Essays in Industrial Organization

Inauguraldissertation

zur Erlangung des akademischen Grades
eines Doktors der Wirtschaftswissenschaften an der
Graduate School: Law, Economics and Society
der Julius-Maximilians-Universität Würzburg

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Würzburg, im August 2015

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[1]This chapter is based on joint research with Matthias Hunold
[2]This chapter is based on joint research with Sebastian Wismer

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Chapter 1

Introduction

With the *more economic approach* in European Competition policy, the use of theoretical and empirical methods in competition policy has increased. The game theoretical models of Industrial Organization have been helpful in identifying potential consumer harm of business practices, but also potential efficiency gains. For example the recent vertical guidelines of the European Commission are clearly informed by economic theory.[1]

The understanding provided by economic theory can be helpful at many stages in the process of competition policy and economic policy in general. Firstly, it can inform the law makers. For example, when a new industry emerges, like the market for search engines, naturally the question arises if law has to be adopted to the new market circumstances or if the exiting law is sufficient. Theory can provide the necessary abstraction to make the similarities and differences to existing market phenomena visible. Making the similarities and differences visible can be sufficient to realize if any changes are necessary.

Secondly, to increase legal certainty courts and authorities often try to make the law more concrete by deriving legal criteria and guidelines. When a court or an authority presents an interpretation of the law that commits it to a certain behavior in the future, it should understand the economic implications.

Finally in case by case decisions, the arguments presented by the parties have to be judged according to soundness and plausibility. Economic theory is particularly helpful in identifying unsound, or - at least - not rigorously founded arguments.

The purpose of this thesis is to contribute to the understanding of specific business practices. The models presented in the thesis aim to identify a context in which a certain business practice is profitable and identify the market outcomes and welfare implications. As a welfare measure the Marshalian consumer and producer surplus is used. This allows to order outcomes according to welfare in partial market setting. It is, however noted, that the normative implications are to be taken with care, since general equilibrium implications and distributional implications are not considered. In general, partial market welfare is consistent with general equi-

[1]See the EU Guidelines on Vertical Restraints (2010/C 130/01)

librium welfare, if all other markets are perfectly competitive.[2] This is, however, often not the case in the real world.

In the second chapter the practice of Resale Price Maintenance (RPM) is investigated. This chapter is motivated by the recent changes in the legal evaluation of RPM in the United States and to a lesser extend in Europe. Also RPM has been discussed in the economic literature for a long time, the context in which it is often observed, when retailers sell products of multiple manufacturers and provide sales efforts, has found little attention. The model aims to fill that gap.

In the third chapter the practice of revenue sharing in platform industries is considered. Many marketplaces primarily generate revenue through revenue based fees. We consider a context where a platform operator, offering a platform to sellers and buyers is also present as seller in the market place. We identify a hold-up problem between the platform operator and the third party sellers and show that revenue based fees can mitigate the problem.

The forth chapter deals with a similar context as the third chapter. A platform is considered that links sellers and buyers. In that context trade between sellers and buyers is considered explicitly in order to understand, how fees are set, if the participation decision of buyers reveals that they have a relatively high valuation. We find that the platform is not able to fully internalize the trade surplus and, hence, does not maximize the social welfare of trade, if buyers know some of transaction value ex-ante. This differs from the predictions typically found in the platform literature e.g. in Rochet and Tirole (2006b).

The fifth chapter is motivated by the observation that pharmaceutical firms often license a product shortly before patent expiration. The licensee is then able to market the product before other generics. The model highlights a rationale for the patentee and provides two micro-foundations for firms strategies that have different implications for the welfare judgment of the practice.

[2] See in Mas-Colell et al. (1995) (p 333 f) for a discussion.

Chapter 2

Resale Price Maintenance and Manufacturer Competition for Retail Services[1]

2.1 Introduction

There is a long debate on whether resale price maintenance (RPM) should be legal. The major concern is that the use of minimum RPM leads to higher consumer prices.[2] The economic literature supports this concern by showing that RPM may facilitate collusion among manufacturers and by arguing that RPM may indirectly support retailer cartels.[3]

If collusion is not a concern, however, the so called *service argument* provides the following explanation of why minimum RPM benefits consumers: As the direct effect of minimum RPM is a higher retail price, which normally reduces demand, a manufacturer only increases the retail margin with minimum RPM if this indirectly increases demand through improved retailer incentives. For example, if retailers can increase demand through costly services, a manufacturer can use minimum RPM to increase the retail margin, which makes it profitable for retailers to provide additional services. As demand only increases if some consumers value the additional services, the use of minimum RPM benefits consumers, at least those who only buy the product as the result of these additional services.

The US Supreme Court assessed these pro-competitive and anti-competitive arguments on minimum RPM in 2007 and overturned the long standing per-se illegality of minimum RPM with the Leegin decision.[4] The court followed the *service argument* by stating in its opinion that *"...in general, the interests of manufacturers and consumers are aligned with respect to retailer profit margins"*. Also the EU

[1]This chapter is based on joint research with Matthias Hunold

[2]Minimum RPM implies that retailers may not sell below specified price and thus imposes a price floor.

[3]See Marvel and McCafferty, 1984; Jullien and Rey, 2007; Rey and Verge, 2010.

[4] Leegin Creative Leather Products, Inc. v. PSKS, Inc., 551 U.S., 2007.

Guidelines on Vertical Restraints of 2010 take a more positive approach towards minimum RPM.[5]

With this paper, we contribute a new theory which shows that the logic of the *service argument* hinges on the assumption of a single manufacturer. We show that in markets where multiple manufacturers sell through common retailers, the interests of each individual manufacturer and the consumers are misaligned with respect to minimum RPM. This result is highly relevant because in many markets minimum RPM is imposed by manufacturers on retailers who carry a wide range of different brands. Examples include books, clothing, contact lenses, hearing devices, and household appliances.[6] Retailers of such products frequently offer services such as pre-sales advice. We show that competing manufacturers individually have incentives to use minimum RPM, but collectively have lower profits when minimum RPM is used by all manufacturers. Minimum RPM results in higher consumer prices without necessarily increasing the service level.[7]

Consider two manufacturers producing products symmetric in demand and competing through two retailers. Suppose that each retailer has a fixed amount of service or influence that can be allocated to one product or the other. Without RPM the equilibrium allocation of service at each retailer is symmetric: each product receives half of each retailer's service. But with RPM the allocation of service is also symmetric, and therefore unchanged. RPM therefore has no impact at all on service levels. But RPM does increase the retail price when competition for retailers' service is intense, because each manufacturer attempts to elicit marginally greater service for its product by raising the retail margin of its product. Protection of the retail margin takes the form, in part, of protecting higher retail prices through minimum RPM. Thus minimum RPM raises retail prices but has no impact on service levels. Minimum RPM is in the individual manufacturer's interest, but in this case makes both manufacturers worse off (a prisoner's dilemma effect). Consumers are unambiguously worse off under RPM because prices are higher with no gain in service.

Although the retailers do not distort their services in favor of any product in

[5]In the EU Vertical Block Exemption, minimum and fixed RPM are still considered core restrictions of competition (Commission Regulation No 330/2010, Article 4a). Yet Par. 223 of the EU Guidelines on Vertical Restraints (2010/C 130/01) states that an efficiency defense in terms of Article 101,3 TFEU is possible also for minimum and fixed RPM. Par. 224 and 225 contain examples of potentially detrimental and beneficial practices.

[6]See Elzinga and Mills (2009) for a discussion of services in the Leegin case. Other recent RPM cases with common retailers and products where pre-sale advice potentially matters include contact lenses (see fine "Bußgeldbescheid B 3 - 123/08," German Federal Cartel Office, September 2009), hearing devices (press release "Bundeskartellamt verhängt Bußgeld gegen Hörgerätehersteller Phonak GmbH," German Federal Cartel Office, October 2009.), and household appliances (see press release "Bundeskartellamt verhängt Bußgelder wegen unzulässiger Preisbindung," German Federal Cartel Office, 2003).

[7]That consumer prices increase with minimum RPM is a non-trivial result as both wholesale and retail prices generally depend on whether RPM is employed. For example, Perry and Besanko (1991) show that minimum RPM may yield lower prices and maximum RPM higher prices compared to no RPM.

equilibrium as the products have the same profitability, the retail margins depend on how willing retailers are to distort services if margins are unequal. If distorting service is sufficiently costly for retailers, manufacturers use maximum RPM instead of minimum RPM to reduce double marginalization.

Besides the pricing and allocational consequences of minimum RPM, we investigate its effects on the equilibrium service investments of retailers. The main point of the existing literature is that with simple contracts, retailers may fail to provide the (privately) efficient levels of service. The literature shows that minimum RPM enables a manufacturer to increase the retail margins above competitive levels and by this induces retailers to provide the adequate levels of service.

We investigate retailers' ex-ante investment incentives in view of manufacturer competition for favorable matching services. By this we mean services that enable retailers to know which product gives a consumer the highest utility and use this to influence consumers' purchase decisions, such as pre-sales advice.We show that when manufacturers use RPM, retailers prefer a lower quality of matching services than without RPM. This relies on the insight that retailers generally benefit from a more intense manufacturer competition for favorable services. This benefit is larger for retailers if manufacturers use RPM, as the latter then determine both wholesale and retail prices, so that they directly compete in retail margins. Without RPM, manufacturers only compete in wholesale prices. A reduction in the wholesale prices typically also leads to a lower retail price when it is determined by competing retailers. This benefits consumers, but makes competition for favorable retail services less effective, so that retailers have less incentives to simulate this competition through a reduced quality in matching services.

Our leading example for service quality is the amount of service personnel in a shop. Depending on the amount of personnel, a sales person has time to present one or two choices to each customer. Showing both products increases the likelihood that the consumer likes one and buys it. When there is only time for presenting one product, the sales person will present the product that is more profitable for the retailer. Manufacturers thus have more incentives to ensure high retail margins when only one product is shown. Retailers accordingly tend to hire less personnel if manufacturers can use minimum RPM because then the gain in retail margin per sale is larger.

Finally, we investigate how the use of minimum RPM is related to a manufacturer's market power. For this, we introduce asymmetric market power by adding a third manufacturer who offers a perfect substitute to the product of one of the manufacturers, say B. Even with RPM, manufacturer B can no more sustain retail margins above the margins resulting without RPM because the third manufacturer would profitably maintain lower retailer prices, which are individually more attractive for the retailers. As a consequence, only manufacturer A can effectively use RPM - due to its market power. The resulting asymmetric use of RPM tends to increase the asymmetry in retail margins and thus services. If retailer competition is strong, consumers are matched with the high priced product too often. Banning RPM can reduce this service distortion.

In the following paragraphs we will briefly point out how our argument relates to the literature. As mentioned before, in most of the literature on service and RPM the authors focus on a *single* manufacturer and argue that minimum RPM allows the retailer to internalize the positive externalities of its services, which can benefit both the manufacturer and competing retailers. The most common source of the externality are spillovers in information or service at the retail level (Telser (1960), Mathewson and Winter (1984)).[8] We adopt the spillover assumption in a simple, but extreme form: service is a public good from the retailers' point of view in that service for a product from one retailer increases demand for the product from both retailers equally.[9] From a social welfare perspective, a monopoly manufacturer may nevertheless induce too little or even too much services, as it aligns services with preferences of marginal consumers rather than the average consumer purchasing. For example, if some consumers do not search for the best price but buy spontaneously, RPM can reduce social welfare even if it increases services to consumers, which is shown by Schulz (2007). The overall conclusion of this literature is that the positive effects of RPM are expected to prevail; see Winter (2009) for a recent discussion. By studying *two* manufacturers and two common retailers who provide *matching services*, we show that *all* consumers can be worse off with RPM.

Perry and Besanko (1991) study how two manufacturers use minimum RPM to compete for exclusive (i.e., single product) retailers. They argue that prices are lower with minimum RPM than with maximum RPM. Their comparison is special, however, in that they compare minimum RPM with franchise fees to maximum RPM with linear wholesale tariffs. Similarly, Shaffer (1994) compares two-part tariffs and no RPM with linear tariffs and RPM in case of one strategic manufacturer. Focusing on linear tariffs, we instead endogenously determine whether manufacturers impose minimum or maximum RPM on common retailers and find that prices are higher if manufacturers use minimum RPM.

We are aware of two articles on RPM in a setting with differentiated manufacturers and common retailers, but neither addresses service. Dobson and Waterson (2007) study bilateral Nash-bargaining between each manufacturer-retailer pair over a linear wholesale price. They find that if retailers have all the bargaining power, retail prices are higher with RPM. If, instead, manufacturers possess all the bargaining power, retail prices are lower with RPM.[10] Rey and Verge (2010) show that the monopolization result of Bernheim and Whinston (1985) with a common retailer and two-part tariffs offered by manufacturers can be extended to competing common retailers if manufacturers can additionally use RPM.

[8] Several articles study RPM in the context of spillovers in case of stock-outs (Deneckere et al., 1997, 1996; Krishnan and Winter, 2007).

[9] Spillovers or free riding are not the only source of distortion in retailer incentives. Distortions also arise with simple contracts if there is a vertical externality, i.e., the retailer does not fully realize the positive effect of services on demand. Winter (1993), for example, traces the incentive for RPM to heterogeneity in consumer time costs combined with assumptions that retail sales effort reduces the time it takes to purchase a product and that shopping or search takes time.

[10] Dobson and Waterson do not analyze cases with intermediate bargaining power and whether manufacturers would like to use RPM. See their footnote 26.

Our result that minimum RPM can impose a prisoner's dilemma on competing manufacturers is to our knowledge new in the literature. In our view it is important for competition policy because it contradicts the gist of the existing literature as well as policy debates that minimum RPM is beneficial for manufacturers, when compared to no RPM.

In the context of single retailer who is paid on a commission basis, Armstrong and Zhou (2011) show that the retailers' ability to make one of the products more prominent gives rise to a prisoners dilemma for manufacturers. Similarly, Raskovich (2007) and Inderst and Ottaviani (2011) highlight that manufacturers may have to compete in commissions for favorable product advice of a monopolistic retailer. However, neither of these articles investigates price competition between multiple retailers nor use of minimum RPM.[11]

2.2 Model

Two symmetric manufacturers ($i = A, B$) sell their differentiated products to two symmetric common retailers ($k = 1, 2$), who in turn sell the products to final consumers. Each consumer is interested in buying only one of the two products, but is initially unaware of which product suits his preferences. Similar to Mathewson and Winter (1984), we assume that consumers rely on the retailers' services to match them with products through recommendations, demonstrations, and general advice.

We assume that more retailer service allocated to product i increases the demand for this product at both retailers. Initially, we will assume that the overall level of services that each retailer can provide is fixed; in part of subsection 2.3 we relax this assumption and endogenize the overall level of services that the retailers offer. If retailer k allocates a fraction $s_k \in [0, 1]$ of his services to product A, the fraction he allocates to product B is $1 - s_k$. If $s_k > (<)\ ^1\!/_2$, retailer k biases his services towards product A (B).

Using $p_{i,k}$ to denote the price retailer k charges for product i, and using $-k$ to denote the rival retailer, we assume that the demand for product i at retailer k is given by

$$D_{i,k} \equiv M_i(s_k, s_{-k})\, d_{i,k}(p_{i,k}, p_{i,-k}), \tag{2.1}$$

where $\frac{\partial d_{i,k}}{\partial p_{i,k}} < 0$, $\frac{\partial d_{i,k}}{\partial p_{i,-k}} > 0$ and $|\frac{\partial d_{i,k}}{\partial p_{i,k}}| > \frac{\partial d_{i,k}}{\partial p_{i,-k}}$.[12] Note that $d_{i,k}(p_{i,k}, p_{i,-k})$ depends only on the prices that the two retailers charge for product i, but is independent of the prices charged for product $-i$. This implies that there is no direct price competition between the manufacturers.[13] This feature of our model ensures that manufacturers' strategic delegation of pricing to retailers does not affect our results as in Bonanno and Vickers (1988) and Rey and Stiglitz (1995), so we can isolate the effects of service competition.

[11] These articles *only* consider RPM (the manufacturers set their retail prices).

[12] We will suppress the arguments of (2.1) when this is unlikely to confuse the reader.

[13] We show in Appendix B that relaxing this assumption yields qualitatively the same results.

The demand structure stated in (2.1) allows us to separate the pricing of the products from the service decisions. One can think about $M_i(s_k, s_{-k})$ as the mass of consumers who consider buying product i given the services that the two retailers allocate to product i, and $d_{i,k}(p_{i,k}, p_{i,-k})$ as the quantity of product i that such a consumer buys from retailer k. The following *Example* contains a micro-foundation for the retailer service that is consistent with this demand system.

Example. In line with Mathewson and Winter (1984), assume that for buying a product, each consumer requires a retailer to present him with that product in order to learn about the product's existence and its characteristics. Assume that consumers initially randomly select one of the retailers to obtain product information. A retailer can only present one product[14] to each consumer and bases the decision about which product to present on noisy information about the consumer's preferences. In particular, each retailer knows a probability with which a particular consumer likes product B (and product A, with complementary probability). Both retailers have the same information about each consumer. Each consumer only buys a product if he was presented with a suitable product. He then chooses at which retailer to buy, based on prices and his preferences over the differentiated retailers.

As each consumer will only consider buying if the product is suitable, a retailer has a natural incentive to present each consumer with the product that is most likely to suit. Each retailer's decision which product to present boils down to choosing a threshold probability $s_k \in [0, 1]$, such that the retailer presents consumers with product B if the probability that the consumer prefers product B is above the threshold level s_k, and with product A if the probability is below s_k. These thresholds s_k define the mass of the consumers that have been presented with their suitable products. If the retailers has imperfect information on preferences, the mass of consumers that have been presented with product A and prefer A over B is a concave function of the thresholds s_k, as additional consumers who see product A become less and less likely to like it.

Here is a parametric example: The total mass of consumers is 4; of them 50% prefer product A, and 50% product B. A consumer preferring a product is only interested in buying that product. The probability estimate of the retailer that a consumer prefers product B is continuously and uniformly distributed between 0 and 1. For product A the mass is $M_A = {}^1\!/_2\, 4 \sum_k \int_0^{s_k} (1 - x)dx = \sum_k (2s_k - s_k^2)$, where x is the probability estimate that a consumer prefers product B. The corresponding mass for B is $M_B = \sum_k (1 - s_k^2)$. A threshold of $s_k = 1$ by both retailers maximizes M_A, which means that all consumers only see product and implies zero mass for product B: $M_B = 0$. The aggregate mass of correctly matched consumers, $M_A + M_B$, is maximized at $s_k = {}^1\!/_2$, which means that a retailer presents each consumer that is most likely a good match according to the retailer's private information.

For our model we assume that consumers know the prices of a product at both

[14] This can, for example, be motivated with decreasing consumer attention and is particularly meaningful if there are more than two products, as is typically the case in practice.

retailers once they have learned about the product. This is in line with the service and RPM literature, such as Mathewson and Winter (1984). We believe that this assumption is even more realistic nowadays with many consumers having smartphones with internet access. Moreover, we assume that a consumer's decision about whereto buy the product is independent of at which retailer she received the matching service.[15] This yields the multiplicative separability between M_i and $d_{i,k}$, as both retailers compete in prices for all consumers who consider buying. Finally, we assume that retailers are differentiated and that total demand for each product is price elastic.

Throughout we maintain the following assumptions for the reduced form demand, which is stated in (2.1):

Assumption 2.1. *M_i is strictly concave with $\frac{\partial M_A}{\partial s_k} > 0 > \frac{\partial M_B}{\partial s_k}$, and symmetric around $1/2$: $M_i(s_1, s_2) = M_{-i}(1 - s_1, 1 - s_2)$.*[16]

The concavity and symmetry of $M_i(s_1, s_2)$ imply that allocating services unevenly to the two products reduces the aggregate $M_A + M_B$.[17] Furthermore,

Assumption 2.2. *The effect of a retailer's service allocation on M_i is independent of the other retailer's service: $\frac{\partial^2 M_i}{\partial s_1 \partial s_2} = 0$, $i = A, B$.*

Finally, to ensure that there is a unique equilibrium when retailers set prices, we assume that the Hessian matrix of $d_{i,k}$ has a negative and dominant main diagonal:

Assumption 2.3. $\frac{\partial^2 d_{i,k}}{(\partial p_{i,k})^2} \leq 0$,$\frac{\partial^2 d_{i,k}}{(\partial p_{i,-k})^2} \leq 0$, $\frac{\partial^2 d_{i,k}}{\partial p_{i,k} \partial p_{i,-k}} \geq 0, \left| \frac{\partial^2 d_{i,k}}{(\partial p_{i,k})^2} \right| \geq \frac{\partial^2 d_{i,k}}{\partial p_{i,k} \partial p_{i,-k}}$.[18]

We assume that services are non-contractible and study how the manufacturers can affect the retail services through RPM. We assume that if manufacturer i imposes RPM and restricts the retail price to p_i, it must be maintained by both retailers.[19] The timing of the game is as follows:

1. Each manufacturer $i \in A, B$ sets a wholesale price w_i, and fixed p_i if RPM is feasible.

[15] This is done for tractability and not driving the results, but can also be plausible in reality. E.g., after having tried out contact lenses from a particular optician for a day, the consumer searches the internet for an attractive offer solely based and price and purchasing effort.

[16] Strict concavity is convenient but our results are also valid as long as $aM_i + bM_{-i}$ is strictly quasi-concave for $a, b > 0$ and $s_1, s_2 \in (0, 1)$. As stated in example 2.2, a uniform distribution of retailer's information about consumers yields a strictly concave M_i. Also, any distribution of retailer's information that has full support on $[0, 1]$ yields $aM_i + bM_{-i}$ being strictly quasi-concave.

[17] The following assumptions on derivatives apply strictly only for the relevant range where M_i and $d_{i,k}$ are positive. Strict concavity of M_i in s_k implies strict concavity of $M_A + M_B$. By symmetry, $M_B(s_1, s_2) = M_A(1 - s_1, 1 - s_2)$. Thus $\frac{\partial}{\partial s_k} (M_A(s_1, s_2) + M_B(s_1, s_2)) = \frac{\partial}{\partial s_k} (M_A(s_1, s_2) + M_A(1 - s_1, 1 - s_2))$. This derivative is zero at $s_k = 0.5$, which by strict concavity is the unique maximizer of $M_A + M_B$.

[18] Assumption 2.3 implies that demand is weakly concave in prices.

[19] We focus here on a symmetric treatment of the retailers, as is common in the literature on RPM. Within the present setting, this is also optimal for each manufacturer.

2. Each retailer $k \in 1, 2$ observes the manufacturers' prices, chooses the service allocation s_k, and sets its own retail prices $p_{i,k}$. Under RPM, $p_{i,k} = p_i$.

3. Demand is realized.

Similar to Inderst and Ottaviani (2011) and Dobson and Waterson (2007), we consider linear wholesale tariffs. This avoids non-existence problems as in Rey and Verge (2010) and avoids to confound our service effects with their common agency effects.[20]

Normalizing all costs of manufacturing and retailing to zero, the profit of manufacturer i is given by

$$\pi_i \equiv w_i \sum_{k=1,2} D_{i,k}, \tag{2.2}$$

and the profit of retailer k by

$$\Pi_k \equiv \sum_{i=A,B} (p_{i,k} - w_i) D_{i,k}. \tag{2.3}$$

In the next section we solve the game for subgame perfect Nash equilibria, without and with RPM. We focus on symmetric equilibria, apart from case when we study the consequences of asymmetric market power.

2.3 Analysis

Equilibrium without resale price maintenance

Assume for this subsection that manufacturers can only set wholesale prices, but cannot use RPM. For given wholesale prices, each retailer k chooses $p_{A,k}$, $p_{B,k}$ and s_k to maximize Π_k. The first order condition (FOC) for the retail price is

$$\frac{\partial \Pi_k}{\partial p_{i,k}} = 0$$

$$\Leftrightarrow d_{i,k} + (p_{i,k} - w_i)\frac{\partial d_{i,k}}{\partial p_{i,k}} = 0. \tag{2.4}$$

The optimal price is independent of s_1 and s_2 by the multiplicative separability of demand. Note also that it is of the wholesale and retail prices of product $-i$. Denote by $p_i^*(w_i)$ the equilibrium retail price for product i. The dominance of the own price effect and the assumption on weak concavity of $d_{i,k}$ imply that the pass through rate, $\partial p_i^* / \partial w_i$, is positive and below one. Hence the retail profitability $(p_i^* - w_i)\ d_{i,k}\,(p_i^*, p_i^*)$ decreases with w_i.

[20]For delegated common agencies Rey and Verge point out that common agency equilibria fail to exist because the binding participation constraint for a retailer to sell the product of a manufacturer can always be profitably undermined by the other manufacturer.

The FOC with respect to s_k is

$$\frac{\partial \Pi_k}{\partial s_k} = \frac{\partial M_i}{\partial s_k}(p_{i,k} - w_i)\, d_{i,k} + \frac{\partial M_{-i}}{\partial s_k}\,(p_{-i,k} - w_{-i})\, d_{-i,k} = 0. \tag{2.5}$$

This FOC together with the strict concavity of M_i (Assumption 2.2) implies that retailer k sets s_k to shift demand towards the more profitable product. If the products are equally profitable, each retailer maximizes profits by maximizing $M_A + M_B$,the mass of attracted consumers. This is the case at $s_k = 1/2$ for each retailer due to the strict concavity and symmetry of M_A and M_B.

We denote the equilibrium service decisions by $s_k^*(w_A, w_B)$, the implied mass of attracted consumers by $M_i^*(w_i, w_{-i}) \equiv M_i\,(s_1^*, s_2^*)$ and summarize in

Lemma 2.4. *Without RPM, there exists a unique equilibrium in each subgame – starting in stage 2 – in which the retailers' decisions are symmetric and*

1. *the retail price p_i^* increases in w_i and is independent of w_{-i} and s_k;*
2. *the retail profitability $(p_i^* - w_i)\; d_{i,k}\,(p_i^*, p_i^*)$ decreases in w_i;*
3. *the equilibrium matches M_i^* decrease in w_i. If $w_A = w_B$, then $s_1^* = s_2^* = 1/2$.*

Proof. All proofs are in Appendix A. □

We now turn to stage 1. Taking the retailer continuation equilibrium into account, each manufacturer solves

$$\max_{w_i} \pi_i = w_i\, M_i^*\,(w_i, w_{-i}) \sum_k d_{i,k}(p_i^*(w_i),\; p_i^*(w_i)), \tag{2.6}$$

facing a trade-off between price and quantity.

Equation (2.6) shows that an increase in w_i increases manufacturer i's margin, but decreases its demand in two ways: First, the retail profitability decreases so that retailers allocate services to product $-i$ and thus attract fewer consumers to product i. Second, the retail prices of product i increase and hence the attracted consumers buy less quantity of that product.

The FOC implied by (2.6), evaluated at symmetric wholesale prices $w_A = w_B = w^N$ and symmetric retail prices $p_{i,k} = p^N$ for both i and k, can be written as

$$w^N \quad = - \quad \frac{d_{i,k}(p^N, p^N)}{\left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}}\right)\frac{\partial p_i^*}{\partial w_i} + \lambda\left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}}\frac{\partial p_i^*}{\partial w_i}\right)}, \tag{2.7}$$

and the FOC (2.4) evaluated at symmetric retail prices as

$$p^N = p_i^*(w^N) = w^N - \frac{d_{i,k}(p^N, p^N)}{\partial d_{i,k}(p^N, p^N)/\partial p_{i,k}}, \tag{2.8}$$

where

$$\lambda \;=\; \frac{\partial M_i/\partial s_k}{M} \times \frac{\partial M_i/\partial s_k}{-\partial^2 M/\partial s_k^2} > 0 \tag{2.9}$$

and $M \equiv M_A(1/2, 1/2) = M_B(1/2, 1/2)$.

Note that λ increases if the mass of consumers for a manufacturer becomes more elastic and less concave in service for that product. Since the retailer sells both products, he maximizes a weighted sum of both M_i. Hence, in analogy to risk aversion, a more concave M (a lower λ) implies that the retailer prefers a more oven allocation of services on products.

Intuitively, the parameter λ measures the opportunity costs of each retailer from focusing service on one product. The lower λ, the more costly it is to have an uneven allocation of services across the two products. Consider the polar case of $\lambda = 0$, which implies that moving s_k away from $1/2$ yields a large loss of consumers mass.[21] For example, a λ of zero results when each retailer perfectly recognizes which product suits which consumer, so that the retailers have no incentive to present unsuitable products, knowing that this would not result in sales. On the contrary, if the retailer has very little information about consumers' preferences, λ is large and each retailer is almost indifferent about which product to present.

As a consequence, for $\lambda = 0$ the products are not substitutable for a retailer and the equilibrium prices implied by (2.7) and (2.8) are as if there were no service decisions by retailers.[22] As λ increases, manufacturer compete for favorable retail services driving the wholesale price w^N towards zero and increasing the retail margin $p^N - w^N$. We sometimes call λ the degree of product substitutability for the retailers.

In what follows, we restrict attention to demand functions that give rise to quasi-concave reduced-form manufacturer profits and a stable interior equilibrium such the FOCs of a manufacturer can be differentiated implicitly.[23]

Proposition 2.5. *In any symmetric equilibrium without RPM, the wholesale prices w^N and retail prices p^N are defined by (2.7) and (2.8), and service is allocated evenly: $s_k^* = 1/2$. The prices w^N and p^N decrease in λ, whereas retail profits increase in λ.*

For the demand parametrization in the Example, the symmetric equilibrium is unique.[24] Note that manufacturer competition for retailers drives the wholesale prices down and also reduces the equilibrium retail prices to the benefit of consumers.

[21] Formally this is the case, if M is very concave (see the denominator in (2.9)) resulting in a lower λ.

[22] Note that $\lambda = 0$ is an illustrative case, but clearly a polar case in which there is typically no interior solution to each retailer's service decision and violates Assumption 1: strict concavity and non-zero derivatives of $M_{i,k}$.

[23] This holds for the parametrization in Example 1.

[24] For linear $d_{i,k}$ (as in the Example) the right hand side of (2.7) is monotone in the price level, which ensures that w^N and p^N are unique.

Equilibrium with resale price maintenance

For this subsection, assume that RPM is enforceable. Both manufacturers have at least weak incentives to use RPM because a manufacturer who unilaterally fixes the retail price can reproduce the equilibrium prices without RPM by setting $p_i^R = p^N$ and $w_i^R = w^N$, and is thus at least as well off as without RPM. Manufacturer i fixing both w_i and p_i faces the following trade-offs:

- increasing w_i increases its own margin, but decreases the retail margin $p_i - w_i$ and thus induces retailers to allocate services away from product i;
- increasing p_i increases the retail margin and thus retailers allocate more services to product i. A higher p_i also implies that attracted consumers buy less quantity of product i.

With the additional control through RPM, a manufacturer can adjust its wholesale price to trade off its own margin and retail service incentives without affecting the retail price. The manufacturer can thus separate the product's optimal retail pricing from the provision of service incentives to the retailers. Hence each manufacturer maximizes joint rents from selling its product by fixing a price of

$$p^M \equiv \arg\max_{p_i} \sum_k p_i \, d_{i,k}(p_i, p_i). \tag{2.10}$$

Manufacturers provide higher retail margins if their products are more substitutable for retailers, i.e., the retail margins increase in λ. Focusing again on demand functions that give rise to interior solutions yields

Proposition 2.6. *There exists a unique symmetric equilibrium with*

$$p^R = p^M, \quad w^R = \frac{p^M}{1+\lambda}. \tag{2.11}$$

Service is allocated symmetrically and the retail margin increases in λ.

With RPM a manufacturer fixes the retail price at the level that maximizes the joint profits of that product along the supply chain. The retail price does not decrease in λ. so that consumers do not benefit from increased competition of the manufacturers for retail services, as it is the case without RPM.

After having characterized the equilibrium prices and service decisions both with and without RPM, we now compare them to evaluate the effects of RPM on profits and welfare.

Competitive effects of resale price maintenance

It is important to understand when RPM increases, and when it decreases the retail price. With RPM, the retail price always equals p^M, whereas without RPM, the retail price p^N depends on both the intensity of manufacturer and of retailer competition. Comparing p^N and p^M yields

Proposition 2.7. *The retail prices under RPM are higher than the prices when no manufacturer uses RPM if and only if*

$$\lambda > \lambda^M \equiv \frac{-\partial\, d_{i,k}(p^M, p^M)}{\partial\, p_{i,k}} \Big/ \frac{\partial\, d_{i,k}(p^M, p^M)}{\partial\, p_{i,-k}} - 1. \tag{2.12}$$

Correspondingly, RPM decreases retail prices if and only if the above inequality is reversed. At $\lambda = \lambda^M$, $p^N = p^M$.[25]

The more intensely retailers compete in prices, the smaller (and closer to zero) is the right hand side of (2.12). The more exchangeable the manufacturers' products are for the retailers when allocating services, the larger is λ on the left hand side of (2.12) and the more intense is the competition of manufacturers for favorable services. If, on balance, the competition among manufacturers and among retailers is both sufficiently intense, the price level without RPM is lower than the level with RPM.

We model RPM as the manufacturer fixing a particular retail price. Yet for competition policy it is important to distinguish the effects of minimum and maximum RPM on retail prices and total surplus. For their distinction it is generally not sufficient to compare the price level with and without RPM and argue that if prices with RPM are higher, then it must be minimum RPM, and maximum RPM otherwise. The reason is that the wholesale prices depend on whether RPM is in place or not. For instance, manufacturers may set lower wholesale prices with RPM than without RPM because they compete more directly for favorable retail services wtih RPM. Manufacturers may fix a retail price that is above the level competing retailers would charge facing the lower wholesale price. It is conceivable that this yields minimum RPM with a lower retail price than without RPM.

If RPM imposes a binding constraint on retailers, it acts either as a price floor or a price ceiling. Whether the manufacturers use minimum or maximum RPM can thus be identified by answering the question: Would a retailer at the equilibrium prices with RPM benefit from reducing or from increasing its retail price? By evaluating a retailer's marginal profit with respect to its retail prices at the equilibrium values $\{w^R, p^R\}$ we obtain

Proposition 2.8. *In equilibrium, manufacturers use minimum RPM if and only if $\lambda > \lambda^M$. Manufacturers use maximum RPM if and only if $\lambda < \lambda^M$. Compared to the regime without RPM, minimum RPM always increases retail prices and maximum RPM always decreases retail prices. The equilibrium allocation of services is not affected by RPM.*

[25] Although the symmetric equilibrium is not necessarily globally unique, at $\lambda = \lambda^M$ equation (2.12) *uniquely* defines the price p^N. Starting from this locally unique symmetric equilibrium, the monotone comparative statics of p^N in λ allow us to compare the symmetric equilibrium prices with and without RPM globally and unambiguously. However, asymmetric equilibria in which one product has a lower wholesale price and higher service and the other has a higher wholesale price and lower services cannot generally be ruled out.

At $\lambda = \lambda^M$, the equilibria with and without RPM coincide ($p^*(w^N) = p^M$, $w^N = w^R$) and RPM is a superfluous instrument. When λ increases above λ^M, the wholesale prices both with and without RPM decrease (Propositions 2.5 and 2.6). In turn, RPM implies a price floor because each retailer individually would prefer to set a price below p^M. Analogously, for $\lambda < \lambda^M$, wholesale prices are higher and maximum RPM restricts the retail price to p^M because each retailer individually would prefer to raise prices further.

We can derive from Proposition 2.8 a simple optimal policy if the retail service level for the product category can be assumed to be fixed: forbid minimum RPM because it unambiguously increases retail prices and leaves the equilibrium service allocation unchanged, but allow maximum RPM as it decreases prices. However, different services efforts and thus qualities may countervail the price effects of RPM. For this reason we investigate the effects of RPM on investments in the overall service level in the next subsection. But before doing so, let us consider the effects of RPM on manufacturer and retailer profits.

Are manufacturers better off when minimum RPM is enforceable? Recall that the unilateral introduction of RPM is always weakly profitable for a manufacturer as it yields direct control over the retail margin. However, this additional control induces manufacturers to compete more fiercely for retail services. Collectively, manufacturers can thus be worse off, even if industry profits increase through RPM. The next remark characterizes this case.

Remark 2.9. The equilibrium profit of a manufacturer under the regime with enforceable RPM is lower than under the regime without RPM if and only if $w^R\, d_{i,k}(p^R, p^R) < w^N\, d_{i,k}(p^N, p^N)$.

The inequality in *Remark 2.9* is independent of M_i, which is the same in any symmetric equilibrium. Minimum RPM implies $p^R > p^N$ (Proposition 2.8) and, in turn $d_{i,k}(p^R, p^R) < d_{i,k}(p^N, p^N)$, because the demand factor $d_{i,k}$ decreases when both retail prices increase. Thus a sufficient condition for minimum RPM to impose a prisoner's dilemma is that the wholesale price is weakly lower with RPM. Unfortunately, with only implicit definitions of w^R and w^N, it is difficult to establish general conditions for $w^R \leq w^N$. However, under the assumption that the demand factor $d_{i,k}$ is linear in prices, we obtain

Proposition 2.10. *Assume that demand is linear in prices. If minimum (maximum) RPM results in equilibrium, the manufacturers' profits are lower (higher) than in the regime without RPM. Retailers benefit from minimum RPM and suffer from maximum RPM.*

Without RPM, a manufacturer has to decrease the wholesale price to increase the retail margin. But when faced with lower input costs, the retailers lower the retail prices, which decreases the retail margin again and thereby affects the product's overall profitability. The manufacturer thus targets two goals with only one instrument, which makes it costly for the manufacturer to induce favorable services. Instead, with minimum RPM as another instrument, a manufacturer can prevent retailers from lowering the retail price. As a result, manufacturers compete more directly, and thus more fiercely, for favorable services when RPM is available.

A caveat applies as the result of a prisoner's dilemma for manufacturers is derived for linear wholesale tariffs. With two-part tariffs, a manufacturer can generally extract retail rents with an upfront-payment, but has to ensure that a retailer prefers carrying its product over exclusively carrying the other product. This trade-off and thus the retailer's outside option to carrying the product generally depend on whether RPM is used in the industry, hence it is an open question whether the dilemma ceases to exist. As explained before in the [intro/lit], obtaining equilibria in these kind of models is still a challenge. We leave this for future research.

Retailers choose the service level

For a fixed overall service level, we have shown that manufacturers use minimum RPM to increase consumer prices when competing for favorable retail services, but nevertheless consumers do not receive better retail services. Conventional wisdom suggests that minimum RPM induces retailers to choose a higher service level. By contrast, we demonstrate how RPM can reduce the incentives of retailers to invest in service, although minimum RPM increases the retail margin.

For this assume that each retailer can choose in an initial stage whether to employ few or a lot of sales personnel. A retailer with a lot of personnel is able to present both products to the consumer, such that each consumer will find his most preferred product. A retailer with few sales personnel can only present one of the products and each retailer has imperfect information on the consumer preferences (as in the Example). Hence the retailer is not able to present the suitable product to all consumers and in turn loses demand.

We show that retailers under RPM may collectively and individually prefer to hire few personnel, whereas without RPM retailers prefer a lot of personnel. Let us first explain the intuition. If both retailers have sufficient personnel to inform all consumers about all products, manufacturers do not compete for preferential treatment by the retailer. For the case of RPM this implies zero retail profits, whereas without RPM retail profits are positive. On the contrary, if both retailers present only one product, retailers typically have larger profits if minimum RPM can be used, see Proposition 2.10. In summary, retailers collectively have more to gain from inducing manufacturer competition when manufacturers use minimum RPM.

To show that retailers under RPM also have stronger *individual* incentives to hire few personnel, we have to characterize retail profits in asymmetric situations, for example when retailer 1 presents one product while retailer 2 presents both products.

For asymmetric situations in which one retailer presents both products, we assume that all consumers use this superior service (recall that consumers split randomly in case of symmetric service). Thus, all consumers are informed about both products. This implies that if a single retailer presents both products, manufacturers do not compete for retail services. To further simplify the argument we abstract from costs for personnel.[26] The implied retailer service game under RPM is summarized in the following pay-off matrix:

Retailer $k/-k$	show 1 product	show 2 products
show 1 product	$\Pi^R(1),\ \Pi^R(1)$	$0, 0$
show 2 products	$0, 0$	$0, 0$

This game has two Nash equilibria.[27] However, under RPM the single equilibrium in undominated strategies and the Pareto-dominant equilibrium is that both retailers show only one product.

Without RPM the game is represented by:

Retailer $k/-k$	show 1 product	show 2 products
show 1 product	$\Pi^N(1),\ \Pi^N(1)$	$\Pi^N(2), \Pi^N(2)$
show 2 products	$\Pi^N(2), \Pi^N(2)$	$\Pi^N(2), \Pi^N(2)$

If $\Pi^N(1) > \Pi^N(2)$, the unique equilibrium is that both retailers present one product. On the contrary, if $\Pi^N(2) > \Pi^N(1)$, three pay-off equivalent equilibria exist, with symmetric pay-offs $\Pi^N(2)$: There are two asymmetric equilibria in which only one retailer presents two products and the other retailer "free-rides" on the service and one symmetric equilibrium in which both retailers present both products. To compare the retailer profits, note that presenting two products yields the same prices as in the case of $\lambda = 0$, i.e., no manufacturer competition. The mass of successfully advised consumers is simply the total mass of consumers M^{max}. The inequality $\Pi^N(2) > \Pi^N(1)$ can be written as

$$M^{max} \cdot (p^N - w^N) \sum_{i=A,B} d_{i,k}(p^N)|_{\lambda=0} \quad > \quad M \cdot (p^N - w^N) \sum_{i=A,B} d_{i,k}(p^N)|_{\lambda>0} \tag{2.13}$$

Note that the mass of matched consumers is larger on the left hand side profit, $M^{max} > M$, whereas the retail profitability of the products is larger on the right hand side and increasing in λ. Intuitively, the inequality holds if presenting two products yields a large increase in consumer mass (i.e. $M^{max} - M$), but the retail profitability is not much lower. For product demand factor d linear and M as given in the Example (uniformly distributed information, $M^{max} = 2$), we can show that (2.13) holds, when presenting two instead of one product, the mass increases more than the profitability decreases.

Proposition 2.11. *If RPM can be used, there always exists a unique equilibrium in undominated strategies in which retailers present only one product. If RPM cannot be used and condition (2.13) holds, there exist only equilibria in which at least one retailer presents both products.*

[26] As an extension, one can assume that retailers can charge service fees to finance a better service.

[27] Only pure strategy equilibria exist.

The proposition implies there are indeed cases in which the retail service quality in the form of product information is lower when minimum RPM is used, when compared to a situation without RPM. That is the case even when better service is costless. Hence minimum RPM can distort efficiency. Note that in the present parametrization with linear demand and a uniform distribution of the retailer information, with RPM 25% less consumers find a suitable product than without RPM. Although this result only highlights a possibility that RPM can reduce service incentives, we would argue that the intuition is more general. With RPM it is generally more profitable for the retailer to increase competition between manufacturers for the retailer, which is not necessarily in the interest of consumers, for example by reducing shelve space.

Asymmetric market power and resale price maintenance

In this subsection we examine the effect of market power on the allocation of services. To this end we assume that product B is produced by two different manufacturers. Without RPM, Bertrand competition between the manufacturers of B forces the wholesale prices of that product to zero. This implies a retail price of $p^*(0)$ for product B (Lemma 2.4).

Manufacturer A earns positive profits by setting a positive wholesale price. As the retail profitability decreases in a product's wholesale price (Lemma 2.4), retailers divert demand to product B in equilibrium. Without RPM, retailers thus allocate more services to product B.

Now assume that RPM is admitted. By nature of perfect competition, the manufacturers of product B cannot effectively increase the retail price with RPM. To see this, assume to the contrary that both manufacturers offer tariffs with wholesale prices of zero and a fixed retail price different from $p^*(0)$, which implies that they effectively use RPM. This cannot be an equilibrium as a manufacturer of product B could profitably offer a contract with a slightly positive wholesale price and let retailers choose the price. Each retailer strictly prefers such an offer as it can play its best response to the other retailer.

Lemma 2.12. *In any equilibrium, $w_B = 0$ and $p_{B,1} = p_{B,2} = p^*(0)$. It is an equilibrium that each manufacturer of product B offers $w_B = 0$ and does not fix the retail price.*[28]

Lemma 2.12 implies that the perfectly substitutable manufacturers of product B cannot effectively use RPM. Hence the retail profitability on product B is not affected by the enforceability of RPM.

The profitability of product A generally depends on whether manufacturer A uses RPM. Faced with the same equilibrium prices on product B independent of whether RPM is feasible, manufacturer A is at least as well off when fixing the

[28] Both manufacturers of B setting $w_B = 0$ and fixing $p_B = p^*(0)$ is not necessarily an equilibrium as one manufacturer could offer $w_B \geq 0$ and fix a much higher p_B and possibly be accepted by both retailers. Moreover, there is no equilibrium in strictly mixed strategies with RPM.

retail price. With RPM, manufacturer A sets $p_A = p^M$ to maximize the overall profitability on product A and sets a positive w_A by trading off the own margin and retailers' service incentives.

To understand the effects of RPM on prices and service allocations, consider two polar cases: retail monopolies vs. close substitutes (i.e., small competitive retail margins). A monopoly retailer faced with input costs of $w_B = 0$ sets the profit maximizing price $p_B = p^M$. Hence in case of retail monopolies, the retail profitability is maximal on product B and strictly smaller on product A as $w_A > w_B = 0$ and $p_A = p_B = p^M$. Thus service is excessively allocated to product B, although double marginalization on product A is reduced. By contrast, in case of fierce retail competition the profitability on product B is arbitrarily low and manufacturer A uses RPM to raise the retail margin and thereby the profitability of A over that of B. Hence service is allocated more to product A in equilibrium. In this case, RPM raises the price level of product A and yields that services are allocated excessively to the more expensive product.

Proposition 2.13. *Assume that two manufacturers sell product B and one manufacturer sells product A. If RPM is not enforceable, service is allocated more to product B than to the more expensive product A. If RPM is enforceable and retailers are close substitutes, product A is more expensive than product B and services are allocated more to product A.*

The case with fierce retail competition and enforceable RPM exhibits that A has a high, manufacturer-maintained retail price and is favorably sold by retailers, whereas product B is both less expensive and less endowed with services, e.g., is less advised or advertised. For instance, A could be a branded product and B a private label which can be produced by several manufacturers. Interestingly, the price-service differential (high price & high service vs. low price & low service) is not caused by different product qualities (vertical differentiation), but by asymmetric market power at the manufacturer level.

2.4 Policy implications

The Leegin decision relies heavily on the traditional service argument that the incentives of a manufacturer and the consumers are broadly aligned with respect to retail margins. We have shown that this is may not be the case if a manufacturer imposes RPM on a retailer who also sells products of other manufacturers. In this case minimum RPM harms consumers through higher prices and can at the same time decrease the quality of the retail services in two important ways. First, if manufacturer have uneven market power, minimum RPM leads to a bias in retailers advice and product presentation towards products of manufacturers that enjoy more market power. Second, to the extant that less overall services intensifies the competition between manufacturers for favorable treatment, retailers can even have an incentive to provide less overall services if manufacturers can use RPM.

Our results have important implications for assessing in individual competition policy cases whether minimum RPM is beneficial. For example, the EU Vertical Guidelines of 2010 state in recital 124 that a manufacturer needs to establish that RPM increases pre-sales services that would otherwise be under-provided and benefit consumers. A likely test is to check whether with RPM the retailers provide more services for the manufacturer's products and sell more of them. Note, however, that if RPM induces retailers to favor that manufacturer's products to the detriment of competing products, the same pattern of increased sales efforts and sales of that product should result. Hence, this test can be misleading for multi-product retailers. The competition authority should carefully assess whether RPM actually leads to more and better services for the consumers. A decrease in the sales of other products could be indicative for an inefficient diversion of services. On the contrary, if RPM is associated with an increase in sales of all products in the same product category, consumer harm is less likely.

Although we present a new and important theory of harm with respect to RPM and retail services, we believe that the general treatment of minimum RPM in competition policy should account for the potential alternative restraints used by manufacturers if RPM is not available. In particular, other vertical restraints may be used as a substitute for RPM and similarly lead to distorted retail services and increased retail prices. As a consequence, an avenue for future research is to further investigate the social trade-offs between different vertical restraints, including exclusive territories, selective distribution systems and fixed fees like slotting allowances. Intuitively vertical restraints that can be used to increase retail prices, like exclusive territories, can have similar negative effects as minimum RPM.

Appendix A: Proofs

Proof of Lemma 2.4. (i.) The FOC for the retail price is given by (2.4) and is independent of w_{-i} and s_k. Evaluating the FOC at symmetric retail prices defines the unique and symmetric equilibrium price $p_{i,1} = p_{i,2} = p_i^*(w_i)$, where uniqueness follows by a contraction mapping argument (dominant diagonal of the Hessian matrix). To obtain the pass trough rate, $\frac{\partial p_i^*}{\partial w_i}$, implicitly differentiate (2.4). The regularity assumptions imposed on $d_{i,k}$ imply $0 < \frac{\partial p_i^*}{\partial w_i} < 1$.

(ii.) Let $\varphi_{i,k}^*(w_i) \equiv (p_i^*(w_i) - w_i)d_{i,k}(p_i^*, p_i^*)$. To see that $\frac{\partial \varphi_{i,k}^*}{\partial w_i} < 0$, note that the retail margin decreases in w_i as $\frac{\partial p_i^*}{\partial w_i} < 1$; moreover, $d_{i,k}$ decreases in w_i because $\frac{\partial p_i^*}{\partial w_i} > 0$ and $\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} < 0$ (own price effect dominates).

(iii.) The equilibrium value s_k^* is defined by the FOC (2.5) evaluated at $p_{i,k} = p_i^*(w_i)\ \forall\ i, k$. By symmetry of the retailers, $s_1^* = s_2^*$. Implicit differentiation of (2.5) yields

$$\frac{\partial s_k^*}{\partial w_i} = -\frac{\partial^2 \Pi_k}{\partial s_k \partial w_i} \Big/ \frac{\partial^2 \Pi_k}{(\partial s_k)^2} = -\frac{\frac{\partial M_i}{\partial s_k}\frac{\partial \varphi_{i,k}^*}{\partial w_i}}{\frac{\partial^2 M_A}{(\partial s_k)^2}\varphi_{A,k}^* + \frac{\partial^2 M_B}{(\partial s_k)^2}\varphi_{B,k}^*}. \tag{2.14}$$

$\frac{\partial^2 \Pi_k}{(\partial s_k)^2} < 0$ holds as M_i is strictly concave. The sign of $\frac{\partial s_k^*}{\partial w_i}$ thus equals the sign of $\frac{\partial^2 \Pi_k}{\partial s_k \partial w_i} = \frac{\partial M_i}{\partial s_k}\frac{\partial \varphi_{i,k}^*}{\partial w_i}$. As shown in part (ii.), $\frac{\partial \varphi_{i,k}^*}{\partial w_i} < 0$. From Assumption 2.1, $\frac{\partial M_A}{\partial s_k} > 0 > \frac{\partial M_B}{\partial s_k}$. Hence, $\frac{\partial s^*}{\partial w_A} < 0$ and $\frac{\partial s^*}{\partial w_B} > 0$. Thus $\frac{\partial M_i^*}{\partial w_i} = \left[\frac{\partial M_i}{\partial s_k} + \frac{\partial M_i}{\partial s_{-k}}\right]\frac{\partial s^*}{\partial w_i} < 0$ as the term in brackets is positive for $i = A$ and negative for $i = B$. Equal wholesale prices $w_A = w_B$ imply equal retail prices $p_A^* = p_B^*$ and thus equal profitabilities $\varphi_{A,k}^* = \varphi_{B,k}^* \equiv \varphi^*$. Hence $s^* = \arg\max_{s_k} M_A \cdot \varphi^* + M_B \cdot \varphi^* = \arg\max_{s_k} M_A + M_B = 1/2$, i.e. service is allocated evenly. □

Proof of Proposition 2.5. Differentiating a manufacturer's profit π_i from (2.6) with respect to w_i yields the FOC

$$M_i^* d_{i,k} + w_i M_i^* \left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}}\right)\frac{\partial p_i^*}{\partial w_i} + w_i d_{i,k}\left[\frac{\partial M_i}{\partial s_k}\frac{\partial s_k^*}{\partial w_i} + \frac{\partial M_i}{\partial s_{-k}}\frac{\partial s_{-k}^*}{\partial w_i}\right] = 0 \tag{2.15}$$

Evaluating the FOC at $w_A = w_B = w^N$, and correspondingly $p_A = p_B = p^N$ and $s_k^* = 1/2 \forall k$, and dividing by M_i yields

$$d_{i,k} + w^N \left[\left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}}\right)\frac{\partial p_i^*}{\partial w_i} + \frac{d_{i,k}}{M_i}\left(\sum_k \frac{\partial M_i}{\partial s_k}\frac{\partial s_k^*}{\partial w_i}\right)\right] = 0. \tag{2.16}$$

Quasi-concavity of $\pi_i(w_i)$ implies that the above condition characterizes the equilibrium wholesale price. Substituting for $\frac{\partial s_k^*}{\partial w_i}$ from (2.14) yields

$$d_{i,k}+w^N\left[\left(\frac{\partial d_{i,k}}{\partial p_{i,k}}+\frac{\partial d_{i,k}}{\partial p_{i,-k}}\right)\frac{\partial p_i^*}{\partial w_i}+d_{i,k}\cdot\frac{\partial \varphi_{i,k}^*}{\partial w_i}/\varphi_{i,k}^*\cdot\left(\frac{-1}{2M_i}\sum_k\left(\frac{\partial M_i}{\partial s_k}\right)^2/\frac{\partial^2 M_i}{(\partial s_k)^2}\right)\right]=0. \tag{2.17}$$

Let

$$\lambda\equiv\frac{-1}{2\,M_i}\sum_k\left(\frac{\partial M_i}{\partial s_k}\right)^2/\left(\frac{\partial^2 M_i}{(\partial s_k)^2}\right). \tag{2.18}$$

Use (2.18) and $d_{i,k}\cdot\frac{\partial\varphi_{i,k}^*}{\partial w_i}/\varphi_{i,k}^*=\frac{\partial d_{i,k}}{\partial p_{i,k}}+\frac{\partial d_{i,k}}{\partial p_{i,-k}}\frac{\partial p_i^*}{\partial w_i}$ (implied by the FOC (2.4)) to reduce (2.17) to

$$w^N\left\{\left(\frac{\partial d_{i,k}}{\partial p_{i,k}}+\frac{\partial d_{i,k}}{\partial p_{i,-k}}\right)\frac{\partial p_i^*}{\partial w_i}+\lambda\left[\frac{\partial d_{i,k}}{\partial p_{i,k}}+\frac{\partial d_{i,k}}{\partial p_{i,-k}}\frac{\partial p_i^*}{\partial w_i}\right]\right\}+d_{i,k}=0. \tag{2.19}$$

Rearranging (2.19) yields (2.7). Note that (2.9) follows from symmetry in k, i.e., at $s^*=1/2$, $\lambda=\frac{-1}{2\,M_i}\sum_k\left(\frac{\partial M_i}{\partial s_k}\right)^2/\left(\frac{\partial^2 M_i}{(\partial s_k)^2}\right)=\frac{\partial M_i/\partial s_k}{M_i}\times\frac{\partial M_i/\partial s_k}{-\partial^2 M_i/(\partial s_k)^2}$. To see that $\frac{\partial w^N}{\partial\lambda}<0$, implicitly differentiate the equilibrium FOC (2.19) to obtain

$$\frac{\partial w^N}{\partial\lambda}=-\frac{\partial^2\pi_i\left(w^N,w^N\right)}{\partial w_i\partial\lambda}/\left(\frac{\partial^2\pi_i\left(w^N,w^N\right)}{\partial w_i\partial w_i}+\frac{\partial^2\pi_i\left(w^N,w^N\right)}{\partial w_i\partial w_{-i}}\right).$$

Local stability implies $\frac{\partial^2\pi_i\left(w^N,w^N\right)}{\partial w_i^2}+\frac{\partial^2\pi_i\left(w^N,w^N\right)}{\partial w_i\partial w_{-i}}<0$. Moreover,

$$\frac{\partial^2\pi_i}{\partial w_i\partial\lambda}=w^N\left[\frac{\partial d_{i,k}(p^N,p^N)}{\partial p_{i,k}}+\frac{\partial d_{i,k}(p^N,p^N)}{\partial p_{i,-k}}\frac{\partial p_i^*}{\partial w_i}\right]<0$$

follows from the assumption that the own price effect dominates and $0<\frac{\partial p_i^*}{\partial w_i}<1$ (Lemma 2.4). Thus $\frac{\partial w^N}{\partial\lambda}<0$. The retail profit decreases in w_i by Lemma 2.4 and hence increases in λ. □

Proof of Proposition 2.6. As argued in the text, using RPM is a dominant strategy for a manufacturer. Given wholesale and retail prices, each retailer chooses $\hat{s}=\arg\max_{s_k}\Pi_k$. Let $\varphi_{i,k}(p_i,w_i)\equiv(p_i-w_i)d_{i,k}(p_i,p_i)$. Implicit differentiation of the FOC $\partial\Pi_k/\partial s_k=0$ yields

$$\frac{\partial\hat{s}}{\partial w_i}=\left(\frac{\partial M_i}{\partial s_k}\,d_{i,k}(p_i,p_i)\right)/\left(\frac{\partial^2 M_i}{\left(\partial s_k\right)^2}\,\varphi_{i,k}+\frac{\partial^2 M_i}{\left(\partial s_k\right)^2}\,\varphi_{-i,k}\right), \tag{2.20}$$

and, analogously,

$$\frac{\partial\hat{s}_k}{\partial p_i}=-\left(\frac{\partial M_i}{\partial s_k}\frac{\partial\varphi_{i,k}}{\partial p_i}\right)/\left(\frac{\partial^2 M_i}{\left(\partial s_k\right)^2}\,\varphi_{i,k}+\frac{\partial^2 M_i}{\left(\partial s_k\right)^2}\,\varphi_{-i,k}\right). \tag{2.21}$$

A manufacturer solves $\max_{w_i,p_i} \pi_i = w_i M_i(s_k, s_{-k}) \sum_k d_{i,k}(p_i, p_i)$, taking the prices w_{-i} and p_{-i} of the other product as given. This yields the FOCs

$$\frac{\partial \pi_i}{\partial w_i} = 2\, d_{i,k}\, M_i + 2\, w_i d_{i,k} \left(\frac{\partial M_i}{\partial s_k} \frac{\partial \hat{s}}{\partial w_i} \right) \quad = \quad 0, \tag{2.22}$$

$$\frac{\partial \pi_i}{\partial p_i} = 2\, w_i \left[\left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} \right) M_i + 2\, d_{i,k} \left(\frac{\partial M_i}{\partial s_k} \frac{\partial \hat{s}}{\partial p_i} \right) \right] \quad = \quad 0. \tag{2.23}$$

Impose symmetry $w^R = w_A = w_B$, substitute for $\frac{\partial \hat{s}}{\partial w_i}$ from (2.20) in (2.22) and substitute λ to obtain

$$\frac{\partial \pi_i}{\partial w_i} = 2\, d_{i,k}\, M_i + 2\, w_i d_{i,k} \left(-\frac{d_{i,k}}{(p-w)d}\, \lambda * M_i \right) \quad = \quad 0 \tag{2.24}$$

$$\Longrightarrow \; w^R \quad = \quad \frac{p^R}{1+\lambda}. \tag{2.25}$$

Condition (2.25) characterizes the relationship between the wholesale price and the equilibrium retail price p^R. To determine p^R, substitute for $\frac{\partial \hat{s}}{\partial p_i}$ from (2.21) in (2.23) to obtain

$$\left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} \right) + \left(\frac{d_{i,k}}{(p^R - w^R)} + \frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} \right) \lambda \quad = \quad 0. \tag{2.26}$$

Substitute for w^R from (2.25) to obtain

$$p^R \quad = \quad \frac{d_{i,k}}{-\left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} \right)}. \tag{2.27}$$

This is the FOC implied by (2.10) which holds if and only if $p_i = p^M$. The wholesale price clearly decreases in λ as p^R is independent of λ. For $\lambda \to \infty$, $w^R \to 0$ and for $\lambda \to 0$, $w^R \to p^M$. $\square$

Proof of Proposition 2.7. The condition $p^N = p^M$ defines a λ such that prices with and without RPM are equal. Substituting for p^N from (2.7) and (2.8), and for p^M from the FOC implied by (2.10), the condition $p^N = p^M$ becomes

$$\frac{-d_{i,k}}{\left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} \right) \frac{\partial p_i^*}{\partial w_i} + \lambda \left(\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} \frac{\partial p_i^*}{\partial w_i} \right)} + \frac{-d_{i,k}}{\frac{\partial d_{i,k}}{\partial p_{i,k}}} = \frac{-d_{i,k}}{\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}}}.$$

Note that all expressions with $d_{i,k}$ are evaluated at prices p^M and $\frac{\partial p_i^*}{\partial w_i}$ at $\left(p^M, w^N\right)$. Isolating λ yields

$$\lambda = \frac{-\partial d_{i,k}(p^M, p^M)/\partial p_{i,k}}{\partial d_{i,k}(p^M, p^M)/\partial p_{i,-k}} - 1 \equiv \lambda^M. \tag{2.28}$$

To see that $\lambda \gtreqless \lambda^M$ implies $p^M \gtreqless p^N$, note that p^M does not depend on λ, whereas w^N decreases in λ by Proposition 2.5 and $\frac{\partial p^N}{\partial w^N} = \frac{\partial p_i^*(w)}{\partial w} > 0$ by Lemma 2.4. $\square$

Proof of Proposition 2.8. Strict concavity of Π_k in $p_{i,k}$ (which follows from weak concavity of $d_{i,k}$) implies that $\frac{\partial \Pi_k}{\partial p_{i,k}}$ is monotone in $p_{i,k}$. Thus if $\frac{\partial \Pi_k}{\partial p_{i,k}}$ is negative (positive) at $w_i = w^R$ and $p_{i,k} = p_{i,-k} = p^M$, each retailer wants to decrease (increase) its price and thus RPM is used as minimum (maximum) RPM. Hence minimum RPM is used if and only if

$$\frac{\partial d_{i,k}(p^M, p^M)}{\partial p_{i,k}} (p^M - w^R) + d_{i,k}(p^M, p^M) < 0.$$

Add $0 = \frac{\partial d_{i,k}}{\partial p_{i,-k}} p^M - \frac{\partial d_{i,k}}{\partial p_{i,-k}} p^M$ on the left hand side to obtain

$$\underbrace{\frac{\partial d_{i,k}}{\partial p_{i,k}} p^M + p^M \frac{\partial d_{i,k}}{\partial p_{i,-k}} + d_{i,k}}_{=0 \text{ at } p^M} - w^R \frac{\partial d_{i,k}}{\partial p_{i,k}} - p^M \frac{\partial d_{i,k}}{\partial p_{i,-k}} < 0.$$

Substitute $w^R = \frac{p^M}{1+\lambda}$ (Proposition 2.6) to get $\lambda > \left[-\frac{\partial d_{i,k}(p^M, p^M)}{\partial p_{i,k}} / \frac{\partial d_{i,k}(p^M, p^M)}{\partial p_{i,-k}} \right] - 1 \equiv \lambda^M$. For $\lambda = \lambda^M$, RPM is not needed as $p^N(w^R) = p^M$; for $\lambda < \lambda^M$, maximum RPM is used in equilibrium. □

Proof of Proposition 2.10. Let $d_{i,k} = 1 - (\beta + \gamma)\, p_{i,k} + \gamma\, p_{i,-k}$ with $\beta, \gamma > 0$. Hence $\frac{\partial d_{i,k}}{\partial p_{i,-k}} = \gamma$, $\frac{\partial d_{i,k}}{\partial p_{i,k}} = -(\beta + \gamma)$, $\frac{\partial d_{i,k}}{\partial p_{i,k}} + \frac{\partial d_{i,k}}{\partial p_{i,-k}} = -\beta$. p_i^* is obtained from substituting the linear demand expressions into (2.5) and letting $p_{i,1} = p_{i,2} = p$. This yields $1 - \beta\, p + \gamma\, p + (p - w)(-\beta) = 0$. Solving for p yields $p_i^* = \frac{1 + w\,(\beta+\gamma)}{2\beta+\gamma}$, $\frac{\partial p_i^*}{\partial w_i} = \frac{\beta+\gamma}{2\beta+\gamma}$, and $d_{i,k}^* = 1 - \beta \frac{1+w\,(\beta+\gamma)}{2\beta+\gamma}$. The demand factor M_i is kept in reduced form, yielding the parameter λ. Equilibrium prices are obtained by plugging the linear-demand analogs into the reduced form expressions (2.10), (2.7), (2.8), and (2.11). This yields $w^N = \frac{1}{2\beta(1+\lambda)}$, $p^N = \frac{1+w^N\,(\beta+\gamma)}{2\beta+\gamma} = \frac{1}{2\beta+\gamma}(1 + \frac{\beta+\gamma}{2\beta(1+\lambda)})$, $p^R = p^M = \frac{1}{2\beta}$, and $w^R = \frac{p^M}{1+\lambda} = \frac{1}{2\beta\,(1+\lambda)}$. Note that $w^N = w^R$, i.e., the wholesale price does not depend on the pricing regime. As argued in the text, this condition implies that manufacturer profits with minimum (maximum) RPM are lower (higher) than without RPM. Retailers benefit from minimum RPM as it maximizes industry profits and manufacturers lose. Retailers lose when maximum RPM is used as input prices remain unchanged, but their margins are lower than is individually optimal for a retailer. □

Proof of Proposition 2.11. Recall from Example 2.2 that the total mass of successfully matched consumers is $M = 3$ if each retailer presents only one product; it is

$M^{max} = 4$ if at least one retailer presents both products because in this case all consumers are matched successfully.

Using the expressions from Example 2.10 for the linear demand factor d yields a profitability of $(p^N - w^N)\, d(p^N) = \frac{(\beta+\gamma)(1+2\lambda)^2}{4(2\beta+\gamma)^2(1+\lambda)^2}$ if each retailer presents only one product and $\frac{(\beta+\gamma)}{4(2\beta+\gamma)^2}$ otherwise, which corresponds to $\lambda = 0$.

Substituting the parametrization of M from Example 2.2 in the definition (2.9) of λ yields $\lambda = \frac{(\partial M_i/\partial s_k)^2}{M \cdot \left(-\partial^2 M/\partial s_k^2\right)} = \frac{(2*(1-2*0.5/2))^2}{M \cdot 2} = \frac{1}{6}$.

Inequality (2.13) reduces to $\frac{\beta+\gamma}{49 \cdot (2\beta+\gamma)^2}$ and is positive as $\beta, \gamma > 0$. Hence without RPM, at least one retailer presents both products. □

Appendix B: Direct inter-brand price competition

The assumption of no direct cross price effects between products A and B simplified the previous exposition, but is certainly not always realistic. In this section we show that also under direct price competition, manufacturers use minimum RPM to increases prices even without any benefit to consumers and minimum RPM can create a prisoner's dilemma to manufacturers.

We allow for cross price effects between the products of the different manufacturers by allowing $d_{i,k}$ to depend on the retail prices of product $-i$. We focus on perfect retailer competition with discrete money. Perfect retail competition ensures that minimum RPM is used in equilibrium and strategic delegation of pricing is not relevant. Discrete money ensures that retailers have a positive equilibrium margin even without RPM so that the service decision is meaningful.[29] Denoting the smallest unit of money by $\triangle > 0$, the competitive retail margins equal $\triangle$.[30] Clearly, the translation from wholesale to retail price is $\frac{\partial p_i^*}{\partial w_i} = 1$.

Formally, because of perfect retail competition the total quantity demanded of product i only depends on the lowest price for each product: $D_i = M_i\, d_i(p_A = \min(p_{A,1}, p_{A,2}), p_B = \min(p_{B,1}, p_{B,2}))$, presumed to satisfy

Assumption 2.14. The own price effect is dominant and the Hessian matrix of d_i has a negative dominant main diagonal.

Assuming that for $p_{i,k} = p_{i,-k}$ the demanded quantity distributes equally over retailers, without RPM each retailer's profit reduces to $1/2 \triangle \sum_i M_i d_i$. A manufacturer can still influence service incentives by lowering its wholesale price as this increases d_i. Yet there is now one additional effect: The demand for product $-i$ increases in the retail prices of product i. Hence retailers shift more services to product $-i$ in response to a retail price increase of product i. Solving for the wholesale

[29] If a retailer makes zero margins on both products, he makes zero profits with every service allocation.

[30] One can equivalently assume that money is continuous so that retailers make zero margins, but that retailers, given that they make zero profits anyway, maximize the quantity of sales.

price analogously to Proposition 2.5 yields

$$w^N = \frac{d_i\left(p^N, p^N\right)}{-\frac{\partial d_i(p^N,p^N)}{\partial p_i}(1+\lambda) + \lambda\frac{\partial d_{-i}(p^N,p^N)}{\partial p_i}} \tag{2.29}$$

with $d_{i,k}$ evaluated at p^N. As $\triangle \to 0$, $p^N \to w^N$. We assume that $\triangle$ is very small and use $p^N \approx w^N$ from now on. Following analogously the steps of the proof to Proposition 2.6 yields $w^R = \frac{p^R}{1+\lambda}$ as before and

$$p^R = \frac{d_{i,k}\left(p^R, p^R\right)(1+\lambda)}{-\frac{\partial d_i(p^R,p^R)}{\partial p_i}(1+\lambda) + \frac{\partial d_{-i}(p^R,p^R)}{\partial p_i}\lambda}. \tag{2.30}$$

Comparing (2.29) and (2.30) and using Assumption 2.14 reveals that $p^R > p^N$ and $w^R < w^N$. Hence RPM implies a prisoner's dilemma for the manufacturers as both manufacturer margins and sales quantities decrease in comparison to the regime without RPM.

Proposition 2.15. *With discrete money, direct price competition between manufacturers, and perfect price competition between retailers, manufacturers always use minimum RPM and, in equilibrium, $p^R > p^N$ and $w^R < w^N$.*

Proof. The derivation of w^N is analogous to the proof of Proposition 2.5. Because of perfect price competition, $\frac{\partial p_i^*}{\partial w_i} = 1$, $d_{i,k} = d_{i,-k} = 1/2 d_i$ and $p_i^*(w_i) - w_i = \Delta$. By the implicit function theorem on the FOC to the problem $\max_{s_k} 1/2\Delta M_i d_i$ we get

$$\frac{\partial s_k}{\partial w_i} = -\frac{\frac{\partial M_i}{\partial s_k}\left(\frac{\partial d_i}{\partial p_i}\right) + \frac{\partial M_{-i}}{\partial s_k}\left(\frac{\partial d_{-i}}{\partial p_i}\right)}{\frac{\partial^2 M_A}{(\partial s_k)^2} d_A + \frac{\partial^2 M_B}{(\partial s_k)^2} d_B}. \tag{2.31}$$

Substituting from (2.31) in the analogue to (2.15) gives us the characterization of w^N in (2.29).

For the equilibrium with RPM, the expressions for $\frac{\partial \hat{s}}{\partial w_i}$, $\frac{\partial \pi_i}{\partial w_i}$ and $\frac{\partial \pi_i}{\partial p_i}$ in the proof to Proposition (2.6) remain analogously valid and $\frac{\partial \hat{s_k}}{\partial p_i}$ changes analogously to $\frac{\partial s_k}{\partial p_i}$ above and is given by

$$\frac{\partial \hat{s_k}}{\partial p_i} = -\frac{\frac{\partial M_i}{\partial s_k}\left(d_{i,k} + (p_i - w_i)\frac{\partial d_i}{\partial p_i}\right) + \frac{\partial M_{-i}}{\partial s_k}\left((p_{-i} - w_{-i})\frac{\partial d_{-i}}{\partial p_i}\right)}{\frac{\partial^2 M_i}{(\partial s_k)^2}\varphi_i + \frac{\partial^2 M_i}{(\partial s_k)^2}\varphi_{-i}}.$$

Using these expressions, w^R and p^R are derived. Noting that the right hand sides of the equations are monotonous in the price level under Assumption 2.14. The comparison of (2.30) and (2.29), implies $p^R > p^N$. Substituting for p^R in the expression for w^R, we obtain

$$w^R = \frac{d_i\left(p^R, p^R\right)}{-\left(\frac{\partial d_i(p^R,p^R)}{\partial p_i}\right)(1+\lambda) + \frac{\partial d_{-i}(p^R,p^R)}{\partial p_i}\lambda} < \frac{d_i\left(p^N, p^N\right)}{-\frac{\partial d_i(p^N,p^N)}{\partial p_i}(1+\lambda) + \frac{\partial d_{-i}(p^N,p^N)}{\partial p_i}\lambda} = w^N$$

which is true because $p^R > p^N$ and again each side of the inequality is decreasing monotonically in the retail price level. □

Chapter 3

Why do Platforms Charge Proportional Fees? Commitment and Seller Participation[1]

3.1 Introduction

Sellers frequently use marketplaces (or trade platforms) to reach consumers. Before they can offer their products on a particular platform, sellers often have to sink platform-specific investment costs, such as development costs. In turn, a platform operator who wants to attract sellers has to guarantee sellers some return on their investment by leaving them a positive margin on sales. However, as the platform operator easily observes sales and, thus, can identify profitable products, he is tempted to cut out the respective sellers, collecting (parts of) their margins just after they established their products on his platform. This generates a particular hold-up problem for platform operators who can offer products themselves.

For example, Amazon is a retailer and, at the same time, provides a platform for sellers to access their customers – the Amazon Marketplace.[2] Similarly, Apple and Google provide their own applications next to third-party applications in their online stores. Using the language of Hagiu (2007), these intermediaries combine the *merchant mode* and the *platform mode*. Therefore, we call this policy "operating under a *dual mode*": for some products, intermediaries act as classical retailers, buying from suppliers and setting prices (merchant mode), while they also allow external sellers access to consumers on their platform for some fee (platform mode).

Interestingly, Amazon primarily charges proportional fees (a fixed share of the revenue) to sellers who use the Amazon Marketplace.[3] Similarly, Apple and Google charge software providers proportional fees for selling their applications in the App-

[1]This chapter is based on joint research with Sebastian Wismer

[2]According to Amazon's reports, sales by third-party sellers reached 36% of unit sales in 2011.

[3]Besides a small membership fee and a fixed per-transaction fee, Amazon charges sellers a proportional fee of about 15% (depending on product category).

Store and on Google Play.[4] Likewise, proportional fees are usually included in franchising arrangements, where the franchisor offers the franchisee a business model (platform) to reach consumers.[5]

In these examples, the platform operator (franchisor) is also a potential competitor to sellers (franchisees) as he often can serve demand himself.

In this paper, we analyze a model with a monopoly intermediary who provides a platform and can be a merchant at the same time. The intermediary can do cherry-picking, selling profitable goods himself after observing sellers' offers. However, this potential competition makes the platform less attractive to sellers in the first place. By choosing a platform tariff, the intermediary shapes competition between himself and sellers, trading off his gains from cherry-picking against platform attractiveness.

We focus on the case in which sellers have to sink investment costs before offering a new product on the platform. Sellers are better informed about product demand than the intermediary. Production costs can differ between sellers and the intermediary, i.e., market conditions are ex ante unknown. In this framework, we firstly analyze "classical" two-part tariffs consisting of fixed (membership) fees and per-transaction fees. Secondly, we examine tariffs that include proportional (per-revenue) fees.

While the extant economic literature concerned with the pricing of (two-sided) platforms has focused on linear and classical two-part tariffs, our analysis departs from this classical approach. In line with the studies of Shy (2011) and Wang and Wright (2014), we thereby account for the fact that proportional fees are often observed in reality. While Shy and Wang (2011) show that proportional fees mitigate double marginalization problems and Wang and Wright (2014) explain that they can be used as a means of price-discrimination, we find that proportional fees allow the intermediary to commit not to compete with sellers, thereby increasing the attractiveness of the platform.

Focussing on classical two-part tariffs first, we find that the intermediary prefers per-transaction fees over membership fees. In contrast to previous results (e.g.Armstrong (2006a)), he is no longer indifferent between both kinds of fees as transaction-based fees create a competitive advantage when the intermediary becomes active as a merchant. Regarding platform attractiveness, we find that an intermediary using classical two-part tariffs enters sellers' markets to undercut their prices whenever he has lower costs. This is to the detriment of the platform's attractiveness to sellers; in particular, if the intermediary is always more efficient than sellers, sellers will be undercut with certainty. Hence, sellers do not join the platform and products are not disclosed. In that case the intermediary would always profit from committing himself not to enter product markets. We find that contracts which include proportional fees allow an intermediary to do so: by increasing the opportunity costs of competition, the use of proportional fees makes it less attractive for an intermediary to compete with sellers as a merchant.

Introducing a dual mode of intermediation into the platform literature, our

[4] Apple and Google charge software developers a proportional fee of 30%.

[5] Cf. e.g.[p. 62ff.]Blair and Lafontaine (2010).

work sheds light on the different impacts of membership fees, per-transaction fees, and proportional fees on market outcomes. It provides a novel explanation why proportional fees are commonly observed in reality.

Related literature

Our paper is most closely related to the literature on platform pricing/two-sided markets and to the work on an intermediary's choice of the optimal intermediation mode.

To the best of our knowledge, the only studies that directly address the question whether an intermediary should take an active role as a (pure) merchant, buying products himself and reselling them to buyers, or a more passive role as a (pure) platform, enabling other sellers to reach potential buyers, are Hagiu (2007) and Hagiu and Wright (2013). Hagiu (2007) finds that under many circumstances a monopoly intermediary prefers the 'merchant mode' to the 'platform mode'. However, he also identifies several factors that affect the intermediary's choice towards the platform mode, e.g. consumers' demand for variety or asymmetric information about product quality between the intermediary and sellers.[6] Hagiu and Wright (2013) illustrate that an intermediary's decision on which intermediation mode to choose may also be driven by a trade-off between coordinating marketing activities as a merchant (taking into account potential externalities across products) and benefiting from sellers internalizing more precise information on individual demand as a platform.

We extend both analyses by explicitly allowing for endogenous seller pricing when the intermediary can become active as a merchant while offering a platform at the same time.

Similar to our work, Jiang et al. (2011) examine the case of an intermediary who both offers a platform and can serve demand himself (dual mode), crowding out sellers. In their framework, the intermediary has to incur fixed costs to enter a market. Better informed sellers fear that the intermediary serves markets with high demand himself to avoid double marginalization. However, by choosing a low service level, sellers can pretend to offer a product whose demand does not suffice to cover the intermediary's fixed costs. Accordingly, the setting also includes moral hazard. Although proportional fees would tackle both the double marginalization problem and the hold-up problem that arises due to screening, Jiang et al. analyze pure per-unit fees only.

During the last decade, several seminal studies on platform pricing/two-sided markets have been published (cf. e.g.[7]Rochet and Tirole (2006b); Armstrong (2006a)). They focus on intermediaries featuring the 'platform mode' and analyze tariff choices in presence of (indirect) network effects under various circumstances. Most stud-

[6]Differently from our model, Hagiu assumes that the merchant has to buy products from a seller who would otherwise sell them on the intermediary's platform (at an exogenous price).

[7]Jullien (2012) offers a comprehensive up-to-date survey on two-sided (B2B) platforms, including a general introduction to two-sided markets.

ies on platform pricing focus on membership fees, per-transaction fees, or two-part tariffs as a combination of both. Furthermore, they usually abstract away explicit payments between the two sides of a market or price setting by sellers. Accordingly, proportional (revenue-based) fees are not discussed.

However, there are several important exceptions who do examine proportional fees. Shy and Wang (2011) analyze a model of a payment card network. They find that profits of the card network are higher under proportional fees than under per-transaction fees as the network faces a double marginalization problem which is mitigated by proportional fees. In their framework, sellers earn lower profits under proportional fees, but consumers are better off and social welfare is higher than under per-transaction fees. Miao (2011) extends the model of Shy and Wang (2011). Allowing for an endogenous number of sellers, he shows that the use of proportional fees results in less seller participation. Consequently, consumer surplus and social welfare may be lower under proportional fees.

Wang and Wright (2014) examine the case of an intermediary who facilitates trade of products that differ in both costs and valuations. They illustrate that a combination of a per-transaction fee and a fee which linearly depends on price can achieve the same profit as third-degree price discrimination, even if the intermediary is uninformed about product attributes.

Hagiu (2006b) studies commitment of two-sided platforms to a tariff system. In contrast to previous studies (which assume that sellers and buyers take their decisions on joining a platform simultaneously), Hagiu analyzes a sequential time structure: he assumes that all sellers arrive at the platform before the first buyer does. He shows that a platform prefers to commit to the access price charged to buyers instead of setting or adapting it after sellers joined the platform under certain circumstances. Although Hagiu does not mention how commitment could be achieved, he points out that platform commitment is an important issue.

Hagiu (2009b) analyzes a platform's tariff decision when sellers compete and consumers value variety. In an extension, he explains that charging variable (proportional) fees can mitigate the aforementioned commitment problem.[8]

Belleflamme and Peitz (2010) analyze how intermediation affects manufacturers' investment incentives. In their model, sellers have to invest in innovation *before* the platforms set their prices. They show that sellers may overinvest in settings where platforms compete. Differing from our model, sellers anticipate the platforms' tariff choices, but, due to the suggested timing in the game, platforms cannot affect investment incentives with their tariff choice. Furthermore, the study differs from our analysis as it focusses on comparing open platforms (no fees) and for-profit platforms that charge *membership* fees only.

[8]However, note that in Hagiu's framework transaction-based fees can create a commitment not to change the buyer fee if buyers join the platform after sellers, while we find that proportional fees relax (potential) competition between the intermediary and sellers.

Our work may also be seen as a contribution to the literature on franchising:[9] by allowing a 'dual mode' of intermediation and analyzing a framework of asymmetric information on demand between sellers (franchisees) and intermediary (franchisor), we provide additional insights into a franchisor's decision on dual distribution/partial vertical integration (cf. e.g.Minkler (1992); Scott (1995); Hendrikse and Jiang (2011)) and on the frequent use of sales revenue royalties.

Taken together, we contribute to the economic literature firstly by introducing a "dual mode" of intermediation. Secondly, in contrast to the majority of the extant studies on two-sided markets, we explicitly account for trade between sellers and buyers, allowing for endogenous seller pricing. Thirdly, we show that an intermediary operating under the dual mode is no longer indifferent between membership fees and transaction-based fees. Fourthly, we identify a hold-up problem created by the threat of competition between the intermediary and sellers which impairs platform attractiveness. Finally, we find that platform tariffs that include proportional fees mitigate this problem, in contrast to "classical" two-part tariffs which previous literature focussed on.

Outline

The remainder of the paper is organized as follows: in section 2 we set up a model of a monopoly intermediary who offers a platform to connect sellers and buyers. In section 3 we solve the model for classical two-part tariffs which consist of membership fees and per-transaction fees. In section 4 we discuss the intermediary's hold-up problem, in particular by illustrating that commitment not to compete with sellers can be profitable. Within section 5 we analyze proportional fees as part of multi-part tariffs. In section 6 we summarize our findings and discuss the results.

3.2 Framework

We consider a market with a monopoly intermediary who offers sellers a platform to reach potential buyers and, at the same time, can offer products himself.

There is a unit mass of sellers. For being able to list a new product on the marketplace, a seller has to incur fixed investment costs I which are sunk after investment. These costs may be interpreted as costs of developing the respective product, or as general costs of sales preparation (e.g. market research, designing an attractive product illustration, or establishing capacities to ensure immediate supply). They are distributed among sellers according to a uniform distribution function: $I \sim U[0, 1]$. We assume products offered by different sellers to be completely independent. Hence, there is no competition between sellers. Taken together, there is a continuum of independent product markets which are characterized by their respective investment costs.[10] For selling their products, sellers incur constant marginal

[9] Blair and Lafontaine (2010) provide a good introduction into the economics of franchising.

[10] Note that our framework also covers the situation of a single seller with unknown investment

costs $c \in (0, r)$, incorporating all per-unit costs except for fees charged by the intermediary. In the following, we simply refer to c as (marginal) *production costs*, although c could also represent costs of purchasing the product from some wholesaler, retailing or transaction costs like payment charges, or the expected costs of product failure.

We assume that each buyer purchases at most one unit of each product. Buyers' gross utilities from consuming a unit of a good are constant over buyers and products and given by $r > c$. Accordingly, we abstract from double marginalization problems and buyer heterogeneity. Hence, the intermediary's tariff decision is neither driven by the effect of mitigating double marginalization unlike Shy and Wang (2011), nor by any price discrimination attempts unlike Wang and Wright (2011).[11] The mass of buyers is normalized to one. Buyers' (as well as sellers') outside options are normalized to zero. Hence, not joining the platform yields a zero payoff to either side. As we will assume that buyers do not have to pay a membership fee, it is a dominant strategy for buyers to join the platform.[12] Hence, for each product the demand function is given as[13]

$$D(p) = \begin{cases} 1, & p \leq r \\ 0, & p > r \end{cases} .$$

In order to guarantee interior solutions for the participation level of sellers we assume that $r - c \leq 1$, i.e., the gross margin does not exceed the highest level of investment costs.

The intermediary chooses a platform tariff system which can comprise different forms of payments by sellers: a membership fee A, a per-transaction fee a, or a proportional fee. For the latter a fixed share α of seller revenues accrues to the platform. All platform costs are normalized to zero.

Additionally, the intermediary can decide to compete with sellers who joined his platform, becoming active as a *merchant* in the respective product markets. In doing so, he either starts selling the same product, purchasing it from some supplier, or he imitates the product that is offered by a seller. More precisely, each product offered by the merchant is not differentiated from the corresponding seller's product.

We assume that the intermediary cannot offer a product if the respective seller did not join the platform.[14] In particular, this assumption captures the following

costs if the cumulative distribution of I is interpreted as a probability instead of the share of the mass of sellers having investment costs below I.

[11]Our results would also generalize to cases of heterogenous product categories with varying market sizes or different gross utilities across markets.

[12]We implicitly rule out trivial equilibria in which no buyer and no seller joins.

[13]We assume that the demand structure for new products is common knowledge. This seems reasonable at least within smaller product categories since the intermediary is supposed to be informed about typical market characteristics, but not about existence of specific products. Note that this might be a rationale for Amazon's discriminating practice of charging different fees across well-defined product categories.

[14]This assumption could be interpreted as a search cost advantages of sellers, Minkler (1992).

situation: the intermediary is ex ante uninformed about existence of new products or corresponding demand. In contrast, more specialized sellers are perfectly informed about existence of demand for products which they may offer. By joining the intermediary's platform, they disclose information. Thereby, the intermediary can easily learn existence of demand for each specific product as platform operator. We emphasize the role of sellers' demand information as we do not include product markets in our model for which the intermediary is informed about demand.

After sellers joined the intermediary's platform, he observes his constant marginal production costs and may pick profitable products, entering markets.[15] We assume that these marginal costs ζ are drawn from a distribution represented by a differentiable distribution function $H(\zeta)$ with support $[\underline{\zeta}, \overline{\zeta}]$. A draw of ζ captures the intermediary's relative bargaining position towards suppliers or his ability in imitating sellers' products; he may have higher or lower production costs than sellers, i.e., $c \in (\underline{\zeta}, \overline{\zeta})$. We assume that the merchant's marginal costs are determined by one single draw, and, hence, are the same for all products. For entering a market that was disclosed by a seller, the intermediary faces infinitesimally small (but positive) costs $\varepsilon > 0$. This assumption is made for two reasons: firstly, the asymmetry between the intermediary's and merchants' investment costs accounts for the fact that the intermediary becomes informed about important product characteristics without bearing any costs. Once a seller disclosed demand and established her product on the platform, it is much less costly to simply imitate the product. Secondly, positive investment costs solve the tie situation that the intermediary would face if he was indifferent with respect to market entry, i.e., in cases he faces higher production costs than the respective seller, and, hence, is not willing to serve any demand.

As the intermediary attains an exclusive information advantage about profitable product markets compared to sellers who are active in other markets, his imitation incentives are much stronger than those ones faced by other sellers. Therefore, we do not allow for sellers imitating each other but focus on potential competition between the intermediary and each individual seller.

Timing

The game has four stages; the timing is given as follows:[16]

1. The intermediary sets the platform tariff.[17]

[15] Again, we use the term "production costs" as representative for any kind of per-transaction costs.

[16] It may be natural to include another period of sales between the second and third stage. In this period, sellers who joined the platform could be active as monopolists. However, this would not affect any of our results.

[17] We implicitly assume that the tariff is contractible, or, at least, that commitment to a tariff system is feasible. Commitment seems plausible: As the tariff system is publicly observable, a reputation for not changing it can be obtained.

2. Sellers' investment costs are realized. Sellers and buyers decide on joining the platform.
3. The intermediary learns the existence of each seller's product and the respective production costs are realized. The intermediary decides whether to enter product markets.
4. In each product market that the intermediary entered he competes with the respective seller in prices; otherwise, sellers take their monopoly pricing decisions.

We assume that the structure of demand as well as all costs, once realized, are common knowledge to sellers and the intermediary. Both sellers and the intermediary are assumed to maximize their expected profits, i.e., they are risk neutral.

In the following, we firstly analyze tariffs that consist of a membership fee and a per-transaction fee charged to sellers. Secondly, we elaborate on the hold-up problem which emerges under those classical two-part tariffs. Finally, we discuss the case of a proportional fee, i.e., revenue sharing between the intermediary and each seller, as a particular component of three-part tariffs.

3.3 Classical two-part tariffs charged to sellers

In this section we consider classical two-part tariffs charged to sellers only. These tariffs combine a membership fee A as fixed transfer and a transaction-based per-unit fee a which increases each seller's perceived marginal costs. We restrict our analysis to non-negative fees: we rule out negative membership fees since they induce a moral hazard problem.[18] Similarly, negative per-transaction fees would create incentives for fictitious transactions.[19]

We solve the game described before by backward induction.

3.3.1 Stage 4: Product pricing decisions

We firstly look at the pricing decisions in one representative product market that a seller disclosed before. The seller paid the membership fee A up front. Hence, A are sunk costs at this stage. However, the seller pays the per-transaction fee a for each unit sold which increases her marginal costs to $c + a$. We can exclude cases where $a > r - c$ as then this stage would never be reached (zero seller participation).

If the intermediary did not enter the market, the seller is a monopolist, charging a price of

$$p^{mon} = r.$$

[18] With a negative A, sellers would list products they do not want to sell. In our setting the platform operator cannot distinguish good products from worthless ones before they are listed; hence, he would have to pay $|A|$ to the seller indiscriminate of the listing value.

[19] We abstract from the provision of free goodies (which could be interpreted as negative fees).

In this case, the seller's profit (before investment costs and membership fee) equals $\pi_s^{mon} = r - (c + a)$.

If the intermediary entered the market in stage 3, he and the seller compete à la Bertrand, with asymmetric costs. However, contrary to standard price competition, the intermediary receives a transfer of a for each unit sold by the seller.

If the intermediary undercuts the seller[20] by setting a price of

$$p_m^{comp}(a) = c + a,$$

his (merchant) profit from this market equals

$$\pi_m(a) = (c + a) - \zeta.$$

If he does not undercut the seller, the (variable) platform revenue that he receives from the seller equals

$$\pi_p(a) = a.$$

He prefers undercutting the seller if and only if

$$\pi_m(a) > \pi_p(a) \;\Leftrightarrow\; \zeta < c. \tag{3.1}$$

Hence, if production costs turn out to be below the seller's costs ($\zeta < c$), the intermediary serves demand himself as a merchant.

If the intermediary had entered the market in stage 3 although $\zeta > c$, the seller would serve the market at a price of

$$p_s^{comp}(a) = \min\{\zeta + a, r\}.$$

The intermediary would not undercut $p_s^{comp}(a)$ by any amount $k > 0$ as he would lose a in platform fees while only gaining merchant profits of $(\zeta + a - k) - \zeta < a$ (assuming that $\zeta + a \leq r$). Charging prices above r is dominated as it results in zero demand. Finally, the case that both are equally efficient ($\zeta = c$) happens with zero probability as the distribution of ζ is atomless.

Lemma 3.1. *[Product pricing under a classical two-part tariff]*

Under a classical two part tariff (A, a), if the intermediary did not enter a market, the respective seller is a monopolist, setting a price of r. If the intermediary entered a market and has lower production costs than the seller ($\zeta < c$), he undercuts the seller by setting a price of $c+a$. If he faces higher production costs ($\zeta > c$), the seller serves demand at a price of $\min\{\zeta + a, r\}$.

Note that competitive prices increase in the per-transaction fee as the increase in seller's perceived marginal costs relaxes competition.

[20] As is standard in the literature on Bertrand competition, we rule out prices below marginal costs (which would lead to "implausible" equilibria of the pricing game) because they are not limits of undominated strategies in discrete approximations of the strategy space.

3.3.2 Stage 3: Intermediary's entry decision

In stage 3, the intermediary decides on entering markets that sellers disclosed by joining the platform, anticipating the pricing decisions just discussed.

The intermediary decides on entry contingent on his production costs. He enters markets only if he serves demand, which is the case when he has lower production costs ($\zeta < c$), as then his merchant profit exceeds his foregone platform revenues, cf. condition (3.1).

If he entered without serving demand, he would lose exactly his entry costs $\varepsilon > 0$, without any gains.

Lemma 3.2. *[Intermediary's entry decision under classical two-part tariffs] Under a classical two-part tariff (A, a), the intermediary enters product markets if and only if his production costs are lower than sellers' costs ($\zeta < c$).*

Note that neither the fixed membership fee nor the per-transaction fee affects the intermediary's entry decision. This is intuitive for the membership fee, but more surprising for the per-transaction fee. The latter increases the platform revenue by a per unit. However, it also increases the competitive price and thus the merchant profit by a per unit. Hence, the per-transaction fee a does not affect the intermediary's trade-off between platform revenue and merchant profit.

3.3.3 Stage 2: Decisions on joining the platform

In stage 2, sellers and buyers simultaneously decide whether to join the platform.

Recall that for buyers joining is a dominant strategy. Hence, all buyers join the platform.[21] Sellers join the platform if they expect to be able to at least recoup their investment costs I.

As argued before, each seller will be a monopolist in her respective product market if $\zeta > c$, but will be undercut if $\zeta < c$. Hence, each seller's expected profit from joining the platform under a two-part tariff (A, a) is given by

$$\pi_s^e(A, a, I) = Pr(\zeta > c) \cdot \{r - (c + a)\} - I - A,$$

where $Pr(\zeta > c) = 1 - H(c)$ represents the probability that the intermediary does not enter as he is less efficient. Defining the critical level of investment costs

$$\widetilde{I}(A, a) \equiv \max\{0,\ (1 - H(c)) \cdot \{r - (c + a)\} - A\}, \tag{3.2}$$

we achieve the following result:

[21]Note that the joining decision would still be homogeneous if buyers had to pay fees as there is no buyer heterogeneity and, hence, each buyer faces the same trade-off. Consequently, there is either zero or full buyer participation, and zero participation can never occur in equilibrium as the intermediary could increase his profit by lowering fees.

Lemma 3.3. *[Decisions on joining the platform under classical two-part tariffs] Under a classical two-part tariff (A, a), all buyers join the platform. Sellers join if their investment costs are below $\widetilde{I}(A, a)$ as defined in (3.2). The mass of sellers joining the platform equals $\widetilde{I}(A, a)$ and it decreases in both A and a.*

Seller participation decreases in the membership fee A and in the per-transaction fee a as both fees decrease seller rents which lowers the maximum level of investment costs that sellers can cover without expecting a negative surplus from joining the platform.

We elaborate on the hold-up problem that evolves from the threat of entry (captured by the probability $1 - H(c)$) in more detail within the next section. Beforehand, we solve the model under two-part tariffs, analyzing the intermediary's tariff decision in the first stage.

3.3.4 Stage 1: Optimal classical two-part tariff

In stage 1 the intermediary sets the membership fee A and the per-transaction fee a.

Recall that under any two-part tariff (A, a) the intermediary will enter product markets as merchant if and only if he has lower production costs than sellers. The respective probability for ζ being below c is given by $H(c)$. Therefore, for each product listed on the marketplace, the intermediary's expected platform profit equals

$$\pi_p^e(A, a) = A + (1 - H(c)) \cdot a, \tag{3.3}$$

and his expected per-product merchant profit (which is independent of the membership fee A) is given by

$$\pi_m^e(a) = H(c) \cdot \{c + a - E[\zeta | \zeta < c]\}. \tag{3.4}$$

His expected overall profit is given by the sum of his platform profit $\pi_p^e(A, a)$ and his merchant profit $\pi_m^e(a)$, times the mass of sellers who joined the platform:

$$\Pi^e(A, a) = \widetilde{I}(A, a) \cdot \{\pi_p^e(A, a) + \pi_m^e(a)\}. \tag{3.5}$$

We observe that if we define the merchant's expected realized cost advantage as

$$\Delta^e(c) \equiv H(c) \cdot \left(c - \frac{1}{H(c)} \int_{\underline{\zeta}}^{c} x dH(x)\right), \tag{3.6}$$

we can rewrite the intermediary's expected overall profit (3.5), inserting (3.3) and (3.4), as

$$\Pi^e(A, a) = \widetilde{I}(A, a) \cdot \{A + (a + \Delta^e(c))\}. \tag{3.7}$$

While the first factor, $\widetilde{I}(A, a)$, is decreasing in A and a (cf. Lemma 3.3), the second factor, i.e., the intermediary's expected profit per market, is increasing in both fees.

Solving the intermediary's first order condition for a yields:

$$a^* = \max\left(\frac{r - c - \Delta^e(c)}{2}, 0\right). \tag{3.8}$$

Proposition 3.4. *[Optimal classical two-part tariff] The optimal two-part tariff consists of a zero membership fee and a positive per-transaction fee a^*, as defined in (3.8). The intermediary's equilibrium profit is given by*

$$\Pi^e(0, a^*) = \begin{cases} (1 - H(c)) \cdot \left(\frac{r-c+\Delta^e(c)}{2}\right)^2, & \Delta^e(c) < r - c \\ (1 - H(c)) \cdot (r - c) \cdot \Delta^e(c), & \Delta^e(c) \geq r - c \end{cases} \tag{3.9}$$

Proof. See appendix, p. 52.

The intuition why the intermediary prefers the per-transaction fee to the membership fee is the following: Starting from any combination of a positive membership fee and a per-transaction fee, increasing the per-transaction fee while lowering the membership fee such that platform revenues remain unchanged (implying a constant level of seller participation) increases the expected merchant profit $\pi_m^e(\cdot)$ by raising the competitive price. □

However, note that it can be optimal for the intermediary to charge no fees at all, i.e., $(A, a) = (0, 0)$. This is the case if $r - c$ is low compared to $\Delta^e(c)$. Under this constellation the intermediary prefers not to charge sellers in order to attain high seller participation; utilizing only the intermediary's natural cost advantage (in cases where $\zeta < c$) is more profitable than charging any tariff with positive platform profits and a higher competitive price but a lower level of seller participation.

In the following, we focus on interior solutions characterized by non-degenerate tariffs and positive platform revenues. Hence, we make the following assumption:

Assumption 3.5. *[Positive platform revenues] The intermediary's expected cost advantage does not exceed sellers' profit margin: $\Delta^e(c) < r - c$.*

3.4 Hold-up problem and commitment

In this section we provide a result showing that with two-part tariffs a hold-up problem always exists – the intermediary chooses to enter markets in too many cases which reduces the incentives for sellers to join the platform. By defining and discussing *full* and *partial commitment*, we illustrate that the intermediary could achieve higher profits if he entered less often, diminishing the threat of (potential) competition.

Firstly, we define *full commitment* as a situation under which the intermediary commits not to enter markets under any circumstances, i.e., taking the role of a pure platform operator ("platform-only"), never becoming active as a merchant. Secondly, we define *partial commitment* as a situation under which the intermediary only competes with sellers as a merchant if he faces a strictly positive cost advantage

$c-\zeta$, i.e., if ζ falls below a certain threshold strictly below c. Although committing to a certain "entry threshold" (measured by the intermediary's costs ζ) seems not realistic, it provides a clear benchmark. However, in the next section, we show that proportional fees imply an indirect commitment to such a threshold.

We illustrate the hold-up problem by showing that the intermediary would always profit if he was able to commit to a certain "entry threshold", measured by his costs ζ (*partial commitment*):

Proposition 3.6. *[Profitability of partial commitment] Under any classical two-part tariff (A, a) that yields positive seller participation, the intermediary strictly benefits from committing not to enter with costs above a threshold $\hat{\zeta} < c$.*

Proof. See appendix, p. 53. □

The reason why it is always profitable to marginally lower the "entry threshold", starting from its original level of c, is that at the margin ($\zeta = c$) the intermediary has no cost advantage and, hence, lowering the entry threshold implies no loss but strictly increases seller participation.

With classical two-part tariffs, the intermediary therefore faces a hold-up problem: he would like to commit to enter markets in less cases. However, as he decides on entry when sellers have already joined the platform, he will enter markets whenever he is more efficient (see Lemma 3.2). Hence, we arrive at the following result:

Corollary 3.7. *[Intermediary's hold-up problem under classical two-part tariffs] Under any two-part tariff consisting of a membership fee and a per-transaction fee, the intermediary faces a hold-up problem: his excessive entry behavior leads to insufficient seller investment incentives as well as poor seller participation and impedes him to open up all profitable product markets.*

In some cases the intermediary would even profit from a commitment never to enter, which we call *full commitment*.[22] Full commitment is profitable if the expected foregone profit of not entering is small. In order to compare the "platform-only" profit to the profit under the dual mode, we observe that under the "platform-only" mode the probability of sellers being undercut by the intermediary becomes zero. Hence, starting from the intermediary's profit (3.7), we can obtain his "platform-only" profit by setting $H(c)$ and $\Delta^e(c)$ equal to zero, yielding the left-hand side of the following condition:

$$\left(\frac{r-c}{2}\right)^2 > (1-H(c)) \cdot \left(\frac{r-c+\Delta^e}{2}\right)^2. \tag{3.10}$$

As the right-hand side of this condition equals the intermediary's profit under the dual mode, we have:

[22] One form of full commitment can be achieved by being a platform only. For example, eBay seems to have committed to the platform-only business model.

Proposition 3.8. *The intermediary profits from full commitment under two-part tariffs if inequality (3.10) holds.*

Intuitively, full commitment is profitable if it is likely that the intermediary has lower costs, but the expected cost advantage is small. However, the intermediary would often prefer to enter markets only if he is much more efficient, while committing not to enter when his cost advantage is small, i.e., *partial commitment* instead of *full commitment*.

3.5 Proportional fees mitigate the hold-up problem

We have shown that for any classical two-part tariff the intermediary always enters a seller's market when he has lower marginal costs than the seller.

Nevertheless, we have argued that an intermediary using only classical two-part tariffs would profit if he committed not to compete with sellers in cases he is more efficient. However, we have not explained *how* an intermediary could achieve such commitment – in fact committing not to compete seems to be hard to achieve (i) in a credible way and (ii) by legal means.[23]

We now consider an intermediary using proportional fees, i.e., tariffs that comprise revenue sharing where the intermediary earns a fraction α of the revenues that sellers realize on his platform. We find that proportional fees allow the intermediary to credibly commit not to compete with sellers even in cases he has lower marginal costs. Therefore, proportional fees help the intermediary to attract more sellers, mitigating the hold-up problem. Furthermore, we show that even if full commitment not to compete with sellers could be achieved without using proportional fees, the intermediary would prefer not to use this option under certain circumstances, while the introduction of a proportional fee is profitable to him.

In the following, we analyze three-part tariffs as combinations of classical two-part tariffs and proportional fees. We again proceed by backward induction. The key insight regarding the intermediary's entry behavior (which is decisive for the hold-up problem) will be given in the second subsection (analysis of third stage). Furthermore, we identify conditions under which the inclusion of an additional proportional fee improves the optimal classical two-part tariff. This gives an explanation for the use of proportional fees by platforms and similar businesses.

3.5.1 Stage 4: Product pricing decisions under a three-part tariff

Along the lines of the analysis under classical two-part tariffs, we have to consider two cases to determine price setting within a (representative) product market that

[23]Note that platforms like Amazon often already have a reputation for acting under the dual mode, i.e., competing with sellers in a variety of existing product markets. Therefore, credible commitment on *not* competing might not be feasible. Furthermore, an announcement not to compete with other sellers may be interpreted as a horizontal collusive agreement.

a seller disclosed under a three-part tariff (A, a, α).

If the intermediary did not enter the market, the seller is a monopolist and earns a profit (before investment costs and membership fee) of $(1 - \alpha) \cdot r - (c + a)$ by setting a price of

$$p^{mon} = r.$$

If the intermediary entered the market as merchant, he competes with the seller in Bertrand fashion. Nevertheless, he might prefer not to serve any demand, even if he earned a positive margin by undercutting the seller, as he would lose the transfer $a + \alpha \cdot p$ that he earns for each transaction conducted by the seller at a price of p.

As before, once entered the market, the intermediary still prefers to serve demand whenever he has lower costs than the seller. This can be seen as follows: at any price p chosen by the seller, the intermediary is tempted to undercut the seller if his merchant profit $p - \zeta$ exceeds his variable platform profit $a + \alpha \cdot p$. Accordingly, serving demand himself at a given price of p is more profitable than acting as platform operator if

$$p - \zeta > a + \alpha \cdot p \;\Leftrightarrow\; p > \frac{\zeta + a}{1 - \alpha}.$$

As the lowest price the seller can offer without obtaining a negative margin equals $\frac{c+a}{1-\alpha}$, the intermediary indeed prefers to undercut the seller by charging a price of

$$p_m^{comp}(a, \alpha) = \frac{c + a}{1 - \alpha}$$

if $\zeta < c$. Then, the intermediary achieves a profit of $\frac{c+a}{1-\alpha} - \zeta$.[24]

If the merchant faces higher production costs than the seller ($\zeta \geq c$), the seller serves demand at a price of

$$p_s^{comp}(a, \alpha) = \min\left\{\frac{\zeta + a}{1 - \alpha}, r\right\}.$$

We summarize our findings in the following result:

Lemma 3.9. *[Pricing decisions under a three-part tariff]*
Under a three-part tariff (A, a, α), if the intermediary did not enter, the seller serves demand at a price equal to r. If the intermediary entered the product market as merchant, he serves demand at a price of $\frac{c+a}{1-\alpha}$ if and only if he has lower costs than sellers ($\zeta < c$); otherwise ($\zeta \geq c$), the seller serves demand at a price of $\min\left\{\frac{\zeta+a}{1-\alpha}, r\right\}$.

Both the per-transaction fee a and the proportional fee α increase competitive prices.

[24] Again, our analysis excludes cases where $\frac{c+a}{1-\alpha} > r$ as these cannot occur (no seller participation).

3.5.2 Stage 3: Intermediary's entry decision under a three-part tariff

After the intermediary's production costs have been realized, he decides on entering product markets. If he faces higher production costs than a (representative) seller ($\zeta \geq c$), he does not enter the market, anticipating the decisions in stage 4: if he entered, he would not serve any demand, but incur entry costs $\varepsilon > 0$. Furthermore, entry would drive down the seller's price by $r - \frac{\zeta + a}{1-\alpha}$. Hence, if the intermediary's tariff includes a positive proportional fee α, the intermediary in addition loses parts of his platform profit by entering the market, even though he does not serve any demand.

The latter logic also applies to the case when the intermediary's production costs turn out to be below the seller's costs: if the intermediary charges a proportional fee, he incurs a direct loss from the reduction in prices which is induced by his market entry. Therefore, the intermediary prefers not to enter even if he has a (small) cost advantage. This can be formalized as follows: the intermediary prefers entry if his merchant profit from undercutting the seller,

$$\pi_m(a, \alpha) = p_m^{comp}(a, \alpha) - \zeta,$$

exceeds his variable platform profit

$$\pi_p(a, \alpha) = a + \alpha \cdot p^{mon};$$

this is the case if

$$\pi_m(a, \alpha) > \pi_p(a, \alpha) \;\Leftrightarrow\; \zeta < \frac{c + a}{1 - \alpha} - \alpha \cdot r - a \equiv \widetilde{\zeta}(a, \alpha). \tag{3.11}$$

The critical threshold $\widetilde{\zeta}(a, \alpha)$ of merchant's production costs generally differs from the seller's marginal costs c. Differently from the analysis under classical two-part tariffs, his entry decision now depends on the difference of production costs, the level of production costs, and the transaction-based tariff components a and α.

Lemma 3.10. *[Intermediary's entry decision under a three-part tariff] Under a three-part tariff (A, a, α), the intermediary enters product markets if and only if $\zeta < \widetilde{\zeta}(a, \alpha)$.*

For a more intuitive illustration of the intermediary's trade-off, we define $\Delta c \equiv c - \zeta$ as the merchant's cost advantage. Then, we have $\pi_m(a, \alpha) = \Delta c + a + \alpha \cdot \left(\frac{c+a}{1-\alpha}\right)$, and condition (3.11) for entry being profitable can be written as

$$\Delta c > \alpha \cdot \left(r - \frac{c + a}{1 - \alpha} \right). \tag{3.12}$$

This inequality exactly corresponds to the reasoning that we made above: if the intermediary enters the market, he incurs a loss from the price reduction caused by competition which is captured by the right-hand side. He only enters if this loss is overcompensated by his cost advantage Δc.

Taking a closer look at the right-hand side of inequality (3.12), we can state the following result:

Proposition 3.11. *[Intermediary's entry decision under a three-part tariff] Under any three-part tariff that yields positive seller participation and comprises a proportional fee $\alpha > 0$, the intermediary only enters product markets if his cost advantage exceeds a strictly positive threshold, i.e., $c - \widetilde{\zeta}(a, \alpha) > 0$.*

Proof. See appendix, p. 53. □

Accordingly, under three-part tariffs that include a positive proportional fee, the intermediary always enters in fewer cases than under any classical two-part tariff. The use of proportional fees creates a credible commitment not to enter product markets for cost advantages $\Delta c < c - \widetilde{\zeta}(a, \alpha)$, and, therefore, mitigates the hold-up problem by reducing the threat of competition.

3.5.3 Stage 2: Sellers' joining decisions under a three-part tariff

Given the critical level of merchant's production costs $\widetilde{\zeta}(a, \alpha)$, a seller's expected profit from joining the intermediary's platform can be written as

$$\pi_s^e(A, a, \alpha, I) = Pr(\zeta \geq \widetilde{\zeta}(a, \alpha)) \cdot \{(1 - \alpha) \cdot r - c - a\} - A - I,$$

where $Pr(\zeta \geq \widetilde{\zeta}(a, \alpha))$ denotes the probability of the intermediary not entering the respective product market, which equals $1 - H(\widetilde{\zeta}(a, \alpha))$. A seller joins the platform if her expected profit $\pi_s^e(A, a, \alpha, I)$ is positive, i.e., if her investment costs are below the critical level

$$\widetilde{I}(A, a, \alpha) \equiv \max\{0,\ \{1 - H(\widetilde{\zeta}(a, \alpha))\} \cdot \{(1 - \alpha) \cdot r - c - a\} - A\}. \tag{3.13}$$

Interestingly, while $\widetilde{I}(A, a, \alpha)$ is decreasing in both A and a, it is strictly increasing in the proportional fee α under certain conditions. For $\alpha = 0$, i.e., classical two-part tariffs, seller participation increases in α if and only if

$$\frac{h(c)}{1 - H(c)} \cdot (r - c - a) > \frac{r}{r - c - a}. \tag{3.14}$$

[25]

While all tariff components, i.e., A, a, and α, strictly reduce sellers' margins from selling their products, the proportional fee α in addition reduces the intermediary's entry incentives, and, in turn, makes sellers more likely to sell their products themselves.

The results are summarized in the following Lemma:

[25] The condition for $\frac{\partial \widetilde{I}}{\partial \alpha}$ being positive in case of $\alpha \neq 0$ can be found in the proof of Lemma 3.12.

Lemma 3.12. *[Sellers' decision to join the platform under a three-part tariff] Under a three-part tariff (A, a, α), the mass of sellers that join the platform equals $\widetilde{I}(A, a, \alpha)$. It decreases in A and a, but the effect of a change in α is ambiguous.*

Proof. See appendix, p. 54. □

Note that the intermediary's platform profit is increasing in α if seller participation increases in α. Furthermore, the increase in platform profits under condition (3.14) overcompensates the reduction of merchant profits, and introducing a proportional fee is profitable to the intermediary, cf. our analysis below.

3.5.4 Stage 1: Intermediary's decision on proportional fees

Given the results derived before, the intermediary's expected per-product platform profit under a three-part tariff (A, a, α) equals

$$\pi_p^e(A, a, \alpha) = A + \{1 - H(\widetilde{\zeta}(a, \alpha))\} \cdot (a + \alpha \cdot r), \tag{3.15}$$

and his expected per-product merchant profit is given by

$$\pi_m^e(a, \alpha) = H(\widetilde{\zeta}(a, \alpha)) \cdot \left\{ \frac{c + a}{1 - \alpha} - E[\zeta | \zeta < \widetilde{\zeta}(a, \alpha)] \right\}. \tag{3.16}$$

His expected overall profit equals the sum of his platform profit $\pi_p^e(A, a, \alpha)$ and his merchant profit $\pi_m^e(a, \alpha)$, multiplied by the mass of sellers who joined the platform:

$$\Pi^e(A, a, \alpha) = \widetilde{I}(A, a, \alpha) \cdot \{\pi_p^e(A, a, \alpha) + \pi_m^e(a, \alpha)\}. \tag{3.17}$$

Substituting (3.15) and (3.16) into (3.17) leads to

$$\Pi^e(A, a, \alpha) = \widetilde{I}(A, a, \alpha) \cdot \left\{ A + \left[a + \alpha \cdot r + \Delta^e \left(\widetilde{\zeta}(a, \alpha) \right) \right] \right\}, \tag{3.18}$$

where

$$\Delta^e \left(\widetilde{\zeta}(a, \alpha) \right) = H(\widetilde{\zeta}(a, \alpha)) \cdot \widetilde{\zeta}(a, \alpha) - \int_{\underline{\zeta}}^{\widetilde{\zeta}(a, \alpha)} x dH(x)$$

as defined in (3.6).[26] Evaluating the partial derivative of the intermediary's profit $\Pi^e(A, a, \alpha)$ with respect to α at the optimal two-part tariff leads to the following result:

Proposition 3.13. *[Proportional fees improve optimal classical two-part tariff] The inclusion of an additional positive proportional fee strictly improves the optimal classical two-part tariff $(0, a^*)$ if*

$$\frac{r - c + \Delta^e(c)}{2} > \frac{H(c) \cdot (1 - H(c))}{h(c)}. \tag{3.19}$$

[26] Note that $\Delta^e(x)$ can only be interpreted as the merchant's expected cost advantage if $x = c$.

Proof. See appendix, p. 54. Introducing a proportional fee is profitable if (new) markets are sufficiently profitable, i.e., $r - c + \Delta^e(c)$ is large enough, and reducing the entry threshold sufficiently increases the probability (or mass) of markets being opened up, i.e., $h(c)$ is large enough. Hence, if the distribution of the intermediary's production costs, represented by $H(\cdot)$, is such that the likelihood of the intermediary having a small cost advantage is sufficiently large (much probability mass just below c), then it is usually profitable to forego the merchant option and commit not to enter, thereby providing sellers additional incentives for joining the platform.[27] □

Given that the intermediary can choose a tariff system that comprises a revenue-based component, he is able to achieve *partial commitment* by introducing a strictly positive proportional fee. In particular, this is profitable under the circumstances given by condition (3.19).

Assuming that the intermediary in certain settings may be able to find a way to fully commit not to compete with sellers – a "platform-only" business model – *without* using proportional fees, under this (full) commitment classical two-part tariffs maximize profits.[28] However, we can identify settings in which the intermediary would reject the option of full commitment while he prefers introducing a proportional fee over operating under a dual mode with classical two-part tariffs only. In particular, partial commitment by proportional fees yields higher profits than classical two-part tariffs while full commitment is not profitable if condition (3.10) does not hold, while condition (3.19) holds:

$$\left(\frac{r-c}{2}\right)^2 < (1-H(c)) \cdot \left(\frac{r-c+\Delta^e}{2}\right)^2,$$

and

$$\frac{r-c+\Delta^e(c)}{2} > \frac{H(c)\cdot(1-H(c))}{h(c)}.$$

Clearly, both inequalities can be met if $h(c)$ is large enough and, at the same time, the intermediary's expected cost advantage $\Delta^e(c)$ is also not too small. Intuitively, if $h(c)$ is large, partial commitment provided by proportional fees is particularly profitable as then it strongly decreases the likelihood of the intermediary competing with sellers, while the expected cost advantage $\Delta^e(\cdot)$ could still be substantial. On the contrary, if $\Delta^e(c)$ was small, full commitment would be profitable as the intermediary then would face low opportunity costs. In summary:

Proposition 3.14. *[Commitment and profitability of proportional fees] Even if full commitment is feasible, the intermediary may prefer to reject the opportunity of full commitment while he strictly benefits from introducing proportional fees.*

[27]Note that condition (3.19) is implied by condition (3.14) for seller participation being increasing in α.

[28]Note that we constructed the model such that the only benefit of proportional fees is commitment; in case of double marginalization or price discrimination there are additional benefits.

3.6 Conclusion

While real world platforms use a mixture of tariff forms, including proportional (per-revenue) fees, the great majority of the economic literature on platform markets has focused on membership fees and per-transaction fees. The extant studies on proportional platform fees highlight the reduction of the double marginalization problem and the ability to price discriminate by using a proportional fee. Analyzing a dual mode of intermediation, we identify the effects of the intermediary's tariff system on competition between sellers and the intermediary and on sellers' investment incentives.

Firstly, we identify a *competition-relaxing effect* of transaction-based fees. Abstracting from double marginalization, the intermediary strictly prefers transaction-based fees to membership fees. The reason is that transaction-based fees increase sellers' marginal costs and, thus, increase prices in case the intermediary competes with a seller. This effect does not occur for a pure platform, and, hence, the operator of a pure platform is indifferent between membership-based and transaction-based tariffs in line with Armstrong (2006b).[29]

If sellers have to sink costs before joining the platform, the threat of competition leads to a hold-up problem: profitable product markets remain unexplored. Sellers' investment incentives are insufficient as sellers do not internalize the profits that the intermediary achieves due to their product. Therefore, the intermediary would like to commit not to compete, foregoing (parts of) his merchant profits to increase investment incentives.

However, even if credible commitment never to enter sellers' markets was feasible, it would not always be profitable. The intermediary would prefer to commit not to enter if his cost advantage is small, but he wants to exercise his merchant option in case of a large cost advantage.[30] We show that proportional (revenue-based) fees can achieve this partial commitment as they change the intermediary's opportunity costs of competition. In particular, the *commitment effect* of proportional fees is such that the intermediary only enters product markets if his cost advantage exceeds a strictly positive threshold. In contrast, under classical two-part tariffs, the intermediary enters if and only if he faces lower production costs than sellers. The reason is that the level of the per-transaction fee does not affect the intermediary's incentives to enter as a change in this platform fee results in an equal change of sellers' perceived costs, affecting merchant profits to the same extent as platform profits.

However, the commitment effect of proportional fees comes at the cost of foregoing cost advantages and a potential reduction of the competitive price. Although proportional fees mitigate the hold-up problem, their profitability depends on the distribution of the intermediary's costs relative to the sellers' costs. If the prob-

[29]The canonical two-sided market models like Armstrong (2006b) abstract from price-setting by sellers and, thereby, also abstract from double marginalization problems.

[30]As the intermediary's costs are rarely verifiable, such behavior seems not to be contractible directly.

ability of the intermediary facing costs slightly below sellers' costs c is large, the introduction of a small proportional fee is always profitable as it significantly reduces the hold-up problem.[31]

Our analysis sheds light on the economics of intermediated markets, in particular markets in which the intermediary does not only organize a marketplace, but can become active in it himself. In addition, the effects we identify could also play a role in the context of franchising and licensing.

[31] Furthermore, if the intermediary's maximal cost advantage is relatively small, the intermediary can achieve credible commitment *never* to enter sellers' markets by charging a proportional fee that implies that the entry threshold $\widetilde{\zeta}(\cdot)$ equals $\underline{\zeta}$.

Appendix: Proofs

Proof of Proposition 3.4

Recall that the intermediary's expected overall profit under a two-part tariff (A, a) can be written as

$$\Pi^e(A, a) = \widetilde{I}(A, a) \cdot \{A + a + \Delta^e(c)\}, \tag{3.20}$$

with $\Delta^e(c)$ being independent of both A and a.

We show that it is always more profitable to charge a higher per-transaction fee instead of a membership fee: a 'compensated' increase in the per-transaction fee a which does not affect seller participation leads to an increase in the intermediary's per-product profit. Starting from an arbitrary tariff scheme (A, a) with $A > 0$, we firstly determine how to adapt the membership fee A such that the critical level of investment costs $\widetilde{I}(A, a)$ remains constant while changing a. Secondly, given this compensation, we show that the effect of a change in the per-transaction fee a overcompensates the effect of the corresponding adaption of the membership fee A.

(i) Given the definition

$$\widetilde{I}(A, a) \equiv \max\{0,\ (1 - H(c)) \cdot \{r - (a + c)\} - A\},$$

we have $\frac{\partial \widetilde{I}(A,a)}{\partial A} = -1$ (we can focus on cases with non-zero seller participation, i.e., $\widetilde{I}(A, a) > 0$). By implicit function theorem it follows that the compensation $A(a)$ has to fulfill $\frac{\partial A(a)}{\partial a} = -\frac{\partial \widetilde{I}/\partial a}{\partial \widetilde{I}/\partial A} = \frac{\partial \widetilde{I}(A,a)}{\partial a}$. Substituting $\frac{\partial \widetilde{I}}{\partial a}$ yields $\frac{\partial A(a)}{\partial a} = -(1 - H(c))$.

(ii) Define $\pi(A, a) \equiv A + a + \Delta^e(c)$. Then, we obtain $\frac{\partial \pi}{\partial A} = \frac{\partial \pi}{\partial a} = 1$. Substituting these derivatives and $\frac{\partial A(a)}{\partial a}$ into the definition of the total differential

$$d\pi = \frac{\partial \pi}{\partial A} dA + \frac{\partial \pi}{\partial a} da$$

leads to $\frac{d\pi}{da} = H(c) > 0$, and the loss from a decrease in A is overcompensated by the corresponding increase in a as the latter creates an additional advantage for the merchant in case of competition (that occurs with probability $H(c)$).

Now, we can focus on pure per-transaction fee tariffs as the optimal membership fee is zero. Differentiating equation (3.7) with respect to a and plugging in $A = 0$ yields the first order condition

$$\frac{\partial \Pi^e(0, a)}{\partial a} = \widetilde{I}(0, a^*) - (1 - H(c)) \cdot (a^* + \Delta^e(c)) = 0.$$

Solving this for a^* yields (3.8).

Note that $\Pi^e(0, a)$ is strictly concave in a. Hence, the first order condition is sufficient for a maximum.

Proof of Proposition 3.6

The intermediary's expected overall profit under commitment not to enter with production costs above $\hat{\zeta}$ is given as follows:

$$\widehat{\Pi}^e(A,a,\hat{\zeta}) = \begin{cases} \hat{I}(A,a,\hat{\zeta}) \cdot \{A + (a + \Delta^e(c,\hat{\zeta}))\}, & \hat{\zeta} \leq c \\ \hat{I}(A,a,\hat{\zeta}) \cdot \{A + (a + \Delta^e(c,c))\}, & \hat{\zeta} > c \end{cases}, \tag{3.21}$$

where

$$\hat{I}(A,a,\hat{\zeta}) = \max\{0,\ (1 - H(\hat{\zeta})) \cdot (r - (c+a)) - A\},$$

and

$$\Delta^e(c,\hat{\zeta}) \equiv H(\hat{\zeta}) \cdot c - \int_{\underline{\zeta}}^{\hat{\zeta}} x dH(x).$$

Assuming positive seller participation, we have $\hat{I}(A,a,c) > 0$. Furthermore, as $r - c \leq 1$, $A \geq 0$, and $a \geq 0$, we have $\hat{I}(A,a,c) < 1$.

Firstly, note that $\hat{\zeta} > c$ are dominated by $\hat{\zeta} = c$. This can be seen as follows: if $\hat{\zeta} > c$, $\hat{\zeta}$ affects the intermediary's profit only through the change in seller participation captured by the change in $\hat{I}(\cdot)$ because it is never profitable for the intermediary to enter with costs $\zeta \in (c, \hat{\zeta})$ (i.e., $\Delta^e(c,c)$ does not depend on $\hat{\zeta}$). For any $\hat{\zeta} > c$, $\hat{I}(A,a,c) > \hat{I}(A,a,\hat{\zeta})$ holds.

Secondly, differentiating $\widehat{\Pi}^e(A,a,\hat{\zeta})$ from below c yields

$$\left.\frac{\partial \widehat{\Pi}^e(A,a,\hat{\zeta})}{\partial \hat{\zeta}}\right|_{\hat{\zeta} \leq c} = \{-h(\hat{\zeta}) \cdot (r - (c+a)\} \cdot \{A + a + \Delta^e(c,\hat{\zeta})\} + \hat{I}(A,a,\hat{\zeta}) \cdot \{h(\hat{\zeta}) \cdot (c - \hat{\zeta})\}.$$

For $\hat{\zeta} = c$, the first term is negative, while the second term equals zero. Hence,

$$\left.\frac{\partial \widehat{\Pi}^e(A,a,\hat{\zeta})}{\partial \hat{\zeta}}\right|_{\hat{\zeta} = c} < 0,$$

and $c > \arg\max_{\hat{\zeta}} \widehat{\Pi}^e(A,a,\hat{\zeta})$.

Proof of Proposition 3.11

The condition $\widetilde{\zeta}(a,\alpha) < c$ is equivalent to $\frac{c+a}{1-\alpha} - \alpha r - a < c$, which can also be written as $c - (1-\alpha) \cdot \alpha \cdot r + \alpha \cdot a < (1-\alpha) \cdot c$, or $\alpha \cdot (c+a) < (1-\alpha) \cdot \alpha \cdot r$. Division by $\alpha > 0$ yields $c + a < (1-\alpha) \cdot r$, a necessary condition for positive seller participation.

Proof of Lemma 3.12

Equation (3.13) defines the critical level of investment costs under a three-part tariff as

$$\widetilde{I}(A,a,\alpha) \equiv \max\{0,\ \{1-H(\widetilde{\zeta}(a,\alpha))\}\cdot((1-\alpha)\cdot r - c - a) - A\}.$$

Since $\widetilde{\zeta}(a,\alpha) \equiv \frac{c+a}{1-\alpha} - \alpha r - a = \frac{c}{1-\alpha} - \alpha r + \frac{\alpha}{1-\alpha}\cdot a$, $\widetilde{I}$ clearly decreases in A and a. Furthermore, if $\widetilde{I}(A,a,\alpha) > 0$, we have

$$\frac{\partial \widetilde{I}(A,a,\alpha)}{\partial \alpha} = -\left\{ \underbrace{(1-H(\widetilde{\zeta}(a,\alpha)))r}_{\text{change of revenue share}} - \underbrace{h(\widetilde{\zeta}(a,\alpha))\left(r - \frac{c+a}{(1-\alpha)^2}\right)\{(1-\alpha)r-(c+a)\}}_{\text{change of entry incentives}} \right\}.$$

This expression is positive if and only if

$$(1-H(\widetilde{\zeta}))\cdot r < h(\widetilde{\zeta})\cdot\left(r - \frac{c+a}{(1-\alpha)^2}\right)\cdot\{(1-\alpha)\cdot r-(c+a)\}.$$

Proof of Proposition 3.13

Firstly, we consider the merchant's expected cost advantage. We observe

$$\begin{aligned}\frac{\partial \Delta^e(\widetilde{\zeta}(a,\alpha))}{\partial \alpha} &= h(\widetilde{\zeta}(a,\alpha))\cdot\frac{\partial\widetilde{\zeta}(a,\alpha)}{\partial\alpha}\cdot\widetilde{\zeta}(a,\alpha) + H(\widetilde{\zeta}(a,\alpha))\cdot\frac{\partial\widetilde{\zeta}(a,\alpha)}{\partial\alpha}\\ &\quad - \left[\widetilde{\zeta}(a,\alpha)\cdot h(\widetilde{\zeta}(a,\alpha))\cdot\frac{\partial\widetilde{\zeta}(a,\alpha)}{\partial\alpha}\right],\end{aligned}$$

where the last term in brackets follows from the Leibniz integral rule. As the first and the last term cancel out, this simplifies to

$$\frac{\partial \Delta^e(\widetilde{\zeta}(a,\alpha))}{\partial \alpha} = H(\widetilde{\zeta}(a,\alpha))\cdot\frac{\partial\widetilde{\zeta}(a,\alpha)}{\partial\alpha} = H(\widetilde{\zeta}(a,\alpha))\cdot\left(\frac{c+a}{(1-\alpha)^2} - r\right).$$

Hence, the derivative of the intermediary's expected profit (3.18) is given by

$$\begin{aligned}\frac{\partial \Pi^e(A,a,\alpha)}{\partial \alpha} &= \widetilde{I}(A,a,\alpha)\cdot\left[r + H(\widetilde{\zeta}(a,\alpha))\left(\frac{c+a}{(1-\alpha)^2} - r\right)\right]\\ &\quad + \left(\frac{\partial\widetilde{I}(A,a,\alpha)}{\partial\alpha}\right)\cdot\left\{A + \left[a + \alpha r + \Delta^e\left(\widetilde{\zeta}(a,\alpha)\right)\right]\right\},\end{aligned}$$

with $\frac{\partial\widetilde{I}(A,a,\alpha)}{\partial\alpha}$ as given in the proof of Lemma 3.12.

Defining

$$\pi(A,a,\alpha) \equiv \left\{A + \left[a + \alpha r + \Delta^e\left(\widetilde{\zeta}(a,\alpha)\right)\right]\right\},$$

we find that $\frac{\partial \Pi^e(A,a,\alpha)}{\partial \alpha}$ is positive if and only if

$$\widetilde{I}(A,a,\alpha) > \pi(A,a,\alpha)\frac{(1-H(\widetilde{\zeta}(a,\alpha)))r + h(\widetilde{\zeta}(a,\alpha))\left(\frac{c+a}{(1-\alpha)^2}-r\right)\{(1-\alpha)r-c-a\}}{r+H(\widetilde{\zeta}(a,\alpha))\left(\frac{c+a}{(1-\alpha)^2}-r\right)}.$$

From Proposition 3.4, we know that the optimal per-transaction fee in case of $\alpha = 0$ is defined by

$$\{1-H(c)\}\cdot \underbrace{(a^* + \Delta^e(c))}_{=\pi(0,a^*,0)} = \widetilde{I}(0,a^*,0) \ (>0, \text{ given Assumption 3.5}).$$

Hence, by envelope theorem, $\frac{\partial \Pi^e(0,a^*,0)}{\partial \alpha} > 0$ holds at the optimal two-part tariff if

$$\begin{aligned}
1-H(c) &> \frac{(1-H(c))r - h(c)\{r-c-a^*\}^2}{r-H(c)\{r-c-a^*\}} \\
\Leftrightarrow -H(c)\{r-c-a^*\} &> \frac{-h(c)\{r-c-a^*\}^2}{1-H(c)} \\
\Leftrightarrow \qquad \frac{h(c)}{1-H(c)} &> \frac{H(c)}{r-c-a^*}.
\end{aligned}$$

Chapter 4

Buyer Sorting and Membership Fees

4.1 Introduction

Offering a trading platform to sellers is an increasingly popular business model in modern retailing. For example Apple offers the Appstore, a platform through which application providers reach Apple users and Amazon invites sellers to use the Marketplace to sell to Amazon's customers. The major difference compared to the classical business model in retailing is that not the retailer (platform), but the third party sellers choose the prices.

The optimal tariffs of platforms have been analyzed in the literature on two-sided markets. This literature emphasizes how platforms internalize the network effects between sellers and buyers. However, this literature is typically does not endogenize the transaction between sellers and buyers, but takes the transaction benefits as given, e.g. Armstrong (2006b). Notable exceptions are Rochet and Tirole (2006a), Hagiu (2006a), and Nocke et al. (2007)). However these papers share the assumption that buyers are homogenous in there transaction values when they decide to join the platform. This assumption is most explicit in the Rochet and Tirole (2006a), where each buyer has the same expected valuation for each transaction with the seller ex-ante, the actual valuation is then drawn after the buyer has joined the platform. However, in many applications buyers differ in their monetary valuations for the goods offered by the other side. For example, because buyers differ in income, which affects their monetary valuations, or in their available leisure time, which affects how much time they can spend, for example, with an application or game.

The goal of this paper is to relax that assumption. We present a model with multiple sellers and buyers. Buyers can differ in their expected value for transactions which defines a buyer type. Each buyer type draws its valuations from a different distribution function but identically for the product of each seller. When a buyer decides to join the platform he trades-off the membership versus the stand-alone and transaction benefits that he expects after joining the platform. This leads to a sorting in the buyers on the platform, as a buyer with higher expected valuation is more likely to join the platform than a buyer with low valuation. This also affects the pricing by sellers and the optimal per transaction fee of the platform. Note that buyers do not only differ in their transaction benefit but also induce different transaction benefits to sellers. The model is thus a micro-foundation for heterogeneous network effects, which also, through the pricing generate a heterogeneous feedback - or indirect - network effect.

We compare two scenarios. Either buyers are ex-ante homogeneous, or they differ in their type, i.e., their expected valuation. The first scenario matches the one presented in Rochet and Tirole (2006b), in that case the platform internalizes the trade surplus fully and subsidizes trade in order to implement the socially optimal transaction prices.

In the second scenario, buyers are sorted by the participation condition. The marginal buyer who joins the platform has a below average valuation for transaction. The platform however only internalizes the surplus of the marginal buyer. Hence, the platform will not fully internalize the trade surplus. In turn the price implemented via the transaction fee will be above the socially optimal price.

These results contrast with the more pessimistic results of Gans (2012a). In a setting with a single seller and no uncertainty about the transaction values Gans (2012b) shows that the sorting-out of low valuation buyers leads to a complete equilibrium break down if the platform charges positive membership fees to buyers. The intuition for this result is that of the "Diamond Paradox" (Diamond, 1971) with a monopolist. Buyers have sunk the search/membership costs when the seller reveals his price. The seller will then skim all surplus. However, a buyer who does not expect any surplus would not pay the membership fee/search costs in the first place. As buyers' expectations cannot be right in equilibrium, all rational expectation equilibria unravel.

In our model, the market does not break down if the seller charges a positive membership fee, since the ex-ante information is still imperfect and thus sorting does not make demand completely inelastic for low prices. A similar result to the one presented in this paper can be achieved by allowing for more than one product in the context of the Gans (2012a) model and imperfect correlation between the valuation of the different products.

This paper provides a novel perspective on the effect of membership fees. In the presence of buyer sorting, membership fees tend to increase the prices sellers ask. If sellers are monopolists this makes them charge excessive prices, which is also to the detriment of platform profits. However, when sellers compete, the platform can use this effect to dampen seller competition. This is particularly useful if dampening seller competition increases sellers' insufficient investment incentives.

For the interested reader, the next section provides a brief overview of related articles.

4.2 Literature review

This paper borrows the general setting from the seminal papers on multi-sided platforms. Armstrong (2006a) investigates platforms that connect sellers and buyers. He allows for heterogeneity of the stand-alone benefit of buyers and sellers but assumes that transaction benefits are homogeneous and exogenous. In the context of this model the platform is indifferent between membership and transaction based fees, both are means to appropriate some of the participants rents but do not affect the value of transaction.

The setting in the present paper is more closely related to the one presented by Rochet and Tirole (2003). They assume that buyers and sellers are only heterogeneous with respect to their valuation of transaction with the other side. Similarly to Armstrong (2006a), Rochet and Tirole (2003) abstract from explicitly modeling the interaction between buyers and sellers.

Price setting by sellers is introduced into the Rochet and Tirole (2003) framework by Hagiu (2006a). Furthermore, Hagiu (2006a) considers that sellers arrive at the platform before buyers. This is a realistic timing when sellers have to make platform-specific investments early on (e.g. game developers for a console). He emphasizes the existence of a hold-up problem when the platform does not commit on buyer fees. He shows that the platform maximizes welfare under the constraint that the transaction fee cannot be too negative. To keep the model tractable he assumes, as is typical for the literature, that buyers are only ex-post heterogeneous, that is, after they have joined the platform. The same assumption is used in Rochet and Tirole (2006a) when they extend the canonical two-sided market setting and allow for an endogenous price in the transaction between sellers and buyers. Rochet and Tirole (2006a) allow the platform to charge membership and transaction fees. In that context they find that if sellers set prices and are imperfectly informed about the heterogeneous valuations of buyers, they charge too high prices. Hence, the platform would like to subsidize trade and recoup the additional surplus through an increase in participation and membership fees. This result stresses the power of two-part tariffs in solving coordination problems in two-sided markets.

I depart from the assumption that buyers only learn their valuation after joining the platform. I follow Gans (2012a) in allowing buyers to know their valuations before joining the platform. Gans (2012a) finds that this stops the platform from charging positive membership fees to buyers. I generalize the setting Gans uses by allowing for multiple sellers. Differing from Gans (2012a) I assume that buyers are not perfectly informed about their valuations when they join the platform.

Papers that analyze multidimensional heterogeneity of platform participants are Weyl (2010) and Veiga and Weyl (2011). However, these focus again on a reduced form of transactions without endogenous price setting. Veiga and Weyl (2011) have a similar interest in the effect of platform organization on the sorting of participants that differ in the value they provide to, and receive from, other participants.

This paper is also related to a recent strand of the literature that asks whether a platform should charge for membership, for transaction or based on revenue, e.g.

Hagiu (2009a), Shy (2011), and also the second Chapter of this dissertation. All these papers argue for the use of revenue sharing arrangements (proportional fees). Hagiu (2009a) follows Hagiu (2006a) in assuming that sellers arrive before buyers, in that context revenue sharing between sellers and platform helps the platform to commit to (partially) internalize the positive effect of participation of buyers on seller profits. Shy (2011) argues that platforms should charge proportional fees to mitigate double marginalization by sellers. However, these models by assumption abstract from membership fees to buyers.

Finally, I follow Amelio and Jullien (2012) in considering the realistic case that platforms cannot charge negative fees, as this would invite opportunistic behavior by participants. Amelio and Jullien (2012) argue that tying "goodies" to the platform membership can be an adequate substitute for negative fees. In that model by assumption each side only cares about the number not the composition of the other side.

4.3 Model

Consider a monopoly platform that is essential for the transaction between agents of two groups, sellers S and buyers B. The platform can charge fees for membership A^i and transaction a.[1] Buyers and sellers derive utility and profits from transactions and the pure membership (potentially also costs) at the platform. Following Rochet and Tirole (2006a) the expected utility of an agent in group $i \in \{S, B\}$, when he decides on joining the platform, depends on the expected surplus from trade with an agent from the other side b^i, the mass of agents on the other side N^j and a stand-alone benefit of cost from joining the platform B^i:

$$u^i = b^i N^j + B^i.$$

To keep the model tractable we assume that B^S and B^B are the same for each seller and buyer respectively. However, the pure membership benefit B^i differs between groups. The expected surplus b^i depends on equilibrium prices and transaction values. For buyers this is the expected surplus of a trade with one seller. For a seller this is the expected profit per buyer. We normalize mass of both sellers and buyers to one. The mass of participation of sellers and buyers N^i then expresses the participation level as a share between 0 and 1. We will focus on situations where there is at least some participation by both groups. Otherwise the market either breaks down or is strictly one-sided and only the stand alone benefit matters.

While the canonical model of two sided markets abstracts from payments between the two groups, we explicitly consider price setting by sellers. Consider that each seller offers a single product. The valuation v of a buyer for the products depends on his type θ. Each buyer considers buying one unit of each product. He

[1] It is without loss of generality to assume that only sellers pay the transaction fee, as it affects the market outcome in the same way independently of who has to pay it. For the exposition we assume that sellers pay the transaction fees and skip the side specific superscript.

draws a valuation v for each product. For each buyer type, v is drawn form a continuous distribution function with conditional density of $f_\theta \equiv f(v|\theta)$. The support of the distribution of valuations is independent of θ and restricted to weakly positive real numbers R^+. Assume that for each buyer type, v is identically and independently drawn for each product. Without loss of generality, the buyer type θ is assumed to be uniformly distributed between 0 and 1. The type θ captures the average valuation of this consumer type for products in the sense of first order stochastic dominance. To that end we assume that

Assumption 4.1. *The type dependent distribution of valuations F_θ has the following properties for all $v \in R^+$ and $\theta \in [0,1]$:*

$$\frac{\partial}{\partial\theta}(1 - F_\theta(v)) > 0,$$

$$\frac{\partial}{\partial\theta}\left(\frac{1 - F_\theta(v)}{f_\theta(v)}\right) > 0,$$

$$\frac{\partial}{\partial v}\left(\frac{1 - F_\theta(v)}{f_\theta(v)}\right) < 0.$$

The first part of the assumption has the implication that θ orders the distributions with respect to their expected value, a higher θ implies a higher expected valuation. This is important for the participation decision of a buyer if he knows his type θ. The second part implies that (expected) demand becomes more elastic, for higher θ. Basically, a consumer with a higher θ is less price elastic at the same price level. The second part ensures that profit maximizing price are defined by first order conditions. Again the intuition is that for each consumer the price elasticity increases with the price, which implies that price increases yield a decreasing return in revenues.

For example if θ strictly orders the distribution functions according to the monotone likelihood ratio property[2] (MLRP) then the first and second part of our assumption are met, the expected value increases in θ and the inverse hazard rate increases in θ.

We consider two differing information structures. In the first case buyers do not know θ. This is the assumption most typically found in the literature for example in the Rochet and Tirole (2006b) model. In that case θ has no influence on the participation decision of buyers. In the second case, buyers know θ before joining the platform.

Denote by $F(v) \equiv \int_0^1 F_\theta(v)d\theta$, the aggregate distribution of valuations over all types and by f the associated density.

The timing is given by the following sequence

1. The platform chooses its tariff scheme

[2]For all$\theta_1 > \theta_0$ and $x > y$: $\frac{f_{\theta 1}(x)}{f_{\theta 0}(x)} > \frac{f_{\theta 1}(y)}{f_{\theta 0}(y)}$.

2. Buyers and seller decide if they want to join the platform
3. Sellers set prices
4. Buyers decide which products to buy

We solve the game for perfect Bayesian equilibria. We are only interested in the equilibria with positive participation by both sides and neglect parameter constellations where it would be optimal to only serve one side.[3]

4.4 Solution

We solve the model in the spirit of backward induction. In the final stage consumer take their buying decisions. In each market i the set of buyers that has joined the platform faces a price p_i.

4.4.1 Stages 3 and 4: Product market decisions

In stage 4 buyers know their willingness to pay for each product. Since the valuation is drawn independently, it can differ for each product and each consumer. A buyer with a higher type θ is more likely to have a high valuation for each product. In each product market i each buyer compares his valuation with the price and buys if $v_i - p_i \geq 0$. From the point of view of the seller the valuation of each buyer is uncertain. Hence, each seller forms expectations about the distribution of valuations he faces. A buyer of type θ has an (expected) demand for each product i of $1 - F_\theta(p_i)$. Denote by Θ the set of buyer types that have joined the platform. This generates an (expected) aggregate demand by all buyers on the platform for product i of $D(p_i)$, which is given by

$$D \equiv \int_\Theta 1 - F_\theta(p_i) d\theta.$$

Note that demand depends on the price of product i but is otherwise symmetric in each market. Each market demand also depends on the set of consumer types that have joined the platform. First note that if all buyer types have joined the platform demand for product i is given by

$$D = \int_0^1 1 - F_\theta(p_i) d\theta = 1 - F(p_i). \tag{4.1}$$

Assumption 4.1 implies that consumers can be ordered by their type. A higher type implies a higher expected valuation and typically a larger incentive to join the

[3] The only situation in which the platform would only serve one side, is if the stand alone benefit of either side is sufficiently negative. For example if sellers investment costs are prohibitively high.

platform. This will lead to a cut-off in the support of buyer types on the platform. Buyers with $\theta < \theta^*$ do not join the platform and $\theta \geq \theta^*$do. If that is the case demand is given by

$$D = \int_{\theta^*}^{1} 1 - F_\theta(p_i) d\theta. \tag{4.2}$$

In stage 3 each seller faces demand D and maximizes $(p_i - c - a)\, D(p_i)$ with respect to p_i. First consider the case that their are no transaction-based fees to pay to the platform $a = 0$. In that case the price set by the seller to maximize his surplus is $p^m \equiv \arg\max_{p_i} (p_i - c)\, D(p_i)$.

If the transaction fee is $a \neq 0$, each seller chooses a price depending on a and is defined by $p^* \equiv \arg\max_p (p_i - c - a)\, D(p_i)$. Note that the maximization problem is identical for each seller.

Proposition 4.2. *The price set by sellers p^* monotonically increases in the per-transaction fee a. If there exists a marginal buyer θ^*, defined such that a buyer joined the platform if and only if $\theta \geq \theta^*$, the price increases in θ^*.*

Proof. For a given transaction fee a the seller maximizes $(p_i - c - a)\, D(p_i)$. The associated first order condition is given by

$$(p_i - c - a) \frac{\partial}{\partial p_i} D(p_i) + D(p_i) = 0. \tag{4.3}$$

The effect of a on p^* can be derived using the implicit function theorem on the first order condition.

$$\frac{dp^*}{da} = -\frac{-\frac{\partial}{\partial p_i} D(p^*)}{(p^* - c - a) \frac{\partial^2}{\partial (p_i)^2} D(p^*) + 2 \frac{\partial}{\partial p_i} D(p^*)} > 0$$

If there exists a marginal buyer θ^* then demand is given by

$$D(p_i) = \int_{\theta^*}^{1} 1 - F_\theta(p_i) d\theta$$

Using the implicit function theorem on (4.3) gives us

$$\frac{dp^*}{d\theta^*} = -\frac{(p^* - c - a) \frac{\partial^2}{\partial p_i \partial \theta^*} D(p^*) + \frac{\partial}{\partial \theta^*} D(p^*)}{(p^* - c - a) \frac{\partial^2}{\partial (p_i)^2} D(p^*) + 2 \frac{\partial}{\partial p_i} D(p^*)}$$

The sign of the derivative depends on the sign of the numerator as the denominator is negative as the second order condition holds. Hence, $\frac{dp^*}{d\theta^*} > 0$ if

$$(p^* - c - a) \frac{\partial^2}{\partial p_i \partial \theta^*} D(p^*) + \frac{\partial}{\partial \theta^*} D(p^*) > 0. \tag{4.4}$$

Note that p^* is defined by (4.3) to be

$$p* = a + c + \frac{\int_{\theta^*}^{1} 1 - F_\theta(p^*)d\theta}{\int_{\theta^*}^{1} f_\theta(p_*)d\theta}, \tag{4.5}$$

and the other terms are

$$\frac{\partial}{\partial\theta^*}D(p^*) = -\left(1 - F_{\theta*}(p^*)\right),$$

and

$$\frac{\partial^2}{\partial p_i \partial\theta^*}D(p_i) = f_{\theta*}(p^*).$$

Substituting the above three expressions into (4.4) and rearranging gives us

$$\frac{\int_{\theta^*}^{1} 1 - F_\theta(p^*)d\theta}{\int_{\theta^*}^{1} f_\theta(p_*)d\theta} > \frac{1 - F_{\theta*}(p^*)}{f_{\theta*}(p^*)}. \tag{4.6}$$

Thus is true under our assumption on F_θ which imply that the inverse hazard rate is decreasing in θ. To see that rewrite (4.6)

$$\frac{\int_{\theta^*}^{1} f_\theta \frac{1-F_\theta(p^*)}{f_\theta(p^*)}d\theta}{\int_{\theta^*}^{1} f_\theta(p_*)d\theta} > \frac{1 - F_{\theta*}(p^*)}{f_{\theta*}(p^*)}$$

and note that $\left[1 - F_\theta(p^*)\right]/f_\theta(p^*)$ on the left hand side is larger then the right hand side term, for any $\theta > \theta^*$ by the assumption that the inverse hazard rate increases in θ.

Interestingly, if buyers know θ when they choose whether to join the platform, this reveals their high willingness to pay. In consequence the price will be larger. □

4.4.2 Stage 2: Platform participation decisions with type not known

We can now use the demand function that we have derived from the micro-foundation to express the reduced form transaction benefits b^i. The transaction benefit b^i is defined such that it measures the value of an additional participant on the other side of the platform.

For sellers, b^S measures the expected value from an additional buyer, expectations are formed when sellers decide whether to join the platform. Hence, it is the average profit a seller can expect from a buyer:

$$b^S = \frac{(p^* - c - a)\,D(p^*)}{N^B} = \frac{(p^* - c - a)\,(1 - F(p^*))}{N^B}.$$

For each buyer the expected transaction benefit is the same if the buyers do not know their θ and depends on the expected price p^e. In equilibrium buyers correctly anticipate the price, such that $p^e = p^*$. Hence, if the type is unknown the expected consumer surplus in a product market is

$$b^B = \int_{p^*}^{+\infty} (v - p^*) f(v) dv = \int_{p^*}^{+\infty} 1 - F(v) dv.$$

The most right hand side expression can be derived using integration by parts and has a similar form to the usual expression of consumer surplus as the area under the demand curve and above the price level.

In stage 2 sellers and buyers decide whether to join the platform. Since the value of joining the platform depends on the presence of users on the other side of the platform there is a potential for coordination failure. We abstract from that coordination failure and select the equilibrium with positive participation levels on both sides.

Since buyers do not know θ, buyers are ex-ante homogeneous. This implies that either all, or none of the buyers will join (for the case that they all indifferent we assume all join the platform). A seller joins the platform if he expects a positive profit. He trades of the participation fee with the profits made from sales. Also sellers are homogeneous and, thus, each seller has an identical decision to take. In a reduced form this trade-offs can be expressed as a condition for joining for the buyers

$$A^B \leq B^B + b^B N^S, \tag{4.7}$$

and for the sellers

$$A^S \leq B^S + b^S N^B. \tag{4.8}$$

These can be expressed more explicitly substituting the transaction benefits we have derived above. Note that if both inequalities are met, all sellers and buyers will join the platform, i.e., $N^S = N^B = 1$.

Proposition 4.3. *If buyers do not know θ ex-ante, all buyers and sellers join the platform if $A^B \leq B^B + \int_{p^*}^{+\infty} 1 - F(v)dv$ and $A^S \leq B^S + (p^* - c - a)(1 - F(p^*))$. Otherwise there will be no participation on at least one side.*

Proof. See discussion above. □

4.4.3 Stage 1: Optimal tariff system with type not known

In stage 1 the platform maximizes profits choosing the membership fees A^B and A^S and the transaction based fee a.

The profit of the platform can be expressed as

$$\Pi = A^B N^B + A^S N^S + a (1 - F(p^*)) N^B N^S.$$

If the platform wants to have participation by both sides, the participation constraints (4.8) and (4.7) have to be met. Both are binding as they are the only constraint stopping the platform from charging infinitely large membership fees. Substituting the binding participation fees and setting $N^S = N^B = 1$ the platform profit can be written as

$$\Pi = B^B + b^B + B^S + b^S + a\left(1 - F(p^*)\right).$$

After substituting the transaction benefits with the explicit expressions and simplifying the profit becomes:

$$\Pi = B^B + B^S + \int_{p^*}^{+\infty} 1 - F(v)dv + (p^* - c)\left(1 - F(p^*)\right). \tag{4.9}$$

Note that a has no direct effect on the platform profit, but recall that a affects p^* monotonically. By choosing a the platform implicitly chooses p^*.

Proposition 4.4. *If buyers do not know θ ex-ante, the platform chooses $a^* < 0$ such that $p^* = c$. The membership fees are $A^{B*} = B^B + \int_c^{+\infty} 1 - F(c)$ and $A^{S*} = B^S + a^*\left(1 - F(c)\right)$. Total surplus from transactions is maximized.*

Proof. The optimal a^* is defined by maximizing (4.9) with respect to a. The associated first order condition is

$$\frac{\partial \Pi}{\partial a} = \frac{dp^*}{da}\left[-\left(1 - F(p^*)\right) + 1 - F(p^*) - f(p^*)(p^* - c)\right] = 0.$$

The first order condition is met, if the term in brackets is zero, i.e., $p^*(a) = c$. The membership fees follow from substituting a^* in and replacing p^* with c. Since p^* is increasing in a and $p^*(0) = p^m > c$, $a^* < 0$. □

The platform fully internalizes the transaction surplus since it can skim all rents with the membership fees. The transaction fee has no direct impact on platform profits but allows it to choose the price. The transaction fee will be negative to make the sellers internalize the price externality in buyers' surplus.

4.4.4 Stage 2: Platform participation decisions if buyers know type.

In stage 2 buyers and sellers decide whether to join the platform. At that stage the information structure matters as buyers base their decision to join on their expected transaction surplus. If buyers know θ, the expected benefit of a buyer from an additional seller, b^B, is type dependent. It is given by

$$b^B(\theta) = \int_{p^*}^{+\infty} 1 - F_\theta(v)dv.$$

The surplus increases in θ which follows directly from the first part of assumption 4.1. A buyer of type θ joins the platform if

$$A^B \leq B^B + b^B(\theta)N^S \tag{4.10}$$

holds.

All terms in (4.10) apart from $b^B(\theta)$ are independent of θ and the same for all buyers (in equilibrium they should expect the same p^* and N^S). As $b(\theta)$ is monotonically increasing in theta, there exist a critical buyer type θ^* for which:

$$A^B = B^B + b^B(\theta^*)N^S. \tag{4.11}$$

4

All buyers with $\theta > \theta^*$ will join the platform. This gives a mass of buyers on the platform of $N^B = 1 - \theta^*$.

Sellers participation in that case does not only depend on the expected mass of buyers N^B but also on the composition, θ^*. Recall that p^* increases θ^*. Still each seller faces the same decision, which is given by

$$A^S \leq B^S + (p^* - c - a)\, D(p^*) = B^S + (p^* - c - a) \int_{\theta^*}^{1} 1 - F_\theta(p^*) d\theta. \tag{4.12}$$

Proposition 4.5. *If buyers know θ ex-ante, all sellers join the platform if $A^S \leq B^S + (p^* - c - a) \int_{\theta^*}^{1} 1 - F_\theta(p^*) d\theta$. The mass of buyers on the platform is $1 - \theta^*$, with the critical buyer defined by (4.11).*

Proof. See above discussion □

4.4.5 Stage 1: Optimal tariff system if buyers know type

In stage 1 the platform chooses the membership and transaction fees.

The profit of the platform can be expressed as

$$\Pi = A^B N^B + A^S N^S + a \frac{\int_{\theta^*}^{1} 1 - F_\theta(p^*) d\theta}{1 - \theta^*} N^B N^S.$$

Since sellers are homogenous either all sellers or no sellers join the platform. Since focus on cases with positive participation, the participation constraint of sellers is binding and can be used to replace A^S in the profit function and consequently $N^S = 1$. Recall that $N^B = 1 - \theta^*(A^B, p^*)$:

$$\Pi = A^B(1 - \theta^*) + B^S + (p^* - c) \int_{\theta^*}^{1} 1 - F_\theta(p^*) d\theta.$$

In the above formulation θ^* is a function of A^B and p^* and p^* depends on θ^* and a. It is helpful to think of the platforms problem as choosing the participation level of the buyers θ^* and the market price p^* instead of membership and transaction fees. This approach is similar to the idea found in Weyl (2010), where the platform problem is expressed in participation levels instead of fees.

[4]There can be corner solution with $\theta^* \in \{0, 1\}$.

Remark 4.6. Instead of choosing A^B and a the problem of the platform can be equivalently expressed as maximizing profits over θ^* and p^*. Note that $A^B(\theta^*)$ can be derived from (4.11), where the right hand side is monotonically increasing in θ^*. The transaction fee a^* has no direct effect on profits and has a monotonic relationship with p^*. For any p^* that the platform wants to implement it can find an appropriate a^*.

In summary the profit of the platform can be expressed as a pure function of p^* and θ^*:

$$\Pi = \left(B^B + \int_{p^*}^{+\infty} 1 - F_{\theta^*}(v)dv \right)(1-\theta^*) + B^S + (p^* - c)\int_{\theta^*}^{1} 1 - F_\theta(p^*)d\theta \quad (4.13)$$

Note that the platform internalizes the consumer surplus of the marginal buyer with type θ^*, the first term in the profit function, instead of the surplus of the average buyer. It fully internalizes the profit if sellers, the second term in the platform profit. For θ^* corner solutions with $\theta^* = 0$ and full participation are possible.

Proposition 4.7. *If buyers know θ ex-ante, the platform does not maximize the total trade surplus. The price is above marginal costs. The membership fee to sellers extracts the profit of sellers. The membership fee of buyers extracts the surplus of the marginal buyer but is below the surplus of the average buyer.*

Proof. The platform maximizes (4.13) choosing θ^* and p^* by choosing an appropriate level of A^B and a .

The first order condition for the optimal price is given by

$$\frac{\partial \Pi}{\partial p^*} = -\left(1 - F_{\theta^*}(p^*)\right)(1-\theta^*) + (p^* - c)\int_{\theta^*}^{1} -f_\theta(p^*) + \int_{\theta^*}^{1} 1 - F_\theta(p^*)d\theta = 0.$$

This can be rewritten as

$$(p^* - c)\frac{-\int_{\theta^*}^{1} f_\theta(p^*)}{1-\theta^*} + \underbrace{\frac{\int_{\theta^*}^{1} 1 - F_\theta(p^*)d\theta}{1-\theta^*} - \left(1 - F_{\theta^*}(p^*)\right)}_{>0} = 0.$$

Observing that the difference between the last two terms is positive, since the average demand per buyer on the platform is larger than the demand of the marginal buyer, implies that $p^* > c$ as the factor at $p^* - c$ is negative.

The derivative of Π with respect to θ^* is given by

$$\frac{\partial \Pi}{\partial \theta^*} = \int_{p^*}^{+\infty} -\frac{\partial}{\partial \theta^*} F_{\theta^*}(p^*)dv(1-\theta^*) - \int_{p^*}^{+\infty} 1 - F_{\theta^*}(v)dv - (p^* - c)\left(1 - F_{\theta^*}(p^*)\right)d\theta = 0.$$

Note that the first therm is positive, while the second and last term are negative. Since our assumptions do not specify the size of $\frac{\partial}{\partial\theta^*}F_{\theta^*}(p^*)$ they are consistent with any $\theta^* \in [0,1]$ as a solution. If $\frac{\partial}{\partial\theta^*}F_{\theta^*}(p^*)$ is small in absolute value, there is corner solution with $\theta^* = 0$. □

Only buyers with relatively high valuations join the platform, sellers anticipate that and charge higher prices. The platform operator internalizes the surplus of sellers perfectly but only the surplus of the marginal buyer who has the lowest θ of all buyers on the platform. The platform, thus, does not maximize total surplus as it cannot extract the rents of infra marginal buyers fully, as a result the price implemented is skewed towards sellers interests and above marginal costs. Note that $A^B > B^B$, the platform extracts more than the stand alone benefit from buyers. This contrasts with the finding by Gans (2012b), that a platform is not able to charge a positive (stand alone benefits is zero) membership fee.

4.4.6 Extension: Non-negative Fees

In many real world world application it is not possible to charge negative fees. For transaction fees a explicit payment may induce unwanted fictitious trade, similarly a negative membership fee induces moral hazard.

In both cases that we have investigated, the optimal level of a^* is negative, since the price the platform wants to implement, is below the price sellers would choose independently. Sellers do not internalize the effect on buyer participation.

If a is restricted, such that $a \geq 0$. Independent of the timing, the optimal price will be $a^* = 0$. Note that $a > 0$ implies a price above the monopoly level, which reduces profits. The resulting price p^* is higher when buyers know their type ex-ante. This follows directly from the fact that p^*increases in θ^*. In the first case, when buyers do not know θ all participate, which is equivalent to $\theta^* = 0$, while in the other case $\theta^* \geq 0$.

Membership fees will be only negative if the stand-alone benefit is negative. This could be the case if seller investment costs are relatively high. If each seller can be subsidized with a negative transaction fee that makes him internalize the positive effect on buyers, negative membership fees would not be necessary. However, if transaction fees are zero, because they are restricted to be non-negative, each seller has a positive externality on buyers. In that case the platform would want to set negative membership fees. If that is not possible there will be suboptimal seller participation. Note that the price in that case is already at the seller profit maximizing level, such that increasing the price, cannot increase profits or seller participation. This would be different if there is competition between sellers.

4.5 Conclusion

The literature on two-sided markets has often framed the platform as a market place where sellers and buyers interact, but typically abstracts from modelling the transactions explicitly. The typical assumptions of the canonical model hold in case that buyers are ex-ante homogenous in there expected transaction surplus, which has been shown by Rochet and Tirole (2006a). However in many applications it seems natural to assume that buyers differ in there monetary valuations for the goods that they can buy on the platform and also have information about their valuations, when they join the platform.

In the model we present, we compare two scenarios. Consumers either have identical information about their valuations, or consumers have different types. A higher type implies both a larger average valuation and a less elastic demand.

In the first scenario the platform will be able to extract the trade surplus fully and thus has incentives to implement to surplus maximizing price. In contrast, in the second scenario, only the surplus of the marginal consumer, who is indifferent ,whether to join the platform, is internalized. This holds in both cases, if negative transaction fees are possible, and also if transaction fees cannot be negative.

The intuition derived from the model is that ex-ante information about valuations leads to a sorting of buyers according to their willingness to pay. A buyer on the platform thus has an above average willingness to pay and hence faces higher prices, if sellers can choose prices freely but also, if the platform can (indirectly) commit to lower prices, since it will not be able to internalize the effect. The presence of informed and heterogeneous buyers reduces platform profits. An interesting effect is that the membership fee to buyers increases type of the marginal consumer and thereby the prices sellers want to set.

Models that allow for heterogeneity are analytically hard to solve. However, especially in the context of platforms it is an interesting question how the composition within the sides affects platform profits and how the platform can affect the composition.

Chapter 5

Licensing an Early Mover Advantage

5.1 Introduction

The majority of theoretical research on licensing has focused on the choices during the patent period, for example, on the question of optimal royalties and optimal number of licenses. Less attention has been given to the effects of licensing on the post patent period. A notable exception is the article by Rockett (1990), in the context of a product patent, she argues that the patentee would prefer to sell a license to a less cost-efficient firm, rather than a more efficient firm in order to face weaker competition in the post-patent period.

The goal of the present paper is to formalize the idea that a license (before patent expiration) gives the licensee a head start over the competition that can only be active in the market after the patent has expired. We present a model in which this head start translates into an early mover advantage for that firm and analyze the market equilibrium (prices, quantities and advertising) and industry equilibrium (the number of firms that enter) with endogenous entry of firms after patent expiration. The idea started out, motivated by the observation of so called pseudo generics in the pharmaceutical industry. In that industry in particular, the patentee faces a large drop in revenues after patent expiration due to the entry of generics. As one strategy to deal with the patent end end the following entry of generics, patent holders have been observed to license their patented product to a third firm that starts marketing the product before the actual generics enter the market. Furthermore, it has been observed in various empirical studies that the the first generic to enter has a persistently larger market share than later entrants (See for example Hollis (2002)).

We provide a general model with symmetric firms. In the model firms' competitive strategies are strategic substitutes and profit depends on the own strategy and the aggregate of the strategies of other firms. Licensing allows the licensee to move before other entrants. Two micro-foundations are provided. Firstly, one can think of firms as choosing quantities in a setting with a homogenous product. Secondly one can interpret strategies as (informative) advertising. For the general model we show that the market outcome in terms of the aggregate level of competitive strategies is independent of licensing. This is due to the identical zero profit condition of endogenous entry. The result generalizes a finding by Economides (1993) for the comparison of Cournot and Stackelberg equilibria with endogenous entry. For the two micro-foundations the result implies that the aggregate quantity is independent of licensing, or, alternatively, the overall share of consumers that are informed about at least one product.

We can show under relatively general conditions (stability) that licensing is always profitable and always reduces the number of firms that are active in equilibrium. A caveat is that we assume that the strategy of the patentee is not affected by his licensing decision. For some cases this may be a reasonable assumption if the strategy of the patentee is tailored to the on patent phase. In general however the patentee may increase or decrease strategy choice with licensing depending on cost and demand assumptions. We return to this question in an extension with quantity competition and linear demand.

While the logic of the models with quantity and advertising are very similar, the welfare implications differ. With quantity competition licensing has no effect on prices. Thus consumers are not affected. Industry profit increases due to a reduction in fixed costs. With advertising as strategic variable the number of consumers who are informed about at least one product is constant, the average price, however is determined by the likelihood that a consumer knows more than one supplier. This overlap between informed consumers depends on the distribution of the individual strategies. It is shown that due to the more uneven distribution of strategies caused by licensing, licensing increases average prices. Thus, in case of advertising as strategic variable, the overall welfare effect of licensing is ambiguous as consumer welfare decreases but producers also save on fixed costs (lower number of firms in the market).

For the main body of the argument we maintained to assumptions that are helpful to derive results without overly restrictive functional form assumptions. First, we focused on a single license, secondly we assumed that the patentee does not adept its quantity to licensing. In an extension we assume that firms compete in quantities and demand is linear. In that context, we can show that the patentee's strategy is independent of licensing and equals the choice under monopoly. This result is related to findings that show that under linear demand the quantity of the Stackelberg leader is independent of the number of followers, as first established in Boyer and Moreaux (1986). We are more general in that we allow for a mixture of Cournot and Stackelberg, that is, allow for more than one firm to move at a time. We can also derive the optimal number of licensees. We find that the optimal number of licenses form the point of view of the patentee increases if fixed costs decrease. The patentee never licenses to all firms in the market and tends to prefer an even number of licensees and non-licensees.

The paper most closely related is Kong and Seldon (2004) who look at the incentives to license in a quantity competition setting also focusing on the post patent entry. They assume a fixed number of firms, and product heterogeneity. They find that deterring entry though licensing may be socially preferable to entry deterrence by the patentee alone. In there model the patentee and the licensee determine their quantities before the entrants decide whether to join the market. As Rodrigues (2006) points out, their findings are not rigorously derived, in particular it is unclear whether licensing increases or decreases welfare.

Rockett (1990) as mentioned above also looks at the effect of licensing on post patent entry. She also assumes that there is fixed number of potential entrants of two that differ in their costs. She shows that it may be profitable to choose less cost efficient firms as licensees in order to face softer competition. Endogenous entry and Stackelberg competition is also the subject of a series of papers by Etro, e.g. Etro (2007). The timing in these papers differs in that the Stackelberg leader is assumed to move before other firms decide on entry. This typically leads to a result of fully deterred entry. The timing in our paper is more similar to that of Economides (1993), firms decide on entry anticipating when they will move in the competitive game. The advantage of this approach is that it does not exclude the possibility for competition in equilibrium although early mover advantages are possible. The timing makes in particular sense if entry decisions precede the decision on market strategies and entail more commitment, for example through the presence of large sunk costs of entry.

The micro-foundation of informative advertising follows the assumptions of Butters (1977) but differs in that advertising is chosen before prices are. This timing is also considered by McAfee (1994). We differ from McAfee (1994) in that we allow firms to price discriminate according to the level of competition. Regan (2008) provide evidence that the market for pharmaceuticals is segmented into competitive and less competitive segments.

The next section introduces the model. Subsequently the results are derived.

5.2 Model

Let us consider a product market with a patented product that is offered by a monopoly incumbent, the patentee. The patent is about to expire and the patentee considers licensing the product. We are interested on the effect of licensing on the market structure and market outcome after the patent has expired. Thus we will look at the effects on the market after patent expiration and neglect the period in which the patentee and its licensees are both active and the patent is still valid.[1]

There is a (sufficiently) large number of potential entrants ready to enter the market such that the number of firms is endogenously characterized by trading of fixed costs (of entry) F with expected operating profits. To fix ideas consider that all active firms offer a homogenous product. The operating profit of firm i is called $\pi_i(s_i, S_{-i})$, where $s_i \geq 0$ is the level of firm i's competitive strategy. For example the quantity it chooses to produce and $S_{-i} \equiv \sum_{j \neq i} s_j$ is the aggregate level of the

[1]There exists a large literature on the market effects when a patentee and licensees compete, the general message of literature is that the patentee can use the licensing terms to manage the competition to his advantage, e.g. Kamien and Tauman (1986); Katz and Shapiro (1986).

strategies chosen by all other firms but firm i. Naturally, for firms j that have not entered the market $s_j = 0$.

Assumption 5.1.

- $\pi_i(s_i, S_{-i})$ is twice continuously differentiable, concave in s_i and decreasing in S_{-i}.
- Choices s_i and s_j are strategic substitutes, or equivalently $s^*(S_{-i}) \equiv \arg\max_{s_i} \pi_i$ is decreasing in S_{-i}.
- Stability:
$$\left|\frac{\partial^2 \pi_i}{\partial s_i \partial S_{-i}}\right| < \left|\frac{\partial^2 \pi_i}{\partial^2 s_i}\right|.$$

To justify the assumption and provide interpretation for s_i we have two examples in mind that are sketched in the following. First, firms choosing quantities, depending on the timing a la Cournot or a la Stackelberg, second, firms choosing informative advertising.[2]

Example 5.2 (Quantity/Capacity Competition)**.** Consider that s_i is the quantity of firm i and denote it by q_i. The properties of the profit functions in assumption 5.1 can be derived in the context of a homogeneous product under fairly natural assumptions on demand and costs. In particular, the assumption holds if inverse demand function satisfies $P'(Q) < 0$ and $P'(Q)Q + P''(Q) \leq 0$, with $Q = \sum_i q_i$ and marginal costs c are constant. Later we will use the specific linear demand function, $P = A - \sum_i q_i$, to derive further results. The linear demand functions also satisfies all the requirements.

That firms choose and commit to some extend to quantities or capacity, following the interpretation of Kreps and Scheinkman (1983), is a natural assumption for manufactured goods that require to set up a specific production facility. Alternatively consider the case of advertising which has for example been considered to be a driver of the first mover effects in pharmaceutical markets (See for example Morton (2000)).

Example 5.3 (Advertising)**.** Consider that s_i is the level of advertising that firm i chooses and denote it by a_i. Advertising could in general be persuasive, altering the actual or perceived utility of buyers, or informative, allowing for a better decision of buyers. To fix ideas let us consider informative advertising.[3] In line with Butters (1977) and Grossman and Shapiro (1984), assume that advertising informs buyers about the existence of the product, such that only informed consumers will be able to buy the product. Consumers will buy from the cheapest supplier they know and

[2]It is also possible to consider s_i as a multidimensional competitive strategy, that is already optimized such that - for example - the level of advertising is optimal for the given quantity and vice versa.

[3]Chioveanu (2008) derives a similar profit function in a setting with persuasive advertising.

depending on the smallest price p according to the demand function $D(p)$. Assume that $D(p)$ is log concave and decreasing in p (strictly if $D > 0$) such that the monopoly profit is concave and the optimal price is well defined. Assume that each seller is able to price discriminate between consumers that only know his product and consumers that know more than one product.[4] Advertising is costly with $C(a_i)$ being convex to ensure concave profits with $C(0) = 0$. Assume that marginal cost are sufficiently large to ensure interior solutions. The level of advertising translates into a probability λ_i that a consumer is informed about the existence of the product. Advertising is non targeted in the sense that the probabilities to be informed about two different suppliers are independent from each other. To make sure that a_i is equivalent to s_i in assumption 5.1, $a_i \equiv -\ln(1-\lambda_i)$.

The patent holder offers a fixed number if licenses to entrants for a fixed fee before the expiration date of the patent. Any licensee would then enjoy a head start compared to the rest of the entrants which might make him willing to pay for the license although the patent is about to expire. Besides the head start, there is no other benefit resulting from the license. The timing of the strategic choices depends on licensing. The idea behind the timing is that licensing takes place before potential entrants decide on entry, and actions are ordered by their degree of long-term commitment.

Assumption 5.4. *The patentee's strategy s_p is not affected by licensing and fixed ex-ante.*

The idea behind this assumption is that the patentee has chosen his strategy to maximize profits during the patent phase and is committed to that. This assumption allows us to derive clear results within the context of the general model. We show for the case of quantity competition and linear demand that the patentee endogenously would not change its strategy from the one that maximizes monopoly profits during the patent phase.

The entry decision is harder to change than capacity and advertising choices, which are less flexible than pricing.

Overall the timing is as follows:

1. The patent holder sets a take it or leave it price for a fixed number L of licenses.

2. Each firm decides to buy or not buy a license.

3. Taking the first two decisions into account, the remaining potential entrants choose whether to enter the market or stay out of it.

[4]Price discrimination may be a reasonable approximation of the way prices are negotiated in the pharmaceutical industry. For the case without price discrimination McAfee (1994) derives a mixed strategy equilibrium in prices and an asymmetric equilibrium in advertising, if firms are symmetric and move simultaneously first in advertising and then in prices.

4. Firms “compete”, Licensees choose their strategies simultaneously and before all other entrants that also move simultaneously.

It is important to note that entry decisions take place before competitive strategies are chosen. Entry entails more commitment than the strategy choice. The alternative timing, i.e., competitive strategies are chosen before the entry decision is for example analyzed in Etro (2007). In equilibrium then, a firm, moving before the others can enter, deters all entry by producing a large quantity.
[5]

The timing is most easily understood in case of a single license and quantity competition. The patentee grants an exclusive license, firms enter until they expect zero profit. Once the entry decisions are made firms engage in quantity competition. The patent holder has already chosen his quantity. The licensee is both: a Stackelberg leader (compared to the other entrants) and follower (compared to the patent holder). The remaining entrants finally set quantities simultaneously after the patent expiration.

We solve this game for subgame perfect equilibria.

5.3 Results

5.3.1 Market Equilibrium: Quantities, Advertising and Prices.

In the spirit of backward induction let us first consider the market outcome and choice for a given number of firms. The choice by the patentee is given exogenously, if there is licensing the licensees move before the other entrants, who again move simultaneously.

Let us first consider the case of quantity competition. If there is no licensing all entrants play a Cournot game on the residual demand after the quantity of the patentee has been subtracted. With licensing, also the quantity of the licensee has to be subtracted from demand. The operating profit of each entrant is $\pi_i = P(q_i + Q_{-i})q_i - cq_i$. Each entrant maximizes profit with respect to own quantity. The associated first order condition is

$$P'\left(q_i + Q_{-i}\right)q_i + P(q_i + Q_{-i}) - c = 0 \tag{5.1}$$

The slope of the reaction function of an entrant with respect to a change in the quantity of the other firms can be derived applying the implicit function theorem on (5.1).

$$\frac{dq_i}{dQ_{-i}} = -\frac{P''\left(q_i + Q_{-i}\right)q_i + P'(q_i + Q_{-i})}{P''\left(q_i + Q_{-i}\right)q_i + 2P'(q_i + Q_{-i})} \tag{5.2}$$

[5]For the pharmaceutical market this timing seems natural, as producers of generics typically decide on entry quite some time before the patent expires, as getting the regulatory permission for market access is time consuming. The empirical analysis of Morton (2000) provides evidence that advertising levels of the patentee are not chosen in order to deter entry of generics which can be interpreted as advertising not being fixed before the entry decision.

Note that given our assumptions (5.2) $-1 < \frac{dq_i}{dQ_{-i}} < 0$, as both the numerator and denominator are negative, since $P\left(q_i + Q_{-i}\right)'' q_i + P'(q_i + Q_{-i}) < 0$ (strategic substitutes) and $P' < 0$ by assumption.

Similarly the reaction of the aggregate equilibrium quantity of entrants can be derived. Applying symmetry to the first order condition and using the implicit function theorem (5.1), denoting the equilibrium symmetric quantity by q_E^* , the number of firms moving simultaneously by m_E and Q_{-E} the quantity of all other firms (moving before them):

$$\frac{dq^*}{dQ_{-E}} = -\frac{m_E P''\left(m_E q_E^* + Q_{-E}\right) q_i + mP'(m_E q_E^* + Q_{-E})}{mP''\left(m_E q_E^* + Q_{-E}\right) q_i + (m+1)\, P'(m_E q_E^* + Q_{-E})} \tag{5.3}$$

Note that also the aggregate reaction has $-1 < \frac{dq^*}{dQ_{-E}} < 0$, which implies that an increase in quantity by the licensee will reduce the aggregate quantity of the entrants by less than one and this yield to an overall increase in quantity.

In fact this argument is also true for the general case

Lemma 5.5. *The aggregate equilibrium strategies S_E^* of entrants react with $-1 < \frac{d}{dS_-} S_E^* < 0$ to an increase ín the strategies they observe S_{-E}.*

Proof. Recall the profit of a firm is $\pi_i(s_i, S_{-i})$. The associated FOC for a firm that has no followers is $\frac{\partial \pi_i(s_i, S_{-i})}{\partial s_i} = 0$. Depending on the number of firms m that move simultaneously with firm i, the equilibrium can be found by applying symmetry to the first order condition. Denoting by S_{-E} the aggregate quantity of firms moving before firm i and by S_E^* the aggregate equilibrium quantity of firms moving simultaneously with i, in the symmetric equilibrium

$$\left|\frac{\partial \pi_i(s, S_{-i})}{\partial s_i}\right|_{s_i = S_E^*/m_E,\, S_{-i} = S_{-E} + S_E^*\left(\frac{m_E - 1}{m_E}\right)} = 0$$

The slope of the reaction function of firm i can be found by applying the implicit function theorem:

$$\frac{dS^*}{dS_{-E}} = -\frac{\frac{\partial^2 \pi_i}{\partial s_i \partial S_{-i}} + (m-1)\frac{\partial^2 \pi_i}{\partial s_i \partial S_{-i}}}{\frac{\partial^2 \pi_i}{\partial (s_i)^2} + (m-1)\frac{\partial^2 \pi_i}{\partial s_i \partial S_{-i}}}$$

Note that all terms in the fraction are negative and recall that by the stability assumption $\frac{\partial^2 \pi_i}{\partial s_i \partial S_{-i}} < \frac{\partial^2 \pi_i}{\partial (s_i)^2}$, thus $-1 < \frac{dS^*}{dS_-} < 0$. $\square$

This result naturally follows from the fact that firms strategies are strategic substitutes and that changing the own strategy has a stronger marginal effect than a change in the other strategies. For the case of quantity competition the latter naturally follows from the fact that other strategies affect the price only, while the own quantity also directly affects revenue. For the case of advertising we will see that own and other advertising have a similar marginal effect on demand, but own advertising has an additional affect on costs.

Consider now that firms choose advertising $a_i = -\ln(1 - \lambda_i)$. The aggregate level of advertising $A = \sum_i a_i$ in that case measures the probability that a consumer knows about at least one firm. To see that note that $A = \sum_i -\ln(1 - \lambda_i)$ can be also expressed as $A = -\ln \prod_i (1 - \lambda_i)$. As the ln is a monotone transformation, A monotonically decreases in $\prod_i (1 - \lambda_i)$ which is the probability that a consumer is not informed about any product. Hence A is positively and monotonically related to the probability that a consumer knows at least one product (which is the counter probability to not knowing any product). A is thus positively related to the informational aspect of consumer welfare. However prices are not uniquely defined by A.

Firms set prices facing three different consumer segments. In the first segment consumers only know the given firm and none of its competitors. In that segment there is no competition and the firm chooses the price $p^m = \arg\max_p (p - c) D(p)$. As $D()$ is price elastic this leads to a dead weight loss. The expected size of the first segment for firm i is $\lambda_i \prod_{j \neq i} (1 - \lambda_j)$. In the second segment the consumer knows at least two firms. Firms compete with identical marginal costs. By standard Bertrand arguments this leads to a price equal to c. Hence, in the second segment firms make zero profits. This is also true for the third segment, the share of consumers that do not know firm i.

In summary firm i has operating profits of $\pi_i = \Phi^m \lambda_i \prod_{j \neq i} (1 - \lambda_j) - C()$, with $\Phi^m \equiv (p^m - c) D(p^m)$. Recalling the definition of a_i, this profit can also be expressed in terms of strategies a_i and a_j as

$$\pi_i(a_i, A_{-i}) = \Phi^m \left(1 - e^{-a_i}\right) e^{-A_{-i}} - C(a_i). \tag{5.4}$$

Note that the profit is decreasing in the aggregate level of advertising of all other firms in the market. From the inspection of (5.4) it is also relatively easy to see that the marginal profit of a_i is decreasing in $A_{-i} \equiv \sum a_j$. As profits are concave, marginal profit of a_i is decreasing in a_i. Lemma (5.5) applies, as the stability condition holds if costs are convex:

$$\left| \frac{\partial^2 \pi_i}{\partial a_i \partial A_{-i}} \right| = \left| -\Phi^m e^{-a_i} e^{-A_{-i}} \right| < \left| -\Phi^m e^{-a_i} e^{-A_{-i}} - C'' \right| = \left| \frac{\partial^2 \pi_i}{\partial \left(a_i\right)^2} \right|.$$

5.3.2 Licensing and Entry Equilibrium

This section deals with equilibrium choices of the strategic variable. In general each firm faces the following problem. It wants to maximize profits taking into account the strategic choices of all firms that moved before it, and taking into account the effect of its choices on all following firms. For any non -licensee there are no followers. Any licensee, however, takes the reaction of all non-licensees into account.

Intuitively, since strategies are strategic substitutes, the more non-licenses observe a licensees choice, the larger the strategy it chooses will be. Formally, a licensee i maximizes $\pi_i(s_i, S_{-i}(s_i))$, where $S_{-i}(s_i)$ is a function of s_i if firm i moves before some other firm. For a non-licensee S_{-i} is a constant. The associated first order condition for a license is

$$\frac{\partial}{\partial s_i}\pi(s_i, S_{-i}(s_i)) + \underbrace{\frac{\partial}{\partial S_{-i}}\pi(s_i, S_{-i}(s_i))}_{<0}\underbrace{\frac{\partial}{\partial s_i}S_{-i}(s_i)}_{<0} = 0. \tag{5.5}$$

Lemma 5.6. *Any any equilibrium, where some non-licensees entered, the strategy chosen by a licensee is larger than that chosen by a non-licensee entrant.*

Proof. Comparing (5.5) with the first order condition of an entrant, which consists only of the first term on the right hand side, yields that a licensee will always choose a larger strategy than a non licensee. Suppose to the contrary that a licensee has $s_L \leq s_E^*$, if that where an equilibrium, aggregate $s_i + S_{-i}$ would have to be the same, however then the first term and the second term would be both positive for the licensee, the FOC holds for the entrant. The second term is strictly positive if there is any firm following the licensee. This excludes $s_L = s_E^*$. Hence the first order condition cannot be met for both licensee and entrant if the licensee does not choose a strictly larger strategy than any entrant.

Note that this does only imply that in a given equilibrium licensees behave more aggressively. Next we look into the entry equilibrium in order to determine the subgame perfect equilibrium choices for the case of licensing and not licensing. The idea is determine the choice of the last entrant using the zero profit condition of endogenous entry. □

Proposition 5.7. *The aggregate level of the competitive strategies is the same independent of licensing. The optimal strategy of the last firm to move s^* is independent of licensing.*

Proof. The last firms entering the market must have a profit of $\pi_i(s_i, S_{-i}) = F$. By strategic substitutability $\partial s^*/\partial S^*_{-i} < 0$. Since the last firm maximizes profits taking the choice of others as given, it must play its best response $s_i = s^*(S^*_{-i})$. Hence, in any endogenous entry equilibrium

$$\pi(s^*(S^*_{-i}), S^*_{-i}) = F \tag{5.6}$$

has to hold. The LHS is monotonically decreasing in S^*_{-i}. This monotonicity can be verified using an envelope theorem argument: $\frac{d}{dS^*_{-i}}\pi = \frac{\partial}{\partial s^*_i}\pi\frac{ds_i}{dS_{-i}} + \frac{\partial}{\partial S^*_{-i}}\pi < 0$, where the first term is zero by optimality. Thus, S^*_{-i} is uniquely defined by (5.6). In turn also $s^* = s^*(S^*_{-i})$ and $S^* = s^* + S^*_{-i}$ are uniquely defined. □

Note that this is a generalization of the argument provided by Economides (1993). Clearly the argument neglects integer constraints for the number of firms.[6]

[6]Strictly speaking the result holds only, if the derived number of firms is "accidentally" integer. In general it would be possible that number of firms is integer with licensing but not without, and the outher ways round. Since the integer property is not systematically linked to licensing, there is no systematic error in neglecting it.

The mechanism behind the result is that more firms will enter as long as its profitable. Profitability only depends on the anticipated aggregate choice of strategies by the rivals. The last firm thus has the same trade-off independent of how many other firms contribute to the aggregate level of strategies.

Finally let us address the question whether licensing is profitable. For the moment we assume that licensing does not affect the strategy choice of the patentee. Since the aggregate level of strategies is constant by proposition (5.7), also the operating profit of the patentee is constant. Hence, the willingness to pay of the licensees.

First let us determine the price of a license. Note that without a license each entrant obtains a sure profit of zero, $\pi^* - F = 0$, either in the market, where the free entry equilibrium ensures that, or outside. Hence, each entrant is willing to pay the difference between the profit of a non-licensee and the profit of a licensee as price for the license. Thus, denoting the price of the license with p^L, the optimal price of the license for the patentee is

$$P^L = \pi_L - \pi^*.$$

As the patentee's operating profit is not affected by licensing, licensing is profitable if and only if the p^L is positive. The profit difference is Lp^L.

Proposition 5.8. *Licensing is always profitable, if there is more than one entrant without licensing. The number of licenses is strictly below the number of firms that would enter without licensing.*

Proof. If the number of licenses is equal to the number of entrants the market game is identical in both situations and hence the profit of a license is the same as the profit of an entrant in case of no licensing and $p^L = 0$.

If L is strictly below the number of entrants, then there will be entry by non-licensees in equilibrium. In that case each licensee will choose a strictly larger s_L than a non-licensee by lemma (5.6). Note that in equilibrium S^* is constant. Alternatively to s_L each licensee is free to choose the same s_i as a non-licensee s^*, in that case S would be strictly below S^* as the reduction of s by a licensee is not compensated fully by entrants, other licensees cannot react at all. Recall (5.5) to verify that $S' \equiv S^* - s_L + \underbrace{\int_{S^*-s^*}^{S^*-s_l} \frac{d}{dS_-} S^*_E dS_{-i}}_{<s^*-s_L} < S^* - s^*$. Thus, a licensee choosing s^* must have $\pi(s^*, S') > \pi(s^*, S^* - s^*) = \pi^*$. Since a license prefers s_L over s^*: $\pi(s_L, S^* - s_L) \geq \pi(s^*, S')) > \pi^*$. Thus, licensing is strictly profitable if there are firms that follow the licensee but does not have an effect on profits if there are no non-licensee entrants. Hence, the optimal number of licenses is smaller then the number of entrants without licensing. □

The intuition for the result is that there exists an early mover advantage for licensees which increases their profits, but does not have a negative effect on the profit of an entrant in equilibrium. The later is the case, since endogenous entry keeps the profit of entrants constant. It is never optimal to license to "all" firms since then the first mover advantage vanishes as there are no followers.

5.4 Welfare

Licensing is profitable to the patentee and thus has to increase the joint profits of patentee and licensees. The profit of non-licensee entrants is constant in all equilibria and equals zero. Hence licensing unambiguously increases producer surplus in the general model. Since the patentee internalizes the change in producer surplus fully, as he can extract the whole profit difference of a licensee compared to a non-licensee, the number of licensees the patentee chooses also maximizes producer surplus. Also the number of firms can be clearly ranked.

Lemma 5.9. *The number of active firms is strictly lower if licensing is strictly profitable and, hence, used.*

Proof. The strategy of the patentee s_P is constant, while each licensee chooses a larger strategy $s_L > s*$. As each non-licensee has $s*$ independent of licensing and $S^* = s_P + Ls_L + m_E s^*$ is also constant, $m_E + L$ in case of licensing has to be lower than m_E in case of no licensing. □

To assess the effect of licensing on consumers we need the more concrete interpretation of the strategic variables and the demand function provided in the examples.

For the case of quantity competition the analysis is straightforward. Since the aggregate quantity and prices are constant in licensing consumers are not affected by licensing. Hence we can summarize for the case of quantity competition

Proposition 5.10. *If firms compete in quantities, licensing increases producer surplus, consumers surplus is constant. The number of licenses chosen by the patentee maximizes total welfare.*

Intuitively, since entry keeps the quantity constant, the only way to generate profit with licensing is to improve the cost efficiency of the industry, that is by saving on fixed cost. This motive is in line with total welfare maximization.

For the case of advertising consumer surplus is affected by the degree of information consumers have and by the price consumers pay. We know that A is independent of licensing implying a constant share of consumer that know at least one product. The average price, however, depends on the distribution of advertising levels in the industry. Consumers face either the monopoly or the competitive price. This depends on whether they know more than one product. Consumers facing the monopoly price induces a welfare loss since demand is elastic. The size of the welfare loss depends on the shape of the demand function. While consumers that face the competitive price are served at the welfare optimal price. Consumer welfare can be measured by the share of consumer that face the monopoly price. This share is a sufficient measure since the overall share of informed consumers is constant and to these consumers only two different prices are offered. Hence it contains all the relevant information. In terms of probabilities λ the share of consumers that know exactly one firm is given by

$$\alpha = \sum_i \lambda_i \prod_{j \neq i} (1 - \lambda_j).$$

It can be equivalently expressed in terms of strategies as $\alpha = e^{-A} \sum_i (e^{a_i} - 1)$.

Note that the first factor is constant in equilibrium. Hence α depends only on the sum $\sum_i (e^{a_i} - 1)$, which is a sum over all firms. Note also that the sum is not affected if non active firms are included with $a_i = 0$. Since $A = \sum_i a_i$ is constant $e^{a_i} - 1$ is a convex transformation of a_i, $\sum_i (e^{a_i} - 1)$ increases for any (mean preserving) spread of a_i. Note that licensing induces a mean preserving spread. For the argument it helpful to fix the number of firms at the level that results under no licensing. For the same number of firms with licensing the distribution of a_i changes in the following way some firms increase a_i (the licensees), some firms decrease a_i (those that leave the market). All other firms do not change a_i, since A is constant this constitutes a mean preserving spread. Hence, α increases in licensing, which means that the average price increases in licensing.

Proposition 5.11. *If firms compete in advertising, licensing increases producer surplus, consumers surplus is lower.*

Proof. See above discussion. □

The total welfare effect can be positive or negative since the dead weight loss from the monopoly price segment could be very large, compared to the savings in fixed costs. The dead weight loss is affected by the shape of the demand function, while the entry equilibrium and licensing decision only depends on demand through the margin $p^m - c$. Note that due to the convexity of $C(a_i)$ there is an additional cost inefficiency caused by the more uneven distribution of a_i due to licensing.

5.5 Linear demand and quantity competition

In this section we endogenize the quantity choice of the patentee. We also determine the optimal number of licenses. In the general model it is impossible to solve the for the equilibrium choices in an explicit way. We turn to a linear demand quantity choice model to derive explicit results.

Suppose that demand is linear with normalized inverse demand $P = A - \sum_i q_i$. Otherwise we maintain the assumptions from (5.2), for example marginal costs are c. We first derive the results of the market game for a given number of firms and then turn to the implications for entry and then licensing.

Within this context we first solve the generalized Stackelberg model, in which more than one firm can move at each stage. There are K stages. At each stage k, a subset of firms chooses their quantities, these subsets form a partition of the set of firms that entered. Denote the number of firms choosing their quantity in stage k with m^k. In general it is possible that all firms move at the same stage (Cournot), or that at each subset contains only one firm (Stackelberg sequence), or anything in between. For the pure Stackelberg sequence Boyer and Moreaux (1986) show that each firm produces the best reply to the total quantity it observes, but behaves as if there are no followers. For example the first firm to move produces the same quantity as a monopolist. We generalize this result.

Proposition 5.12. *In the generalized linear Stackelberg model, firms at each stage produce the Cournot equilibrium quantity on the residual demand, after quantities produced in earlier stages Q^{k-} are subtracted. The aggregate quantity in each stage is defined by:*

$$Q^k = \frac{m^k}{1+m^k}\left(A - c - Q^{k-}\right).$$

Proof. See Appendix □

A direct implication of this result is that the patentee will produce the monopoly quantity independent of licensing.

The total quantity produced and the quantity produced by a non-licensee, in any endogenous entry equilibrium with fixed costs F, can be determined by solving the pure Cournot game.

Proposition 5.13. *In the generalized linear Stackelberg model, the total quantity produced in an endogenous entry equilibrium is given by*

$$\overline{Q} = A - c - \sqrt{F}$$

The quantity produced by one of the firms moving in the last stage is $q^* = \sqrt{F}$.

Proof. See Appendix □

Finally we can determine the optimal number of licenses for the patentee. Independent of licensing, the total quantity is always $\overline{Q}$. Without licensing there are two stages in which quantities are chosen. In stage 1 only the patentee moves and in stage two all other firms.

If there is at least on licensee, there are three stages in which quantities are chosen. In stage one the patentee moves, in stage two all licensees, and in stage three all non licensees.

Proposition 5.14. *The optimal number of licenses L^* is such that in equilibrium the number of licenses equals the number of non-licensees that enter $L^* = E\left(L^*\right)$.*

Proof. See Appendix □

L^*	N^N	N^L
1	4	3
2	9	5
3	16	7
4	25	9

Table 5.1: Number of licenses and firms

Note that this implies that there is neither full

The following table compares the equilibrium number of firms with N^L and without licensing N^N for parameter constellations such that L^* is an integer between 1 and 4. The table indicates that there can be a substantial reduction in the number of active firms if licensing provides an early mover advantage.

5.6 Firm Heterogeneity

We assume in the model that firms are symmetric. This assumption is not harmless, since the results that the aggregate of the strategies is constant relies on it. Consider for example that firms can be ordered with respect to their fixed costs. Then those in the market should have lower fixed costs than those outside. If licensing changes the number of active firms, also the fixed costs of the last firm that entered would change. If in both cases the zero profit condition holds, the overall level of strategies must be higher if there are less firms active, as the last firm would otherwise make positive profits due to lower fixed costs. Recall from the proof of proposition 5.7 that profits are a monotone function of the aggregate strategy. This implies that there is an additional social benefit to licensing (quantity or advertising increases), which however is not fully aligned with the interest of the patentee.

Firm heterogeneity can also affect the patentee's ability to extract the profits of its licensee. If there is asymmetric information, even with an auction mechanism, the patentee would be typically only able to extract the willingness to pay of the license with the second highest profit due to licensing. Furthermore, since then, also non-licensees could have positive profits, the outside option to buying a license is positive and depends on the concrete mechanism by which the licenses are sold. Because of this effects the patentee will not internalize the industry profits fully and the licensing decision is thus not necessarily fully aligned with industry profit maximization.

5.7 Concluding Remarks

The model illustrates the idea that licensing a head start can be profitable. In a model with endogenous entry we find that the free entry condition holds the aggregate level of strategies fixed, independent of licensing. Depending on the interpretation for the strategies this has different implications. in case of quantity competition licensing does not affect consumer surplus but increases producer surplus and cost efficiency. For the case of advertising the overall welfare effect is unclear, since consumers are harmed by higher prices, while firms profit and fixed costs are saved.

The results contribute to the discussion of the effects of pseudo generics. First the results imply that it is possible that pseudo generics licensing is profitable without harming consumers. Secondly, if the first mover advantage is likely to come through marketing, the results of advertising case however tend to support the idea that it is possible that consumers are harmed through higher prices.

The results are also generally interesting for industries in which licensing takes place not primarily to affect the post patent market outcome. Still in many industries the licensee will have a first mover advantage after the patent expires. One aspect is that the patentee can price this effect into the price for the license and will have an additional incentive to not license to all potential entrants, as then the first mover effect vanishes.

Within a more concrete example of linear demand quantity competition, it can be shown that allowing the patentee to adept its competitive strategy to licensing does not generally alter the results. However, if it is likely that the patentee adapts its strategy. We can not exclude generally that licensing may not be profitable as it may undermine the commitment power of the patentee. If licensing is profitable the above mentioned results are likely to hold.

Finally we want to point out that the model is rather general in its functional form assumptions but is limited to the case of symmetric firms. For example, allowing for heterogeneity in fixed cost would alter the results, as then the free entry condition may hold for a different firm depending on licensing. This would have implications for both the profitability of licensing as well as the welfare properties.

Appendix: Proofs

Proof of Proposition 5.12

A firm moving at stage k in that game maximizes profits choosing quantity q^k. It faces the total quantity of all firms that have moved before it Q^{k-} or move at the same time $Q^k - q^k$ and anticipates the reaction of all firms following to its quantity $Q^{k+}(q_k)$.

$$\pi^k = \big(A - Q^{k-} - Q^k - Q^{k+}(q^k) - c\big)\, q^k$$

The first order condition is given by

$$A - Q^{k-} - Q^k - q^k - Q^{k+} - \frac{d}{dq^k} Q^{k+} qk - c = 0$$

$$A - Q^{k-} - m^k q^k - 1q^k - Q^{k+} - \frac{d}{dq_k} Q^{k+} q^k - c = 0$$

$$A - Q^{k-} - \left(1 + m^k + \frac{dQ^{k+}}{dq^k}\right) q^k - Q^{k+}(q^k) - c = 0$$

$$q^k = \frac{A - Q^{k-} - Q^{k+}(q^k) - c}{1 + m^k + \frac{dQ^{k+}}{dq^k}}$$

$$q^k = \frac{A - Q^{k-} - c - Q^{k+}(q^k)}{1 + m^k + \frac{dQ^{k+}}{dq^k}}$$

$$\left(1 + \frac{dQ^{k+}}{dq^k}\right) q^k = \big(A - Q^{k-} - Q^k - c\big) - Q^{k+}$$

We can work backward from stage K

The quantity of each firm in the last stage K is the Cournot quantity on the residual demand:

$$q^K = \frac{A - Q^{K-} - c}{1 + m^K}$$

from this the reaction to the total previous quantity can be extracted

$$\frac{dQ^K}{dQ^{K-}} = -\frac{m^K}{1 + m^K}$$

Thus, in stage $K - 1$, the quantity is defined by:

$$A - Q^{K-} - c - \left(1 + \frac{dQ^K}{dq^{K-1}}\right) q^{K-1} - Q^K(q^{K-1}) = 0$$

$$A - Q^{K-} - c - \left(1 - \frac{m^K}{1+m^K}\right) q^{K-1} - \frac{m^K}{1+m^K} \left(A - Q^{K-} - c\right) = 0$$

$$\left(1 - \frac{m^K}{1+m^K}\right) \left(A - Q^{K-} - c\right) - \left(1 - \frac{m^K}{1+m^K}\right) q^{K-1} = 0$$

$$\left(1 - \frac{m^K}{1+m^K}\right) \left(A - Q^{(K-1)-} - m^{K-1} q^{K-1} - c\right) - \left(1 - \frac{m^K}{1+m^K}\right) q^{K-1} = 0$$

$$\left(1 - \frac{m^K}{1+m^K}\right) \left(A - Q^{(K-1)-} - c\right) - \left(1 + m^{K-1}\right) \left(1 - \frac{m^K}{1+m^K}\right) q^{K-1} = 0$$

$$q^{K-1} = \frac{A - Q^{(K-1)-} - c}{1 + m^{K-1}}$$

$$Q^{K-1} = \frac{m^{K-1}}{1+m^{K-1}} \left(A - Q^{(K-1)-} - c\right)$$

By analogy

$$Q^k = \frac{m^k}{1+m^k} \left(A - c - Q^{k-}\right)$$

The quantity produced in stage k equals the Cournot equilibrium output, in a Cournot game on the residual demand, that is left after all quantities from previous stages are taken into account.

Proof to Proposition 5.13

When n^* is the number of firms entering in that case, the total quantity is

$$\overline{Q} = \frac{n^*}{1+n^*} \left(A - c\right).$$

The quantity of a single firm

$$q^* = \frac{A-c}{1+n^*}.$$

Hence, the profit of the last firm entering is

$$\pi^* = \left(A - c - \frac{n^*}{1+n^*} \left(A - c\right)\right) \frac{A-c}{1+n^*} = \left(\frac{A-c}{1+n^*}\right)^2$$

Thus, n^* is defined by

$$\left(\frac{A-c}{1+n^*}\right)^2 = F$$

$$n^* = \frac{A-c}{\sqrt{F}} - 1$$

Given n^* the equilibrium quantities can be calculated

$$q^* = \frac{A-c}{1+\frac{A-c}{\sqrt{F}}-1} = \sqrt{F}$$

$$\overline{Q} = \frac{\frac{A-c}{\sqrt{F}}-1}{1+\frac{A-c}{\sqrt{F}}-1}(A-c) = A-c-\sqrt{F}$$

Proof to Proposition 5.14

The quantity of the patentee is always the same $Q^1(1) = \frac{1}{2}(A-c)$. The total quantity of the licensees is $Q^2(L) = \frac{L}{2+2L}(A-c)$. The quantity of the non-licensees is $Q^3(n)$ and depends on the number of non-licensees. The number of non-licensees E is defined by

$$\overline{Q} - Q^1(1) - Q^2(L) = Q^3(E)$$

$$A-c-\sqrt{F} - \frac{1}{2}(A-c)(1+\frac{L}{1+L}) = Q^3(E)$$

$$A-c-\sqrt{F} - (A-c)\frac{1+2L}{2+2L} = Q^3(E)$$

$$(A-c)\left(1-\frac{1+2L}{2+2L}\right) - \sqrt{F} = Q^3(E)$$

$$(A-c)\frac{1}{2+2L} - \sqrt{F} = E\sqrt{F}$$

$$E = \frac{A-c}{2\sqrt{F}}\frac{1}{1+L} - 1 \tag{5.7}$$

In order to maximize total surplus the patentee minimizes the total number of firms in the market

$$\min_L L + E(L) = \min_L L + \frac{A-c}{2\sqrt{F}}\frac{1}{1+L} - 1$$

The associated first order condition is given by

$$1 - \frac{A-c}{2\sqrt{F}}\frac{1}{(1+L)^2} = 0.$$

Solving this yields

$$L^* = \sqrt{\frac{A-c}{2\sqrt{F}}} - 1.$$

Substituting L^* in (5.7) yields $L^* = E(L^*)$.

Chapter 6

Summary

The dissertation deals with the market and welfare effects of different business practices and the firm's incentives to use them: resale price maintenance, revenue sharing of a platform operator, membership fees to buyers using a platform and patent licensing.

In the second chapter we investigate the incentives of two manufacturers with common retailers to use resale price maintenance (RPM). Retailers provide product specific services that increase demand and manufacturers use minimum RPM to compete for favorable services for their products. Minimum RPM increases consumer prices and can create a prisoner's dilemma for manufacturers without increasing, and possibly even decreasing the overall service level. If manufacturer market power is asymmetric, minimum RPM tends to distort the allocation of sales services towards the high-priced products of the manufacturer with more market power. These results challenge the service argument as an efficiency defense for minimum RPM.

The third chapter deals with trade platforms whose operators not only allow third party sellers to offer their products to consumers, but also offer products themselves. In this context, the platform operator faces a hold-up problem if he uses classical two-part tariffs only (which previous literature on two-sided markets has focused on) as potential competition between the platform operator and sellers reduces platform attractiveness. Since some sellers refuse to join the platform, some products that are not known to the platform operator will not be offered at all. We discuss the effects of different platform tariffs on this hold-up problem. We find that revenue-based fees lower the platform operator's incentives to compete with sellers, increasing platform attractiveness. Therefore, charging such proportional fees can be profitable, what may explain why several trade platforms indeed charge proportional fees.

The fourth chapter investigates the optimal tariff system in a model in which buyers are heterogeneous. A platform model is presented in which transactions are modeled explicitly and buyers can differ in their expected valuations when they decide to join the platform. The main effect that the model identifies is that the participation decision sorts buyers according to their expected valuations.

This affects the pricing of sellers. Furthermore diffing form the usual approach, in which buyers are ex-ante homogenous, the platform does not internalize the full transaction surplus. Hence it does not implement the socially efficient price on the platform, also it has control of the price with the transaction fee.

The fifth chapter investigates the effects of licensing on the market outcome after the patent has expired. In a setting with endogenous entry, a licensee has a head start over the competition which translated into a first mover advantage if strategies are strategic substitutes. As competitive strategies quantities and informative advertising are considered explicitly. We find that although licensing increases the joint profit of the patentee and licensee, this does not necessarily come from a reduction in consumer surplus or other firms profits. For the case of quantity competition we show that licensing is welfare improving. For the case of informative advertising, however, we show that licensing increases prices and is thus detrimental to consumer surplus.

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