CP Violation in Production and Decay of Supersymmetric Particles

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Contents

Prologue

Supersymmetry and the search for new particles

Elementary particle physics has made enormous progress in the last decades. The electroweak and strong interactions of the fundamental building blocks of matter, the quarks and leptons, are now described by the so called Standard Model (SM) of particle physics.

The development of the SM was possible due to intensive efforts and successful achievements in experiment and theory. On the one side, theoreticians have provided the physical models and the mathematical techniques necessary to define and calculate observables. On the other side, experiments with particle accelerators and detectors have not only allowed to find new fundamental particles, but also high precision measurements have made it possible to test the models.

However, there are general arguments which point towards the existence of a theory beyond the SM. One of the most attractive candidates for such a more fundamental theory is Supersymmetry (SUSY). SUSY transformations change the spin of a particle field, and thus bosonic and fermionic degrees of freedom get related to each other. Therefore, new particles are predicted in SUSY models like in the minimal supersymmetric extension of the SM. If SUSY is realized in nature, the supersymmetric partners of the SM particles have to be discovered.

The next future colliders, like the Large Hadron Collider (LHC) at CERN or a planned International Linear Collider (ILC), are designed to find these particles. Their properties will be measured with high precision in production and decay processes. The underlying physical model can then be determined by a comparison of experimental and phenomenological studies.

Chapter 1

Introduction

1.1 Motivation: Symmetries and models

Symmetries in physics have always played an important role in understanding the structure of the underlying theories. For instance, the existence of conservation laws can be explained by specific symmetry transformations under which a theory is invariant. Energy, momentum and angular momentum are conserved in field theories with continuous spacetime symmetries.

High energy models of elementary particle interactions have to be invariant under the transformations of the Poincaré group, which are relativistic generalizations of the spacetime symmetries. The interactions of the particles, the strong and electroweak forces, can be understood as a consequence of the so called gauge symmetries.

But not only such continuous symmetries are crucial. The discrete symmetries

- Charge conjugation C: interchange of particles with antiparticles
- Parity P: transformation of the space coordinates $\mathbf{x} \to -\mathbf{x}$
- Time reversal T: transformation of the parameter time $t \to -t$

are also essential for the formulation of relativistic quantum field theories. In particular, any Poincaré invariant local field theory has to be symmetric under the combined transformation of C, P and T, which is called CPT invariance.

The discovery in the fifties that weak interactions violate C and P maximally, was noted with substantial belief that a particle theory apparently still conserves the combined symmetry CP. However, a few years later, in 1964, CP violation was confirmed in the K-meson system.

This lead to a powerful prediction in 1972. The implementation of CP violation made it necessary to include a phase in the quark mixing matrix, the so called Kobayashi-Maskawa Matrix [1]. Such a phase is only CP violating if the matrix is at least of dimension three, and thus a third generation of quarks and leptons was required. This prediction of a third family was made long before the final member of the second family, the charm quark, was found.

Current experiments with B-mesons verify the existence of one CP phase in the quark mixing matrix of the SM. However, one phase alone cannot explain the observed baryon asymmetry of the universe, as shown in [2]. The fact that further sources of CP violation are needed leads to the crucial prediction of CP violating phases in theories beyond the SM.

Apart from that, a further important symmetry of physics was born in 1974: Supersymmetry (SUSY), which relates fermionic and bosonic degrees of freedom. SUSY models, like the the Minimal Supersymmetric Standard Model (MSSM), are one of the most attractive theories beyond the SM. They give a natural solution to the hierarchy problem and provide neutralinos as dark matter candidates. Furthermore, SUSY allows for grand unifications and for theories, which might include also gravity.

1.2 CP violating phases and electric dipole moments

The MSSM might have several complex parameters, which cause CP violating effects. In the neutralino and chargino sector these are the Higgsino mass parameter $\mu = |\mu| e^{i\varphi_\mu}$ and the $U(1)$ gaugino mass parameter $M_1 = |M_1| e^{i\varphi_{M_1}}$ [3]. The $SU(2)$ gaugino mass parameter M_2 can be made real by redefining the fields. In the sfermion sector of the MSSM, also the trilinear scalar coupling parameter A_f of the sfermion $\tilde{f}$ can be complex, $A_f = |A_f| e^{i\varphi_{A_f}}$.

The CP violating phases are constrained by electric dipole moments (EDMs) [4] of electron e, neutron n, ^{199}Hg and ^{205}Tl atoms. Their upper bounds are, respectively:

$$|d_e| < 4.3 \times 10^{-27}\, e\,\mathrm{cm}\ [5], \tag{1.1}$$

$$|d_n| < 6.3 \times 10^{-26}\, e\,\mathrm{cm}\ [6], \tag{1.2}$$
$$|d_{Hg}| < 2.1 \times 10^{-28}\, e\,\mathrm{cm}\ [7], \tag{1.3}$$
$$|d_{Tl}| < 1.3 \times 10^{-24}\, e\,\mathrm{cm}\ [8]. \tag{1.4}$$

The CP phase in the quark sector of the SM give contributions to EDMs, which generally arise at two loop level, and respect the bounds of the EDMs. In SUSY models, however, neutralino and chargino contributions to the electron EDM can occur at one loop level, see Fig. 1.1. For the neutron EDM in addition also gluino exchange contributions are present due to a phase of the gluino mass parameter. The phases of the SUSY parameters are thus constrained by the experimental upper limits of the EDMs. In the literature, three solutions are being proposed [9]:

- The SUSY phases are severely suppressed [10, 11].
- SUSY particles of the first two generations are rather heavy, with masses of the order of a TeV [12].
- There are strong cancellations between the different SUSY contributions to the EDMs, allowing a SUSY particle spectrum of the order of a few 100 GeV [13–15].

Due to such cancellations, for example, in the constrained MSSM [14], the phase φ_{M_1} is not restricted but the phase of μ is still constrained with $|\varphi_\mu| \lesssim 0.1\pi$ [14]. If lepton flavor violating terms are included [15], also the restriction on φ_μ may disappear.

The restrictions on the SUSY phases are thus very model dependent. Independent methods for their measurements are desirable, in order to clarify the situation. In order to determine the phases unambiguously, measurements of CP sensitive observables are necessary. Such observables are non-zero only if CP is violated, i.e. they are proportional to the sine of the phases.

Figure 1.1: SUSY contributions to the electron EDM.

1.3 Methods for analyzing CP violating phases

1.3.1 Neutralino and chargino polarizations

For measuring SUSY phases, the study of neutralino and chargino production at an e^+e^- linear collider with longitudinally polarized beams [16] will play an important role. By measurements of the chargino masses and production cross sections, a method has been developed [17–19] to determine $\cos(\varphi_\mu)$, in addition to the other parameters M_2, $|\mu|$ and $\tan\beta$ of the chargino sector. For neutralino production analogous methods have been proposed in [19–22] to determine also $\cos(\varphi_{M_1})$ and M_1, besides $\cos(\varphi_\mu)$, M_2, $|\mu|$ and $\tan\beta$.

However, in order to determine also the sign of φ_μ and φ_{M_1}, the transverse neutralino and chargino polarizations perpendicular to the production plane have to be taken into account [17, 18, 23]. They are only present if there are CP violating phases in the neutralino/chargino sector, and if a pair of different neutralinos/charginos is produced. At tree level, their polarizations lead to triple-product asymmetries of the decay products [24, 25]. Energy distributions and polar angle distributions of the neutralinos and charginos or their decay products are not CP sensitive at tree level, since they do not depend on the transverse neutralino or chargino polarizations, see e.g. [26] for neutralino production.

In order to include the particle polarizations in our calculations, we use the spin density matrix formalism of [27]. For an introduction into this formalism and for our conventions and definitions used, see Appendices C and D.

1.3.2 T-odd and CP-odd triple-product asymmetries

The SUSY phases give rise to T-odd and CP-odd observables which involve triple products of momenta [28]. They allow us to define various T and CP asymmetries which are sensitive to the different SUSY phases. On the one hand, these observables can be large because they are present at tree level. On the other hand, they also allow a determination of the sign of the phases, which is impossible if only CP-even observables were studied.

We consider neutralino or chargino production

$$e^+ + e^- \;\to\; \tilde{\chi}_i + \tilde{\chi}_j \tag{1.5}$$

followed by the two-body decay of one neutralino or chargino into a SM particle A (e.g. lepton or W, Z boson) and a SUSY particle $\tilde{X}$ (e.g. slepton or $\tilde{\chi}_1^0$):

$$\tilde{\chi}_i \to A + \tilde{X}. \tag{1.6}$$

The momenta of electron, chargino (or neutralino), and particle A define the triple product

$$\mathcal{T} = (\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_i}) \cdot \mathbf{p}_A, \tag{1.7}$$

which is T-odd, i.e. changes sign under time reversal. The T-odd asymmetry of the cross section σ of production (1.5) and decay (1.6) is then defined as

$$\mathcal{A}^{\mathrm{T}} = \frac{\sigma(\mathcal{T} > 0) - \sigma(\mathcal{T} < 0)}{\sigma(\mathcal{T} > 0) + \sigma(\mathcal{T} < 0)}. \tag{1.8}$$

The asymmetry can be expressed by the angular distribution of particle A

$$\mathcal{A}^{\mathrm{T}} = \frac{\int_1^0 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_0^{-1} \frac{d\sigma}{d\cos\theta} d\cos\theta}{\int_1^0 \frac{d\sigma}{d\cos\theta} d\cos\theta + \int_0^{-1} \frac{d\sigma}{d\cos\theta} d\cos\theta} = \frac{N_+ - N_-}{N_+ + N_-}, \tag{1.9}$$

where

$$\cos\theta := \frac{\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_i}}{|\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_i}|} \cdot \frac{\mathbf{p}_A}{|\mathbf{p}_A|}, \tag{1.10}$$

and thus $\mathcal{A}^{\mathrm{T}}$ is the difference of the number of events with particle A above (N_+) and below (N_-) the production plane, defined by $\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_i}$, normalized by the total number of events $N = N_+ + N_-$.

The T-odd asymmetry is not only sensitive to CP phases, but also to absorptive contributions, which could enter via s-channel resonances or final state interactions at loop level. Although the absorptive contributions are a higher order effect, and thus expected to be small, they do not signal CP violation. However, they can be eliminated in the CP-odd asymmetry

$$\mathcal{A}^{\mathrm{CP}} = \frac{1}{2}(\mathcal{A}^{\mathrm{T}} - \bar{\mathcal{A}}^{\mathrm{T}}), \tag{1.11}$$

where $\bar{\mathcal{A}}^{\mathrm{T}}$ denotes the asymmetry for the CP-conjugated process.

Note that the triple product $\mathcal{T}$ (1.7) requires the identification of the neutralino (or chargino) momentum $\mathbf{p}_{\chi_i}$, which could be reconstructed by measuring the decay of the other $\tilde{\chi}_j$. Therefore, the masses of the neutralinos/charginos as well as the masses of their decay products have to be known.

To avoid the reconstruction of $\mathbf{p}_{\chi_i}$, we can also define triple products in which $\mathbf{p}_{\chi_i}$ is replaced by a momentum of the decay products of particles A, if A is a W or Z boson, or $\tilde{X}$, if $\tilde{X}$ is a slepton. In the first case, $A = W$ or Z and $\tilde{X} = \tilde{\chi}_1^0$, the decay of the boson into two quarks

$$A \to q + q', \tag{1.12}$$

defines the triple product

$$\mathcal{T} = (\mathbf{p}_{e^-} \times \mathbf{p}_q) \cdot \mathbf{p}_{q'}. \tag{1.13}$$

In the second case, $A = \ell$ and $\tilde{X} = \tilde{\ell}$, the decay

$$\tilde{X} \to \ell + \tilde{\chi}_1^0, \tag{1.14}$$

defines the triple product

$$\mathcal{T} = (\mathbf{p}_{e^-} \times \mathbf{p}_A) \cdot \mathbf{p}_{\ell}. \tag{1.15}$$

These triple products define then corresponding T or CP asymmetries, which do not require the momentum reconstruction of $\tilde{\chi}_i$. However, for these triple products the leptons have to be distinguished by their energy distributions [29], and the quarks have to be distinguished by flavor tagging [30–32].

Triple-product asymmetries can also be defined and analyzed for three-body decays of neutralinos [23, 24, 33, 34] and charginos [25, 33, 35].

1.3.3 Statistical error and significance

The T-odd and CP-odd asymmetries, as defined in (1.8) and (1.11), could be measured in neutralino and chargino production at future linear collider experiments, and would allow us to determine the values of the SUSY phases. In order to decide whether an asymmetry, and thus a CP phase can be measured, we have to calculate its statistical error. Also, we have to consider the statistical significance of the asymmetry.

The relative statistical error of the asymmetry is given by

$$\delta\mathcal{A} := \frac{\Delta\mathcal{A}}{|\mathcal{A}|} = \frac{1}{|\mathcal{A}|\sqrt{N}}, \tag{1.16}$$

with the number of events $N = \mathcal{L}\cdot\sigma$, where $\mathcal{L}$ is the integrated luminosity of the linear collider. Formula (1.16) follows from (1.9), with the estimate $\Delta N_\pm = \sqrt{N_\pm} \approx \sqrt{N/2}$.

The statistical significance of the asymmetry (1.8) is then defined as

$$S = |\mathcal{A}^{\mathrm{T}}|\sqrt{\mathcal{L}\cdot\sigma}. \tag{1.17}$$

For $S = 1$, the asymmetry can be measured at the 68% confidence level (CL), for $S = 1.96$ at the 95% CL, etc. The significance for the CP-odd asymmetry (1.11) is given by

$$S = |\mathcal{A}^{\mathrm{CP}}|\sqrt{2\mathcal{L}\cdot\sigma}, \tag{1.18}$$

since $\Delta\mathcal{A}^{\mathrm{CP}} = \Delta\mathcal{A}^{\mathrm{T}}/\sqrt{2}$, which follows from (1.11).

Also background and detector simulations have to be taken into account to predict the expected accuracies for the asymmetries, see e.g. [36]. However, this would imply detailed Monte Carlo studies, which is beyond the scope of the present work.

1.4 Organization of the work

The goal of the thesis is to analyze CP violating effects of MSSM phases in production and/or two-body decay processes of neutralinos, charginos and sfermions. We will therefore define and calculate T-odd and CP-odd asymmetries for the different supersymmetric processes.

We study neutralino and chargino production in electron-positron collisions at a future linear collider (LC) with a center of mass energy of 500 GeV to 800 GeV, high luminosity and longitudinally polarized beams. A LC of this kind is an ideal tool for measuring the properties of SUSY particles with high precision.

Finally, we address the question, whether the phases can be constrained at the LC. We thus calculate the statistical significances for measuring the asymmetries. Our analyses will also have particular emphasis on the beam-polarization dependence of the asymmetries and cross sections.

In most of the numerical examples we choose $\varphi_{M_1} = \pm\pi/2$, $\varphi_\mu = 0$, which is allowed by the constraints from the electron and neutron EDMs. In order to show the full phase dependences of the asymmetries in some examples we study their φ_μ behavior in the whole φ_μ range, relaxing in this case the restrictions from the EDMs. This is justified e.g. in theories with lepton flavor violation [15], where the constraints on φ_μ disappear.

- Chapter 2 contains neutralino production $e^+e^- \to \tilde{\chi}_i^0\, \tilde{\chi}_j^0$ and decay:
  - In Section 2.1 we discuss neutralino decay into sleptons: $\tilde{\chi}_i^0 \to \ell\, \tilde{\ell}$ for $\ell = e, \mu, \tau$.
  - In Section 2.2 we discuss neutralino decay into a stau-tau pair: $\tilde{\chi}_i^0 \to \tau\, \tilde{\tau}$, including the τ polarization.
  - In Section 2.3 we discuss neutralino decay into a Z boson: $\tilde{\chi}_i^0 \to \tilde{\chi}_n^0\, Z$.
- Chapter 3 deals with chargino production $e^+e^- \to \tilde{\chi}_i^\pm\, \tilde{\chi}_j^\mp$ and decay:
  - In Section 3.1 we discuss chargino decay into sneutrinos: $\tilde{\chi}_i^+ \to \ell^+\, \tilde{\nu}_\ell$, $\ell = e, \mu, \tau$.
  - In Section 3.2 we discuss chargino decay into a W boson: $\tilde{\chi}_i^+ \to \tilde{\chi}_n^0\, W^+$.
- In chapter 4 we analyze CP violation in the two-body decay chain of a sfermion: $\tilde{f} \to f\, \tilde{\chi}_j^0,\ \tilde{\chi}_j^0 \to \tilde{\chi}_1^0\, Z,\ Z \to f\, \bar{f}$.
- Chapter 5 contains a summary and conclusions.
- In the Appendices we give a short account on the MSSM, with emphasis on its complex parameters. We discuss details of particle kinematics and phase space and give the analytical formulae for the production and decay amplitudes squared. Finally, we give useful spin-formulae for fermions and bosons and a formulary of our definitions and conventions, used for our numerical calculations.

Chapter 2

CP violation in production and decay of neutralinos

Overview

We study neutralino production with longitudinally polarized beams $e^+ e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ with the subsequent leptonic decay of one neutralino $\tilde{\chi}_i^0 \to \tilde{\ell}\,\ell$; $\tilde{\ell} \to \tilde{\chi}_1^0\,\ell$, for $\ell = e, \mu, \tau$, [29] or the decay into the Z boson $\tilde{\chi}_i^0 \to \chi_n^0 Z$; $Z \to \ell\bar{\ell}(q\bar{q})$ [37]. These decay modes allow the definition of CP observables which are sensitive to the phases φ_{M_1} and φ_μ.

For the leptonic decay of the neutralino into the tau $\tilde{\chi}_i^0 \to \tilde{\tau}^\pm \tau^\mp$, we propose the transverse $\tau^\mp$ polarization as a CP sensitive observable [38]. This asymmetry is also sensitive to the phase φ_{A_τ}.

We present numerical results for the asymmetries, cross sections and branching ratios for a linear electron-positron collider in the 500 GeV - 800 GeV range. The asymmetries can go up to 60% and we estimate the event rates which are necessary to observe the asymmetries. Polarized electron and positron beams can significantly enhance the asymmetries and cross sections.

2.1 T-odd asymmetries in neutralino production and decay into sleptons

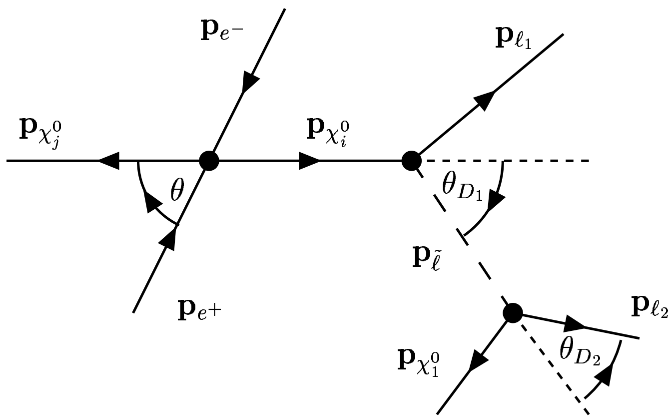


Figure 2.1: Schematic picture of the neutralino production and decay process.

For neutralino production

$$e^+ + e^- \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0 \tag{2.1}$$

with longitudinally polarized beams and the subsequent leptonic two-body decay of one of the neutralinos

$$\tilde{\chi}_i^0 \to \tilde{\ell} + \ell_1, \tag{2.2}$$

we introduce the triple-product

$$\mathcal{T}_I = (\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_i^0}) \cdot \mathbf{p}_{\ell_1}, \tag{2.3}$$

and define the T-odd asymmetry

$$\mathcal{A}_I^{\mathrm{T}} = \frac{\sigma_I(\mathcal{T}_I > 0) - \sigma_I(\mathcal{T}_I < 0)}{\sigma_I(\mathcal{T}_I > 0) + \sigma_I(\mathcal{T}_I < 0)}, \tag{2.4}$$

where σ_I is the cross section for reactions (2.1) and (2.2).

With the subsequent leptonic decay of the slepton

$$\tilde{\ell} \to \tilde{\chi}_1^0 + \ell_2; \quad \ell = e, \mu, \tau, \tag{2.5}$$

we can construct a further asymmetry which does not require the identification of the neutralino momentum. We replace the neutralino momentum $\mathbf{p}_{\chi_i^0}$ in (2.3) by the momentum $\mathbf{p}_{\ell_2}$ of the lepton from the slepton decay

$$\mathcal{T}_{II} \;=\; (\mathbf{p}_{e^-} \times \mathbf{p}_{\ell_2}) \cdot \mathbf{p}_{\ell_1} \tag{2.6}$$

and define the asymmetry

$$\mathcal{A}_{II}^{\mathrm{T}} = \frac{\sigma_{II}(\mathcal{T}_{II} > 0) - \sigma_{II}(\mathcal{T}_{II} < 0)}{\sigma_{II}(\mathcal{T}_{II} > 0) + \sigma_{II}(\mathcal{T}_{II} < 0)}, \tag{2.7}$$

where σ_{II} is the cross section for reactions (2.1) - (2.5).

These T-odd observables in the production of neutralinos at tree level are due to spin effects. Only if there are CP-violating phases φ_{M_1} and φ_μ in the neutralino sector and if two different neutralinos are produced, each of them has a polarization perpendicular to the production plane [20, 23, 39]. This polarization leads to asymmetries in the angular distributions of the decay products, as defined in (2.4) and (2.7).

2.1.1 Cross sections

In order to calculate the production and decay amplitudes, we use the spin density matrix formalism of [27, 39], see Appendix C. For neutralino production (2.1) and decay (2.2), the amplitude squared can be written as

$$|T_I|^2 \;=\; |\Delta(\tilde{\chi}_i^0)|^2 \sum_{\lambda_i \lambda_i'} \; \rho_P(\tilde{\chi}_i^0)^{\lambda_i \lambda_i'} \rho_{D_1}(\tilde{\chi}_i^0)_{\lambda_i' \lambda_i}, \tag{2.8}$$

with the neutralino propagator $\Delta(\tilde{\chi}_i^0)$, the spin-density matrix of neutralino production $\rho_P(\tilde{\chi}_i^0)$, the decay matrix $\rho_{D_1}(\tilde{\chi}_i^0)$, and the neutralino helicities λ_i, λ_i'. Inserting the expansions of the density matrices $\rho_P(\tilde{\chi}_i^0)$, see (C.10), and $\rho_{D_1}(\tilde{\chi}_i^0)$, see (C.28), into (2.8) gives

$$|T_I|^2 \;=\; 4\; |\Delta(\tilde{\chi}_i^0)|^2 \; (P D_1 + \vec{\Sigma}_P \vec{\Sigma}_{D_1}). \tag{2.9}$$

Analogously, the amplitude squared for the complete process of neutralino production, followed by the two-body decays (2.2) and (2.5), can be written as

$$|T_{II}|^2 = |\Delta(\tilde{\chi}_i^0)|^2 \, |\Delta(\tilde{\ell})|^2 \sum_{\lambda_i \lambda_i'} \rho_P(\tilde{\chi}_i^0)^{\lambda_i \lambda_i'} \rho_{D_1}(\tilde{\chi}_i^0)_{\lambda_i' \lambda_i} \, D_2 \tag{2.10}$$

$$= 4 \, |\Delta(\tilde{\chi}_i^0)|^2 \, |\Delta(\tilde{\ell})|^2 (PD_1 + \vec{\Sigma}_P \vec{\Sigma}_{D_1}) \, D_2, \tag{2.11}$$

where D_2 is the factor for the slepton decay, given in (C.35).

The cross sections and distributions in the laboratory system are then obtained by integrating the squared amplitudes

$$d\sigma_{I,II} = \frac{1}{2s} |T_{I,II}|^2 d\mathrm{Lips}_{I,II} \tag{2.12}$$

over the Lorentz invariant phase space elements

$$d\mathrm{Lips}_I := d\mathrm{Lips}(s; p_{\chi_j^0}, p_{\ell_1}, p_{\tilde{\ell}}), \tag{2.13}$$

$$d\mathrm{Lips}_{II} := d\mathrm{Lips}(s; p_{\chi_j^0}, p_{\ell_1}, p_{\ell_2}, p_{\chi_1^0}), \tag{2.14}$$

given in (B.22) and (B.23), respectively.

The contributions of the spin correlation terms $\vec{\Sigma}_P \vec{\Sigma}_{D_1}$ to the total cross section vanish. Their contributions to the energy distributions of the leptons ℓ_1 and ℓ_2 from decay (2.2) and (2.5) vanish due to the Majorana properties of the neutralinos [40] if CP is conserved. In our case of CP violation, they vanish to leading order perturbation theory [40], and thus the contributions can be neglected since they are proportional to the widths of the exchanged particles.

2.1.2 T-odd asymmetries

Inserting the cross sections (2.12) in the definitions of the asymmetries (2.4) and (2.7) we obtain

$$\mathcal{A}_{I,II}^{\mathrm{T}} = \frac{\int \mathrm{Sign}[\mathcal{T}_{I,II}] |T|^2 d\mathrm{Lips}_{I,II}}{\int |T_{I,II}|^2 d\mathrm{Lips}_{I,II}} = \frac{\int \mathrm{Sign}[\mathcal{T}_{I,II}] \Sigma_P^2 \Sigma_{D_1}^2 d\mathrm{Lips}_{I,II}}{\int PD_1 d\mathrm{Lips}_{I,II}}, \tag{2.15}$$

where we have used the narrow width approximation for the propagators. In the numerator only the spin-correlation term $\Sigma_P^2 \Sigma_{D_1}^2$ remains, since only this term contains the triple products (2.3) or (2.6). Thus, the contributions to $\mathcal{A}^{\rm T}_{I,II}$ are directly proportional to the neutralino polarization Σ_P^2 perpendicular to the production plane.

In case the neutralino decays into a scalar tau, we take stau mixing into account and the asymmetries are reduced due to their dependence on the $\tilde{\chi}_i^0$-$\tilde{\tau}_k$-τ couplings

$$\mathcal{A}^{\rm T}_{I,II} \propto \frac{|a^{\tilde{\tau}}_{ki}|^2 - |b^{\tilde{\tau}}_{ki}|^2}{|a^{\tilde{\tau}}_{ki}|^2 + |b^{\tilde{\tau}}_{ki}|^2}, \tag{2.16}$$

which can be seen from the expressions of D_1 and $\Sigma_{D_1}^2$, given in Appendix (C.2). Since the asymmetries are proportional to the absolute values of $a^{\tilde{\tau}}_{ki}, b^{\tilde{\tau}}_{ki}$, they are not sensitive to the phase φ_{A_τ} of A_τ. As an observable which is sensitive to φ_{A_τ}, we will consider in Section 2.2 an asymmetry which involves the transverse τ polarization.

2.1.3 Numerical results

We analyze the dependence of the asymmetries $\mathcal{A}^{\rm T}_{I}$ and $\mathcal{A}^{\rm T}_{II}$, the neutralino production cross sections $\sigma_P(e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0)$ and the branching ratios ${\rm BR}(\tilde{\chi}_i^0 \to \tilde{\ell}\ell)$ on the parameters $\mu = |\mu|\, e^{i\,\varphi_\mu}$, $M_1 = |M_1|\, e^{i\,\varphi_{M_1}}$ and M_2 for $\tan\beta = 10$. In order to reduce the number of parameters, we assume $|M_1| = 5/3 M_2 \tan^2\theta_W$ and fix the universal scalar mass parameter $m_0 = 100$ GeV. The renormalization group equations for the slepton masses are given in (A.56) and (A.57). Since the pair production of equal neutralinos is not CP sensitive, we discuss the lightest pairs $\tilde{\chi}_1^0\,\tilde{\chi}_2^0$, $\tilde{\chi}_1^0\,\tilde{\chi}_3^0$ and $\tilde{\chi}_2^0\,\tilde{\chi}_3^0$, for which we choose a center of mass energy of $\sqrt{s} = 500$ GeV and longitudinal beam polarization $P_{e^-} = 0.8$ and $P_{e^+} = -0.6$.

- ***Production of*** $\tilde{\chi}_1^0\,\tilde{\chi}_2^0$

In Fig. 2.2a we show the cross section for $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production for $\varphi_\mu = 0$ and $\varphi_{M_1} = 0.5\,\pi$ in the $|\mu|$–M_2 plane. The cross section reaches values up to 300 fb. For $|\mu| \lesssim 250$ GeV the right selectron exchange dominates so that our choice of polarizations $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$ enhances the cross section by a factor up to 2.5 compared to the unpolarized case. For $|\mu| \gtrsim 300$ GeV the left selectron exchange dominates because of the larger $\tilde{\chi}_2^0 - \tilde{e}_L$ coupling. In this region a sign reversal of both polarizations, i.e. $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$, would enhance the cross section up to a factor of 20.

The branching ratio $\mathrm{BR}(\tilde{\chi}^0_2 \to \tilde{\ell}_R \ell_1)$ for the neutralino two-body decay into right selectrons and smuons, summed over both signs of charge, is shown in Fig. 2.2b. It reaches values up to 64% and decreases with increasing $|\mu|$ when the two-body decays into the lightest neutral Higgs boson H^0_1 and/or the Z boson are kinematically allowed. The decays $\tilde{\chi}^0_2 \to \tilde{\chi}^\pm_1 W^\mp$ are not allowed. With our choice $m_0 = 100$ GeV, the decays into left selectrons and smuons can be neglected because these channels are either not open or the branching ratio is smaller than 1%. As we assume that the squarks and the other Higgs bosons are heavy, the decay into the stau is competing, and dominates for $M_2 \lesssim 200$ GeV in our scenario, see Fig. 2.4a, which is discussed below. The resulting cross section is shown in Fig. 2.2c.

Fig. 2.2d shows the $|\mu|$–M_2 dependence of the asymmetry $\mathcal{A}^{\mathrm{T}}_{II}$ for $\varphi_{M_1} = 0.5\,\pi$ and $\varphi_\mu = 0$. In the region $|\mu| \lesssim 250$ GeV, where the right selectron exchange dominates, the asymmetry reaches 9.5% for our choice of beam polarization. This enhances the asymmetry up to a factor of 2 compared to the case of unpolarized beams. With increasing $|\mu|$ the asymmetry decreases and finally changes sign. This is due to the increasing contributions of the left selectron exchange which contributes to the asymmetry with opposite sign and dominates for $|\mu| \gtrsim 300$ GeV. In this region the asymmetry could be enhanced up to a factor 2 by reversing the sign of both beam polarizations.

The sensitivity of the cross section σ and the asymmetry $\mathcal{A}^{\mathrm{T}}_{II}$ on the CP phases is shown by contour plots in the φ_μ–φ_{M_1} plane for $|\mu| = 240$ GeV and $M_2 = 400$ GeV (Fig. 2.3). In our scenario the variation of the cross section, Fig. 2.3a, is more than 100%. In addition to the CP sensitive observables, the cross section may serve to constrain the phases. For unpolarized beams, the cross section would be reduced by a factor 0.4. The asymmetry $\mathcal{A}^{\mathrm{T}}_{II}$ (Fig. 2.3b) varies between -8.9% and 8.9%. It is remarkable that these maximal values are not necessarily obtained for maximal CP phases. In our scenario the asymmetry is much more sensitive to variations of the phase φ_{M_1} around 0. The reason is that $\mathcal{A}^{\mathrm{T}}_{II}$ is proportional to a product of a CP odd (Σ^2_P) and a CP even factor ($\Sigma^2_{D_1}$), see (2.15). The CP odd (CP even) factor has as sine-like (cosine-like) dependence on the phases. Thus the maximum of $\mathcal{A}^{\mathrm{T}}_{II}$ is shifted towards $\varphi_{M_1} = 0$ in Fig. 2.3b. On the other hand, the asymmetry is rather insensitive to φ_μ. For unpolarized beams this asymmetry would be reduced roughly by a factor 0.33.

The statistical significance for measuring each asymmetry is given by $S = |\mathcal{A}^{\mathrm{T}}|\sqrt{N}$ (1.17), with $N = \mathcal{L}\sigma$ is the number of events with $\mathcal{L}$ the total integrated luminosity. We show the contour lines for $S = 3$ and 5 for $\mathcal{A}^{\mathrm{T}}_{II}$ in Fig. 2.3c with $\mathcal{L} = 500$ fb^{-1}.

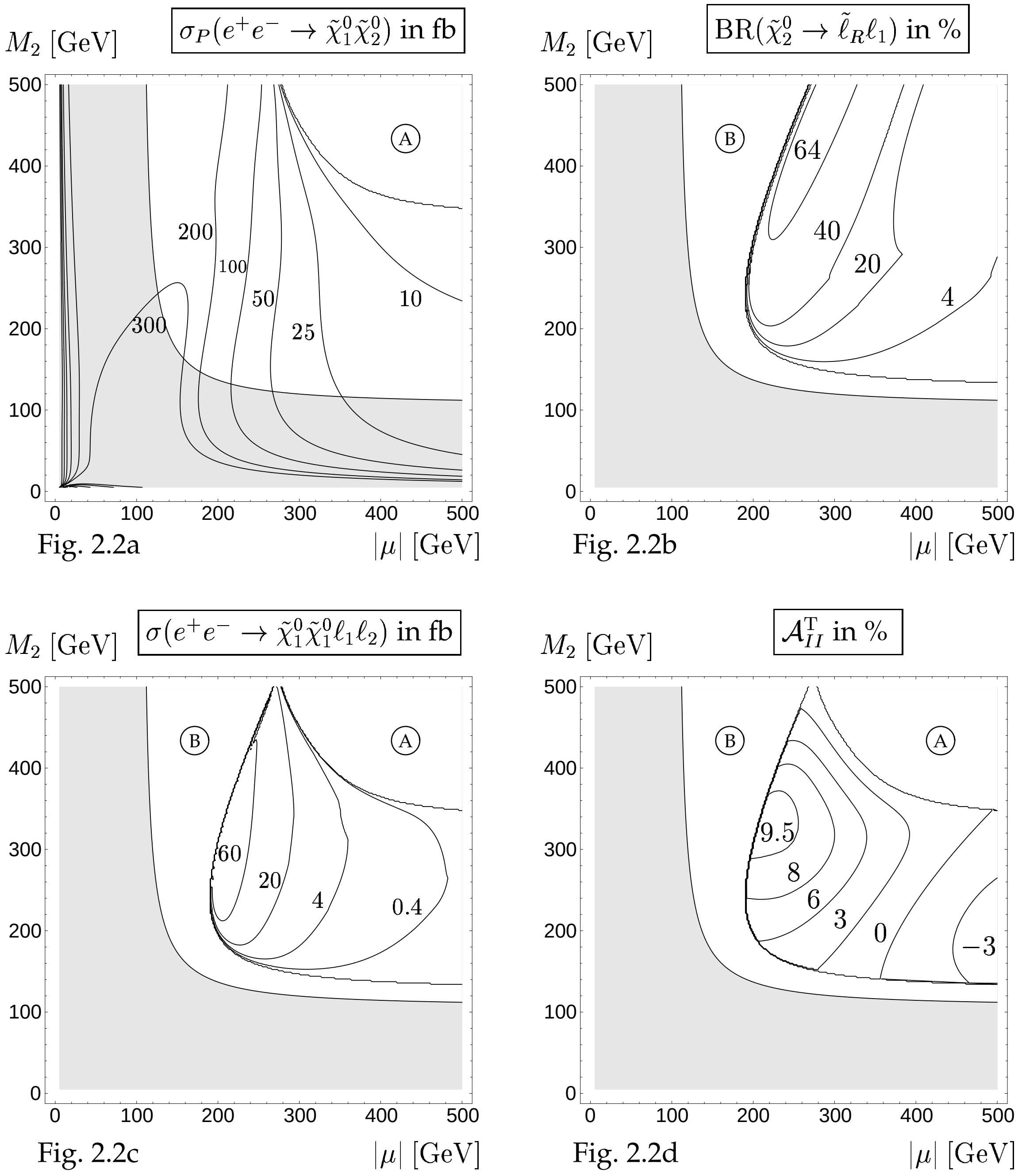


Figure 2.2: Contour lines of 2.2a: $\sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$, 2.2b: $\mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\ell}_R\ell_1)$, $\ell = e, \mu$, 2.2c: $\sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\ell}_R\ell_1) \times \mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}_1^0\ell_2)$ with $\mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}_1^0\ell_2) = 1$, 2.2d: the asymmetry $\mathcal{A}_{II}^{\mathrm{T}}$, in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 100$ GeV, $A_\tau = -250$ GeV, $\sqrt{s} = 500$ GeV and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. The area A (B) is kinematically forbidden by $m_{\chi_1^0} + m_{\chi_2^0} > \sqrt{s}$ ($m_{\tilde{\ell}_R} > m_{\chi_2^0}$). The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

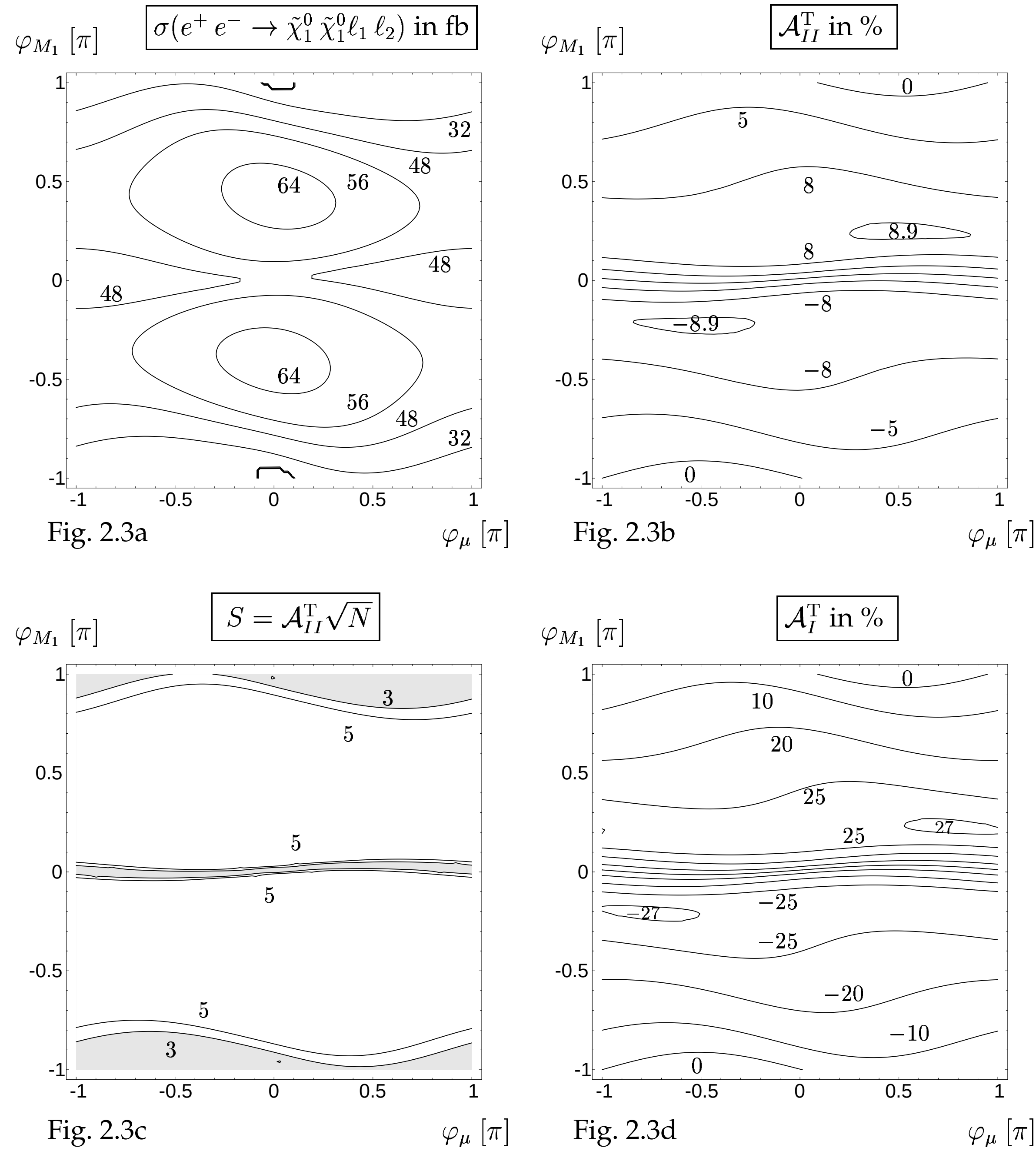


Figure 2.3: Contour lines of 2.3a: $\sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to \tilde{\ell}_R\ell_1) \times \mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}_1^0\ell_2)$ with $\mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}_1^0\ell_2) = 1$, 2.3b: the asymmetry $\mathcal{A}_{II}^{\mathrm{T}}$, 2.3c: the significance S, 2.3d: the asymmetry $\mathcal{A}_{I}^{\mathrm{T}}$, in the φ_μ–φ_{M_1} plane for $M_2 = 400$ GeV, $|\mu| = 240$ GeV, $\tan\beta = 10$, $m_0 = 100$ GeV, $A_\tau = -250$ GeV, $\sqrt{s} = 500$ GeV and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. In the gray shaded area of 2.3c we have $S < 3$. For $\varphi_{M_1}, \varphi_\mu = 0$ we have $m_{\tilde{\ell}_R} = 221$ GeV, $m_{\chi_1^0} = 178$ GeV and $m_{\chi_2^0} = 243$ GeV.

In Fig. 2.3d we also show the asymmetry $\mathcal{A}_I^{\rm T}$ which is a factor 2.9 larger than $\mathcal{A}_{II}^{\rm T}$, because in $\mathcal{A}_{II}^{\rm T}$ the CP-violating effect from the production is partly washed out by the kinematics of the slepton decay. However, for a measurement of $\mathcal{A}_I^{\rm T}$ the reconstruction of the $\tilde{\chi}_2^0$ momentum is necessary. The asymmetry $\mathcal{A}_I^{\rm T}$ shows a similar dependence on the phases as $\mathcal{A}_{II}^{\rm T}$ because both are due to the non vanishing neutralino polarization perpendicular to the production plane. It is interesting to note that the asymmetries can be sizable for small values of φ_μ, which is suggested by the EDM constraints, see Section 1.2.

Next we comment on the neutralino decay into the scalar tau and discuss the main differences from the decay into the selectron and smuon. In some regions of the parameter space, the decay of the neutralino into the lightest stau $\tilde{\tau}_1$ may dominate over that into the right selectron and smuon, and may even be the only decay channel. In Fig. 2.4a we show contour lines of the branching ratio $BR(\tilde{\chi}_2^0 \to \tilde{\tau}_1\tau)$ in the $|\mu|$–M_2 plane for $A_\tau = -250$ GeV, $\varphi_{M_1} = 0.5\,\pi$ and $\varphi_\mu = 0$. For $M_2 < 200$ GeV the branching ratio $BR(\tilde{\chi}_2^0 \to \tilde{\tau}_1\tau)$ is larger than 80%. However, due to the mixing in the stau sector the asymmetry $\mathcal{A}_{II}^{\rm T}$, Fig. 2.4b, is reduced compared to that in the selectron and smuon channels, see Fig. 2.2d. The reason is the suppression factor $(|a_{ki}^{\tilde{\tau}}|^2 - |b_{ki}^{\tilde{\tau}}|^2)/(|a_{ki}^{\tilde{\tau}}|^2 + |b_{ki}^{\tilde{\tau}}|^2)$ (2.16), which may be small or even be zero.

- ***Production of*** $\tilde{\chi}_1^0\tilde{\chi}_3^0$

We show in Fig. 2.5a and b contour lines of the cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0) \times {\rm BR}(\tilde{\chi}_3^0 \to \tilde{\ell}_R\ell_1) \times {\rm BR}(\tilde{\ell}_R \to \tilde{\chi}_1^0\ell_2)$ with ${\rm BR}(\tilde{\ell}_R \to \tilde{\chi}_1^0\ell_2) = 1$, and of the asymmetry $\mathcal{A}_{II}^{\rm T}$, respectively. The cross section with polarized beams reaches more than 100 fb, which is up to a factor 2.5 larger than for unpolarized beams. The asymmetry $\mathcal{A}_{II}^{\rm T}$, shown in Fig. 2.5b, reaches -9.5%. For unpolarized beams this value would be reduced by a factor 0.75. For our choice of parameters the cross section and the asymmetry for $\tilde{\chi}_1^0\tilde{\chi}_3^0$ production and decay show a similar dependence on M_2 and $|\mu|$ as for $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production, however, the kinematically allowed regions are different. We also studied the φ_μ dependence of $\mathcal{A}_{II}^{\rm T}$. For $\varphi_\mu = 0.5\pi(0.1\pi)$ and $\varphi_{M_1} = 0$, the maximal values of $\mathcal{A}_{II}^{\rm T}$ in the M_2–$|\mu|$ plane are $|\mathcal{A}_{II}^{\rm T}| < 3\%(1\%)$.

- ***Production of*** $\tilde{\chi}_2^0\tilde{\chi}_3^0$

The production of the neutralino pair $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_3^0$ could make it easier to reconstruct the production plane because both neutralinos decay. This allows one to determine also asymmetry $\mathcal{A}_I^{\rm T}$, which is a factor 2-3 larger than $\mathcal{A}_{II}^{\rm T}$. We discuss the decay of the heavier neutralino $\tilde{\chi}_3^0$, which has a larger kinematically allowed region in the $|\mu|$–M_2 plane than that of $\tilde{\chi}_2^0$. In Fig. 2.6 we show the pro-

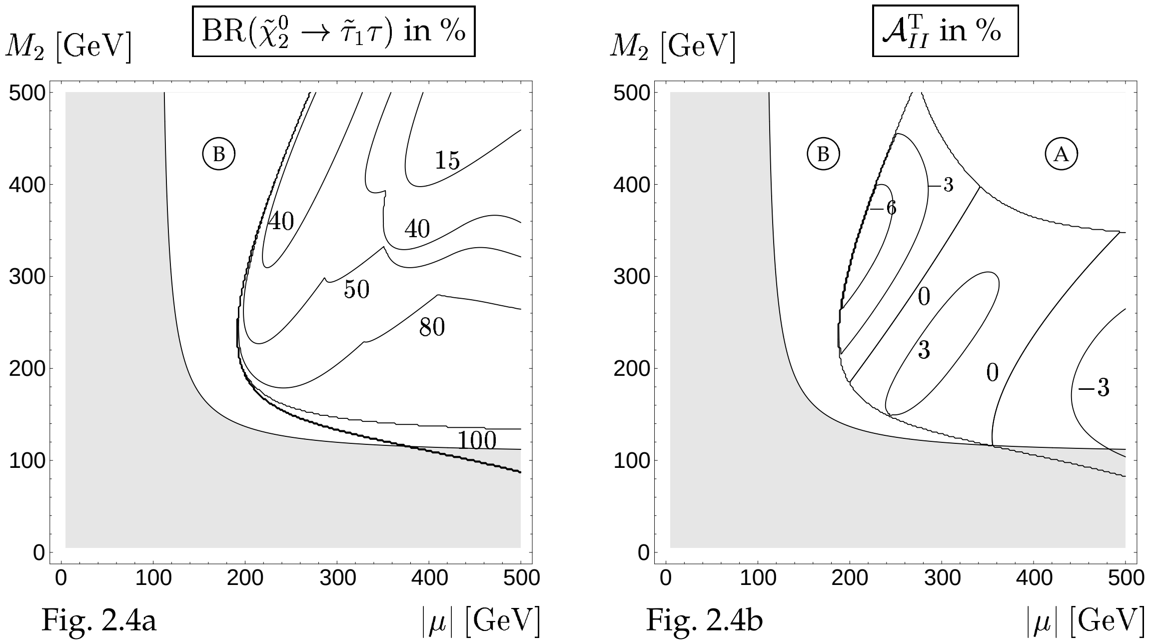


Figure 2.4: Contour lines of 2.4a: $\mathrm{BR}(\tilde{\chi}^0_2 \to \tilde{\tau}_1\tau)$ and 2.4b: the asymmetry $\mathcal{A}^{\mathrm{T}}_{II}$, in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $A_\tau = -250$ GeV, $\tan\beta = 10$, $m_0 = 100$ GeV, $\sqrt{s} = 500$ GeV and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. The area A (B) is kinematically forbidden by $m_{\chi^0_1} + m_{\chi^0_2} > \sqrt{s}$ ($m_{\tilde{\tau}_1} > m_{\chi^0_2}$). The gray area is excluded by $m_{\chi^\pm_1} < 104$ GeV.

duction cross section $\sigma_P(e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_3)$ which reaches 100 fb. The cross section $\sigma_P(e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_3) \times \mathrm{BR}(\tilde{\chi}^0_3 \to \tilde{\ell}_R\ell_1) \times \mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}^0_1\ell_2)$ with $\mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}^0_1\ell_2) = 1$, is shown in Fig. 2.6b. The asymmetry $\mathcal{A}^{\mathrm{T}}_{II}$ is shown in Fig. 2.6d. As to the φ_μ dependence of $\mathcal{A}^{\mathrm{T}}_{I}$, we found that for $\varphi_\mu = 0.5\pi(0.1\pi)$ and $\varphi_{M_1} = 0$, $|\mathcal{A}^{\mathrm{T}}_{I}|$ can reach 25% (2%) in the $|\mu|$–M_2 plane.

■ *Energy distributions of the leptons*

In order to measure the asymmetries $\mathcal{A}^{\mathrm{T}}_{I}$ (2.4) and $\mathcal{A}^{\mathrm{T}}_{II}$ (2.7), the two leptons ℓ_1 and ℓ_2 from the neutralino (2.2) and slepton decay (2.5) have to be distinguished. We therefore calculate the energy distributions of the leptons from the first and second decay vertex in the laboratory system, see Appendix B.2.4. One can distinguish between the two leptons event by event, if their energy distributions do not overlap. If their energy distributions do overlap, only those leptons can be distinguished, whose energies are not both in the overlapping region.

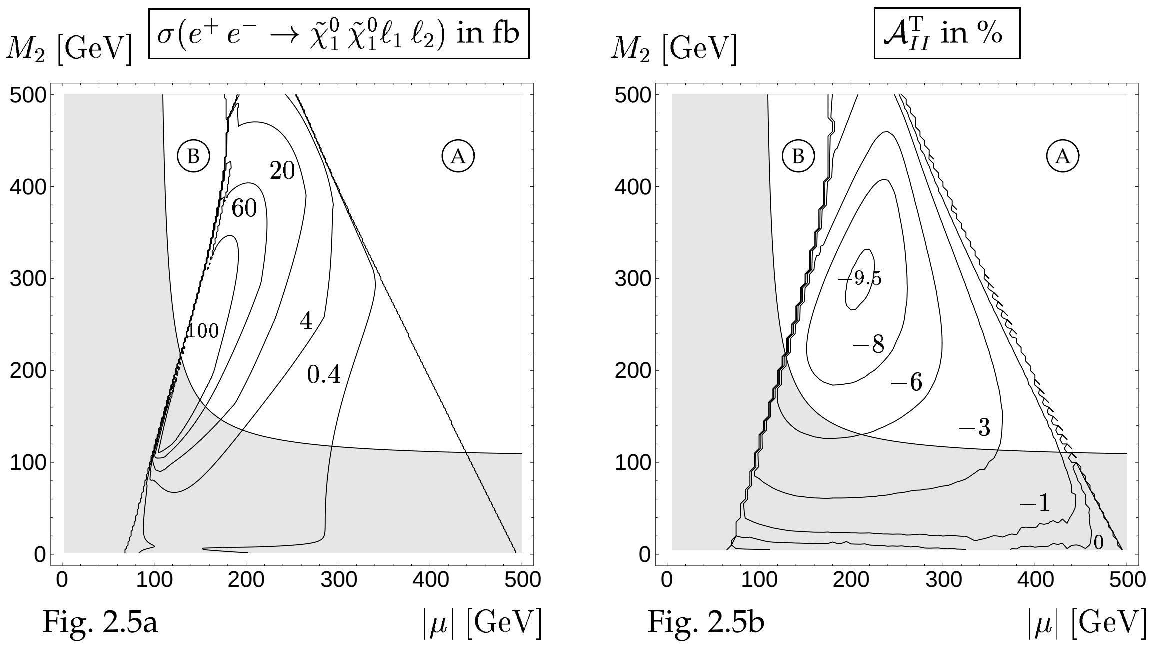


Figure 2.5: Contour lines of 2.5a: $\sigma_P(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3) \times \mathrm{BR}(\tilde{\chi}^0_3 \to \tilde{\ell}_R\ell_1) \times \mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}^0_1\ell_2)$ with $\mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}^0_1\ell_2) = 1$ and $\ell = e, \mu$, 2.5b: the asymmetry $\mathcal{A}^{\mathrm{T}}_{II}$, in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 100$ GeV, $A_\tau = -250$ GeV, $\sqrt{s} = 500$ GeV and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. The area A (B) is kinematically forbidden by $m_{\chi^0_1} + m_{\chi^0_3} > \sqrt{s}$ ($m_{\tilde{\ell}_R} > m_{\chi^0_3}$). The gray area is excluded by $m_{\chi^\pm_1} < 104$ GeV.

We show in Figs. 2.7a - c different types of energy distributions for lepton ℓ_1 (dashed line), and lepton ℓ_2 (solid line), $\ell = e, \mu$, for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ and the subsequent decays $\tilde{\chi}^0_2 \to \tilde{\ell}\ell_1$ and $\tilde{\ell} \to \tilde{\chi}^0_1\ell_2$. The parameters $\tan\beta = 10$, $M_2 = 300$ GeV, $\varphi_\mu = 0$, $\varphi_{M_1} = 0.5\pi$, and for $|\mu| = 200$, 300 and 500 GeV, are chosen such that the slepton mass $m_{\tilde{\ell}_R} = 180$ GeV is constant, the LSP mass $m_{\chi^0_1} = 140, 145, 150$ GeV is almost constant whereas the neutralino mass $m_{\chi^0_2} = 185, 240, 300$ GeV is increasing. The mass difference between $\tilde{\ell}_R$ and $\tilde{\chi}^0_1$ decreases ($\Delta m = 40, 35, 30$ GeV), whereas the mass difference between $\tilde{\chi}^0_2$ and $\tilde{\ell}_R$ increases ($\Delta m = 5, 60, 120$ GeV). The endpoints of the energy distributions of the decay leptons depend on these mass differences. Thus, in Fig. 2.7a, the second lepton is more energetic than the first lepton. The energy distributions do not overlap and thus the two leptons can be distinguished by measuring their energies. This also holds for Fig. 2.7c, where the first lepton is more energetic than the second one. In Fig. 2.7b the two distributions overlap because the mass differences between $\tilde{\chi}^0_1$, $\tilde{\ell}_R$ and $\tilde{\chi}^0_2$ are similar. One has to apply cuts to distinguish the leptons, which reduce the number of events.

2.1.4 Summary of Section 2.1

We have considered two triple-product asymmetries in neutralino production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and the subsequent leptonic two-body decay chain of one neutralino $\tilde{\chi}_i^0 \to \tilde{\ell}\ell$, $\tilde{\ell} \to \tilde{\chi}_1^0\ell$ for $\ell = e, \mu, \tau$. These asymmetries are present already at tree level and are due to spin effects in the production and decay process of two different neutralinos. The asymmetries are sensitive to CP-violating phases of the gaugino and Higgsino mass parameters M_1 and/or μ in the neutralino production process.

For the process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and neutralino decay into a right slepton $\tilde{\chi}_2^0 \to \tilde{\ell}_R\ell$, we have shown that the asymmetries can be as large as 25%. They can be enhanced using polarized beams, and can be sizable even for a small phases, $\varphi_\mu, \varphi_{M_1} \approx 0.1\pi$, which is suggested by the experimental limits on EDMs.

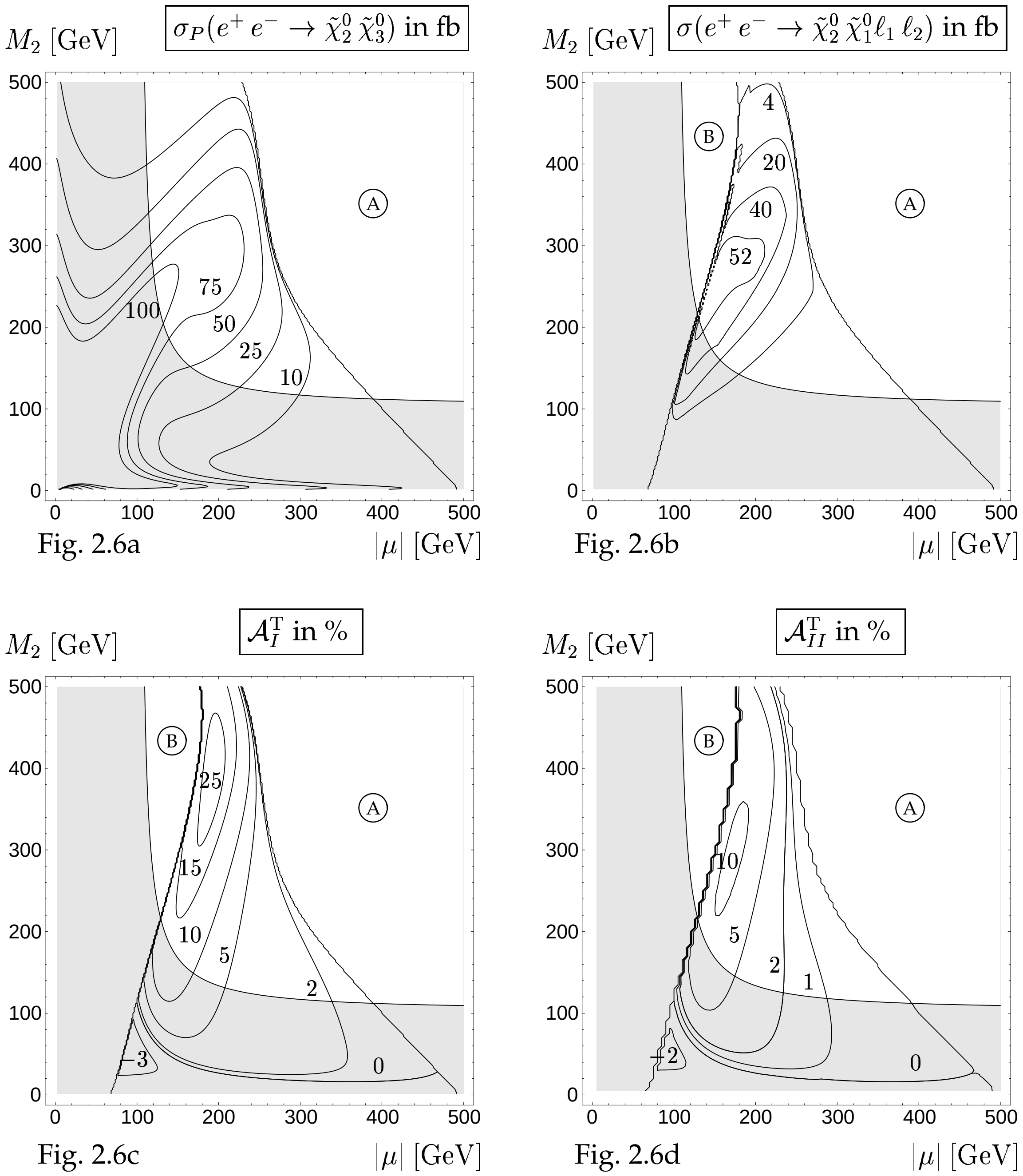


Figure 2.6: Contour lines of 2.6a: $\sigma_P(e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_3)$, 2.6b: $\sigma_P(e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_3) \times \mathrm{BR}(\tilde{\chi}^0_3 \to \tilde{\ell}_R\ell_1) \times \mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}^0_1\ell_2)$ for $\ell = e, \mu$, and $\mathrm{BR}(\tilde{\ell}_R \to \tilde{\chi}^0_1\ell_2) = 1$, 2.6c: the asymmetry $\mathcal{A}^{\mathrm{T}}_{I}$, 2.6d: the asymmetry $\mathcal{A}^{\mathrm{T}}_{II}$, in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 100$ GeV, $A_\tau = -250$ GeV, $\sqrt{s} = 500$ GeV and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. The area A (B) is kinematically forbidden by $m_{\chi^0_2} + m_{\chi^0_3} > \sqrt{s}$ ($m_{\tilde{\ell}_R} > m_{\chi^0_3}$). The gray area is excluded by $m_{\chi^\pm_1} < 104$ GeV.

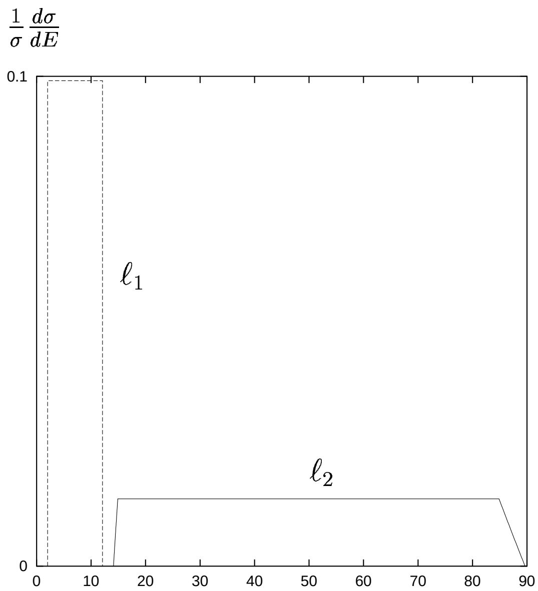


Fig. 2.7a E [GeV]

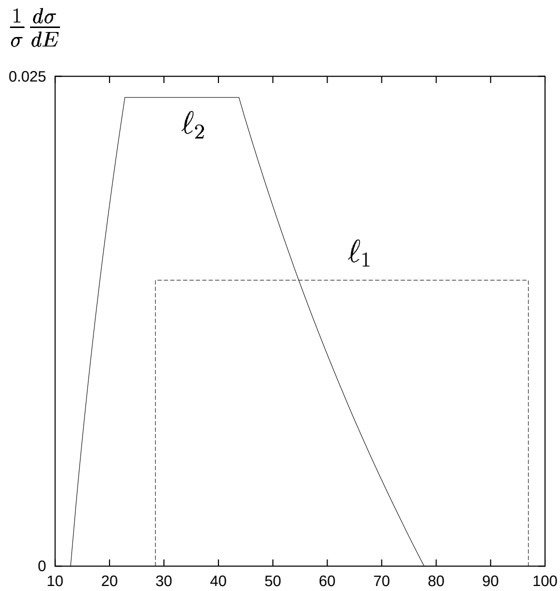


Fig. 2.7b E [GeV]

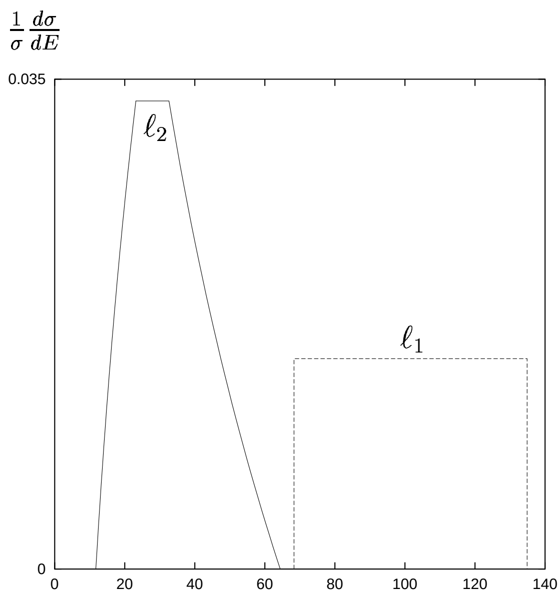


Fig. 2.7c E [GeV]

Figure 2.7: Energy distributions in the laboratory system for ℓ_1 (dashed line) and ℓ_2 (solid line) for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and the subsequent decays $\tilde{\chi}_2^0 \to \tilde{\ell}_R\ell_1$ and $\tilde{\ell}_R \to \tilde{\chi}_1^0\ell_2$, for $M_2 = 300$ GeV, $m_{\tilde{\ell}_R} = 180$ GeV, $\tan\beta = 10$ and $\{|\mu|, m_{\chi_1^0}, m_{\chi_2^0}\}/\mathrm{GeV} = \{200, 140, 185\},\ \{300, 145, 240\},\ \{500, 150, 300\}$ in a, b, c respectively.

2.2 CP asymmetry in neutralino production and decay into polarized taus

For the two-body decays of neutralinos into sleptons, where the lepton polarizations are summed, we have shown in the last section that the asymmetries have only CP-odd contributions from the neutralino production process. For neutralino decay into a tau, the tau polarization allows to define an asymmetry, which has also CP-odd contributions from the neutralino decay process. This is particularly interesting since an asymmetry can be defined, which is sensitive to the CP phase of the trilinear scalar coupling parameter A_τ.

We consider neutralino production

$$e^+ + e^- \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0; \quad i, j = 1, \ldots, 4, \tag{2.17}$$

and the subsequent two-body decay of one neutralino into a tau

$$\tilde{\chi}_i^0 \to \tilde{\tau}_m^\pm + \tau^\mp; \quad m = 1, 2. \tag{2.18}$$

The τ^- polarization is given by [41]

$$\mathbf{P} = \frac{\mathrm{Tr}(\rho \boldsymbol{\sigma}\,)}{\mathrm{Tr}(\rho)}, \tag{2.19}$$

with ρ the hermitean spin density matrix of the τ^- and σ_i the Pauli matrices. The component P_3 of the polarization vector $\mathbf{P} = (P_1, P_2, P_3)$ is the longitudinal polarization, P_1 is the transverse polarization in the plane and P_2 is the transverse polarization perpendicular to the plane defined by the momenta $\mathbf{p}_\tau$ and $\mathbf{p}_{e^-}$. The transverse polarization P_2 is proportional to the triple product

$$\mathcal{T}_\tau = \mathbf{s}_\tau \cdot (\mathbf{p}_\tau \times \mathbf{p}_{e^-}), \tag{2.20}$$

where $\mathbf{s}_\tau$ is the τ^- spin 3-vector. For its definition in the laboratory system, see B.21. In order to eliminate absorptive phases, we define the CP-odd asymmetry

$$\mathcal{A}_{\mathrm{CP}} = \frac{1}{2}(P_2 - \bar{P}_2), \tag{2.21}$$

where $\bar{\mathbf{P}}$ denotes the τ^+ polarization in the charge conjugated process $\tilde{\chi}_i^0 \to \tilde{\tau}_m^- \tau^+$. The asymmetry $\mathcal{A}_{\rm CP}$ is sensitive to the phase φ_{A_τ} in the stau sector, as well as to the phases φ_μ and φ_{M_1} in the neutralino sector.

Note, that for the two-body decay (2.18), the transverse τ polarization P_2 is the only observable which is sensitive to φ_{A_τ}. The asymmetries defined in Section 2.1, where the τ polarization is summed, are only sensitive to CP violation due to φ_μ and φ_{M_1} in the production process.

2.2.1 Tau spin-density matrix and cross section

In the spin density matrix formalism, see Appendix C, the unnormalized spin-density matrix of the τ^- can be written as

$$\rho_P(\tau^-)^{\lambda_k\lambda_k'} = |\Delta(\tilde{\chi}_i^0)|^2 \sum_{\lambda_i,\lambda_i'} \rho_P(\tilde{\chi}_i^0)^{\lambda_i\lambda_i'}\, \rho_D(\tilde{\chi}_i^0)_{\lambda_i'\lambda_i}^{\lambda_k\lambda_k'}. \tag{2.22}$$

It is composed of the neutralino propagator $\Delta(\tilde{\chi}_i^0)$, the spin density matrices $\rho_P(\tilde{\chi}_i^0)$ for neutralino production (2.17) and $\rho_D(\tilde{\chi}_i^0)$ for neutralino decay (2.18). The $\tilde{\chi}_i^0$ helicities are denoted by λ_i and λ_i', and the τ^- helicities are denoted by λ_k and λ_k'. The neutralino production matrix $\rho_P(\tilde{\chi}_i^0)$ is defined in (C.10) and the neutralino decay matrix $\rho_D(\tilde{\chi}_i^0)$ in (C.38). Inserting these density matrices into (2.22) gives

$$\rho_P(\tau^-)^{\lambda_k\lambda_k'} = 4|\Delta(\tilde{\chi}_i^0)|^2\, [(PD + \Sigma_P^a\Sigma_D^a)\delta_{\lambda_k\lambda_k'} + (PD^b + \Sigma_P^a\Sigma_D^{ab})\sigma^b_{\lambda_k\lambda_k'})]. \tag{2.23}$$

The last term of the coefficient Σ_D^{ab}, see (C.44), contains for $b = 2$ the triple product (2.20). This term is proportional to the product of the $\tilde{\chi}_i^0$-$\tilde{\tau}_k$-τ couplings $\mathrm{Im}(b_{mi}^{\tilde{\tau}}{}^* a_{mi}^{\tilde{\tau}})$ and is therefore sensitive to the phases φ_{A_τ}, φ_μ and φ_{M_1}.

The amplitude squared is obtained by summing over the τ helicities in (2.23)

$$|T|^2 = 4|\Delta(\tilde{\chi}_i^0)|^2\, [P\,(2D) + \Sigma_P^a\,(2\Sigma_D^a)], \tag{2.24}$$

where the CP sensitive term Σ_D^{ab} drops out. The cross section is then given by

$$d\sigma = \frac{1}{2s}|T|^2 d\mathrm{Lips}(s; p_{\chi_j^0}, p_\tau, p_{\tilde{\tau}}), \tag{2.25}$$

with the phase space element dLips as defined in (B.22).

2.2.2 Transverse tau polarization and CP asymmetry

From (2.23) we obtain for the transverse τ^- polarization (2.19)

$$P_2 = \frac{\int \Sigma_P^a \Sigma_D^{a2} \, d\text{Lips}}{\int PD \, d\text{Lips}}, \tag{2.26}$$

which follows since we have used the narrow width approximations for the propagators and in the numerator $\int |\Delta(\tilde{\chi}_i^0)|^2 \; PD^2 \; d\text{Lips} = 0$ and in the denominator $\int |\Delta(\tilde{\chi}_i^0)|^2 \; \Sigma_P^a \Sigma_D^a \; d\text{Lips} = 0$.

As can be seen from (2.26), P_2 is proportional to the spin correlation term Σ_D^{a2} (C.44), which contains the CP-sensitive part $\text{Im}({b_{mi}^{\tilde{\tau}}}^* a_{mi}^{\tilde{\tau}}) \epsilon_{\mu\nu\rho\sigma} \, p_\tau^\mu \, p_{\tilde{\chi}_i^0}^\nu \, s_{\chi_i^0}^{a,\rho} \, s_\tau^{b,\sigma}$. In order to study the dependence of P_2 on the parameters, we expand for $\tilde{\tau}_1$

$$\begin{aligned} \text{Im}({b_{1i}^{\tilde{\tau}}}^* a_{1i}^{\tilde{\tau}}) &= g^2 \cos^2\theta_{\tilde{\tau}} Y_\tau \text{Im}(f_{Li}^\tau N_{i3}) + g^2 \sin^2\theta_{\tilde{\tau}} Y_\tau \sqrt{2} \tan\theta_W \text{Im}(N_{i1} N_{i3}) \\ &+ g^2 \sin^2\theta_{\tilde{\tau}} \cos^2\theta_{\tilde{\tau}} [Y_\tau^2 \text{Im}(N_{i3} N_{i3} e^{i\varphi_{\tilde{\tau}}}) + g^2 \sqrt{2} \tan\theta_W \text{Im}(f_{Li}^\tau N_{i1} e^{-i\varphi_{\tilde{\tau}}})], \end{aligned} \tag{2.27}$$

using the definitions of the couplings in the stau sector, see Appendix A.2.

For $\varphi_\mu, \varphi_{M_1} = 0$, we find from (2.27) that $P_2 \propto \sin 2\theta_{\tilde{\tau}} \sin\varphi_{\tilde{\tau}}$. We note that the dependence of $\varphi_{\tilde{\tau}}$ on φ_{A_τ} is weak if $|A_\tau| \ll |\mu| \tan\beta$, see (A.49). Thus, we expect that P_2 increases with increasing $|A_\tau|$.

In order to measure P_2 and the CP asymmetry $\mathcal{A}_{\text{CP}}$ (2.21), the τ^- from the neutralino decay $\tilde{\chi}_i^0 \to \tilde{\tau}_m^+ \tau^-$ and the τ^+ from the subsequent $\tilde{\tau}_m^+$ decay $\tilde{\tau}_m^+ \to \tilde{\chi}_1^0 \tau^+$ have to be distinguished. This can be accomplished by their different energy distributions, see Appendix B.2.4.

2.2.3 Numerical results

We present numerical results for $e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$ and the subsequent decay of the neutralino into the lightest stau $\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau$ for a linear collider with $\sqrt{s} = 500$ GeV and longitudinally polarized beams with $(P_{e^-}, P_{e^+}) = (\pm 0.8, \mp 0.6)$. This choice favors right or left selectron exchange in the neutralino production process, respectively.

We study the dependence of the asymmetry $\mathcal{A}_{\text{CP}}$ and the production cross sections $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0) \times \text{BR}(\tilde{\chi}_2^0 \to \tilde{\tau}_1^+ \tau^-)$ on the parameters φ_μ, $|\mu|$, φ_{M_1}, $|M_1|$, φ_{A_τ},

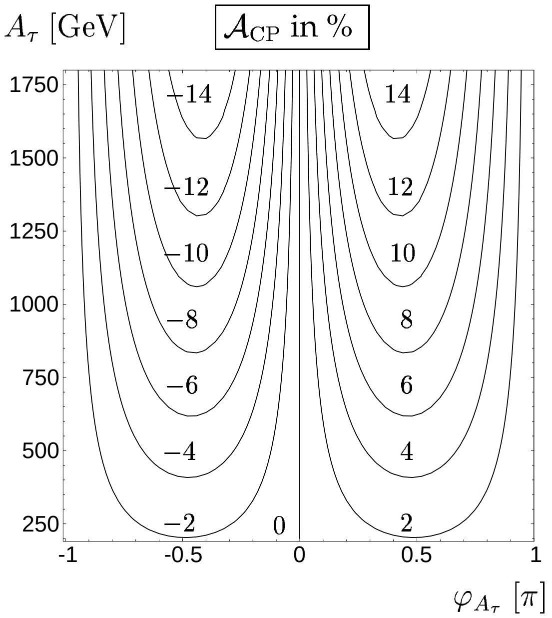


Figure 2.8: Contour lines of $\mathcal{A}_{\rm CP}$ for $M_2 = 200$ GeV, $|\mu| = 250$ GeV, $\tan\beta = 5$, $\varphi_{M_1} = \varphi_\mu = 0$ and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$.

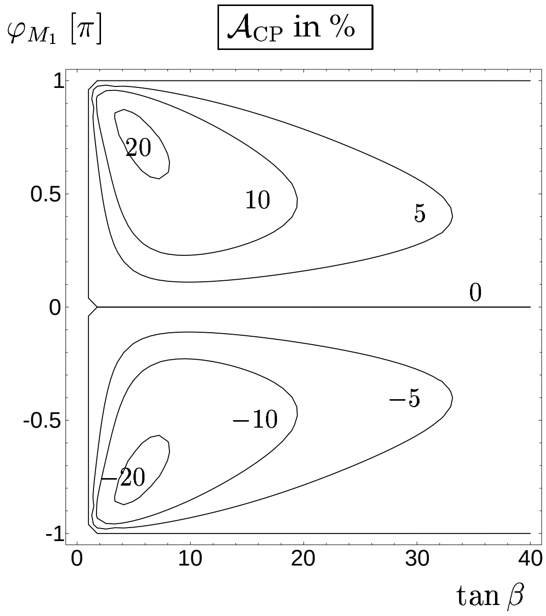


Figure 2.9: Contour lines of $\mathcal{A}_{\rm CP}$ for $A_\tau = 1$ TeV, $M_2 = 300$ GeV, $|\mu| = 250$ GeV, $\varphi_{A_\tau} = \varphi_\mu = 0$ and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$.

$|A_\tau|$ and $\tan\beta$. We assume $|M_1| = 5/3\ M_2 \tan^2\theta_W$, and use $m_0 = 100$ GeV for the universal scalar mass parameter in the renormalization group equations of the selectron masses, see (A.46) and (A.47). We take into account the restrictions on $|A_\tau|$ due to the tree-level vacuum stability conditions [42].

For the calculation of the branching ratio ${\rm BR}(\tilde\chi^0_2 \to \tilde\tau_1^+ \tau^-)$ we include the two-body decays

$$\tilde\chi^0_2 \to \tilde\tau_m \tau,\ \tilde\ell_{R,L}\ell,\ \tilde\chi^0_1 Z,\ \tilde\chi^\mp_n W^\pm,\ \tilde\chi^0_1 H^0_1, \quad \ell = e, \mu, \quad m, n = 1, 2, \tag{2.28}$$

with $m_A = 1$ TeV, such that the neutralino decays into the charged Higgs bosons $\tilde\chi^0_2 \to \tilde\chi^\pm_n H^\mp$, as well as decays into the heavy neutral Higgs bosons $\tilde\chi^0_2 \to \tilde\chi^0_1\ H^0_{2,3}$, are excluded in our scenarios.

In Fig. 2.8 we show the contour lines of $\mathcal{A}_{\rm CP}$ in the φ_{A_τ}–$|A_\tau|$ plane. The asymmetry $\mathcal{A}_{\rm CP}$ is proportional to $\sin 2\theta_{\tilde\tau} \sin\varphi_{\tilde\tau}$, and increases with increasing $|A_\tau| \gg |\mu|\tan\beta$, which is expected from (2.27). Furthermore, in the parameter region shown the cross section σ varies between 20 fb and 30 fb.

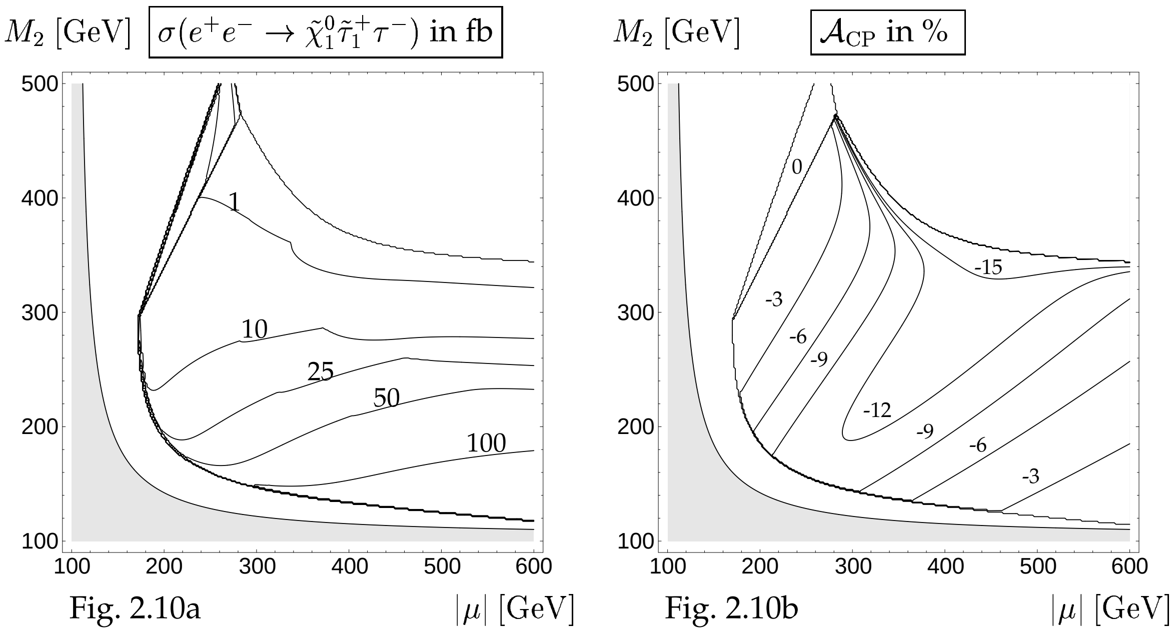


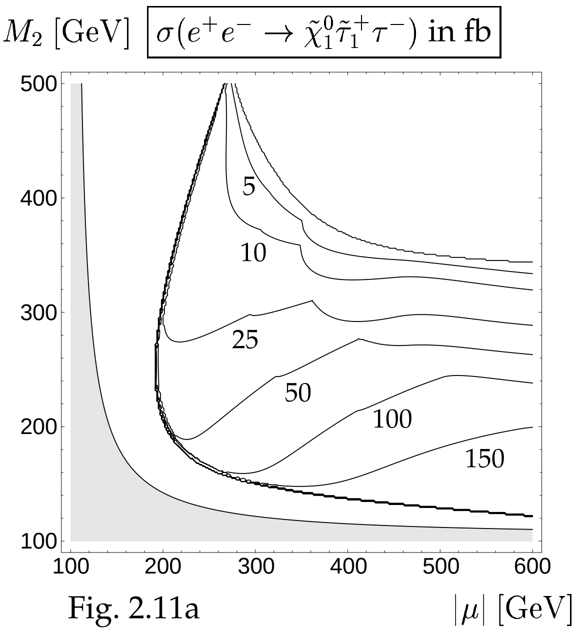
Figure 2.10: Contour lines of σ and $\mathcal{A}_{\rm CP}$ in the $|\mu|$–M_2 plane for $\varphi_{A_\tau} = 0.5\pi$, $\varphi_{M_1} = \varphi_\mu = 0$, $A_\tau = 1$ TeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\chi_1^0} + m_{\chi_2^0}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\chi_2^0}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

In Fig. 2.9 we show the dependence of $\mathcal{A}_{\rm CP}$ on $\tan\beta$ and φ_{M_1}. Large values up to $\pm 20\%$ are obtained for $\tan\beta \approx 5$. Note that these values are obtained for $\varphi_{M_1} \approx \pm 0.8\pi$ rather than for maximal $\varphi_{M_1} \approx \pm 0.5\pi$. This is due to the interplay of CP-even and CP-odd contributions to the spin correlation terms in (2.26). In the region shown in Fig. 2.9, the cross section σ varies between 10 fb and 30 fb.

Figs. 2.10a and 2.10b show, for $\varphi_{A_\tau} = 0.5\pi$ and $\varphi_{M_1} = \varphi_\mu = 0$, the $|\mu|$–M_2 dependence of the cross section σ and the asymmetry $\mathcal{A}_{\rm CP}$, respectively. The asymmetry reaches values up to -15% due to the large value of $|A_\tau| = 1$ TeV and the choice of the beam polarization $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. This choice also enhances the cross section, which reaches values of more than 100 fb.

For $\varphi_{M_1} = 0.5\pi$ and $\varphi_\mu = \varphi_{A_\tau} = 0$ we show in Figs. 2.11a,b the contour lines of σ and $\mathcal{A}_{\rm CP}$, respectively, in the $|\mu|$-M_2 plane. It is remarkable that in a large region the asymmetry is larger than -10% and reaches values up to -40%. Unpolarized beams would reduce $\mathcal{A}_{\rm CP}$ only marginally, however the largest values of σ would be reduced by a factor 3.

For $|\mu| = 300$ GeV and $M_2 = 400$ GeV, we show in Figs. 2.12a,b contour lines of σ



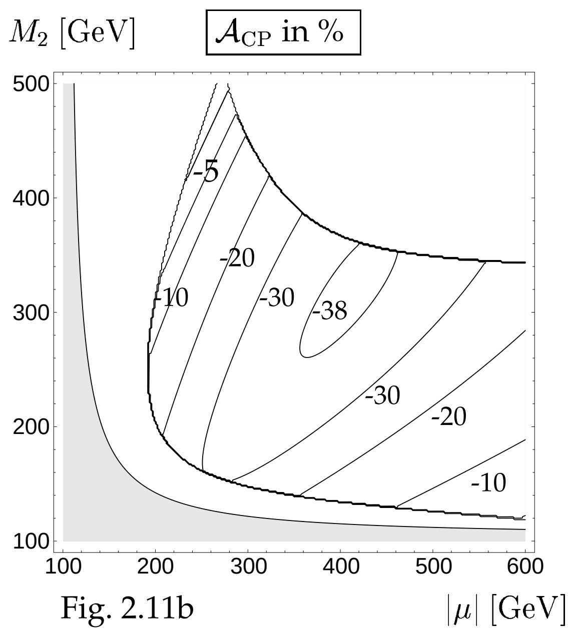


Figure 2.11: Contour lines of σ and $\mathcal{A}_{\rm CP}$ in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_{A_\tau} = \varphi_\mu = 0$, $A_\tau = 250$ GeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\chi_1^0} + m_{\chi_2^0}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\chi_2^0}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

and $\mathcal{A}_{\rm CP}$, respectively, in the φ_μ–φ_{M_1} plane. The asymmetry $\mathcal{A}_{\rm CP}$ is very sensitive to variations of the phases φ_{M_1} and φ_μ. Even for small phases, e.g. $\varphi_\mu, \varphi_{M_1} \approx 0.1$, we have $\mathcal{A}_{\rm CP} \approx 15\%$.

The polarization of the τ can be analyzed through its decay distributions. The sensitivities for measuring the polarization of the τ for the various decay modes are given in [43]. The numbers quoted there are for an ideal detector and for longitudinal τ polarization and it is expected that the sensitivities for transversely polarized τ leptons are somewhat smaller. Combining informations from all τ decay modes a sensitivity of $S = 0.35$ [44] has been obtained. Following [43], the relative statistical error of P_2 (and of $\bar{P}_2$ analogously) can be calculated as $\delta P_2 = \Delta P_2/|P_2| = \sigma^s/(S|P_2|\sqrt{N})$, for σ^s standard deviations, and $N = \sigma\mathcal{L}$ events for the integrated luminosity $\mathcal{L}$ and the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times {\rm BR}(\tilde{\chi}_2^0 \to \tilde{\tau}_1^+\tau^-)$. Then for $\mathcal{A}_{\rm CP}$ (2.21), it follows $\Delta\mathcal{A}_{\rm CP} = \Delta P_2/\sqrt{2}$. We show in Fig. 2.13 the contour lines of the sensitivity $S = \sqrt{2}/(|\mathcal{A}_{\rm CP}|\sqrt{N})$ which is needed to measure $\mathcal{A}_{\rm CP}$ at 95% CL ($\sigma^s = 2$) for $\mathcal{L} = 500$ fb^{-1}, for $\varphi_{A_\tau} = 0.2\pi$ and $\varphi_{M_1} = \varphi_\mu = 0$. In Fig. 2.14 we show the contour lines of the sensitivity S for $\varphi_{M_1} = 0.2\pi$ and $\varphi_\mu = \varphi_{A_\tau} = 0$. It is interesting to note that in a large region in the $|\mu|$–M_2 plane in Figs. 2.13 and 2.14 we

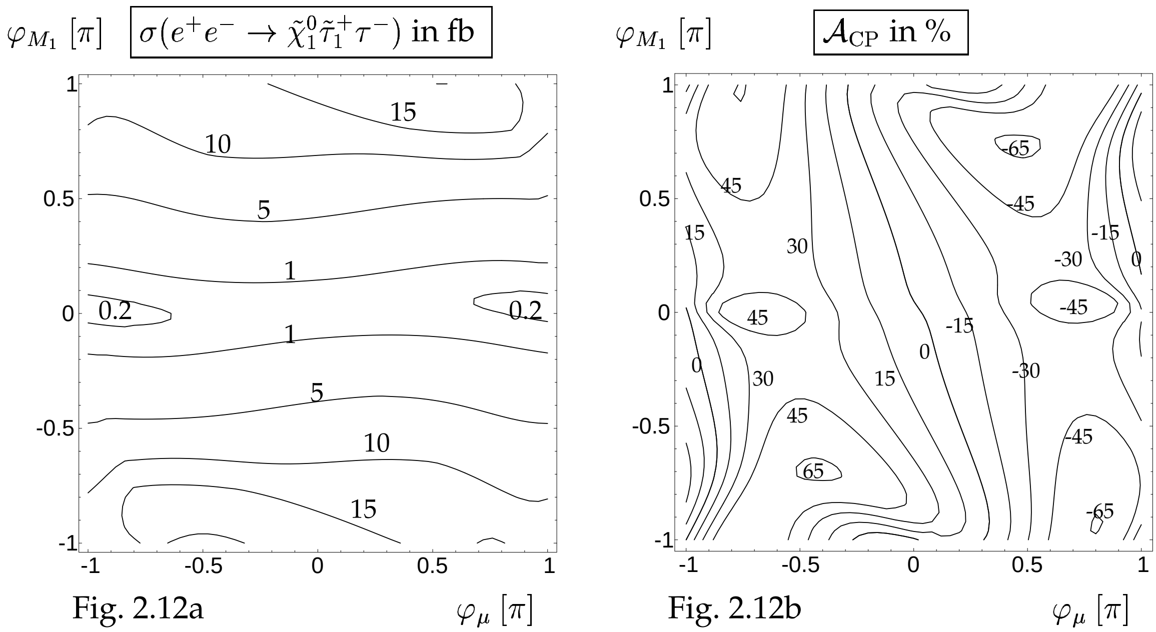


Fig. 2.12a Fig. 2.12b

Figure 2.12: Contour lines of σ and $\mathcal{A}_{\rm CP}$ in the φ_μ–φ_{M_1} plane for $M_2 = 400$ GeV, $|\mu| = 300$ GeV, $\tan\beta = 5$, $\varphi_{A_\tau} = 0$, $A_\tau = 250$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

obtain a sensitivity $S < 0.35$, which means that the asymmetries can be measured at 95% CL.

2.2.4 Summary of Section 2.2

We have defined and analyzed a CP odd asymmetry $\mathcal{A}_{\rm CP}$ of the transverse τ polarization in neutralino production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and subsequent two-body decay $\tilde{\chi}_i^0 \to \tilde{\tau}_k^\pm \tau^\mp$. The asymmetry is sensitive to CP-violating phases of the the trilinear scalar coupling parameter A_τ and the gaugino and Higgsino mass parameters M_1, μ. The asymmetry occurs already at tree level and is due to spin effects in the neutralino production and decay process. In a numerical study for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and neutralino decay $\tilde{\chi}_2^0 \to \tilde{\tau}_1^\pm\tau^\mp$ we have shown that the asymmetry can be as large as 60%. It can be sizable even for small phases of μ and M_1, suggested by the experimental limits on EDMs.

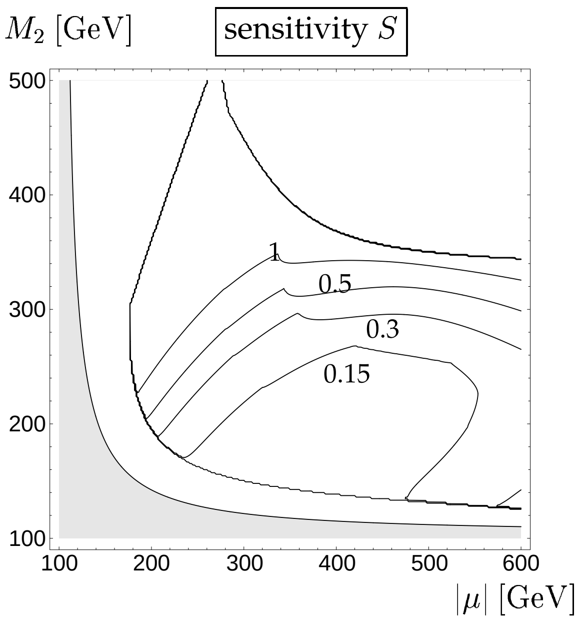


Figure 2.13: Contour lines of S for $\varphi_{A_\tau} = 0.2\pi$, $\varphi_{M_1} = \varphi_\mu = 0$, $A_\tau = 1$ TeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\chi_1^0} + m_{\chi_2^0}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\chi_2^0}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

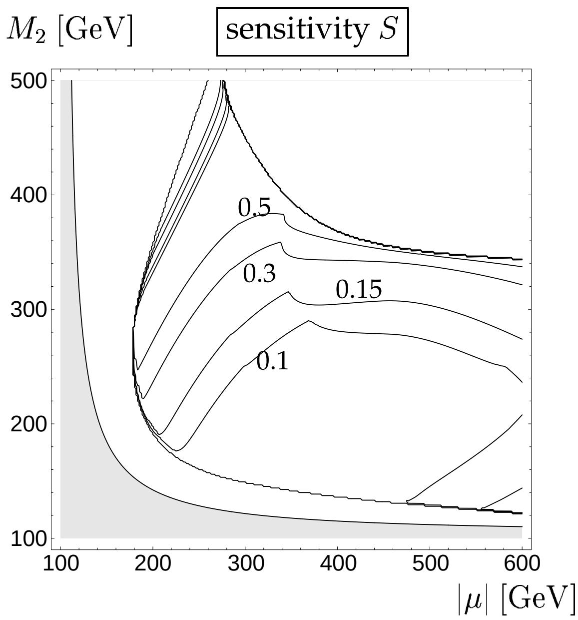


Figure 2.14: Contour lines of S for $\varphi_{M_1} = 0.2\pi$, $\varphi_{A_\tau} = \varphi_\mu = 0$, $A_\tau = 250$ GeV, $\tan\beta = 5$ and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The blank area outside the area of the contour lines is kinematically forbidden since here either $\sqrt{s} < m_{\chi_1^0} + m_{\chi_2^0}$ or $m_{\tilde{\tau}_1} + m_\tau > m_{\chi_2^0}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

2.3 T-odd observables in neutralino production and decay into a Z boson

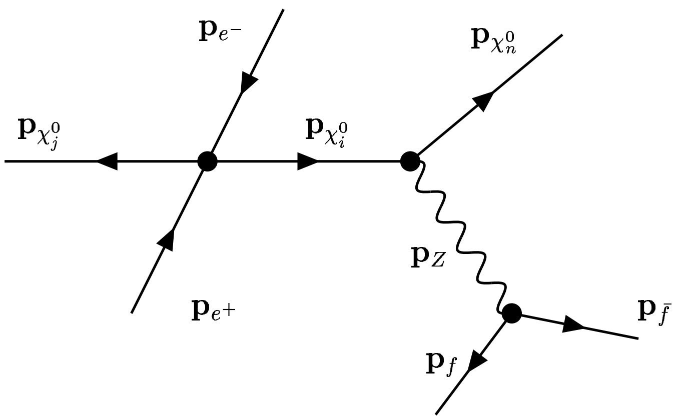


Figure 2.15: Schematic picture of the neutralino production and decay process.

A further possibility to study CP violation in the neutralino sector is the two-body decay of the neutralino into a Z boson. Due to spin correlations of the neutralino and the Z boson, observables can be defined which have not only CP-odd contributions from the neutralino production, but also from its decay.

We study CP violation in neutralino production

$$e^+ + e^- \to \tilde{\chi}^0_i + \tilde{\chi}^0_j; \quad i, j = 1, \ldots, 4, \tag{2.29}$$

with the subsequent two-body decay of one neutralino into a Z boson

$$\tilde{\chi}^0_i \to \chi^0_n + Z; \quad n < i, \tag{2.30}$$

followed by the decay of the Z boson

$$Z \to f + \bar{f}; \quad f = \ell, q, \quad \ell = e, \mu, \tau, \quad q = c, b. \tag{2.31}$$

For a schematic picture of the neutralino production and decay process see Fig. 2.15. If CP is violated, the phases φ_{M_1} and φ_μ lead to CP sensitive elements of the Z

boson density matrix. They involve CP-odd asymmetries $\mathcal{A}_f$ in the angular distribution of the decay fermions

$$\mathcal{A}_f = \frac{\sigma(\mathcal{T}_f > 0) - \sigma(\mathcal{T}_f < 0)}{\sigma(\mathcal{T}_f > 0) + \sigma(\mathcal{T}_f < 0)}, \tag{2.32}$$

of the triple product

$$\mathcal{T}_f = \mathbf{p}_{e^-} \cdot (\mathbf{p}_f \times \mathbf{p}_{\bar{f}}), \tag{2.33}$$

and the cross section σ of neutralino production (2.29) and subsequent decay chain (2.30)-(2.31). Due to the correlations between the $\tilde{\chi}_i^0$ polarization and the Z boson polarization, there are CP-odd contributions to the Z boson density matrix and to the asymmetries $\mathcal{A}_f$ from the production (2.29) and from the decay process (2.30).

In Section (2.1) we have studied asymmetries for neutralino decay into sleptons $\tilde{\chi}_i^0 \to \tilde{\ell}\ell$. We have shown that these asymmetries have in contrast only contributions from the neutralino production process, since the neutralino decay is a two-body decay into scalars.

Note that if we would replace the triple product $\mathcal{T}_f$ by $\mathcal{T}_f' = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\chi_i^0} \times \mathbf{p}_Z)$, and would calculate the corresponding asymmetry, where the Z boson polarization is summed, all spin correlations and thus this asymmetry would vanish identically because of the Majorana properties of the neutralinos.

2.3.1 Cross section

For the calculation of the cross section for the combined process of neutralino production (2.29) and the subsequent two-body decays (2.30), (2.31) of $\tilde{\chi}_i^0$ we use the same spin-density matrix formalism as in [27, 39]. The (unnormalized) spin-density matrix of the Z boson

$$\rho_P(Z)^{\lambda_k \lambda_k'} = |\Delta(\tilde{\chi}_i^0)|^2 \sum_{\lambda_i, \lambda_i'} \rho_P(\tilde{\chi}_i^0)^{\lambda_i \lambda_i'} \, \rho_{D_1}(\tilde{\chi}_i^0)_{\lambda_i' \lambda_i}^{\lambda_k \lambda_k'}, \tag{2.34}$$

is composed of the neutralino propagator $\Delta(\tilde{\chi}_i^0)$, the spin-density production matrix $\rho_P(\tilde{\chi}_i^0)$, defined in (C.3), and the decay matrix $\rho_{D_1}(\tilde{\chi}_i^0)$, defined in (C.46). The amplitude squared for the complete process $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$; $\tilde{\chi}_i^0 \to \tilde{\chi}_n^0 Z$; $Z \to f\bar{f}$ can now be written

$$|T|^2 = |\Delta(Z)|^2 \sum_{\lambda_k,\lambda'_k} \rho_P(Z)^{\lambda_k\lambda'_k}\, \rho_{D_2}(Z)_{\lambda'_k\lambda_k}, \tag{2.35}$$

with the decay matrix $\rho_{D_2}(Z)$ for the Z decay, defined in (C.49). Inserting the density matrices $\rho_P(\tilde{\chi}_i^0)$ (C.10) and $\rho_{D_1}(\tilde{\chi}_i^0)$ (C.63) into (2.34) leads to

$$\rho_P(Z)^{\lambda_k\lambda'_k} = 4\,|\Delta(\tilde{\chi}_i^0)|^2 \left[PD_1\,\delta^{\lambda_k\lambda'_k} + \Sigma_P^a\,{}^c\Sigma_{D_1}^a\,(J^c)^{\lambda_k\lambda'_k} + P\,{}^{cd}D_1\,(J^{cd})^{\lambda_k\lambda'_k}\right], \tag{2.36}$$

summed over a, c, d. Here the Z production matrix $\rho_P(Z)$ is decomposed into contributions of scalar (first term), vector (second term) and tensor parts (third term). Inserting then $\rho_P(Z)$ (2.36) and $\rho_{D_2}(Z)$ (C.64) into (2.35) leads finally to

$$|T|^2 = 4\,|\Delta(\tilde{\chi}_i^0)|^2\,|\Delta(Z)|^2 \left[3PD_1D_2 + 2\Sigma_P^a\,{}^c\Sigma_{D_1}^a\,{}^cD_2 + 4P({}^{cd}D_1^{cd}D_2 - \tfrac{1}{3}\,{}^{cc}D_1\,{}^{dd}D_2)\right]. \tag{2.37}$$

The differential cross section is then given by

$$d\sigma = \frac{1}{2s}|T|^2 d\mathrm{Lips}(s; p_{\chi_j^0}, p_{\chi_n^0}, p_f, p_{\bar{f}}), \tag{2.38}$$

where dLips is the Lorentz invariant phase-space element defined in (B.40).

2.3.2 Z boson polarization

The mean polarization of the Z boson is given by its 3×3 density matrix $<\rho(Z)>$ with $\mathrm{Tr}\{<\rho(Z)>\} = 1$. We obtain $<\rho(Z)>$ in the laboratory system by integrating (2.36) over the Lorentz invariant phase-space element $d\mathrm{Lips}(s; p_{\chi_j^0}, p_{\chi_n^0}, p_Z)$ (B.39), and normalizing by the trace

$$<\rho(Z)^{\lambda_k\lambda'_k}> = \frac{\int \rho_P(Z)^{\lambda_k\lambda'_k}\, d\mathrm{Lips}}{\int \mathrm{Tr}\{\rho_P(Z)^{\lambda_k\lambda'_k}\}\, d\mathrm{Lips}} = \tfrac{1}{3}\delta^{\lambda_k\lambda'_k} + V_c\,(J^c)^{\lambda_k\lambda'_k} + T_{cd}\,(J^{cd})^{\lambda_k\lambda'_k}. \tag{2.39}$$

The components V_c of the vector polarization and T_{cd} of the tensor polarization are given by

$$V_c = \frac{\int \Sigma_P^a \, {}^c\Sigma_{D_1}^a \; d\mathrm{Lips}}{3 \int 2 \, P D_1 \; d\mathrm{Lips}}, \quad T_{cd} = T_{dc} = \frac{\int P \, {}^{cd}D_1 \; d\mathrm{Lips}}{3 \int P D_1 \; d\mathrm{Lips}}, \tag{2.40}$$

where we have used the narrow width approximation for the neutralino propagator. The tensor components T_{12} and T_{23} vanish due to phase-space integration. The density matrix in the helicity basis, see Appendix F.2, is given by

$$< \rho(Z)^{--} > \; = \; \tfrac{1}{2} - V_3 + T_{33}, \tag{2.41}$$
$$< \rho(Z)^{00} > \; = \; -2T_{33}, \tag{2.42}$$
$$< \rho(Z)^{-0} > \; = \; \tfrac{1}{\sqrt{2}}(V_1 + iV_2) - \sqrt{2}\,T_{13}, \tag{2.43}$$
$$< \rho(Z)^{-+} > \; = \; T_{11}, \tag{2.44}$$
$$< \rho(Z)^{0+} > \; = \; \tfrac{1}{\sqrt{2}}(V_1 + iV_2) + \sqrt{2}\,T_{13}, \tag{2.45}$$

where we have used $T_{11} + T_{22} + T_{33} = -\frac{1}{2}$ and $T_{12} = T_{23} = 0$.

2.3.3 T-odd asymmetry

From (2.37) we obtain for the asymmetry (2.32)

$$\mathcal{A}_f = \frac{\int \mathrm{Sign}[\mathcal{T}_f] |T|^2 \; d\mathrm{Lips}}{\int |T|^2 \; d\mathrm{Lips}} = \frac{\int \mathrm{Sign}[\mathcal{T}_f] \; 2 \; \Sigma_P^a \, {}^c\Sigma_{D_1}^a \, {}^cD_2 \; d\mathrm{Lips}}{\int 3 \; P D_1 D_2 \; d\mathrm{Lips}}, \tag{2.46}$$

and $d\mathrm{Lips}(s; p_{\chi_j^0}, p_{\chi_n^0}, p_f, p_{\bar{f}})$ is the Lorentz invariant phase-space element defined in (B.40), where we have already used the narrow width approximation for the propagators. In the numerator only the vector part of $|T|^2$ remains which contains the triple product $\mathcal{T}_f = \mathbf{p}_{e^-} \cdot (\mathbf{p}_f \times \mathbf{p}_{\bar{f}})$. In the denominator the vector and tensor parts of $|T|^2$ vanish, since the complete phase-space integration eliminates the spin correlations. Due to the correlations $\Sigma_P^a \, {}^c\Sigma_{D_1}^a$ between the $\tilde{\chi}_i^0$ and the Z boson polarization, there are CP-odd contributions to the asymmetry $\mathcal{A}_f$ from both the neutralino production process (2.29), and from the neutralino decay process (2.30). The contribution from the production is given by the term with $a = 2$ in (2.46) and is proportional to the transverse polarization Σ_P^2 (C.17) of the neutralino perpendicular to the production plane. For the production of a pair of equal neutralinos,

$e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_i^0$, we have $\Sigma_P^2 = 0$. The contributions from the decay, given by the terms with $a = 1, 3$ in (2.46), are proportional to

$$ {}^c\Sigma_{D_1}^a\, {}^cD_2 \supset -8m_{\chi_n^0}(ImO_{ni}^{''L})(ReO_{ni}^{''L})(R_f^2 - L_f^2)(t_Z^c \cdot p_{\bar{f}})\epsilon_{\mu\nu\rho\sigma}s_{\chi_i^0}^{a,\mu}p_{\chi_i^0}^{\nu}p_Z^{\rho}t_Z^{c,\sigma}, \tag{2.47} $$

see last term of (C.62), which contains the ϵ-tensor. Thus $\mathcal{A}_f$ may be enhanced (reduced) if the contributions from production and decay have the same (opposite) sign.

Note that the contributions from the decay would vanish for a two-body decay of the neutralino into a scalar particle, as discussed in Section (2.1). We have found in this case, that only contributions to $\mathcal{A}_f$ from the production remain, which are multiplied by a decay factor $\propto (|R|^2 - |L|^2)$, and thus $\mathcal{A}_f \propto (|R|^2 - |L|^2)/(|R|^2 + |L|^2)$, see (2.16), where R and L are the right and left couplings of the scalar particle to the neutralino.

For the measurement of $\mathcal{A}_f$ the charges and the flavors of f and $\bar{f}$ have to be distinguished. For $f = e, \mu$ this will be possible on an event by event basis. For $f = \tau$ it will be possible after taking into account corrections due to the reconstruction of the τ momentum. For $f = q$ the distinction of the quark flavors should be possible by flavor tagging in the case $q = b, c$ [30, 31]. However, in this case the quark charges will be distinguished statistically for a given event sample only. Note that $\mathcal{A}_q$ is always larger than $\mathcal{A}_\ell$, due to the dependence of $\mathcal{A}_f$ on the Z-$\bar{f}$-f couplings, which follows from (C.65), (C.66) and (2.46):

$$ \mathcal{A}_f \propto \frac{R_f^2 - L_f^2}{R_f^2 + L_f^2} \quad \Rightarrow \mathcal{A}_{b(c)} = \frac{R_\ell^2 + L_\ell^2}{R_\ell^2 - L_\ell^2}\frac{R_{b(c)}^2 - L_{b(c)}^2}{R_{b(c)}^2 + L_{b(c)}^2}\,\mathcal{A}_\ell \simeq 6.3\ (4.5) \times \mathcal{A}_\ell, \tag{2.48} $$

compare also Section 4.2.

The significance for measuring the asymmetry is given by $S_f = |\mathcal{A}_f|\sqrt{N}$, see (1.17). Note that S_f is larger for $f = b, c$ than for $f = \ell = e, \mu, \tau$ with $S_b \simeq 7.7 \times S_\ell$ and $S_c \simeq 4.9 \times S_\ell$, which follows from (2.48) and from $\mathrm{BR}(Z \to b\bar{b}) \simeq 1.5 \times \mathrm{BR}(Z \to \ell\bar{\ell})$, $\mathrm{BR}(Z \to c\bar{c}) \simeq 1.2 \times \mathrm{BR}(Z \to \ell\bar{\ell})$ [45].

2.3.4 Numerical results

We study the dependence of the Z density matrix $< \rho(Z) >$ (2.39), the asymmetry $\mathcal{A}_\ell(\ell = e, \mu, \tau)$ (2.32), and the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0) \times \mathrm{BR}(\tilde{\chi}_i^0 \to$

$\tilde{\chi}_1^0 Z) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ on the MSSM parameters $\mu = |\mu|\, e^{i\,\varphi_\mu}$ and $M_1 = |M_1|\, e^{i\,\varphi_{M_1}}$ for $\tan\beta = 10$. In order to reduce the number of parameters, we assume the relation $|M_1| = 5/3 M_2 \tan^2\theta_W$ and use the renormalization group equations for the sfermion masses, Appendix A.3.4, with $m_0 = 300$ GeV. For the branching ratio $Z \to \ell\bar{\ell}$, summed over $\ell = e, \mu, \tau$, we take $\mathrm{BR}(Z \to \ell\bar{\ell}) = 0.1$ [45]. The values for $\mathcal{A}_{b,c}$ are given by (2.48). We choose a center of mass energy of $\sqrt{s} = 800$ GeV and longitudinally polarized beams with beam polarizations $(P_{e^-}, P_{e^+}) = (\pm 0.8, \mp 0.6)$.

For the calculation of the neutralino widths $\Gamma_{\chi_i^0}$ and branching ratios $\mathrm{BR}(\tilde{\chi}_i^0 \to \tilde{\chi}_1^0 Z)$, see Appendix E.1, we include the following two-body decays, if kinematically allowed,

$$\tilde{\chi}_i^0 \to \tilde{e}_{R,L} e,\ \tilde{\mu}_{R,L}\mu,\ \tilde{\tau}_m \tau,\ \tilde{\nu}_\ell \bar{\nu}_\ell,\ \tilde{\chi}_n^0 Z,\ \tilde{\chi}_m^\mp W^\pm,\ \tilde{\chi}_n^0 H_1^0,\ \ell = e, \mu, \tau,\ m = 1, 2,\ n < i \tag{2.49}$$

and neglect three-body decays. The Higgs parameter is chosen $m_A = 1$ TeV and in the stau sector, we fix the trilinear scalar coupling parameter $A_\tau = 250$ GeV.

- ***Production of*** $\tilde{\chi}_1^0\,\tilde{\chi}_2^0$

In Fig. 2.16a we show the cross section for $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production in the $|\mu|$–M_2 plane for $\varphi_\mu = 0$ and $\varphi_{M_1} = 0.5\pi$. For $|\mu| \gtrsim 250$ GeV the left selectron exchange dominates due to the larger $\tilde{\chi}_2^0$–$\tilde{e}_L$ coupling, such that the polarization $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ enhances the cross section to values of more than 110 fb. The branching ratio $\mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0)$, see Fig. 2.16b, can even be 100% and decreases with increasing $|\mu|$ and M_2, when the two-body decays into sleptons and/or into the lightest neutral Higgs boson are kinematically allowed. The cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$, see Fig. 2.16c, does however not exceed 7 fb, due to the small $\mathrm{BR}(Z \to \ell\bar{\ell}) = 0.1$. Fig. 2.16d shows the $|\mu|$–M_2 dependence of the asymmetry $\mathcal{A}_\ell$ for $\varphi_{M_1} = 0.5\pi$ and $\varphi_\mu = 0$. The asymmetry $|\mathcal{A}_\ell|$ can reach a value of 1.6%. The (positive) contributions from the production cancel the (negative) contributions from the decay on the contour $\mathcal{A}_\ell = 0$. We also studied the φ_μ dependence of $\mathcal{A}_\ell$. In the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0$ and $\varphi_\mu = 0.5\pi$ we found $|\mathcal{A}_\ell| < 0.5\%$.

In Fig. 2.17 we show the φ_μ–φ_{M_1} dependence of $\mathcal{A}_\ell$ for $|\mu| = 400$ GeV and $M_2 = 250$ GeV. The asymmetry $\mathcal{A}_\ell$ is more sensitive to φ_{M_1} than to φ_μ. It is remarkable that the maximal phases of $\varphi_{M_1}, \varphi_\mu = \pm\pi/2$ do not lead to the highest values of $\mathcal{A}_\ell \approx \pm 1.4\%$, which are reached for $(\varphi_{M_1}, \varphi_\mu) \approx (\pm 0.3\pi, 0)$. The reason for this is that the spin-correlation terms $\Sigma_P^a\, {}^c\Sigma_{D_1}^a\, {}^cD_2$ in the numerator of $\mathcal{A}_f$ (2.46), are products of CP-odd and CP-even factors. The CP-odd (CP-even) factors have a sine-like

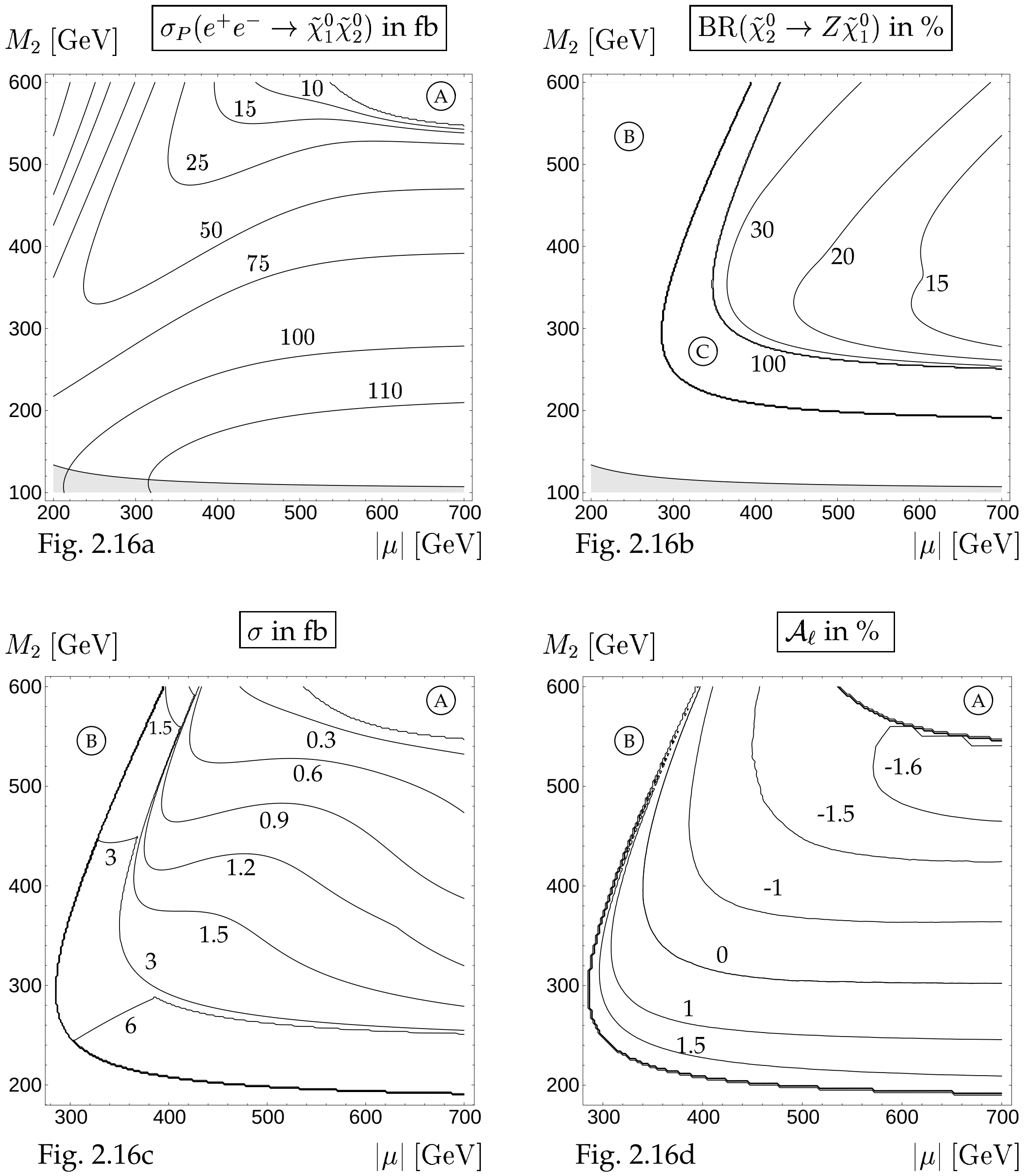


Figure 2.16: Contour plots for 2.16a: $\sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$, 2.16b: $\mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0)$, 2.16c: $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ with $\mathrm{BR}(Z \to \ell\bar{\ell}) = 0.1$, 2.16d: the asymmetry $\mathcal{A}_\ell$, in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A (B) is kinematically forbidden by $m_{\tilde{\chi}_1^0} + m_{\tilde{\chi}_2^0} > \sqrt{s}$ ($m_Z + m_{\tilde{\chi}_1^0} > m_{\tilde{\chi}_2^0}$). In area C of plot 2.16b: $\mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0) = 100\%$. The gray area is excluded by $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

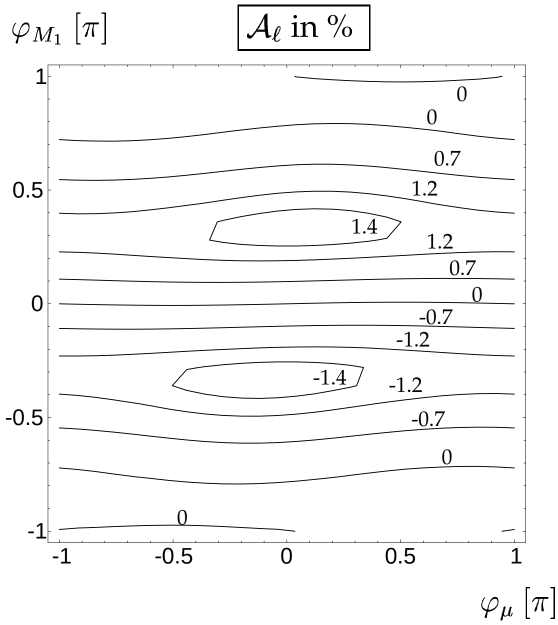


Figure 2.17: Contour lines of the asymmetry $\mathcal{A}_\ell$ for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0; \tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0; Z \to \ell\bar{\ell}(\ell = e, \mu, \tau)$, in the φ_μ–φ_{M_1} plane for $M_2 = 250$ GeV, $|\mu| = 400$ GeV, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

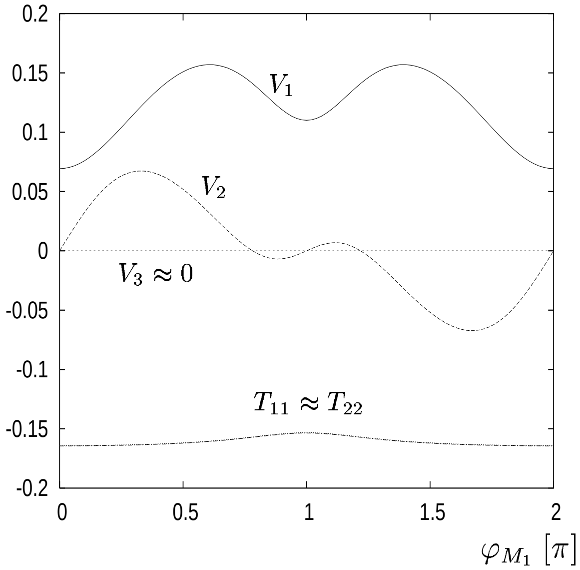


Figure 2.18: Vector (V_i) and tensor (T_{ii}) components of the Z density matrix for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0; \tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0$, for $M_2 = 250$ GeV, $|\mu| = 400$ GeV, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

(cosine-like) phase dependence. Therefore, the maximum of the CP asymmetry $\mathcal{A}_f$ is shifted from $\varphi_{M_1}, \varphi_\mu = \pm\pi/2$ to a smaller or larger value.

In the φ_μ–φ_{M_1} region shown in Fig. 2.17 also the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ with $\mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0) = 1$ and $\mathrm{BR}(Z \to \ell\bar{\ell}) = 0.1$, is rather insensitive to φ_μ and varies between 7 fb ($\varphi_{M_1} = 0$) and 14 fb ($\varphi_{M_1} = \pm\pi$). The statistical significance for measuring the asymmetry for the leptonic decay of the Z is given by $S_\ell = |\mathcal{A}_\ell|\sqrt{\mathcal{L}\cdot\sigma}$, see Section 2.3.3. For $\mathcal{L} = 500$ fb^{-1}, we have $S_\ell < 1$ in the scenario of Fig. 2.17 and thus $\mathcal{A}_\ell$ cannot be measured at the 68% confidence level ($S_\ell = 1$). For hadronic decays into $b(c)$ quarks, however, the significance is larger $S_{b(c)} = 7.7(4.9)S_\ell$, as discussed Section 2.3.3. For $\mathcal{L} = 500$ fb^{-1} and $(\varphi_{M_1}, \varphi_\mu) = (\pm 0.3\pi, 0)$ in Fig. 2.17 we find $S_{b(c)} = 8(5)$ and thus $\mathcal{A}_{b(c)}$ can be measured.

In Fig. 2.18 we show the φ_{M_1} dependence of the vector (V_i) (V_i) and tensor (T_{ii}) components of the Z boson polarization. The components T_{11}, T_{22} and V_1 have a CP-even dependence on φ_{M_1}. The component V_2 is CP-odd and is not only zero for $\varphi_{M_1} = 0$ and $\varphi_{M_1} = \pi$, but also for $\varphi_{M_1} \approx (1 \pm 0.2)\pi$, due to the destructive interference of the contributions from CP violation in production and decay. The interference of the contributions from the CP-even effects in production and decay cause the two maxima of V_1. As discussed in Appendix C.4, the tensor components T_{11} and T_{22} are almost equal. Compared to V_1 and V_2, they have the same order of magnitude but their dependence on φ_{M_1} is rather weak. The components $T_{13}, V_3 < 10^{-6}$ are small, and thus the density matrix $< \rho(Z) >$ is almost symmetric. In the CP conserving case, e.g. for $\varphi_{M_1} = \varphi_\mu = 0$, $M_2 = 250$ GeV, $|\mu| = 400$ GeV, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ it reads

$$< \rho(Z) >= \begin{pmatrix} 0.329 & 0.049 & 0.0003 \\ 0.049 & 0.343 & 0.049 \\ 0.0003 & 0.049 & 0.329 \end{pmatrix}. \tag{2.50}$$

In the CP violating case, e.g. for $\varphi_{M_1} = 0.5\pi$ and the other parameters as above, $< \rho(Z) >$ has imaginary parts due to a non-vanishing V_2

$$< \rho(Z) >= \begin{pmatrix} 0.324 & 0.107 + 0.037i & 0.0003 \\ 0.107 - 0.037i & 0.352 & 0.107 + 0.037i \\ 0.0003 & 0.107 - 0.037i & 0.324 \end{pmatrix}. \tag{2.51}$$

Imaginary elements of $< \rho(Z) >$ are thus an indication of CP violation. Note that also the CP even diagonal elements are changed for $\varphi_{M_1} \neq 0$ (and also for $\varphi_\mu \neq 0$). This fact has been exploited in [46] as a possibility to determine the CP violating phases. The $\varphi_{M_1}, \varphi_\mu$ dependence of the Z-density matrix elements has also been studied in [47], for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_3$ followed by $\tilde{\chi}^0_3 \to Z\tilde{\chi}^0_1$.

■ ***Production of*** $\tilde{\chi}^0_2\tilde{\chi}^0_2$

In Fig. 2.19a we show the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_2) \times \mathrm{BR}(\tilde{\chi}^0_2 \to Z\tilde{\chi}^0_1) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ in the $|\mu|$–M_2 plane for $\varphi_\mu = 0$ and $\varphi_{M_1} = 0.5\pi$. The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_2)$, which is not shown, is enhanced by the choice $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ and reaches values up to 130 fb. The branching ratio $\mathrm{BR}(\tilde{\chi}^0_2 \to Z\tilde{\chi}^0_1)$ can be 100%, see Fig. 2.16b, however, due to $\mathrm{BR}(Z \to \ell\bar{\ell}) = 0.1$, σ is not larger than 13 fb, see Fig. 2.19a.

If two equal neutralinos are produced, the CP sensitive transverse polarization of the neutralinos perpendicular to the production plane vanishes, $\Sigma^2_P = 0$ in (2.46). However, the asymmetry $\mathcal{A}_f$ obtains CP sensitive contributions from the neutralino decay process, terms with $a = 1, 3$ in (2.47). In Fig. 2.19b we show for

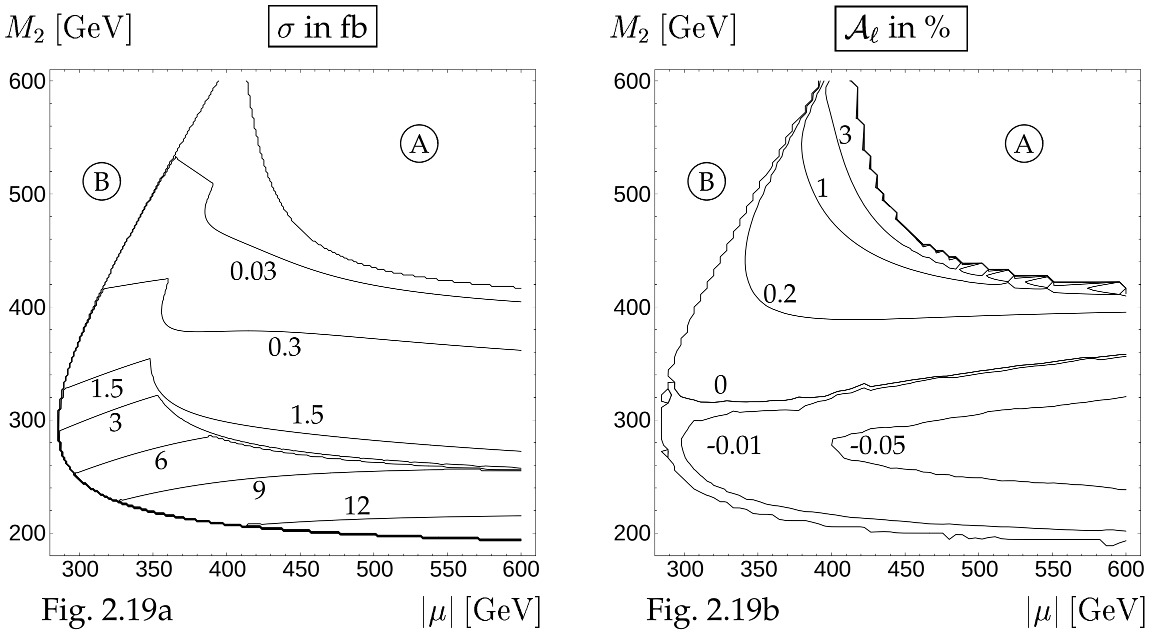


Figure 2.19: Contour lines of $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ (2.19a), and the asymmetry $\mathcal{A}_\ell$ (2.19b) in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A (B) is kinematically forbidden by $m_{\tilde{\chi}_2^0} + m_{\tilde{\chi}_2^0} > \sqrt{s}$ $(m_Z + m_{\tilde{\chi}_1^0} > m_{\tilde{\chi}_2^0})$.

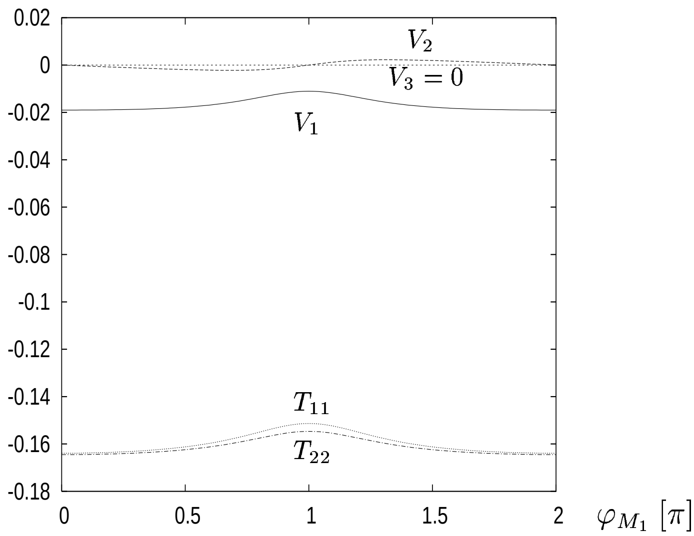


Figure 2.20: Vector (V_i) and tensor (T_{ii}) components of the Z density matrix for $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0$; $\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0$, for $M_2 = 250$ GeV, $|\mu| = 400$ GeV, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

$\varphi_{M_1} = 0.5\pi$ and $\varphi_\mu = 0$ the $|\mu|$ and M_2 dependence of the asymmetry $\mathcal{A}_\ell$, which reaches more than 3%. Along the contour $\mathcal{A}_\ell = 0$ in Fig. 2.19b the contribution to $\mathcal{A}_\ell$ which is proportional to Σ_P^1, see 2.46, cancels that which is proportional to Σ_P^3. As the largest values of $\mathcal{A}_\ell \gtrsim 0.2\%$ and $\mathcal{A}_q \gtrsim 1\%$ lie in a region of the $|\mu|$–M_2 plane where $\sigma \lesssim 0.3$ fb, it will be difficult to measure $\mathcal{A}_f$ in a statistically significant way. We also studied the φ_μ dependence of $\mathcal{A}_\ell$. In the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0$ and $\varphi_\mu = 0.5\pi$ we found $|\mathcal{A}_\ell| < 0.5\%$, and thus the influence of φ_μ is also small.

In Fig. 2.20 we show the φ_{M_1} dependence of the vector (V_i) and tensor (T_{ii}) components of the Z boson polarization. Because there are only CP sensitive contributions from the neutralino decay process, V_2 is only zero at $\varphi_{M_1} = 0, \pi$ and V_1 has one maximum at $\varphi_{M_1} = \pi$, compared to the components for $\tilde{\chi}_1^0\,\tilde{\chi}_2^0$ production, shown in Fig. 2.18. In addition, the vector components V_1 and V_2 in Fig. 2.20 are much smaller than the tensor components $T_{11} \approx T_{22}$. The smallness of V_2 accounts for the smallness of the asymmetry $|\mathcal{A}_\ell| < 0.05\%$. Furthermore, the other components are small, i.e. $T_{13} < 10^{-6}$ and $V_3 = 0$.

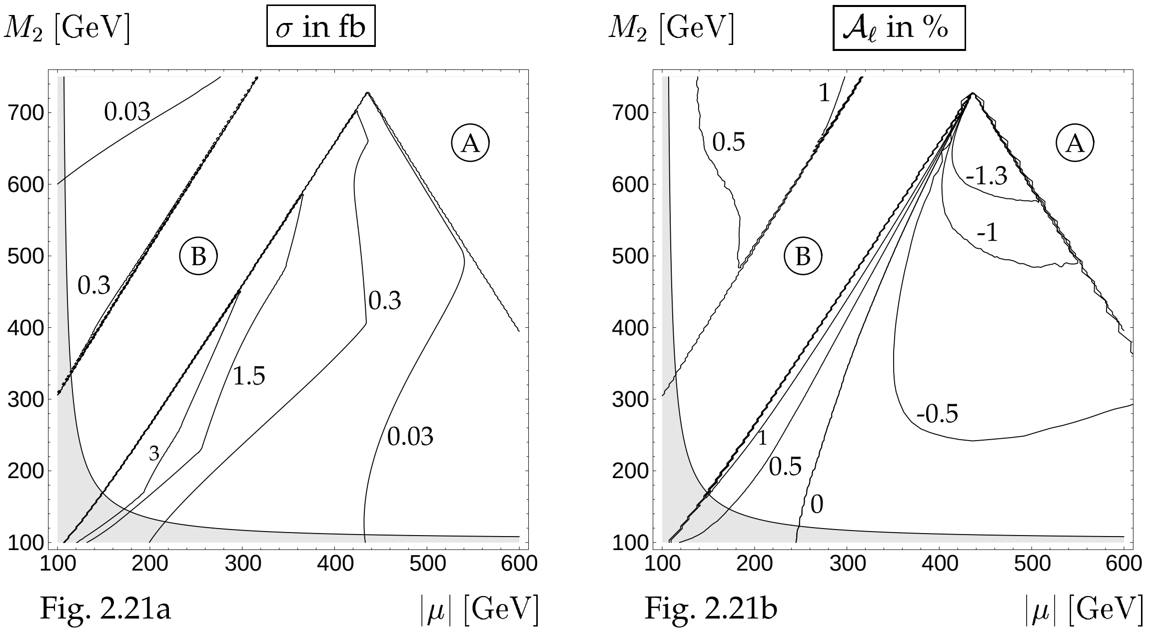


Figure 2.21: Contour lines of $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0) \times \mathrm{BR}(\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ (2.21a), and the asymmetry $\mathcal{A}_\ell$ (2.21b) in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. The area A (B) is kinematically forbidden by $m_{\tilde{\chi}_1^0} + m_{\tilde{\chi}_3^0} > \sqrt{s}$ $(m_Z + m_{\tilde{\chi}_1^0} > m_{\tilde{\chi}_3^0})$. The gray area is excluded by $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

■ *Production of* $\tilde{\chi}_1^0\,\tilde{\chi}_3^0$

In Fig. 2.21a we show the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0) \times \mathrm{BR}(\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ in the $|\mu|$–M_2 plane for $\varphi_\mu = 0$ and $\varphi_{M_1} = 0.5\pi$. The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0)$, which is not shown, is enhanced by the choice $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$ and reaches up to 50 fb. The branching ratio $\mathrm{BR}(\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0)$, which is not shown, can be 100%, however, due to $\mathrm{BR}(Z \to \ell\bar{\ell}) = 0.1$, the cross section shown in Fig. 2.21a does not exceed 5 fb. In Fig. 2.21b we show the $|\mu|$–M_2 dependence of the asymmetry $\mathcal{A}_\ell$. The asymmetry $|\mathcal{A}_\ell|$ reaches 1.3% at its maximum, however in a region, where $\sigma < 0.3$ fb, the asymmetry $\mathcal{A}_\ell$ thus cannot be measured. In the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0$ and $\varphi_\mu = 0.5\pi$ we found $|\mathcal{A}_\ell| < 0.7\%$.

■ *Production of* $\tilde{\chi}_2^0\,\tilde{\chi}_3^0$

For the process $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_3^0$ we discuss the decay $\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0$ of the heavier neutralino which has a larger kinematically allowed region than that for $\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0$. Similar to $\tilde{\chi}_1^0\,\tilde{\chi}_3^0$ production and decay, the cross section $\sigma_P(e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_3^0)$ reaches

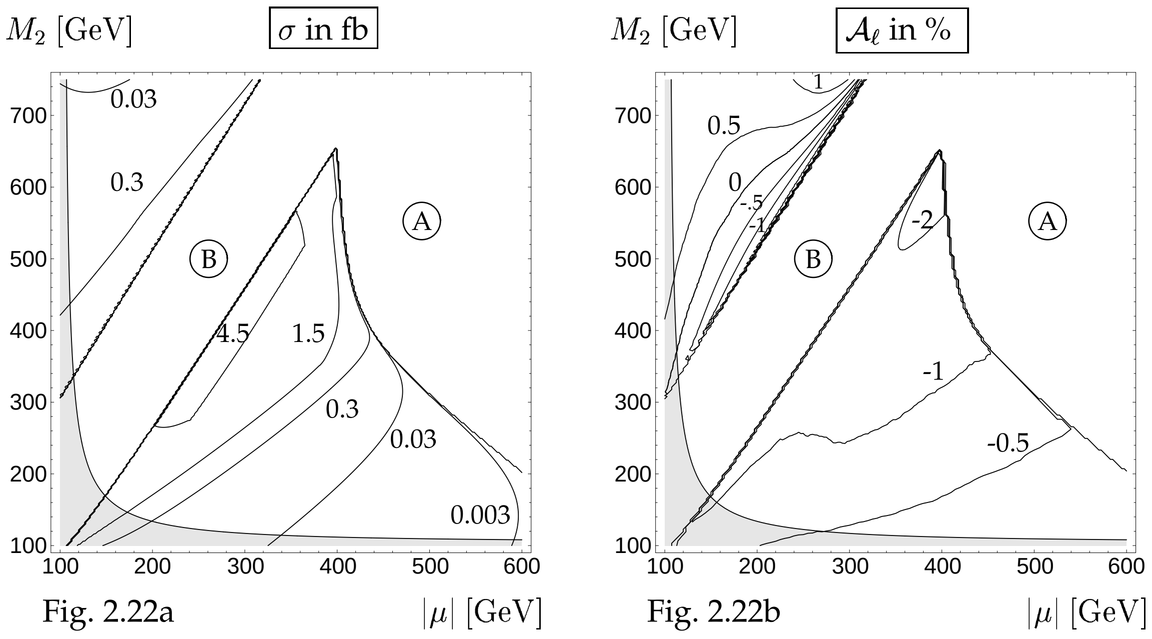


Figure 2.22: Contour lines of $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_3^0) \times \mathrm{BR}(\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ (2.22a), and the asymmetry $\mathcal{A}_\ell$ (2.22b) in the $|\mu|$–M_2 plane for $\varphi_{M_1} = 0.5\pi$, $\varphi_\mu = 0$, $\tan\beta = 10$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$. The area A (B) is kinematically forbidden by $m_{\tilde{\chi}_2^0} + m_{\tilde{\chi}_3^0} > \sqrt{s}$ ($m_Z + m_{\tilde{\chi}_1^0} > m_{\tilde{\chi}_3^0}$). The gray area is excluded by $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

values up to 50 fb for a beam polarization of $(P_{e^-}, P_{e^+}) = (0.8, -0.6)$ and that for the complete process $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_3^0) \times \mathrm{BR}(\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\bar{\ell})$ attains values up to 5 fb in the investigated regions of the $|\mu|$–M_2 plane in Fig. 2.22a.

The asymmetry $\mathcal{A}_\ell$, Fig. 2.22b, is somewhat larger than that for $\tilde{\chi}_1^0\,\tilde{\chi}_3^0$ production and decay, and reaches at its maximum 2%. However, it will be difficult to measure $\mathcal{A}_\ell$, since e.g. for $|\mu| = 380$ GeV, $M_2 = 560$ GeV and $(\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0)$, we found $S_\ell \approx 1$, for $\mathcal{L} = 500$ fb^{-1}. For the hadronic decays of the Z boson we have $S_{b(c)} \approx 8(5)$ and thus $\mathcal{A}_{b(c)}$ is accessible for $\tilde{\chi}_1^0\,\tilde{\chi}_3^0$ production. For $\varphi_\mu = 0.5\pi$ and $\varphi_{M_1} = 0$ we found that $|\mathcal{A}_\ell| \lesssim 1\%$ in regions of the $|\mu|$–M_2 plane where $\sigma \lesssim 0.5$ fb, and $|\mathcal{A}_\ell| \lesssim 0.4\%$ in regions where $\sigma \lesssim 5$ fb.

2.3.5 Summary of Section 2.3

We have analyzed CP sensitive observables in neutralino production $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ and the subsequent two-body decay of one neutralino into a Z boson $\tilde{\chi}_i^0 \to \tilde{\chi}_n^0 Z$, followed by the decay $Z \to \ell\bar{\ell}$ for $\ell = e, \mu, \tau$, or $Z \to q\bar{q}$ with $q = c, b$. The CP sensitive observables are defined by the vector component V_2 of the Z boson density matrix and the CP asymmetry $\mathcal{A}_{\ell(q)}$, which involves the triple product $\mathcal{T}_{\ell(q)} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\ell(q)} \times \mathbf{p}_{\bar{\ell}(\bar{q})})$. The tree level contributions to these observables are due to correlations of the neutralino $\tilde{\chi}_i^0$ spin and the Z boson spin. In a numerical study of the MSSM parameter space with complex M_1 and μ for $\tilde{\chi}_1^0\tilde{\chi}_2^0$, $\tilde{\chi}_2^0\tilde{\chi}_2^0$, $\tilde{\chi}_1^0\tilde{\chi}_3^0$ and $\tilde{\chi}_2^0\tilde{\chi}_3^0$ production, we have shown that the asymmetry $\mathcal{A}_\ell$ can go up to 3%. For the hadronic decays of the Z boson, larger asymmetries are obtained with $\mathcal{A}_{c(b)} \simeq 6.3(4.5) \times \mathcal{A}_\ell$.

Chapter 3

CP violation in production and decay of charginos

Overview

We study chargino production with longitudinally polarized beams $e^+\,e^- \to \tilde{\chi}_i^+\,\tilde{\chi}_j^-$ with the subsequent leptonic decay of one chargino $\tilde{\chi}_i^+ \to \ell^+\tilde{\nu}_\ell$ for $\ell = e, \mu, \tau$ [48]. This decay mode allows the definition of a CP asymmetry which is sensitive to the phase φ_μ and probes CP violation in the chargino production process. For chargino decay into a W boson $\tilde{\chi}_i^+ \to W^+\chi_n^0$ [49], CP observables can be obtained which are also sensitive to φ_{M_1}. We present numerical results for the asymmetries, W polarizations, cross sections and branching ratios at a linear electron-positron collider with $\sqrt{s} = 800$ GeV.

3.1 CP asymmetry in chargino production and decay into a sneutrino

We study chargino production

$$e^+ + e^- \to \tilde{\chi}_i^+ + \tilde{\chi}_j^-; \quad i,j = 1,2, \tag{3.1}$$

with longitudinally polarized beams and the subsequent two-body decay of one of the charginos into a sneutrino

$$\tilde{\chi}_i^+ \to \ell^+ + \tilde{\nu}_\ell; \quad \ell = e, \mu, \tau. \tag{3.2}$$

We define the triple product

$$\mathcal{T}_\ell = (\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_i^+}) \cdot \mathbf{p}_\ell \tag{3.3}$$

and the T-odd asymmetry

$$\mathcal{A}_\ell^{\mathrm{T}} = \frac{\sigma(\mathcal{T}_\ell > 0) - \sigma(\mathcal{T}_\ell < 0)}{\sigma(\mathcal{T}_\ell > 0) + \sigma(\mathcal{T}_\ell < 0)}, \tag{3.4}$$

of the cross section σ for chargino production (3.1) and decay (3.2). The asymmetry $\mathcal{A}_\ell^{\mathrm{T}}$ is not only sensitive to the phase φ_μ, but also to absorptive contributions, which are eliminated in the CP asymmetry

$$\mathcal{A}_\ell = \frac{1}{2}(\mathcal{A}_\ell^{\mathrm{T}} - \bar{\mathcal{A}}_\ell^{\mathrm{T}}), \tag{3.5}$$

where $\bar{\mathcal{A}}_\ell^{\mathrm{T}}$ is the CP conjugated asymmetry for the process $e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+$; $\tilde{\chi}_i^- \to \ell^- \bar{\tilde{\nu}}_\ell$. In this context it is interesting to note that in chargino production it is not possible to construct a triple product and a corresponding asymmetry by using transversely polarized e^+ and e^- beams [17,50], therefore, one has to rely on the transverse polarization of the produced chargino.

3.1.1 Cross section

For the calculation of the cross section for the combined process of chargino production (3.1) and the subsequent two-body decay of $\tilde{\chi}_i^+$ (3.2), we use the spin-density matrix formalism as in [27, 51]. The amplitude squared,

$$|T|^2 = |\Delta(\tilde{\chi}_i^+)|^2 \sum_{\lambda_i,\lambda_i'} \rho_P(\tilde{\chi}_i^+)^{\lambda_i\lambda_i'} \rho_D(\tilde{\chi}_i^+)_{\lambda_i'\lambda_i}, \tag{3.6}$$

is composed of the (unnormalized) spin-density production matrix $\rho_P(\tilde{\chi}_i^+)$, defined in (D.8), and the decay matrix $\rho_D(\tilde{\chi}_i^+)$, defined in (D.36), with the helicity indices λ_i and λ_i' of the chargino. Inserting the density matrices into (3.6) leads to

$$|T|^2 = 4\,|\Delta(\tilde{\chi}_i^+)|^2 \,(PD + \Sigma_P^a \Sigma_D^a), \tag{3.7}$$

where we sum over a. The cross section and distributions are then obtained by integrating $|T|^2$ over the Lorentz invariant phase space element $d\mathrm{Lips}$, defined in (B.22):

$$d\sigma = \frac{1}{2s}|T|^2 d\mathrm{Lips}(s; p_{\chi_j^-}, p_\ell, p_{\tilde{\nu}_\ell}). \tag{3.8}$$

3.1.2 CP asymmetries

Inserting the cross section (3.8) into the definition of the asymmetry (3.4) we obtain

$$\mathcal{A}_\ell^{\mathrm{T}} = \frac{\int \mathrm{Sign}[\mathcal{T}_\ell]\ |T|^2\ d\mathrm{Lips}}{\int |T|^2\ d\mathrm{Lips}} = \frac{\int \mathrm{Sign}[\mathcal{T}_\ell]\ \Sigma_P^2 \Sigma_D^2\ d\mathrm{Lips}}{\int PD\ d\mathrm{Lips}}, \tag{3.9}$$

where we have already used the narrow width approximation for the chargino propagator. In the numerator of (3.9) only the CP sensitive contribution $\Sigma_P^2\Sigma_D^2$ from chargino polarization perpendicular to the production plane remains, since only this term contains the triple product $\mathcal{T}_\ell = (\mathbf{p}_{e^-} \times \mathbf{p}_{\chi_i^+}) \cdot \mathbf{p}_\ell$ (3.3). In the denominator only the term PD remains, since all spin correlations $\Sigma_P^a\Sigma_D^a$ vanish due to the integration over the complete phase space.

The coefficient Σ_P^2 is non-zero only for production of an unequal pair of charginos, $e^+e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$, and obtains contributions from Z-exchange and Z-$\tilde{\nu}$ interference only, see (D.17). The contribution to Σ_P^2 from Z-exchange, see (D.27), is non-zero only for $\varphi_\mu \neq 0, \pi$, whereas the Z-$\tilde{\nu}$ interference term, see (D.28), obtains also absorptive contributions due to the finite Z-width which do not signal CP violation. These, however, will be eliminated in the asymmetry $\mathcal{A}_\ell$ (3.5).

For chargino decay into a tau sneutrino, $\tilde{\chi}_i^+ \to \tau^+ \tilde{\nu}_\tau$, the asymmetry $\mathcal{A}_\tau^{\mathrm{T}} \propto (|V_{i1}|^2 - Y_\tau^2 |U_{i2}|^2)/(|V_{i1}|^2 + Y_\tau^2 |U_{i2}|^2)$ is reduced, which follows from the expressions for D and Σ_D^2, given in (D.39) and (D.40).

3.1.3 Numerical results

We present numerical results for the asymmetries $\mathcal{A}_\ell$ (3.5), for $\ell = e, \mu$, and the cross sections $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+ \tilde{\nu}_\ell)$. We study the dependence of the asymmetries and cross sections on the MSSM parameters $\mu = |\mu|\, e^{i\varphi_\mu}$, M_2 and $\tan\beta$. We choose a center of mass energy of $\sqrt{s} = 800$ GeV and longitudinally polarized beams with beam polarizations $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$, which enhance $\tilde{\nu}_e$ exchange in the production process. This results in larger cross sections and asymmetries.

We study the decays of the lighter chargino $\tilde{\chi}_1^+$. For the calculation of the chargino widths $\Gamma_{\chi_1^+}$ and the branching ratios $\mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+ \tilde{\nu}_\ell)$ we include the following two-body decays,

$$\tilde{\chi}_1^+ \to W^+ \tilde{\chi}_n^0,\ e^+ \tilde{\nu}_e,\ \mu^+ \tilde{\nu}_\mu,\ \tau^+ \tilde{\nu}_\tau,\ \tilde{e}_L^+ \nu_e,\ \tilde{\mu}_L^+ \nu_\mu,\ \tilde{\tau}_{1,2}^+ \nu_\tau, \tag{3.10}$$

and neglect three-body decays. In order to reduce the number of parameters, we assume the relation $|M_1| = 5/3\ M_2 \tan^2\theta_W$. For all scenarios we fix the sneutrino and slepton masses, $m_{\tilde{\nu}_\ell} = 185$ GeV, $\ell = e, \mu, \tau$, $m_{\tilde{\ell}_L} = 200$ GeV, $\ell = e, \mu$. These values are obtained from the renormalization group equations (A.57) and (A.58), for $M_2 = 200$ GeV, $m_0 = 80$ GeV and $\tan\beta = 5$. In the stau sector, see Appendix A.3.3, we fix the trilinear scalar coupling parameter to $A_\tau = 250$ GeV. The stau masses are fixed to $m_{\tilde{\tau}_1} = 129$ GeV and $m_{\tilde{\tau}_2} = 202$ GeV.

In Fig. 3.1a we show the contour lines of the cross section for chargino production and decay $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+ \tilde{\nu}_\ell)$ in the M_2–φ_μ plane for $|\mu| = 400$ GeV and $\tan\beta = 5$. The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_2^-)$ can attain values from 10 fb to 150 fb and $\mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+ \tilde{\nu}_\ell)$, summed over $\ell = e, \mu$, can be as large as 50%. Note that σ is very sensitive to φ_μ, which has been exploited in [17, 18] to constrain $\cos(\varphi_\mu)$.

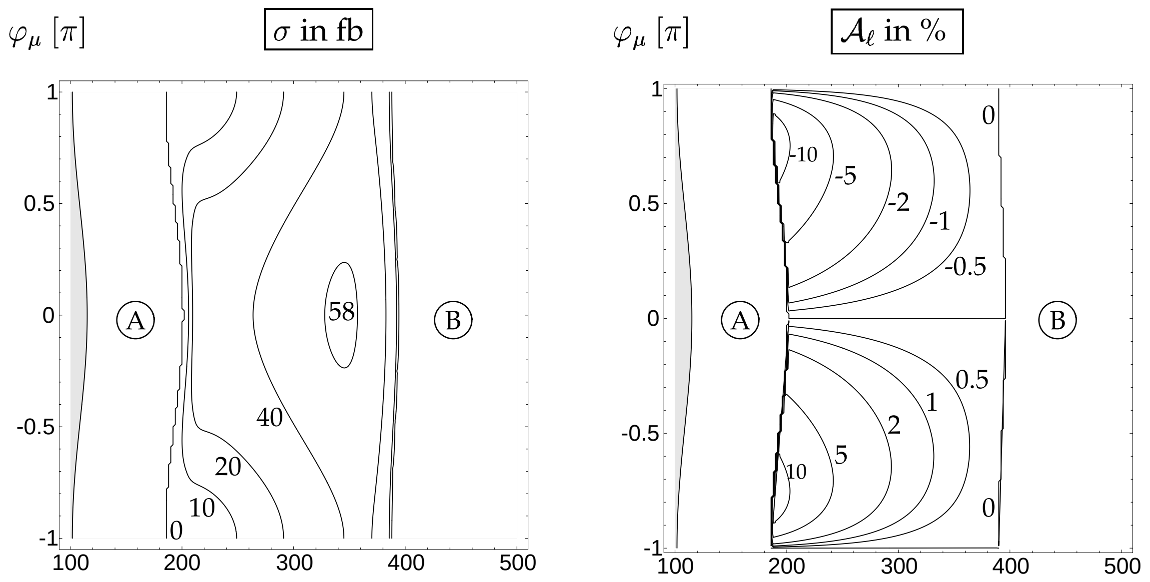


Fig. 3.1a M_2 [GeV] Fig. 3.1b M_2 [GeV]

Figure 3.1: Contour lines of $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+\tilde{\nu}_\ell)$, summed over $\ell = e, \mu$, (3.1a), and the asymmetry $\mathcal{A}_\ell$ for $\ell = e$ or μ (3.1b), in the M_2–φ_μ plane for $|\mu| = 400$ GeV, $\tan\beta = 5$, $m_{\tilde{\nu}_\ell} = 185$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV. The area A is kinematically forbidden by $m_{\tilde{\nu}_\ell} + m_{\chi_1^0} > m_{\chi_1^+}$. The area B is kinematically forbidden by $m_{\chi_1^+} + m_{\chi_2^-} > \sqrt{s}$.

The M_2–φ_μ dependence of the CP asymmetry $\mathcal{A}_\ell$ for $\ell = e$ or μ is shown in Fig. 3.1b. The asymmetry can be as large as 10% and it does, however, not attain maximal values for $\varphi_\mu = \pm 0.5\pi$. The reason is that $\mathcal{A}_\ell$ is proportional to a product of a CP-odd (Σ_P^2) and a CP-even factor (Σ_D^2), see (3.9). The CP-odd (CP-even) factor has as sine-like (cosine-like) dependence on φ_μ. Thus the maximum of $\mathcal{A}_\ell$ is shifted towards $\varphi_\mu = \pm\pi$ in Fig. 3.1b. Phases close to the CP conserving points, $\varphi_\mu = 0, \pm\pi$, are favored by the experimental upper limits on the EDMs, as discussed in Section 1.2.

For $M_2 = 200$ GeV, we show the $\tan\beta$–φ_μ dependence of σ and $\mathcal{A}_\ell$ in Figs. 3.2a,b. The asymmetry can reach values up to 30% and shows a strong $\tan\beta$ dependence and decreases with increasing $\tan\beta$. The feasibility of measuring the asymmetry depends also on the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+\tilde{\nu}_\ell)$, Fig. 3.2a, which attains values up to 20 fb.

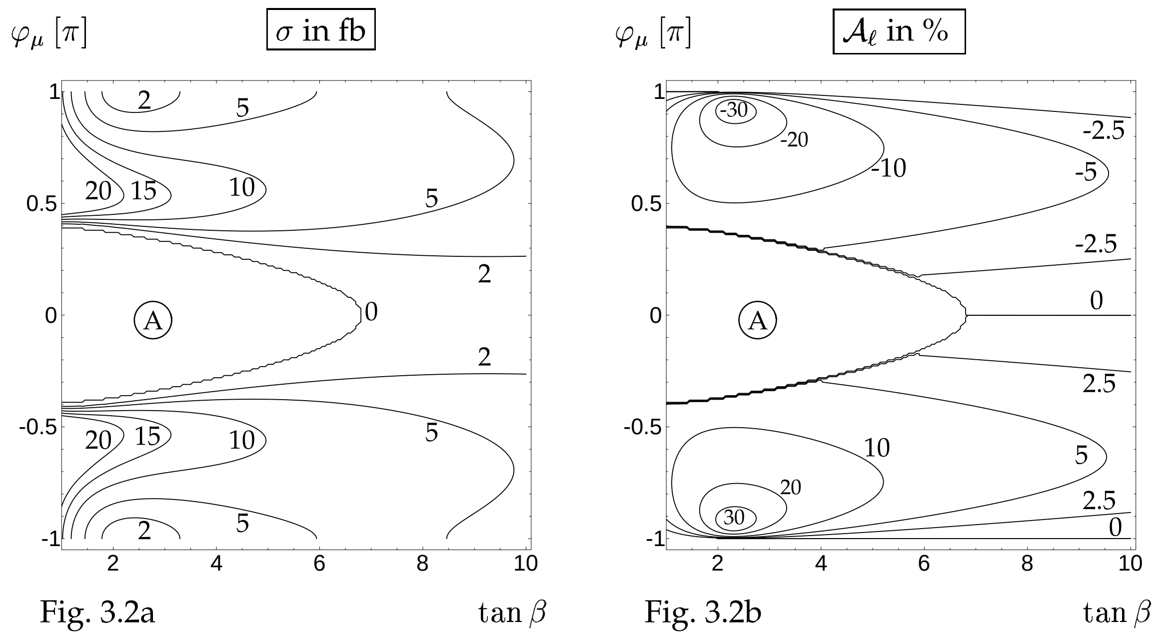


Figure 3.2: Contour lines of $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+\tilde{\nu}_\ell)$, summed over $\ell = e, \mu$, (3.2a), and the asymmetry $\mathcal{A}_\ell$ for $\ell = e$ or μ (3.2b), in the $\tan\beta$–φ_μ plane for $M_2 = 200$ GeV, $|\mu| = 400$ GeV, $m_{\tilde{\nu}_\ell} = 185$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A is kinematically forbidden by $m_{\tilde{\nu}_\ell} + m_{\chi_1^0} > m_{\chi_1^+}$.

For the phase $\varphi_\mu = 0.9\pi$ and $\tan\beta = 5$, we study the beam polarization dependence of $\mathcal{A}_\ell$, which can be strong as shown in Fig. 3.3a. An electron beam polarization $P_{e^-} > 0$ and a positron beam polarization $P_{e^+} < 0$ enhance the channels with $\tilde{\nu}_e$ exchange in the chargino production process. For e.g. $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ the asymmetry can attain -7%, Fig. 3.3a, with $\sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \approx 10$ fb and $\mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+\tilde{\nu}_\ell) \approx 50\%$, summed over $\ell = e, \mu$. The cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+\tilde{\nu}_\ell)$ ranges between 2.3 fb for $(P_{e^-}, P_{e^+}) = (0, 0)$ and 6.8 fb for $(P_{e^-}, P_{e^+}) = (-1, 1)$. The statistical significance of $\mathcal{A}_\ell$, given by $S_\ell = |\mathcal{A}_\ell|\sqrt{2\mathcal{L}\cdot\sigma}$, is shown in Fig. 3.3b for $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. We have $S_\ell \approx 5$ for $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$, and thus $\mathcal{A}_\ell$ could be accessible at a linear collider, even for $\varphi_\mu = 0.9\pi$, by using polarized beams.

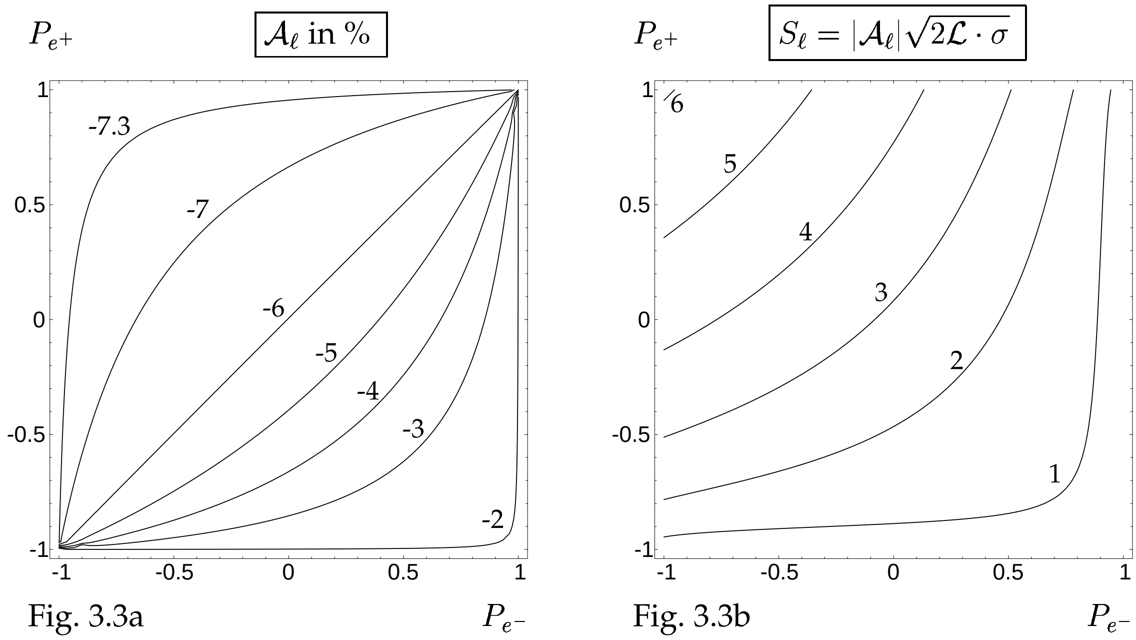


Figure 3.3: Contour lines of the asymmetry $\mathcal{A}_\ell$ for $\ell = e$ or μ (3.3a), and the significance S_ℓ (3.3b), for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$; $\tilde{\chi}_1^+ \to \ell^+\tilde{\nu}_\ell$ in the P_{e^-}–P_{e^+} plane for $\varphi_\mu = 0.9\pi$, $|\mu| = 400$ GeV, $M_2 = 200$ GeV, $\tan\beta = 5$, $m_{\tilde{\nu}_\ell} = 185$ GeV, $\sqrt{s} = 800$ GeV and $\mathcal{L} = 500~\mathrm{fb}^{-1}$.

3.1.4 Summary of Section 3.1

We have studied CP violation in chargino production with longitudinally polarized beams, $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$, and subsequent two-body decay of one chargino into the sneutrino $\tilde{\chi}_i^+ \to \ell^+\tilde{\nu}_\ell$. We have defined the T-odd asymmetries $\mathcal{A}_\ell^{\mathrm{T}}$ of the triple product $(\mathbf{p}_{e^-} \times \mathbf{p}_{\tilde{\chi}_i^+}) \cdot \mathbf{p}_\ell$. The CP-odd asymmetries $\mathcal{A}_\ell = \frac{1}{2}(\mathcal{A}_\ell^{\mathrm{T}} - \bar{\mathcal{A}}_\ell^{\mathrm{T}})$, where $\bar{\mathcal{A}}_\ell^{\mathrm{T}}$ denote the CP conjugated of $\mathcal{A}_\ell^{\mathrm{T}}$, are sensitive to the phase φ_μ of the Higgsino mass parameter μ. At tree level, the asymmetries have large CP sensitive contributions from spin-correlation effects in the production of an unequal pair of charginos. In a numerical discussion for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ production, we have found that $\mathcal{A}_\ell$ for $\ell = e$ or μ can attain values up to 30%. By analyzing the statistical errors, we have shown that, even for e.g. $\varphi_\mu \approx 0.9\pi$, the asymmetries could be accessible in future e^+e^- collider experiments in the 800 GeV range with high luminosity and longitudinally polarized beams.

3.2 CP violation in chargino production and decay into a W boson

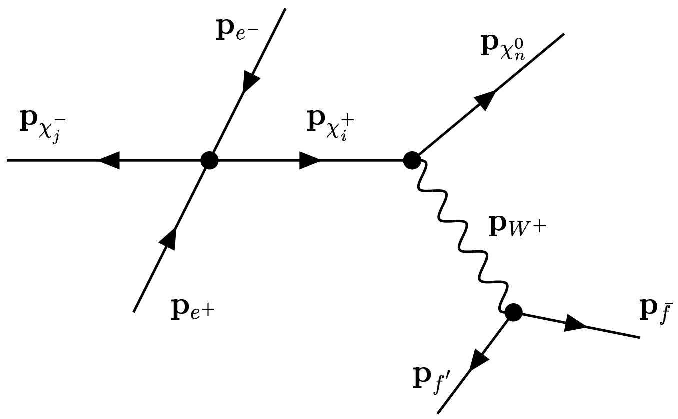


Figure 3.4: Schematic picture of the chargino production and decay process.

CP violation in the chargino sector can be studied also by the two-body decay of the chargino into a W boson. In contrast to the decay into a sneutrino, the spin correlations of chargino and W lead to CP observables, which are also sensitive to the phase of M_1. Since these observables have in addition contributions from the decay, they do not necessarily vanish for $e^+e^- \to \tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\mp}$ production. In this mode CP violation could be established, even if the production of the heavier pair $\tilde{\chi}_1^{\pm}\tilde{\chi}_2^{\mp}$ is not yet accessible.

We study chargino production

$$e^+ + e^- \to \tilde{\chi}_i^+ + \tilde{\chi}_j^-; \quad i, j = 1, 2, \tag{3.11}$$

with longitudinally polarized beams and the subsequent two-body decay

$$\tilde{\chi}_i^+ \to W^+ + \tilde{\chi}_n^0. \tag{3.12}$$

We define the triple product

$$\mathcal{T}_I = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\chi_i^+} \times \mathbf{p}_W) \tag{3.13}$$

and the T-odd asymmetry

$$\mathcal{A}_I^{\mathrm{T}} = \frac{\sigma(\mathcal{T}_I > 0) - \sigma(\mathcal{T}_I < 0)}{\sigma(\mathcal{T}_I > 0) + \sigma(\mathcal{T}_I < 0)}, \tag{3.14}$$

with σ the cross section of chargino production (3.11) and decay (3.12). The asymmetry $\mathcal{A}_I^{\mathrm{T}}$ is sensitive to the CP violating phase φ_μ.

In order to probe also the phase φ_{M_1}, which enters in the chargino decay process (3.12), we consider the subsequent hadronic decay of the W boson

$$W^+ \to c + \bar{s}. \tag{3.15}$$

The correlations between the $\tilde{\chi}_i^+$ polarization and the W boson polarization lead to CP sensitive elements of the W boson density matrix. The triple product

$$\mathcal{T}_{II} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_c \times \mathbf{p}_{\bar{s}}), \tag{3.16}$$

which includes the momenta of the W decay products, and thus probes the W polarization, defines a second T-odd asymmetry

$$\mathcal{A}_{II}^{\mathrm{T}} = \frac{\sigma(\mathcal{T}_{II} > 0) - \sigma(\mathcal{T}_{II} < 0)}{\sigma(\mathcal{T}_{II} > 0) + \sigma(\mathcal{T}_{II} < 0)}. \tag{3.17}$$

Here, σ is the cross section of production (3.11) and decay of the chargino (3.12) followed by that of the W boson (3.15). Owing to the spin correlations, $\mathcal{A}_{II}^{\mathrm{T}}$ has CP sensitive contributions from φ_μ due to the chargino production process (3.11) and contributions due to φ_μ and φ_{M_1} from the chargino decay process (3.12).

The T-odd asymmetries $\mathcal{A}_I^{\mathrm{T}}$ and $\mathcal{A}_{II}^{\mathrm{T}}$ have also absorptive contributions from s-channel resonances or final-state interactions, which are eliminated in the CP-odd asymmetries

$$\mathcal{A}_I = \frac{1}{2}(\mathcal{A}_I^{\mathrm{T}} - \bar{\mathcal{A}}_I^{\mathrm{T}}), \quad \mathcal{A}_{II} = \frac{1}{2}(\mathcal{A}_{II}^{\mathrm{T}} - \bar{\mathcal{A}}_{II}^{\mathrm{T}}), \tag{3.18}$$

where $\bar{\mathcal{A}}_{I,II}^{\mathrm{T}}$ are the CP conjugated asymmetries for the processes $e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+; \tilde{\chi}_i^- \to W^- \tilde{\chi}_n^0$ and $e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+; \tilde{\chi}_i^- \to W^- \tilde{\chi}_n^0; W^- \to \bar{c}s$, respectively.

3.2.1 Spin density matrix of the W boson

For the calculation of the amplitudes squared for the combined process of chargino production (3.11) and the subsequent two-body decays (3.12) and (3.15) of $\tilde{\chi}_i^+$ we use the same spin-density matrix formalism as in [27, 51]. The (unnormalized) spin-density matrix of the W boson

$$\rho_P(W^+)^{\lambda_k\lambda'_k} = |\Delta(\tilde{\chi}_i^+)|^2 \sum_{\lambda_i,\lambda'_i} \rho_P(\tilde{\chi}_i^+)^{\lambda_i\lambda'_i}\, \rho_{D_1}(\tilde{\chi}_i^+)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i}, \tag{3.19}$$

is composed of the chargino propagator $\Delta(\tilde{\chi}_i^+)$, the spin-density production matrix $\rho_P(\tilde{\chi}_i^+)$, defined in (D.8), and the decay matrix $\rho_{D_1}(\tilde{\chi}_i^+)$, defined in (D.42). The amplitude squared for the complete process $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$; $\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0$; $W^+ \to f'\bar{f}$ can now be written

$$|T|^2 = |\Delta(W^+)|^2 \sum_{\lambda_k,\lambda'_k} \rho_P(W^+)^{\lambda_k\lambda'_k}\, \rho_{D_2}(W^+)_{\lambda'_k\lambda_k}, \tag{3.20}$$

with the decay matrix for W decay $\rho_{D_2}(W^+)$, defined in (D.45). Inserting the density matrices $\rho_P(\tilde{\chi}_i^+)$ (D.8) and $\rho_{D_1}(\tilde{\chi}_i^+)$ (D.62) into (3.19) gives

$$\begin{aligned}\rho_P(W^+)^{\lambda_k\lambda'_k} = {} & 4\,|\Delta(\tilde{\chi}_i^+)|^2\, [(PD_1 + \Sigma_P^a\Sigma_{D_1}^a)\, \delta^{\lambda_k\lambda'_k} + (P\,{}^cD_1 + \Sigma_P^a\,{}^c\Sigma_{D_1}^a)\,(J^c)^{\lambda_k\lambda'_k} \\ & + (P\,{}^{cd}D_1 + \Sigma_P^a\,{}^{cd}\Sigma_{D_1}^a)\,(J^{cd})^{\lambda_k\lambda'_k}],\end{aligned} \tag{3.21}$$

summed over a, c, d. We have thus decomposed the W production matrix $\rho_P(W^+)$ into contributions of scalar (first term), vector (second term), and tensor parts (third term). Inserting then $\rho_P(W^+)$ (3.21) and $\rho_{D_2}(W^+)$ (D.63) into (3.20) gives

$$\begin{aligned}|T|^2 = {} & 4\,|\Delta(\tilde{\chi}_i^+)|^2\,|\Delta(W^+)|^2\{3(PD_1 + \Sigma_P^a\Sigma_{D_1}^a)D_2 + 2(P\,{}^cD_1 + \Sigma_P^a\,{}^c\Sigma_{D_1}^a)\,{}^cD_2 \\ & + 4[(P\,{}^{cd}D_1 + \Sigma_P^a\,{}^{cd}\Sigma_{D_1}^a)\,{}^{cd}D_2 - \tfrac{1}{3}(P\,{}^{cc}D_1 + \Sigma_P^a\,{}^{cc}\Sigma_{D_1}^a)\,{}^{dd}D_2]\}.\end{aligned} \tag{3.22}$$

3.2.2 W boson polarization

The mean polarization of the W bosons in the laboratory system is given by the 3×3 density matrix $< \rho(W^+) >$, obtained by integrating (3.21) over the Lorentz invariant phase-space element $d\mathrm{Lips}(s; p_{\chi_j^-}, p_{\chi_n^0}, p_W)$, see (B.39), and normalizing by the trace

$$< \rho(W^+)^{\lambda_k\lambda'_k} >= \frac{\int \rho_P(W^+)^{\lambda_k\lambda'_k}\, d\mathrm{Lips}}{\int \mathrm{Tr}\{\rho_P(W^+)^{\lambda_k\lambda'_k}\}\, d\mathrm{Lips}} = \tfrac{1}{3}\delta^{\lambda_k\lambda'_k} + V_c\,(J^c)^{\lambda_k\lambda'_k} + T_{cd}\,(J^{cd})^{\lambda_k\lambda'_k}. \quad (3.23)$$

The components V_c of the vector polarization and T_{cd} of the tensor polarization are

$$V_c = \frac{\int (P\,{}^cD_1 + \Sigma_P^a\,{}^c\Sigma_{D_1}^a)\, d\mathrm{Lips}}{3\int\, PD_1\, d\mathrm{Lips}}, \quad (3.24)$$

$$T_{cd} = T_{dc} = \frac{\int (P\,{}^{cd}D_1 + \Sigma_P^a\,{}^{cd}\Sigma_{D_1}^a)\, d\mathrm{Lips}}{3\int\, PD_1\, d\mathrm{Lips}}, \quad (3.25)$$

where we have already used the narrow width approximation for the chargino propagator. The density matrix elements in the helicity basis (B.38) are given by

$$< \rho(W^+)^{--} > = \tfrac{1}{2} - V_3 + T_{33}, \quad (3.26)$$

$$< \rho(W^+)^{00} > = -2\,T_{33}, \quad (3.27)$$

$$< \rho(W^+)^{-0} > = \tfrac{1}{\sqrt{2}}(V_1 + iV_2) - \sqrt{2}\,(T_{13} + iT_{23}), \quad (3.28)$$

$$< \rho(W^+)^{-+} > = T_{11} - T_{23} + 2iT_{12}, \quad (3.29)$$

$$< \rho(W^+)^{0+} > = \tfrac{1}{\sqrt{2}}(V_1 + iV_2) + \sqrt{2}\,(T_{13} + iT_{23}), \quad (3.30)$$

where we have used $T_{11} + T_{22} + T_{33} = -\frac{1}{2}$.

3.2.3 T-odd asymmetries

From (3.21) we obtain for asymmetry $\mathcal{A}_I^{\mathrm{T}}$ (3.14):

$$\mathcal{A}_I^{\mathrm{T}} = \frac{\int \mathrm{Sign}[\mathcal{T}_I]\mathrm{Tr}\{\rho_P(W^+)^{\lambda_k\lambda'_k}\}\, d\mathrm{Lips}}{\int \mathrm{Tr}\{\rho_P(W^+)^{\lambda_k\lambda'_k}\}\, d\mathrm{Lips}} = \frac{\int \mathrm{Sign}[\mathcal{T}_I]\Sigma_P^2\,\Sigma_{D_1}^2 d\mathrm{Lips}}{\int PD_1\, d\mathrm{Lips}}, \quad (3.31)$$

with $d\mathrm{Lips}(s; p_{\chi_j^-}, p_{\chi_n^0}, p_W)$ given in (B.39), where we have used the narrow width approximation. In the numerator of (3.31), only the spin correlations $\Sigma_P^2\,\Sigma_{D_1}^2$ perpendicular to the production plane remain, since only this term contains the triple product $\mathcal{T}_I = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\chi_i^+} \times \mathbf{p}_W)$. In the denominator only the term PD_1 remains, and all spin correlations vanish due to the integration over the complete phase space. Note that $\mathcal{A}_I^{\rm T} \propto \Sigma^2_{D_1} \propto (|O^R_{ni}|^2 - |O^L_{ni}|^2)$, see (D.59), and thus $\mathcal{A}_I^{\rm T}$ may be reduced for $|O^R_{ni}| \approx |O^L_{ni}|$. Moreover, $\mathcal{A}_I^{\rm T}$ will be small for $m^2_{\chi_i^+} - m^2_{\chi_n^0} \approx 2m^2_W$, see (D.59).

For the asymmetry $\mathcal{A}_{II}^{\rm T}$ (3.17), we obtain from (3.22):

$$\mathcal{A}_{II}^{\rm T} = \frac{\int {\rm Sign}[\mathcal{T}_{II}]|T|^2 d{\rm Lips}}{\int |T|^2 d{\rm Lips}} = \frac{\int {\rm Sign}[\mathcal{T}_{II}] 2\Sigma_P^a\, {}^c\Sigma^a_{D_1}\, {}^cD_2\, d{\rm Lips}}{\int 3PD_1D_2\, d{\rm Lips}}, \tag{3.32}$$

with $d{\rm Lips}(s; p_{\chi_j^-}, p_{\chi_n^0}, p_{f'}, p_{\bar f})$, defined in (B.40), where we have used the narrow width approximations for the propagators. In the numerator only the vector part of $|T|^2$ remains because only the vector part contains the triple product $\mathcal{T}_{II} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_c \times \mathbf{p}_{\bar s})$. In the denominator the vector and tensor parts of $|T|^2$ vanish due to phase-space integration. Owing to the correlations $\Sigma_P^a\, {}^c\Sigma^a_{D_1}$ between the $\tilde\chi_i^+$ and the W boson polarization, there are contributions to the asymmetry $\mathcal{A}_{II}^{\rm T}$ from the chargino production process (3.11), and/or from the chargino decay process (3.12). The contribution from the production is given by the term with $a = 2$ in (3.32) and it is proportional to the transverse polarization of the chargino perpendicular to the production plane Σ_P^2, see Appendix D.1.2. For the production of a pair of equal charginos, $e^+e^- \to \tilde\chi_i^+\tilde\chi_i^-$, we have $\Sigma_P^2 = 0$. The contributions from the decay, given by terms with $a = 1, 3$ in (3.32), are proportional to

$${}^c\Sigma^a_{D_1}\, {}^cD_2 \supset -2g^4 m_{\chi_n^0} Im(O^{R*}_{ni} O^L_{ni})(t^c_W \cdot p_{\bar f})\epsilon_{\mu\nu\rho\sigma}\, s^{a,\mu}_{\chi_i^+}\, p^\nu_{\chi_i^+}\, p^\rho_W\, t^{c,\sigma}_W, \tag{3.33}$$

see last term of (D.60), which contains the ϵ-tensor. Thus $\mathcal{A}_{II}^{\rm T}$ can be enhanced (reduced) if the contributions from production and decay have the same (opposite) sign. Note that the contributions from the decay vanish for a two-body decay of the chargino into a scalar particle instead of a W boson. In order to measure $\mathcal{A}_I$ the momentum of $\tilde\chi_i^+$, i.e. the production plane, has to be determined. This could be accomplished by measuring the decay of the other chargino $\tilde\chi_j^-$. For the measurement of $\mathcal{A}_{II}$, the flavors of the quarks c and $\bar s$ have to be distinguished, which will be possible by flavor tagging of the c-quark [30, 32]. In principle, for the decay $W \to u\,\bar d$ also an asymmetry similar to $\mathcal{A}_{II}$ can be considered, if it is possible to distinguish between the u and $\bar d$ jet, for instance, by measuring the average charge.

3.2.4 Numerical results

We study the dependence of $\mathcal{A}_I$, $\mathcal{A}_{II}$, and the density matrix $< \rho(W^+) >$ on the MSSM parameters $\mu = |\mu|\, e^{i\,\varphi_\mu}$, $M_1 = |M_1|\, e^{i\,\varphi_{M_1}}$, $\tan\beta$ and the universal scalar mass parameter m_0. The feasibility of measuring the asymmetries depends also on the cross sections $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-) \times \mathrm{BR}(\tilde{\chi}_i^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$, which we will discuss in our scenarios. We choose a center of mass energy of $\sqrt{s} = 800$ GeV and longitudinally polarized beams with $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$. This choice enhances sneutrino exchange in the chargino production process, which results in larger cross sections and asymmetries. For the calculation of the branching ratios $\mathrm{BR}(\tilde{\chi}_i^+ \to W^+\tilde{\chi}_1^0)$ and widths $\Gamma_{\chi_1^+}$, we include the two-body decays

$$\tilde{\chi}_1^+ \to W^+\tilde{\chi}_n^0,\ \tilde{e}_L^+\nu_e,\ \tilde{\mu}_L^+\nu_\mu,\ \tilde{\tau}_{1,2}^+\nu_\tau,\ e^+\tilde{\nu}_e,\ \mu^+\tilde{\nu}_\mu,\ \tau^+\tilde{\nu}_\tau, \tag{3.34}$$

and neglect three-body decays. For the W boson decay we take the experimental value $\mathrm{BR}(W^+ \to c\bar{s}) = 0.31$ [45]. In order to reduce the number of parameters, we assume the relation $|M_1| = 5/3\, M_2 \tan^2\theta_W$ and use the renormalization group equations for the slepton and sneutrino masses, see Appendix A.3.4. In the stau sector, see Appendix A.3.3, we fix the trilinear scalar coupling parameter $A_\tau = 250$ GeV.

■ *Production of* $\tilde{\chi}_1^+\,\tilde{\chi}_1^-$

For the production $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_i^-$ of a pair of charginos the polarization perpendicular to the production plane vanishes, and thus $\mathcal{A}_I = 0$. However, $\mathcal{A}_{II}$ need not to be zero and is sensitive to φ_μ and φ_{M_1}, because this asymmetry has contributions from the chargino decay process. For $(\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0)$ we show in Fig. 3.5a the $|\mu|$–M_2 dependence of $\mathcal{A}_{II}$, which can reach values of $5\% - 7\%$ for $M_2 \gtrsim 400$ GeV. We also studied the φ_μ dependence of $\mathcal{A}_{II}$ in the $|\mu|$–M_2 plane. For $\varphi_{M_1} = 0$, $\varphi_\mu = 0.1\pi(0.5\pi)$ and the other parameters as given in the caption of Fig. 3.5, we find $|\mathcal{A}_{II}| < 2\%(7\%)$.

In Fig. 3.5b we show the contour lines of the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$ in the $|\mu|$–M_2 plane for $(\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0)$. The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ reaches more than 400 fb. For our choice of $m_0 = 300$ GeV, $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$ is the only allowed two-body decay channel.

In Fig. 3.6a we plot the contour lines of $\mathcal{A}_{II}$ for $|\mu| = 350$ GeV and $M_2 = 400$ GeV in the φ_μ–φ_{M_1} plane. Fig. 3.6a shows that $\mathcal{A}_{II}$ is essentially depending on the sum $\varphi_\mu + \varphi_{M_1}$. However, maximal phases of $\varphi_{M_1} = \pm 0.5\pi$ and $\varphi_\mu = \pm 0.5\pi$ do not lead to the highest values of $|\mathcal{A}_{II}| \gtrsim 6\%$, which are reached for $(\varphi_{M_1}, \varphi_\mu) \approx (\pm 0.8\pi, \pm 0.6\pi)$. The reason for this is that the spin-correlation terms $\Sigma_P^a\,{}^c\Sigma_{D_1}^a\,{}^cD_2$ in the numerator of $\mathcal{A}_{II}$ (3.32) are products of CP-odd and CP-even factors. The CP-odd (CP-even)

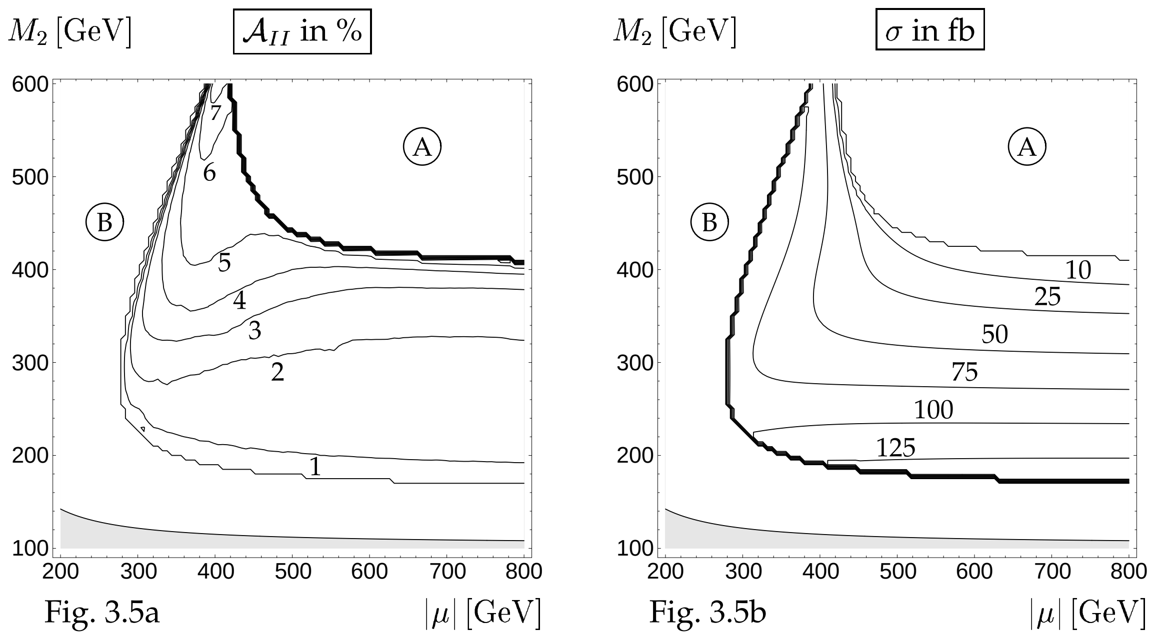


Figure 3.5: Contour lines of the asymmetry $\mathcal{A}_{II}$ (3.5a) and $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$ (3.5b), in the $|\mu|$–M_2 plane for $(\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0)$, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A (B) is kinematically forbidden by $m_{\chi_1^+} + m_{\chi_1^-} > \sqrt{s}$ ($m_W + m_{\chi_1^0} > m_{\chi_1^+}$). The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

factors have a sine-like (cosine-like) phase dependence. Therefore, the maximum of the CP asymmetry $\mathcal{A}_{II}$ may be shifted to smaller or larger values of the phases. In the φ_μ–φ_{M_1} region shown in Fig. 3.6a the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$ with $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 W^+) = 1$ does not depend on φ_{M_1} and ranges between 74 fb for $\varphi_\mu = 0$ and 66 fb for $\varphi_\mu = \pi$.

In Fig. 3.6b we show the contour lines of the significance $S_{II} = |\mathcal{A}_{II}|\sqrt{2\mathcal{L}\cdot\sigma}$, defined in (1.18). For $\mathcal{L} = 500$ fb^{-1} and for e.g. $(\varphi_{M_1}, \varphi_\mu) \approx (\pi, 0.1\pi)$ we have $S_{II} \approx 8$ and thus $\mathcal{A}_{II}$ should be measured even for small φ_μ.

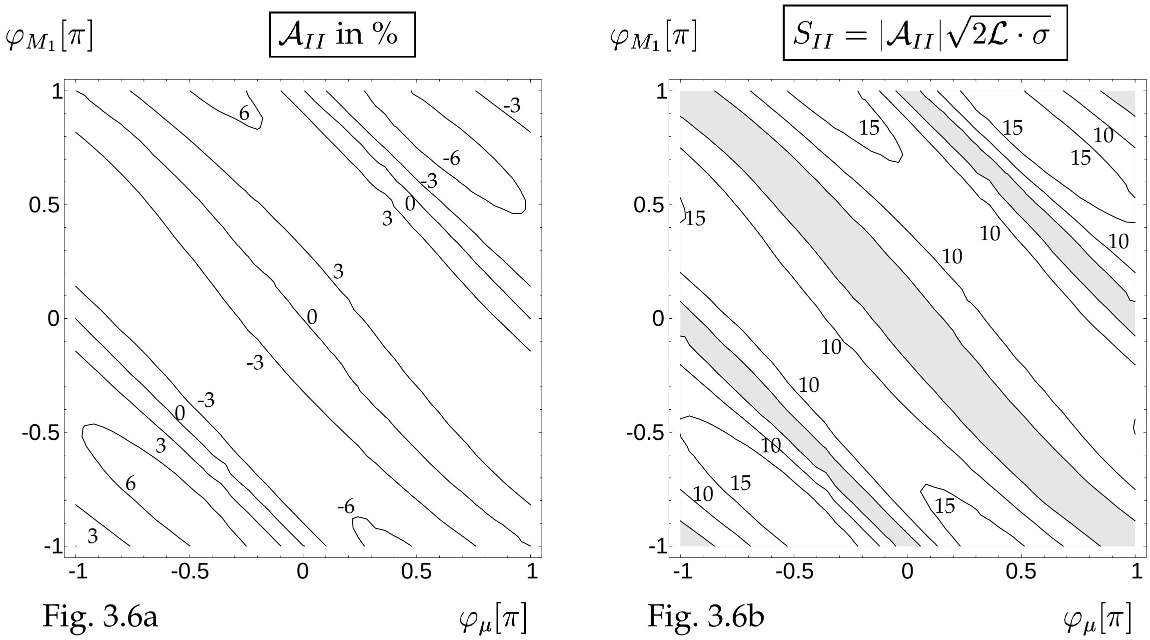


Figure 3.6: Contour lines of and the asymmetry $\mathcal{A}_{II}$ (3.6a) and the significance S_{II} (3.6b) for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$; $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$; $W^+ \to c\bar{s}$, in the φ_μ–φ_{M_1} plane for $|\mu| = 350$ GeV, $M_2 = 400$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ and $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. In the gray shaded area of Fig. 3.6b we have $S_{II} < 5$.

In Figs. 3.7a,b we show the $\tan\beta$–m_0 dependence of $\mathcal{A}_{II}$ and σ for $(\varphi_{M_1}, \varphi_\mu) = (0.7\pi, 0)$. The asymmetry is rather insensitive to m_0 and shows strong dependence on $\tan\beta$ and decreases with increasing $\tan\beta \gtrsim 2$. The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ increases with increasing m_0 and decreasing $\tan\beta$. For $m_0 \lesssim 200$ GeV, the branching ratio $\mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) < 1$, since the decay channels of $\tilde{\chi}_1^+$ into sleptons and/or sneutrinos open, see (3.34).

In Fig. 3.8a we show the φ_μ dependence of the vector (V_i) and tensor (T_{ij}) components of the density matrix $< \rho(W^+) >$ for $\varphi_{M_1} = \pi$. In Fig. 3.8b we plot their φ_{M_1} dependence for $\varphi_\mu = 0$. In both figures, the element V_2 is CP-odd, while T_{13}, T_{11}, T_{22} and V_1, V_3 show a CP-even behavior. As discussed in Appendix D.3, the components T_{11} and T_{22} are almost equal and have the same order of magnitude as V_1 and V_3, whereas $T_{12}, |T_{23}| < 10^{-5}$ are small. For CP conserving phases $(\varphi_{M_1}, \varphi_\mu) = (0, 0)$ the density matrix reads

$$< \rho(W^+) >= \begin{pmatrix} <\rho^{--}> & <\rho^{-0}> & <\rho^{-+}> \\ <\rho^{0-}> & <\rho^{00}> & <\rho^{0+}> \\ <\rho^{+-}> & <\rho^{+0}> & <\rho^{++}> \end{pmatrix} = \begin{pmatrix} 0.200 & -0.010 & -0.001 \\ -0.010 & 0.487 & 0.137 \\ -0.001 & 0.137 & 0.313 \end{pmatrix}, \quad (3.35)$$

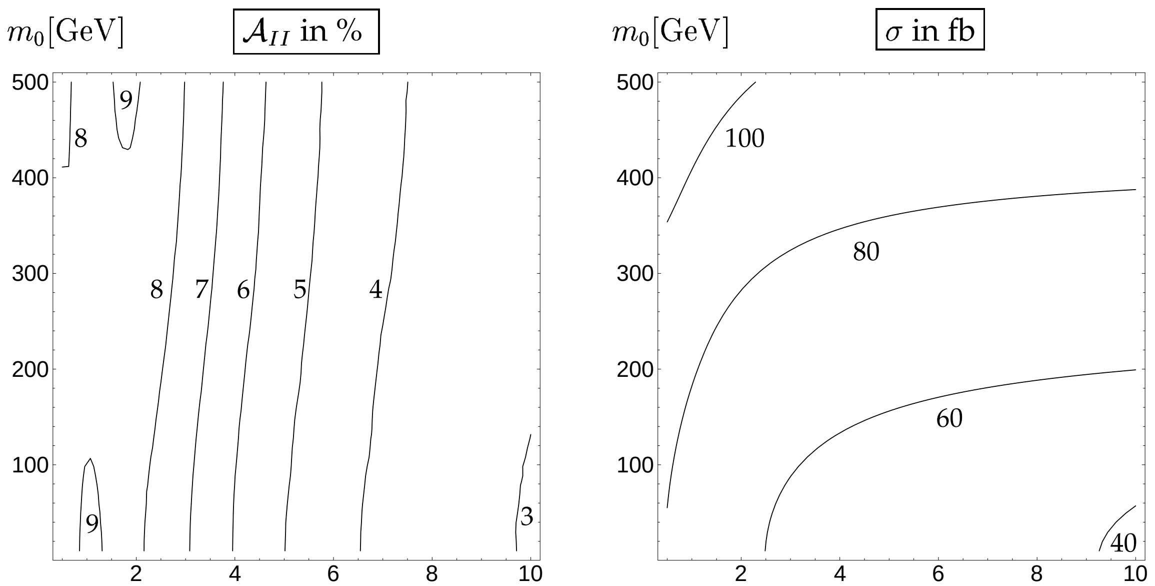


Fig. 3.7a $\tan\beta$ Fig. 3.7b $\tan\beta$

Figure 3.7: Contour lines of the asymmetry $\mathcal{A}_{II}$ (3.7a) and $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$ (3.7b), in the $\tan\beta$–m_0 plane for $(\varphi_{M_1}, \varphi_\mu) = (0.7\pi, 0)$, $M_2 = 400$ GeV, $|\mu| = 350$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

for $M_2 = 400$ GeV, $|\mu| = 350$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ For CP violating phases, e.g. $(\varphi_{M_1}, \varphi_\mu) = (0.7\pi, 0)$ and the other parameters as above, the density matrix has imaginary parts due to a non-vanishing V_2:

$$< \rho(W^+) >= \begin{pmatrix} 0.219 & -0.010 + 0.025i & 0.002 \\ -0.010 - 0.025i & 0.405 & 0.171 + 0.025i \, . \\ 0.002 & 0.171 - 0.025i & 0.376 \end{pmatrix} \tag{3.36}$$

Imaginary parts of the density matrix are thus an indication of CP violation.

- ***Production of*** $\tilde{\chi}_1^+ \tilde{\chi}_2^-$

For the production of an unequal pair of charginos, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$, their polarization perpendicular to the production plane is sensitive to the phase φ_μ, which leads to a non-vanishing asymmetry $\mathcal{A}_I$ (3.31). We will study the decay of the

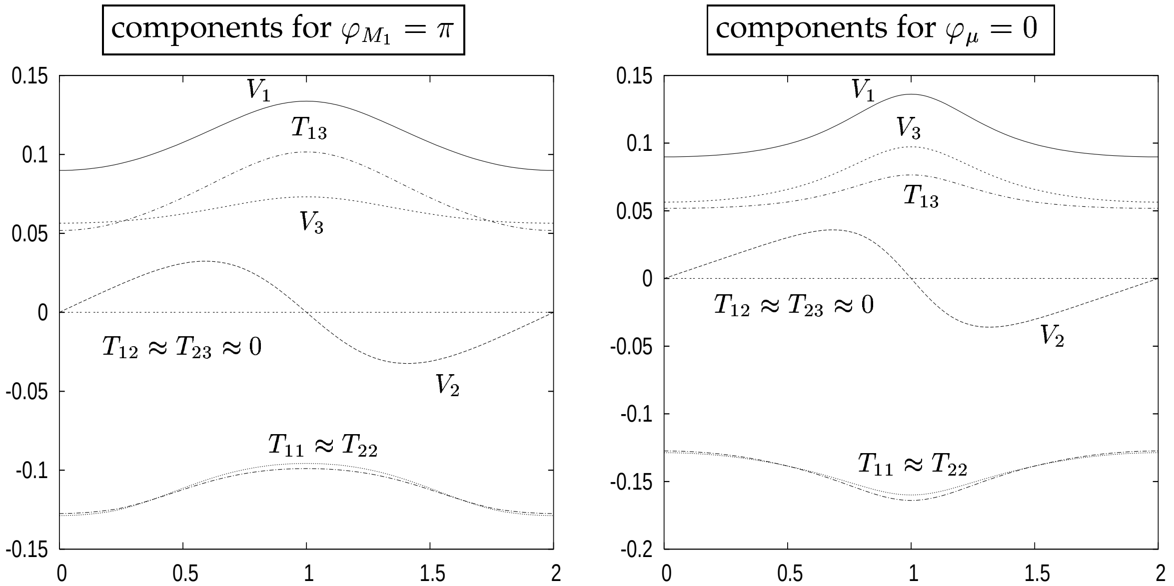


Fig. 3.8a Fig. 3.8b

Figure 3.8: Dependence of vector (V_i) and tensor (T_{ij}) components of the W^+ density matrix on φ_μ (3.8a) and on φ_{M_1} (3.8b), for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$; $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$, for $|\mu| = 350$ GeV, $M_2 = 400$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

lighter chargino $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$. For $|M_2| = 250$ GeV and $\varphi_{M_1} = 0$, we show in Fig. 3.9a the $|\mu|$–φ_μ dependence of $\mathcal{A}_I$, which attains values up to 4%. Note that $\mathcal{A}_I$ is not maximal for $\varphi_\mu = 0.5\pi$, but is rather sensitive for phases in the regions $\varphi_\mu \in [0.7\pi, \pi]$ and $\varphi_\mu \in [-0.7\pi, -\pi]$. As discussed in Section 1.2, values of φ_μ close to the CP conserving points $\varphi_\mu = 0, \pm\pi$ are suggested by EDM analyses. For $\varphi_\mu = 0.9\pi$ and $|\mu| = 350$ GeV the statistical significance is $S_I = |\mathcal{A}_I|\sqrt{2\mathcal{L}\cdot\sigma} \approx 1.5$ with $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. Thus $\mathcal{A}_I$ could be measured at a confidence level larger than 68% $(S_I = 1)$.

In Fig. 3.9b we show contour lines of the corresponding cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0)$ in the $|\mu|$–φ_μ plane for the parameters as above. The cross section shows a CP even behavior, which has been used in [17, 18, 50] to constrain $\cos\varphi_\mu$. In our scenario we have considered the decay of the lighter chargino $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$ since for our choice $m_0 = 300$ GeV we have $\mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) = 1$. For the decay of $\tilde{\chi}_2^+$, one would have to take into account also the decays into the Z boson and the lightest neutral Higgs, which would reduce $\mathrm{BR}(\tilde{\chi}_2^+ \to W^+\tilde{\chi}_1^0) \approx 0.2$.

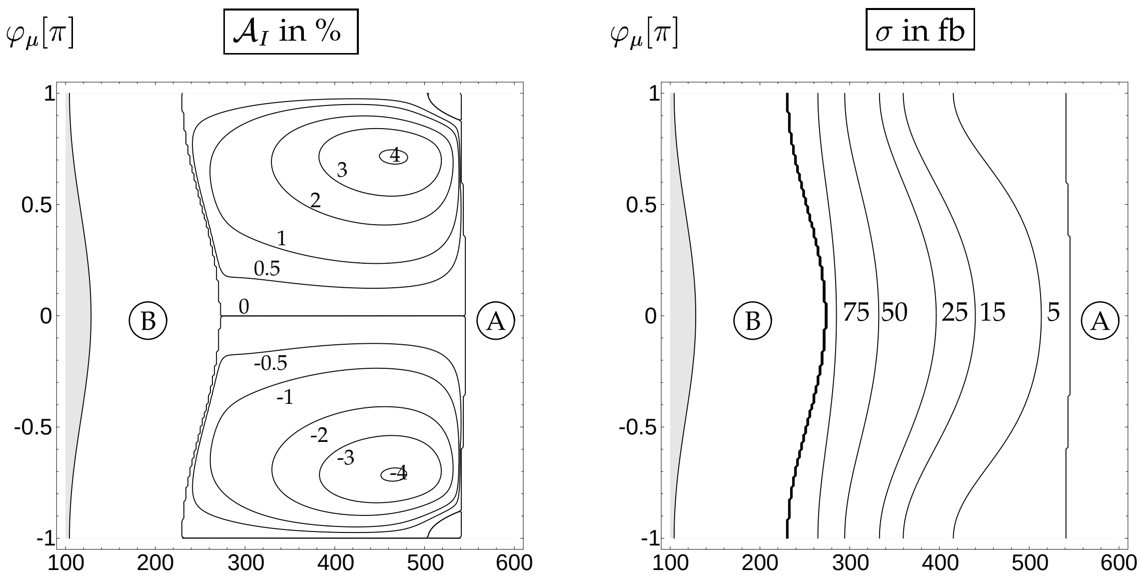


Fig. 3.9a $|\mu|$ [GeV] Fig. 3.9b $|\mu|$ [GeV]

Figure 3.9: Contour lines of the asymmetry $\mathcal{A}_I$ (3.9a) and $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0)$ (3.9b), in the $|\mu|$–φ_μ plane for $\varphi_{M_1} = 0$, $M_2 = 250$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A (B) is kinematically forbidden by $m_{\chi_2^+} + m_{\chi_1^-} > \sqrt{s}$ ($m_W + m_{\chi_1^0} > m_{\chi_1^+}$). The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

The asymmetry $\mathcal{A}_{II}$ is also sensitive to the phase φ_{M_1}. We show the φ_μ–φ_{M_1} dependence of $\mathcal{A}_{II}$, choosing the parameters as above, in Fig. 3.10a. In Fig. 3.10b we show the contour lines of the significance $S_{II} = |\mathcal{A}_{II}|\sqrt{2\mathcal{L}\cdot\sigma}$ for $\mathcal{L} = 500$ fb^{-1}. For $(\varphi_{M_1}, \varphi_\mu) \approx (\pi, 0.1\pi)$ we have $S_{II} \approx 2.4$ and thus $\mathcal{A}_{II}$ could be accessible even for small phases by using polarized beams.

3.2.5 Summary of Section 3.2

We have analyzed CP sensitive observables in chargino production, $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$, with subsequent two-body decay, $\tilde{\chi}_i^+ \to W^+\chi_n^0$. We have defined the CP asymmetry $\mathcal{A}_I$ of the triple product $\mathbf{p}_{e^-}\cdot(\mathbf{p}_{\tilde{\chi}_i^+}\times\mathbf{p}_W)$. In the MSSM with complex parameters μ and M_1, we have shown that $\mathcal{A}_I$ can reach 4% and that even for $\varphi_\mu \approx 0.9\pi$ the asymmetry could be accessible in the process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$. Further we have analyzed the CP sensitive density-matrix elements of the W boson. The phase φ_{M_1}

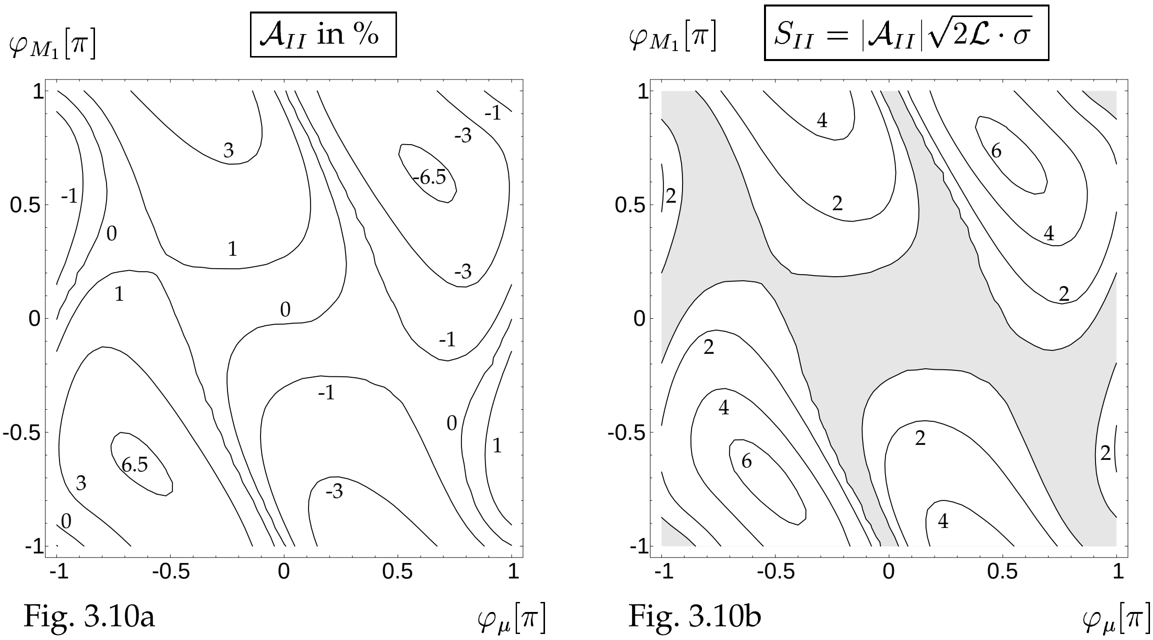


Fig. 3.10a

Fig. 3.10b

Figure 3.10: Contour lines of and the asymmetry $\mathcal{A}_{II}$ (3.10a) and the significance S_{II} (3.10b) for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$; $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$; $W^+ \to c\bar{s}$, in the φ_μ–φ_{M_1} plane for $|\mu| = 350$ GeV, $M_2 = 250$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ and $\mathcal{L} = 500\,\mathrm{fb}^{-1}$. In the gray shaded area of Fig. 3.10b we have $S_{II} < 1$.

enters in the decay $\tilde{\chi}_i^+ \to W^+\chi_n^0$ due to correlations of the chargino and the W boson spins, which can be probed via the hadronic decay $W^+ \to c\bar{s}$. Moreover the triple product $\mathbf{p}_{e^-}\cdot(\mathbf{p}_c \times \mathbf{p}_{\bar{s}})$ defines the CP asymmetry $\mathcal{A}_{II}$, which can be as large as 7% for $\tilde{\chi}_1^+\tilde{\chi}_1^-$ or $\tilde{\chi}_1^+\tilde{\chi}_2^-$ production. By analyzing the statistical errors of $\mathcal{A}_I$ and $\mathcal{A}_{II}$ we found that the phases φ_μ and φ_{M_1} could be strongly constrained at a future e^+e^- collider with $\sqrt{s} = 800$ GeV, high luminosity and longitudinally polarized beams.

Chapter 4

CP violation in sfermion decays

For the sfermion decays

$$\tilde{f} \to f + \tilde{\chi}_j^0; \quad \tilde{\chi}_j^0 \to Z + \tilde{\chi}_1^0; \quad Z \to \ell + \bar{\ell} \quad \ell = e, \mu, \tau, \tag{4.1}$$

$$\tilde{f} \to f + \tilde{\chi}_j^0; \quad \tilde{\chi}_j^0 \to Z + \tilde{\chi}_1^0; \quad Z \to q + \bar{q}, \quad q = b, c, \tag{4.2}$$

schematically shown in Fig. (4.1), the triple product of the momenta of the outgoing leptons

$$\mathcal{T}_\ell = \mathbf{p}_f \cdot (\mathbf{p}_\ell \times \mathbf{p}_{\bar{\ell}}), \tag{4.3}$$

and that of the outgoing quarks,

$$\mathcal{T}_q = \mathbf{p}_f \cdot (\mathbf{p}_q \times \mathbf{p}_{\bar{q}}), \tag{4.4}$$

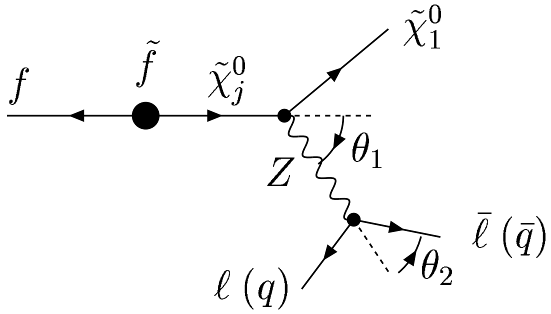


Figure 4.1: Schematic picture of the sfermion decay process.

define T-odd asymmetries

$$\mathcal{A}^{\mathrm{T}}_{\ell,q} = \frac{\Gamma(\mathcal{T}_{\ell,q} > 0) - \Gamma(\mathcal{T}_{\ell,q} < 0)}{\Gamma(\mathcal{T}_{\ell,q} > 0) + \Gamma(\mathcal{T}_{\ell,q} < 0)} \tag{4.5}$$

of the partial sfermion decay width Γ for the process (4.1). The asymmetries [52] are sensitive to correlations between the $\tilde{\chi}^0_j$ polarization and the Z boson polarization, which are encoded in the momenta of the final leptons or quarks. The correlations thus would vanish if a scalar particle in place of the Z boson is exchanged. The tree-level contribution to the asymmetries (4.5) are proportional to the imaginary part of a product of the $\tilde{\chi}^0_j$-Z-$\tilde{\chi}^0_1$ couplings. However, they are not sensitive to the phase φ_{A_f} of the trilinear scalar coupling parameter A_f, since the decay $\tilde{f} \to f\tilde{\chi}^0_j$ (4.1) is a two-body decay of a scalar particle. As an observable in the process (4.1) which is sensitive to φ_{A_f}, one would have to measure the transverse polarization of the fermion f, which is possible for $f = \tau, t$ [53]. Also three-body decays of the sfermion $\tilde{f}$ can be studied alternatively [54].

In the numerical study we estimate the event rates necessary to measure the asymmetries, which can reach 3% for leptonic decays $\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_1\ \ell\ \bar{\ell}$, and 20% for semi-leptonic decays like $\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_1\ b\ \bar{b}$.

The triple product (4.3) was proposed in [55] and the size of the asymmetry was calculated for the decay $\tilde{\mu} \to \mu\ \tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ \mu\ \ell\ \bar{\ell}$, however, for a specific final state configuration only. We extend the work of [55] by calculating the asymmetries (4.5) in the entire phase space.

4.1 Sfermion decay width

For the calculation of the amplitude squared of the subsequent two-body decays of the sfermion (4.1), we use the spin-density matrix formalism of [27]:

$$|T|^2 = |\Delta(\tilde{\chi}^0_j)|^2\ |\Delta(Z)|^2 \sum_{\lambda_j,\lambda'_j,\lambda_k,\lambda'_k} \rho_{D_1}(\tilde{f})_{\lambda_j\lambda'_j}\ \rho_{D_2}(\tilde{\chi}^0_j)^{\lambda'_j\lambda_j}_{\lambda_k\lambda'_k}\ \rho_{D_3}(Z)^{\lambda'_k\lambda_k}. \tag{4.6}$$

The amplitude squared is composed of the propagators $\Delta(\tilde{\chi}^0_j), \Delta(Z)$, the unnormalized spin density matrices $\rho_{D_1}(\tilde{f})$, $\rho_{D_2}(\tilde{\chi}^0_j)$ and $\rho_{D_3}(Z)$, with the helicity indices

λ_j, λ'_j of the neutralino and/or the helicity indices λ_k, λ'_k of the Z boson. Introducing a set of spin four-vectors $s^a_{\chi^0_j}$, $a = 1, 2, 3$, for the neutralino $\tilde{\chi}^0_j$, see Appendix (B.49), the density matrices can be expanded in terms of the Pauli matrices

$$\rho_{D_1}(\tilde{f})_{\lambda_j \lambda'_j} = \delta_{\lambda_j \lambda'_j}\, D_1 + \sigma^a_{\lambda_j \lambda'_j}\, \Sigma^a_{D_1}, \tag{4.7}$$

$$\rho_{D_2}(\tilde{\chi}^0_j)^{\lambda'_j, \lambda_j}_{\lambda_k, \lambda'_k} = \left[\delta_{\lambda'_j \lambda_j}\, D_2^{\mu\nu} + \sigma^b_{\lambda'_j \lambda_j}\, \Sigma^{b\,\mu\nu}_{D_2}\right] \varepsilon^{\lambda_k *}_{\mu} \varepsilon^{\lambda'_k}_{\nu}, \tag{4.8}$$

$$\rho_{D_3}(Z)^{\lambda'_k \lambda_k} = D_3^{\rho\sigma}\, \varepsilon^{\lambda'_k *}_{\sigma} \varepsilon^{\lambda_k}_{\rho}. \tag{4.9}$$

The polarization vectors $\varepsilon^{\lambda_k}_{\mu}$ of the Z boson obey $p^{\mu}_{Z} \varepsilon^{\lambda_k}_{\mu} = 0$ and the completeness relation $\sum_{\lambda_k} \varepsilon^{\lambda_k *}_{\mu} \varepsilon^{\lambda_k}_{\nu} = -g_{\mu\nu} + p_{Z\mu} p_{Z\nu}/m^2_Z$. The expansion coefficients of the density matrices (4.7)-(4.9) are

$$D_1 = (|a^{\tilde{f}}_{kj}|^2 + |b^{\tilde{f}}_{kj}|^2)(p_{f'} \cdot p_{\chi^0_j}), \tag{4.10}$$

$$\Sigma^a_{D_1} = \pm m_{\chi^0_j} (|a^{\tilde{f}}_{kj}|^2 - |b^{\tilde{f}}_{kj}|^2)(p_{f'} \cdot s^a_{\chi^0_j}), \tag{4.11}$$

$$\begin{aligned} D_{2\rho\sigma} = \frac{4\, g^2}{\cos^2\theta_W} \Big\{ & g_{\rho\sigma} \left[2\, \mathrm{Re}(O''^{L}_{1j} O''^{R*}_{1j}) m_{\chi^0_1} m_{\chi^0_j} - (|O''^{L}_{1j}|^2 + |O''^{R}_{1j}|^2)(p_{\chi^0_1} \cdot p_{\chi^0_j}) \right] \\ & + (|O''^{L}_{1j}|^2 + |O''^{R}_{1j}|^2)(p_{\chi^0_j \rho} p_{\chi^0_1 \sigma} + p_{\chi^0_j \sigma} p_{\chi^0_1 \rho}) \Big\}, \end{aligned} \tag{4.12}$$

$$\begin{aligned} \Sigma^a_{D_2\rho\sigma} = \frac{4\, i\, g^2}{\cos^2\theta_W} \Big\{ & 2\, m_{\chi^0_1} \mathrm{Im}(O''^{L}_{1j} O''^{R*}_{1j}) (p_{\chi^0_j \rho}\, s^a_{\chi^0_j \sigma} - p_{\chi^0_j \sigma}\, s^a_{\chi^0_j \rho}) \\ & - \varepsilon_{\rho\sigma\mu\nu}\, p^{\mu}_{\chi^0_1} s^{a\nu}_{\chi^0_j} m_{\chi^0_j} (|O''^{L}_{1j}|^2 + |O''^{R}_{1j}|^2) \\ & + 2\, \varepsilon_{\rho\sigma\mu\nu}\, p^{\mu}_{\chi^0_j} s^{a\nu}_{\chi^0_j} m_{\chi^0_1} \mathrm{Re}(O''^{L}_{1j} O''^{R*}_{1j}) \Big\}, \end{aligned} \tag{4.13}$$

$$\begin{aligned} D_3^{\rho\sigma} = \frac{2\, g^2}{\cos^2\theta_W} \Big\{ & -\, g^{\rho\sigma} (L^2_f + R^2_f)(p_f \cdot p_{\bar{f}}) + (p^{\rho}_f\, p^{\sigma}_{\bar{f}} + p^{\rho}_{\bar{f}}\, p^{\sigma}_f)(L^2_f + R^2_f) \\ & - i\, (R^2_f - L^2_f)\, \varepsilon^{\rho\sigma\mu\nu}\, p_{f\mu}\, p_{\bar{f}\nu} \Big\}, \end{aligned} \tag{4.14}$$

with $\varepsilon_{0123} = 1$ and the couplings as defined in Appendix A.2. The negative sign in (4.11) holds for the decay of a negatively charged sfermion. In (4.10) and (4.11), f' denotes the fermion from the first decay $\tilde{f} \to f'\, \tilde{\chi}^0_j$ in (4.1). Inserting the density matrices (4.7)-(4.9) in (4.6), we obtain for the amplitude squared

$$|T|^2 = 2\, |\Delta(\tilde{\chi}^0_j)|^2\, |\Delta(Z)|^2\, \{D_1\, D_{2\rho\sigma} + \Sigma^a_{D_1}\, \Sigma^a_{D_2\rho\sigma}\} D_3^{\rho\sigma}. \tag{4.15}$$

The $\tilde{f}$ decay width for the decay chain (4.1) is then given by

$$\Gamma(\tilde{f} \to f' \, \tilde{\chi}_1^0 \, f \, \bar{f}) = \frac{1}{2m_{\tilde{f}}} \int |T|^2 d\text{Lips}(m_{\tilde{f}}^2; p_{f'}, p_{\chi_1^0}, p_{\bar{f}}, p_f), \tag{4.16}$$

with the phase-space element dLips defined in Appendix B.3.2.

4.2 T-odd asymmetry

In the following we present in some detail the calculation of the T-odd asymmetry (4.5) for the slepton decays $\tilde{\ell} \to \ell \, \tilde{\chi}_j^0 \to \ell \, \tilde{\chi}_1^0 \, Z \to \ell \, \tilde{\chi}_1^0 \, f \, \bar{f}$. The replacements that must be made to obtain the asymmetry for $\tilde{q}$ decays are obvious. From (4.5) and (4.15) we find

$$\mathcal{A}_{\ell,q}^{\mathrm{T}} = \frac{\int \text{Sign}[\mathcal{T}_{\ell,q}] \Sigma_{D_1}^a \; \Sigma_{D_2 \rho\sigma}^a D_3^{\rho\sigma} d\text{Lips}}{\int D_1 \; D_{2\rho\sigma} D_3^{\rho\sigma} d\text{Lips}}, \tag{4.17}$$

where we have already used the narrow width approximation for the propagators. In the numerator we have used $\int \text{Sign}[\mathcal{T}_{\ell,q}] D_1 \; D_{2\rho\sigma} D_3^{\rho\sigma} d\text{Lips} = 0$ and in the denominator $\int \Sigma_{D_1}^a \; \Sigma_{D_2\rho\sigma}^a D_3^{\rho\sigma} d\text{Lips} = 0$. Among the spin correlation terms $\Sigma_{D_1}^a \; \Sigma_{D_2\rho\sigma}^a D_3^{\rho\sigma}$ only those contribute to $\mathcal{A}_{\ell,q}^{\mathrm{T}}$, which are proportional to the triple product $\mathcal{T}_{\ell,q}$

$$\Sigma_{D_2\rho\sigma}^a D_3^{\rho\sigma} \supset 32 \, m_{\chi_1} \text{Im}(O_{1j}''^L O_{1j}''^{R*})(R_f^2 - L_f^2) \varepsilon^{\rho\sigma\mu\nu} \, p_{\chi_j^0 \rho} \, s_{\chi_j^0 \sigma}^a \, p_{f\mu} \, p_{\bar{f}\nu}, \tag{4.18}$$

see first term of (4.13) and the last term of (4.14). From the explicit representations of the neutralino spin vector (B.49) and the lepton momentum vector (B.46), we find in the sfermion rest frame $(p_\ell \cdot s_{\chi_j^0}^a) = 0$ for $a = 1, 2$ so that $\Sigma_{D_1}^{1,2} = 0$ in (4.11). Thus only $\Sigma_{D_2\rho\sigma}^3$ contributes and the momentum dependent part of (4.18) can be written as

$$\varepsilon^{\rho\sigma\mu\nu} \, p_{\chi_j^0 \rho} \, s_{\chi_j^0 \sigma}^3 \, p_{f\mu} \, p_{\bar{f}\nu} = m_{\chi_j^0} \, \hat{\mathbf{p}}_\ell \cdot (\mathbf{p}_f \times \mathbf{p}_{\bar{f}}), \tag{4.19}$$

with $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$.

The dependence of $\mathcal{A}_f^{\mathrm{T}}$ on the $\tilde{\ell}_k$-ℓ-$\tilde{\chi}_j^0$ couplings $a_{kj}^{\tilde{\ell}}, b_{kj}^{\tilde{\ell}}$, on the Z-$\bar{f}$-f couplings L_f, R_f and on the Z-$\tilde{\chi}_1^0$-$\tilde{\chi}_j^0$ couplings $O_{1j}^{\prime\prime L,R}$ follows from (4.17) and (4.18):

$$\mathcal{A}_f^{\mathrm{T}} \propto \frac{|a_{kj}^{\tilde{\ell}}|^2 - |b_{kj}^{\tilde{\ell}}|^2}{|a_{kj}^{\tilde{\ell}}|^2 + |b_{kj}^{\tilde{\ell}}|^2} \frac{L_f^2 - R_f^2}{L_f^2 + R_f^2} \,\mathrm{Im}(O_{1j}^{\prime\prime L} O_{1j}^{\prime\prime R*}). \tag{4.20}$$

Due to the first factor $\frac{|a_{kj}^{\tilde{\ell}}|^2 - |b_{kj}^{\tilde{\ell}}|^2}{|a_{kj}^{\tilde{\ell}}|^2 + |b_{kj}^{\tilde{\ell}}|^2}$, the asymmetry will be strongly suppressed for $|a_{kj}^{\tilde{\ell}}| \approx |b_{kj}^{\tilde{\ell}}|$ and maximally enhanced for vanishing mixing in the slepton sector. Due to the second factor $\frac{L_f^2 - R_f^2}{L_f^2 + R_f^2}$, the asymmetry $\mathcal{A}_{b(c)}^{\mathrm{T}}$ for hadronic decays of the Z boson is larger than the asymmetry $\mathcal{A}_\ell^{\mathrm{T}}$ for leptonic decays:

$$\mathcal{A}_{b(c)}^{\mathrm{T}} = \frac{L_\ell^2 + R_\ell^2}{L_\ell^2 - R_\ell^2} \frac{L_{b(c)}^2 - R_{b(c)}^2}{L_{b(c)}^2 + R_{b(c)}^2} \,\mathcal{A}_\ell^{\mathrm{T}} \simeq 6.3\ (4.5) \times \mathcal{A}_\ell^{\mathrm{T}}. \tag{4.21}$$

For the measurement of $\mathcal{A}_f^T$ the charges and the flavors of f and $\bar{f}$ have to be distinguished. For $f = e, \mu$ this will be possible on an event by event basis. For $f = \tau$ one has to take into account corrections due to the reconstruction of the τ momentum. For $f = b, c$ the distinction of the quark flavors should be possible by flavor tagging [30, 31]. However, in this case the quark charges will be distinguished statistically for a given event sample only.

4.3 Numerical results

We assume that $\tilde{\tau}_1$ is the lightest sfermion and study the decay chain $\tilde{\tau}_1 \to \tau \tilde{\chi}_j^0;\ \tilde{\chi}_j^0 \to \tilde{\chi}_1^0 Z;\ Z \to \ell \bar{\ell}$ for the two cases $\tilde{\tau}_1 \to \tau \tilde{\chi}_2^0$ and $\tilde{\tau}_1 \to \tau \tilde{\chi}_3^0$ separately. We present numerical results for the T-odd asymmetry $\mathcal{A}_\ell^{\mathrm{T}}$ (4.5) and the branching ratios $\mathrm{BR}(\tau_1 \to \tau\, \tilde{\chi}_1^0\, \ell\, \bar{\ell}) := \mathrm{BR}(\tilde{\tau}_1 \to \tau\, \tilde{\chi}_j^0) \times \mathrm{BR}(\tilde{\chi}_j^0 \to Z \tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\, \bar{\ell})$. The size of the asymmetry $\mathcal{A}_{b,c}^{\mathrm{T}}$ for hadronic decays may be obtained from (4.21).

The relevant MSSM parameters are $|\mu|, \varphi_\mu, |M_1|, \varphi_{M_1}, M_2, \tan\beta, |A_\tau|, \varphi_{A_\tau}, m_{\tilde{\tau}_1}$ and $m_{\tilde{\tau}_2}$. We fix $\tan\beta = 10$, $|A_\tau| = 1$ TeV, $\varphi_{A_\tau} = 0$, $m_{\tilde{\tau}_1} = 300$ GeV, $m_{\tilde{\tau}_2} = 800$ GeV and use the relation $|M_1| = 5/3\, M_2 \tan^2\theta_W$ in order to reduce the number of parameters.

Table 4.1: Masses and widths for various combinations of φ_μ and φ_{M_1}, for $|\mu| = 300$ GeV, $M_2 = 280$ GeV, $\tan\beta = 10$, $A_\tau = 1$ TeV, $m_{\tilde{\tau}_1} = 300$ GeV, $m_{\tilde{\tau}_2} = 800$ GeV.

φ_μ	φ_{M_1}	$m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}$ [GeV]	$\Gamma_{\chi_2^0}$ [MeV]	$\Gamma_{\tilde{\tau}_1}$ [MeV]
0	0	135, 234, 306, 358	4.06	527
0	$\frac{\pi}{2}$	137, 233, 308, 357	1.79	550
0	π	138, 231, 309, 356	0.09	573
$\frac{\pi}{2}$	0	137, 239, 307, 353	5.43	487
$\frac{\pi}{2}$	$\frac{\pi}{2}$	138, 238, 309, 352	2.89	511
$\frac{\pi}{2}$	π	137, 237, 311, 351	1.49	529
π	0	138, 245, 309, 347	7.25	448
π	$\frac{\pi}{2}$	137, 244, 311, 346	5.78	466
π	π	136, 243, 313, 345	4.32	484

For the calculation of the branching ratios $\mathrm{BR}(\tilde{\tau}_1 \to \tau\, \tilde{\chi}_j^0)$ and $\mathrm{BR}(\tilde{\chi}_j^0 \to Z\tilde{\chi}_1^0)$, we include the decays

$$\tilde{\tau}_1 \to \tau\, \tilde{\chi}_j^0,\ \tilde{\chi}_j^- \nu_\tau, \tag{4.22}$$

$$\tilde{\chi}_j^0 \to Z\tilde{\chi}_1^0,\ \tilde{\chi}_m^\mp W^\pm,\ \tilde{\chi}_n^0 H_1^0,\ m = 1,2,\ n < j. \tag{4.23}$$

We fix the Higgs mass parameter $m_A = 800$ GeV so that the decays of the neutralino into charged Higgs bosons $\tilde{\chi}_j^0 \to \tilde{\chi}_m^\pm H^\mp$, as well as decays into heavy neutral Higgs bosons $\tilde{\chi}_j^0 \to \tilde{\chi}_n^0\, H_{2,3}^0$ are forbidden. The decays via sleptons $\tilde{\chi}_j^0 \to \ell\, \tilde{\ell}$ are forbidden due to our assumption that $\tilde{\tau}_1$ is the lightest sfermion.

4.3.1 Decay chain via $\tilde{\tau}_1 \to \tau\, \tilde{\chi}_2^0$

We study $\mathcal{A}_\ell^{\mathrm{T}}$ for the decay chain $\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0;\ \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z;\ Z \to \ell\bar{\ell}$ for $\ell = e, \mu, \tau$. In Fig. 4.2a we show the contour lines for the branching ratio $\mathrm{BR}(\tau_1 \to \tau\, \tilde{\chi}_1^0\, \ell\, \bar{\ell}) = \mathrm{BR}(\tilde{\tau}_1 \to \tau\, \tilde{\chi}_2^0) \times \mathrm{BR}(\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0) \times \mathrm{BR}(Z \to \ell\, \bar{\ell})$, summed over $\ell = e, \mu, \tau$, in the M_2-$|\mu|$ plane for $\varphi_{M_1} = \pi/2$ and $\varphi_\mu = 0$. The $\tilde{\tau}_1$-τ-$\tilde{\chi}_2^0$ coupling $|a_{12}^{\tilde{\tau}}|$ is larger, which implies a larger $\mathrm{BR}(\tilde{\tau}_1 \to \tau\, \tilde{\chi}_2^0)$ if we choose $M_{\tilde{E}} > M_{\tilde{L}}$. We use the usual notation $M_{\tilde{E}} \equiv M_{R\tilde{\tau}}$, $M_{\tilde{L}} \equiv M_{L\tilde{\tau}}$, see (A.46) and (A.47). The ordering $M_{\tilde{E}} > M_{\tilde{L}}$ is suggested in some scenarios with non-universal scalar mass parameters at the GUT scale [56]. Furthermore, in (A.46) and (A.47) one could have $M_{\tilde{\tau}_{RR}} > M_{\tilde{\tau}_{LL}}$ in extended models with additional D-terms [57].

than for $M_{\tilde{E}} < M_{\tilde{L}}$. In a large region of the parameter space $\mathrm{BR}(\tilde{\chi}^0_2 \to Z\tilde{\chi}^0_1) = 1$. The asymmetry $\mathcal{A}^{\mathrm{T}}_{\ell}$ is shown in Fig. 4.2b. The dependence of $\mathcal{A}^{\mathrm{T}}_{\ell}$ on M_2 and $|\mu|$ is dominantly determined by $\mathrm{Im}(O''^{L}_{12}O''^{R*}_{12})$, as expected from (4.20).

In Fig. 4.3 we show the φ_{M_1} and φ_{μ} dependence of $\mathrm{BR}(\tilde{\tau}_1 \to \tau\ \chi^0_1\ \ell\ \bar{\ell})$ and of $\mathcal{A}^{\ell}_{\mathrm{T}}$ for $|\mu| = 300$ GeV and $M_2 = 280$ GeV. For these parameters, we also give in Table 4.3.1 the neutralino masses and the total neutralino and stau widths for various phase combinations. Note that maximal phases $\varphi_{\mu}, \varphi_{M_1} = \pm\pi/2$ do not lead to the highest value of $\mathcal{A}^{\mathrm{T}}_{\ell}$, since the asymmetry is proportional to a product of CP even $(\Sigma^3_{D_1})$ and CP odd terms $(\Sigma^3_{D_2\rho\sigma}D^{\rho\sigma}_3)$, see (4.17). We give a lower bound on the number N of $\tilde{\tau}_1$'s to be produced at a linear collider, in order to measure $\mathcal{A}^{\mathrm{T}}_{\ell}$ at 1σ. We estimate $N = [(\mathcal{A}^{\mathrm{T}}_{\ell})^2 \times \mathrm{BR}]^{-1}$ from the relative statistical error of the asymmetry, see (1.16), with $\mathrm{BR} = \mathrm{BR}(\tau_1 \to \tau\ \tilde{\chi}^0_1\ \ell\ \bar{\ell})$. For the point $\varphi_{\mu} = \pi/2$ and $\varphi_{M_1} = \pi/2$, marked by $\bullet$ in Fig. 4.3, $\mathrm{BR} \approx 2.5\%$ and $|\mathcal{A}^{\mathrm{T}}_{\ell}| \approx 3\%$, so that $N \approx 4.4 \times 10^5$. For the decay $\tilde{\tau}_1 \to b\bar{b}\tilde{\chi}^0_1\tau$, however, $\mathrm{BR} \approx 3.6\%$ and $|\mathcal{A}^{\mathrm{T}}_{b}| \approx 19\%$, so that only $N \approx 7.7 \times 10^2$ $\tilde{\tau}_1$'s are needed. We obtain almost the same results for smaller CP phases $\varphi_{\mu} = 0$ and $\varphi_{M_1} = -0.3\pi$, marked by $\otimes$ in Fig. 4.3. In these two examples $\mathcal{A}^{\mathrm{T}}_{\ell,q}$ should be measurable at an e^+e^- linear collider with $\sqrt{s} = 800$ GeV and an integrated luminosity of 500 fb^{-1}.

4.3.2 Decay chain via $\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_3$

We discuss the decay chain $\tilde{\tau}_1 \to \tau\tilde{\chi}^0_3;\ \tilde{\chi}^0_3 \to \tilde{\chi}^0_1 Z;\ Z \to \ell\bar{\ell}$ for $\ell = e, \mu, \tau$. The decay $\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_3$ can be distinguished from $\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_2$ by the τ energy. In Fig. 4.4a we show the contour lines of $\mathrm{BR}(\tau_1 \to \tau\ \tilde{\chi}^0_1\ \ell\ \bar{\ell}) = \mathrm{BR}(\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_3) \times \mathrm{BR}(\tilde{\chi}^0_3 \to Z\tilde{\chi}^0_1) \times \mathrm{BR}(Z \to \ell\ \bar{\ell})$ in the $|\mu|$–M_2 plane for $\varphi_{M_1} = \pi/2$ and $\varphi_{\mu} = 0$. We choose $M_{\tilde{E}} < M_{\tilde{L}}$ since the $\tilde{\tau}_1$-τ-$\tilde{\chi}^0_3$ coupling $|a^{\tilde{\tau}}_{13}|$ is larger, thus $\mathrm{BR}(\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_3)$ is larger than for $M_{\tilde{E}} > M_{\tilde{L}}$. However, the total branching ratio is smaller than for the previous decay chain due to the small $\mathrm{BR}(\tilde{\tau}_1 \to \tau\ \tilde{\chi}^0_3) < .75(0.05)$ in the upper (lower) part of Fig. 4.4a.

The asymmetry $\mathcal{A}^{\mathrm{T}}_{\ell}$, shown in Fig. 4.4b, vanishes on contours where either $|a^{\tilde{\tau}}_{13}| = |b^{\tilde{\tau}}_{13}|$ or $\mathrm{Im}(O''^{L}_{13}O''^{R*}_{13}) = 0$, see (4.20). Along the contour $\mathcal{A}^{\mathrm{T}}_{\ell} = 0$ in the lower part of Fig. 4.4b we have $|a^{\tilde{\tau}}_{13}| = |b^{\tilde{\tau}}_{13}|$, whereas along the contour line 0 in the upper part of Fig. 4.4b we have $\mathrm{Im}(O''^{L}_{13}O''^{R*}_{13}) = 0$. Furthermore, there is a sign change of $\mathrm{Im}(O''^{L}_{13}O''^{R*}_{13})$ between the upper and the lower part of Fig. 4.4b (area A). The first factor in (4.20) increases for $|\mu|/M_2 \to 0$, since the gaugino component of $\tilde{\chi}^0_3$ gets enhanced, resulting in $|b^{\tilde{\tau}}_{13}|/|a^{\tilde{\tau}}_{13}| \to 0$.

In Fig. 4.5 we show the φ_{M_1}, φ_{μ} dependence of $\mathrm{BR}(\tau_1 \to \tau\ \chi^0_1\ \ell\ \bar{\ell})$ and $\mathcal{A}^{\mathrm{T}}_{\ell}$, for $|\mu| = 150$ GeV and $M_2 = 450$ GeV. Two points of level crossing appear at $(\varphi_{M_1}, \varphi_{\mu}) \approx$

$(\pm 0.95\pi, \pm 0.7\pi)$ in Fig. 4.5b. Neutralino masses and the neutralino and stau widths are given in Table 4.2. Comparing Fig. 4.3b and Fig. 4.5b, one can see the common and strong φ_{M_1} dependence of the asymmetries. In a good approximation $\mathrm{Sign}[\mathcal{A}_\ell^{\mathrm{T}}] \approx \mathrm{Sign}[\varphi_{M_1}]$ in Fig. 4.3b and $\mathrm{Sign}[\mathcal{A}_\ell^{\mathrm{T}}] \approx -\mathrm{Sign}[\varphi_{M_1}]$ in Fig. 4.5b, due to the different phase dependence of $\mathrm{Im}(O''^L_{12} O''^{R*}_{12})$ and $\mathrm{Im}(O''^L_{13} O''^{R*}_{13})$.

4.4 Summary of Chapter 4

We have considered a T-odd asymmetry in the sequential decay of a sfermion $\tilde{f} \to f' \; \tilde{\chi}^0_j \to f' \; \tilde{\chi}^0_1 \; Z \to f' \; \tilde{\chi}^0_1 \; f \bar{f}$. The asymmetry is sensitive to the phases in the neutralino sector. In a numerical study for stau decay $\tilde{\tau}_1 \to \tau \tilde{\chi}^0_1 \, f \bar{f}$, we have shown that the asymmetry can be of the order of 3% for leptonic final states $\tau \; \tilde{\chi}^0_1 \; \bar{\ell}\ell$, and is larger by a factor 6.3 for the semi-leptonic final state $\tau \; \tilde{\chi}^0_1 \; \bar{b}b$. The number of produced $\tilde{\tau}$'s which are necessary to observe the asymmetry is at least of the order 10^5 for leptonic final states, and 10^3 for semi-leptonic final states, such that the phases in the neutralino sector may be accessible at future collider experiments.

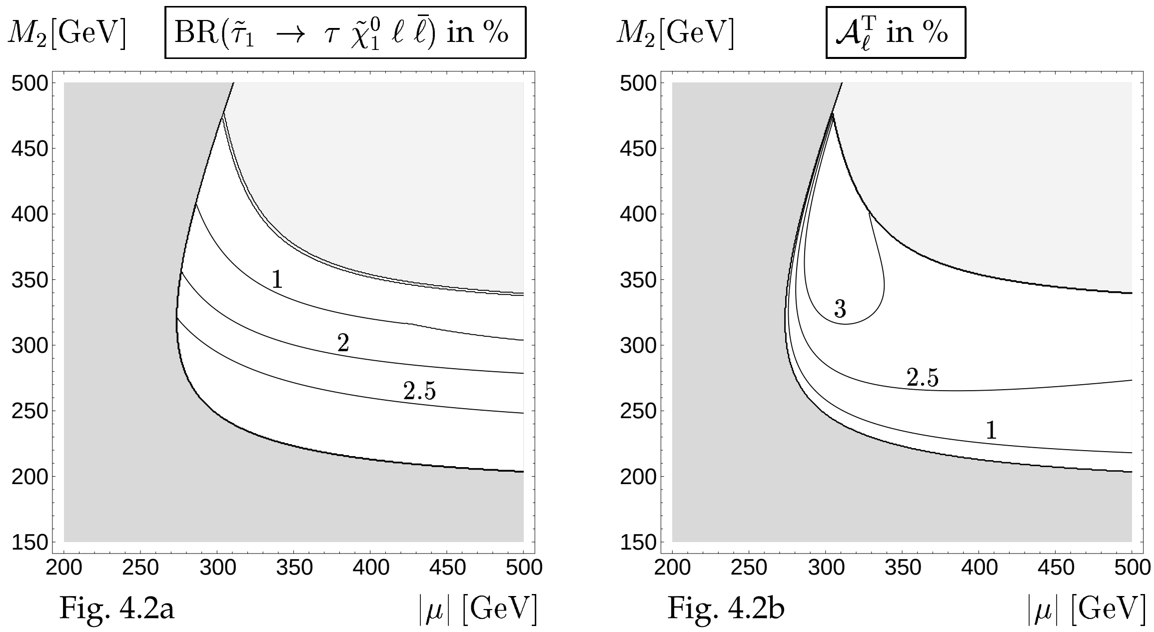


Figure 4.2: Contour lines of the branching ratio for $\tilde\tau_1 \to \tilde\chi_1^0 \tau\ell\bar\ell$ and asymmetry $\mathcal{A}_\ell^{\mathrm{T}}$ in the μ–M_2 plane for $\varphi_{M_1} = \pi/2$ and $\varphi_\mu = 0$, taking $\tan\beta = 10$, $A_\tau = 1$ TeV, $m_{\tilde\tau_1} = 300$ GeV, $m_{\tilde\tau_2} = 800$ GeV for $M_{\tilde E} > M_{\tilde L}$. The gray areas are kinematically forbidden by $m_{\tilde\tau_1} < m_{\chi_2^0} + m_\tau$ (light gray) or $m_{\chi_2^0} < m_{\chi_1^0} + m_Z$ (dark gray).

Table 4.2: Masses and widths for various combinations of φ_μ and φ_{M_1}, for $|\mu| = 150$ GeV, $M_2 = 450$ GeV, $\tan\beta = 10$, $A_\tau = 1$ TeV, $m_{\tilde\tau_1} = 300$ GeV, $m_{\tilde\tau_2} = 800$ GeV.

φ_μ	φ_{M_1}	$m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}$ [GeV]	$\Gamma_{\chi_3^0}$ [MeV]	$\Gamma_{\tilde\tau_1}$ [MeV]
0	0	128, 156, 238, 467	59.0	362
0	$\frac{\pi}{2}$	132, 153, 238, 466	68.2	359
0	π	141, 145, 238, 466	75.5	356
$\frac{\pi}{2}$	0	131, 158, 237, 466	41.5	356
$\frac{\pi}{2}$	$\frac{\pi}{2}$	136, 154, 237, 466	49.4	353
$\frac{\pi}{2}$	π	142, 145, 240, 465	73.8	360
π	0	135, 159, 236, 465	27.7	351
π	$\frac{\pi}{2}$	137, 154, 239, 465	47.5	357
π	π	143, 144, 242, 464	71.0	364

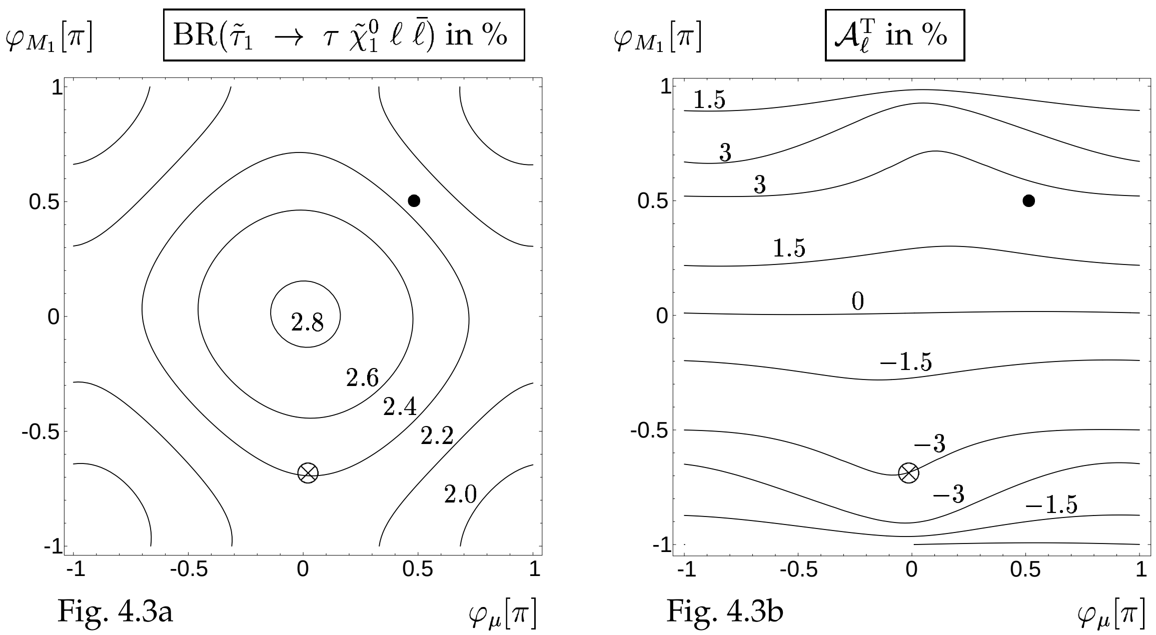


Figure 4.3: Contour lines of the branching ratio for $\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau \ell \bar{\ell}$ and asymmetry $\mathcal{A}_\ell^{\mathrm{T}}$ in the φ_μ–φ_{M_1} plane for $|\mu| = 300$ GeV, $M_2 = 280$ GeV, taking $\tan\beta = 10$, $A_\tau = 1$ TeV, $m_{\tilde{\tau}_1} = 300$ GeV, $m_{\tilde{\tau}_2} = 800$ GeV for $M_{\tilde{E}} > M_{\tilde{L}}$.

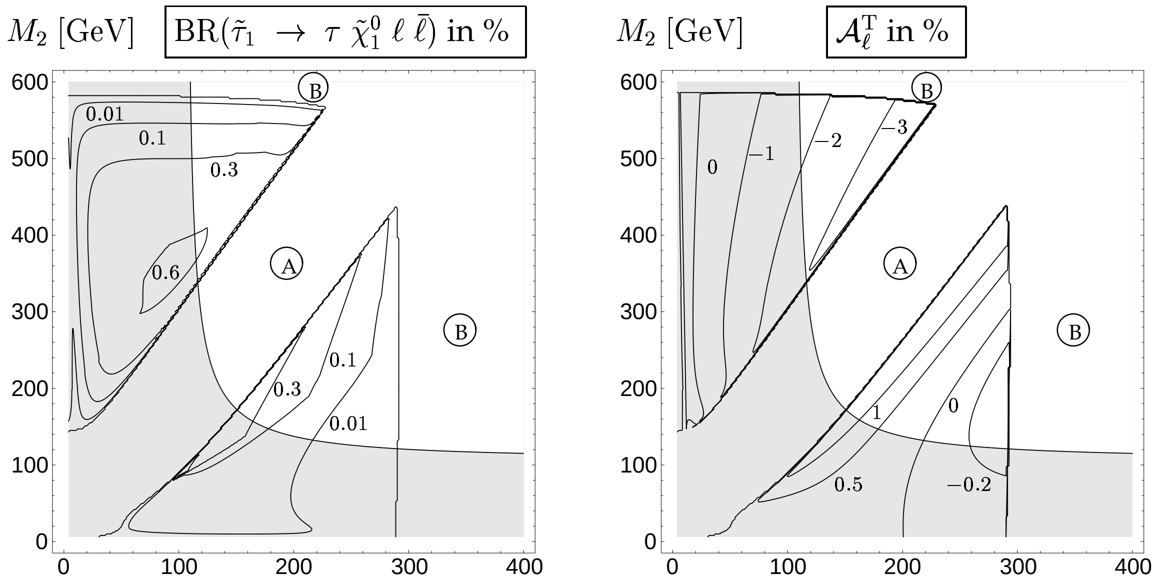


Fig. 4.4a Fig. 4.4b

Figure 4.4: Contour lines of the branching ratio for $\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau \ell \bar{\ell}$ and asymmetry $\mathcal{A}_\ell^{\rm T}$ in the $|\mu|$–M_2 plane for $\varphi_{M_1} = \pi/2$, $\varphi_\mu = 0$, $\tan\beta = 10$, $A_\tau = 1$ TeV, $m_{\tilde{\tau}_1} = 300$ GeV, $m_{\tilde{\tau}_2} = 800$ GeV and $M_{\tilde{E}} < M_{\tilde{L}}$. The area A (B) is kinematically forbidden by $m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_1^0} + m_Z$ $(m_{\tilde{\tau}_1} < m_{\tilde{\chi}_3^0} + m_\tau)$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

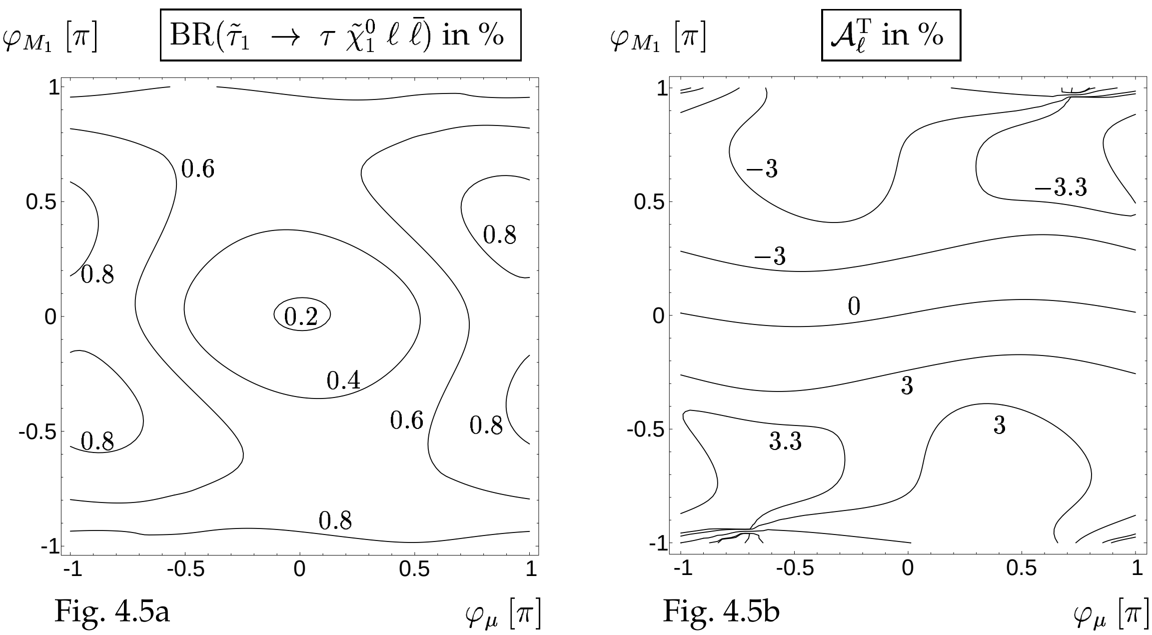


Figure 4.5: Contour lines of the branching ratio for $\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau \ell \bar{\ell}$ and asymmetry $\mathcal{A}_\ell^{\mathrm{T}}$ in the φ_μ–φ_{M_1} plane for $|\mu| = 150$ GeV, $M_2 = 450$ GeV, $\tan\beta = 10$, $A_\tau = 1$ TeV, $m_{\tilde{\tau}_1} = 300$ GeV, $m_{\tilde{\tau}_2} = 800$ GeV and $M_{\tilde{E}} < M_{\tilde{L}}$.

Chapter 5

Summary and conclusions

5.1 Summary

In supersymmetric (SUSY) extensions of the Standard Model (SM), several parameters can be complex. In the neutralino sector of the Minimal extended Supersymmetric Standard Model (MSSM), these are the Higgsino mass parameter μ and the gaugino mass parameter M_1. In addition, in the sfermion sector also the trilinear scalar coupling parameter A_f can be complex. The imaginary parts of these parameters imply CP violating effects in the production and decay of SUSY particles.

In this thesis we have analyzed the implications of φ_μ, φ_{M_1} and φ_{A_τ} in neutralino and chargino production and decay in electron-positron collisions. For the decays of sfermions we have analyzed the effects of φ_μ and φ_{M_1}. We have analyzed T-odd and CP-odd asymmetries of triple products of particle momenta or spins. Such asymmetries are non-zero only if CP is violated. Their measurements at future colliders allow a determination of the phases, in particular also their signs.

The asymmetries involve angular distributions of the neutralino, chargino and sfermion decay products. The tree-level calculations in the spin-density formalism include the complete spin correlations between production and decay. Modular FORTRAN codes have been programmed for numerical analyses.

For neutralino production, $e^+e^- \to \tilde{\chi}_i^0\, \tilde{\chi}_j^0$, we can summarize as follows:

- For the leptonic two-body decay chain of one of the neutralinos $\tilde{\chi}_i^0 \to \tilde{\ell}\ell_1, \tilde{\ell} \to \tilde{\chi}_1^0\ell_2$ for $\ell = e, \mu, \tau$, we have analyzed the asymmetries of two triple products $\mathcal{T}_I = (\mathbf{p}_{e^-} \times \mathbf{p}_{\tilde{\chi}_i^0}) \cdot \mathbf{p}_{\ell_1}$ and $\mathcal{T}_{II} = (\mathbf{p}_{e^-} \times \mathbf{p}_{\ell_2}) \cdot \mathbf{p}_{\ell_1}$, which are sensitive to φ_μ and φ_{M_1}. In a numerical study for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ and subsequent neutralino decay into a right slepton $\tilde{\chi}_2^0 \to \tilde{\ell}_R\ell$ we have shown that the asymmetry $\mathcal{A}_I$ can be as large as 25%. The asymmetry $\mathcal{A}_{II}$, which does not require the identification of the neutralino momentum, can reach 10%. Asymmetries of the same order are obtained for the processes $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ and $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_3^0$.

- For the two-body decay of one neutralino into a stau-tau pair, $\tilde{\chi}_i^0 \to \tilde{\tau}_k\tau$, we have analyzed the asymmetry of the triple product $\mathcal{T}_\tau = \mathbf{s}_\tau \cdot (\mathbf{p}_\tau \times \mathbf{p}_{e^-})$, which includes the transverse τ polarization $\mathbf{s}_\tau$. The asymmetry is sensitive to the phases φ_μ, φ_{M_1} and φ_{A_τ} and can attain values up to 60% for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$.

- For the neutralino decay into a Z boson, $\tilde{\chi}_i^0 \to Z\tilde{\chi}_n^0$, followed by the decay $Z \to \ell\bar{\ell}$ for $\ell = e, \mu, \tau$, or $Z \to q\bar{q}$ for $q = c, b$, we have defined and analyzed the asymmetry $\mathcal{A}_{\ell(q)}$ of the triple product $\mathcal{T}_{\ell(q)} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\ell(q)} \times \mathbf{p}_{\bar{\ell}(\bar{q})})$, which is sensitive to φ_μ and φ_{M_1}. We have also identified the CP sensitive elements of the Z spin-density matrix. For $\tilde{\chi}_1^0\tilde{\chi}_2^0$, $\tilde{\chi}_2^0\tilde{\chi}_2^0$, $\tilde{\chi}_1^0\tilde{\chi}_3^0$ and $\tilde{\chi}_2^0\tilde{\chi}_3^0$ production, the asymmetry $\mathcal{A}_\ell$ can go up to 3%. For the hadronic decays of the Z boson, larger asymmetries $\mathcal{A}_{c(b)} \simeq 6.3(4.5) \times \mathcal{A}_\ell$ are obtained.

The results for chargino production, $e^+e^- \to \tilde{\chi}_i^\pm\, \tilde{\chi}_j^\mp$, can be summarized as follows:

- For the two-body decay of one of the charginos into a sneutrino $\tilde{\chi}_i^+ \to \ell^+\tilde{\nu}_\ell$, the asymmetry of the triple product $(\mathbf{p}_{e^-} \times \mathbf{p}_{\tilde{\chi}_i^+}) \cdot \mathbf{p}_\ell$ is sensitive to φ_μ and can be as large as 30% for $e^+e^- \to \tilde{\chi}_1^\pm\, \tilde{\chi}_2^\mp$.

- For the decay $\tilde{\chi}_i^+ \to W^+\chi_n^0$ the triple product $\mathcal{T}_I = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\tilde{\chi}_i^+} \times \mathbf{p}_W)$ defines the asymmetry $\mathcal{A}_I$, which is sensitive to φ_μ. The asymmetry $\mathcal{A}_I$ can reach 4% for $e^+e^- \to \tilde{\chi}_1^\pm\, \tilde{\chi}_2^\mp$ production. Further, we have analyzed the CP sensitive elements of the W boson spin-density matrix. The phase φ_{M_1} enters in the decay $\tilde{\chi}_i^+ \to W^+\chi_n^0$ due to correlations of the chargino and the W boson polarizations, which can be probed via the hadronic decay $W^+ \to c\bar{s}$. Moreover the triple product $\mathcal{T}_{II} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_c \times \mathbf{p}_{\bar{s}})$ defines the φ_μ and φ_{M_1} sensitive asymmetry $\mathcal{A}_{II}$, which can be as large as 7% for $\tilde{\chi}_1^+\tilde{\chi}_1^-$ or $\tilde{\chi}_1^+\tilde{\chi}_2^-$ production.

We note in addition that if the neutralinos or charginos decay into scalar particles, like sleptons or sneutrinos, the asymmetries probe CP violation in the production process only. The asymmetries are then caused by the neutralino or chargino polarizations perpendicular to the production plane, which are non-vanishing only for the production of a non-diagonal pair of neutralinos $e^+e^- \to \tilde{\chi}_i^0\,\tilde{\chi}_j^0, i \neq j$ or charginos $e^+e^- \to \tilde{\chi}_1^{\pm}\,\tilde{\chi}_2^{\mp}$. If the neutralinos and charginos decay into particles with spin, like τ or W and Z bosons, we have found that also the CP phases which enter in the decay can be probed, in addition to the CP contributions from the production. This is due to the spin correlations between production and decay.

For the two-body decay chain of a sfermion $\tilde{f} \to f\tilde{\chi}_j^0 \to f\tilde{\chi}_1^0\,Z \to f'\tilde{\chi}_1^0\,l\,\bar{l}\,(f'\tilde{\chi}_1^0\,q\,\bar{q})$ we have obtained the following results:

- We have defined the asymmetries $\mathcal{A}_{\ell,q}$ of the triple products of the momenta of the outgoing leptons $\mathcal{T}_\ell = \mathbf{p}_f \cdot (\mathbf{p}_\ell \times \mathbf{p}_{\bar{\ell}})$ or quarks $\mathcal{T}_q = \mathbf{p}_f \cdot (\mathbf{p}_q \times \mathbf{p}_{\bar{q}})$, which are sensitive to φ_μ and φ_{M_1}. In a numerical study of stau decay $\tilde{\tau}_1 \to \tau\chi_1^0\,f\bar{f}$ we found that the asymmetry $\mathcal{A}_\ell$ can be of the order of 3% percent for leptonic final states. The number of produced $\tilde{\tau}$'s necessary to observe $\mathcal{A}_\ell$ is at least of the order 10^5, which may be accessible at future collider experiments. For the semi-leptonic final state $\tilde{\tau}_1 \to \tau\,\tilde{\chi}_1^0\,b\,\bar{b}$ the asymmetry $\mathcal{A}_b$ is larger by a factor 6.3 and could be measured if the number of produced $\tilde{\tau}$'s is of the order 10^3.

- The asymmetries are not sensitive to the phase φ_{A_f}, since the decay $\tilde{f} \to f\,\tilde{\chi}_j^0$ is a two-body decay of a scalar particle. As an observable sensitive to φ_{A_f}, one would have to measure the transverse polarization of the fermion f, or study asymmetries in three-body decays of the sfermion $\tilde{f}$.

5.2 Conclusions

We have shown that in all the processes studied, triple-product asymmetries can be defined which are sensitive to the CP phases. Especially promising for measuring the phases are leptonic decays of charginos and neutralinos, where the asymmetries can be as large as 30%. For neutralino decays into a stau-tau pair, the asymmetry of the τ polarization may even reach 60%. The asymmetries of neutralino and chargino decays into a Z or W boson may be as large as 18% and 7%, respectively. For the neutralino and chargino production processes, we have found that longitudinal polarization of both beams can significantly enhance the asymmetries and cross sections.

By analyzing the statistical significances, we have shown that the asymmetries are accessible in future electron-positron linear collider experiments in the 500 GeV - 800 GeV range with high luminosity and polarized beams. The MSSM phases φ_μ, φ_{M_1} and φ_{A_τ} could thus be strongly constrained.

5.3 Outlook

In further investigations our analysis of CP-odd asymmetries in the production and decay of neutralinos and charginos should be extended to electron-positron collisions with transverse beam polarizations. For chargino production the asymmetries of triple products including the transverse beam polarization are only non-vanishing if the decay of both particles is included.

Moreover, triple products which involve the momenta of the decay products of both neutralinos (or charginos) could be sensitive to correlations between their polarizations. These spin-spin correlations would yield additional information on the CP phases.

Chapter 6

Zusammenfassung und Schlussfolgerungen

6.1 Zusammenfassung

Einige Parameter in supersymmetrischen (SUSY) Erweiterungen des Standardmodells (SM) können komplex sein. Im Neutralinosektor des minimal erweiterten supersymmetrischen Standardmodells (MSSM) sind dies der Higgsino-Massenparameter μ und der Gaugino-Massenparameter M_1. Im Sfermionsektor ist der trilineare skalare Kopplungsparameter A_f im Allgemeinen komplex. Die Imaginärteile dieser Parameter verursachen CP-verletzende Effekte in Produktion und Zerfall von SUSY Teilchen.

In dieser Arbeit wurde der Einfluss der Phasen φ_μ, φ_{M_1} und φ_{A_τ} auf die Produktion und den Zerfall von Neutralinos und Charginos in Elektron-Positron-Kollisionen untersucht. Für Sfermionzerfälle wurden die Effekte der Phasen φ_μ und φ_{M_1} studiert. Es wurden auf Spatprodukten basierende T- und CP-ungerade Asymmetrien definiert und berechnet. Solche Asymmetrien verschwinden nur im Falle der CP-Erhaltung und signalisieren daher die Existenz von CP-Phasen. Messungen der Asymmetrien an zukünftigen Beschleunigern erlauben nicht nur eine Bestimmung der Phasen, sonderen auch eine Bestimmung ihrers Vorzeichens.

Die Asymmetrien erfordern Berechnungen von Winkelverteilungen der Zerfallsprodukte von Neutralinos, Charginos und Sfermionen. Die Rechnungen auf Baum-Graphen-Niveau im Spin-Dichtematrix-Formalismus beinhalten die vollständigen Spinkorrelationen zwischen Produktion und Zerfall. Für numerische Analysen wurden modulare Fortran-Programme erstellt.

Die Ergebnisse für Neutralino-Produktion $e^+e^- \to \tilde{\chi}_i^0\, \tilde{\chi}_j^0$ und -Zerfall können folgendermaßen zusammengefasst werden:

- Für den leptonischen Zerfall des Neutralinos $\tilde{\chi}_i^0 \to \tilde{\ell}\ell_1$, $\tilde{\ell} \to \tilde{\chi}_1^0\ell_2$ mit $\ell = e, \mu, \tau$, wurden die Asymmetrien der zwei Spatprodukte $\mathcal{T}_I = (\mathbf{p}_{e^-} \times \mathbf{p}_{\tilde{\chi}_i^0}) \cdot \mathbf{p}_{\ell_1}$ und $\mathcal{T}_{II} = (\mathbf{p}_{e^-} \times \mathbf{p}_{\ell_2}) \cdot \mathbf{p}_{\ell_1}$ betrachtet, die auf die Phasen φ_μ und φ_{M_1} sensitiv sind. In der numerischen Analyse für $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ und Zerfall des Neutralinos in ein rechtshändiges Slepton $\tilde{\chi}_2^0 \to \tilde{\ell}_R\ell$ erreicht $\mathcal{A}_I$ Werte von 25%. Die Asymmetrie $\mathcal{A}_{II}$ erreicht 10%, erfordert jedoch nicht die Rekonstruktion des Neutralinoimpulses. Die Asymmetrien haben für die Prozesse $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_3^0$ und $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_3^0$ ähnliche Werte.

- Für den Zerfall des Neutralinos in ein Stau-Tau Paar, $\tilde{\chi}_i^0 \to \tilde{\tau}_k\tau$, wurde die Asymmetry des Spatprodukts $\mathcal{T}_\tau = \mathbf{s}_\tau \cdot (\mathbf{p}_\tau \times \mathbf{p}_{e^-})$ untersucht, welches die transversale τ-Polarisation $\mathbf{s}_\tau$ enthält. Die Asymmetrie ist auf die Phasen φ_μ, φ_{M_1} und φ_{A_τ} sensitiv und kann für den Prozess $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ bis zu 60% betragen.

- Für den Zerfall des Neutralinos in ein Z-Boson, mit anschließendem Zerfall $Z \to \ell\bar{\ell}$ mit $\ell = e, \mu, \tau$, bzw. $Z \to q\bar{q}$ mit $q = c, b$, wurde eine Asymmetrie $\mathcal{A}_{\ell(q)}$ des Spatprodukts $\mathcal{T}_{\ell(q)} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\ell(q)} \times \mathbf{p}_{\bar{\ell}(\bar{q})})$ definiert und untersucht. Sie ist sensitiv auf die Phasen φ_μ und φ_{M_1}. Des Weiteren wurden die CP-sensitiven Elemente der Dichtematrix des Z-Bosons identifiziert und berechnet. Die Asymmetrie $\mathcal{A}_\ell$ kann für die Neutralinoproduktionen $\tilde{\chi}_1^0\tilde{\chi}_2^0$, $\tilde{\chi}_2^0\tilde{\chi}_2^0$, $\tilde{\chi}_1^0\tilde{\chi}_3^0$ und $\tilde{\chi}_2^0\tilde{\chi}_3^0$ Werte bis 3% erreichen. Für die hadronischen Zerfälle des Z-Bosons sind die Asymmetrien größer $\mathcal{A}_{c(b)} \simeq 6.3(4.5) \times \mathcal{A}_\ell$.

Die Ergebnisse für Chargino-Produktion $e^+e^- \to \tilde{\chi}_i^\pm\, \tilde{\chi}_j^\mp$ und -Zerfall können folgendermaßen zusammengefasst werden:

- Der Zerfall des Charginos in ein Sneutrino, $\tilde{\chi}_i^+ \to \ell^+\tilde{\nu}_\ell$, definiert eine Asymmetrie des Spatprodukts $\mathcal{T}_\ell = (\mathbf{p}_{e^-} \times \mathbf{p}_{\tilde{\chi}_i^+}) \cdot \mathbf{p}_\ell$. Die Asymmetrie ist φ_μ-sensitiv und erreicht für $e^+e^- \to \tilde{\chi}_1^\pm\, \tilde{\chi}_2^\mp$ Werte bis 30%.

- Der Zerfall in ein W-Boson, $\tilde{\chi}_i^+ \to W^+\chi_n^0$, definiert die φ_μ-sensitive Asymmetrie $\mathcal{A}_I$ des Spatprodukts $\mathcal{T}_I = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\tilde{\chi}_i^+} \times \mathbf{p}_W)$. Sie kann im Prozess $e^+e^- \to \tilde{\chi}_1^\pm\, \tilde{\chi}_2^\mp$ bis zu 4% erreichen. Durch die Spinkorrelationen zwischen Chargino und W-Boson ist der Zerfall $\tilde{\chi}_i^+ \to W^+\chi_n^0$ auch sensitiv auf die

Phase φ_{M_1}. Die CP-sensitiven Dichtematrixelemente des W-Bosons wurden identifiziert, welche durch den hadronischen Zerfall $W^+ \to c\bar{s}$ untersucht werden können. Das Spatprodukt $\mathcal{T}_{II} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_c \times \mathbf{p}_{\bar{s}})$ definiert die φ_μ- und φ_{M_1}-sensitive Asymmetrie $\mathcal{A}_{II}$, welche 7% in der $\tilde{\chi}_1^+\tilde{\chi}_1^-$- und $\tilde{\chi}_1^+\tilde{\chi}_2^-$-Produktion erreicht.

Die Untersuchungen der Neutralino- und Chargino-Zerfälle haben gezeigt, dass die Asymmetrien für Zerfälle in skalare Teilchen, wie z.B. Sleptonen und Sneutrinos, ein Maß für CP-Verletzung im Produktionsprozess darstellen. Diese Asymmetrien sind sensitiv auf die Neutralino- und Chargino-Polarisation senkrecht zur Produktionsebene, welche nur für die Produktion verschiedener Neutralinos $e^+e^- \to \tilde{\chi}_i^0\,\tilde{\chi}_j^0, i \neq j$, oder Charginos $e^+e^- \to \tilde{\chi}_1^\pm\,\tilde{\chi}_2^\mp$ nicht verschwinden. Falls die Neutralinos oder Charginos in Teilchen mit Spin, also z.B. in τ's, W- oder Z-Bosonen zerfallen, erhält man aufgrund der Spinkorrelationen zusätzliche Information über die CP-Verletzung im Zerfall.

Die Ergebnisse für Sfermionzerfälle $\tilde{f} \to f\tilde{\chi}_j^0 \to f\tilde{\chi}_1^0\,Z \to f'\tilde{\chi}_1^0\,l\,\bar{l}\;(f'\tilde{\chi}_1^0\,q\,\bar{q})$ können folgendermaßen zusammengefasst werden:

- Die Spatprodukte $\mathcal{T}_\ell$ und $\mathcal{T}_q$, bestehend aus den Impulsen der auslaufenden Leptonen $\mathcal{T}_\ell = \mathbf{p}_f \cdot (\mathbf{p}_\ell \times \mathbf{p}_{\bar{\ell}})$ oder Quarks $\mathcal{T}_q = \mathbf{p}_f \cdot (\mathbf{p}_q \times \mathbf{p}_{\bar{q}})$, definieren die Asymmetrien $\mathcal{A}_{\ell,q}$, die sensitiv auf die Phasen φ_μ und φ_{M_1} sind. Für den leptonischen Stau-Zerfall $\tilde{\tau}_1 \to \tau\chi_1^0\,l\,\bar{l}$ wurde in einer numerischen Studie gezeigt, dass die Asymmetrie $\mathcal{A}_\ell$ 3% erreichen kann. Für die Messung von $\mathcal{A}_\ell$ werden etwa 10^5 Stau-Zerfälle benötigt. Solche Ereignisraten können bei zukünftigen Beschleunigerexperimenten erreicht werden. Für den semileptonischen Zerfall $\tilde{\tau}_1 \to \tilde{\chi}_1^0\,\tau\,\bar{b}\,b$ ist die Asymmetrie $\mathcal{A}_b$ um einen Faktor 6.3 größer. Zu ihrer Messung werden daher etwa nur 10^3 Staus benötigt.
- Da es sich bei dem untersuchten Prozess um einen Zwei-Körper-Zerfall eines skalaren Teilchens handelt, sind die Asymmetrien nicht auf die Phase φ_{A_f} sensitiv. Um eine solche Sensitivität zu erhalten, ist die Messung der transversalen Polarisation des Fermions f erforderlich. Alternativ können auch Drei-Körper-Zerfälle des Sfermions $\tilde{f}$ untersucht werden.

6.2 Schlussfolgerungen

Es wurde gezeigt, dass in allen betrachteten Prozessen geeignete Spatprodukte und die zugehörigen Asymmetrien definiert werden konnten, um die CP-Phasen des MSSM zu messen. Besonders geeignet sind die Asymmetrien in leptonischen Zerfällen der Neutralinos und Charginos, da Werte bis zu 30% erreicht werden. Die Asymmetrien der τ-Polarisation können sogar 60% erreichen. In den Zerfällen der Neutralinos und Charginos in Z- und W-Bosonen haben die Asymmetrien Werte von 18% bzw. 7%.

Durch eine Analyse der statistischen Signifikanz konnte gezeigt werden, dass die Asymmetrien – und damit die komplexen MSSM Phasen φ_μ, φ_{M_1} und φ_{A_τ} – bei zukünftigen Elektron-Positron-Beschleunigerexperimenten mit Strahlenergien von 500 GeV bis 800 GeV und hoher Luminosität messbar sein werden bzw. stark eingeschränkt werden können. Eine beidseitige Strahlpolarisation erhöht dabei erheblich die Asymmetrien und Wirkungsquerschnitte.

6.3 Ausblick

In weiteren Untersuchungen von CP-Asymmetrien in Produktion und Zerfall von Neutralinos und Charginos durch Elektron-Positron-Kollisionen sollte auch die transversale Strahlpolarisation einbezogen werden. In der Chargino-Produktion sind Asymmetrien von Spatprodukten, welche die transversale Strahlpolarisation beinhalten, nur dann nichtverschwindend, wenn die Zerfälle beider Teilchen berücksichtig werden.

Des Weiteren könnten aber auch Asymmetrien von Spatprodukten definiert werden, welche die Impulse der Zerfallsprodukte beider Neutralinos bzw. Charginos enthalten. Es bleibt zu klären ob dadurch eine Sensitivität auf die CP-Phasen in den Spin-Spin-Korrelationen erlangt wird, welche die Korrelationen der Polarisationen beider Neutralinos bzw. Charginos beschreiben.

Appendix A

Basics of the Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is characterized by the following properties:

- A minimal gauge group: $SU(3) \times SU(2) \times U(1)$.
- A minimal content of fields: 3 generations of leptons and quarks, 12 gauge bosons, 2 Higgs doublets, and their super partners, see Table A.1.
- Explicit SUSY breaking parametrized by soft breaking terms.
- R-parity conservation.

In the following we give a short account on the relevant Lagrangians and couplings, and mixing of neutralinos, charginos and staus.

In terms of superfields, the field content and the parameter content of the MSSM is well visible, and the Lagrangian can be written in an elegant and short way. We thus give its electroweak part in terms of superfields in Section A.1, where we also define the parameters.

For calculations of interaction amplitudes, however, it is more convenient to expand the MSSM Lagrangian in component fields, which is albeit a more complex procedure with a longish result. We thus restrict our discussions on the neutralino, chargino and stau sector, and give the mass matrices in Section A.3. In Section A.2 we give the relevant parts of the Lagrangian.

For detailed reviews of the MSSM and its Lagrangians, see e.g., [58–60]. For a detailed study of CP violating sources in the MSSM, see [61].

Table A.1: Particle spectrum of the MSSM

SM particles	SUSY-partners			
	weak eigenstates		mass eigenstates	
$q_u = u, c, t$ $q_d = d, s, b$	$\tilde{q}_L, \tilde{q}_R$	squarks	$\tilde{q}_1, \tilde{q}_2$	squarks
$\ell = e, \mu, \tau$ $\nu = \nu_e, \nu_\mu, \nu_\tau$	$\tilde{\ell}_L, \tilde{\ell}_R$ $\tilde{\nu}_\ell$	sleptons sneutrinos	$\tilde{\ell}_1, \tilde{\ell}_2$ $\tilde{\nu}_\ell$	sleptons sneutrinos
g	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^\pm$ (H_1^+, H_2^-)	$\tilde{W}^\pm$ $\tilde{H}_1^+, \tilde{H}_2^-$	wino Higgsinos	$\tilde{\chi}^\pm_{1,2}$	charginos
γ Z^0 $H_1^0, (H_2^0)$	$\tilde{\gamma}$ $\tilde{Z}^0$ $\tilde{H}_1^0, \tilde{H}_2^0$	photino zino Higgsinos	$\tilde{\chi}^0_{1,\ldots,4}$	neutralinos

A.1 MSSM Lagrangian in terms of superfields

We give the electroweak (EW) part of the MSSM Lagrangian in the superfield formalism, which consists of a supersymmetric part and terms which break SUSY softly

$$\mathcal{L}_{EW} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}. \tag{A.1}$$

A.1.1 Superfield content

The left-handed lepton superfields are arranged in $SU(2)$ doublets and the right-handed ones in $SU(2)$ singlets,

$$\hat{L} = \begin{pmatrix} \hat{\nu}_l(x,\theta,\bar{\theta}) \\ \hat{l}(x,\theta,\bar{\theta}) \end{pmatrix}_L, \qquad \hat{R} = \hat{l}_R(x,\theta,\bar{\theta}), \tag{A.2}$$

where the generation indices of e, μ and τ have been suppressed. The superfields are functions on the superspace, with spacetime coordinates x^μ and anticommuting Grassmann variables θ_α. The lepton superfield contains both bosonic and fermionic degrees of freedom. It can be expanded

$$\begin{aligned}\hat{L} \;=\; & \tilde{L}(x) + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\,\tilde{L}(x) - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^{\mu}\partial_{\mu}\,\tilde{L}(x) \;+ \\ & +\sqrt{2}\theta\,L^{(2)}(x) + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}\,L^{(2)}(x) + \theta\theta\,F_L(x),\end{aligned} \tag{A.3}$$

in terms of the spin-0 slepton field $\tilde{L}$, the spin-1/2 Weyl field $L^{(2)}$ and a spin-0 auxiliary field F_L, which however can be removed by the Euler-Lagrange equations. The right-handed field $\hat{R}$ is expanded similarly to $\hat{L}$.

There are two doublets of chiral Higgs superfields

$$\hat{H}_1 = \begin{pmatrix} \hat{H}_1^1(x,\theta,\bar{\theta}) \\ \hat{H}_1^2(x,\theta,\bar{\theta}) \end{pmatrix}, \qquad \hat{H}_2 = \begin{pmatrix} \hat{H}_2^1(x,\theta,\bar{\theta}) \\ \hat{H}_2^2(x,\theta,\bar{\theta}) \end{pmatrix}, \tag{A.4}$$

where the upper index denotes the $SU(2)$ index. The component field expansions of the Higgs fields is similar to that of the lepton field $\hat{L}$ (A.3).

The $U(1)$ and $SU(2)$ gauge vector superfields are respectively given by

$$\hat{V}' = \mathbf{Y}\;\hat{v}'(x,\theta,\bar{\theta}), \qquad \hat{V} = \mathbf{T}^a\;\hat{V}^a(x,\theta,\bar{\theta}), \tag{A.5}$$

with sum over $a = 1,2,3$ and $\mathbf{Y}$ and $\mathbf{T}^a$ are the $U(1)$ and $SU(2)$ generators. The gauge vector superfield contains a bosonic (spin 1) gauge field V_μ^a, and a fermionic (spin 1/2) gaugino Weyl field λ^a

$$\hat{V}^a(x,\theta,\bar{\theta}) \;=\; -\,\theta\sigma^{\mu}\bar{\theta}\;V_\mu^a(x) + i\theta\theta\bar{\theta}\;\bar{\lambda}^a(x) - i\bar{\theta}\bar{\theta}\theta\;\lambda^a(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\;D^a(x). \tag{A.6}$$

The auxiliary spin-1/2 field D^a can be removed by the Euler-Lagrange equations. The gauge field $\hat{v}'$ is expanded similarly to $\hat{V}^a$.

A.1.2 The supersymmetric Lagrangian

The supersymmetric Lagrangian contains the kinetic terms for chiral and vector superfields, as well as a Higgs part

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Lepton} + \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs}. \tag{A.7}$$

The lepton and gauge parts are given by

$$\mathcal{L}_{Lepton} = \int d^4\theta \, [\hat{L}^\dagger \, e^{2g\hat{V}+g'\hat{V}'} \hat{L} + \hat{R}^\dagger \, e^{2g\hat{V}+g'\hat{V}'} \hat{R}], \tag{A.8}$$

$$\mathcal{L}_{Gauge} = \frac{1}{4} \int d^4\theta \, [W^{a\alpha} W^a_\alpha + W'^\alpha W'_\alpha] \, \delta^2(\bar{\theta}) + h.c., \tag{A.9}$$

where g and g' are the $SU(2)$ and $U(1)$ gauge couplings. The $SU(2)$ and $U(1)$ field strengths are defined by

$$W_\alpha = -\frac{1}{8g} \bar{D}\bar{D} e^{-2g\hat{V}} D_\alpha e^{2g\hat{V}}, \qquad W'_\alpha = -\frac{1}{4} DD\bar{D}_\alpha \hat{V}', \tag{A.10}$$

where D are covariant derivatives. The Higgs part

$$\mathcal{L}_{Higgs} = \int d^4\theta \left[\hat{H}_1^\dagger e^{2g\hat{V}} + g'\hat{V}'\hat{H}_1 + \hat{H}_2^\dagger e^{2g\hat{V}} + g'\hat{V}'\hat{H}_2 + W\delta^2(\bar{\theta}) + \bar{W}\delta^2(\theta) \right] \tag{A.11}$$

contains the superpotential

$$W = W_H + W_Y. \tag{A.12}$$

The Higgs (H) and Yukawa (Y) parts are

$$W_H = \mu \, \epsilon^{ij} \hat{H}_1^i \hat{H}_2^j, \qquad W_Y = \epsilon^{ij} [f \hat{H}_1^i \hat{L}^j \hat{R} + f_1 \hat{H}_1^i \hat{Q}^j \hat{D} + f_2 \hat{H}_2^j \hat{Q}^i \hat{U}], \tag{A.13}$$

with μ the Higgsino mass parameter, f_i the Yukawa couplings, with the generation index suppressed, and the antisymmetric tensor $\epsilon^{11} = \epsilon^{22} = 0$, $\epsilon^{12} = -\epsilon^{21} = 1$. Note that in order to be renormalizable, the superpotential can only be cubic in the superfields at its maximum.

By construction, the Lagrangians given in this section are gauge invariant and invariant under supersymmetry transformations. In addition, they are R-parity conserving, if trilinear terms in $W|_{\theta=0}$ are disregarded. The Higgsino mass parameter μ and the Yukawa couplings f can have physical CP-phases.

A.1.3 The soft SUSY breaking Lagrangian

The most general Lagrangian, which breaks SUSY softly, i.e., which does not lead to quadratical divergencies, can be classified in scalar mass terms (S) and gaugino mass terms (G) [62]:

$$\mathcal{L}_{soft} = \mathcal{L}_S + \mathcal{L}_G. \tag{A.14}$$

Note that trilinear scalar interaction terms are not R-invariant and will thus be neglected. The scalar mass term reads (neglecting squark fields)

$$\begin{aligned}\mathcal{L}_S \ = \ & -\int d^4\theta \left[M_L^2 \hat{L}^\dagger \hat{L} + m_R^2 \hat{R}^\dagger \hat{R} + m_1^2 \hat{H}_1^\dagger \hat{H}_1 + m_2^2 \hat{H}_2^\dagger \hat{H}_2 + \right. \\ & \left. + m_3^2 \epsilon^{ij} (\hat{H}_1^i \hat{H}_2^j + h.c.)\right] \delta^4(\theta, \bar{\theta}),\end{aligned} \tag{A.15}$$

with the abbreviation $M_L^2 \hat{L}^\dagger \hat{L} = m_{\tilde{\nu}}^2 \hat{\nu}^\dagger + m_L^2 \hat{l}_L^\dagger \hat{l}_L$. The gaugino mass term

$$\mathcal{L}_G = \frac{1}{2} \int d^4\theta \left[(M_1 W'^\alpha W'_\alpha + M_2 W^{a\alpha} W^a_\alpha) + h.c.\right] \delta^4(\theta, \bar{\theta}) \tag{A.16}$$

includes the $U(1)$ and $SU(2)$ gaugino mass parameters M_1 and M_2, respectively. By redefining the fields, one parameter, usually M_2, can be made real, while M_1 can have a CP violating phase.

A.2 MSSM Lagrangian in component fields

The MSSM Lagrangians relevant for chargino production and decay are [58]:

$$\mathcal{L}_{Z^0 \ell \bar{\ell}} = -\frac{g}{\cos\theta_W} Z_\mu \bar{\ell} \gamma^\mu [L_\ell P_L + R_\ell P_R] \ell, \tag{A.17}$$

$$\mathcal{L}_{\gamma \tilde{\chi}_j^+ \tilde{\chi}_i^-} = -e A_\mu \bar{\tilde{\chi}}_i^+ \gamma^\mu \tilde{\chi}_j^+ \delta_{ij}, \quad e > 0, \tag{A.18}$$

$$\mathcal{L}_{Z^0 \tilde{\chi}_j^+ \tilde{\chi}_i^-} = \frac{g}{\cos\theta_W} Z_\mu \bar{\tilde{\chi}}_i^+ \gamma^\mu [O'^L_{ij} P_L + O'^R_{ij} P_R] \tilde{\chi}_j^+, \tag{A.19}$$

$$\mathcal{L}_{W^- \tilde{\chi}_i^+ \tilde{\chi}_k^0} = g W_\mu^- \bar{\tilde{\chi}}_k^0 \gamma^\mu [O^L_{ki} P_L + O^R_{ki} P_R] \tilde{\chi}_i^+ + \text{h.c.}, \tag{A.20}$$

$$\mathcal{L}_{\ell \tilde{\nu} \tilde{\chi}_i^+} = -g U_{i1}^* \bar{\tilde{\chi}}_i^+ P_L \nu \tilde{\ell}_L^* - g V_{i1}^* \bar{\tilde{\chi}}_i^{+C} P_L \ell \tilde{\nu}^* + \text{h.c.}, \quad \ell = e, \mu, \tag{A.21}$$

$$\mathcal{L}_{\tau \tilde{\nu}_\tau \tilde{\chi}_i^+} = -g \bar{\tilde{\chi}}_i^{+C} (V_{i1}^* P_L - Y_\tau U_{i2} P_R) \tau \tilde{\nu}_\tau^* + \text{h.c.}, \tag{A.22}$$

Table A.2: Electric charge e_ℓ and weak isospin $T_{3\ell}$ of fermions

	d	u	e_L	e_R	ν
e_ℓ	$-\frac{1}{3}$	$+\frac{2}{3}$	-1	-1	0
$T_{3\ell}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$

and the terms relevant also for neutralino production and decay are [58]:

$$\mathcal{L}_{Z^0\tilde{\chi}^0_m\tilde{\chi}^0_n} = \frac{1}{2}\frac{g}{\cos\theta_W}Z_\mu\bar{\tilde{\chi}}^0_m\gamma^\mu[O''^L_{mn}P_L + O''^R_{mn}P_R]\tilde{\chi}^0_n, \tag{A.23}$$

$$\mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}^0_k} = gf^L_{\ell k}\bar{\ell}P_R\tilde{\chi}^0_k\tilde{\ell}_L + gf^R_{\ell k}\bar{\ell}P_L\tilde{\chi}^0_k\tilde{\ell}_R + \text{h.c.}, \tag{A.24}$$

$$\mathcal{L}_{\nu\tilde{\nu}\tilde{\chi}^0_k} = gf^L_{\nu k}\bar{\nu}P_R\tilde{\chi}^0_k\tilde{\nu}_L + \text{h.c.}, \tag{A.25}$$

with the couplings

$$L_\ell = T_{3\ell} - e_\ell\sin^2\theta_W, \quad R_\ell = -e_\ell\sin^2\theta_W, \tag{A.26}$$

$$O'^L_{ij} = -V_{i1}V^*_{j1} - \frac{1}{2}V_{i2}V^*_{j2} + \delta_{ij}\sin^2\theta_W, \tag{A.27}$$

$$O'^R_{ij} = -U^*_{i1}U_{j1} - \frac{1}{2}U^*_{i2}U_{j2} + \delta_{ij}\sin^2\theta_W, \tag{A.28}$$

$$O^L_{ki} = -1/\sqrt{2}\Big(\cos\beta N_{k4} - \sin\beta N_{k3}\Big)V^*_{i2} + \Big(\sin\theta_W N_{k1} + \cos\theta_W N_{k2}\Big)V^*_{i1}, \tag{A.29}$$

$$O^R_{ki} = +1/\sqrt{2}\Big(\sin\beta N^*_{k4} + \cos\beta N^*_{k3}\Big)U_{i2} + \Big(\sin\theta_W N^*_{k1} + \cos\theta_W N^*_{k2}\Big)U_{i1}, \tag{A.30}$$

$$O''^L_{mn} = -\frac{1}{2}(N_{m3}N^*_{n3} - N_{m4}N^*_{n4})\cos 2\beta - \frac{1}{2}(N_{m3}N^*_{n4} + N_{m4}N^*_{n3})\sin 2\beta, \tag{A.31}$$

$$O''^R_{mn} = -O''^{L*}_{mn}, \tag{A.32}$$

$$f^L_{\ell k} = -\sqrt{2}\Big[\frac{1}{\cos\theta_W}(T_{3\ell} - e_\ell\sin^2\theta_W)N_{k2} + e_\ell\sin\theta_W N_{k1}\Big], \tag{A.33}$$

$$f^R_{\ell k} = -\sqrt{2}e_\ell\sin\theta_W\Big[\tan\theta_W N^*_{k2} - N^*_{k1}\Big], \tag{A.34}$$

$$f^L_{\nu k} = -\sqrt{2}\frac{1}{\cos\theta_W}T_{3\nu}N_{k2}, \tag{A.35}$$

with $i,j = 1,2$ and $k,m,n = 1,\ldots,4$. The charge e_ℓ and the third component of the weak isospin $T_{3\ell}$ of each fermion is given in Tab. (A.2). The τ-Yukawa coupling is given by $Y_\tau = m_\tau/(\sqrt{2}m_W\cos\beta)$.

For the neutralino decay into staus $\tilde{\chi}^0_i \to \tilde{\tau}_k \tau$, we take stau mixing into account and write for the Lagrangian [63]:

$$\mathcal{L}_{\tau\tilde{\tau}\chi_i} = g\tilde{\tau}_k \bar{\tau}(a^{\tilde{\tau}}_{ki} P_R + b^{\tilde{\tau}}_{ki} P_L)\chi^0_i + \text{h.c.}\,, \quad k = 1,2;\; i = 1,\ldots,4, \tag{A.36}$$

with

$$a^{\tilde{\tau}}_{kj} = (\mathcal{R}^{\tilde{\tau}}_{kn})^* \mathcal{A}^{\tau}_{jn}, \quad b^{\tilde{\tau}}_{kj} = (\mathcal{R}^{\tilde{\tau}}_{kn})^* \mathcal{B}^{\tau}_{jn}, \quad \ell^{\tilde{\tau}}_{kj} = (\mathcal{R}^{\tilde{\tau}}_{kn})^* \mathcal{O}^{\tau}_{jn}, \quad (n = L, R), \tag{A.37}$$

$$\mathcal{A}^{\tau}_j = \begin{pmatrix} f^L_{\tau j} \\ h^R_{\tau j} \end{pmatrix}, \quad \mathcal{B}^{\tau}_j = \begin{pmatrix} h^L_{\tau j} \\ f^R_{\tau j} \end{pmatrix}, \quad \mathcal{O}^{\tau}_j = \begin{pmatrix} -U_{j1} \\ Y_\tau U_{j2} \end{pmatrix}, \tag{A.38}$$

$$h^L_{\tau j} = (h^R_{\tau j})^* = -Y_\tau (N^*_{j3}\cos\beta + N^*_{j4}\sin\beta), \tag{A.39}$$

$$Y_\tau = m_\tau/(\sqrt{2} m_W \cos\beta), \tag{A.40}$$

with $\mathcal{R}^{\tilde{\tau}}_{kn}$ given in (A.50) and $f^{L,R}_{\tau j}$ given in (A.33), (A.34).

A.3 Mass matrices

A.3.1 Neutralino mass matrix

The complex symmetric mass matrix of the neutral gauginos and Higgsinos in the basis $(\tilde{\gamma}, \tilde{Z}, \tilde{H}^0_a, \tilde{H}^0_b)$ is given by

$$Y = \begin{pmatrix} M_2\sin^2\theta_W + M_1\cos^2\theta_W & (M_2 - M_1)\sin\theta_W\cos\theta_W & 0 & 0 \\ (M_2 - M_1)\sin\theta_W\cos\theta_W & M_2\cos^2\theta_W + M_1\sin^2\theta_W & m_Z & 0 \\ 0 & m_Z & \mu\sin 2\beta & -\mu\cos 2\beta \\ 0 & 0 & -\mu\cos 2\beta & -\mu\sin 2\beta \end{pmatrix}, \tag{A.41}$$

with μ the Higgsino mass parameter and $\tan\beta = \frac{v_2}{v_1}$, where $v_{1,2}$ are the vacuum expectation values of the two neutral Higgs fields. The mass matrix $Y_{\alpha\beta}$ (A.41) can be diagonalized by a complex, unitary 4×4 matrix N_{ij} [58],

$$N^*_{i\alpha}\, Y_{\alpha\beta}\, N^\dagger_{\beta k} = m_{\tilde{\chi}^0_i}\, \delta_{ik}, \tag{A.42}$$

with the neutralino masses $m_{\tilde{\chi}_i^0} > 0$. Then the weak eigenstates $(\tilde{\gamma}, \tilde{Z}, \tilde{H}_a^0, \tilde{H}_b^0)$ mix to the neutralino mass eigenstates $(\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0)$.

The diagonalization of the neutralino matrix is achieved with the singular value decomposition [61]. Let Z be a complex $n \times n$ matrix, then:

- The matrices $ZZ^\dagger$ and $Z^\dagger Z$ are selfadjoint and have the same real eigenvalues $\lambda_i \geq 0$.
- The eigenvectors $\hat{e}_i$ connected to the eigenvalues λ_i built an orthonormal system. If some of the $\lambda_i = 0$, the eigenvectors can be completed to built an orthonormal system.
- Let $\hat{e}_i$ be an eigenvector of $Z^\dagger Z$ with eigenvalue $\lambda_i \neq 0$, that is $Z^\dagger Z\, \hat{e}_i = \lambda_i\, \hat{e}_i$, then the vectors $\hat{e}'_i := \frac{1}{\sqrt{\lambda_i}} Z\, \hat{e}_i$ also built an orthonormal system with $Z^\dagger\, \hat{e}'_i = \sqrt{\lambda_i}\, \hat{e}_i$.

Thus Z can be decomposed into its singular values: $\tilde{Z} = (\hat{e}'_1 \ldots \hat{e}'_2)^\dagger Z (\hat{e}_1 \ldots \hat{e}_2) = diag(\sqrt{\lambda_i})$.

A.3.2 Chargino mass matrix

The complex chargino mass matrix is given by

$$X = \begin{pmatrix} M_2 & m_W\sqrt{2}\sin\beta \\ m_W\sqrt{2}\cos\beta & \mu \end{pmatrix}. \tag{A.43}$$

It can be diagonalized by two complex unitary 2×2 matrices U_{mn} and V_{mn} [58],

$$U^*_{m\alpha}\, X_{\alpha\beta}\, V^{-1}_{\beta n} = m_{\tilde{\chi}_i^+}\, \delta_{mn}, \tag{A.44}$$

with the chargino masses $m_{\tilde{\chi}_i^+} > 0$. The chargino-mass eigenstates $\tilde{\chi}_i^+ = \binom{\chi_i^+}{\bar{\chi}_i^-}$ are defined by $\chi_i^+ = V_{i1} w^+ + V_{i2} h^+$ and $\chi_j^- = U_{j1} w^- + U_{j2} h^-$ with $w^\pm$ and $h^\pm$ the two-component spinor fields of the W-ino and the charged Higgsinos, respectively. We diagonalize the chargino mass matrix with the singular value decomposition, see Section A.3.1.

A.3.3 Stau mass matrix

The masses and couplings of the τ-sleptons follow from the hermitian 2×2 $\tilde{\tau}_L - \tilde{\tau}_R$ mass matrix [63]:

$$\mathcal{L}_M^{\tilde{\tau}} = -(\tilde{\tau}_L^*,\, \tilde{\tau}_R^*) \begin{pmatrix} M^2_{\tilde{\tau}_{LL}} & e^{-i\varphi_{\tilde{\tau}}}|M^2_{\tilde{\tau}_{LR}}| \\ e^{i\varphi_{\tilde{\tau}}}|M^2_{\tilde{\tau}_{LR}}| & M^2_{\tilde{\tau}_{RR}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}, \tag{A.45}$$

with

$$M^2_{\tilde{\tau}_{LL}} = M^2_{\tilde{L}} + (-\frac{1}{2} + \sin^2\theta_W)\cos 2\beta\, m_Z^2 + m_\tau^2, \tag{A.46}$$

$$M^2_{\tilde{\tau}_{RR}} = M^2_{\tilde{E}} - \sin^2\theta_W \cos 2\beta\, m_Z^2 + m_\tau^2, \tag{A.47}$$

$$M^2_{\tilde{\tau}_{RL}} = (M^2_{\tilde{\tau}_{LR}})^* = m_\tau(A_\tau - \mu^* \tan\beta), \tag{A.48}$$

$$\varphi_{\tilde{\tau}} = \arg[A_\tau - \mu^* \tan\beta], \tag{A.49}$$

where A_τ is the complex trilinear scalar coupling parameter and $M_{\tilde{L}}$, $M_{\tilde{E}}$ are the other soft SUSY–breaking parameters of the $\tilde{\tau}_i$ system. In order to reduce the number of MSSM parameters, we will often use the renormalization group equations [64], $M^2_{\tilde{L}} = m_0^2 + 0.79M_2^2$ and $M^2_{\tilde{E}} = m_0^2 + 0.23M_2^2$. The $\tilde{\tau}$ mass eigenstates are $(\tilde{\tau}_1, \tilde{\tau}_2) = (\tilde{\tau}_L, \tilde{\tau}_R)\mathcal{R}^{\tilde{\tau}^T}$, with

$$\mathcal{R}^{\tilde{\tau}} = \begin{pmatrix} e^{i\varphi_{\tilde{\tau}}}\cos\theta_{\tilde{\tau}} & \sin\theta_{\tilde{\tau}} \\ -\sin\theta_{\tilde{\tau}} & e^{-i\varphi_{\tilde{\tau}}}\cos\theta_{\tilde{\tau}} \end{pmatrix}, \tag{A.50}$$

with the mixing angle

$$\cos\theta_{\tilde{\tau}} = \frac{-|M^2_{\tilde{\tau}_{LR}}|}{\sqrt{|M^2_{\tilde{\tau}_{LR}}|^2 + (m^2_{\tilde{\tau}_1} - M^2_{\tilde{\tau}_{LL}})^2}}, \quad \sin\theta_{\tilde{\tau}} = \frac{M^2_{\tilde{\tau}_{LL}} - m^2_{\tilde{\tau}_1}}{\sqrt{|M^2_{\tilde{\tau}_{LR}}|^2 + (m^2_{\tilde{\tau}_1} - M^2_{\tilde{\tau}_{LL}})^2}}, \tag{A.51}$$

and the mass eigenvalues

$$m^2_{\tilde{\tau}_{1,2}} = \frac{1}{2}\left[(M^2_{\tilde{\tau}_{LL}} + M^2_{\tilde{\tau}_{RR}}) \mp \sqrt{(M^2_{\tilde{\tau}_{LL}} - M^2_{\tilde{\tau}_{RR}})^2 + 4|M^2_{\tilde{\tau}_{LR}}|^2}\,\right]. \tag{A.52}$$

A.3.4 First and second generation sfermion masses

The off-diagonal terms of the sfermion mass matrices are proportional to the fermion mass. For fermions of the first and second generation, whose masses are small compared to SUSY masses, their mass matrices are diagonal to a good approximation. For these sfermions we will assume the approximate solutions to the renormalization group equations [64]:

$$m^2_{\tilde{f}_{L,R}} = m_f^2 + m_0^2 + C(\tilde{f})M_2^2 \pm m_Z^2 \cos 2\beta (T_{3f} - e_f \sin^2\theta_W). \tag{A.53}$$

where T_{3f} is the third component of weak isospin, m_0 is the common scalar mass parameter at the GUT-scale, and $C(\tilde{f})$ depends on the sfermion

$$C(\tilde{\ell}_L) \approx 0.79, \qquad C(\tilde{\ell}_R) \approx 0.23, \tag{A.54}$$

$$C(\tilde{q}_L) \approx 10.8, \qquad C(\tilde{q}_R) \approx 10.1. \tag{A.55}$$

For the slepton and sneutrino masses we have

$$m^2_{\tilde{\ell}_R} = m_0^2 + 0.23 M_2^2 - m_Z^2 \cos 2\beta \sin^2\theta_W, \tag{A.56}$$

$$m^2_{\tilde{\ell}_L} = m_0^2 + 0.79 M_2^2 + m_Z^2 \cos 2\beta (-\frac{1}{2} + \sin^2\theta_W), \tag{A.57}$$

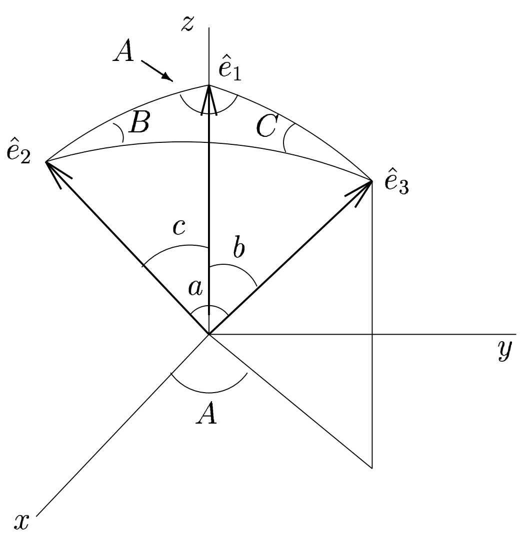
$$m^2_{\tilde{\nu}_\ell} = m_0^2 + 0.79 M_2^2 + \frac{1}{2} m_Z^2 \cos 2\beta. \tag{A.58}$$

Appendix B

Kinematics and phase space

B.1 Spherical trigonometry

For the parametrization of the phase space one often needs the following relations from spherical trigonometry. Consider the following spherical triangle with sides a, b, c and angles A, B, C.



The unit vectors of the triangle sides are given by

$$\hat{e}_1 = (0, 0, 1), \tag{B.1}$$

$$\begin{aligned} \hat{e}_2 &= (\sin c, 0, \cos c), && \text{(B.2)} \\ \hat{e}_3 &= (\sin b \cos A, \sin b \sin A, \cos b). && \text{(B.3)} \end{aligned}$$

In the following we give formulas relating the sides and the angles of the triangle [65, 66]:

- law of sines

  $$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}. \tag{B.4}$$

- law of cosines for sides (cosine theorem)

  $$\cos a = \cos b \cos c + \sin b \sin c \cos A. \tag{B.5}$$

  Similar formulae for the other sides may be obtained by cyclical permutations.

- law of cosines for angles

  $$\cos A = -\cos B \cos C + \sin B \sin C \cos a, \tag{B.6}$$

  etc, cyclically.

- products of functions from sides and angles

  $$\begin{aligned} \sin a \cos B &= \cos b \sin c - \sin b \cos c \cos A, && \text{(B.7)} \\ \sin a \cos C &= \cos c \sin b - \sin c \cos b \cos A. && \text{(B.8)} \end{aligned}$$

  The products $\sin b \cos C$ and $\sin c \cos A$ are obtained from (B.7) by cyclical permutations. The products $\sin b \cos A$ and $\sin c \cos B$ are obtained from (B.8) by cyclical permutations.

  $$\begin{aligned} \sin A \cos b &= \cos B \sin C + \sin B \cos C \cos a, && \text{(B.9)} \\ \sin A \cos c &= \cos C \sin B + \sin C \cos B \cos a. && \text{(B.10)} \end{aligned}$$

  The products $\sin B \cos c$ and $\sin C \cos a$ are obtained from (B.9) by cyclical permutations. The products $\sin B \cos a$ and $\sin C \cos b$ are obtained from (B.10) by cyclical permutations.

B.2 Kinematics of neutralino/chargino production and decay

B.2.1 Momenta and spin vectors of the production process

We choose a coordinate frame in the laboratory system (center of mass system) such that the momentum of the neutralino $\tilde{\chi}^0_j$ or chargino $\tilde{\chi}^-_j$, denoted by $\mathbf{p}_{\chi_j}$, points in the z-direction (in our definitions we follow closely [39, 51]). The scattering angle is $\theta\angle(\mathbf{p}_{e^-}, \mathbf{p}_{\chi_j})$ and the azimuth ϕ can be chosen zero. The momenta are

$$p^\mu_{e^-} = E_b(1, -\sin\theta, 0, \cos\theta), \quad p^\mu_{e^+} = E_b(1, \sin\theta, 0, -\cos\theta), \tag{B.11}$$

$$p^\mu_{\chi_i} = (E_{\chi_i}, 0, 0, -q), \quad p^\mu_{\chi_j} = (E_{\chi_j}, 0, 0, q), \tag{B.12}$$

with the beam energy $E_b = \sqrt{s}/2$ and

$$E_{\chi_i} = \frac{s + m^2_{\chi_i} - m^2_{\chi_j}}{2\sqrt{s}}, \quad E_{\chi_j} = \frac{s + m^2_{\chi_j} - m^2_{\chi_i}}{2\sqrt{s}}, \quad q = \frac{\lambda^{\frac{1}{2}}(s, m^2_{\chi_i}, m^2_{\chi_j})}{2\sqrt{s}}, \tag{B.13}$$

where $\lambda(x, y, z) = x^2+y^2+z^2-2(xy+xz+yz)$. For the description of the polarization of the neutralino $\tilde{\chi}^0_i$ or chargino $\tilde{\chi}^+_i$ we choose three spin vectors

$$s^{1,\mu}_{\chi_i} = (0, -1, 0, 0), \quad s^{2,\mu}_{\chi_i} = (0, 0, 1, 0), \quad s^{3,\mu}_{\chi_i} = \frac{1}{m_{\chi_i}}(q, 0, 0, -E_{\chi_i}). \tag{B.14}$$

Together with $p^\mu_{\chi_i}/m_{\chi_i}$ they form an orthonormal set

$$s^a_{\chi_i} \cdot s^b_{\chi_i} = -\delta^{ab}, \quad s^a_{\chi_i} \cdot p_{\chi_i} = 0. \tag{B.15}$$

B.2.2 Momenta and spin vectors of leptonic decays

If the neutralino or chargino decays into a lepton, $\tilde{\chi}^0_i \to \ell_1 \tilde{\ell}$ or $\tilde{\chi}^+_i \to \ell_1 \tilde{\nu}_\ell$, in short $\tilde{\chi}_i \to \ell_1 \tilde{\xi}$, it is suitable to parametrize in terms of the angle $\theta_1 = \angle(\mathbf{p}_{\ell_1}, \mathbf{p}_{\chi_i})$:

$$p^\mu_{\ell_1} = (E_{\ell_1}, -|\mathbf{p}_{\ell_1}|\sin\theta_1\cos\phi_1, |\mathbf{p}_{\ell_1}|\sin\theta_1\sin\phi_1, -|\mathbf{p}_{\ell_1}|\cos\theta_1), \tag{B.16}$$

$$E_{\ell_1} = |\mathbf{p}_{\ell_1}| = \frac{m^2_{\chi_i} - m^2_{\tilde{\xi}}}{2(E_{\chi_i} - q\cos\theta_1)}. \tag{B.17}$$

For the subsequent slepton decay $\tilde{\ell} \to \ell_2 \tilde{\chi}_1^0$ we define $\theta_2 = \angle(\mathbf{p}_{\ell_2}, \mathbf{p}_{\chi_i})$ and write

$$p^\mu_{\ell_2} = (E_{\ell_2}, -|\mathbf{p}_{\ell_2}| \sin\theta_2 \cos\phi_2, |\mathbf{p}_{\ell_2}| \sin\theta_2 \sin\phi_2, -|\mathbf{p}_{\ell_2}| \cos\theta_2), \tag{B.18}$$

$$E_{\ell_2} = |\mathbf{p}_{\ell_2}| = \frac{m^2_{\tilde{\ell}} - m^2_{\chi^0_1}}{2(E_{\tilde{\ell}} - |\mathbf{p}_{\chi_i} - \mathbf{p}_{\ell_1}| \cos\theta_{D_2})}, \tag{B.19}$$

with $\theta_2 = \angle(\mathbf{p}_{\ell_2}, \mathbf{p}_{\chi_i})$, the decay angles $\theta_{D_2}\angle(\mathbf{p}_{\tilde{\ell}}, \mathbf{p}_{\ell_2})$, $\theta_{D_1}\angle(\mathbf{p}_{\chi_i}, \mathbf{p}_{\tilde{\ell}})$ and

$$\cos\theta_{D_2} = \cos\theta_{D_1}\cos\theta_2 - \sin\theta_{D_1}\sin\theta_2\cos(\phi_2 - \phi_1), \quad \cos\theta_{D_1} = \frac{\mathbf{p}_{\chi_i}(\mathbf{p}_{\chi_i} - \mathbf{p}_{\ell_1})}{|\mathbf{p}_{\chi_i}|\,|\mathbf{p}_{\chi_i} - \mathbf{p}_{\ell_1}|}. \tag{B.20}$$

If the neutralino decays into a stau, $\tilde{\chi}^0_i \to \tilde{\tau}_m \tau$, $m = 1, 2$, the τ spin vectors are chosen by

$$s^1_\tau = \left(0, \frac{\mathbf{s}_2 \times \mathbf{s}_3}{|\mathbf{s}_2 \times \mathbf{s}_3|}\right), \quad s^2_\tau = \left(0, \frac{\mathbf{p}_\tau \times \mathbf{p}_{e^-}}{|\mathbf{p}_\tau \times \mathbf{p}_{e^-}|}\right), \quad s^3_\tau = \frac{1}{m_\tau}\left(|\mathbf{p}_\tau|, \frac{E_\tau}{|\mathbf{p}_\tau|}\mathbf{p}_\tau\right). \tag{B.21}$$

B.2.3 Phase space for leptonic decays

For neutralino/chargino production $e^+e^- \to \tilde{\chi}_i \tilde{\chi}_j$ and subsequent leptonic decay $\tilde{\chi}^0_i \to \ell_1 \tilde{\ell}$ or $\tilde{\chi}^+_i \to \ell_1 \tilde{\nu}_\ell$, in short $\tilde{\chi}_i \to \ell_1 \tilde{\xi}$, the Lorentz invariant phase-space element can be decomposed into two-body phase-space elements [65]:

$$d\mathrm{Lips}(s; p_{\chi_j}, p_{\ell_1}, p_{\tilde{\xi}}) = \frac{1}{2\pi}\, d\mathrm{Lips}(s; p_{\chi_i}, p_{\chi_j})\; ds_{\chi_i}\; d\mathrm{Lips}(s_{\chi_i}; p_{\ell_1}, p_{\tilde{\xi}}). \tag{B.22}$$

For $\tilde{\xi} = \tilde{\ell}$, we can include the subsequent selectron decay $\tilde{\ell} \to \ell_2 \tilde{\chi}^0_1$ and have for the complete process $e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j$; $\tilde{\chi}^0_i \to \ell_1 \tilde{\ell}$; $\tilde{\ell} \to \ell_2 \tilde{\chi}^0_1$:

$$d\mathrm{Lips}(s; p_{\chi_j}, p_{\ell_1}, p_{\ell_2}, p_{\chi^0_1}) = \frac{1}{2\pi}\, d\mathrm{Lips}(s; p_{\chi_j}, p_{\ell_1}, p_{\tilde{\ell}})\; ds_{\tilde{\ell}}\; d\mathrm{Lips}(s_{\tilde{\ell}}; p_{\ell_2}, p_{\chi^0_1},). \tag{B.23}$$

The several parts of the phase space elements are

$$d\mathrm{Lips}(s; p_{\chi_i}, p_{\chi_j}) = \frac{q}{8\pi\sqrt{s}}\, \sin\theta\, d\theta, \tag{B.24}$$

$$d\mathrm{Lips}(s_{\chi_i}; p_{\tilde{\xi}}, p_{\ell_1}) = \frac{1}{2(2\pi)^2}\, \frac{|\mathbf{p}_{\ell_1}|^2}{m^2_{\chi_i} - m^2_{\tilde{\xi}}}\, d\Omega_1, \tag{B.25}$$

$$d\mathrm{Lips}(s_{\tilde{\ell}}; p_{\ell_2}, p_{\chi^0_1}) = \frac{1}{2(2\pi)^2}\, \frac{|\mathbf{p}_{\ell_2}|^2}{m^2_{\tilde{\xi}} - m^2_{\chi_1}}\, d\Omega_2, \tag{B.26}$$

with $s_{\chi_i} = p^2_{\chi_i}$, $s_{\tilde{\xi}} = p^2_{\tilde{\xi}}$ and $d\Omega_i = \sin\theta_i \; d\theta_i \; d\phi_i$. We use the narrow width approximation for the propagators $\int |\Delta(\tilde{\chi}_i)|^2 \, ds_{\chi_i} = \frac{\pi}{m_{\chi_i}\Gamma_{\chi_i}}$, $\int |\Delta(\tilde{\ell})|^2 ds_{\tilde{\ell}} = \frac{\pi}{m_{\tilde{\ell}}\Gamma_{\tilde{\ell}}}$. The approximation is justified for $(\Gamma_{\chi_i}/m_{\chi_i})^2 \ll 1$, and $(\Gamma_{\tilde{\ell}}/m_{\tilde{\ell}})^2 \ll 1$, which holds in our case with $\Gamma_{\chi_i} \lesssim \mathcal{O}(1\mathrm{GeV})$ and $\Gamma_{\tilde{\ell}} \lesssim \mathcal{O}(1\mathrm{GeV})$.

B.2.4 Energy distributions of the decay leptons

For neutralino production $e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j$ and the subsequent leptonic decay $\tilde{\chi}^0_i \to \ell_1 \tilde{\ell}$, and $\tilde{\ell} \to \ell_2 \tilde{\chi}^0_1$, the two decay leptons ℓ_1 and ℓ_2 can be distinguished by their different energy distributions. The energy distribution of lepton ℓ_1 in the laboratory system has the form of a box with the endpoints

$$E_{\ell_1,min,max} = \frac{m^2_{\chi^0_i} - m^2_{\tilde{\ell}}}{2(E_{\chi^0_i} \pm q)}, \tag{B.27}$$

with q the neutralino momentum. The energy distribution of the second lepton ℓ_2 is obtained by integrating over the energy $E_{\tilde{\ell}}$ of the decaying slepton

$$\frac{1}{\sigma}\frac{d\sigma}{dE_{\ell_2}} = \frac{m^2_{\tilde{\ell}}\, m^2_{\chi^0_i}}{q[m^2_{\chi^0_i} - m^2_{\tilde{\ell}}][m^2_{\tilde{\ell}} - m^2_{\chi^0_1}]} \times \begin{cases} ln\frac{E_{\ell_2}}{A} & ; \quad A \le E_{\ell_2} \le a \\ ln\frac{a}{A} & ; \quad a \le E_{\ell_2} \le b \\ ln\frac{B}{E_{\ell_2}} & ; \quad b \le E_{\ell_2} \le B \end{cases} \tag{B.28}$$

with

$$A, B = \frac{m^2_{\tilde{\ell}} - m^2_{\chi^0_1}}{2m^2_{\tilde{\ell}}}\left(E_{\tilde{\ell},max} \mp \sqrt{E^2_{\tilde{\ell},max} - m^2_{\tilde{\ell}}}\right) \tag{B.29}$$

$$a, b = \frac{m^2_{\tilde{\ell}} - m^2_{\chi^0_1}}{2m^2_{\tilde{\ell}}}\left(E_{\tilde{\ell},min} \mp \sqrt{E^2_{\tilde{\ell},min} - m^2_{\tilde{\ell}}}\right) \tag{B.30}$$

$$E_{\tilde{\ell},max,min} = \frac{E_{\chi^0_i}(m^2_{\chi^0_i} + m^2_{\tilde{\ell}}) \pm (m^2_{\chi^0_i} - m^2_{\tilde{\ell}})\sqrt{E^2_{\chi^0_i} - m^2_{\chi^0_i}}}{2m^2_{\chi^0_i}}. \tag{B.31}$$

B.2.5 Momenta and spin vectors of bosonic decays

For the bosonic two-body decays of neutralino $\tilde{\chi}_i^0 \to Z^0\tilde{\chi}_n^0$ or chargino $\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0$, in short $\tilde{\chi}_i \to B\tilde{\chi}_n^0$, we define the decay angle between neutralino (or chargino) and the boson B as $\theta_1 \angle(\mathbf{p}_{\chi_i}, \mathbf{p}_B)$. The angle is constrained by $\sin\theta_1^{\max} = q'/q$ for $q > q'$, where $q' = \lambda^{\frac{1}{2}}(m_{\chi_i}^2, m_B^2, m_{\chi_n^0}^2)/2m_B$ is the neutralino (chargino) momentum if the boson B is produced at rest. In this case there are two solutions

$$|\mathbf{p}_B^{\pm}| = \frac{(m_{\chi_i}^2 + m_B^2 - m_{\chi_n^0}^2)q\cos\theta_1 \pm E_{\chi_i}\sqrt{\lambda(m_{\chi_i}^2, m_B^2, m_{\chi_n^0}^2) - 4q^2\, m_B^2\,(1-\cos^2\theta_1)}}{2q^2(1-\cos^2\theta_1) + 2m_{\chi_i}^2}. \tag{B.32}$$

For $q' > q$, the angle θ_1 is not constrained and only the physical solution $|\mathbf{p}_B^+|$ is left. We parametrize the momenta of the decay $B \to f\bar{f}$ in the laboratory system as

$$p_B^{\pm,\mu} = (E_B^{\pm}, -|\mathbf{p}_B^{\pm}|\sin\theta_1\cos\phi_1, |\mathbf{p}_B^{\pm}|\sin\theta_1\sin\phi_1, -|\mathbf{p}_B^{\pm}|\cos\theta_1), \tag{B.33}$$

$$p_{\bar{f}}^{\mu} = (E_{\bar{f}}, -|\mathbf{p}_{\bar{f}}|\sin\theta_2\cos\phi_2, |\mathbf{p}_{\bar{f}}|\sin\theta_2\sin\phi_2, -|\mathbf{p}_{\bar{f}}|\cos\theta_2), \tag{B.34}$$

$$E_{\bar{f}}^{\mu} = |\mathbf{p}_{\bar{f}}| = \frac{m_B^2}{2(E_B^{\pm} - |\mathbf{p}_B^{\pm}|\cos\theta_{D_2})}, \tag{B.35}$$

with $\theta_2\angle(\mathbf{p}_{\chi_i}, \mathbf{p}_{\bar{f}})$ and the decay angle $\theta_{D_2}\angle(\mathbf{p}_B, \mathbf{p}_{\bar{f}})$ given by

$$\cos\theta_{D_2} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_2 - \phi_1). \tag{B.36}$$

The three spin vectors t_B^c of the boson $B = Z^0, W^+$ are in the laboratory system

$$t_B^{1,\mu} = \left(0, \frac{\mathbf{t}_B^2 \times \mathbf{t}_B^3}{|\mathbf{t}_B^2 \times \mathbf{t}_B^3|}\right), \quad t_B^{2,\mu} = \left(0, \frac{\mathbf{p}_{e^-} \times \mathbf{p}_B}{|\mathbf{p}_{e^-} \times \mathbf{p}_B|}\right), \quad t_B^{3,\mu} = \frac{1}{m_B}\left(|\mathbf{p}_B|, E_B\frac{\mathbf{p}_B}{|\mathbf{p}_B|}\right). \tag{B.37}$$

Together with p_B^{μ}/m_B they form an orthonormal set. The polarization four-vectors ε^{λ_k} for helicities $\lambda_k = -1, 0, +1$ of the boson are defined by

$$\varepsilon^- = \tfrac{1}{\sqrt{2}}(t_B^1 - i t_B^2); \quad \varepsilon^0 = t_B^3; \quad \varepsilon^+ = -\tfrac{1}{\sqrt{2}}(t_B^1 + i t_B^2). \tag{B.38}$$

B.2.6 Phase space for bosonic decays

For neutralino/chargino production $e^+e^- \to \tilde{\chi}_i\tilde{\chi}_j$ and subsequent decay of the neutralino $\tilde{\chi}_i^0 \to Z^0\tilde{\chi}_n^0$ or chargino $\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0$, in short $\tilde{\chi}_i \to B\tilde{\chi}_n^0$, the Lorentz invariant phase-space element can be decomposed into two-body phase-space elements [65]:

$$d\mathrm{Lips}(s; p_{\chi_j}, p_{\chi_n^0}, p_B) = \frac{1}{2\pi}\, d\mathrm{Lips}(s; p_{\chi_i}, p_{\chi_j})\; ds_{\chi_i} \sum_{\pm} d\mathrm{Lips}(s_{\chi_i}; p_{\chi_n^0}, p_B^{\pm}). \tag{B.39}$$

If we include the subsequent decay $B \to f\bar{f}$ we have

$$d\mathrm{Lips}(s; p_{\chi_j}, p_{\chi_n^0}, p_f, p_{\bar{f}}) = \frac{1}{2\pi}\, d\mathrm{Lips}(s; p_{\chi_j}, p_{\chi_n^0}, p_B)\; ds_B\; d\mathrm{Lips}(s_B; p_f, p_{\bar{f}}). \tag{B.40}$$

The several parts of the phase space elements are given by

$$d\mathrm{Lips}(s; p_{\chi_i}, p_{\chi_j}) = \frac{q}{8\pi\sqrt{s}} \sin\theta\; d\theta, \tag{B.41}$$

$$d\mathrm{Lips}(s_{\chi_i}; p_{\chi_n^0}, p_B^{\pm}) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_B^{\pm}|^2}{2|E_B^{\pm}\; q\cos\theta_1 - E_{\chi_i}\; |\mathbf{p}_B^{\pm}||}\, d\Omega_1, \tag{B.42}$$

$$d\mathrm{Lips}(s_B; p_f, p_{\bar{f}}) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_{\bar{f}}|^2}{m_B^2}\, d\Omega_2, \tag{B.43}$$

with $s_{\chi_i} = p_{\chi_i}^2$, $s_B = p_B^2$ and $d\Omega_i = \sin\theta_i\; d\theta_i\; d\phi_i$. We use the narrow width approximation for the propagators $\int |\Delta(\tilde{\chi}_i)|^2\; ds_{\chi_i} = \frac{\pi}{m_{\chi_i}\Gamma_{\chi_i}}$, $\int |\Delta(B)|^2 ds_B = \frac{\pi}{m_B\Gamma_B}$. The approximation is justified for $(\Gamma_{\chi_i}/m_{\chi_i})^2 \ll 1$, which holds in our case with $\Gamma_{\chi_i} \lesssim \mathcal{O}(1\mathrm{GeV})$.

B.3 Kinematics of sfermion decays

B.3.1 Momenta and spin vectors

We consider the slepton decay chain $\tilde{\ell} \to \ell\tilde{\chi}_j^0$, $\tilde{\chi}_j^0 \to \tilde{\chi}_1^0 Z$, $Z \to f\bar{f}$. The substitutions which must be made for similar decay chains of a squark are obvious. The momenta in the slepton rest frame are

$$
\begin{aligned}
p_Z^\mu &= (E_Z, 0, 0, |\mathbf{p}_Z^\pm|), & \text{(B.44)}\\
p_{\chi_j^0}^\mu &= |\mathbf{p}_{\chi_j^0}|(E_{\chi_j^0}/|\mathbf{p}_{\chi_j^0}|, \sin\theta_1, 0, \cos\theta_1), & \text{(B.45)}\\
p_{\bar f}^\mu &= |\mathbf{p}_{\bar f}|(E_{\bar f}/|\mathbf{p}_{\bar f}|, \sin\theta_2\cos\phi_2, \sin\theta_2\sin\phi_2, \cos\theta_2), & \text{(B.46)}\\
|\mathbf{p}_{\chi_j^0}| &= \frac{m_{\tilde\ell}^2 - m_{\chi_j^0}^2}{2\, m_{\tilde\ell}}, \qquad |\mathbf{p}_{\bar f}| = \frac{m_Z^2}{2(E_Z - |\mathbf{p}_Z|\cos\theta_2)}. & \text{(B.47)}
\end{aligned}
$$

There are two solutions for $|\mathbf{p}_Z^\pm|$, see (B.32), if the decay angle $\theta_1 = \angle(\mathbf{p}_Z, \mathbf{p}_{\chi_i})$ is constrained by

$$
\sin\theta_1^{\max} = \frac{m_{\tilde\ell}}{m_Z}\frac{\lambda^{\frac{1}{2}}(m_{\chi_j}^2, m_Z^2, m_{\chi_1}^2)}{(m_{\tilde\ell}^2 - m_{\chi_j}^2)} \le 1. \tag{B.48}
$$

The $\tilde\chi_j^0$ spin vectors in the $\tilde\ell$ rest frame are

$$
s_{\chi_j^0}^{1,\mu} = \left(0, \frac{\mathbf{s}_{\chi_j^0}^2 \times \mathbf{s}_{\chi_j^0}^3}{|\mathbf{s}_{\chi_j^0}^2 \times \mathbf{s}_{\chi_j^0}^3|}\right), \; s_{\chi_j^0}^{2,\mu} = \left(0, \frac{\mathbf{p}_{\chi_j^0} \times \mathbf{p}_Z}{|\mathbf{p}_{\chi_j^0} \times \mathbf{p}_Z|}\right), \; s_{\chi_j^0}^{3,\mu} = \frac{1}{m_{\chi_j^0}}\left(|\mathbf{p}_{\chi_j^0}|, E_{\chi_j^0}\frac{\mathbf{p}_{\chi_j^0}}{|\mathbf{p}_{\chi_j^0}|}\right). \tag{B.49}
$$

Together with $p_{\chi_j^0}^\mu/m_{\chi_j^0}$ they form an orthonormal set.

B.3.2 Phase space for sfermion decays

The Lorentz invariant phase-space element for the decay chain $\tilde\ell \to \ell\tilde\chi_j^0$, $\tilde\chi_j^0 \to \tilde\chi_1^0 Z$, $Z \to f\bar f$ can be written in the rest frame of $\tilde\ell$ as

$$
\begin{aligned}
&d\mathrm{Lips}(m_{\tilde\ell}^2; p_\ell, p_{\chi_1^0}, p_{\bar f}, p_f) = \\
&\frac{1}{(2\pi)^2}\, d\mathrm{Lips}(m_{\tilde\ell}^2; p_\ell, p_{\chi_j^0})\, ds_{\chi_j^0} \sum_\pm d\mathrm{Lips}(s_{\chi_j^0}; p_{\chi_1^0}, p_Z^\pm)\, ds_Z\, d\mathrm{Lips}(s_Z; p_{\bar f}, p_f), \qquad \text{(B.50)}
\end{aligned}
$$

$$d\mathrm{Lips}(m_{\tilde{\ell}}^2; p_\ell, p_{\chi_j^0}) = \frac{1}{8(2\pi)^2}\left(1 - \frac{m_{\chi_j^0}^2}{m_{\tilde{\ell}}^2}\right) d\Omega, \tag{B.51}$$

$$d\mathrm{Lips}(s_{\chi_j^0}; p_{\chi_1^0}, p_Z^\pm) = \frac{1}{4(2\pi)^2}\,\frac{|\mathbf{p}_Z^\pm|^2}{|E_Z^\pm\,|\mathbf{p}_{\chi_j^0}|\cos\theta_1 - E_{\chi_j^0}\,|\mathbf{p}_Z^\pm||}\,d\Omega_1, \tag{B.52}$$

$$d\mathrm{Lips}(s_Z; p_{\bar{f}}, p_f) = \frac{1}{8(2\pi)^2}\frac{m_Z^2}{(E_Z^\pm - |\mathbf{p}_Z^\pm|\cos\theta_2)^2}\,d\Omega_2, \tag{B.53}$$

with $s_{\chi_j^0} = p_{\chi_j^0}^2$, $s_Z = p_Z^2$ and $d\Omega_i = \sin\theta_i\; d\theta_i\; d\phi_i$. We use the narrow width approximation for the propagators $\int|\Delta(\tilde{\chi}_j^0)|^2\; ds_{\chi_j^0} = \frac{\pi}{m_{\chi_j^0}\Gamma_{\chi_j^0}}$, $\int|\Delta(Z)|^2 ds_Z = \frac{\pi}{m_Z\Gamma_Z}$. The approximation is justified for $(\Gamma_{\chi_j^0}/m_{\chi_j^0})^2 \ll 1$, which holds in our case with $\Gamma_{\chi_j^0} \lesssim \mathcal{O}(1\mathrm{GeV})$.

Appendix C

Spin-density matrices for neutralino production and decay

We give the analytic formulae for the squared amplitudes for neutralino production, $e^+\ e^- \to\ \tilde{\chi}^0_i\ \tilde{\chi}^0_j$, with longitudinally polarized beams, and for different subsequent two-body decay chains of one neutralino. We use the spin density matrix formalism as in [27, 39, 66]. The amplitude squared can then be written

$$|T|^2 \ =\ |\Delta(\tilde{\chi}^0_i)|^2 \sum_{\lambda_i \lambda_i'} \rho_P(\tilde{\chi}^0_i)^{\lambda_i \lambda_i'}\ \rho_D(\tilde{\chi}^0_i)_{\lambda_i' \lambda_i}, \tag{C.1}$$

with $\rho_P(\tilde{\chi}^0_i)$ the spin density production matrix of neutralino $\tilde{\chi}^0_i$, the propagator $\Delta(\tilde{\chi}^0_i) = i/[s_{\chi^0_i} - m^2_{\chi^0_i} + i m_{\chi^0_i}\Gamma_{\chi^0_i}]$ and the neutralino decay matrix $\rho_D(\tilde{\chi}^0_i)$.

C.1 Neutralino production

For the production of neutralinos

$$e^+ + e^- \ \to\ \tilde{\chi}^0_i(p_{\chi^0_i}, \lambda_i) + \tilde{\chi}^0_j(p_{\chi^0_j}, \lambda_j), \tag{C.2}$$

with momentum p and helicity λ, the unnormalized spin-density matrix of neutralino $\tilde{\chi}^0_i$ is defined as

$$\rho_P(\tilde{\chi}^0_i)^{\lambda_i \lambda_i'} \ =\ \sum_{\lambda_j} T_P^{\lambda_i \lambda_j} T_P^{\lambda_i' \lambda_j *}. \tag{C.3}$$

The helicity amplitudes are [39,66]:

$$
\begin{aligned}
T_P^{\lambda_i\lambda_j}(s,Z) &= \frac{g^2}{\cos^2\theta_W}\Delta^s(Z)\bar{v}(p_{e^+})\gamma^\mu(L_eP_L+R_eP_R)u(p_{e^-}) \\
&\quad \times\bar{u}(p_{\chi_j^0},\lambda_j)\gamma_\mu(O''^L_{ji}P_L+O''^R_{ji}P_R)v(p_{\chi_i^0},\lambda_i), \qquad \text{(C.4)}\\
T_P^{\lambda_i\lambda_j}(t,\tilde{e}_L) &= -g^2f^L_{ei}f^{L*}_{ej}\Delta^t(\tilde{e}_L)\bar{v}(p_{e^+})P_Rv(p_{\chi_i^0},\lambda_i)\bar{u}(p_{\chi_j^0},\lambda_j)P_Lu(p_{e^-}), \qquad \text{(C.5)}\\
T_P^{\lambda_i\lambda_j}(t,\tilde{e}_R) &= -g^2f^R_{ei}f^{R*}_{ej}\Delta^t(\tilde{e}_R)\bar{v}(p_{e^+})P_Lv(p_{\chi_i^0},\lambda_i)\bar{u}(p_{\chi_j^0},\lambda_j)P_Ru(p_{e^-}), \qquad \text{(C.6)}\\
T_P^{\lambda_i\lambda_j}(u,\tilde{e}_L) &= g^2f^{L*}_{ei}f^{L}_{ej}\Delta^u(\tilde{e}_L)\bar{v}(p_{e^+})P_Rv(p_{\chi_j^0},\lambda_j)\bar{u}(p_{\chi_i^0},\lambda_i)P_Lu(p_{e^-}), \qquad \text{(C.7)}\\
T_P^{\lambda_i\lambda_j}(u,\tilde{e}_R) &= g^2f^{R*}_{ei}f^{R}_{ej}\Delta^u(\tilde{e}_R)\bar{v}(p_{e^+})P_Lv(p_{\chi_j^0},\lambda_j)\bar{u}(p_{\chi_i^0},\lambda_i)P_Ru(p_{e^-}). \qquad \text{(C.8)}
\end{aligned}
$$

The propagators are

$$
\Delta^s(Z)=\frac{i}{s-m_Z^2+im_Z\Gamma_Z},\quad \Delta^t(\tilde{e}_{R,L})=\frac{i}{t-m^2_{\tilde{e}_{R,L}}},\quad \Delta^u(\tilde{e}_{R,L})=\frac{i}{u-m^2_{\tilde{e}_{R,L}}}, \qquad \text{(C.9)}
$$

with $s=(p_{e^-}+p_{e^+})^2$, $t=(p_{e^-}-p_{\chi_j^0})^2$ and $u=(p_{e^-}-p_{\chi_i^0})^2$. The Feynman diagrams are shown in Fig. C.1.

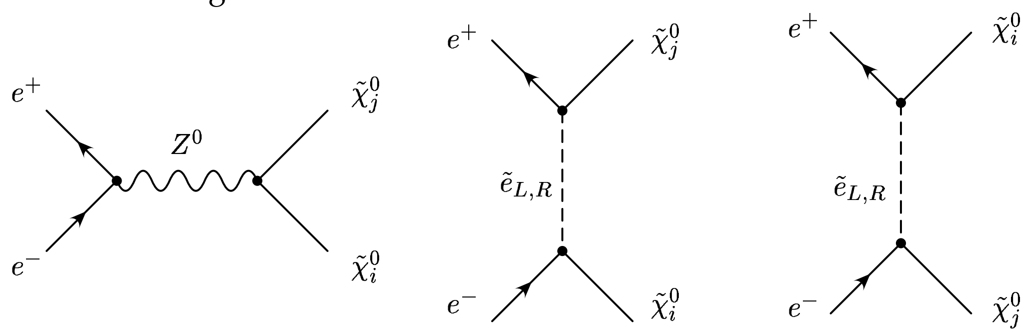


Figure C.1: Feynman diagrams for neutralino production

For the polarization of the decaying neutralino $\tilde{\chi}_i^0$ with momentum $p_{\chi_i^0}$ we have introduced three space like spin vectors $s^a_{\chi_i^0}$ (B.14). Then the neutralino production matrix (C.3) can be expanded in terms of the Pauli matrices, see Appendix F.1:

$$
\rho_P(\tilde{\chi}_i^0)^{\lambda_i\lambda_i'} = 2(\delta_{\lambda_i\lambda_i'}P+\sigma^a_{\lambda_i\lambda_i'}\Sigma^a_P), \qquad \text{(C.10)}
$$

where we sum over a and the factor 2 is due to the summation of the helicities of the second neutralino $\tilde{\chi}_j^0$, whose decay will not be considered. With our choice of the spin vectors, Σ_P^3/P is the longitudinal polarization of neutralino $\tilde{\chi}_i^0$, Σ_P^1/P is the transverse polarization in the production plane and Σ_P^2/P is the polarization perpendicular to the production plane. Only if there are non-vanishing CP phases φ_{M_1} and/or φ_μ in the neutralino sector, and only if two different neutralinos are produced, $e^+ e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$, $i \neq j$, the polarization Σ_P^2/P perpendicular to the production plane is non-zero. Thus it is a probe for CP violation in the production of an unequal pair of neutralinos. Note that Σ_P^2 also gets contributions from the finite Z width, which however do not signal CP violation.

We give the analytical formulae for P and $\Sigma_P^1, \Sigma_P^2, \Sigma_P^3$ in the laboratory system [66] in the following sections. Lorentz invariant expressions for these functions can be found in [39, 66].

C.1.1 Neutralino polarization independent quantities

The coefficient P is independent of the neutralino polarization. It can be composed into contributions from the different production channels

$$P = P(ZZ) + P(Z\tilde{e}_R) + P(\tilde{e}_R\tilde{e}_R) + P(Z\tilde{e}_L) + P(\tilde{e}_L\tilde{e}_L), \tag{C.11}$$

with

$$\begin{aligned}
P(ZZ) &= 2\frac{g^4}{\cos^4\theta_W}|\Delta^s(Z)|^2(R_e^2 c_R + L_e^2 c_L)E_b^2 \\
&\times\Big\{|O_{ij}^{''R}|^2(E_{\chi_i^0}E_{\chi_j^0} + q^2\cos^2\theta) - [(ReO_{ij}^{''R})^2 - (ImO_{ij}^{''R})^2]m_{\chi_i^0}m_{\chi_j^0}\Big\}, &\text{(C.12)}\\
P(Z\tilde{e}_R) &= \frac{g^4}{\cos^2\theta_W}R_e c_R E_b^2 Re\Big\{\Delta^s(Z) \\
&\times\Big[-(\Delta^{t*}(\tilde{e}_R)f_{ei}^{R*}f_{ej}^{R}O_{ij}^{''R*} + \Delta^{u*}(\tilde{e}_R)f_{ei}^{R}f_{ej}^{R*}O_{ij}^{''R})m_{\chi_i^0}m_{\chi_j^0} \\
&+(\Delta^{t*}(\tilde{e}_R)f_{ei}^{R*}f_{ej}^{R}O_{ij}^{''R} + \Delta^{u*}(\tilde{e}_R)f_{ei}^{R}f_{ej}^{R*}O_{ij}^{''R*})(E_{\chi_i^0}E_{\chi_j^0} + q^2\cos^2\theta) \\
&-(\Delta^{t*}(\tilde{e}_R)f_{ei}^{R*}f_{ej}^{R}O_{ij}^{''R} - \Delta^{u*}(\tilde{e}_R)f_{ei}^{R}f_{ej}^{R*}O_{ij}^{''R*})2E_b q\cos\theta\Big]\Big\}, &\text{(C.13)}\\
P(\tilde{e}_R\tilde{e}_R) &= \frac{g^4}{4}c_R E_b^2\Big\{|f_{ei}^R|^2|f_{ej}^R|^2 \times \\
&\Big[(|\Delta^t(\tilde{e}_R)|^2 + |\Delta^u(\tilde{e}_R)|^2)(E_{\chi_i^0}E_{\chi_j^0} + q^2\cos^2\theta) \\
&-(|\Delta^t(\tilde{e}_R)|^2 - |\Delta^u(\tilde{e}_R)|^2)2E_b q\cos\theta\Big] \\
&-Re\{(f_{ei}^{R*})^2(f_{ej}^R)^2\Delta^u(\tilde{e}_R)\Delta^{t*}(\tilde{e}_R)\}2m_{\chi_i^0}m_{\chi_j^0}\Big\}. &\text{(C.14)}
\end{aligned}$$

To obtain the quantities $P(Z\tilde{e}_L), P(\tilde{e}_L\tilde{e}_L)$ one has to exchange in (C.13) and (C.14)

$$\begin{aligned}&\Delta^t(\tilde{e}_R) \to \Delta^t(\tilde{e}_L), \quad \Delta^u(\tilde{e}_R) \to \Delta^u(\tilde{e}_L), \quad c_R \to c_L\\ &R_e \to L_e, \quad O''^R_{ij} \to O''^L_{ij}, \quad f^R_{ei} \to f^L_{ei}, \quad f^R_{ej} \to f^L_{ej}.\end{aligned} \tag{C.15}$$

The longitudinal beam polarizations are included in the weighting factors

$$c_L = (1 - P_{e^-})(1 + P_{e^+}), \quad c_R = (1 + P_{e^-})(1 - P_{e^+}). \tag{C.16}$$

Generally the contributions from the exchange of $\tilde{e}_R$ ($\tilde{e}_L$) is enhanced and that of $\tilde{e}_L$ ($\tilde{e}_R$) is suppressed for $P_{e^-} > 0, P_{e^+} < 0$ ($P_{e^-} < 0, P_{e^+} > 0$).

C.1.2 Neutralino polarization

The coefficients Σ^a_P, which describe the polarization of the neutralino $\tilde{\chi}^0_i$, decompose into

$$\Sigma^a_P = \Sigma^a_P(ZZ) + \Sigma^a_P(Z\tilde{e}_R) + \Sigma^a_P(\tilde{e}_R\tilde{e}_R) + \Sigma^a_P(Z\tilde{e}_L) + \Sigma^a_P(\tilde{e}_L\tilde{e}_L). \tag{C.17}$$

- The contributions to the transverse polarization in the production plane are

$$\begin{aligned}
\Sigma^1_P(ZZ) &= 2\frac{g^4}{\cos^4\theta_W}|\Delta^s(Z)|^2 E_b^2 \sin\theta (R_e^2 c_R - L_e^2 c_L)\\
&\quad \times\Big[|O''^R_{ij}|^2 m_{\chi^0_i} E_{\chi^0_j} - [(ReO''^R_{ij})^2 - (ImO''^R_{ij})^2] m_{\chi^0_j} E_{\chi^0_i}\Big], && \text{(C.18)}\\
\Sigma^1_P(Z\tilde{e}_R) &= \frac{-g^4}{\cos^2\theta_W} R_e c_R E_b^2 \sin\theta\\
&\quad \times\Big[- Re\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^{R*}_{ij} \Delta^{u*}(\tilde{e}_R) + f^{R*}_{ei} f^R_{ej} O''^R_{ij} \Delta^{t*}(\tilde{e}_R)] m_{\chi^0_i} E_{\chi^0_j}\}\\
&\quad -Re\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^{R*}_{ij} \Delta^{u*}(\tilde{e}_R) - f^{R*}_{ei} f^R_{ej} O''^R_{ij} \Delta^{t*}(\tilde{e}_R)] m_{\chi^0_i} q\cos\theta\}\\
&\quad +Re\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^R_{ij} \Delta^{u*}(\tilde{e}_R) + f^{R*}_{ei} f^R_{ej} O''^{R*}_{ij} \Delta^{t*}(\tilde{e}_R)] m_{\chi^0_j} E_{\chi^0_i}\}\Big], && \text{(C.19)}\\
\Sigma^1_P(\tilde{e}_R\tilde{e}_R) &= \frac{g^4}{4} c_R E_b^2 \sin\theta \Big\{ |f^R_{ei}|^2 |f^R_{ej}|^2\\
&\quad \times\Big[(|\Delta^t(\tilde{e}_R)|^2 + |\Delta^u(\tilde{e}_R)|^2) m_{\chi^0_i} E_{\chi^0_j} - (|\Delta^t(\tilde{e}_R)|^2 - |\Delta^u(\tilde{e}_R)|^2) m_{\chi^0_i} q\cos\theta\Big]\\
&\quad -2Re\{(f^{R*}_{ei})^2 (f^R_{ej})^2 \Delta^u(\tilde{e}_R)\Delta^{t*}(\tilde{e}_R)\} m_{\chi^0_j} E_{\chi^0_i}\Big\}. && \text{(C.20)}
\end{aligned}$$

To obtain the expressions for $\Sigma_P^1(Z\tilde{e}_L)$ and $\Sigma_P^1(\tilde{e}_L\tilde{e}_L)$ one has to apply the exchanges (C.15) in (C.19) and (C.20) and to change the overall sign of the right hand side of (C.19) and (C.20).

- The contributions to the transverse $\tilde{\chi}_i^0$ polarization perpendicular to the production plane are

$$\begin{aligned}
\Sigma_P^2(ZZ) &= \frac{4g^4}{\cos^4\theta_W}|\Delta^s(Z)|^2(R_e^2 c_R - L_e^2 c_L)\, m_{\chi_j^0} q E_b^2 \sin\theta Re(O''^R_{ij}) Im(O''^R_{ij}), \qquad (C.21)\\
\Sigma_P^2(Z\tilde{e}_R) &= \frac{g^4}{\cos^2\theta_W} R_e c_R m_{\chi_j^0} E_b^2 q \sin\theta\\
&\quad\times Im\Big\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^R_{ij}\Delta^{u*}(\tilde{e}_R) - f^{R*}_{ei} f^R_{ej} O''^{R*}_{ij}\Delta^{t*}(\tilde{e}_R)]\Big\}, \qquad (C.22)\\
\Sigma_P^2(\tilde{e}_R\tilde{e}_R) &= -\frac{g^4}{2} c_R m_{\chi_j^0} E_b^2 q \sin\theta\, Im\Big\{(f^{R*}_{ei})^2 (f^R_{ej})^2 \Delta^u(\tilde{e}_R)\Delta^{t*}(\tilde{e}_R)\Big\}. \qquad (C.23)
\end{aligned}$$

To obtain the expressions for $\Sigma_P^2(Z\tilde{e}_L)$ and $\Sigma_P^2(\tilde{e}_L\tilde{e}_L)$ one has to apply the exchanges (C.15) in (C.22) and (C.23).

- The contributions to the longitudinal $\tilde{\chi}_i^0$ polarization are

$$\begin{aligned}
\Sigma_P^3(ZZ) &= \frac{2g^4}{\cos^4\theta_W}|\Delta^s(Z)|^2(L_e^2 c_L - R_e^2 c_R) E_b^2 \cos\theta\\
&\quad\times\Big[|O''^R_{ij}|^2(E_{\chi_i^0}E_{\chi_j^0} + q^2) - [(ReO''^R_{ij})^2 - (ImO''^{R*}_{ij})^2] m_{\chi_i^0} m_{\chi_j^0}\Big], \qquad (C.24)\\
\Sigma_P^3(Z\tilde{e}_R) &= \frac{-g^4}{\cos^2\theta_W} R_e c_R E_b^2\\
&\quad\times\Big[Re\Big\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^{R*}_{ij}\Delta^{u*}(\tilde{e}_R) - f^{R*}_{ei} f^R_{ej} O''^R_{ij}\Delta^{t*}(\tilde{e}_R)] E_{\chi_j^0} q\Big\}\\
&\quad + Re\Big\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^{R*}_{ij}\Delta^{u*}(\tilde{e}_R) + f^{R*}_{ei} f^R_{ej} O''^R_{ij}\Delta^{t*}(\tilde{e}_R)](E_{\chi_i^0}E_{\chi_j^0} + q^2)\cos\theta\Big\}\\
&\quad - Re\Big\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^R_{ij}\Delta^{u*}(\tilde{e}_R) + f^{R*}_{ei} f^R_{ej} O''^{R*}_{ij}\Delta^{t*}(\tilde{e}_R)] m_{\chi_i^0} m_{\chi_j^0}\cos\theta\Big\}\\
&\quad + Re\Big\{\Delta^s(Z)[f^R_{ei} f^{R*}_{ej} O''^{R*}_{ij}\Delta^{u*}(\tilde{e}_R) - f^{R*}_{ei} f^R_{ej} O''^R_{ij}\Delta^{t*}(\tilde{e}_R)] E_{\chi_i^0} q \cos^2\theta\Big\}\Big], \qquad (C.25)\\
\Sigma_P^3(\tilde{e}_R\tilde{e}_R) &= -\frac{g^4}{4} c_R E_b^2\Big[|f^R_{ei}|^2 |f^R_{ej}|^2\\
&\quad\times\Big\{[|\Delta^u(\tilde{e}_R)|^2 - |\Delta^t(\tilde{e}_R)|^2] E_{\chi_j^0} q + [|\Delta^u(\tilde{e}_R)|^2 - |\Delta^t(\tilde{e}_R)|^2] q E_{\chi_i^0}\cos^2\theta\\
&\quad + [|\Delta^t(\tilde{e}_R)|^2 + |\Delta^u(\tilde{e}_R)|^2](E_{\chi_i^0}E_{\chi_j^0} + q^2)\cos\theta\Big\}\\
&\quad - 2Re\{(f^{R*}_{ei})^2 (f^R_{ej})^2 \Delta^u(\tilde{e}_R)\Delta^{t*}(\tilde{e}_R)\} m_{\chi_i^0} m_{\chi_j^0}\cos\theta\Big]. \qquad (C.26)
\end{aligned}$$

To obtain the expressions for $\Sigma^3_P(Z\tilde{e}_L)$ and $\Sigma^3_P(\tilde{e}_L\tilde{e}_L)$ one has to apply the exchanges (C.15) in (C.25) and (C.26) and to change the overall sign of the right hand side of (C.25) and (C.26).

C.2 Neutralino decay into sleptons

For neutralino two-body decay into sleptons

$$\tilde{\chi}^0_i(p_{\chi^0_i}, \lambda_i) \;\to\; \tilde{\ell} + \ell_1; \quad \ell = e, \mu, \tau, \tag{C.27}$$

the neutralino decay matrix (2.8) is given by

$$\rho_{D_1}(\tilde{\chi}^0_i)_{\lambda'_i\lambda_i} = \delta_{\lambda'_i\lambda_i} D_1 + \sigma^a_{\lambda'_i\lambda_i}\Sigma^a_{D_1}, \tag{C.28}$$

where we sum over a. For the decay into right sleptons $\tilde{\chi}^0_i \to \tilde{\ell}^{\mp}_R\,\ell^{\pm}_1$, $\ell = e, \mu$, the expansion coefficients are

$$D_1 \;=\; \frac{g^2}{2}|f^R_{\ell i}|^2(m^2_{\chi^0_i} - m^2_{\tilde{\ell}}), \tag{C.29}$$
$$\Sigma^a_{D_1} \;=\; \pm g^2|f^R_{\ell i}|^2 m_{\chi^0_i}(s^a_{\chi^0_i}\cdot p_{\ell_1}). \tag{C.30}$$

For the decay into the left sleptons $\tilde{\chi}^0_i \to \tilde{\ell}^{\mp}_L\,\ell^{\pm}_1$, $\ell = e, \mu$, the coefficients are

$$D_1 \;=\; \frac{g^2}{2}|f^L_{\ell i}|^2(m^2_{\chi^0_i} - m^2_{\tilde{\ell}}), \tag{C.31}$$
$$\Sigma^a_{D_1} \;=\; \mp g^2|f^L_{\ell i}|^2 m_{\chi^0_i}(s^a_{\chi^0_i}\cdot p_{\ell_1}). \tag{C.32}$$

For the decay into the stau $\tilde{\chi}^0_i \to \tilde{\tau}^{\mp}_k\,\tau^{\pm}$, $k = 1, 2$, one obtains

$$D_1 \;=\; \frac{g^2}{2}(|a^{\tilde{\tau}}_{ki}|^2 + |b^{\tilde{\tau}}_{ki}|^2)(m^2_{\chi^0_i} - m^2_{\tilde{\tau}_k}), \tag{C.33}$$
$$\Sigma^a_{D_1} \;=\; \mp g^2(|a^{\tilde{\tau}}_{ki}|^2 - |b^{\tilde{\tau}}_{ki}|^2) m_{\chi^0_i}(s^a_{\chi^0_i}\cdot p_{\ell_1}). \tag{C.34}$$

The factor for the subsequent slepton decays $\tilde{\ell}_{R,L} \to \ell_2\tilde{\chi}^0_1$, $\ell = e, \mu$, is given by

$$D_2 \;=\; g^2|f^{R,L}_{\ell 1}|^2(m^2_{\tilde{\ell}} - m^2_{\chi^0_1}), \tag{C.35}$$

and that for stau decay $\tilde{\tau}_k \to \tau\tilde{\chi}^0_1$ by

$$D_2 \;=\; g^2(|a^{\tilde{\tau}}_{k1}|^2 + |b^{\tilde{\tau}}_{k1}|^2)(m^2_{\tilde{\tau}_k} - m^2_{\chi^0_1}). \tag{C.36}$$

C.3 Neutralino decay into staus

For neutralino two-body decay into staus

$$\tilde{\chi}_i^0(p_{\chi_i^0}, \lambda_i) \to \tilde{\tau}_m^{\pm}(p_{\tilde{\tau}_m}) + \tau^{\mp}(p_\tau, \lambda_k); \quad m = 1, 2, \tag{C.37}$$

the decay matrix is

$$\rho_D(\tilde{\chi}_i^0)^{\lambda_k \lambda'_k}_{\lambda'_i \lambda_i} = \delta_{\lambda'_i \lambda_i} D^{\lambda_k \lambda'_k} + \sum_a \sigma^a_{\lambda'_i \lambda_i} (\Sigma^a_D)^{\lambda_k \lambda'_k}. \tag{C.38}$$

With the spin basis vectors s^b_τ for the $\tau^{\mp}$, given in (B.21), we can expand

$$D^{\lambda_k \lambda'_k} = \delta_{\lambda_k \lambda'_k} D + \sigma^b_{\lambda_k \lambda'_k} D^b, \tag{C.39}$$

$$(\Sigma^a_D)^{\lambda_k \lambda'_k} = \delta_{\lambda_k \lambda'_k} \Sigma^a_D + \sigma^b_{\lambda_k \lambda'_k} \Sigma^{ab}_D. \tag{C.40}$$

The expansion coefficients are given by

$$D = g^2 \mathrm{Re}(b^{\tilde{\tau}\,*}_{mi} a^{\tilde{\tau}}_{mi}) m_\tau m_{\chi_i^0} + \frac{g^2}{2}(|b^{\tilde{\tau}}_{mi}|^2 + |a^{\tilde{\tau}}_{mi}|^2)(p_\tau \cdot p_{\chi_i^0}), \tag{C.41}$$

$$D^b = \pm \frac{g^2}{2} m_\tau (|b^{\tilde{\tau}}_{mi}|^2 - |a^{\tilde{\tau}}_{mi}|^2)(p_{\chi_i^0} \cdot s^b_\tau), \tag{C.42}$$

$$\Sigma^a_D = \pm \frac{g^2}{2} m_{\chi_i^0} (|a^{\tilde{\tau}}_{mi}|^2 - |b^{\tilde{\tau}}_{mi}|^2)(p_\tau \cdot s^a_{\chi_i^0}), \tag{C.43}$$

$$\begin{aligned}
\Sigma^{ab}_D = {} & g^2 \mathrm{Re}(b^{\tilde{\tau}\,*}_{mi} a^{\tilde{\tau}}_{mi})(p_\tau \cdot s^a_{\chi_i^0})(p_{\chi_i^0} \cdot s^b_\tau) \\
& - g^2 (s^a_{\chi_i^0} \cdot s^b_\tau)[\frac{1}{2}(|b^{\tilde{\tau}}_{mi}|^2 + |a^{\tilde{\tau}}_{mi}|^2) m_\tau m_{\chi_i^0} + \mathrm{Re}(b^{\tilde{\tau}\,*}_{mi} a^{\tilde{\tau}}_{mi})(p_\tau \cdot p_{\chi_i^0})] \\
& \mp g^2 \mathrm{Im}(b^{\tilde{\tau}\,*}_{mi} a^{\tilde{\tau}}_{mi}) \epsilon_{\mu\nu\rho\sigma}\, p^\mu_\tau\, p^\nu_{\tilde{\chi}_i^0}\, s^{a,\rho}_{\chi_i^0}\, s^{b,\sigma}_\tau, \quad (\epsilon_{0123} = 1).
\end{aligned} \tag{C.44}$$

C.4 Neutralino decay into the Z boson

For the neutralino two-body decay into the Z boson

$$\tilde{\chi}_i^0(p_{\chi_i^0}, \lambda_i) \to \chi_n^0(p_{\chi_n^0}, \lambda_n) + Z(p_Z, \lambda_k); \quad n < i, \tag{C.45}$$

the decay matrix is given by

$$\rho_{D_1}(\tilde{\chi}_i^0)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i} = \sum_{\lambda_n} T^{\lambda_n\lambda_k}_{D_1,\lambda_i} T^{\lambda_n\lambda'_k *}_{D_1,\lambda'_i}, \tag{C.46}$$

with the helicity amplitude

$$T^{\lambda_n\lambda_k}_{D_1,\lambda_i} = \bar{u}(p_{\chi_n^0},\lambda_n)\gamma^\mu \frac{g}{\cos\theta_W}[O''^L_{ni}P_L + O''^R_{ni}P_R]u(p_{\chi_i^0},\lambda_i)\varepsilon^{\lambda_k *}_\mu. \tag{C.47}$$

For the subsequent decay of the Z boson

$$Z(p_Z,\lambda_k) \to f(p_f,\lambda_f) + \bar{f}(p_{\bar{f}},\lambda_{\bar{f}}); \quad f = \ell, q, \tag{C.48}$$

the decay matrix is

$$\rho_{D_2}(Z)_{\lambda'_k\lambda_k} = \sum_{\lambda_f,\lambda_{\bar{f}}} T^{\lambda_f\lambda_{\bar{f}}}_{D_2,\lambda_k} T^{\lambda_f\lambda_{\bar{f}} *}_{D_2,\lambda'_k}, \tag{C.49}$$

and the helicity amplitude

$$T^{\lambda_f\lambda_{\bar{f}}}_{D_2,\lambda_k} = \bar{u}(p_f,\lambda_f)\gamma^\mu \frac{g}{\cos\theta_W}[L_f P_L + R_f P_R]v(p_{\bar{f}},\lambda_{\bar{f}})\varepsilon^{\lambda_k}_\mu. \tag{C.50}$$

The polarization vectors of the Z boson $\varepsilon^{\lambda_k}_\mu$, $\lambda_k = 0, \pm 1$, are given in (B.38). With the set of neutralino spin vectors $s^a_{\chi_i^0}$, given in (B.14), we obtain for the neutralino decay matrix

$$\rho_{D_1}(\tilde{\chi}_i^0)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i} = (\delta_{\lambda'_i\lambda_i} D_1^{\mu\nu} + \sigma^a_{\lambda'_i\lambda_i}\Sigma^{a\ \mu\nu}_{D_1})\varepsilon^{\lambda_k *}_\mu \varepsilon^{\lambda'_k}_\nu, \tag{C.51}$$

and for the Z decay matrix

$$\rho_{D_2}(Z)_{\lambda'_k\lambda_k} = D_2^{\mu\nu}\varepsilon^{\lambda_k}_\mu \varepsilon^{\lambda'_k *}_\nu, \tag{C.52}$$

with

$$D_1^{\mu\nu} = \frac{2g^2}{\cos^2\theta_W}\Big\{[2\,p^\mu_{\chi^0_i}p^\nu_{\chi^0_i} - (p^\mu_{\chi^0_i}p^\nu_Z + p^\nu_{\chi^0_i}p^\mu_Z) - \tfrac{1}{2}(m^2_{\chi^0_i} + m^2_{\chi^0_n} - m^2_Z)g^{\mu\nu}]|O''^L_{ni}|^2$$
$$-g^{\mu\nu}m_{\chi^0_i}m_{\chi^0_n}[(ReO''^L_{ni})^2 - (ImO''^L_{ni})^2]\Big\}, \quad \text{(C.53)}$$

$$\Sigma^{a\,\mu\nu}_{D_1} = \frac{2ig^2}{\cos^2\theta_W}\Big\{ - m_{\chi^0_i}\epsilon^{\mu\alpha\nu\beta}s^a_{\chi^0_i,\alpha}(p_{\chi^0_i,\beta} - p_{Z,\beta})|O''^L_{ni}|^2$$
$$+2m_{\chi^0_n}(s^{a,\mu}_{\chi^0_i}p^\nu_{\chi^0_i} - s^{a,\nu}_{\chi^0_i}p^\mu_{\chi^0_i})(ImO''^L_{ni})(ReO''^L_{ni})$$
$$-m_{\chi^0_n}\epsilon^{\mu\alpha\nu\beta}s^a_{\chi^0_i,\alpha}p_{\chi^0_i,\beta}[(ReO''^L_{ni})^2 - (ImO''^L_{ni})^2]\Big\}; \quad (\epsilon_{0123} = 1), \quad \text{(C.54)}$$

and

$$D_2^{\mu\nu} = \frac{2g^2}{\cos^2\theta_W}\Big\{(-2\,p^\mu_{\bar f}p^\nu_{\bar f} + p^\mu_Z p^\nu_{\bar f} + p^\mu_{\bar f}p^\nu_Z - \tfrac{1}{2}m^2_Z g^{\mu\nu})(L^2_f + R^2_f)$$
$$-i\epsilon^{\mu\alpha\nu\beta}p_{Z,\alpha}p_{\bar f,\beta}(L^2_f - R^2_f)\Big\}. \quad \text{(C.55)}$$

Due to the Majorana character of the neutralinos, $D_1^{\mu\nu}$ is symmetric and $\Sigma^{a\,\mu\nu}_{D_1}$ is antisymmetric under interchange of μ and ν. In (C.51) and (C.52) we use the expansion (F.12) for the Z polarization vectors

$$\varepsilon^{\lambda_k}_\mu\varepsilon^{\lambda'_k *}_\nu = \tfrac{1}{3}\delta^{\lambda'_k\lambda_k}I_{\mu\nu} - \frac{i}{2m_Z}\epsilon_{\mu\nu\rho\sigma}p^\rho_Z t^{c,\sigma}_Z(J^c)^{\lambda'_k\lambda_k} - \tfrac{1}{2}t^c_{Z,\mu}t^d_{Z,\nu}(J^{cd})^{\lambda'_k\lambda_k}, \quad \text{(C.56)}$$

summed over c, d. The decay matrices can be expanded in terms of the spin matrices J^c and J^{cd}, given in Appendix F.2. The first term of the decay matrix ρ_{D_1} (C.51), which is independent of the neutralino polarization, then gives

$$D_1^{\mu\nu}\varepsilon^{\lambda_k *}_\mu\varepsilon^{\lambda'_k}_\nu = D_1\delta^{\lambda_k\lambda'_k} + {}^cD_1(J^c)^{\lambda_k\lambda'_k} + {}^{cd}D_1(J^{cd})^{\lambda_k\lambda'_k}, \quad \text{(C.57)}$$

with

$$D_1 = \frac{g^2}{\cos^2\theta_W}\Big\{\Big[m^2_{\chi^0_n} - \tfrac{1}{3}m^2_{\chi^0_i} - m^2_Z + \frac{4}{3}\frac{(p_{\chi^0_i}\cdot p_Z)^2}{m^2_Z}\Big]|O''^L_{ni}|^2$$
$$+2m_{\chi^0_i}m_{\chi^0_n}[(ReO''^L_{ni})^2 - (ImO''^L_{ni})^2]\Big\}, \quad \text{(C.58)}$$

$${}^{cd}D_1 = -\frac{g^2}{\cos^2\theta_W}\Big\{\Big[2(t^c_Z\cdot p_{\chi^0_i})(t^d_Z\cdot p_{\chi^0_i}) + \tfrac{1}{2}(m^2_{\chi^0_i} + m^2_{\chi^0_n} - m^2_Z)\delta^{cd}\Big]\,|O''^L_{ni}|^2$$
$$+\delta^{cd}m_{\chi^0_i}m_{\chi^0_n}[(ReO''^L_{ni})^2 - (ImO''^L_{ni})^2]\Big\}, \quad \text{(C.59)}$$

and ${}^{c}D_1 = 0$ due to the Majorana character of the neutralinos. As a consequence of the completeness relation (F.14), the diagonal coefficients are linearly dependent

$$ {}^{11}D_1 + {}^{22}D_1 + {}^{33}D_1 \;=\; -\tfrac{3}{2}D_1. \tag{C.60}$$

For large three momentum $\mathbf{p}_{\chi_i^0}$, the Z boson will mainly be emitted into the forward direction with respect to $\mathbf{p}_{\chi_i^0}$, i.e. $\hat{\mathbf{p}}_{\chi_i^0} \approx \hat{\mathbf{p}}_Z$, with $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$, so that $(t_Z^{1,2} \cdot p_{\chi_i^0}) \approx 0$ in (C.59). Therefore, for high energies ${}^{11}D_1 \approx {}^{22}D_1$, and the contributions of the non-diagonal coefficients ${}^{cd}D_1 (c \neq d)$ will be small.

For the second term of ρ_{D_1} (C.51), which depends on the polarization of the decaying neutralino, we obtain

$$ \Sigma^{a\,\mu\nu}_{D_1} \varepsilon^{\lambda_k *}_{\mu} \varepsilon^{\lambda'_k}_{\nu} \;=\; \Sigma^{a}_{D_1} \delta^{\lambda_k\lambda'_k} + {}^{c}\Sigma^{a}_{D_1} (J^c)^{\lambda_k\lambda'_k} + {}^{cd}\Sigma^{a}_{D_1} (J^{cd})^{\lambda_k\lambda'_k}, \tag{C.61}$$

with

$$\begin{aligned} {}^{c}\Sigma^{a}_{D_1} \;=\; & \frac{2g^2}{m_Z \cos^2\theta_W} \Big\{ \Big[|O''^{L}_{ni}|^2 m_{\chi_i^0} + [(ReO''^{L}_{ni})^2 - (ImO''^{L}_{ni})^2] m_{\chi_n^0} \Big] \\ & \times \Big[(s^a_{\chi_i^0} \cdot p_Z)(t^c_Z \cdot p_{\chi_i^0}) - (s^a_{\chi_i^0} \cdot t^c_Z)(p_Z \cdot p_{\chi_i^0}) \Big] + |O''^{L}_{ni}|^2 m_{\chi_i^0} m_Z^2 (s^a_{\chi_i^0} \cdot t^c_Z) \\ & - 2(ImO''^{L}_{ni})(ReO''^{L}_{ni}) m_{\chi_n^0} \epsilon_{\mu\nu\rho\sigma} s^{a,\mu}_{\chi_i^0} p^{\nu}_{\chi_i^0} p^{\rho}_Z t^{c,\sigma}_Z \Big\}, \end{aligned} \tag{C.62}$$

and $\Sigma^{a}_{D_1} \;=\; {}^{cd}\Sigma^{a}_{D_1} \;=\; 0$ due to the Majorana character of the neutralinos. Inserting (C.57) and (C.61) into (C.51), we obtain the expansion of the neutralino decay matrix

$$ \rho_{D_1}(\tilde{\chi}^0_i)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i} \;=\; \delta_{\lambda'_i\lambda_i} D_1 \,\delta^{\lambda_k\lambda'_k} + \sigma^a_{\lambda'_i\lambda_i} \,{}^{c}\Sigma^{a}_{D_1}\, (J^c)^{\lambda_k\lambda'_k} + \delta_{\lambda'_i\lambda_i} \,{}^{cd}D_1\, (J^{cd})^{\lambda_k\lambda'_k}, \tag{C.63}$$

into the scalar (first term), vector (second term) and tensor part (third term).

A similar expansion for the Z decay matrix (C.52) results in

$$ \rho_{D_2}(Z)_{\lambda'_k\lambda_k} \;=\; D_2\, \delta^{\lambda'_k\lambda_k} + {}^{c}D_2\, (J^c)^{\lambda'_k\lambda_k} + {}^{cd}D_2\, (J^{cd})^{\lambda'_k\lambda_k}, \tag{C.64}$$

with

$$ D_2 \;=\; \frac{2g^2}{3\cos^2\theta_W}(R_f^2 + L_f^2) m_Z^2, \tag{C.65}$$

$$ cD_2 \;=\; \frac{2g^2}{\cos^2\theta_W}(R_f^2 - L_f^2) m_Z (t^c_Z \cdot p_{\bar{f}}), \tag{C.66}$$

$$ {}^{cd}D_2 \;=\; \frac{g^2}{\cos^2\theta_W}(R_f^2 + L_f^2) \Big[2(t^c_Z \cdot p_{\bar{f}})(t^d_Z \cdot p_{\bar{f}}) - \tfrac{1}{2} m_Z^2 \delta^{cd} \Big]. \tag{C.67}$$

As a consequence of the completeness relation (F.14), the diagonal coefficients are linearly dependent

$$^{11}D_2 + {}^{22}D_2 + {}^{33}D_2 = -\tfrac{3}{2}D_2. \tag{C.68}$$

For large three-momentum $\mathbf{p}_Z$, the fermion $\bar{f}$ will mainly be emitted into the forward direction with respect to $\mathbf{p}_Z$, i.e. $\hat{\mathbf{p}}_Z \approx \hat{\mathbf{p}}_{\bar{f}}$, so that $(t_Z^{1,2} \cdot p_{\bar{f}}) \approx 0$ in (C.67). Therefore, for high energies ${}^{11}D_2 \approx {}^{22}D_2$, and the contributions for the non-diagonal coefficients ${}^{cd}D_2(c \neq d)$ will be small.

Appendix D

Spin-density matrices for chargino production and decay

We give the analytic formulae for the squared amplitudes for chargino production $e^+ \, e^- \to \tilde{\chi}_i^+ \, \tilde{\chi}_j^-$, with longitudinally polarized beams and for different subsequent two-body decay chains of one chargino. We use the spin density matrix formalism as in [27, 51, 66]. The amplitude squared can then be written

$$|T|^2 \;=\; |\Delta(\tilde{\chi}_i^+)|^2 \sum_{\lambda_i \lambda_i'} \rho_P(\tilde{\chi}_i^+)^{\lambda_i \lambda_i'} \, \rho_D(\tilde{\chi}_i^+)_{\lambda_i' \lambda_i}, \tag{D.1}$$

with $\rho_P(\tilde{\chi}_i^+)$ the spin density production matrix of chargino $\tilde{\chi}_i^+$, the propagator $\Delta(\tilde{\chi}_i^+) = i/[s_{\chi_i^+} - m_{\chi_i^+}^2 + i m_{\chi_i^+} \Gamma_{\chi_i^+}]$ and the chargino decay matrix $\rho_D(\tilde{\chi}_i^+)$.

D.1 Chargino production

For the production of charginos

$$e^+ + e^- \;\to\; \tilde{\chi}_i^+(p_{\chi_i^+}, \lambda_i) + \tilde{\chi}_j^-(p_{\chi_j^-}, \lambda_j), \tag{D.2}$$

with momentum p and helicity λ, the unnormalized spin-density matrix of chargino $\tilde{\chi}_i^+$ is defined as

$$\rho_P(\tilde{\chi}_i^+)^{\lambda_i \lambda_i'} \;=\; \sum_{\lambda_j} T_P^{\lambda_i \lambda_j} T_P^{\lambda_i' \lambda_j *}. \tag{D.3}$$

The helicity amplitudes are [51, 66]:

$$T_P^{\lambda_i\lambda_j}(\gamma) = -e^2\Delta(\gamma)\delta_{ij}\bar{v}(p_{e^+})\gamma^\mu u(p_{e^-})\bar{u}(p_{\chi_i^+},\lambda_i)\gamma_\mu v(p_{\chi_j^-},\lambda_j), \tag{D.4}$$

$$T_P^{\lambda_i\lambda_j}(Z) = -\frac{g^2}{\cos^2\theta_W}\Delta(Z)\bar{v}(p_{e^+})\gamma^\mu(L_eP_L+R_eP_R)u(p_{e^-}) \times\bar{u}(p_{\chi_i^+},\lambda_i)\gamma_\mu(O_{ij}'^LP_L+O_{ij}'^RP_R)v(p_{\chi_j^-},\lambda_j), \tag{D.5}$$

$$T_P^{\lambda_i\lambda_j}(\tilde{\nu}) = -g^2V_{i1}V_{j1}^*\Delta(\tilde{\nu})\bar{v}(p_{e^+})P_Rv(p_{\chi_i^+},\lambda_i)\bar{u}(p_{\chi_j^-},\lambda_j)P_Lu(p_{e^-}), \tag{D.6}$$

with the propagators

$$\Delta(\gamma) = \frac{i}{p_\gamma^2}, \quad \Delta(Z) = \frac{i}{p_Z^2-m_Z^2+im_Z\Gamma_Z}, \quad \Delta(\tilde{\nu}) = \frac{i}{p_{\tilde{\nu}}^2-m_{\tilde{\nu}}^2}. \tag{D.7}$$

The Feynman diagrams are shown in Fig. D.1.

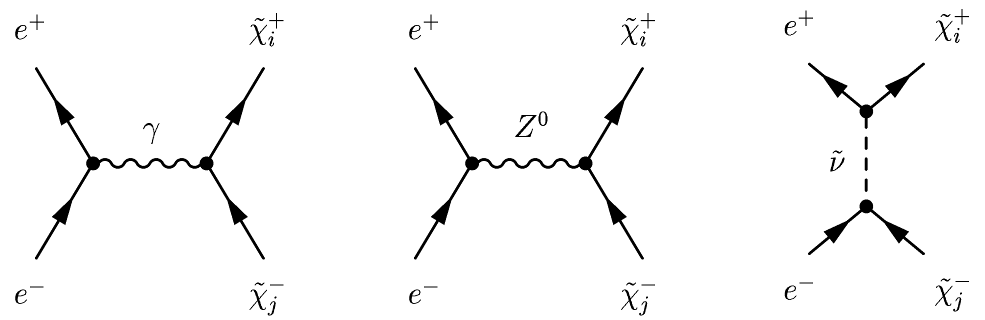


Figure D.1: Feynman diagrams for chargino production

For the polarization of the decaying chargino $\tilde{\chi}_i^+$ with momentum $p_{\chi_i^+}$ we have introduced three space like spin vectors $s^a_{\chi_i^+}$ (B.14). Then the chargino production matrix (D.1) can be expanded in terms of the Pauli matrices, see Appendix F.1:

$$\rho_P(\tilde{\chi}_i^+)^{\lambda_i\lambda_i'} = 2(\delta_{\lambda_i\lambda_i'}P + \sigma^a_{\lambda_i\lambda_i'}\Sigma_P^a), \tag{D.8}$$

where we sum over a. The factor 2 in (D.8) is due to the summation of the helicities of the second chargino $\tilde{\chi}_j^-$, whose decay will not be considered. With our choice of the spin vectors, Σ_P^3/P is the longitudinal polarization of chargino $\tilde{\chi}_i^+$, Σ_P^1/P

is the transverse polarization in the production plane and Σ_P^2/P is the polarization perpendicular to the production plane. Only if there is a non-vanishing CP phase φ_μ in the chargino sector, and only if two different charginos are produced, $e^+ e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$, the polarization Σ_P^2/P perpendicular to the production plane is non-zero. Thus it is a probe for CP violation in the production of an unequal pair of charginos. Note that Σ_P^2 also gets contributions from the finite Z width, which however do not signal CP violation.

We give the analytical formulae for P and $\Sigma_P^1, \Sigma_P^2, \Sigma_P^3$ in the laboratory system in the following sections. Lorentz invariant expressions for these functions can be found in [51, 66].

D.1.1 Chargino polarization independent quantities

The coefficient P is independent of the chargino polarization. It can be composed into contributions from the different production channels

$$P = P(\gamma\gamma) + P(\gamma Z) + P(\gamma\tilde{\nu}) + P(ZZ) + P(Z\tilde{\nu}) + P(\tilde{\nu}\tilde{\nu}) \tag{D.9}$$

which read

$$\begin{aligned}
P(\gamma\gamma) &= \delta_{ij} 2e^4 |\Delta(\gamma)|^2 (c_L + c_R) E_b^2 (E_{\chi_i^+} E_{\chi_j^-} + m_{\chi_i^+} m_{\chi_j^-} + q^2 \cos^2\theta), && \text{(D.10)}\\
P(\gamma Z) &= \delta_{ij} 2 \frac{e^2 g^2}{\cos^2\theta_W} E_b^2 Re\Big\{\Delta(\gamma)\Delta(Z)^* \Big[(L_e c_L - R_e c_R)(O_{ij}^{\prime R*} - O_{ij}^{\prime L*}) 2E_b q \cos\theta \\
&\quad + (L_e c_L + R_e c_R)(O_{ij}^{\prime L*} + O_{ij}^{\prime R*})(E_{\chi_i^+} E_{\chi_j^-} + m_{\chi_i^+} m_{\chi_j^-} + q^2\cos^2\theta)\Big]\Big\}, && \text{(D.11)}\\
P(\gamma\tilde{\nu}) &= \delta_{ij} e^2 g^2 E_b^2 c_L Re\Big\{V_{i1}^* V_{j1} \Delta(\gamma)\Delta(\tilde{\nu})^*\Big\} \\
&\quad \times (E_{\chi_i^+} E_{\chi_j^-} + m_{\chi_i^+} m_{\chi_j^-} - 2E_b q\cos\theta + q^2\cos^2\theta), && \text{(D.12)}\\
P(ZZ) &= \frac{g^4}{\cos^4\theta_W} |\Delta(Z)|^2 E_b^2 \Big[(L_e^2 c_L - R_e^2 c_R)(|O_{ij}^{\prime R}|^2 - |O_{ij}^{\prime L}|^2) 2E_b q\cos\theta \\
&\quad + (L_e^2 c_L + R_e^2 c_R)(|O_{ij}^{\prime L}|^2 + |O_{ij}^{\prime R}|^2)(E_{\chi_i^+} E_{\chi_j^-} + q^2\cos^2\theta) \\
&\quad + (L_e^2 c_L + R_e^2 c_R) 2Re\{O_{ij}^{\prime L} O_{ij}^{\prime R*}\} m_{\chi_i^+} m_{\chi_j^-}\Big], && \text{(D.13)}\\
P(Z\tilde{\nu}) &= \frac{g^4}{\cos^2\theta_W} L_e c_L E_b^2 Re\Big\{V_{i1}^* V_{j1} \Delta(Z)\Delta(\tilde{\nu})^* \\
&\quad \times [O_{ij}^{\prime L}(E_{\chi_i^+} E_{\chi_j^-} - 2E_b q\cos\theta + q^2\cos^2\theta) + O_{ij}^{\prime R} m_{\chi_i^+} m_{\chi_j^-}]\Big\}, && \text{(D.14)}\\
P(\tilde{\nu}\tilde{\nu}) &= \frac{g^4}{4} c_L |V_{i1}|^2 |V_{j1}|^2 |\Delta(\tilde{\nu})|^2 E_b^2 (E_{\chi_i^+} E_{\chi_j^-} - 2E_b q\cos\theta + q^2\cos^2\theta). && \text{(D.15)}
\end{aligned}$$

The longitudinal beam polarizations are included in the weighting factors

$$c_L = (1 - P_{e^-})(1 + P_{e^+}), \quad c_R = (1 + P_{e^-})(1 - P_{e^+}). \tag{D.16}$$

Sneutrino exchange is enhanced for $P_{e^-} < 0$ and $P_{e^+} > 0$.

D.1.2 Chargino polarization

The coefficients Σ_P^a, which describe the polarization of the chargino $\tilde{\chi}_i^+$, decompose into

$$\Sigma_P^a = \Sigma_P^a(\gamma\gamma) + \Sigma_P^a(\gamma Z) + \Sigma_P^a(\gamma\tilde{\nu}) + \Sigma_P^a(ZZ) + \Sigma_P^a(Z\tilde{\nu}) + \Sigma_P^a(\tilde{\nu}\tilde{\nu}). \tag{D.17}$$

- The contributions to the transverse polarization in the production plane are

$$\begin{aligned}
\Sigma_P^1(\gamma\gamma) &= \delta_{ij} 2e^4 |\Delta(\gamma)|^2 (c_R - c_L) E_b^2 \sin\theta (m_{\chi_i^+} E_{\chi_j^-} + m_{\chi_j^-} E_{\chi_i^+}), && \text{(D.18)}\\
\Sigma_P^1(\gamma Z) &= \delta_{ij} 2\frac{e^2 g^2}{\cos^2\theta_W} E_b^2 \sin\theta Re\Big\{\Delta(\gamma)\Delta(Z)^* \\
&\quad \times\Big[-(L_e c_L + R_e c_R)(O_{ij}^{\prime R*} - O_{ij}^{\prime L*}) m_{\chi_i^+} q\cos\theta \\
&\quad +(R_e c_R - L_e c_L)(O_{ij}^{\prime L*} + O_{ij}^{\prime R*})(m_{\chi_i^+} E_{\chi_j^-} + m_{\chi_j^-} E_{\chi_i^+})\Big]\Big\}, && \text{(D.19)}\\
\Sigma_P^1(\gamma\tilde{\nu}) &= -\delta_{ij} e^2 g^2 c_L E_b^2 \sin\theta Re\Big\{V_{i1}^* V_{j1} \Delta(\gamma)\Delta(\tilde{\nu})^*\Big\} \\
&\quad \times[m_{\chi_i^+}(E_{\chi_j^-} - q\cos\theta) + m_{\chi_j^-} E_{\chi_i^+}], && \text{(D.20)}\\
\Sigma_P^1(ZZ) &= \frac{g^4}{\cos^4\theta_W} |\Delta(Z)|^2 E_b^2 \sin\theta \Big[(L_e^2 c_L + R_e^2 c_R)(|O_{ij}^{\prime L}|^2 - |O_{ij}^{\prime R}|^2) m_{\chi_i^+} q\cos\theta \\
&\quad +(R_e^2 c_R - L_e^2 c_L) 2Re\Big\{O_{ij}^{\prime L} O_{ij}^{\prime R*}\Big\} m_{\chi_j^-} E_{\chi_i^+} \\
&\quad +(R_e^2 c_R - L_e^2 c_L)(|O_{ij}^{\prime R}|^2 + |O_{ij}^{\prime L}|^2) m_{\chi_i^+} E_{\chi_j^-}\Big], && \text{(D.21)}\\
\Sigma_P^1(Z\tilde{\nu}) &= -\frac{g^4}{\cos^2\theta_W} L_e c_L E_b^2 \sin\theta Re\Big\{V_{i1}^* V_{j1} \Delta(Z)\Delta(\tilde{\nu})^* \\
&\quad \times[O_{ij}^{\prime L} m_{\chi_i^+}(E_{\chi_j^-} - q\cos\theta) + O_{ij}^{\prime R} m_{\chi_j^-} E_{\chi_i^+}]\Big\}, && \text{(D.22)}\\
\Sigma_P^1(\tilde{\nu}\tilde{\nu}) &= -\frac{g^4}{4} c_L |V_{i1}|^2 |V_{j1}|^2 |\Delta(\tilde{\nu})|^2 E_b^2 \sin\theta m_{\chi_i^+}(E_{\chi_j^-} - q\cos\theta). && \text{(D.23)}
\end{aligned}$$

- The contributions to the transverse $\tilde{\chi}_i^+$ polarization perpendicular to the production plane are

$$\begin{aligned}
\Sigma_P^2(\gamma\gamma) &= \Sigma_P^2(\tilde{\nu}\tilde{\nu}) \;=\; 0, && \text{(D.24)}\\
\Sigma_P^2(\gamma Z) &= \delta_{ij} 2\frac{e^2 g^2}{\cos^2\theta_W}(R_e c_R - L_e c_L) Im\Big\{\Delta(\gamma)\Delta(Z)^*(O_{ij}^{\prime R*} - O_{ij}^{\prime L*})\Big\} \\
&\quad \times E_b^2 m_{\chi_j^-} q \sin\theta, && \text{(D.25)}\\
\Sigma_P^2(\gamma\tilde{\nu}) &= \delta_{ij} e^2 g^2 c_L Im\Big\{V_{i1}^* V_{j1}\Delta(\gamma)\Delta(\tilde{\nu})^*\Big\} E_b^2 m_{\chi_j^-} q \sin\theta, && \text{(D.26)}\\
\Sigma_P^2(ZZ) &= 2\frac{g^4}{\cos^4\theta_W}|\Delta(Z)|^2 (R_e^2 c_R - L_e^2 c_L) Im\Big\{O_{ij}^{\prime L} O_{ij}^{\prime R*}\Big\} E_b^2 m_{\chi_j^-} q \sin\theta, && \text{(D.27)}\\
\Sigma_P^2(Z\tilde{\nu}) &= \frac{g^4}{\cos^2\theta_W} L_e c_L Im\Big\{V_{i1}^* V_{j1} O_{ij}^{\prime R}\Delta(Z)\Delta(\tilde{\nu})^*\Big\} E_b^2 m_{\chi_j^-} q \sin\theta. && \text{(D.28)}
\end{aligned}$$

- The contributions to the longitudinal $\tilde{\chi}_i^+$ polarization are

$$\begin{aligned}
\Sigma_P^3(\gamma\gamma) &= \delta_{ij} 2 e^4 |\Delta(\gamma)|^2 (c_L - c_R) E_b^2 \cos\theta (q^2 + E_{\chi_i^+} E_{\chi_j^-} + m_{\chi_i^+} m_{\chi_j^-}), && \text{(D.29)}\\
\Sigma_P^3(\gamma Z) &= \delta_{ij} 2\frac{e^2 g^2}{\cos^2\theta_W} E_b^2 Re\Big\{\Delta(\gamma)\Delta(Z)^* \\
&\quad \times\Big[(L_e c_L - R_e c_R)(O_{ij}^{\prime R*} + O_{ij}^{\prime L*})(q^2 + E_{\chi_i^+} E_{\chi_j^-} + m_{\chi_i^+} m_{\chi_j^-})\cos\theta \\
&\quad + (L_e c_L + R_e c_R)(O_{ij}^{\prime R*} - O_{ij}^{\prime L*}) q (E_{\chi_j^-} + E_{\chi_i^+}\cos^2\theta)\Big]\Big\}, && \text{(D.30)}\\
\Sigma_P^3(\gamma\tilde{\nu}) &= -\delta_{ij} e^2 g^2 c_L E_b^2 Re\Big\{V_{i1}^* V_{j1}\Delta(\gamma)\Delta(\tilde{\nu})^*\Big\} \\
&\quad \times[q E_{\chi_j^-} - (q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta + q E_{\chi_i^+}\cos^2\theta - m_{\chi_i^+} m_{\chi_j^-}\cos\theta], && \text{(D.31)}\\
\Sigma_P^3(ZZ) &= \frac{g^4}{\cos^4\theta_W}|\Delta(Z)|^2 E_b^2\Big[(L_e^2 c_L + R_e^2 c_R)(|O_{ij}^{\prime R}|^2 - |O_{ij}^{\prime L}|^2) q (E_{\chi_j^-} + E_{\chi_i^+}\cos^2\theta) \\
&\quad + (L_e^2 c_L - R_e^2 c_R) 2 Re\Big\{O_{ij}^{\prime L} O_{ij}^{\prime R*}\Big\} m_{\chi_i^+} m_{\chi_j^-}\cos\theta \\
&\quad + (L_e^2 c_L - R_e^2 c_R)(|O_{ij}^{\prime L}|^2 + |O_{ij}^{\prime R}|^2)(q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta\Big], && \text{(D.32)}\\
\Sigma_P^3(Z\tilde{\nu}) &= \frac{g^4}{\cos^2\theta_W} L_e c_L E_b^2 Re\Big\{V_{i1}^* V_{j1}\Delta(Z)\Delta(\tilde{\nu})^*[O_{ij}^{\prime R} m_{\chi_i^+} m_{\chi_j^-}\cos\theta \\
&\quad - O_{ij}^{\prime L}(q E_{\chi_j^-} - (q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta + q E_{\chi_i^+}\cos^2\theta)]\Big\}, && \text{(D.33)}\\
\Sigma_P^3(\tilde{\nu}\tilde{\nu}) &= -\frac{g^4}{4} c_L |V_{i1}|^2 |V_{j1}|^2 |\Delta(\tilde{\nu})|^2 E_b^2 \\
&\quad \times[q E_{\chi_j^-} - (q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta + q E_{\chi_i^+}\cos^2\theta]. && \text{(D.34)}
\end{aligned}$$

D.2 Chargino decay into sneutrinos

For chargino two-body decay into sneutrinos

$$\tilde{\chi}_i^+(p_{\chi_i^+}, \lambda_i) \rightarrow \ell^+ + \tilde{\nu}_\ell; \quad \ell = e, \mu, \tau, \tag{D.35}$$

the chargino decay matrix is given by

$$\rho_D(\tilde{\chi}_i^+)_{\lambda_i'\lambda_i} = \delta_{\lambda_i'\lambda_i} D + \sigma^a_{\lambda_i'\lambda_i} \Sigma^a_D. \tag{D.36}$$

For the chargino decay into an electron or muon sneutrino the coefficients are

$$D = \frac{g^2}{2}|V_{i1}|^2(m^2_{\chi_i^+} - m^2_{\tilde{\nu}_\ell}), \tag{D.37}$$

$$\Sigma^a_D = \underset{(+)}{-} g^2 |V_{i1}|^2 m_{\chi_i^+}(s^a_{\chi_i^+} \cdot p_\ell); \quad \text{for } \ell = e, \mu, \tag{D.38}$$

where the sign in parenthesis holds for the conjugated process $\tilde{\chi}_i^- \rightarrow \ell^- \bar{\tilde{\nu}}_\ell$. For the decay into the tau sneutrino the coefficients are given by

$$D = \frac{g^2}{2}(|V_{i1}|^2 + Y_\tau^2 |U_{i2}|^2)(m^2_{\chi_i^+} - m^2_{\tilde{\nu}_\tau}), \tag{D.39}$$

$$\Sigma^a_D = \underset{(+)}{-} g^2(|V_{i1}|^2 - Y_\tau^2 |U_{i2}|^2) m_{\chi_i^+}(s^a_{\chi_i^+} \cdot p_\tau), \tag{D.40}$$

where $Y_\tau = m_\tau/(\sqrt{2} m_W \cos\beta)$ is the τ Yukawa coupling, and the sign in parenthesis holds for the conjugated process $\tilde{\chi}_i^- \rightarrow \tau^- \bar{\tilde{\nu}}_\tau$.

D.3 Chargino decay into the W boson

For the chargino two-body decay into the W boson

$$\tilde{\chi}_i^+(p_{\chi_i^+}, \lambda_i) \rightarrow \tilde{\chi}_n^0(p_{\chi_n^0}, \lambda_n) + W^+(p_W, \lambda_k), \tag{D.41}$$

the decay matrix is given by

$$\rho_{D_1}(\tilde{\chi}_i^+)^{\lambda_k \lambda_k'}_{\lambda_i' \lambda_i} = \sum_{\lambda_n} T^{\lambda_n \lambda_k}_{D_1, \lambda_i} T^{\lambda_n \lambda_k' *}_{D_1, \lambda_i'}, \tag{D.42}$$

with helicity amplitude

$$T_{D_1,\lambda_i}^{\lambda_n\lambda_k} = ig\bar{u}(p_{\chi_n^0},\lambda_n)\gamma^\mu[O_{ni}^L P_L + O_{ni}^R P_R]u(p_{\chi_i^+},\lambda_i)\varepsilon_\mu^{\lambda_k *}. \tag{D.43}$$

For the subsequent decay of the W boson

$$W^+(p_W,\lambda_k) \to f^{'}(p_{f'},\lambda_{f'}) + \bar{f}(p_{\bar{f}},\lambda_{\bar{f}}), \tag{D.44}$$

the decay matrix is

$$\rho_{D_2}(W^+)_{\lambda_k'\lambda_k} = \sum_{\lambda_{f'},\lambda_{\bar{f}}} T_{D_2,\lambda_k}^{\lambda_{f'}\lambda_{\bar{f}}} T_{D_2,\lambda_k'}^{\lambda_{f'}\lambda_{\bar{f}}*} \tag{D.45}$$

and the helicity amplitude

$$T_{D_2,\lambda_k}^{\lambda_{f'}\lambda_{\bar{f}}} = i\frac{g}{\sqrt{2}}\bar{u}(p_{f'},\lambda_{f'})\gamma^\mu P_L v(p_{\bar{f}},\lambda_{\bar{f}})\varepsilon_\mu^{\lambda_k}. \tag{D.46}$$

The W polarization vectors $\varepsilon_\mu^{\lambda_k}, \lambda_k = 0, \pm 1$, are defined in (B.38). With the set of chargino spin vectors $s^a_{\chi_i^+}$, given in (B.14), we obtain for the chargino decay matrix

$$\rho_{D_1}(\tilde{\chi}_i^+)_{\lambda_i'\lambda_i}^{\lambda_k\lambda_k'} = (\delta_{\lambda_i'\lambda_i} D_1^{\mu\nu} + \sigma^a_{\lambda_i'\lambda_i}\Sigma_{D_1}^{a\;\mu\nu})\varepsilon_\mu^{\lambda_k *}\varepsilon_\nu^{\lambda_k'} \tag{D.47}$$

and for the W boson decay matrix

$$\rho_{D_2}(W^+)_{\lambda_k'\lambda_k} = D_2^{\mu\nu}\varepsilon_\mu^{\lambda_k}\varepsilon_\nu^{\lambda_k' *}. \tag{D.48}$$

The expansion coefficients are

$$\begin{aligned} D_1^{\mu\nu} &= g^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)[2p^\mu_{\chi_i^+}p^\nu_{\chi_i^+} - (p^\mu_{\chi_i^+}p^\nu_W + p^\nu_{\chi_i^+}p^\mu_W) - \tfrac{1}{2}(m^2_{\chi_i^+} + m^2_{\chi_n^0} - m_W^2)g^{\mu\nu}] \\ &\quad + 2g^2 Re(O_{ni}^{R*}O_{ni}^L)m_{\chi_i^+}m_{\chi_n^0}g^{\mu\nu} \overset{+}{\underset{(-)}{}} ig^2(|O_{ni}^R|^2 - |O_{ni}^L|^2)\epsilon^{\mu\alpha\nu\beta}\, p_{\chi_i^+,\alpha}\, p_{W,\beta}, \qquad \text{(D.49)} \\ \Sigma_{D_1}^{a\;\mu\nu} &= \overset{+}{\underset{(-)}{}} g^2(|O_{ni}^R|^2 - |O_{ni}^L|^2)m_{\chi_i^+}[s^{a,\mu}_{\chi_i^+}(p^\nu_{\chi_i^+} - p^\nu_W) + s^{a,\nu}_{\chi_i^+}(p^\mu_{\chi_i^+} - p^\mu_W) + (s^a_{\chi_i^+}\cdot p_W)g^{\mu\nu}] \\ &\quad - ig^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)m_{\chi_i^+}\epsilon^{\mu\alpha\nu\beta}s^a_{\chi_i^+,\,\alpha}(p_{\chi_i^+,\,\beta} - p_{W,\beta}) \\ &\quad + 2ig^2 Re(O_{ni}^{R*}O_{ni}^L)m_{\chi_n^0}\epsilon^{\mu\alpha\nu\beta}s^a_{\chi_i^+,\alpha}\, p_{\chi_i^+,\beta} \\ &\quad - 2ig^2 Im(O_{ni}^{R*}O_{ni}^L)m_{\chi_n^0}(s^{a,\mu}_{\chi_i^+}p^\nu_{\chi_i^+} - s^{a,\nu}_{\chi_i^+}p^\mu_{\chi_i^+}); \quad (\epsilon_{0123} = 1), \qquad \text{(D.50)} \end{aligned}$$

and

$$D_2^{\mu\nu} = g^2(-2p_{\bar{f}}^{\mu}p_{\bar{f}}^{\nu} + p_W^{\mu}p_{\bar{f}}^{\nu} + p_{\bar{f}}^{\mu}p_W^{\nu} - \tfrac{1}{2}m_W^2 g^{\mu\nu}) \underset{(+)}{-} i g^2 \epsilon^{\mu\alpha\nu\beta} p_{W,\alpha}\, p_{\bar{f},\beta}, \tag{D.51}$$

where here, and in the following, the signs in parenthesis hold for the charge conjugated processes, $\tilde{\chi}_i^- \to W^-\tilde{\chi}_n^0$ and $W^- \to \bar{f}' f$, respectively. In (D.47) and (D.48) we use the expansion (F.12) for the W polarization vectors

$$\varepsilon_\mu^{\lambda_k}\varepsilon_\nu^{\lambda_k'*} = \tfrac{1}{3}\delta^{\lambda_k'\lambda_k} I_{\mu\nu} - \frac{i}{2m_W}\epsilon_{\mu\nu\rho\sigma}p_W^{\rho}t_W^{c,\sigma}(J^c)^{\lambda_k'\lambda_k} - \tfrac{1}{2}t_{W,\mu}^c t_{W,\nu}^d (J^{cd})^{\lambda_k'\lambda_k}. \tag{D.52}$$

The decay matrices can be expanded in terms of the spin matrices J^c and J^{cd}, given in Appendix F.2. The first term of the decay matrix ρ_{D_1} (D.47), which is independent of the chargino polarization, then is

$$D_1^{\mu\nu}\varepsilon_\mu^{\lambda_k *}\varepsilon_\nu^{\lambda_k'} = D_1\delta^{\lambda_k\lambda_k'} + {}^cD_1(J^c)^{\lambda_k\lambda_k'} + {}^{cd}D_1(J^{cd})^{\lambda_k\lambda_k'}, \tag{D.53}$$

with

$$\begin{aligned} D_1 &= \tfrac{1}{6}g^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)\Big[m_{\chi_i^+}^2 + m_{\chi_n^0}^2 - 2m_W^2 + \frac{(m_{\chi_i^+}^2 - m_{\chi_n^0}^2)^2}{m_W^2}\Big] \\ &\quad -2g^2 Re(O_{ni}^{R*}O_{ni}^L)m_{\chi_i^+}m_{\chi_n^0}, & \text{(D.54)} \\ {}^cD_1 &= \underset{(-)}{+} g^2(|O_{ni}^R|^2 - |O_{ni}^L|^2)m_W(t_W^c\cdot p_{\chi_i^+}), & \text{(D.55)} \\ {}^{cd}D_1 &= -g^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)\left[(t_W^c\cdot p_{\chi_i^+})(t_W^d\cdot p_{\chi_i^+}) + \tfrac{1}{4}(m_{\chi_i^+}^2 + m_{\chi_n^0}^2 - m_W^2)\delta^{cd}\right] \\ &\quad +g^2 Re(O_{ni}^{R*}O_{ni}^L)m_{\chi_i^+}m_{\chi_n^0}\delta^{cd}. & \text{(D.56)} \end{aligned}$$

As a consequence of the completeness relation (F.14), the diagonal coefficients are linearly dependent

$${}^{11}D_1 + {}^{22}D_1 + {}^{33}D_1 = -\tfrac{3}{2}D_1. \tag{D.57}$$

For large chargino momentum $\mathbf{p}_{\chi_i^+}$, the W boson will mainly be emitted into the forward direction with respect to $\mathbf{p}_{\chi_i^+}$, i.e. $\hat{\mathbf{p}}_{\chi_i^+} \approx \hat{\mathbf{p}}_W$, with $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. Therefore, for high energies we have $(t_W^{1,2}\cdot p_{\chi_i^+}) \approx 0$ in (D.56), and in ${}^{11}D_1 \approx {}^{22}D_1$.

For the second term of ρ_{D_1} (D.47), which depends on the polarization of the decaying chargino, we obtain

$$\Sigma^{a\ \mu\nu}_{D_1}\varepsilon^{\lambda_k *}_{\mu}\varepsilon^{\lambda'_k}_{\nu} = \Sigma^{a}_{D_1}\delta^{\lambda_k\lambda'_k} + {}^{c}\Sigma^{a}_{D_1}(J^c)^{\lambda_k\lambda'_k} + {}^{cd}\Sigma^{a}_{D_1}(J^{cd})^{\lambda_k\lambda'_k}, \tag{D.58}$$

with

$$\Sigma^{a}_{D_1} = \overset{+}{\scriptstyle(-)}\tfrac{2}{3}g^2(|O^R_{ni}|^2 - |O^L_{ni}|^2)m_{\chi^+_i}(s^a_{\chi^+_i}\cdot p_W)[\frac{m^2_{\chi^+_i} - m^2_{\chi^0_n}}{2m^2_W} - 1], \tag{D.59}$$

$$\begin{aligned} {}^{c}\Sigma^{a}_{D_1} = {}& \frac{g^2}{m_W}\left[(|O^R_{ni}|^2 + |O^L_{ni}|^2)m_{\chi^+_i} - 2Re(O^{R*}_{ni}O^L_{ni})m_{\chi^0_n}\right]\times \\ & \left[(t^c_W\cdot p_{\chi^+_i})(s^a_{\chi^+_i}\cdot p_W) + \tfrac{1}{2}(t^c_W\cdot s^a_{\chi^+_i})(m^2_{\chi^0_n} - m^2_{\chi^+_i} + m^2_W)\right] \\ & + \frac{2g^2}{m_W}Im(O^{R*}_{ni}O^L_{ni})m_{\chi^0_n}\epsilon_{\mu\nu\rho\sigma}\, s^{a,\mu}_{\chi^+_i}\, p^{\nu}_{\chi^+_i}\, p^{\rho}_W\, t^{c,\sigma}_W, \end{aligned} \tag{D.60}$$

$$\begin{aligned} {}^{cd}\Sigma^{a}_{D_1} = {}& \overset{+}{\scriptstyle(-)}\tfrac{1}{2}g^2(|O^R_{ni}|^2 - |O^L_{ni}|^2)m_{\chi^+_i}\times \\ & \left[(s^a_{\chi^+_i}\cdot p_W)\delta^{cd} - (t^c_W\cdot p_{\chi^+_i})(t^d_W\cdot s^a_{\chi^+_i}) - (t^d_W\cdot p_{\chi^+_i})(t^c_W\cdot s^a_{\chi^+_i})\right]. \end{aligned} \tag{D.61}$$

Inserting (D.53) and (D.58) into (D.47), we obtain the expansion of the chargino decay matrix in the scalar (first term), vector (second term) and tensor part (third term):

$$\begin{aligned} \rho_{D_1}(\tilde{\chi}^+_i)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i} = {}& (\delta_{\lambda'_i\lambda_i}\ D_1 + \sigma^a_{\lambda'_i\lambda_i}\ \Sigma^a_{D_1})\,\delta^{\lambda_k\lambda'_k} + \\ & (\delta_{\lambda'_i\lambda_i}\ {}^{c}D_1 + \sigma^a_{\lambda'_i\lambda_i}\ {}^{c}\Sigma^a_{D_1})\,(J^c)^{\lambda_k\lambda'_k} + \\ & (\delta_{\lambda'_i\lambda_i}\ {}^{cd}D_1 + \sigma^a_{\lambda'_i\lambda_i}\ {}^{cd}\Sigma^a_{D_1})\,(J^{cd})^{\lambda_k\lambda'_k}. \end{aligned} \tag{D.62}$$

A similar expansion for the W decay matrix (D.48), results in

$$\rho_{D_2}(W^+)_{\lambda'_k\lambda_k} = D_2\,\delta^{\lambda'_k\lambda_k} + {}^{c}D_2\,(J^c)^{\lambda'_k\lambda_k} + {}^{cd}D_2\,(J^{cd})^{\lambda'_k\lambda_k}, \tag{D.63}$$

with

$$D_2 = \tfrac{1}{3}g^2 m^2_W, \tag{D.64}$$

$${}^{c}D_2 = \overset{-}{\scriptstyle(+)}g^2 m_W(t^c_W\cdot p_{\bar f}), \tag{D.65}$$

$${}^{cd}D_2 = g^2\left[(t^c_W\cdot p_{\bar f})(t^d_W\cdot p_{\bar f}) - \tfrac{1}{4}m^2_W\delta^{cd}\right]. \tag{D.66}$$

The diagonal coefficients are linearly dependent

$${}^{11}D_2 + {}^{22}D_2 + {}^{33}D_2 = -\tfrac{3}{2}D_2. \tag{D.67}$$

Appendix E

Neutralino and Chargino two-body decay widths

For the two-body decay of a massive particle in its rest frame

$$a \to b + c \tag{E.1}$$

the decay width of particle a is

$$\Gamma(a \to b\ c) = \frac{|\mathbf{p}_b|}{32\ \pi^2\ m_a^2} \int |T|^2 d\Omega = \frac{\sqrt{\lambda(m_a^2, m_b^2, m_c^2)}}{16\ \pi\ m_a^3} |T|^2, \tag{E.2}$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$ and $|T|^2$ the amplitude squared for decay (E.1), where we average over the spins of particle a and sum over the spins of particles b,c.

E.1 Neutralino decay widths

We give the tree-level formulae for the neutralino two-body decay widths $\Gamma_{\chi_i^0}$ for the decays

$$\tilde{\chi}_i^0 \to \tilde{e}_{R,L} e,\ \tilde{\mu}_{R,L}\mu,\ \tilde{\tau}_m\tau,\ \tilde{\nu}_\ell\bar{\nu}_\ell,\ \tilde{\chi}_n^0 Z,\ \tilde{\chi}_m^{\mp} W^{\pm},\ \tilde{\chi}_n^0 H_1^0;\ \ell = e, \mu, \tau;\ m = 1, 2. \tag{E.3}$$

- Neutralino decay into right selectrons or smuons: $\tilde{\chi}_i^0 \to \tilde{\ell}_R^+ + \ell^-;\ \ell = e, \mu$

$$|T|^2(\tilde{\chi}_i^0 \to \tilde{\ell}_R^+ \ell^-) \;=\; \frac{g^2}{2}|f_{\ell i}^R|^2(m_{\chi_i^0}^2 - m_{\tilde{\ell}_R}^2), \tag{E.4}$$

$$\Gamma(\tilde{\chi}_i^0 \to \tilde{\ell}_R^+ \ell^-) \;=\; \frac{(m_{\chi_i^0}^2 - m_{\tilde{\ell}_R}^2)^2}{32\,\pi\, m_{\chi_i^0}^3} g^2 |f_{\ell i}^R|^2. \tag{E.5}$$

- Neutralino decay into left selectrons or smuons: $\tilde{\chi}_i^0 \to \tilde{\ell}_L^+ + \ell^-;\ \ell = e, \mu$

$$|T|^2(\tilde{\chi}_i^0 \to \tilde{\ell}_L^+ \ell^-) \;=\; \frac{g^2}{2}|f_{\ell i}^L|^2(m_{\chi_i^0}^2 - m_{\tilde{\ell}_L}^2), \tag{E.6}$$

$$\Gamma(\tilde{\chi}_i^0 \to \tilde{\ell}_L^+ \ell^-) \;=\; \frac{(m_{\chi_i^0}^2 - m_{\tilde{\ell}_L}^2)^2}{32\,\pi\, m_{\chi_i^0}^3} g^2 |f_{\ell i}^L|^2. \tag{E.7}$$

- Neutralino decay into staus: $\tilde{\chi}_i^0 \to \tilde{\tau}_m^+ + \tau^-;\ m = 1, 2$

$$|T|^2(\tilde{\chi}_i^0 \to \tilde{\tau}_m^+ \tau^-) \;=\; \frac{g^2}{2}(|a_{mi}^{\tilde{\tau}}|^2 + |b_{mi}^{\tilde{\tau}}|^2)(m_{\chi_i^0}^2 - m_{\tilde{\tau}_m}^2), \tag{E.8}$$

$$\Gamma(\tilde{\chi}_i^0 \to \tilde{\tau}_m^+ \tau^-) \;=\; \frac{(m_{\chi_i^0}^2 - m_{\tilde{\tau}_m}^2)^2}{32\,\pi\, m_{\chi_i^0}^3} g^2 (|a_{mi}^{\tilde{\tau}}|^2 + |b_{mi}^{\tilde{\tau}}|^2). \tag{E.9}$$

- Neutralino decay into sneutrinos: $\tilde{\chi}_i^0 \to \tilde{\nu}_\ell + \bar{\nu}_\ell;\ \ell = e, \mu, \tau$

$$|T|^2(\tilde{\chi}_i^0 \to \tilde{\nu}_\ell \bar{\nu}_\ell) \;=\; \frac{g^2}{2}|f_{\nu i}^L|^2(m_{\chi_i^0}^2 - m_{\tilde{\nu}_\ell}^2), \tag{E.10}$$

$$\Gamma(\tilde{\chi}_i^0 \to \tilde{\nu}_\ell \bar{\nu}_\ell) \;=\; \frac{(m_{\chi_i^0}^2 - m_{\tilde{\nu}_\ell}^2)^2}{32\,\pi\, m_{\chi_i^0}^3} g^2 |f_{\nu i}^L|^2. \tag{E.11}$$

- Neutralino decay into Z boson: $\tilde{\chi}_i^0 \to Z + \tilde{\chi}_n^0$

$$|T|^2(\tilde{\chi}_i^0 \to Z\tilde{\chi}_n^0) = \frac{g^2}{\cos^2\theta_W}\Big\{6\, m_{\chi_i^0} m_{\chi_n^0}[(ReO''^L_{ni})^2 - (ImO''^L_{ni})^2] +$$

$$+ |O''^L_{ni}|^2\Big[m^2_{\chi_i^0} + m^2_{\chi_n^0} - 2m_Z^2 + \frac{(m^2_{\chi_i^0} - m^2_{\chi_n^0})^2}{m_Z^2}\Big]\Big\}, \tag{E.12}$$

$$\Gamma(\tilde{\chi}_i^0 \to Z\tilde{\chi}_n^0) = \frac{\sqrt{\lambda(m^2_{\chi_i^0}, m^2_{\chi_n^0}, m_Z^2)}}{16\,\pi\, m^3_{\chi_i^0}}|T|^2(\tilde{\chi}_i^0 \to Z\tilde{\chi}_n^0). \tag{E.13}$$

- Neutralino decay into W boson: $\tilde{\chi}_i^0 \to W^+ + \tilde{\chi}_j^-$

$$|T|^2(\tilde{\chi}_i^0 \to W^+\tilde{\chi}_j^-) = -6\, g^2 m_{\chi_i^0} m_{\chi_j^-} Re(O^{R*}_{ij} O^L_{ij}) +$$

$$+ \frac{g^2}{2}(|O^R_{ij}|^2 + |O^L_{ij}|^2)\Big[m^2_{\chi_i^0} + m^2_{\chi_j^-} - 2m_W^2 + \frac{(m^2_{\chi_i^0} - m^2_{\chi_j^-})^2}{m_W^2}\Big], \tag{E.14}$$

$$\Gamma(\tilde{\chi}_i^0 \to W^+\tilde{\chi}_j^-) = \frac{\sqrt{\lambda(m^2_{\chi_i^0}, m^2_{\chi_j^-}, m_W^2)}}{16\,\pi\, m^3_{\chi_i^0}}|T|^2(\tilde{\chi}_i^0 \to W^+\tilde{\chi}_j^-). \tag{E.15}$$

- Neutralino decay into Higgs boson: $\tilde{\chi}_i^0 \to H_1^0 + \tilde{\chi}_n^0$

$$|T|^2(\tilde{\chi}_i^0 \to H_1^0\tilde{\chi}_n^0) \;=\; 2g^2 m_{\chi_i^0} m_{\chi_n^0}[Re(H^L_{ni})Re(H^R_{ni}) + Im(H^L_{ni})Im(H^R_{ni})] \;+$$

$$\frac{g^2}{2}(m^2_{\chi_i^0} + m^2_{\chi_n^0} - m^2_{H_1^0})(|H^L_{ni}|^2 + |H^R_{ni}|^2), \tag{E.16}$$

$$\Gamma(\tilde{\chi}_i^0 \to H_1^0\tilde{\chi}_n^0) \;=\; \frac{\sqrt{\lambda(m^2_{\chi_i^0}, m^2_{\chi_n^0}, m^2_{H_1^0})}}{16\,\pi\, m^3_{\chi_i^0}}|T|^2(\tilde{\chi}_i^0 \to H_1^0\tilde{\chi}_n^0), \tag{E.17}$$

with $H^L_{ij} = Q''^*_{ij}\cos\alpha - S''^*_{ij}\sin\alpha$, $H^R_{ij} = H^{L*}_{ij}$ and

$$Q''_{ij} \;=\; \frac{1}{2\cos\theta_W}\left[(N_{i3}\cos\beta + N_{i4}\sin\beta)N_{j2} + (N_{j3}\cos\beta + N_{j4}\sin\beta)N_{i2}\right], \tag{E.18}$$

$$S''_{ij} \;=\; \frac{1}{2\cos\theta_W}\left[(N_{i4}\cos\beta - N_{i3}\sin\beta)N_{j2} + (N_{j4}\cos\beta - N_{j3}\sin\beta)N_{i2}\right]. \tag{E.19}$$

The Higgs mixing angle α for small $\tan\beta$ can be obtained approximately by diagonalization of the neutral Higgs mass matrix

$$M^H = \begin{pmatrix} m_Z^2\cos^2\beta + m_A^2\sin^2\beta & -(m_Z^2+m_A^2)\cos\beta\sin\beta \\ -(m_Z^2+m_A^2)\cos\beta\sin\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta + \delta_t \end{pmatrix}, \qquad \text{(E.20)}$$

which includes the largest term (top-loop) of the one-loop radiative corrections

$$\delta_t = \frac{3g^2 m_t^4}{16\,\pi^2 m_W^2 \sin^2\beta}\log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right). \qquad \text{(E.21)}$$

We obtain for the Higgs masses

$$(m_{H_1^0})^2 = \frac{1}{2}\left[M_{11}^H + M_{22}^H - \sqrt{(M_{11}^H - M_{22}^H)^2 + 4\,(M_{12}^H)^2}\,\right], \qquad \text{(E.22)}$$

$$(m_{H_2^0})^2 = \frac{1}{2}\left[M_{11}^H + M_{22}^H + \sqrt{(M_{11}^H - M_{22}^H)^2 + 4\,(M_{12}^H)^2}\,\right]. \qquad \text{(E.23)}$$

For the mixing angle we obtain

$$\cos\alpha = \frac{-M_{12}^H}{\sqrt{(M_{12}^H)^2 + [M_{11}^H - (m_{H_1^0})^2]^2}}, \qquad \text{(E.24)}$$

$$\sin\alpha = \frac{M_{11}^H - (m_{H_1^0})^2}{\sqrt{(M_{12}^H)^2 + [M_{11}^H - (m_{H_1^0})^2]^2}}. \qquad \text{(E.25)}$$

If we choose a large Higgs mass parameter, e.g. $m_A = 1$ TeV, we have very approximately $m_{H_1^0} \approx 115 - 130$ GeV and $m_{H_2^0} \approx M_A$, which follows from (E.22) and (E.23). In addition, explicit CP violation is not relevant for the lightest Higgs state [67].

E.2 Chargino decay widths

We give the tree-level formulae for the chargino two-body decay widths $\Gamma_{\chi_i^+}$ for the decays

$$\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0,\ \tilde{e}_L^+\nu_e,\ \tilde{\mu}_L^+\nu_\mu,\ \tilde{\tau}_{1,2}^+\nu_\tau,\ e^+\tilde{\nu}_e,\ \mu^+\tilde{\nu}_\mu,\ \tau^+\tilde{\nu}_\tau. \qquad \text{(E.26)}$$

For the heavy chargino $\tilde{\chi}_2^+$ also the decays into the lightest neutral Higgs boson H_1^0 and the Z boson are possible

$$\tilde{\chi}_2^+ \to \tilde{\chi}_1^+ Z,\ \tilde{\chi}_1^+ H_1^0. \tag{E.27}$$

- Chargino decay into W boson: $\tilde{\chi}_i^+ \to W^+ + \tilde{\chi}_n^0$

$$|T|^2(\tilde{\chi}_i^+ \to W^+ \tilde{\chi}_n^0) = \frac{g^2}{2}(|O_{ni}^R|^2 + |O_{ni}^L|^2)\Big[m_{\chi_i^+}^2 + m_{\chi_n^0}^2 - 2m_W^2 + \frac{(m_{\chi_i^+}^2 - m_{\chi_n^0}^2)^2}{m_W^2}\Big]$$
$$-6g^2 m_{\chi_i^+} m_{\chi_n^0} Re(O_{ni}^{R*} O_{ni}^L), \tag{E.28}$$

$$\Gamma(\tilde{\chi}_i^+ \to W^+ \tilde{\chi}_n^0) = \frac{\sqrt{\lambda(m_{\chi_i^+}^2, m_{\chi_n^0}^2, m_W^2)}}{16\,\pi\, m_{\chi_i^+}^3}|T|^2(\tilde{\chi}_i^+ \to W^+ \tilde{\chi}_n^0). \tag{E.29}$$

- Chargino decay into selectrons or smuons: $\tilde{\chi}_i^+ \to \tilde{\ell}_L^+ + \nu_\ell;\ \ell = e, \mu$

$$|T|^2(\tilde{\chi}_i^+ \to \tilde{\ell}_L^+ \nu_\ell) = \frac{g^2}{2}|U_{i1}|^2(m_{\chi_i^+}^2 - m_{\tilde{\ell}}^2), \tag{E.30}$$

$$\Gamma(\tilde{\chi}_i^+ \to \tilde{\ell}_L^+ \nu_\ell) = \frac{(m_{\chi_i^+}^2 - m_{\tilde{\ell}}^2)^2}{32\,\pi\, m_{\chi_i^+}^3} g^2 |U_{i1}|^2. \tag{E.31}$$

- Chargino decay into staus: $\tilde{\chi}_i^+ \to \tilde{\tau}_m^+ + \nu_\tau;\ m = 1, 2$

$$|T|^2(\tilde{\chi}_i^+ \to \tilde{\tau}_m^+ \nu_\tau) = \frac{g^2}{2}|\ell_{mi}^{\tilde{\tau}}|^2(m_{\chi_i^+}^2 - m_{\tilde{\tau}_m}^2), \tag{E.32}$$

$$\Gamma(\tilde{\chi}_i^+ \to \tilde{\tau}_m^+ \nu_\tau) = \frac{(m_{\chi_i^+}^2 - m_{\tilde{\tau}_m}^2)^2}{32\,\pi\, m_{\chi_i^+}^3} g^2 |\ell_{mi}^{\tilde{\tau}}|^2, \tag{E.33}$$

  and $\ell_{mi}^{\tilde{\tau}}$ defined in (A.37).

- Chargino decay into electron or muon sneutrinos: $\tilde{\chi}_i^+ \to \ell^+ + \tilde{\nu}_\ell;\ \ell = e, \mu$

$$|T|^2(\tilde{\chi}_i^+ \to \ell^+ \tilde{\nu}_\ell) = \frac{g^2}{2}|V_{i1}|^2(m_{\chi_i^+}^2 - m_{\tilde{\nu}}^2), \tag{E.34}$$

$$\Gamma(\tilde{\chi}_i^+ \to \ell^+ \tilde{\nu}_\ell) = \frac{(m_{\chi_i^+}^2 - m_{\tilde{\nu}}^2)^2}{32\,\pi\, m_{\chi_i^+}^3} g^2 |V_{i1}|^2. \tag{E.35}$$

- Chargino decay into tau sneutrino: $\tilde{\chi}_i^+ \to \tau^+ + \tilde{\nu}_\tau$

$$|T|^2(\tilde{\chi}_i^+ \to \tau^+ \tilde{\nu}_\tau) = \frac{g^2}{2}(|V_{i1}|^2 + Y_\tau^2 |U_{i2}|^2)(m_{\chi_i^+}^2 - m_{\tilde{\nu}}^2), \tag{E.36}$$

$$\Gamma(\tilde{\chi}_i^+ \to \tau^+ \tilde{\nu}_\tau) = \frac{(m_{\chi_i^+}^2 - m_{\tilde{\nu}}^2)^2}{32\,\pi\, m_{\chi_i^+}^3} g^2(|V_{i1}|^2 + Y_\tau^2 |U_{i2}|^2), \tag{E.37}$$

and Y_τ defined in (A.40).

- Chargino decay into Z boson: $\tilde{\chi}_2^+ \to Z + \tilde{\chi}_1^+$

$$|T|^2(\tilde{\chi}_2^+ \to Z\tilde{\chi}_1^+) = \frac{g^2}{\cos^2\theta_W}\Big\{ - 6\, m_{\chi_1^+} m_{\chi_2^+} Re(O_{12}^{R*} O_{12}^L) +$$

$$+ \frac{1}{2}(|O_{12}^R|^2 + |O_{12}^L|^2)\Big[m_{\chi_2^+}^2 + m_{\chi_1^+}^2 - 2m_Z^2 + \frac{(m_{\chi_2^+}^2 - m_{\chi_1^+}^2)^2}{m_Z^2}\Big]\Big\}, \tag{E.38}$$

$$\Gamma(\tilde{\chi}_2^+ \to Z\tilde{\chi}_1^+) = \frac{\sqrt{\lambda(m_{\chi_2^+}^2, m_{\chi_1^+}^2, m_Z^2)}}{16\,\pi\, m_{\chi_2^+}^3} |T|^2(\tilde{\chi}_2^+ \to Z\tilde{\chi}_1^+). \tag{E.39}$$

- Chargino decay into Higgs boson: $\tilde{\chi}_2^+ \to H_1^0 + \tilde{\chi}_1^+$

$$|T|^2(\tilde{\chi}_2^+ \to H_1^0\tilde{\chi}_1^+) = 2g^2 m_{\chi_1^+} m_{\chi_2^+}[Re(F_{12}^L)Re(F_{12}^R) + Im(F_{12}^L)Im(F_{12}^R)] + \frac{g^2}{2}(m_{\chi_1^+}^2 + m_{\chi_2^+}^2 - m_{H_1^0}^2)(|F_{12}^L|^2 + |F_{12}^R|^2), \tag{E.40}$$

$$\Gamma(\tilde{\chi}_2^+ \to H_1^0\tilde{\chi}_1^+) = \frac{\sqrt{\lambda(m_{\chi_2^+}^2, m_{\chi_1^+}^2, m_{H_1^0}^2)}}{16\,\pi\, m_{\chi_2^+}^3} |T|^2(\tilde{\chi}_2^+ \to H_1^0\tilde{\chi}_1^+), \tag{E.41}$$

with $F_{ij}^L = \frac{1}{\sqrt{2}}(U_{i2}^* V_{j1}^* \sin\alpha - U_{i1}^* V_{j2}^* \cos\alpha)$ and $F_{ij}^R = F_{ji}^{L*}$.

The Higgs mixing angle α is given in (E.24) and (E.25).

Appendix F

Spin formalism for fermions and bosons

F.1 Bouchiat-Michel formulae for spin $\frac{1}{2}$ particles

For the calculation of cross sections we expand the spin density matrices in terms of the Pauli matrices, see e.g. (C.1), (C.10) for neutralinos. This expansion is straight forward if for the neutralinos or charginos a set of spin-basis vectors $s^{a,\mu}$ has been introduced, see (B.14). Together with $\hat{p}^\mu = p^\mu/m$ they form an orthonormal set

$$\hat{p} \cdot s^a = 0, \tag{F.1}$$
$$s^a \cdot s^b = -\delta^{ab}, \tag{F.2}$$
$$s^a_\mu s^a_\nu = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}, \quad (\text{sum over } a). \tag{F.3}$$

The helicity spinors are normalized by

$$\bar{u}(p,\lambda)\, u(p,\lambda') = 2\, m\, \delta_{\lambda\lambda'}, \tag{F.4}$$
$$\bar{v}(p,\lambda)\, v(p,\lambda') = -2\, m\, \delta_{\lambda\lambda'}. \tag{F.5}$$

The Bouchiat-Michel formulae for massive spin $1/2$ particles are then [68]

$$u(p,\lambda')\, \bar{u}(p,\lambda) = \frac{1}{2}\, [\delta_{\lambda\lambda'} + \gamma_5\; \not{s}^a \sigma^a_{\lambda\lambda'}](\not{p} + m), \tag{F.6}$$
$$v(p,\lambda')\, \bar{v}(p,\lambda) = \frac{1}{2}\, [\delta_{\lambda'\lambda} + \gamma_5\; \not{s}^a \sigma^a_{\lambda'\lambda}](\not{p} - m), \quad (\text{sum over } a). \tag{F.7}$$

F.2 Spin formulae for spin 1 particles

The Bouchiat-Michel formulae for spin 1/2 particles can be generalized for higher spins [69]. In order to describe the polarization states of a spin 1 boson, we have introduced a set of spin vectors t^a_μ, see (B.37). Note that they are are not helicity eigenstates like the polarization vectors $\varepsilon^{\lambda_k}_\mu$, defined in (B.38). The spin vectors t^a_μ and $\hat{k}^\mu = k^\mu/m$ form an orthonormal set:

$$\hat{k} \cdot t^a = 0, \tag{F.8}$$

$$t^a \cdot t^b = -\delta^{ab}, \tag{F.9}$$

$$t^a_\mu \, t^a_\nu = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}, \quad (\text{sum over } a). \tag{F.10}$$

The 3×3 spin 1 matrices J^c obey $[J^c, J^d] = i\epsilon_{cde} J^e$ and are given below. We can define six further matrices

$$J^{cd} = J^c J^d + J^d J^c - \tfrac{4}{3}\delta^{cd}, \tag{F.11}$$

with $J^{11} + J^{22} + J^{33} = 0$. They are the components of a symmetric, traceless tensor. We now can expand [69]

$$\varepsilon^{\lambda_k}_\mu \varepsilon^{\lambda'_k *}_\nu = \tfrac{1}{3}\delta^{\lambda'_k \lambda_k} I_{\mu\nu} - \frac{i}{2m}\epsilon_{\mu\nu\rho\sigma} p^\rho_Z t^{c,\sigma} (J^c)^{\lambda'_k \lambda_k} - \tfrac{1}{2} t^c_\mu t^d_\nu (J^{cd})^{\lambda'_k \lambda_k}, \tag{F.12}$$

summed over c, d. The tensor

$$I_{\mu\nu} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2} \tag{F.13}$$

guarantees the completeness relation of the polarization vectors

$$\sum_{\lambda_k} \varepsilon^{\lambda_k *}_\mu \varepsilon^{\lambda_k}_\nu = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}. \tag{F.14}$$

The second term of (F.12) describes the vector polarization and the third term describes the tensor polarization of the boson.

In the linear basis (B.37) the spin-1 matrices are defined as $(J_L^c)^{jk} = -i\epsilon_{cjk}$:

$$J_L^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad J_L^2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad J_L^3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{F.15}$$

$$J_L^{11} = \begin{pmatrix} -\frac{4}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix}, \quad J_L^{22} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{4}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix}, \quad J_L^{33} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & -\frac{4}{3} \end{pmatrix}, \tag{F.16}$$

$$J_L^{12} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; J_L^{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \; J_L^{13} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{F.17}$$

The matrices J^c and J^{cd} in the circular basis, see (B.38), are obtained by the unitary transformations $J^c = A^\dagger \cdot J_L^c \cdot A$ and $J^{cd} = A^\dagger \cdot J_L^{cd} \cdot A$, respectively, with

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}; \quad A^\dagger = A^{-1}, \tag{F.18}$$

$$J^1 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \; J^2 = \begin{pmatrix} 0 & \frac{i}{\sqrt{2}} & 0 \\ -\frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & -\frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \; J^3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{F.19}$$

$$J^{11} = \begin{pmatrix} -\frac{1}{3} & 0 & 1 \\ 0 & \frac{2}{3} & 0 \\ 1 & 0 & -\frac{1}{3} \end{pmatrix}, \quad J^{22} = \begin{pmatrix} -\frac{1}{3} & 0 & -1 \\ 0 & \frac{2}{3} & 0 \\ -1 & 0 & -\frac{1}{3} \end{pmatrix}, \quad J^{33} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{4}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix}, \tag{F.20}$$

$$J^{12} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \; J^{23} = \begin{pmatrix} 0 & -\frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & -\frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \; J^{13} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}. \tag{F.21}$$

In calculating products of density matrices, the following relations are helpful

$$\mathrm{Tr}\{J^a\} = 0, \quad \mathrm{Tr}\{J^{ab}\} = 0, \quad \mathrm{Tr}\{J^a J^{bc}\} = 0, \tag{F.22}$$

$$\mathrm{Tr}\{J^a J^b\} = 2\delta^{ab}, \quad \mathrm{Tr}\{J^a J^b J^c\} = i\epsilon^{abc}, \quad \mathrm{Tr}\{J^a J^b J^c J^d\} = \delta^{ab}\delta^{cd} + \delta^{ad}\delta^{bc}, \tag{F.23}$$

$$\mathrm{Tr}\{J^{ab} J^{cd}\} = -\tfrac{4}{3}\delta^{ab}\delta^{cd} + 2\delta^{ad}\delta^{bc} + 2\delta^{ac}\delta^{bd}. \tag{F.24}$$

Appendix G

Definitions and conventions

We use natural units $c = 1$, $h/2\pi = 1$.

The metric tensor

$$(g_{\mu\nu}) = (g^{\mu\nu}) := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{G.1}$$

defines scalar products

$$(a \cdot b) := g_{\mu\nu} a^\mu b^\nu = a^\mu b_\mu = a^0 b^0 - \mathbf{ab} \tag{G.2}$$

between covariant and contravariant four-vectors

$$a^\mu := (a^0, a^1, a^2, a^3) = (a^0, \mathbf{a}), \quad a_\mu := g_{\mu\nu} a^\nu = (a_0, a_1, a_2, a_3) = (a_0, -\mathbf{a}). \tag{G.3}$$

The total antisymmetric ϵ-tensor is defined as

$$\epsilon_{\mu\nu\rho\sigma} = -\epsilon^{\mu\nu\rho\sigma} := \begin{cases} +1 \ , & \text{if } \mu\nu\rho\sigma \text{ is an even permutation of } 0123, \\ -1 \ , & \text{if } \mu\nu\rho\sigma \text{ is an odd permutation,} \\ \ \ 0 \ , & \text{if any two indices are the same.} \end{cases} \tag{G.4}$$

The analog definition in three dimensions, the *Levi-Cevita-Tensor* is

$$\epsilon_{ijk} = \epsilon^{ijk} := \begin{cases} +1 \ , & \text{if } ijk \text{ is an even permutation of } 123, \\ -1 \ , & \text{if } ijk \text{ is an odd permutation,} \\ \ \ 0 \ , & \text{if any two indices are the same.} \end{cases} \tag{G.5}$$

Useful relations of the ϵ-tensor

$$-\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\rho\sigma} = 2(\delta^\mu_\rho\delta^\nu_\sigma - \delta^\mu_\sigma\delta^\nu_\rho), \quad -\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\mu\rho} = 6\,\delta^\nu_\rho, \quad -\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\mu\nu} = 24, \tag{G.6}$$

with $\delta^\mu_\rho = g^{\mu\nu}g_{\nu\rho}$.

The Pauli matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{G.7}$$

The Dirac matrices obey the commutation relations

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \tag{G.8}$$

In the Dirac representation they read

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix}, \qquad j = 1, 2, 3, \tag{G.9}$$

and $\gamma_5 := -i\gamma_0\gamma_1\gamma_2\gamma_3 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^5$.

Trace theorems:

$$\begin{aligned} &P_{L,R} := \tfrac{1}{2}(1 \mp \gamma_5), \qquad \not{a} := \gamma^\mu a_\mu, \qquad \mathrm{Tr}\{P_{L,R}\} = 2, \\ &\mathrm{Tr}\{\not{a}\, P_{L,R}\} = 0, \quad \mathrm{Tr}\{\not{a}\,\not{b}\, P_{L,R}\} = 2\,(a\cdot b), \qquad \mathrm{Tr}\{\not{a}\,\not{b}\,\not{c}\, P_{L,R}\} = 0, \\ &\mathrm{Tr}\{\not{a}\,\not{b}\,\not{c}\,\not{d}\, P_{L,R}\} = 2\,[(a\cdot b)(c\cdot d) - (a\cdot c)(b\cdot d) + (a\cdot d)(b\cdot c)] \mp 2i[a,b,c,d], \end{aligned} \tag{G.10}$$

and $[a,b,c,d] := \epsilon_{\mu\nu\rho\sigma}a^\mu b^\nu c^\rho d^\sigma$.

For numerical calculations we have used the values

$$\begin{array}{rll} \alpha & = 1/128 & \text{fine}-\text{structure constant at } 500\text{ GeV} \\ \sin^2\theta_W & = 0.2315 & \text{weak mixing angle} \\ m_W & = 80.41\text{ GeV} & W \text{ boson mass} \\ \Gamma_W & = 2.12\text{ GeV} & W \text{ boson width} \\ m_Z & = 91.187\text{ GeV} & Z \text{ boson mass} \\ \Gamma_Z & = 2.49\text{ GeV} & Z \text{ boson width} \end{array} \tag{G.11}$$

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3. CP Sensitive Observables in $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ and Neutralino Decay into Z Boson
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4. Impact of CP Phases on SUSY Particle Production and Decays
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5. CP Violation in $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$
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13. Masses of Flavor Singlet Hybrid Baryons
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3. CP Violation in Neutralino Production
   International School on CP Violation and Heavy Quarks, Prerow, Germany, September 26, 2003
4. CP Violation in $e^+ \, e^- \to \tilde{\chi}_i^0 \, \tilde{\chi}_j^0$
   67. Tagung der Deutschen Physikalischen Gesellschaft, Aachen, Germany, March 12, 2003
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