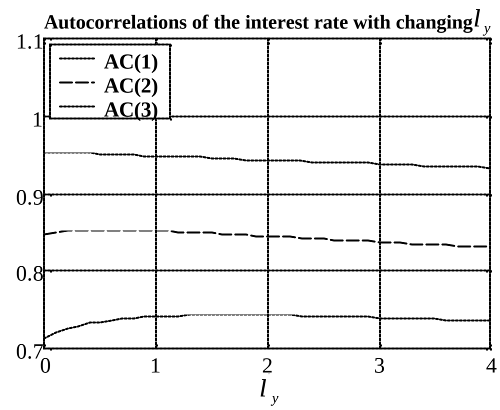
**Figure 36:** Autocorrelations of Inflation with Changing $V^{US} = [l_p \quad l_y \quad l_{\Delta i} \quad g_f \quad v_y]$ *

An increasing degree of forward lookingness in the IS-curve strengthens the correlation between past changes in the interest rate and today's inflation rate.

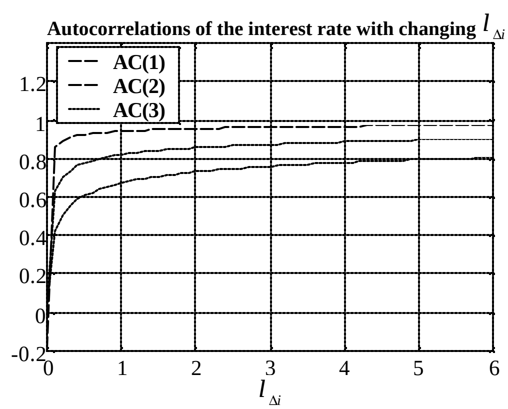
(a)



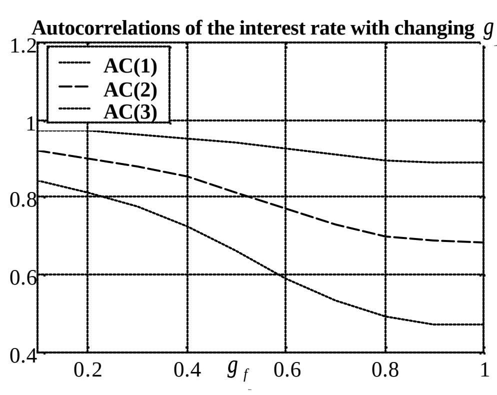

(b)



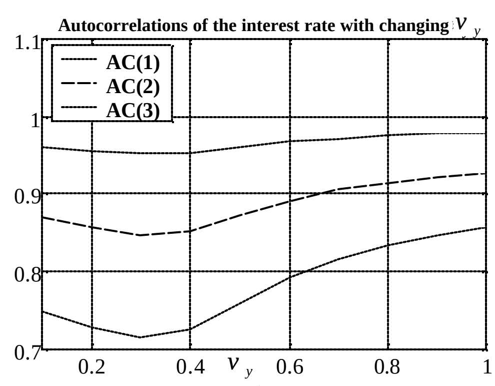

(c)




(d)




(e)

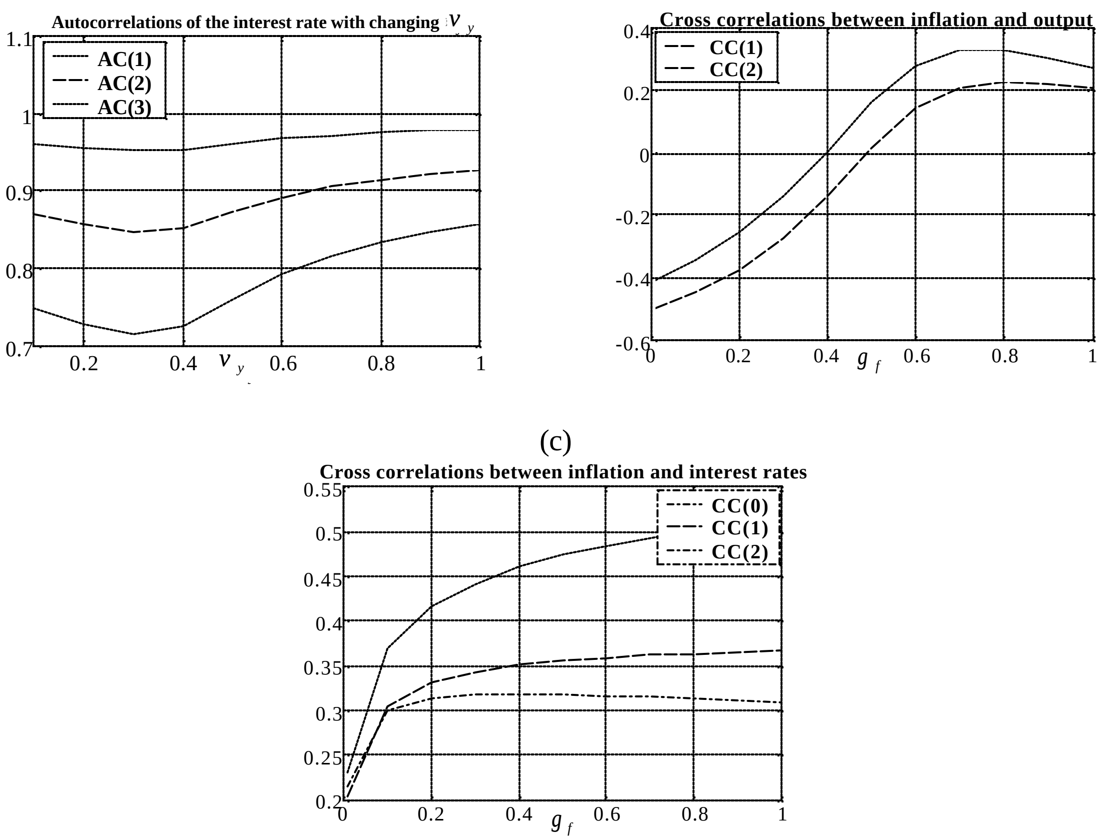



*setting $v_r = 1$ dominated setting $v_r = 0$ in terms of the chosen criterion, therefore we kept $v_r = 1$ for all possible specifications.

**Figure 37:** Autocorrelations of the Interest Rate $V^{US} = [l_p \quad l_y \quad l_{\Delta i} \quad g_f \quad m_y]$*

This result can be interpreted as a faster ‘pass-through’ effect running from interest rates to the inflation rate. Figure 36 (c) depicts the cross-correlation between the current output gap $y_t$ and past changes in the interest rate $\Delta i_t, \Delta i_{t-1}$ and $\Delta i_{t-2}$. Figure 38 shows that the model needs a critical mass of rule-of-thumb setters otherwise the prediction of the purely forward-looking IS-equation will dominate according to which high interest rates today will foreshadow an economic boom (see equation (144)). Hence given the preference vector $V = \begin{bmatrix} 1 & 0.15 & 1.85 & 0.4 & 0.4 & 1 \end{bmatrix}$ there is a restriction on the set of reasonable parameter constellations $v_y$. The model needs at least 60% of economic agents that are forward looking. The same holds true for the cross correlation between the inflation rate and the output gap. Unless we do not have a critical mass of 60% of economic agents that are backward-looking the cross correlation will predict that periods of high inflation were driven by periods of low output gaps.

(a) (b)




*setting $v_r = 1$ dominated setting $v_r = 0$ in terms of the chosen criterion, therefore we kept $v_r = 1$ for all possible specifications.

**Figure 38:** Selected Cross Correlations with Changing $V^{US} = [l_p \quad l_y \quad l_{\Delta i} \quad v_p \quad v_y]$

### 4.3.4 Some Conclusions

Within this chapter we proposed a calibration technique, which explicitly takes the variances, covariances, autocorrelations and cross-correlations into account. Based on this technique we present evidence that around 60% of the pricing and consumption decisions are not made by optimising agents but by rule of thumb setters. This result is in line with earlier studies and underlines that purely forward-looking Phillips curves and IS-equations are unable to match the persistence present in the data. The finding that a majority of households and firms do not seem to optimise but base their decisions on heuristics may be a fruitful area for future research. We have indicated that 'conventional wisdom' which states that estimated coefficients are smaller than those retrieved by means of optimal control may be spurious. The analysis of the level of variances present in the data as well as the evaluation of selected cross-correlations clearly indicates that some degree of forward lookingness is necessary to fit the facts. If monetary policy opts for a stable and unique rule, the job of monetary policy makers is much easier as it would be in a purely backward-looking system. Thus due to the implied self stabilizing properties of forward looking systems grounded on peoples expectations on stabilizing monetary policy itself (self-fulfilling expectations) the disequilibrium dynamics are less pronounced. The evaluation of some selected cross correlations served as a useful benchmark to put restrictions on the degree of forward and backward-lookingness in the data in the Phillips curve and the IS-equation. The identified preference vector of monetary policy indicates that the dominant goal of US monetary policy is the stabilization of the inflation rate around the inflation target. Output-gap stabilization as an independent goal of monetary policy only seems to play a minor role for the conduct of monetary policy.

## 4.4 The Stability and Growth Pact: Time to Rebuild! *

In this section we will apply the open economy model to address the SGP. With the launch of the third stage EMU the member countries have embarqued to unknown territory. Once more

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[20] * The chapter benefited from presentation in Berlin (9th Workshop Macroeconomics and Macroeconomic Policies-Alternatives to the Orthodoxy, Alternative Macroeconomic Policies, 2005), Göttingen (Workshop International Economics, 2005), Geneva (x[th] Spring Meeting of Young Economists, 2005) and Dresden (Annual Meeting of the German Economic Association 2004). The authors would like to thank the session participants for valuable comments, in particular Alina Barnett (Warwick University).

the economic momentum served as a vehicle to link irreversibly the fate of the member countries as it was from the outset of the union. As a consequence the twelve member states had to rethink and rebuild a new common European macroeconomic architecture- The SGP- that enshrined the views on monetary and fiscal policy interaction in the euro area. Unfortunately, the grandfathers of the SGP were not ignited by the challenge to restructure but mostly guided by cautioness. Or put differently the new architecture is a child of German “Angst”. Given the current problems and shortcomings of the SGP it is time to rethink and rebuild.

The unique feature of a currency area is given by the fact that the different macroeconomic agents, the ECB, national governments and labour unions focus on different levels of target variables. The common central bank whose policy we assume to be conducted according to the notion of inflation targeting (Svensson (1999)) focuses on union wide aggregates. It sets nominal interest rates for the currency area consistent with its inflation target while equally having a concern for economic activity. This means in particular that the interest rate policy of the ECB will be indifferent against mean preserving distributions of macroeconomic outcomes across member states. In contrast governments basically focus on national aggregates. In a monetary union that is subject to asymmetric shocks fiscal policy serves as a buffer to block idiosyncratic shocks from spreading to other member countries. Of course fiscal policy might equally be itself the source of destabilisation as incentives for free-rider behaviour are present. Therefore a monetary union calls for a renaissance of fiscal stabilization policy[21]. Obviously this calls for rules which neatly balance the chances and perils that are nested in monetary and fiscal policy interaction in a currency union with decentralised fiscal authorities.

This section is structured as follows: In a first step we aim to identify a small scale symmetric two country macromodel for the euro area that realistically describes the data. To specify a model we need to identify the monetary and fiscal policy rules that describe the current macroeconomic paradigm in reign in Europe. Thereby we stress the view taken by the European Commission on fiscal policy rules and the view taken by the ECB- given its high status of legal independence- on monetary policy rules.

[21] In a closed large economy it is somewhat a consensus that active demand management should be conducted by the central bank as these have heavily improved over the last decades when it comes to stabilize economic fluctuations. Taylor (2000) comes to the following conclusion: "In the current context of the U.S economy, it seems best to let fiscal policy have its main countercyclical impact through the automatic stabilizers. U.S. monetary policy has been doing a good job in the recent decades at keeping aggregate demand close to its potential GDP, partly because this is consistent with the Fed's inflation objective and partly because it is viewed as a good policy in its own light. "

Once we have calibrated the model we perform a battery of tests. In particular we want to evaluate the impact of symmetric and asymmetric fiscal spending shocks to underline the need for rules as fiscal policy instrumentalised by myopic politicians inflicts substantial damage on the rest of the union. This holds in particular as the ECB will only take care on idiosyncratic events insofar as they have an impact on the overall European averages. In a second step we analyse how effective fiscal policy is in stabilising economic cycles by computing the impact of varying degrees of automatic stabilizers on the correlation structure of the model. In particular we will evaluate whether fiscal policy can reduce the persistence nested in the output gap and the inflation rate. We will state the case that the rules as laid down in the SGP are of little use. The implicit assumption of the SGP that "high deficits lead to high inflation rates" has generated a malfunctioning alarming system. The 3% deficit criterion impairs the ability of fiscal policy to effectively stabilize the cycle. We finish the chapter by giving some proposals along which we think the SGP should be reformed.

### 4.4.1 Specifying a Symmetric Two Country Model for the Euro Area

In this section we will shortly highlight the current macroeconomic interaction in the euro-area as enshrined in the treaty of Maastricht and the SGP in order to identify a realistic model for the euro area. As we will see the current macroeconomic paradigm in reign in Europe was highly shaped by the view that one needs stringent fiscal policy rules to safeguard the de-facto independence of the ECB[22]. The European Council feared that an unsustainable fiscal policy at the national level causes negative spill over effects in the form of higher inflation rates and real interest rates for the rest of the union, and in the worst case scenario ultimately calls for a bail out as a consequence of unsustainable debt to GDP ratios. In conclusion one can say that the current SGP was strongly shaped by the view that wet nosed governments could ultimately inflate Europe. In the following section we put the focus on identifying monetary and fiscal policy rules that were designed to prevent such developments.

[22] The European Council stated: "The European Council underlines the importance of safeguarding sound government finances as a means to strengthening the conditions for price stability and for strong sustainable growth conducive to employment creation. It is also necessary to ensure that national budgetary policies support stability oriented monetary policies. " (European Council (2003)).

#### 4.4.1.1 The Current Setting of the ECB

Despite its official strategy (ECB (2004)) it is common practice to specify the objective function of the ECB by the following loss function, although it is typically related to a regime of inflation targeting:

$$L_t = E_0 \sum_{t=0}^{\infty} b^t \left\{ l_p p_t^2 + l_y y_t^2 + l_{\Delta i} i_t^2 \right\}.^{23} \qquad (4.35)$$

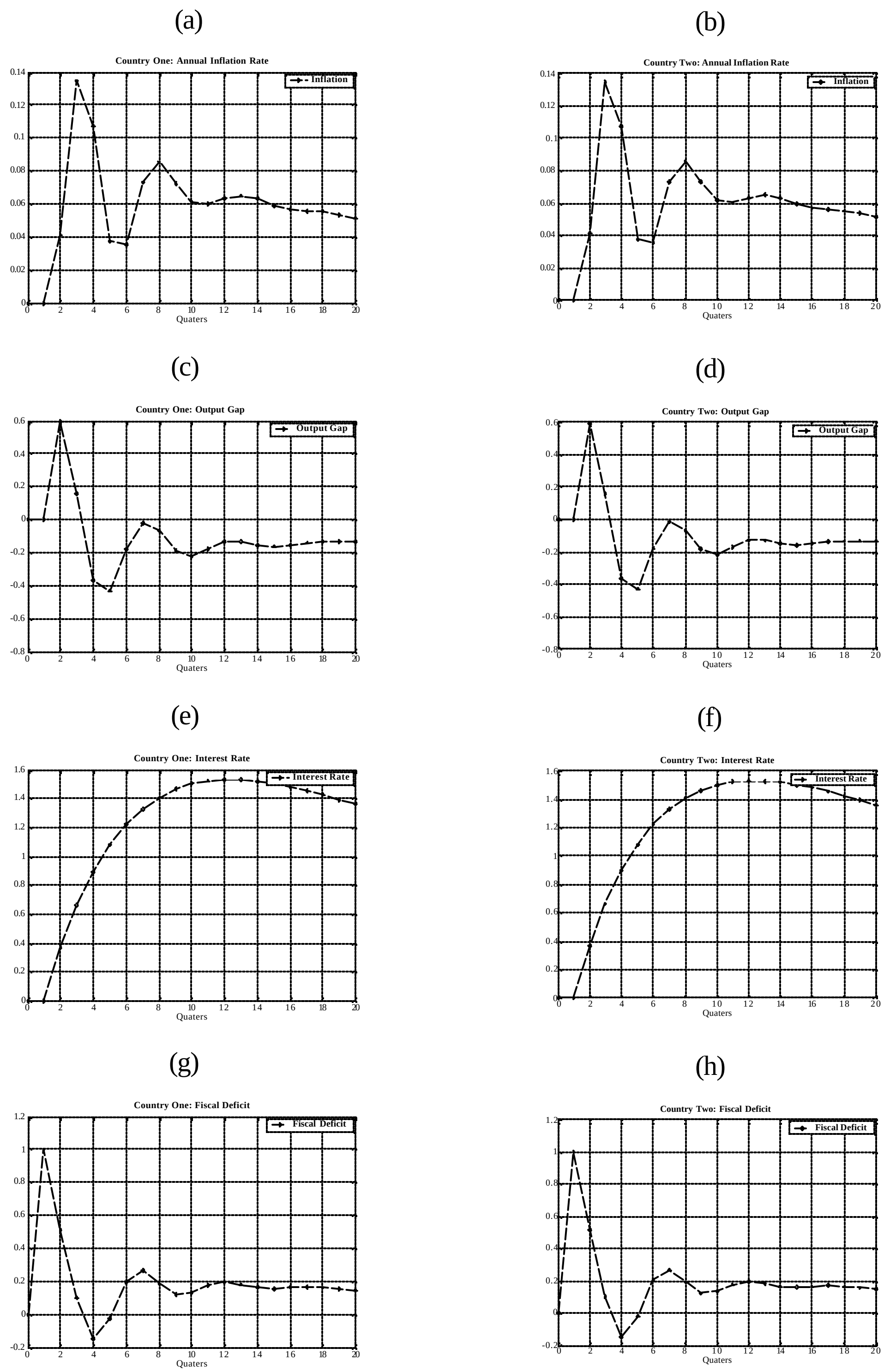
According to equation (4.35) the ECB tries to reduce aggregate price dispersion across the currency area while equally having a concern for stabilising economic activity. The preference parameter $l_y$ depicts the weight monetary policy attaches to stabilise the output gap versus stabilising the inflation rate. Additionally Woodford (2003, ch. 6) has shown that equation (4.35) can be derived as a second order approximation to a households expected utility problem in a New Keynesian macromodel (see 2.1.3.2). In order to achieve its targets the ECB sets the interest rate in response to exogenous disturbances and consistent with the structural equations of the model so that the loss function $L_t$ is minimised. Note that the ECB only targets at euro area wide averages, whereas it does not take care on the dispersion of goal variables across member states. In other words the ECB does not consider the spread as a problem as long as it is mean preserving. Additionally we assume that the ECB implements its desired target rate only gradually. As indicated interest rate smoothing can be rationalized by a broad range of arguments. Among them are for instance that the ECB does not want to disrupt financial markets. Additionally gradualism can be a direct result of uncertainties to which a monetary policy maker is exposed (Brainard uncertainty, model uncertainty, data uncertainty (Martin and Salmon Chris (1999)). From a theoretic perspective interest rate smoothing is a device of making use of private sector expectations of further interest rate steps in the same direction in a forward looking environment (Lansing and Bharat (2003)).

[23] Note that throughout our exposition we will make use of the fact that after scaling the intertemporal loss function by $(1-b)$ the intertemporal loss function approaches the weighted average of the unconditional variances of the individual goal variables: $\lim_{b \longrightarrow 1} (1-b) L_t = Var[p_t] + l_y Var[y_t] + l_{\Delta i} Var[\Delta i_t]$.

### 4.4.1.2 On The Need for Stringent Fiscal Policy Rules in a Monetary Union

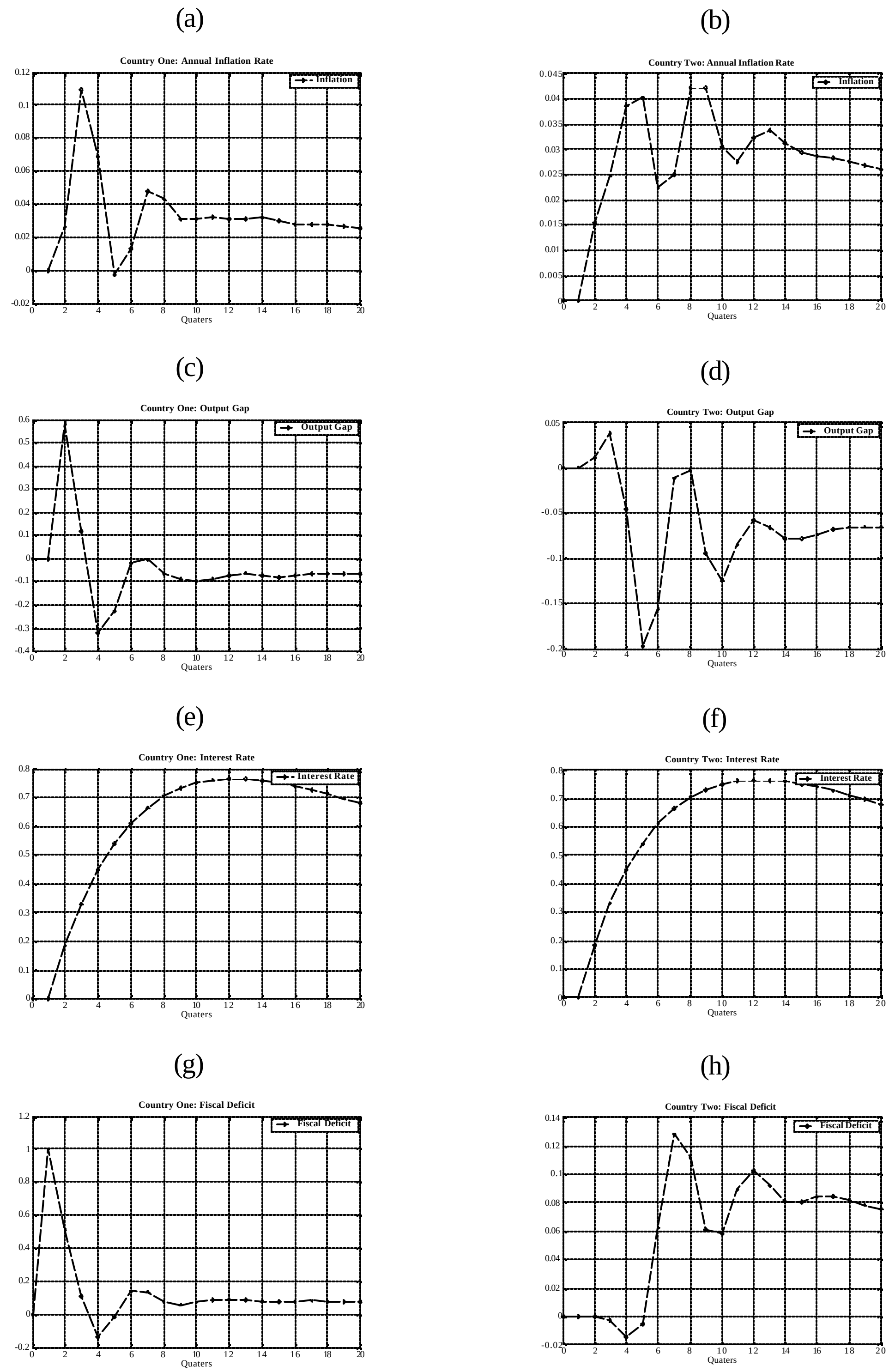
In the proceeding section we have identified a symmetric two-country model for the euro area that generates a stable and unique rational-expectations equilibrium. Based on this model we will now provide the basic rationale for stringent rules in a currency area. In order to understand how fiscal policy functions in a monetary union and why there is a need for stringent rules we evaluate the impulse response functions with respect to symmetric and asymmetric fiscal spending shocks.

As can be seen from Figure 39 a symmetric fiscal spending shock induces persistent deviations of the inflation rate from the inflation target of the central bank. Due to the boost in fiscal spending economic activity starts to accelerate and output is above its potential. As monetary policy aims at stabilising inflation as well as the output gap the central bank will increase short term interest rates which depresses economic activity. Quite naturally the impulse response pattern is similar to that one would observe in the case of a demand shock. In sum the impulse-response function depicts that fiscal spending shocks can induce persistent swings in all key variables. Quite obviously as known from VAR-analysis persistence is a very common theme and not specific to a monetary union. Potential conflicts are nested in asymmetric spending shocks as they potentially generate dispersion in macroeconomic aggregates across the currency areas (see ch. 3). Therefore the need for rules prevails for the case of asymmetric fiscal spending shocks. Let us assume that fiscal authorities in country one trigger an unexpected fiscal expansion. The impulse-response functions illustrate that a central bank that is indifferent against mean preserving spreads can hardly operate conveniently in such an environment. The key problem for the ECB prevails in the graphs (a)/(b)/(c)/(d). The persistent deviations in all target variables are remarkable. The spending shock in country one boosts its own inflation rate as well as the output gap if both blocs are of equal size. The ECB only reacts modestly compared to a symmetric spending shock as the short term nominal rate is only raised round about half of the size one could observe for a symmetric shock. The ECB faces the fundamental problem that economic activity in country one is fuelled by the domestic spending shock whereas country two exhibits cyclical swings, as the ECB increases short term interest rates. Thus the ECB can not punish individual member states by raising average real interest rates which clearly shows that stringent rules are a necessary prerequisite for the well functioning of a monetary union, to prevent free rider behaviour and negative spill over effects for other member states. The depression in economic activity in country two is somewhat dampened as our model allows for direct demand spill over effects. Additionally we can already see that automatic stabilizers in country two prevent swings in the output gap from becoming more persistent. Hence the impulse response functions show that automatic stabilisers serve as a useful instrument to cushion the consequences of an unsustainable policy in other member states.



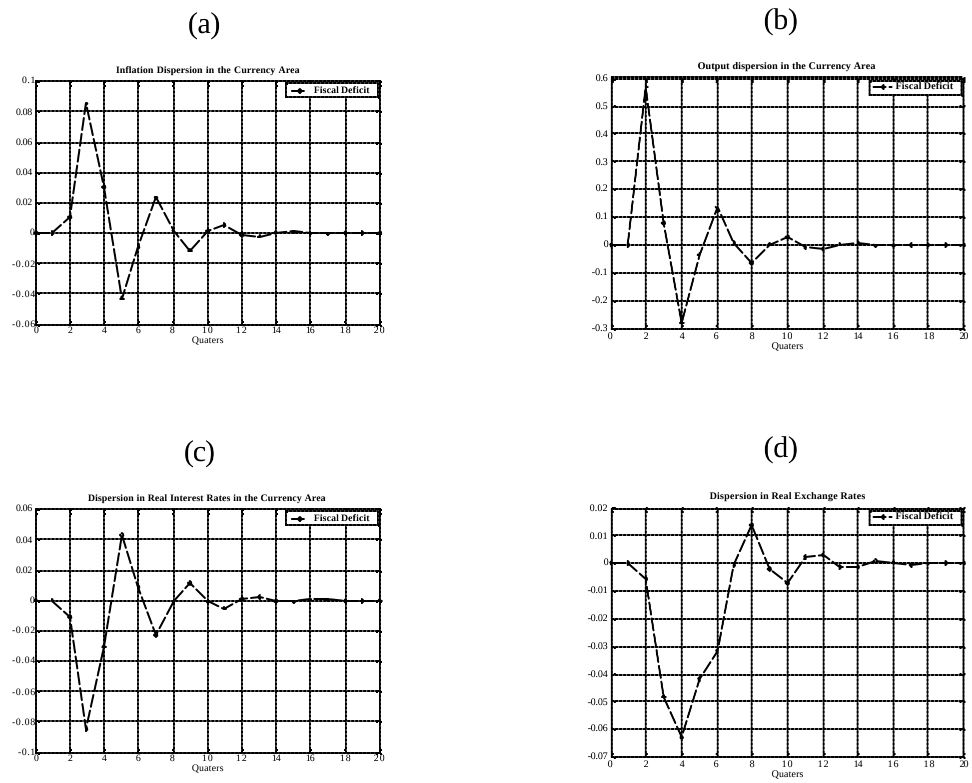

*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 39:** Symmetric Fiscal Spending Shock*




*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 40:** Asymmetric Fiscal Spending Shock




*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 41:** Measuring some Indicators for Dispersion

This simple illustration clearly indicates that there is a need to safeguard the ECB by stringent fiscal rules. In the case of asymmetric fiscal spending shocks fiscal authorities can be a source of destabilisation. The link between fiscal deficits and high inflation rates that were of great concern to the fathers of the SGP is clearly present. The two causal mechanisms in our model that govern the divergence in real conditions are the wedges in the real interest rate and the wedge in the intra-european competitiveness. Ceteris paribus the real rate effects will be more pronounced in relatively closed economies whereas with an increasing degree of openness the real exchange rate effect is likely to decrease in importance.

### 4.4.1.3 On the Deficiencies of the SGP

In the previous section we have stressed the need for stringent policy rules that combine long run sustainability with short run flexibility. Unfortunately the current SGP is inappropriate to achieve this task. The main construction error of the SGP is its underlying assumption that countries with high deficits have produced high inflation rates. As shown in Bofinger (2003) by regression analysis this is generally not true for the euro area. This fundamental construction error implies that the SGP has at least two series deficiencies which we will discuss in term. In particular it potentially impairs the ability of fiscal policy to effectively stabilise the cycle. Additionally the 3% deficit criterion has created a malfunctioning alarm-system which needs to be reformed.

#### 4.4.1.3.1 The Impact of Automatic Stabilizers on the Correlation Structure

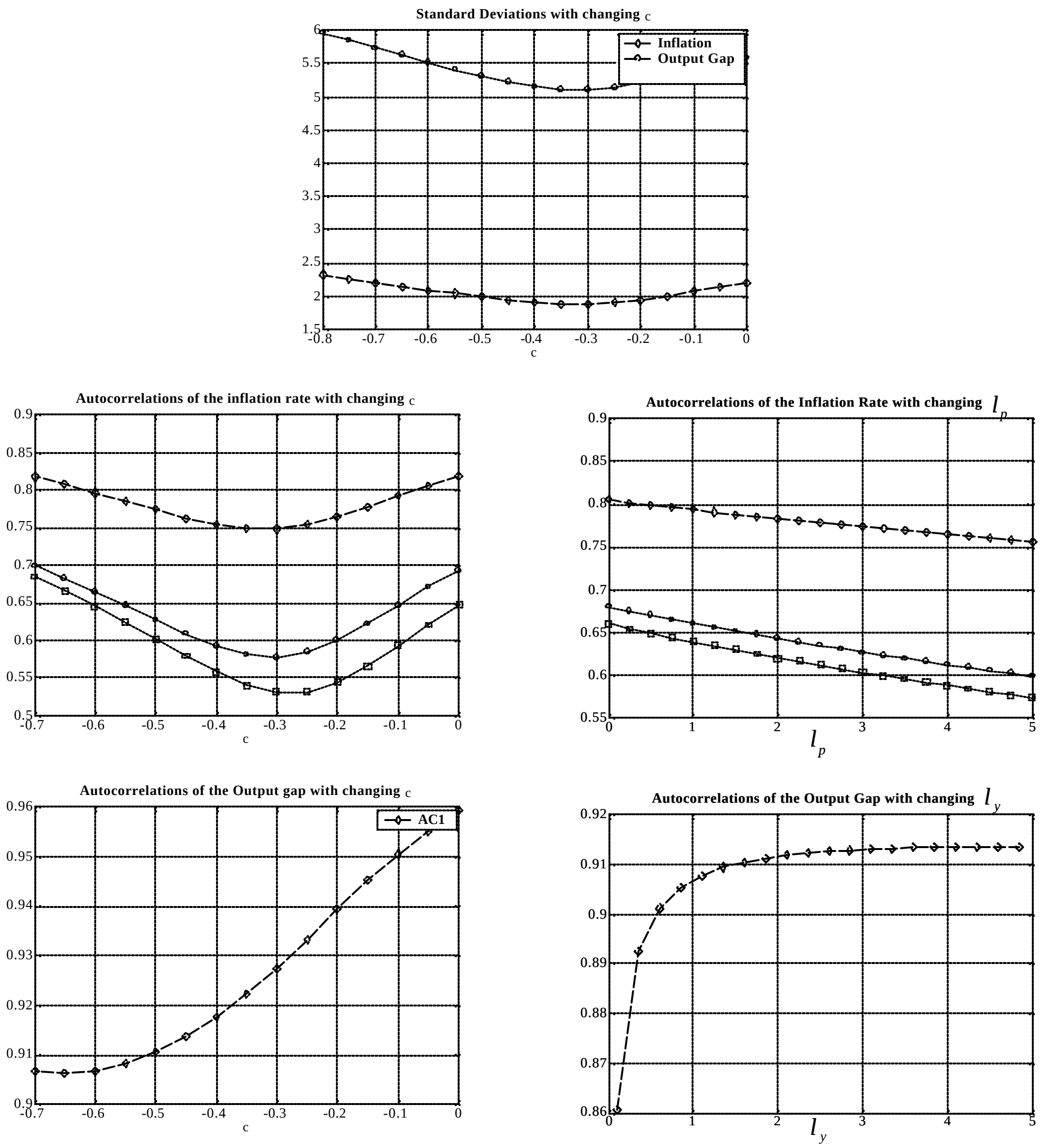
In this section we analyse how powerful fiscal policy is. To get a deeper understanding we evaluate the mechanics of the model. In particular we will take a look at the sensitivity of the variances and autocorrelations with respect to changes in the stance of fiscal policy as measured by $|c|$ (see equation (4.312)). Remember throughout our model we have assumed that fiscal policy is conducted according to the following rule:

$$g_{1,t} = \overline{g}_1 - c\, y_{1,t} + \breve{l}\, g_{1,t-1} + e^{g}_{1,t}\,. \tag{4.36}$$

In order to evaluate the power of fiscal policy we analyse the impact of fiscal policy on the standard deviation of the inflation rate, the output gap and the interest rate if fiscal policy engages more actively in dampening cycles (increasing $|c|$). Figure 42 shows that a symmetric increase in fiscal activity (increasing $|c|$) leads to a drop in the standard deviations of the aggregate inflation rate and the output gap. Nevertheless overstabilization of the output gap leads to an increase in inflation volatility as national fiscal policy runs counter to a restrictive monetary stance in the case of a supply shock.

Not surprisingly the variance in the output gap drops as fiscal policy becomes more active, as it smoothes out the impact of demand shocks (Figure 42). As positive side effect a more stable output gap translates into less persistent fluctuations of the inflation rate. Hence a more active fiscal policy does not only succeed in stabilising output but also serves as an instrument to bring the inflation rate closer to a white noise process.

In order to compare the stabilisation properties of fiscal and monetary policy Figure 42 depicts the ability to reduce the persistence of the inflation rate and the output gap respectively. Quite remarkably fiscal policy –guided by a simple rule- is very effective in stabilising the inflation rate as persistence sharply drops in response to a more active fiscal stance (up to a level of $|c| = 0.3$). The ability to reduce inertia in output is comparatively better than the one of monetary policy, although the output gap seems to exhibit a high degree of persistence. Hence a passive fiscal policy can hardly be rationalised from a stabilisation perspective given the sound evidence on the ability of fiscal policy to stabilise economic activity.




*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 42:** Standard Deviations

The preceding two sections clearly present evidence that a currency area needs fiscal policy rules. On the one hand rules are necessary in order to prevent individual member states from free rider behaviour. On the other hand fiscal policy is potentially a powerful tool from a stabilisation perspective. Therefore we conclude that a tying hands policy that would prevent fiscal policy from being used actively to fight cycles can hardly be rationalised. Such a trade-off constellation clearly calls for stringent rules that liberate the potential benefits while equally dampening the potential harms. This was of course exactly what the fathers of the SGP had in mind when trying to combine short run flexibility (operate within 3%-deficits) with long run sustainability (debt to GDP< 60%), but unfortunately as we will illustrate in the next section the rules designed were flawed from the outset as they are based on the faulty assumption that high inflation rates mirror high deficits.

### 4.4.1.3.2 Deficiency 1: The 3% Deficit Criterion Impairs the Ability of Fiscal Policy to Stabilise Economic Activity

The current SGP assumes that 'sound budgetary positions' are the dominant strategy to safeguard price stability. Therefore the 3% deficit criterion limits the ability of short run flexibility as the SGP assumes that excessive deficits might cause inflation and unsustainable debt dynamics in the long run. Based on the following simulations:

$$X_t = MX_{t-1} + v_t \,. \qquad (4.37)$$

$$X_{1t} = \left\{p_{i,t}, p_{i,t-1}, p_{i,t-2}, p_{i,t-3}, y_{i,t}, y_{i,t-1}, g_{i,t-1}, i_{i,t-1}, i_{i,t-2}, i_{i,t-3}, p_{-i,t}, p_{-i,t-1}, p_{-i,t-2}, p_{-i,t-3}, y_{-i,t}, y_{-i,t-1}, g_{-i,t}\right\}'$$

$$n_{1t} = \left\{e_{i,t}, 0,0,0, h_{i,t}, 0, e^g_{i,t}, 0,0,0, e_{-i,t}, 0,0,0, h_{-i,t}, 0, e^g_{-i,t}\right\}',$$ [24]

we either assume asymmetric supply or demand shocks that hit country one.

Table 13 shows the beneficial impact of automatic stabilisers. The ratios indicate the relationship of the aggregate variable with and without automatic stabilisation. As indicated the ratios are all well below one which clearly signals the beneficial impact of automatic stabilisation. Additionally the results show that within the New Keynesian framework output stabilisation is a valuable device to keep the inflation rate on track, as it helps to smooth out the impact on demand shocks as well as the consequences of fiscal spending shocks itself. E.g., in the case of a symmetric demand shock the ratio of output gap variability with

[24] Note that the error vector is based on the variance covariance matrix as given in appendix A.3.

stabilization and without stabilization drops round about 22%. Additionally the variability of inflation sharply drops as the main source of inflation variability is smoothed. The same applies for the case of a supply shock where the reduction in inflation variability translates into less output variability as the central bank does not need to make a rigorous use of the real interest rate.

| COUNTRY I | |
|---|---|
| RATIO ($Y_{c=0.3}/Y_{c=0}$) | RATIO ($Y_{c=0.5}/Y_{c=0}$) |
| SYMMETRIC SHOCK | |
| OUTPUT GAP | |
| 0.84 | 0.93 |
| INFLATION RATE | |
| 0.74 | 0.86 |
| SYMMETRIC SUPPLY SHOCK | |
| OUTPUT GAP | |
| 0.84 | 0.96 |
| INFLATION RATE | |
| 0.76 | 0.90 |
| SYMMETRIC DEMAND SHOCK | |
| OUTPUT GAP | |
| 0.78 | 0.68 |
| INFLATION RATE | |
| 0.40 | 0.84 |
| SYMMETRIC FISCAL SHOCK | |
| OUTPUT GAP | |
| 1.75 | 1.49 |
| INFLATION RATE | |
| 0.98 | 0.49 |

**Table 13**: Switch from an Active Fiscal Stance $c = 0.3$ to a Passive $c = 0$ one.

#### 4.4.1.3.3 Deficiency 2: A Defective Alarming System

In this section we will show that the SGP is a defective alarming system that is likely to trigger the monitoring procedure even if fiscal policy is conducted in a sustainable fashion. To state the case the starting point is the following reduced form:

$$X_t = MX_{t-1} + v_t . \tag{4.38}$$

$$X_{1t}=\{p_{i,t},p_{i,t-1},p_{i,t-2},p_{i,t-3},y_{i,t},y_{i,t-1},g_{i,t},i_{i,t-1},i_{i,t-2},i_{i,t-3},p_{-i,t},p_{-i,t-1},p_{-i,t-2},p_{-i,t-3},y_{-i,t},y_{-i,t-1},g_{-i,t}\}'$$

$$n_{1t}=\{e_{i,t},0,0,0,h_{i,t},0,e^{g}_{it},0,0,0,e_{-i,t},0,0,0,h_{-i,t},0,e^{g}_{-it}\}'$$

Fiscal policy is conducted according to the notion:

$$g_{1,t}=\bar{g}_1-c\,y_{1,t}+\breve{l}\,g_{1t-1}+e^{g}_{1t}\ . \tag{4.39}$$

To make realistic inferences we have calibrated $\bar{g}_1=-1.8\%$ which is approximately the average from (1996 - 2005) and is also equal to the structural balance projected for 2006 for the euro area on average by the OECD. Clearly a structural balance equal to -1.8% would not be in line with the strict view the Commission takes but it would well be consistent with the 60% (debt/GDP) ratio in the long run (see (De Grauwe (2000)). Therefore we model by assumption a sustainable fiscal stance. The 3%-deficit criterion can only be breached within our simulation if the fiscal stance parameter is driven by large demand, supply or fiscal spending shocks itself. In the following we want to illustrate with the help of a simulation that the alarm system nested in the SGP is not reliable.

To make realistic inferences within our small scale macro model we have taken care of the fact that the structural supply and demand shocks hitting the individual member countries are correlated (see Angeloni and Ehrmann (1999), Karman and Weimann (2004)). The structural variance covariance matrix was set equal to the following baseline calibration which is based on the assumption that the structural shocks are correlated by 0.2 (See Appendix 4.G).

Based on this reduced form we have simulated the model over a hypothetical period of 100,000 quarters[25]. Table 14 shows the results of the simulation. Hence the Maastricht criteria are too strict as in none of the identified outcomes the long run sustainability is endangered as we have modelled by definition a sustainable fiscal stance. Therefore as long as the violation of the 3%-deficit criterion stems from the size of exogenous shocks and not from a fiscal policy that is conducted in an unsustainable fashion the violation of the Maastricht criteria is a necessary precondition to let automatic stabilisers freely operate. Of course it is equally possible that fiscal spending shocks itself might be responsible for "excessive deficits" (e.g. a

[25] Note that their is quite some discussion whether the very introduction of a currency area has altered the correlation structure of shocks. Karman and Weimann (2004) argue that there is evidence from bivariate VAR-analysis that shocks effecting the demand and supply side of the economies in Europe have converged to a degree of correlation of round about 0.5 for both types of shocks respectively. Within our simulation we have chosen lower correlations which were reported by Angeloni and Ehrmann (1999).The correlation of fiscal spending shocks was set equal to null. Bruneau et al (1999) report modest negative correlations of -0.11 prior to the introduction of the monetary union.

natural disaster like in Germany's flooding events 2002). But as long as these shocks are symmetrically distributed and centered around null they are unproblematic.

Based on this analysis we have found that the unconditional probability for an individual member country to be driven above the 3% deficit criterion by a large shock is approximately 9%. A related study by Hallet and McAdam estimates the probability to be equal to 8%. This in itself might seem an acceptable probability. Nevertheless the currency area consists of twelve member states. Let us for the sake of simplicity assume that the member countries are of equal size and that the unconditional probabilities are uncorrelated. Then we see that the probability of triggering the alarming mechanism on error dramatically increases well above 50% which does not seem to be sustainable from a political perspective. Hence with the help of the proceeding analysis we indicated that the current setting implied that the 3%-deficit criterion is unreliable as a warning system to identify an unsustainable fiscal stance. The assumed causal relationship between high inflation rates driven by fiscal policy and the fiscal balance is everything but exhaustive. Additionally one should keep in mind that historically a direct relationship between the fiscal balance and the inflation rate was only given if fiscal policy could borrow directly from the central bank. But this is explicitly ruled out by Art. 102 as the ECB is prohibited to borrow funds to fiscal authorities.

| CALIBRATION | PROB. FOR INDIVIDUAL COUNTRY I | PROB. THAT AT LEAST ONE OUT OF 12 | WAS INFLATION ABOVE 2% |
|---|---|---|---|
| Fitted | 0.0934 | 0.69 | 0.179 |
| Hallet; McAdam (2003) | 0.0800 | 0.63 | |

**Table 14** Simulation: Percentage of Quarters when g is below 3%

Therefore we propose to reform the three percent deficit criterion. In particular we think that triggering of the monitoring procedure should be conditioned on additional macroeconomic variables. In particular we would like to propose to test whether the actual inflation rate is above 2%. The logic is quite simple. Only if the inflation rate and the deficit criterion are violated simultaneously fiscal policy might undermine the credibility of the central bank in the medium to long run. In other scenarios high deficits are likely to simply mirrow weak economic growth. Under such settings fiscal spending cuts are procyclical. Conditioning the monitoring procedure on the criterion whether the inflation rate was in the target range of the ECB dramatically reduces the risk of triggering the 3% deficit criterion on error.

### 4.4.2 Some Propositions

In the previous sections we have outlined that the current institutional setting impairs the ability of the euro area members to use fiscal policy as an effective stabilisation tool. What one effectively needs to understand is that limiting the functioning of fiscal policy effectively adds more fluctuations on other parts of the economy (e.g., the output gap). Quite clearly today's rules in reign were shaped by the views on the functioning of the economy we as economists had at the outset of the 1990's. This view was still heavily shaped by myopic politicians that aimed at cheating the public. In that respect De Grauwe (2002) states:

"The stability pact is a vote of no confidence by the European authorities in the strength of the democratic institutions in the member countries. It is quite surprising that EU-countries have allowed this to happen, and that they have agreed to be subjected to control by European institutions that even the International Monetary Fund does not impose on banana republics."

Generally the relationship between the cyclical stance of fiscal policy and the level of indebtness is not yet well understood. In particular the question which needs to be addressed is to which extent can the CAB be negative on average but still consistent with a long run sustainable debt to GDP ratio. Today's answer seems to be zero which is clearly at odds with for instance the golden rule that states that long term public investments that generates yields over many periods to come should be financed by debt. Nevertheless there seems to be some evidence that a high level of existing debt seems to induce procyclical movements in the fiscal stance during downturns (OECD 2002).

Since fiscal policy rules are essential for the functioning of a monetary union, the analysis of this chapter calls for a reform of the SGP. While the current framework with its focus on inflation is clearly too one-dimensional, it could be relatively easily supplemented with an additional dimension which takes care of the mix between the common monetary policy and national fiscal policies. This chapter can only give some general suggestions. Since the ECB has a very strong interest in preventing excessive inflation at the national level, it would be useful to base the assessment of fiscal policy on forecasts for the national rate and their compatibility with the ECB's inflation target.

As long as the majority of forecasts show that a country's inflation rate will remain within the ECB's target range of "below 2%", there would be presumption that the overall policy mix of national fiscal policy and the national real interest rate is adequate. In this situation, a fiscal deficit exceeding the 3% threshold would not pose a problem for the common monetary policy. Of course, it would be necessary to make an additional assessment whether this fiscal policy stance could threaten the overall solidity of a country's public finances. E.g., in the present situation of Germany such a risk could be clearly excluded.

If the majority of forecasts shows an inflation rate that exceeds the ECB's target range by a certain margin (e.g. one percentage point), there is a presumption that the policy mix is inadequate. If in this situation the deficit exceeds 3%, there is a strong indication that the national fiscal policy is not compatible with an adequate policy mix and an excessive deficit procedure would be warranted. If the forecasts show that the national rate will exceed the ECB's inflation target by a wider margin (e.g. two percentage points), one can think of imposing sanctions for fiscal policy even if the deficit is below three percent or even if it is in a much better position. The main advantage of this inflation targeting framework, which would of course need much discussion in detail, is that it provides the flexibility that national fiscal policy needs in a monetary union in order to cope with idiosyncratic shocks. At the same time, it would set more stringent fiscal limits for high inflation countries than envisaged in the SGP.

In sum, the main flaw of the SGP is its neglect of the interplay of national fiscal policy and national monetary conditions in a monetary union. Although, as the example of Portugal shows, an "excessive deficit" can be caused by fiscal laxness, it can also be due to a self-aggravating process of below average growth, subdued nominal wage increases, below average inflation and an above average real interest rate. Thus, the SGP's one dimensional focus on the deficit-inflation nexus can be misleading. A strict application of the SGP can have the consequence that a country is forced to abandon its only macroeconomic stabiliser and even to pursue a procyclical fiscal policy. Together with above average real interest rates such a policy mix entails a high risk of deflation and of a further widening of monetary conditions within EMU. As monetary policy would become very difficult under such conditions, the ECB should also have a strong interest in avoiding such risks. Since fiscal policy rules are necessary in a monetary union, the SGP should be supplemented in a way that it sanctions fiscal policies only if a country's overall macroeconomic policy stance is inflationary, i.e. if forecasts show that its inflation rate will exceed the ECB's target rate by one or more percentage points. Such an "inflation targeting" approach would not only provide a better policy mix in countries with weak growth, since the 3 % threshold would not be binding. It would also improve the policy mix in above inflation countries since one could think of sanctions whenever the fiscal policy stance contributes to inflation beyond the ECB's target range.

# APPENDIX : THE GENERAL MODEL SETUP

## 4. A: STATE SPACE NOTATION

**US-model:** Closely following Söderlind, Söderström and Vredin (2005) we can rewrite our basic equation in state space form as follows. In a first step we lead our model one period ahead and solve for the rational expectations variables $E_t p_{t+4}$ and $\mathrm{E}_t y_{t+2}$ with the highest time index:

$$
\begin{aligned}
\frac{g_f}{4} E_t p_{t+4} = \left(1-\frac{g_f}{4}\right) E_t p_{t+1} - \frac{g_f}{4} E_t p_{t+2} - \frac{g_f}{4} E_{t+3} p_{t+3} \\
-\left(1-g_f\right)\left[a_{p1} p_t + a_{p2} p_{t-1} + a_{p3} p_{t-3} + a_{p4} p_{t-4}\right] - a_y y_t
\end{aligned}
\tag{4.A.1}
$$

$$
\begin{aligned}
v_y E_t y_{t+2} + \frac{b_r v_r}{4} E_t p_{t+4} = E_t y_{t+1} - \left(1-v_y\right)\left[b_{y1} y_t + b_{y2} y_{t-1}\right] \\
+ b_r v_r \left[i_t - \frac{1}{4} E_t\left(p_{t+1} + p_{t+2} + p_{t+3}\right)\right] \\
+ \frac{b_r\left(1-v_r\right)}{4}\left[i_t + i_{t-1} + i_{t-2} + i_{t-3} - p_t - p_{t-1} - p_{t-2} - p_{t-3}\right]
\end{aligned}
\tag{4.A.2}
$$

Hence we can rewrite the general model in state space form as:

$$
A_0 \begin{bmatrix} X_{1t+1} \\ E_t X_{2t+1} \end{bmatrix} = A \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + B i_t + \begin{bmatrix} e_{t+1} \\ 0_{n2x1} \end{bmatrix}. \tag{4.A.3}
$$

$$X_{1t} = \{p_t, p_{t-1}, p_{t-2}, p_{t-3}, y_t, y_{t-1}, i_{t-1}, i_{t-2}, i_{t-3}\}'$$

$$X_{2t} = \{E_t p_{t+3}, E_t p_{t+2}, E_t p_{t+1}, E_t y_{t+1}\}'$$

$$n_{1t} = \{e_t, 0, 0, 0, h_t, 0, 0, 0, 0\}'$$

Where $X_{1t}$ is a $9\times1$ vector of predetermined state variables $X_{2t}$ is a $4\times1$ vector of forward looking variables and $n_{1t}$ is a vector of shocks. Following Söderlind, Söderström and Vredin (2005) we have made use of the fact that $p_{t+1} = E_t p_{t+1} + e_{t+1}$ and that $y_{t+1} = E_t y_{t+1} + h_{t+1}$.

$$
A_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g_f}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{b_r v_r}{4} & 0 & 0 & v_y
\end{bmatrix}
$$

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(1-g_f)a_{p_1} & (1-g_f)a_{p_2} & (1-g_f)a_{p_3} & (1-g_f)a_{p_4} & -a_{p_5} & 0 & 0 & 0 & 0 & -\frac{g_f}{4} & -\frac{g_f}{4} & \left(1-\frac{g_f}{4}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{-b_r(1-v_r)}{4} & \frac{-b_r(1-v_r)}{4} & \frac{-b_r(1-v_r)}{4} & \frac{-b_r(1-v_r)}{4} & b_{y1}(1-v_r) & b_{y2}(1-v_r) & \frac{b_r(1-v_p)}{4} & \frac{b_r(1-v_p)}{4} & \frac{b_r(1-v_p)}{4} & -\frac{b_r v_r}{4} & -\frac{b_r v_r}{4} & -\frac{b_r v_r}{4} & 1
\end{bmatrix}
$$

$$
B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & b_r v_r + b_r(1-v_r)/4 \end{bmatrix}
$$

**Euro-Area model:** Applying the same apparatus as beforehand we lead our model one period ahead and solve for the rational expectations variables $E_t p_{t+4}$ and $\mathrm{E}_t y_{t+2}$ with the highest time index. Then for country i the Phillips curve and the IS equation can be stated as follows:

$$
\begin{aligned}
\frac{g_f}{4} E_t p_{i,t+4} = {} & \left(1-\frac{g_f}{4}\right) E_t p_{i,t+1} - \frac{g_f}{4} E_t p_{i,t+2} - \frac{g_f}{4} E_{t+3} p_{i,t+3} \\
& -(1-g_f)\left[a_{p1} p_{i,t} + a_{p2} p_{i,t-1} + a_{p3} p_{i,t-3} + a_{p4} p_{i,t-4}\right] - a_y y_{i,t} \\
& -x\left(\frac{1}{4}\left(p_{-i,t} + p_{-i,t} + p_{-i,t} + p_{-i,t}\right)\right)
\end{aligned}
\qquad (4.A.4)
$$

$$
\begin{aligned}
w_y E_t y_{i,t+2} + \frac{b_r w_r}{4} E_t p_{i,t+4} = {} & E_t y_{i,t+1} - (1-w_y)\left[ b_{y1} y_{i,t} + b_{y2} y_{i,t-1} \right] \\
& + b_r w_r \left[ i_t - \frac{1}{4} E_t \left( p_{i,t+1} + p_{i,t+2} + p_{i,t+3} \right) \right] \\
& - i \left( \frac{1}{4} \left( p_{-i,t} + p_{-i,t-1} + p_{-i,t-2} + p_{-i,t-3} - p_{i,t} - p_{i,t-1} - p_{i,t-2} - p_{i,t-3} \right) \right) \qquad (4.A.5) \\
& - f \left( \frac{-c}{2} \left( y_{i,t} - y_{i,t-1} \right) + z\, g_{i,t-1} \right) \\
& - ex^{d}
\end{aligned}
$$

Hence we can rewrite the general model in state space form as:

$$
A_0 \begin{bmatrix} X_{1t+1} \\ E_t X_{2t+1} \end{bmatrix} = A \begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + B i_t + \begin{bmatrix} e_{t+1} \\ 0_{n2x1} \end{bmatrix}. \qquad (4.A.6)
$$

$$
X_{1t} = \left\{ p_{i,t}, p_{i,t-1}, p_{i,t-2}, p_{i,t-3}, y_{i,t}, y_{i,t-1}, g_{i,t}, i_{i,t-1}, i_{i,t-2}, i_{i,t-3}, p_{-i,t}, p_{-i,t-1}, p_{-i,t-2}, p_{-i,t-3}, y_{-i,t}, y_{-i,t-1}, g_{-i,t} \right\}'
$$

$$
X_{2t} = \left\{ E_t p_{i,t+3}, E_t p_{i,t+2}, E_t p_{i,t+1}, E_t y_{i,t+1}, E_t p_{-i,t+3}, E_t p_{-i,t+2}, E_t p_{-i,t+1}, E_t y_{-i,t+1} \right\}'
$$

$$
n_{1t} = \left\{ e_{i,t}, 0,0,0, h_{i,t}, 0, e^{g}_{it}, 0,0,0, e_{-i,t}, 0,0,0, h_{-i,t}, 0, e^{g}_{-it} \right\}'
$$

Where $X_{1t}$ is a $17\times1$ vector of predetermined state variables $X_{2t}$ is a $8\times1$ vector of forward looking variables and $n_{1t}$ is a vector of shocks. Following Söderlind et al. (2005) we have made use of the fact that $p_{t+1} = E_t p_{t+1} + e_{t+1}$ and that $y_{t+1} = E_t y_{t+1} + h_{t+1}$.

$$
A_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g_f}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{b_r m_r}{4} & 0 & 0 & m_y & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g_f}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{b_r m_r}{4} & 0 & 0 & m_y
\end{bmatrix}
$$

Note that due to the specific structure of the matrix $A_0$ it holds that $A_0^{-1} v_t = v_t$.

$$A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{c}{2} & -\frac{c}{2} & z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-c}{2} & \frac{-c}{2} & z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-(1-g_f)a_{p_1} & -(1-g_f)a_{p_2} & -(1-g_f)a_{p_3} & -(1-g_f)a_{p_4} & -a_{p_5} & 0 & 0 & 0 & 0 & 0 & \frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & 0 & 0 & 0 & -\frac{m_p}{4} & -\frac{m_p}{4} & \left(1-\frac{m_p}{4}\right) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-b_r(1-m_r)}{4}+\frac{t}{4} & \frac{-b_r(1-m_r)}{4}+\frac{t}{4} & \frac{-b_r(1-m_r)}{4}+\frac{t}{4} & \frac{-b_r(1-m_r)}{4}+\frac{t}{4} & -b_{y1}(1-m_y)+\left(f\left(\frac{c}{2}\right)\right) & -b_{y1}(1-m_y)+\left(f\left(\frac{c}{2}\right)\right) & -fz & \frac{b_r(1-m_r)}{4} & \frac{b_r(1-m_r)}{4} & 0 & \frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & \frac{-j}{2} & \frac{-j}{2} & 0 & -\frac{b_r m_r}{4} & -\frac{b_r m_r}{4} & -\frac{b_r m_r}{4} & 1 & 0 & 0 & 0 & 0 \\
\frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & 0 & 0 & 0 & 0 & 0 & 0 & -(1-g_f)a_{p_1} & -(1-g_f)a_{p_2} & -(1-g_f)a_{p_3} & -(1-g_f)a_{p_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{m_p}{4} & -\frac{m_p}{4} & \left(1-\frac{m_p}{4}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & \frac{-i}{4} & 0 & 0 & 0 & \frac{b_r(1-m_p)}{4} & \frac{b_r(1-m_p)}{4} & \frac{b_r(1-m_p)}{4} & \frac{-b_r(1-m_y)}{4}+\frac{t}{4} & \frac{-b_r(1-m_y)}{4}+\frac{t}{4} & \frac{-b_r(1-m_y)}{4}+\frac{t}{4} & \frac{-b_r(1-m_y)}{4}+\frac{t}{4} & b_{y1}(1-m_r)-\left(f\left(\frac{c}{2}\right)\right) & b_{y1}(1-m_r)-\left(f\left(\frac{c}{2}\right)\right) & -fz & 0 & 0 & 0 & 0 & -\frac{b_r m_r}{4} & -\frac{b_r m_r}{4} & -\frac{b_r m_r}{4} & 1
\end{bmatrix}$$

$$B_1 = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad b_r m_r + b_r\left(1-m_r\right)/4 \quad 0 \quad 0 \quad 0 \quad b_r m_r + b_r\left(1-m_r\right)/4\right]'$$

## 4. B: Defining the Measurement Equation

**US-model:** Let us define a vector $Y_t$ of measurement variables in which the monetary policy maker is interested in. We assume that the goal variables are given by:

$$Y_t = [p_t \;\; \bar{p}_t \;\; p_{t-1} \;\; p_{t-2} \;\; p_{t-3} \;\; y_t \;\; y_{t-1} \;\; i_t \;\; i_{t-1} \;\; i_{t-2} \;\; i_{t-3} \;\; \Delta i_t \;\; \Delta i_{t-1} \;\; \Delta i_{t-2} \;\; \Delta p_t \;\; \Delta p_{t-1} \;\; \Delta p_{t-2} \;\; \Delta y]$$

We can define the target variables as a function of the state variables and the interest rate.

$$Y_t = (C_{X1} \;\; C_{X2}) X_t + C_i i_t \tag{4.B.1}$$

$$C_{X_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0{,}25 & 0{,}25 & 0{,}25 & 0{,}25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{C}_{\mathrm{X}2} = 0_{18x4}$$

$$C_i = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]'$$

**Euro-Area Model:** Applying the same procedure as beforehand let us define a vector $Y_t$ as target variables in which the ECB is interested. We assume that the common central bank is only interested in aggregate variables:

$$Y_t = \begin{bmatrix} p_t & \overline{p}_t & p_{t-1} & p_{t-2} & p_{t-3} & y_t & y_{t-1} & i_t & i_{t-1} & i_{t-2} & i_{t-3} & \Delta i_t & \Delta i_{t-1} & \Delta i_{t-2} & \Delta p_t & \Delta p_{t-1} & \Delta p_{t-2} & \Delta y_t \end{bmatrix}$$

We can define the target variables as a function of the state variables with the help of equation (4.B.1).

$$(C_{X1} \quad C_{X2}) = \begin{bmatrix}
0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.125 & 0.125 & 0.125 & 0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0.125 & 0.125 & 0.125 & 0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$C_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$$

# 4. C: The Linear Quadratic Control Problem

The starting point of the linear quadratic control problem is the following value function:

$$V(X,t) = \min_{i_t}\left\{\sum_{t=0}^{T} b^t\left(\tfrac{1}{2}X_t'Q_tX_t + i_t'Ri_t\right) + \tfrac{1}{2}X_{T+1}'W_{T+1}X_{T+1}\right\}. \tag{4.C.1}$$

Subject to the constraint:

$$\begin{bmatrix} X_{1t+1} \\ E_tX_{2t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} i_t + \begin{bmatrix} v_{t+1} \\ 0_{n2\times 1} \end{bmatrix}. \tag{4.C.2}$$

Premultiplying by $A_0$ yields the standard state space form:

$$\begin{bmatrix} X_{1t+1} \\ E_tX_{2t+1} \end{bmatrix} = A\begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} + Bi_t + v_{t+1}. \tag{4.C.3}$$

With $A = A_0^{-1}A_1$ and $B = A_0^{-1}B_1$. Given the specific structure of the matrix $A_0$ it holds that: $A_0^{-1}v_{t+1} = v_{t+1}$. The variance covariance matrix will be given by:

$$\Sigma_{v1} = A_0^{-1}v_t\left(A_0^{-1}v_t\right)' = A_0^{-1}v_tv_t'A_0^{-1}. \tag{4.C.4}$$

Consequently it holds that the variance-covariance matrix stays a diagonal matrix with the following diagonal elements: $diag\left\{s_e^2 \quad 0 \quad 0 \quad 0 \quad s_h^2 \quad 0 \quad 0 \quad 0 \quad 0\right\}$

The value function has to satisfy in each period the following Bellman equation:

$$V(X_t) = \min_{i_t}\left\{X_t'QX_t + i_t'Ri_t + bV\left(X_{t+1}\right)\right\}. \tag{4.C.5}$$

A cornerstone assumption in order to solve the model is to postulate a (linear) way according to which expectations are formed. We make the fundamental assumption that expectations are built as follows:

$$E_t X_{2,t+1} = C_{t+1} E_t X_{1,t+1} \,. \tag{4.C.6}$$

As every distinct policy rule is linked to a different C matrix the approach takes care of the well-known Lucas critique. The policy maker cannot take expectations as given when changing the policy rule. With this assumption at hand one can arrive at a value function, which is only expressed in terms of predetermined variables:

$$V(t) = X_{1t}^{'} Q_t^{*} X_{1t} + r_t^{'} R_t^{*} r_t + b\, E_t (V_{t+1}) \tag{4.C.7}$$

Taking the F.O.C we arrive again at expressions for the optimal feedback rule as well as for the Ricatti-matrix V. Nevertheless contrasting the backward looking case our solution algorithm is quite different, as we do not only lack the matrix V but also the matrix C. Therefore the algorithm functions as follows. With an initial guess for $V_0$ and $C_0$ at hand we can iterate on the respective matrix equation until some matrix norm $\|C_{t+1} - C_t\| < e$ and $\|V_{t+1} - V_t\| < e$ has converged.

The (converged) time invariant solution can be written as:

| **TIME INVARIANT SOLUTIONS IN THE BACKWARD LOOKING MODEL** |
|---|
| $X_{1t+1} = (A_{11} + A_{12}C - B_1 F) X_{1t}$ <br> $X_{2t} = CX_{1t}$ <br> $F = -\left(R_t^{*} + b\, B_t^{*} V_{t+1} B_t^{*}\right)^{-1} \left(U_t^{*'} + b\, B_t^{*'} V_{t+1} A_t^{*}\right) X_{1t}$ |

The solution nicely depicts the expectational feedback, as the variable C does not only determine the forward looking variables $X_{2t}$ but also influences the predetermined variables $X_{1t}$.

# 4. D: The Individual Calibration Criterion

**Criterion 1:** $$Crit1 = (1/4)\begin{bmatrix} abs((stdv(p) - stdv(p_{87-02}))/stdv(p_{87-02})) \\ +abs((AC1(p) - AC1(p_{87-02}))/AC1(p_{87-02})) \\ +abs((AC2(p) - AC2(p_{87-02}))/AC2(p_{87-02})) \\ +abs((AC3(p) - AC3(p_{87-02}))/AC3(p_{87-02})) \end{bmatrix}^2$$

**Criterion 2:** $$Crit2 = (1/2)\begin{bmatrix} abs((sdtv(y) - stdv(y)_{87-02})/stdv(y)_{87-02}) \\ +abs((AC1(y) - AC1(y)_{87-02}))/stdv(y)_{87-02}) \end{bmatrix}^2$$

**Criterion 3:** $$Crit3 = (1/4)\begin{bmatrix} \left(abs(stdv(i) - stdv(i)_{87-02} / stdv(i)_{87-02}\right) \\ +\left(abs(AC1(i) - AC1(i)_{87-02}) / AC1(i)_{87-02}\right) \\ +\left(abs(AC2(i) - AC2(i)_{87-02}) / AC2(i)_{87-02}\right) \\ +\left(abs(AC3(i) - AC3(i)_{87-02}) / AC3(i)_{87-02}\right) \end{bmatrix}^2$$

**Criterion 4:** $$Crit4 = (1/3)\begin{bmatrix} abs\left(stdv(d(p)) - stdv(d(p)_{87-02}) / stdv(d(p)_{87-02})\right) \\ abs(AC1(d(p)) - AC1(d(p_{87-02}) / AC1(d(p)_{87-02})) \\ abs(AC2(d(p)) - AC2(d(p_{87-02}) / AC2(d(p)_{87-02})) \end{bmatrix}^2$$

**Criterion 5:** $$Crit5 = \left[abs((sdtv(d(y)) - stdv(d(y))_{87-02}) / stdv(d(y))_{87-02})\right]^2$$

**Criterion 6:** $$Crit6 = (1/3)\begin{bmatrix} \left(abs(stdv(d(i)) - stdv(d(i))_{87-02} / stdv(d(i))_{87-02}\right) \\ +\left(abs(AC1(d(i)) - AC1(d(i))_{87-02}) / AC1(d(i))_{87-02}\right) \\ +\left(abs(AC2(d(i)) - AC2(d(i))_{87-02}) / AC2(d(i))_{87-02}\right) \end{bmatrix}^2$$

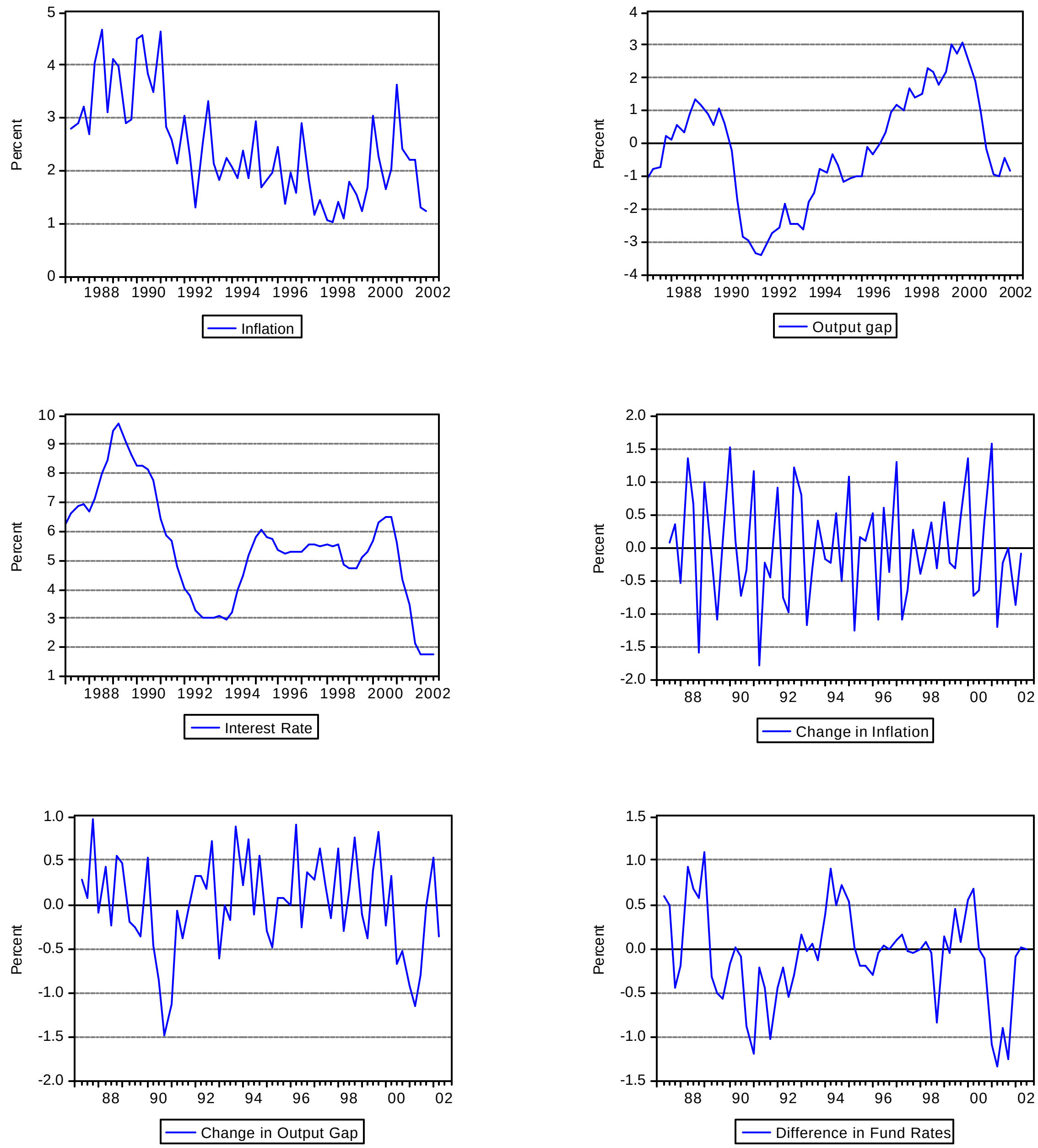
# 4.E. TIME SERIES PROPERTIES OF THE TEN BEST ESTIMATES

**Table 15** Time series properties: Simulated and actual data: (1987:4-2002:1)

| RANK | LEVELS | | | | ONE-QUATER-CHANGES | | | |
|---|---|---|---|---|---|---|---|---|
| $\psi_i$ | STANDARD DEVIATION | AC(1) | AC(2) | AC(3) | STANDARD DEVIATION | AC(1) | AC(2) | AC(3) |
| | | | | **Inflation** | | | | |
| **Data** | **0.9794** | **0.649** | **0.514** | **0.585** | **0.8105** | **-0.32** | **-0.283** | **0.101** |
| 1 | 1.4678 | 0.6325 | 0.4487 | 0.4069 | 1.2584 | -0.2501 | -0.193 | |
| 2 | 1.4688 | 0.632 | 0.4473 | 0.4042 | 1.2601 | -0.2491 | -0.1924 | |
| 3 | 1.4588 | 0.627 | 0.4406 | 0.3983 | 1.26 | -0.2502 | -0.1931 | |
| 4 | 1.4485 | 0.6217 | 0.4336 | 0.3921 | 1.2599 | -0.2513 | -0.1939 | |
| 5 | 1.4377 | 0.6161 | 0.4261 | 0.3856 | 1.2598 | -0.2525 | -0.1947 | |
| 10 | 1.4668 | 0.67328 | 0.49702 | 0.49383 | 1.1857 | -0.23026 | -0.2649 | |
| | | | | **Output Gap** | | | | |
| **Data** | **1.6953** | **0.945** | **0.865** | **0.755** | **0.5462** | **0.28** | **0.278** | **0.049** |
| 1 | 1.6026 | 0.8256 | | | 0.9464 | | | |
| 2 | 1.613 | 0.8278 | | | 0.9467 | | | |
| 3 | 1.6085 | 0.8265 | | | 0.9475 | | | |
| 4 | 1.6038 | 0.8252 | | | 0.9484 | | | |
| 5 | 1.5988 | 0.8237 | | | 0.9493 | | | |
| 10 | 1.4906 | 0.78226 | | | 0.98367 | | | |
| | | | | **Federal Funds Rate** | | | | |
| **Data** | **1.9326** | **0.930** | **0.814** | **0.671** | **0.5365** | **0.58** | **0.303** | **0.191** |
| 1 | 1.7351 | 0.58 | 0.303 | 0.191 | 0.6093 | 0.6042 | 0.2837 | |
| 2 | 1.7262 | 0.58 | 0.303 | 0.191 | 0.5999 | 0.6136 | 0.296 | |
| 3 | 1.7332 | 0.58 | 0.303 | 0.191 | 0.6075 | 0.6109 | 0.2916 | |
| 4 | 1.7411 | 0.58 | 0.303 | 0.191 | 0.6159 | 0.608 | 0.2869 | |
| 5 | 1.7501 | 0.58 | 0.303 | 0.191 | 0.6253 | 0.6049 | 0.2818 | |
| 10 | 1.5113 | 0.58 | 0.303 | 0.191 | 0.4626 | 0.5873 | 0.2810 | |

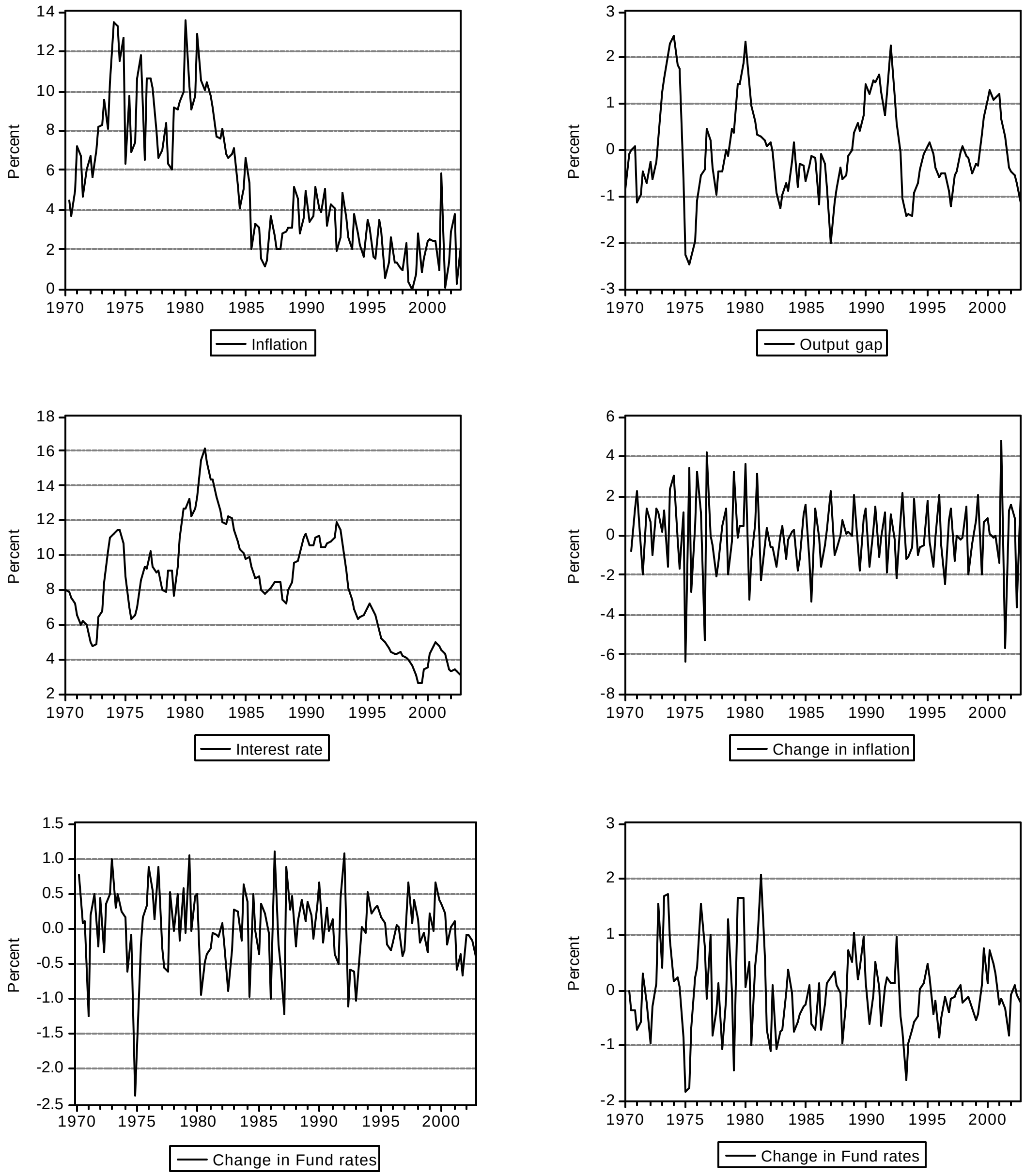
# 4. F: Stylized Time Series Properties: Levels and Differences

**Figure 4.A.1:** US-data




All data were taken from: [http://research.stlouisfed.org/ fred/](http://research.stlouisfed.org/fred/)

**Figure 4.A.2:** Euro-Area-data



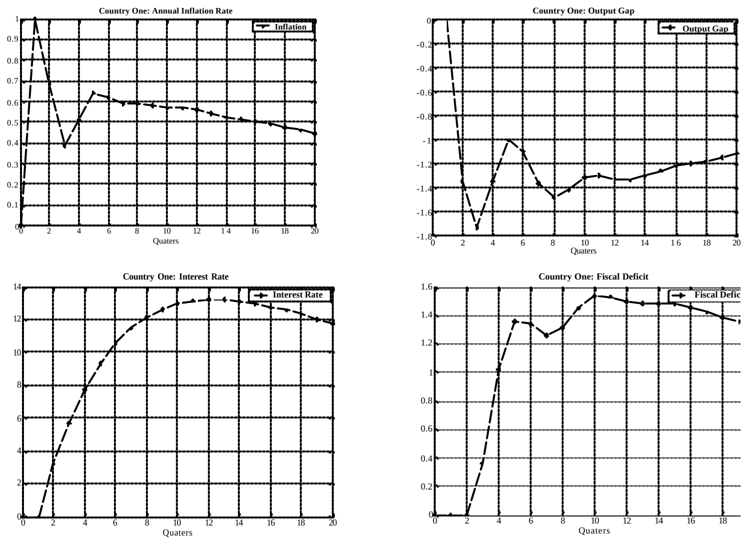

Data was taken from Fagan, Jerome and Ricardo (2002).

# 4. G: Variance-Covariance Matrix

$$\Sigma = \begin{bmatrix}
1.89 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & 0.64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.32 & 0 & 0 \\
 & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & 1.89 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & & & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & & & & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & & & & & 0.64 & 0 & 0 \\
 & & & & & & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & & & & & 0 & 1
\end{bmatrix}$$

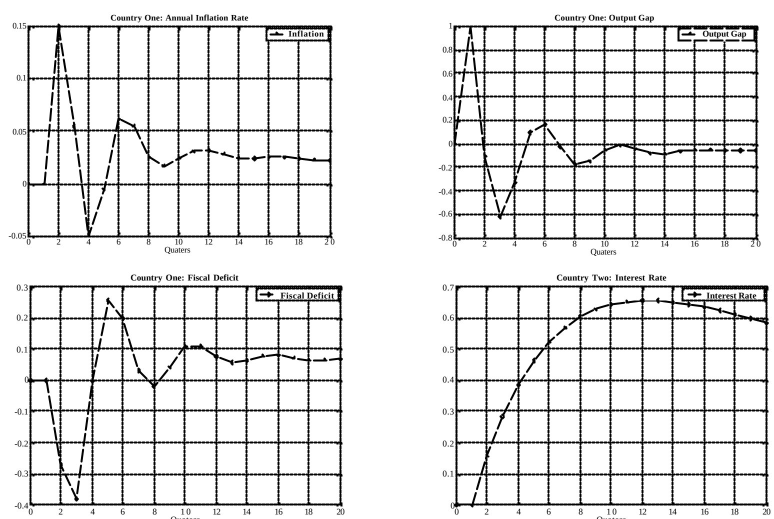
# 4. H: IMPULSE RESPONSE FUNCTIONS

**Figure 4.H.1:** Symmetric Supply Shock Hitting the Currency Area*



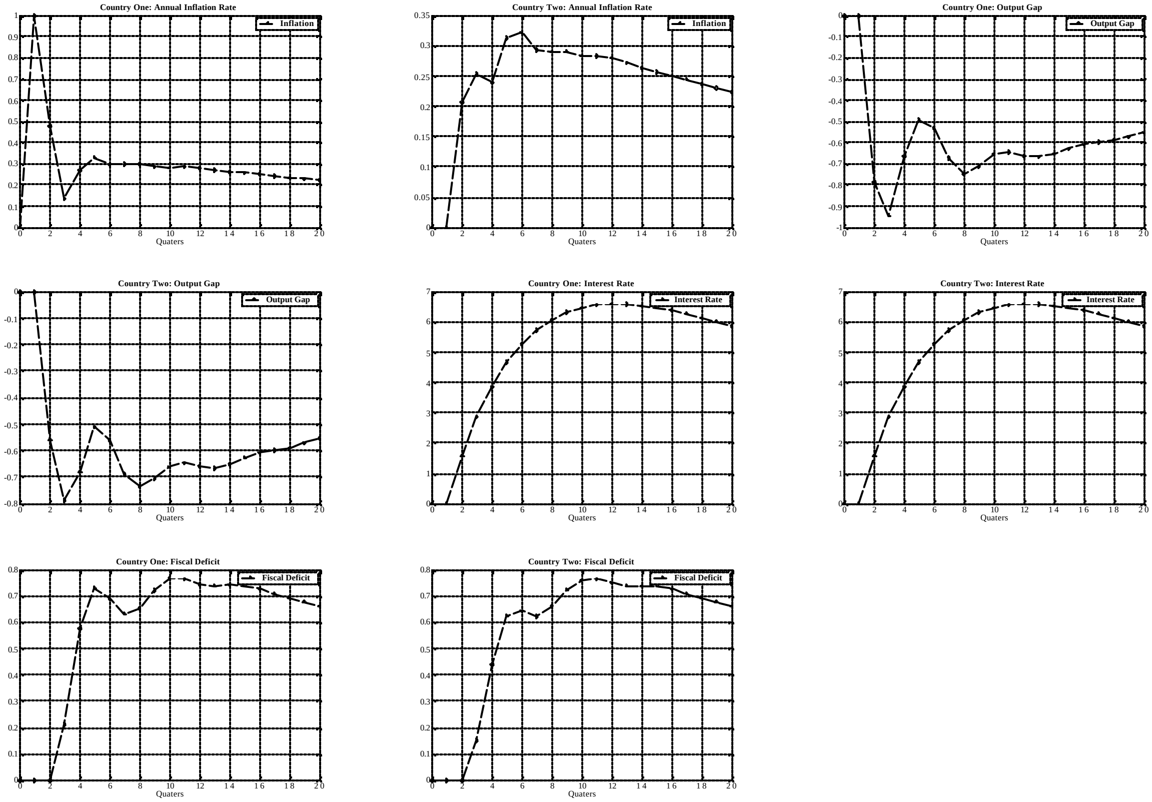

*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 4.H.2:** Symmetric Demand Shock Hitting the Currency Area*



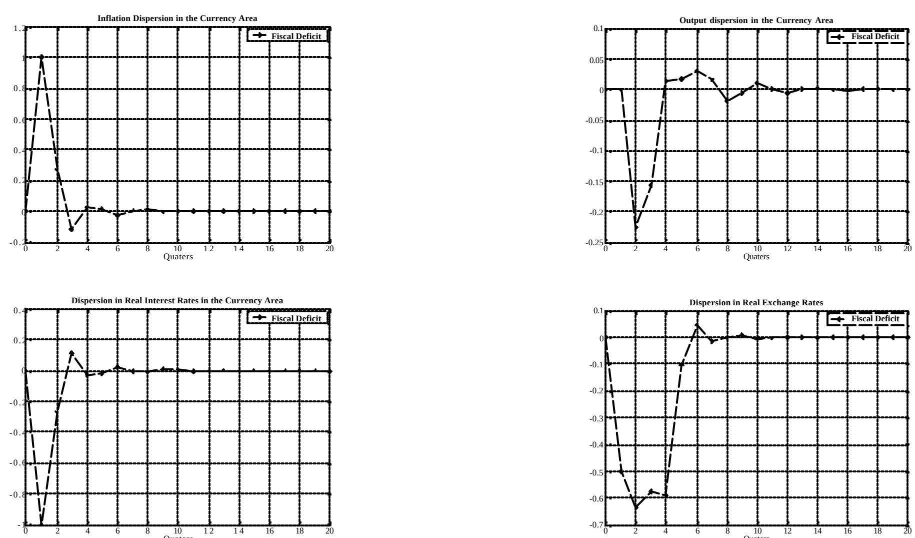

*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 4.H.3:** Asymmetric Supply Shock Hitting the Currency Area*



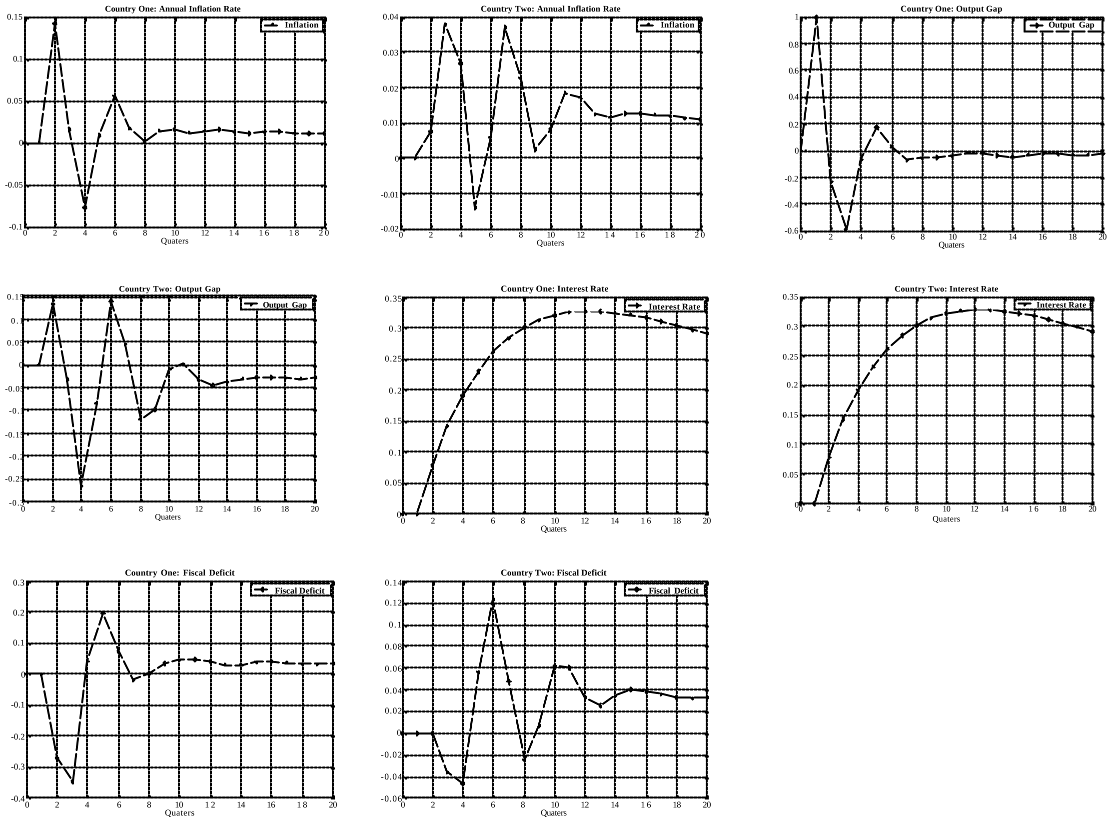

*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 4.H.4:** Asymmetric Supply Shock: Dispersion Indicators *



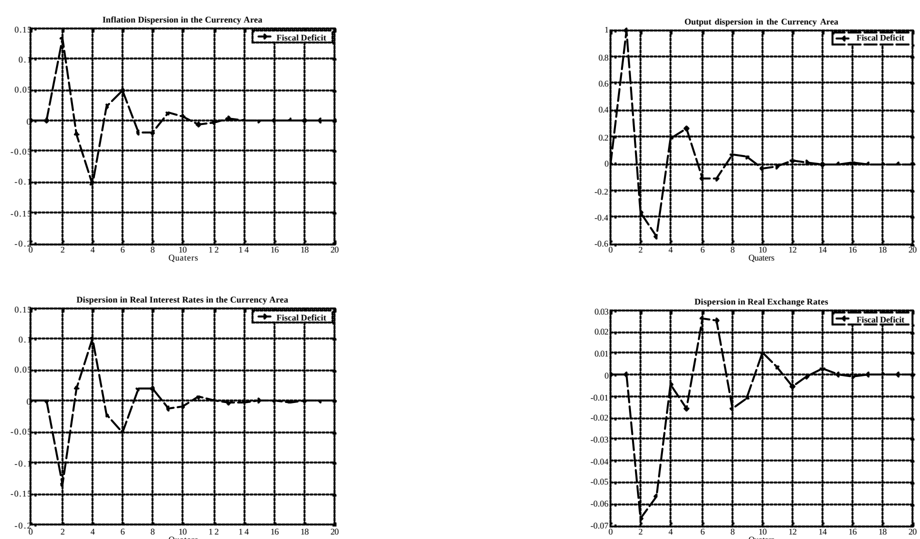

*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 4.H.5:** Asymmetric Demand Shock Hitting the Currency Area*



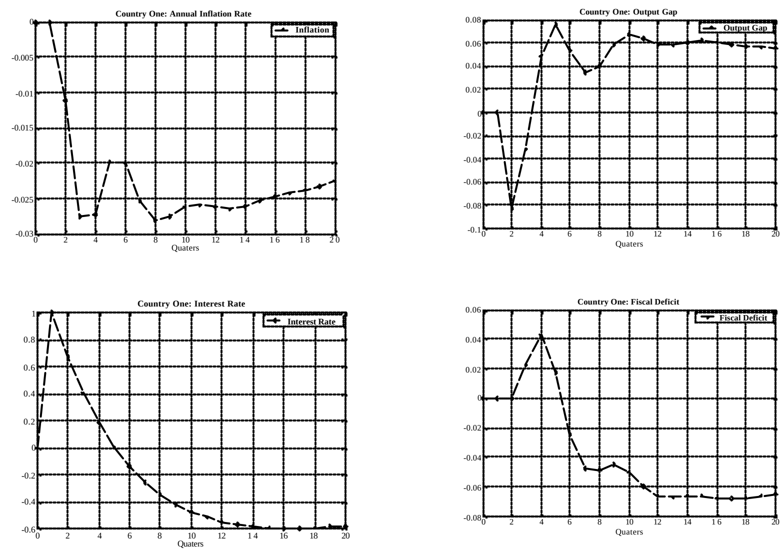

*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 4.H.6:** Asymmetric Demand Shock: Dispersion Indicators*




*The dotted line plots the impulse response function for the estimated baseline calibration.

**Figure 4.H.7:** Interest Rate Shock Hitting the Currency Area*




*The dotted line plots the impulse response function for the estimated baseline calibration.

# 5 Concluding Remarks

We have shown in depth that New Keynesian macroeconomics makes it possible to think micro founded and Keynesian at the same time. Based on a highly stylized economy New Keynesian macroeconomics can be reduced to a system of three equations: An intertemporal IS-equation, a NKPC and a relationship depicting the conduct of monetary policy. The insight that there exists nominal inertia in the economy states nothing but that the central bank can steer the real short term interest rate by manipulating the nominal short term interest rate according to its preferences. Following these 'new insights' monetary policy serves as an insurance company that promises to smooth out macroeconomic fluctuations. Quite impressively it offers these services to society at negligible costs. Theoretical evidence seems to suggest that exogenous shocks might be detrimental in terms of output and price volatility if monetary policy would not dampen economic cycles (Canzoneri, Cumby, Dina (2004)). Not surprisingly the concrete question whether an economy is good natured in terms of stabilization properties depends critically on the people populating the economy. The more households and firms believe in the New Keynesian model, the trend growth path and the inflation target, the easier the economy can be steered by the central bank. In a backward looking environment it becomes more difficult to manoeuver the economy as we do not observe the self-stabilizing properties of forward looking systems. Economic agents do not react on impulse to signals set by the central bank but just on macroeconomic outcomes. This lengthens the link between monetary impulses and the reaction in the goal variables. Hence, if monetary policy opts for an instrument rule that generates a stable and unique rational expectations equilibrium, the conduct of monetary policy is much easier as it would be in a purely backward looking economy, due to the implied self-stabilizing properties of forward looking systems grounded on people's expectations on stabilizing monetary policy itself (self-fulfilling expectations). We have shown in depth in chapter 2.2 that it is possible to map the New Keynesian framework into a simple three equation model that preserves the main insights of the theory while equally being powerful enough to discuss central issues like inflation targeting and issues of credibility.

Given this New Keynesian apparatus of mind we have analyzed the interaction between monetary and fiscal policy in a monetary union. Unfortunately a dominating stream in literature (Dixit and Lambertini (2003)) focuses on game theoretic interactions based on the

Barro-Gordon (1983) framework. Unfortunately Dixit and Lambertini entirely focus on the issue of time consistency. Thereby they neglect to analyze the beneficial impact of stabilization policy if the union is hit by symmetric or asymmetric shocks. Therefore we extend Dixits and Lambertinis joint commitment solution to the case where the common monetary union is hit by symmetric or asymmetric supply and demand shocks. In contrast to Dixit and Lambertini (2001, 2003) we apply the New Keynesian framework to analyze a currency area. We have shown that asymmetric shocks can drive a wedge between macroeconomic outcomes even if the central bank is guided by a state of the art strategy of inflation targeting. Largely neglected in related literature we have evaluated the impact of diverging real interest rates in a currency area. In principle, fiscal policy guided by a loss function can significantly reduce cyclical variations in macroeconomic aggregates. Therefore we have proposed to reform the current SGP. A monetary union calls for a renaissance of fiscal policy from a stabilization perspective. With the launch of third stage EMU the member countries have embarqued to unknown territory to tighten the political vision of a common European future. While the current macroeconomic framework with its focus on the defcit criterion is clearly too one-dimensional, it could be relatively easy supplemented with an additional dimension which takes care on the mix between the common monetary policy and national fiscal policies. We have proposed that as long as the majority of inflation forecasts shows that a countries inflation rate will remain within the ECB's target range there would be the presumption that the overall policy mix of national fiscal policy and the national real interest rate is adequate. In this situation, a fiscal deficit exceeding the 3% threshold would not pose a problem from the perspective of the common monetary policy.

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