The use of the Hypercube Queueing Model for the location optimization decision of Emergency Medical Service systems

Dissertation

zur Erlangung des akademischen Grades Dr. rer. pol.
von der Wirtschaftswissenschaftlichen Fakultät der
Julius-Maximilians-Universität Würzburg

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Master of Science
aus Würzburg

2021

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Acknowledgements

This dissertation would not have been possible without the continued support and encouragement by many people over the last few years.

First, I want to thank my first supervisor, Prof. Dr. Ronald Bogaschewsky, who supported my research during every step. I am extremly grateful for the academic and personal freedom he provided. Additionally, I want to thank my second supervisor, Prof. Dr. Rainer Lasch, for the fast preparation of the second opinion on this thesis.

Furthermore, I want to give a very special thank you to two colleagues at the chair for business management and industrial management. Many thanks to Jennifer Keidel who not only was an excellent project partner but also provided me with the necessary freedom to write this dissertation. A very special thank you also to Benjamin Siller who offered tremendous support and was available for any discussion at any time, not always necessarily on optimization topics.

Additionally, I want to thank Michelle Kraus for the hourless library sessions and the mental support, especially during the last stages. I would also like to take the chance to express my gratitude to my parents, Dagmar and Reinhard, who layed the foundation for my academic career.

Last, but not least, I want to express my gratefulness to my beloved wife, Lena. Thank you so much for the countless hours of proof-reading this dissertation, your endless support and for always standing by my side. I dedicate this work to you!

Contents

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List of Abbreviations

A-HQM	Approximate Hypercube Queueing Model by LARSON
AT-HQM	Approximate Hypercube Queueing Model by ATKINSON ET AL. (2006)
AT-HQM2	Approximate Hypercube Queueing Model by ATKINSON ET AL. (2008)
AYA-HQM	Approximate Hypercube Queueing Model by ANSARI, YOON AND ALBERT
ACO	Ant-Colony-Optimization
ACPE	Average calculations per experiment
ADPOS	Average deviation to ‘pseudo-optimal’ solution
AERT	Average Expected Response Time
AIBSF	Average iteration best solution found
ALS	Advanced Life Support
B-HQM	Approximate Hypercube Queueing Model by BATTA, DOLAN AND KRISHNAMURTHY
BIE-HQM	Approximate Hypercube Queueing Model by BUDGE, INGOLFSSON AND ERKUT
BLS	Basic Life Support
D	Deterministic
DSM	Double Standard Model
E_K	Erlang distribution with K as shape parameter
EMS	Emergency Medical Service
FCFS	First-Come-First-Served
FSFP	First-Server-Free-Percentage
FLEET	Facility Location Equipment Emplacement Technique

G-HQM	Approximate Hypercube Queueing Model by GEROLIMINIS, KEPAPTSOGLOU AND KARLAFTIS
G	General
GA	Genetic Algorithm
GD	General Distribution
H_K	Mixture of K exponentials
HQM	Hypercube Queueing Model
K-HQM	Approximate Hypercube Queueing Model by KARIMI, GENDREAU AND VERTER
LCFS	Last-Come-First-Served
LRT	Longest Response Time
M	Exponential distribution
MALP	Maximum Availability Location Problem
MCLP	Maximal Covering Location Problem
MEXCLP	Maximal Expected Covering Location Problem
MR	Model Robustness
MSLM	Maximum Survivability Location Model
NCSO	Newly calculated solutions per optimization
NHS	National Health Service
OF	Objective Function
PH	Phase Type
PR	Priority
PSCLP	Partial Set Covering Location Problem
RGP	Robust Goal Programming
RQ	Research Question

RSS	Random Selection
RTCov	Percent of demand points that can be covered under 9 minutes
SCLP	Set Covering Location Problem
SR	Solution Robustness
TEAM	Tandem Equipment Allocation Model
USO	Unique solutions per optimization
WBM	Workload-Balance-Metric

Directory of Variables

The directory of variables lists the used variables with respect to the chapter in which they are initially introduced.

Chapter 2.2.1

λ	Arrival rate
μ	Service rate
$N(t)$	Stochastic process at time t
$Pr\{x\}$	Probability of event x
A	Constant
T	Time until an event occurs
p_n	Probability mass function (of state n)
$p_n(t)$	Probability that n events have occurred at time t
X_n	State n
p_{ij}	Probability of transition between states i and j
p_{ij}^n	Probability of transition between states i and j for step n
$a(t)$	Interarrival times
$b(t)$	Service times
L	Number of customers
L_q	Number of customers in queue
W	Average waiting time
W_q	Average time delay

Chapter 2.2.2

Sets

$n \in N$	Set of servers to be located
$j \in J$	Set of demand points
$B_a \in N_B^n$	Set of states in which server n is unavailable

Variables

C_N	Vertex of the N-dimensional hypercube
f_j	Fraction of demand point j on the overall demand
p_{nj}	Fraction of dispatches server n sends to demand point j
t_{nj}	Travel time of server n to demand point j
w_n	Workload of server n
λ_{ab}	Upward transition rate between state a and state b
μ_{ab}	Downward transition rate between state a and state b
B_a, B_b	State a and state b of the N-dimensional hypercube
$P\{B_a\}$	Steady-state probability of state a
d^+_{ab}, d^-_{ab}	Upward and downward hamming distances between states a and b
$PLoss_j$	Loss probabilities for demand point j
PLoss	System-wide loss probabilities
$AERT_j$	Average expected response time for demand point j
AERT	System-wide average expected response time

Chapter 3.1

Sets

$d \in D$	Level of districting
$j \in L_{ab}$	Set of demand points that can lead to a downward transition from state a to state b

Variables

T_j	Ordered vector for demand point j with length of D (preference list)
w_l and w_k	Nearest and d-nearest server
D^d_{lk}	Sub area for districting level d and the combination of server l and k

λ_{lk}^{d}	Demand of the D_{lk}^{d} sub area
λ_p and μ_p	Term p of the weighted average transition rate formulation
$\emptyset'_{nj}$	Incident handling rate of server n for demand point j
T	Time period
$\emptyset$	Incident handling rate
t_{nj}	Travel time of server n to demand point j
f_j	Fraction of demand for demand point j

Chapter 3.2

Sets

$i \in I$	Set of candidate server locations
$n \in N$	Set of servers to be located
$i \in W_j$	Set of servers covering demand point j

Variables

y_j	Binary variable that shows whether demand point j is covered
y_j^{cov}	Binary variable that shows whether demand point j can be covered within a pre-defined time constraint

Decision Variable

x_i	Binary variable that shows whether candidate server location i is chosen

Chapter 4.1

Goal Programming

Sets

$g \in G$	Set of objectives considered

Variables

Goal Programming Notation

W_g^+, W_g^-	Positive and negative weight of objective g
δ_g^+, δ_g^-	Positive and negative deviations of objective g
d_g	Deviation of objective g
g_g	Aspiration level of objective g
$f_g(x)$	Realization of objective g

Goal Programming with Satisfaction Functions

$F_g^+(\delta_g^+), F_g^-(\delta_g^-)$	Value of the satisfaction function for the positive and negative deviation of objective g
$\alpha_{gv}^+, \alpha_{gv}^-$	Positive and negative veto thresholds for objective g
δ	Deviation
α_1, α_2	Threshold and veto values

Performance Metrics

Sets

$B_a \in N_j$	Set of states in which no server is available for demand j
$B_a \in E_{nj}$	Set of states in which server n is the nearest-available for demand point j
$j \in N^T$	Set of demand points j that can be covered subject to a time constraint

Variables

$PLoss_j$	Loss probabilities of demand point j
p_{nj}	Fraction of dispatches server n sends to demand point j
t_{nj}	Travel time from server n to demand point j
$ytime_j^{high}$	Binary variable that shows whether demand point j

	can be covered subject to a time constraint
RTCov	Fraction of demand points that can be covered under a time constraint
LRT	Longest response time
WBM	Workload-Balance-Metric

Chapter 4.2

Sets

$\Omega = \{1, 2, \ldots, S\}$	Set of scenarios
$j \in J^s$	Set of scenario-dependent demand points
$i \in W_j^s$	Set of servers covering demand point j for scenario s
$B_a \in E_{nj}^s$	Set of states in which server n is the nearest-available for demand point j for scenario s
$B_a \in N_j^s$	Set of states in which no server is available for demand point j for scenario s

Variables

X	Vector of decision variables
Y	Vector of control variables
A, B, C	Parameter matrices
b, e	Vector of parameters
d_s, B_s, C_s, e_s	Scenario-dependent realizations of parameters
p_s	Probability of scenario s
ϑ	Weighting parameter for solution robustness
ψ_s	Scenario-dependent realization of the objective function for scenario s
θ_s	Scenario-dependent deviation for scenario s
ω	Weighting parameter for the model robustness
T_j^s	Ordered vector for demand point j with length of D (preference list) for scenario s
w_l^s, w_k^s	Nearest and d-nearest server for scenario s

$D_{lk}^{d,s}$	Sub area for districting level d and the combination of server l and k for scenario s
$\lambda_{lk}^{d,s}$	Demand of the $D_{lk}^{d,s}$ sub area for scenario s
λ^{s}	Demand for scenario s
λ_{ab}^{s}	Upward transition rate between state a and state b for scenario s
T^{s}	Time period in scenario s
$\phi_{nj}^{',s}$	Incident handling rate of server n for demand point j and scenario s
t_{nj}^{s}	Travel time for server n to demand point j for scenario s
L_{ab}^{s}	Set of demand points that can lead to a downward transition from state a to state b for scenario s
$\phi_{nj}^{',s}$	Incident handling rate of server n and demand point j for scenario s
f_{j}^{s}	Fraction of demand for demand point j and scenario s
μ_{ab}^{s}	Downward transition rate between state a and state b for scenario s
$P^{s}\{B_a\}$, $P^{s}\{B_b\}$	Steady-state probability of state a and b for scenario s
$Ploss_{j}^{s}$	Loss percentages of incoming demand calls for demand point j and scenario s
$y_{j}^{cov,s}$	Binary variable that shows whether demand point j can be covered within a pre-defined time constraint for scenario s
p_{nj}^{s}	Fraction of dispatches server n sends to demand point j in scenario s
δ_{s}	Violation parameter for model robustness and scenario s

Chapter 4.3

Sets

$j \in N^{Ts}$	Set of demand points j that can be covered subject to

	a time constraint for scenario s

Variables

$AELP^s$	Average expected loss probability for scenario s
$AERT^s$	Average expected response time for scenario s
$RTCov^s$	Fraction of demand points that can be covered under a time constraint for scenario s
$ytime_j^s$	Binary variable that shows whether demand point j can be covered subject to a time constraint for scenario s
LRT^s	Longest response time for scenario s
p_n^s	Workload of server n for scenario s
WBM^s	Workload-Balance-Metric for scenario s

Robust Goal Programming Notation

W_{gs}^{+}, W_{gs}^{-}	Positive and negative weight of objective g for scenario s
$\delta_{gs}^{+}, \delta_{gs}^{-}$	Positive and negative deviations of objective g for scenario s
g_{gs}	Aspiration level of objective g for scenario s
$F_g^{+}(\delta_{gs}^{+}), F_g^{-}(\delta_{gs}^{-})$	Value of the satisfaction function for the positive and negative deviation of objective g for scenario s
$f_g(x^s)$	Realization of objective g for scenario s
$\alpha_{gvs}^{+}, \alpha_{gvs}^{-}$	Positive and negative veto thresholds for objective g and scenario s

Chapter 6.1

Sets

$i \in I$	Set of solution components (candidate server locations)
$s_{\mathbf{n}} \in S$	Set of candidate solutions (base population)
$s_n \in S^{New}$	Set of candidate solutions (extended population)

$g \in G$	Set of generations
$c_i \in N$	Number of solution components required (server locations)

Variables

fit_s	Fitness value of solution s
p_s	Probability of candidate solution s to be chosen for reproduction
p_c	Crossover probability
p_m	Mutation probability

Chapter 6.2

Sets

$i \in I$	Set of solution components (candidate server locations)
$s_a \in S$	Set of candidate solutions
$s_a \in S_{upd}$	Set of best solutions that are used to update the probabilities
$c_i \in N$	Number of solution components required (server locations)

Variables

τ_i	Probability of candidate server location i
ρ	Evaporation percentage
ω_s	Weight of solution s
F(s)	Quality function of solution s

Chapter 6.3

Sets

$x_i, x_i^s \in I$	Set of decision variables (candidate server locations)

$x_i^s \in X_{s^{new}}^{Mem}$	Set of cached decision variables
$s \in S^{Mem}$	Set of cached solutions in the memory
$p \in P^s$	Set of performance metrics for solution s
$pm_p^s \in PM_p^s$	Set of cached performance metrics for solution s

Variables

s^{new}	Newly proposed solution
$pm_p^{s^{new}}$	Newly calculated performance metric p for solution s^{new}
x^{skip}	Binary variable that shows whether the current solution is already in the memory

German summary

Die strategische Planung von medizinischen Notfallsystemen steht in unmittelbarem Zusammenhang mit der Überlebenswahrscheinlichkeit betroffener Patienten. Die Standortplanung hat einen wesentlich Einfluss auf wichtige Kenngrößen solcher Systeme. Die akademische Forschung hat daher in den letzten Jahren zahlreiche modelltheoretische Ansätze entwickelt, die eine wissenschaftlich gestützte Planung und Operation unterstützen sollen. Gleichermaßen unterliegen die meisten dieser Systeme, aufgrund ihrer Verantwortlichkeit für eine schnelle und zielgerichtete Notfallversorgung, strengen gesetzlichen Vorgaben. Auch an dieser Stelle hat die Forschung zahlreiche Kenngrößen und Evaluationsparameter entworfen, die zur Bewertung verwendet werden können. Darunter fallen beispielsweise die Reaktionszeit, die Systemauslastung, diverse Wartezeitenparameter sowie der Anteil der Nachfrage, der nicht unmittelbar bedient werden kann.

Aufgrund der deutlich zunehmenden verfügbaren Rechenkapazität aktueller Computer sowie der Entwicklung von realitätsnäheren Modelltheorien können moderne Ansätze die Gegebenheiten von medizinischen Notfallsystemen in einem höheren Maße berücksichtigen. Dies beinhaltet beispielsweise die Berücksichtigung von Unwägbarkeiten und Unsicherheiten. Dabei ist das Hypercube Queueing Model eines der am häufigsten verwendeten Modelle. Mit Hilfe von Prinzipien der Warteschlangentheorie kann es verwendet werden um deskriptive Charaktereigenschaften des zu analysierenden Systems zu berechnen. Aufgrund seines theoretischen Hintergrundes und der damit verbundenen hohen notwendigen Rechenzeiten wurde das Hypercube Queueing Model erst in der jüngeren Vergangenheit häufiger zur Optimierung von medizinischen Notfallsystemen verwendet. Gleichermaßen wurden nur wenige Systemparameter mit Hilfe des Modelles berechnet und das volle Potenzial demnach noch nicht ausgeschöpft. Ebenso berücksichtigen die vorhandenen Studien nur Praxisbeispiele von mittlerer Größe. Die vorliegende Arbeit soll daher einen Beitrag zur Beantwortung der nachgenannten Forschungsfragen liefern.

Forschungsfrage 1: Welche Kriterien und Ziele sind relevant für die Gestaltung von medizinischen Notfallsystemen und welchen Einfluss haben diese auf die resultierende Entscheidung?

Forschungsfrage 2: Wie kann ein ganzheitlicher Ansatz formuliert sein und welche zusätzlichen Einblicke kann er bieten?

Forschungsfrage 3: Welche Maßnahmen können unternommen werden, um die Rechenbelastungen durch das Hypercube Queueing Model signifikant zu reduzieren?

In Kapitel 2 werden die notwendigen Hintergründe von medizinischen Notfallsystemen besprochen sowie die relevante Literatur mit Blick auf das Hypercube Queueing Model, allgemeine Modelle zur Standortplanung von medizinischen Notfallsystemen sowie Modelle zur Berücksichtigung von multi-kriterialen Entscheidungen und Entscheidungen unter Unsicherheit vorgestellt. Kapitel 3 beinhaltet die Charakterisierung und Konkretisierung des Hypercube Queueing Models. In Kapitel 4 werden die Modelltheorien des Goal Programmings, der robusten Optimierung sowie des robusten Goal Programmings unter Einbettung eines Hypercube Queueing Models erweitert. Kapitel 5 stellt die verwendete Fallstudie vor. In Kapitel 6 werden der genetische Algorithmus sowie die Ant-Colony-Optimization als Lösungstechniken auf das Optimierungsproblem angepasst sowie eine Strategie zur dynamischen Sicherung von bereits berechneten Lösungen entwickelt. Kapitel 7 beschreibt die computergestützten Auswertungen der entwickelten Modelle und geht dabei auf die unterschiedlichen Lösungstechniken sowie die entwickelten Modelle des vierten Kapitels ein.

Die entwickelten Forschungsfragen lassen sich aufgrund der gewonnenen Erkenntnisse wie folgt beantworten:

Forschungsfrage 1:

Die meisten der bereits vorhandenen Studien im Bereich der Optimierung unter Zuhilfenahme eines Hypercube Queueing Modells nutzen die zu erwartende Reaktionszeit des Systems als Zielparameter. Obwohl die Verwendung von diesem eine zumeist ausgeglichene Systemkonfiguration zur Folge hat, wurden andere Zielparameter identifiziert. Durch diese können, in Abhängigkeit der Präferenzen des Entscheidungsträgers, zahlreiche Vorteile in der Systemkonfiguration erzielt werden. Weitergehend wurden unterschiedliche Zielkonflikte zwischen den einzelnen Parametern identifiziert und analysiert.

Forschungsfrage 2:

Die Verwendung des Hypercube Queueing Modells in den Modellen der robusten Optimierung sowie des robusten Goal Programmings haben versucht einen ganzheitlicheren Blick, durch die Verwendung von unterschiedlichen Tageszeiten, zu offerieren. Dabei hat sich gezeigt, dass das Verhalten von medizinischen Notfallsystemen sowie die Parameter stark von diesen abhängen. Daher sollte die Analyse und gegebenenfalls Optimierung dieser Systeme unterschiedliche Verteilungen der Nachfrage, in Abhängigkeit ihrer Menge und räumlichen Verteilung, unbedingt berücksichtigen um eine möglichst ganzheitliche Entscheidungsgrundlage zu garantieren.

Forschungsfrage 3:

Die Rechenzeiten des Hypercube Queueing Models bleiben eine der am meisten limitierenden Faktoren bei dessen Verwendung. Insbesondere durch die Verwendung eines szenario-basierten Ansatzes sind hohe Rechenzeiten zu erwarten. Durch den Vergleich der eingesetzten Lösungstechniken konnte die Ant-Colony-Optimization als ein hilfreiches Werkzeug identifiziert werden, da sie im Durchschnitt zu mindestens vergleichbare Lösungen in weniger Rechenzeit zur Folge hat. Die Strategie zur dynamischen Sicherung von bereits berechneten Lösungen hat sich darüber hinaus als hilfreich erwiesen, um die hohen Rechenzeiten kontrollierbar zu halten.

English summary

The strategic planning of Emergency Medical Service systems is directly related to the probability of surviving of the affected humans. The facility location planning has a direct influence on key performance metrics of such systems. Academic research has developed numerous models and theories over the past years which are aimed at supporting the scientific planning and operation. Most of these systems are subject to legally binding guidelines due to their responsibility for fast and effective emergency services. Academic research has also contributed to the evaluation of these systems by defining a variety of key performance metrics. The average response time, the workload of the system, several waiting time parameters as well as the fraction of demand that cannot immediately be served are among the most important examples.

Modern approaches can consider the specific conditions of Emergency Medical Service systems to a higher degree due to the significantly rising computational power of modern computers as well as the development of more realistic models. This for example includes the inclusion of uncertainty concerns. The Hypercube Queueing Model is one of the most applied models in this field. With the help of Queueing Theory principles, it can be employed to compute descriptive parameters of the analyzed system. Due to its theoretical background and the implied high computational times, the Hypercube Queueing Model has only been recently used for the optimization of Emergency Medical Service systems. Likewise, only a few system performance metrics were calculated with the help of the model and the full potential therefore has not yet been reached. Moreover, existing studies only consider case studies of medium size. This study aims at contributing to the research questions mentioned below:

Research Question 1: Which criteria and objectives are relevant for the design of an EMS system and which influence do they exercise on the resulting decision?

Research Question 2: How can a holistic approach be formulated and which additional insights can it offer?

Research Question 3: Which measures can be taken in order to significantly reduce the computational burdens of the HQM in a location optimization study?

Chapter 2 discusses the necessary backgrounds of Emergency Medical Service systems

as well as the relevant literature with regard to the Hypercube Queueing Model, common models for the facility location problem of Emergency Medical Service systems as well as models that include uncertainty concerns as well as multi-objective facility location models. Chapter 3 characterizes the Hypercube Queueing Model. The Hypercube Queueing Model is embedded into the Goal Programming, Robust Optimization as well as Robust Goal Programming in chapter 4. The case study is described in chapter 5. In chapter 6 the Genetic Algorithm as well as the Ant-Colony-Optimization are applied to the optimization problem. Additionally, the Dynamic Caching Strategy is developed that makes use of already calculated solutions. The computational results of the developed models are given in chapter 7 and consider the different solution techniques as well as the models of chapter 4.

With regard to the derived findings, the research questions can be answered as follows:

Research Question 1:

Most of the existing studies in the field of optimization with the help of a Hypercube Queueing Model apply the expected response time of the system as their objective function. While it leads to oftentimes balanced system configurations, other influencing factors were identified. Through their use and in conjunction with the preferences of the decision maker, numerous advantages in the system configuration can be reached. Furthermore, several conflicting goals between the individual parameters were identified and analyzed.

Research Question 2:

The embedding of the Hypercube Queueing Model in the Robust Optimization as well as the Robust Goal Programming intended to offer a more holistic view through the use of different day times. It was shown that the behavior of Emergency Medical Service systems as well as the corresponding parameters are highly subjective to them. The analysis and optimization of such systems should therefore consider the different distributions of the demand, with regard to their quantity and location, in order to derive a holistic basis for the decision-making.

Research Question 3:

The computational times of the Hypercube Queueing Model remain one of the most limiting factors during its use. Especially the use of a scenario-based approach leads to high computational times. The comparison of the applied solution techniques has identified the Ant-Colony-Optimization as a helpful tool, because it could on average offer at least comparable solutions in less computational time. The Dynamic Caching Strategy has furthermore contributed towards controlling the high computational times.

1. Introduction

The design, planning and subsequent management of Emergency Medical Service (EMS) systems has gained considerable attention from both the academic and the practitioner perspective over the last few years. As for other fields, the strategic location decision has a significant influence on the behavior and performance of such systems. Academic literature has contributed to this problem by developing a large amount of models aimed at the facility location problem. Those models are usually solved by using either quantitative or qualitative research methods from the broad field of operations research.[1]

EMS systems are considered to be essential lifelines during the initial phase of emergency response to emergency (medical) demands. Due to the important nature of a fast and reliable emergency response, government authorities intend to ensure a minimum level of service to be provided by these systems at any given time. Therefore, legally binding minimum threshold criteria for the operation of EMS systems are defined and effective in most countries.[2] For example, the National Health Service (NHS) of the United Kingdom requires that life-threatening emergency demands be responded to within 8 minutes in 75 % of the cases.[3] An extra 60 seconds are allowed for potentially life-threatening incidents.[4]

Normally, a number of different criteria has to be met in the design of EMS systems. The trade-offs between the individual criteria then exercise significant influence on the decisions that are taken at the initial design and planning phase. The most common examples for such criteria are the average response time of the system to an emergency medical demand, the utilization of the system, the fraction of demand that cannot (immediately) be met, the waiting time, equity-related concerns or, with less relevance due to the scope of EMS systems, cost and other budget related constraints.[5]

The recent advances in computational power as well as in the development of more sophisticated models allow for a more detailed and realistic approach when it comes to the strategic location decision for EMS systems. It has to be considered that the location decision for EMS systems frequently incorporates both strategical and tactical concerns as

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[1] Cf. FARAHANI, R. Z. ET AL., 2019, p. 1
[2] Cf. AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018, p. 134
[3] Cf. NHS, 2017
[4] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 188
[5] Cf. FARAHANI, R. Z. ET AL., 2019, p. 1; AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018, pp. 134–135; ENAYATI, S. ET AL., 2019, p. 2

noted by BÉLANGER, RUIZ AND SORIANO[6] and REUTER-OPPERMANN, VAN DEN BERG AND VILE[7]. While the very early models focus on deterministic inputs, modern approaches also account for the unpredictability and uncertainty of emergency demands in quantity and location.[8] One of the most common approaches for the simulation of the behavior of an EMS system with respect to varying sets of inputs is the Hypercube Queueing Model (HQM) by LARSON.[9] The main benefit of applying Queueing Theory is the ability to define and calculate several system-specific performance metrics, like the response time or the loss behavior of the system. Additionally, it can be used to incorporate relatively complex policies for the operation of an EMS system.[10] ARINGHIERI ET AL. describe the HQM as "… a viable alternative to simulation.".[11] Due to its Queueing Theory based approach, the HQM is often criticized as being be computationally prohibitive. This is especially relevant in a location optimization approach for which a higher number of solution candidates have to be calculated in order to find an adequate solution. Nonetheless, the HQM has been embedded in optimization procedures in more recent approaches by, for example, IANNONI, MORABITO AND SAYDAM[12], GEROLIMINIS, KARLAFTIS AND SKABARDONIS[13], TORO-DÍAZ ET AL.[14], AKDOĞAN, BAYINDIR AND IYIGUNG[15] and ENAYATI ET AL.[16].

This work intends to contribute to the field of HQM-based location optimization decisions for EMS systems by developing three separate models that cannot only be used for their optimization potential alone but also for gaining significant insights into the behavior of the system with respect to various scenarios. The use of the HQM is especially motivated by the already mentioned possibility to calculate reliable performance metrics that can then be used in the optimization process. Another motive is given by the fact that the HQM has not yet been applied in a larger-scale case study. The former allows for the meaningful definition and incorporation of various objectives into an EMS system location optimization study while the latter can offer new insights into the behavior of a larger-scale EMS system. In order to overcome this existing research gap, this study will use the

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6 Cf. BÉLANGER, V.; RUIZ, A.; SORIANO, P., 2019, p. 5
7 Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 193
8 Cf. GEROLIMINIS, N.; KEPAPTSOGLOU, K.; KARLAFTIS, M. G., 2011, p. 288
9 Cf. LARSON, R. C., 1974
10 Cf. CHIYOSHI, F. Y.; IANNONI, A. P.; MORABITO, R., 2011, p. 272
11 ARINGHIERI, R. ET AL., 2017, p. 356
12 Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008
13 Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009
14 Cf. TORO-DÍAZ, H. ET AL., 2013
15 Cf. AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018
16 Cf. ENAYATI, S. ET AL., 2019

city of Munich as a real-world case.

In the following three research questions (RQs) are formulated in order to guide the reader through this work.

RQ1: Which criteria and objectives are relevant for the design of an EMS system and which influence do they exercise on the resulting decision?

RQ2: How can a holistic approach be formulated and which additional insights can it offer?

RQ3: Which measures can be taken in order to significantly reduce the computational burdens of the HQM in a location optimization study?

The study is organized as follows:

A brief overview over the relevant fundamentals of EMS systems and their design and operation is provided in chapter 2. Moreover, chapter 2 contains a review of the literature on basic location optimization models, multi-objective optimization models and uncertainty in EMS system location optimization models. Additionally, some relevant Queueing Theory fundamentals as well as the basic HQM are discussed. Then, the literature on HQM models is reviewed, and the research gap and research contribution are characterized.

The HQM, which will subsequently be embedded into several optimization techniques, is developed in chapter 3. The extensions for the Hypercube Queueing Location Models are presented in chapter 4. The Goal Programming using an embedded HQM is discussed in chapter 4.1, the Robust Optimization using an embedded HQM in chapter 4.2, and the joint Robust Goal Programming approach in chapter 4.3. The case study and the relevant study areas are introduced in chapter 5.

The Ant-Colony-Optimization (ACO) is introduced as an alternative metaheuristic to the Genetic Algorithm (GA) in chapter 6. Additionally, a caching strategy is provided that seeks to make use of already calculated solutions.

In order to validate the results of the ACO against the GA, several metaheuristic-related experiments are conducted in chapter 7.1. The computational results for the three models discussed in chapter 4 and the Munich case study are given in chapters 7.2 to 7.6.

A summary of this work is provided in chapter 8. Also, its relevance is discussed with respect to the academic and practitioner community. Additionally, the relevant drawbacks and caveats are given and some areas of future research are identified.

2. Literature Review

2.1 Emergency Medical Service Systems

The following chapter treats the general definition (chapter 2.1.1), the planning levels for EMS systems (chapter 2.1.2), the operational characteristics of EMS systems (chapter 2.1.3), the relevant objectives and performance metrics (chapter 2.1.4) as well as the concepts of 'districting' (chapter 2.1.5) and 'dispatch policies' (chapter 2.1.6) relevant for this work.

2.1.1 Definition

The EMS system describes a system that provides medical services to people in immediate need. In a medical context, a definition of EMS sometimes includes the direct patient treatment in a hospital setting and is not limited to the on-scene treatment in the case of an incident or the subsequent transportation. SORIYA AND COLWELL[17] describe it as "… an all-encompassing term for acute patient care in a variety of settings, including the emergency department, urgent care clinic, pre-hospital setting, and disaster situations.". The NATIONAL HIGHWAY TRAFFIC SAFETY ADMINISTRATION state: "Emergency Medical Services, more commonly known as EMS, is a system that provides emergency medical care. Once it is activated by an incident that causes serious illness or injury, the focus of EMS is emergency medical care of the patient(s).". [18]

In the broader context of operations research, the term EMS mostly refers to services that are provided until the patient enters the hospital. "EMS refers to the provision of out-of-hospital acute medical care and the transport of patients to hospital for definitive care", as described in the definition of INGOLFSSON[19]. MAYORGA ET AL. aim at the responsibility of EMS systems for the overall patient survival: "The major focus of (a) Emergency Medical Service (EMS) system is to save lives and to minimize the effects of emergency health incidents.".[20] TORO-DÍAZ ET AL. characterize EMS as "… a public service that provides out-of-hospital acute care and transport to a place of definitive care, to patients with illnesses and injuries that constitute a medical emergency."[21]. CHANTA ET AL. include the first response character in their definition: "EMS systems are public services that often

[17] SORIYA, G.; COLWELL, C., 2012, p. 839
[18] NATIONAL TRAFFIC HIGHWAY SAFETY ADMINISTRATION, 2020
[19] INGOLFSSON, A., 2013, p. 105
[20] MAYORGA, M. E.; BANDARA, D.; MCLAY, L. A., 2013, p. 39
[21] TORO-DÍAZ, H. ET AL., 2013, p. 917

provide the first line of response to urgent health care needs within a community.”.[22] ARINGHIERI ET AL.[23] choose a broader definition: “Emergency medical service is one of the most important health care services as it plays a vital role in saving people’s lives and reducing the rate of mortality and morbidity”. For the general field of emergency services, which, for example, also includes fire stations, AKDOĞAN, BAYINDIR AND IYIGUN formulated the following definition: “Emergency services are systems that exist to provide a rapid response to people in need of services such as rescue, fire fighting, on-site medical care or transportation to definitive care.”.[24]

As seen from the definitions above, the term EMS mostly refers to medical services that are provided in the case of a serious illness or injury up until the respective patient is transported to any downstream medical facility. For the current work, EMS shall be defined as services that are provided to emergency medical incidents from the first response at the scene of the incident up until the potential subsequent transportation to a hospital or other medical facility. It should be noted that the abbreviation “EMS” is sometimes used with the meaning ‘Emergency Medical System’ in the academic literature. In this work, the corresponding term shall be ”EMS system” (i.e. Emergency Medical Services System).

2.1.2 Planning Levels for Emergency Medical Service Systems

As for other organizations, the planning levels for EMS systems can be subdivided into strategic, tactical and operational planning. All three levels are briefly characterized below.

Strategic Level

Decisions taken at the strategic level usually define the long-term future of the respective organization and can, if at all, only be changed under extensive use of resources. Therefore, the strategic level consists of decisions regarding the long-term structure of the respective organization, with a planning horizon of at least one year. In the case of EMS systems, the strategic level includes decisions regarding the construction of new locations, the assignment of (geographical) areas of responsibility to the locations, emergency vehicles and staff hiring as well as a long-term demand forecast and analysis.[25]

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[22] CHANTA, S.; MAYORGA, M. E.; MCLAY, L. A., 2014, p. 133
[23] ARINGHIERI, R. ET AL., 2017, p. 349
[24] AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018, p. 134
[25] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 193

Tactical Level

Decisions taken at the tactical level have a planning horizon of several months up to one year. Therefore, they can be changed, if necessary, more easily than decisions on the strategic level. For the planning of EMS systems, tactical decisions could be the identification of the current number of necessary emergency vehicles, the crew scheduling and rostering or shift assignments. As noted by REUTER-OPPERMANN, VAN DEN BERG AND VILE, the EMS system location problem often includes both strategic and tactical concerns.[26]

Operational Level

Decisions taken at the operational level usually have a daily planning horizon or have to be taken in real-time. For the planning of EMS systems, operational decisions can include the relocation of emergency vehicles, the assignment of crews to the emergency vehicles or the dispatching of emergency vehicles to incoming demands. Additionally, operational decisions for EMS systems usually have to take into account the congestion of the system and the resulting potential unavailability of certain or all emergency units and act accordingly.[27]

Table 1 below summarizes the strategic, tactical and operational planning levels for EMS systems.

[26] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 193
[27] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 193

Planning Levels for Emergency Medical Service Systems		
Planning Level	**Time Horizon**	**Decisions**
Strategic	One year or longer	Location Areas of responsibility Staff hiring Demand forecast and analysis
Tactical	Monthly, up to one year	Number of emergency vehicles Crew scheduling and rostering Shift assignments
Operational	Daily or real-time	Relocation of emergency vehicles Crew assignment Dispatch of emergency vehicles to demand calls

Table 1: Planning levels for EMS systems based on REUTER-OPPERMANN, VAN DEN BERG AND VILE[28]

2.1.3 Operating Characteristics for Emergency Medical Service Systems

As noted by DICK[29] and BÉLANGER ET AL.[30], EMS systems can be divided into two main groups: Anglo-American and Franco-German. While the first group concentrates mostly on the transportation of the patient to a medical facility, the second group also provides extensive medical services directly at the scene of the incident. AL-SHAQSI characterized the Anglo-American system as 'load and go' and the Franco-German system as 'delay and treat'. [31] Note that the following remarks fit for both categories of EMS systems. An EMS system usually consists of call centers, dispatchers, emergency vehicles and paramedics.[32] Accordingly, the process of serving incoming demands follows the procedure of call reception, confirmation and prioritization, emergency unit assignment and subse-

[28] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 194
[29] Cf. DICK, W. F., 2003, pp. 29–35
[30] Cf. BÉLANGER, V.; RUIZ, A.; SORIANO, P., 2019, p. 1
[31] Cf. AL-SHAQSI, S., 2010, p. 320
[32] Cf. GHOLAMI-ZANJANI, S. M.; PISHVAEE, M. S.; TORABI, S. A., 2018, p. 395

quent dispatch and, after that, an update to the available resources, i.e. the available emergency units or vehicles that can respond to a future demand for emergency help. Note that the EMS system decision maker is usually limited to make decisions that affect the prioritization, the dispatch as well as the corresponding structure of the overall system since demand and service (on-scene) cannot be influenced. The process is illustrated in figure 1 below.

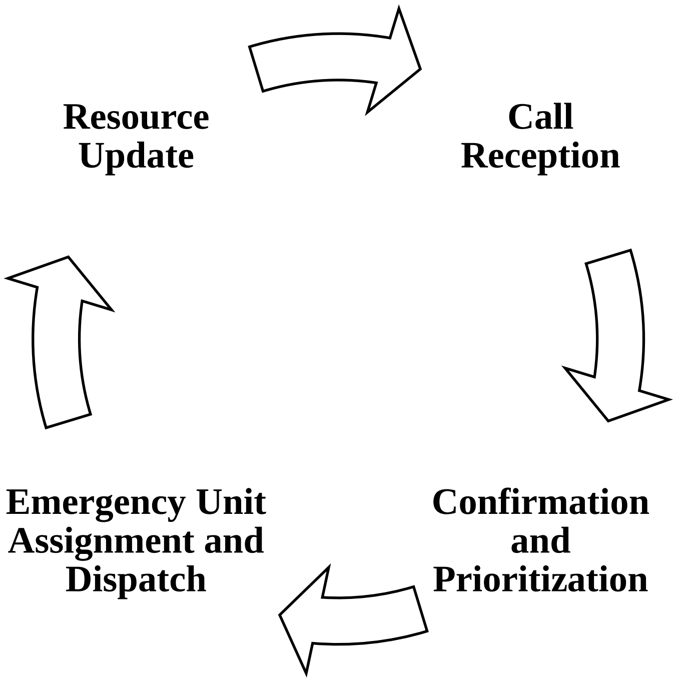

Figure 1: EMS system call process

In order to respond to emergency medical incidents with varying causes, EMS systems may accommodate different types of emergency vehicles. Generally, they can be divided into air and ground units as well as basic life support (BLS) and advanced life support (ALS). BLS units usually cannot offer higher-level treatments or transportation capabilities and their sole use in an EMS system can result in inferior survival rates for example in the case of cardiac incidents. Therefore, they are often paired with ALS units that can offer a more comprehensive level of service and more trained personnel as well as transportation capabilities.[33]

German EMS systems use three different types of emergency vehicles – two types of ambulances for patient rescue and general patient transportation with varying levels of equipment as well as one special vehicle for emergency doctors. The emergency doctor

[33] Cf. BLACKWELL, T. H., 2017, p. 2390

can be part of an ambulance or use a special vehicle in which case he is met by the corresponding ambulance at the emergency scene. This procedure is called 'rendez-vous-system'. In general, one emergency vehicle can only transport one patient and is usually stationed at a hospital or an emergency service station.[34]

The response process for an individual demand call for emergency help may vary with regard to different countries or EMS systems, but usually consists of the following elements according to REPEDE AND BERNARDO[35], SUNG AND LEE[36] and REUTER-OPPERMANN, VAN DEN BERG AND VILE[37]:

Prioritization and emergency unit assignment

After the demand call for emergency help is received, it is prioritized and an emergency unit is assigned.

Chute time and dispatch

Before the emergency unit can be dispatched, usually some (demand-specific) preparations have to be made which are summarized as 'chute time' here. After completion, the respective emergency unit is dispatched.

Travel time to demand origin

After the dispatch the emergency unit has to travel to the origin of the demand for emergency help.

On-scene service time

After arriving on the scene of the incident, the emergency unit starts its service. Note that the actual service time is highly dependent on the nature of the emergency and can also be influenced by events that occur after arrival.

Travel time to medical facility

On completion of the on-scene service, the medical team decides whether a transport to a medical facility, e.g. a hospital, is necessary.

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[34] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 189
[35] Cf. REPEDE, J. F.; BERNARDO, J. J., 1994
[36] Cf. SUNG, I.; LEE, T., 2012
[37] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 191

Turnaround time and travel time to base

After the emergency unit arrives at the medical facility, it hands over the patient. Subsequently, the emergency unit is restocked to be able to attend another call for emergency help. This process is either done directly at the medical facility or at its base location. Note that there is no travel time back to the base if the medical facility itself is the base of the unit.

The response process as well as the composition of different time periods that are relevant for EMS systems are illustrated in figure 2 below. Note that the actual occupation of the respective emergency unit is dependent on whether it has to travel back to its base to be able to be redeployed again.

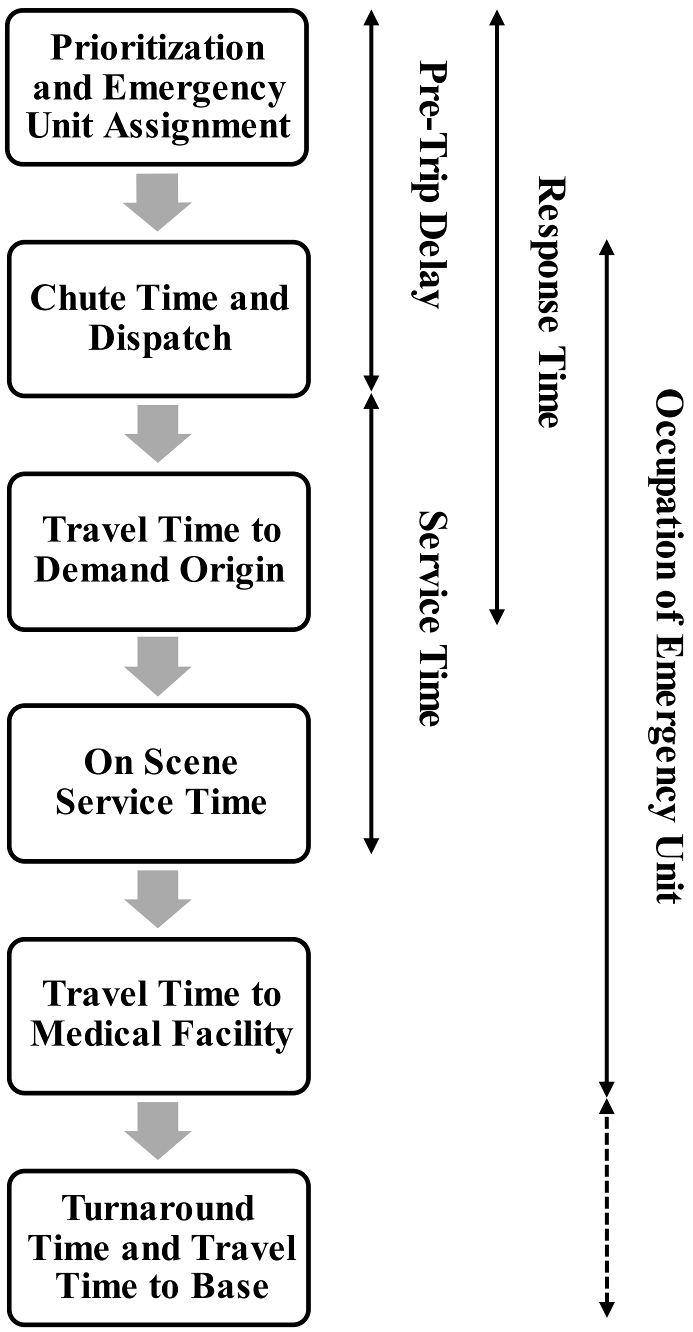

Figure 2: EMS response process extended from REPEDE AND BERNARDO[38]

2.1.4 Performance Metrics

The overall goal of any EMS system is to preserve human lives. GUSTAFSSON ET AL. describe the objectives of EMS systems as: “A primary purpose of emergency medical service systems is to ensure that the needs of the emergency victim are matched with the

[38] Cf. REPEDE, J. F.; BERNARDO, J. J., 1994, p. 568

appropriate form of care, and to do so as quickly as possible so that the victim's illness or injury is not exacerbated.".[39] McLay and Mayorga state them as follows: "The primary objective of emergency medical service (EMS) systems is to respond to emergency medical 911 calls and to save patient´s lives.".[40] Toro-Díaz et al. choose a simpler and shorter definition: "The ultimate goal of EMS systems is to save lives.".[41] Sudtachat et al. include the pre-hospital (or, in general, medical facility)-character: "The goal of emergency medical service (EMS) systems is to save the lives of out-of-hospital patients.".[42]

Since EMS systems operate in a highly sensitive environment and are, directly or indirectly, responsible for human lives, the defined objectives have always to be achieved to a very high degree, desirably to its full extent. There is a rather large body of literature by both academics and practitioners on how the achievement of the objectives of an EMS system can be measured. Generally, they can be divided into external and internal performance measures as noted by Iannoni, Morabito and Saydam.[43] While the first group is valuable both to the public and to system managers, the value of the second group is limited to the system managers for the most part or exclusively. The key performance metrics for the evaluation of EMS systems are listed and described as follows:[44]

Response time

The response time is the most used performance metric and is an important external as well as internal performance criterion. It denotes the timespan from the moment the call for emergency help is initially received until the emergency unit arrives at the scene of the incident. The actual time threshold defined as objective for the response time can vary significantly with regard to the respective EMS system. In Germany, each of the 16 federal states has its own EMS regulations and defines guidelines and objectives on its own. Therefore, the time threshold for the response time varies from 8 to 15 minutes and usually has to be met in 95 percent of the cases. For the United Kingdom, very high priority demands have to be served within 8 minutes in 75 percent of the cases while for the Netherlands a 15 minute threshold has to be achieved in 95 percent.[45]

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[39] Gustafson, D. H. et al., 1993, p. 132
[40] McLay, L. A.; Moore, H., 2012, p. 380
[41] Toro-Díaz, H. et al., 2013, p. 917
[42] Sudtachat, K. et al., 2020
[43] Cf. Iannoni, A. P.; Morabito, R.; Saydam, C., 2008, pp. 219–220
[44] The calculations of the performance metrics relevant to this work, are given in chapter 3.2.
[45] Cf. Reuter-Oppermann, M.; van den Berg, P. L.; Vile, J. L., 2017, p. 189

The response time is often calculated as an average value in a study period or, in the case of simulation and optimization studies, as the expected (weighted) average for all districts and, if valid, groups of demands or parts of the region that are served by the EMS system groups. In order to gain additional insight, the response time can also be calculated for certain groups of demand or regions. For example, CAPAR ET AL. calculated separate response times for the emergency units that are first at the scene of the incident and, if necessary, for those who arrive last.[46] ENAYATI ET AL. compute demand group specific response times.[47]

Coverage

The coverage criterion is one of the most used performance metrics to evaluate the performance of a given set of locations for an emergency unit, especially in the early EMS system studies. Usually, it indicates the fraction of the study area that can be covered (with respect to a time threshold). It can be extended to include backup coverages in the analysis.[48] Coverage is an important external as well as internal performance criterion.

There are several variants of the coverage criterion used in simulation and optimization studies. The early studies mostly used a 'standard coverage' value that abstracts from the probability that one or more emergency units may be unavailable in the case of an incoming demand. Therefore, more modern approaches, as used, for example, by INGOLFSSON, BUDGE AND ERKUT[49] and ENAYATI ET AL.[50], use 'expected coverage' that explicitly considers the potential congestion of the system.

Loss probabilities

The loss probabilities indicate the degree to which incoming demands can be served immediately or, if they can be served later, be served at all. Like the response time and the coverage criterion, it can be used for external as well as internal communication.

The loss probabilities are highly dependent on the level of backup that is provided by the EMS system, the overall congestion as well as the size of the geographical area that has to be served.[51] Especially the first aspect has a rather high influence due to the ability of

[46] Cf. ÇAPAR, I.; MELOUK, S. H.; KESKIN, B. B., 2017, p. 792
[47] Cf. ENAYATI, S. ET AL., 2019, p. 423
[48] For more details on backup coverages, see chapter 2.1.6.
[49] Cf. INGOLFSSON, A.; BUDGE, S.; ERKUT, E., 2008, p. 267
[50] Cf. ENAYATI, S. ET AL., 2019, p. 423
[51] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008, pp. 211–212

downstream units to respond to incoming demands. As already mentioned above, this may also affect the (expected) response time of the system and other performance metrics. The loss probabilities can be given separately with respect to a given group of demand or region.

Workloads

The workloads of the emergency units show the actual congestion of the system. They may be given system-wide or specifically for each emergency unit.[52] The value is mostly useful for the internal analysis of the EMS system.

A balanced workload among all emergency units can generally be viewed as beneficial for the availability of capacities in the case of incoming demands.

Survivability rate

The survivability rate denotes which fraction of patients survives after an immediate response and the subsequent treatment at a medical facility. The latter, depending on the medical characteristics of the individual demand, cannot be fully influenced by an EMS system. Therefore, most survivability rates in the context of EMS systems concern cardiac arrests[53] and are calculated using the response time of the system. The survivability rate can be used for external as well as internal communication.

Equity

Equity concerns the degree to which equal service can be provided for different groups of demand, regions or even among emergency units. Usually, some kind of trade-off between equity and efficiency, in terms of response time, coverage or loss probabilities, has to be faced if equity concerns are introduced into the optimization of EMS systems. It can be considered as more of an internal performance metric.

One common way to consider equity is the calculation of a Gini coefficient which is generally used in economics to measure disparities among different groups. Applied to EMS systems, the Gini coefficient, for example, calculates the difference in response time or loss probabilities if a differentiation into different demand groups is made.[54]

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[52] Cf. LARSON, R. C., 1974
[53] Cf. ERKUT, E.; INGOLFSSON, A.; ERDOGAN, G., 2008, p. 49
[54] Cf. ENAYATI, S. ET AL., 2019, p. 415–416

Cost/Budget

Like most other organizations, EMS systems have to be operated under a certain cost or budget limitation. Monetary concerns are often of secondary nature in the existing EMS system literature due to the focus on survival aspects in a broader sense. Cost and budget concerns can be used for external communication as well as internal analysis of the existing or available resources.

In the literature on the optimization of EMS systems, monetary aspects are often used as a constraint or expressed through other factors like a maximum number of emergency unit locations or redeployments.

2.1.5 Districting

Each EMS system is responsible for a pre-defined geographical area. This area may consist of solely urban or rural areas or a combination of both. Obviously, the equal, undifferentiated assignment of responsibilities to emergency units or stations in the whole area can lead to inferior outcomes. One concept to improve the service quality, is the subdivision of the overall area into smaller zones[55]. This concept is referred to as 'districting'. MAYORGA, BANDARA AND MCLAY defined it as "… the idea of partitioning the response area into districts or response areas, where the vehicles assigned to a district are to provide primary response to all zones in that district.".[56] KALCSICS AND RÍOS-MERCADO classify districting as "… the problem of grouping smaller geographical areas, called basic units or basic areas, into larger geographic clusters, called districts or territories, in a way that the latter are acceptable according to relevant planning criteria."[57]

The definition of districts and resulting areas of responsibility for each emergency unit can be considered as a strategic decision and is especially relevant for EMS systems that are responsible for larger geographical areas. The districting problem refers to the grouping of individual demand zones or points into larger districts. Those districts can then be easily differentiated since they have physical borders.[58] There are three different basic criteria that should be considered during the districting decision according to KALCSICS AND RÍOS-MERCADO: Balance, contiguity and compactness.[59]

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[55] Cf. DASKIN, M. S.; STERN, E. H., 1981, p. 138; RAJAGOPALAN, H. K.; SAYDAM, C.; XIAO, J., 2008, p. 817
[56] MAYORGA, M. E.; BANDARA, D.; MCLAY, L. A., 2013, p. 39
[57] KALCSICS, J.; RÍOS-MERCADO, R. Z., 2019, p. 705
[58] Cf. ENAYATI, S. ET AL., 2019, p. 427
[59] Cf. KALCSICS, J.; RÍOS-MERCADO, R. Z., 2019, pp. 715–725

Balance

The balance criterion refers to the definition of districts with relatively equal size which can be seen as beneficial with regard to performance metrics for the district as well as for the overall EMS system. Note that the balance criterion can in some cases be violated by the definition of certain coverage requirements.

Contiguity

A district, consisting of adjacent demand zones, is called contiguous if these zones can be travelled through without leaving the total area covered, i.e. the district.

Compactness

The compactness criterion requires a district to be geographically compact, i.e. to be in some geographical form and to show no holes (for the most part) . As with the balance criterion, the compactness can theoretically be violated if, for example, high incident demand zones are located in the middle of the district. In this case it might be beneficial to define a district exclusively for this demand zone which then results in a hole in the encircling district.

Most of the early location optimization models for EMS systems[60] solely concentrated on the location decision but did not define the corresponding districts for each location or emergency unit. Generally, it can be stated that the districting decision problem has been explored to a smaller degree than its location optimization part in the field of EMS.[61] SMITH was the first to introduce the determination of response areas in an intra-district response time minimization study.[62] Some early studies on the districting problem by, for example, SANTONE AND GEOFFREY[63] and SCHNEIDER[64] assumed responses to be all intra-district. A district in their work is defined by all elements that are closest to the respective location; this assumption has remained valid to date. As noted by CARTER, CHAIKEN AND IGNALL, the limitation of the (primary) response area of an emergency unit can contribute to the improvement in key performance criteria of the EMS system, like the average response time.[65] LARSON first used the actual term ‘districting’ in the definition of response

[60] For more details on coverage models in general, see chapter 2.4.
[61] Cf. MAYORGA, M. E.; BANDARA, D.; MCLAY, L. A., 2013, p. 41
[62] Cf. SMITH, R. D., 1961
[63] Cf. SANTONE, L. C.; GEOFFREY, N. B., 1969
[64] Cf. SCHNEIDER, J. B., 1971
[65] Cf. CARTER, G. M.; CHAIKEN, J. M.; IGNALL, E., 1972, p. 592

areas and showed the influence of the determination of districts on balancing the workloads between emergency units. [66]

In the following section some relevant recent studies on districting decisions for EMS systems are briefly presented.

PATEL, WATERS AND GHALI visualize districts in a cardiac incident survivability study that discusses the use of different transportation modes for faster accessibility.[67]

IANNONI, MORABITO AND SAYDAM[68] as well as IANNONI, MORABITO AND SAYDAM[69] developed a districting and location model that simultaneously determines locations and the corresponding districts. The latter study allows for emergency vehicles to be located anywhere in the study area.

GEROLIMINIS, KARLAFTIS AND SKABARDONIS combine the districting and EMS system location optimization problem in a HQM-based study.[70]

MAYORGA, BANDARA AND MCLAY aim for a simultaneous decision of districts and dispatch policies.[71] [72]

TORO-DÍAZ ET AL. simultaneously decide districting and dispatch policies in a HQM-based EMS system location study.[73]

STEINER ET AL. use a districting approach for the partitioning of a health care system while also determining locations.[74]

ANSARI, MCLAY AND MAYORGA extend the maximum coverage location problem for district design.[75]

ENAYATI ET AL. extend the approach of TORO-DÍAZ ET AL. for multiple classes of demand.[76]

Usually, the districting decision is also determined to provide for backup coverages in

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[66] Cf. LARSON, R. C., 1974, p. 67
[67] Cf. PATEL, A. B.; WATERS, N. M.; GHALI, W. A., 2007
[68] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008
[69] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2009
[70] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009
[71] Cf. MAYORGA, M. E.; BANDARA, D.; MCLAY, L. A., 2013
[72] For more details with regard to ‘dispatch policies’, see chapter 2.1.6.
[73] Cf. TORO-DÍAZ, H. ET AL., 2013
[74] Cf. STEINER, M. T. A. ET AL., 2015
[75] Cf. ANSARI, S.; MCLAY, L. A.; MAYORGA, M. E., 2017
[76] Cf. ENAYATI, S. ET AL., 2019

case the primary emergency unit is unavailable to respond. Thereby, primary as well as lower-tier areas of responsibility are defined for each emergency unit.[77] The decision to which degree backup coverages are introduced is subject to the preferences of the decision maker as well as other components, like the overall (expected) congestion of the system and the geographical size and nature of the area to be served by the EMS system. As noted by GEROLIMINIS, KARLAFTIS AND SKABARDONIS[78] and ENAYATI ET AL.[79], there are also trade-offs to be considered between higher levels of backup coverages and key performance metrics of the system. So, for example, due to a more thorough service that is provided for even remote areas the (expected) response time of the system may increase with a higher level of backup.

2.1.6 Dispatch Policies

A dispatch policy includes pre-defined assignments of emergency units to incoming demands for emergency help. BÉLANGER, RUIZ AND SORIANO classify them as follows: "Dispatching decisions are made to determine which vehicle to assign to an emergency call. They have a significant impact on response times, and thus greatly affect the system's performance."[80] Dispatching decisions are highly interconnected to the decision taken at the districting decision level. On the other hand, the construction of the districts does not necessarily result in a dispatch policy since the latter can be adapted to represent preferences of the decision maker, to yield gains in performance metrics as well as to put emphasis on serving certain demand groups with a higher priority.

The process of making a dispatching decision has to be made as fast as possible to avoid any unnecessary delays to the emergency response. The most common, in practice as well as in the academic literature, dispatch policy is the 'nearest-available'[81] dispatch policy which dispatches, without differentiating according to the urgencies of demands, always the nearest-available emergency unit.[82] This is done for two main reasons: First, the policy is easy to implement and to communicate, and second, it guarantees a fair and (rather) rapid response for all types of emergency demands.

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[77] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009, p. 800
[78] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009, p. 802
[79] Cf. ENAYATI, S. ET AL., 2019, p. 426
[80] BÉLANGER, V.; RUIZ, A.; SORIANO, P., 2019, p. 32
[81] Also called 'closest-idle'.
[82] Cf. BÉLANGER, V.; RUIZ, A.; SORIANO, P., 2019, p. 33

However, a rather significant amount of existing academic literature showed that the dispatch of the nearest-available emergency unit to all incidents can result in inferior performance metrics for the overall system as well as for specific sub groups. CARTER, CHAIKEN AND IGNALL demonstrate that the nearest-available dispatch policy does not always result in the lowest average response time.[83] In a more recent study, SCHMID showed that a flexible dispatch policy, which allows for dynamic relocation of the respective emergency unit, can result in improved performance.[84] Moreover, it was demonstrated that the nearest-available policy is not optimal for less urgent demands. SCHMID[85] and BANDARA, MAYORGA AND MCLAY[86] conclude that the nearest-available policy may lead to unnecessary congestion of the system and therefore to less flexibility for incoming future demands that may potentially be of high priority. JAGTENBERG, VAN DEN BERG AND VAN DER MEI show that the nearest-available dispatch policy can result in up to 2,7 times higher late arrivals if compared to other policies.[87] As MCLAY AND MAYORGA have shown, the drawbacks of the nearest-available policy can even increase if equity concerns between different demand groups, regions or emergency units are considered.[88]

There are other dispatch policies proposed in the academic literature. The main policies are briefly described in the following, in alphabetical order:

Cost minimization

Each dispatch of an emergency unit can lead to costs regarding the potential unavailability of other units or the transfer of the incoming call to another EMS system. Therefore, GENDREAU, LAPORTE AND SEMET study a dispatch policy that minimizes the cost of relocation for the emergency units due to potential unavailability. They formulate a relocation plan based on the costs resulting from the dispatch of different emergency units. The dispatching decision is then made according to this plan.[89]

Decision variable

TORO-DÍAZ ET AL. define their dispatch policy as decision variable. Therefore, their model

[83] Cf. CARTER, G. M.; CHAIKEN, J. M.; IGNALL, E., 1972, p. 592
[84] Cf. SCHMID, V., 2012, p. 620
[85] Cf. SCHMID, V., 2012, p. 620
[86] Cf. BANDARA, D.; MAYORGA, M. E.; MCLAY, L. A., 2014, p. 584
[87] Cf. JAGTENBERG, C. J.; VAN DEN BERG, P. L.; VAN DER MEI, R. D., 2017, p. 725
[88] Cf. MCLAY, L. A.; MAYORGA, M. E., 2013, p. 22
[89] Cf. GENDREAU, M.; LAPORTE, G.; SEMET, F., 2001, p. 1645

simultaneously processes the location, the districting and the dispatching decision.[90] Enayati et al. extend this procedure for different demand groups.[91] The mentioned models define the dispatch policy for every defined district or demand zone. The definition as decision variable allows for neighboring districts to have divergent dispatch policies. This can lead to ethical as well as legal issues due to a potential favoritism for one district or the other.

Non-nearest-available (reservation) policy

The non-nearest-available policy reserves emergency units for future incoming demands to ensure future coverage. The policy may lead to ethical and legal issues since the nearest-available unit is not dispatched even when it is available.[92] Repede and Bernardo used this policy in the case the primary unit was unavailable to respond and stated an over thirty percent decrease in response time.[93] Andersson and Värbrand used the non-nearest-available policy for lower priority demands and state reduced waiting periods for the patients.[94]

Priority differentiation

Since, by nature, not all demands for emergency medical help are of the same severity, one obvious possibility to adapt the dispatch policies is to differentiate into priorities. In the process potential legal and ethical issues regarding the differentiation have to be mitigated. Nicholl et al. show the use of a priority differentiation dispatch strategy yielding large benefits for high priority patients. They draw the conclusion that low priority patients should be responded to as soon as possible rather than immediately.[95] Bandara, Mayorga and McLay state that the differentiation approach can lead to an increase in the average expected survivability of the patients.[96] McLay and Mayorga discuss optimal dispatch strategies for multiple priorities of patients in the case of classification errors. They conclude that the nearest-available dispatch strategy is not always optimal, even for high priority patients if there is a rather high fraction of classification errors.[97] Sudtachat, Mayorga and McLay analyze optimal dispatch strategies with multiple

[90] Cf. Toro-Díaz, H. et al., 2013, p. 920
[91] Cf. Enayati, S. et al., 2019, p. 421
[92] Cf. Lim, C. S.; Mamat, R.; Braunl, T., 2011, p. 626
[93] Cf. Repede, J. F.; Bernardo, J. J., 1994, p. 575
[94] Cf. Andersson, T.; Värbrand, P., 2007, p. 197
[95] Cf. Nicholl, J. et al., 1999
[96] Cf. Bandara, D.; Mayorga, M. E.; Mclay, L. A., 2014
[97] Cf. McLay, L. A.; Mayorga, M. E., 2013

types of emergency units and call priorities. They show that dispatching the nearest-available unit is the optimal strategy for high priority demands while the workloads can be balanced by dispatching more distant units to lower priority demands.[98] The differentiation policy is a powerful tool in order to reach benefits for one group of EMS system patients while (slightly) disadvantaging the other groups.

Regionalized response

The regionalized response dispatch policy is highly interconnected with the nearest-available dispatch policy. It dispatches the respective emergency unit to any incidents in its region. If, and only if, it is unavailable for other incoming demands of its area of responsibility, the nearest-available emergency unit from another region may be dispatched to those incidents. Thereby, it can differentiate from the nearest-available dispatch policy by dispatching emergency units from the respective region to incidents in which an emergency unit from another region is closer. SWOVELAND ET AL. use regionalized-response dispatch policy and state an about twenty percent decrease in response time if compared to the nearest-available policy.[99]

Reroute dispatch

The reroute dispatch describes the rerouting of an emergency unit due to incoming higher priority demand calls for emergency help. This may lead to the original deployment to be entirely cancelled which then results in significantly longer waiting times for low priority demands as well as in ethical and legal issues. LIM, MAMAT AND BRAUNL state a significant decrease in response time for high priority demands if the reroute-dispatch is applied.[100]

2.2 Queueing Theory

The following chapter will explain some relevant Queueing Theory fundamentals (chapter 2.2.1) and provide a basic description and illustration of the HQM (chapter 2.2.2). The functioning of the latter is demonstrated by a numerical example.

2.2.1 Queueing Theory Fundamentals

The origins of Queueing Theory date back to the beginning of the twentieth century. In 1909, ERLANG published the first paper on a concept that nowadays would be referred to

[98] Cf. SUDTACHAT, K.; MAYORGA, M. E.; MCLAY, L. A., 2014
[99] Cf. SWOVELAND, C. ET AL., 1973
[100] Cf. LIM, C. S.; MAMAT, R.; BRAUNL, T., 2011

as Queueing Theory. He studied the congestion in telephone networks and formulated and solved several key practical problems in this field. Additionally, he introduced assumptions and formulas that have been used in the analysis of queueing networks ever since.[101]

Queueing Theory is used in a variety of operations research fields like healthcare, banking or industrial applications. Usually, it has been applied to gain insights into the behavior of a system with respect to the key parameters λ and μ. λ is the arrival rate of customers into the system while μ denotes the service rate. In simple terms, λ describes the demand for services that are provided by the respective system while μ describes the service that can be provided. Queueing systems can usually be described with the following characteristics according to BHAT[102], SHORTLE ET AL.[103] and STEWART[104]:

Arrival pattern of customers

The arrival process of customers into a queueing system is usually stochastic. Therefore, it is necessary to know the corresponding distribution. Often a Poisson arrival is assumed, which will be described later in this chapter. Moreover, it is necessary to establish whether the arrival process changes over time and can be considered stationary (time-independent) or non-stationary (time-dependent).

Service pattern of servers

As for the arrival process, the service pattern is usually assumed to be of stochastic nature as well, and a distribution has to be specified. The service that is provided by the system is highly interlinked with its real-life characteristics. Most basic queueing systems assume that one customer is served by one server, but it is also possible that one server serves multiple customers and vice versa. The word ‘server’ describes any serving unit in a queueing system. As for the arrival patterns, the service pattern can also be considered stationary or non-stationary.

Number of servers

The number of servers is a key characteristic of any queueing system and represents the trade-off between increased service and costs. The servers in a system can be homogenous

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[101] Cf. BANDI, C.; BERTSIMAS, D., 2012, p. 33
[102] Cf. BHAT, U. N., 2015, p. 1–3
[103] Cf. SHORTLE, J. F. ET AL., 2018, pp. 4–7
[104] Cf. STEWART, W. J., 2009, p. 385–388

or non-homogenous. In the case of the latter, more specified servers are used for special demands.

System capacity

Some queueing systems have a natural limit to the number of customers they can accommodate. If so, the respective system does not allow any new customers to enter until enough space becomes available again. Note that some systems do not allow for customers to be queued and transfer them to a neighboring system. Therefore, the capacity is also dependent on whether queueing is generally allowed or not.

Queue discipline

The term queue discipline refers to the manner in which customers are served after having entered the system. The most common discipline is 'first-come-first-served'. Additionally, a common practice is to differentiate into the priority of the customers in a way that customers with a higher priority are served earlier.

Number of service stages

A queueing system can have either one or multiple stages in which the customer has to pass through all stages to complete the service. Note that in multiple-stage queueing systems the individual stages are heavily interlinked with each other and the overall service depends on each stage.

As there is a variety of queueing systems, a common notation was established by KENDALL to describe the basic characteristics of each system.[105] The queueing process is described by a series of symbols and slashes that denote the queueing processes applied and are usually given in the form A/B/X/Y/Z. A denotes the interarrival time distribution, B denotes the service-time distribution, X denotes the number of parallel servers, Y denotes the system capacity and Z denotes the queue discipline. Table 4 below summarizes some of the key distributions and parameters for the queueing notation.[106]

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[105] Cf. KENDALL, D. G., 1953, p. 340
[106] Cf. SHORTLE, J. F. ET AL., 2018, p. 8; STEWART, W. J., 2009, pp. 396–397

Characteristic	Symbol	Explanation
Interarrival time distribution (A) Service-time distribution (B)	M	Exponential
	D	Deterministic
	E_K	Erlang type
	H_K	Mixture of k exponentials
	PH	Phase type
	G	General
Parallel servers (X)	1, …, ∞	
System capacity (Y)	1, …, ∞	
Queue discipline (Z)	FCFS	First-come-first-served
	LCFS	Last-come-first-served
	RSS	Random selection
	PR	Priority
	GD	General discipline

Table 2: Queueing system notation

So, for example, the M/M/5/∞/FCFS notation describes a system with exponential interarrival and service times, five servers, infinite capacity and a first-come-first-served policy. It is common practice, to omit the system capacity (Y) if no restriction is imposed, and the queue discipline (Z) if it is assumed to be first-come-first-served.

In the following section, the interarrival and service time distributions are briefly described.[107] Since it is of particular relevance to this work, the exponential distribution is described to greater detail.

Exponential (M)

M describes the use of an exponential distribution. The exponential distribution is highly dependent on the common assumption of Poisson and Markov processes, which will be described further down this chapter. Some of the key definitions and theorems relevant to the exponential distribution are briefly characterized in the following.[108]

[107] The parallel servers, the system capacity and the queue discipline are not described to further detail here with respect to the scope of this work.
[108] Note that there are additional theorems relevant to exponential distributions, as for the combination of variables, which are not of particular relevance to this work. The reader is referred to SHORTLE ET AL., p. 38 for further details.

Definition I: An exponential random variable is a continuous random variable with the probability density function:[109]

(2.2.1.1) $f(t)= \lambda e^{-\lambda t} \quad (t \geq 0)$

For the field of Queueing Theory, an exponential random variable usually characterizes a quantity of time while λ represents a rate that has a certain number of units per time.

One of the key properties, which also explains the relevance of the exponential distribution, is its memoryless property.[110]

Definition II: An exponential distribution has a memoryless property if:

(2.2.1.2) $\Pr\{T > t + s \mid T > s\} = \Pr\{T > t\} \ (s, t \geq 0)$

T represents the time until an event occurs. The memoryless property states that if one has already waited s time units for an event to occur (T > s) then the conditional probability of waiting at least t more time units $\Pr\{T > t + s \mid T > s\}$ is equal to waiting t units from the start $\Pr\{T > t\}$. In practical terms, this means that the fact that one has already waited a certain amount of time does not lead to the event to occur with a higher likelihood. A memoryless process continually starts all over again and the time until an event occurs does not depend on the time already spent waiting for it to occur.

Theorem I: An exponential random variable also features the memoryless property:[111]

$$(2.2.1.3)\ \Pr\{T > t + s \mid T > s\} = \frac{\Pr\{T > t + s \mid T > s\}}{\Pr\{T > s\}} = \frac{\Pr\{T > t + s\}}{\Pr\{T > s\}}$$

$$= \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} = e^{-\lambda t}$$

Note that the exponential distribution is the only continuous distribution that possesses the memoryless property.

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[109] Cf. SHORTLE, J. F. ET AL., 2018, p. 36; STEWART, W. J., 2009, p. 136; BHAT, U. N., 2015, p. 17
[110] Cf. SHORTLE, J. F. ET AL., 2018, p. 36–37; STEWART, W. J., 2009, p. 136–138; BHAT, U. N., 2015, p. 18
[111] Cf. SHORTLE, J. F. ET AL., 2018, p. 36; STEWART, W. J., 2009, p. 138

Deterministic (D)

D describes the use of constant interarrival and service times.[112]

Erlang Type (E_K)

E_K describes the use of the Erlang type distribution which was introduced as a specified type of a gamma distribution. K is called the shape parameter and, in contrast to the gamma distribution, has to be an integer. If K is set to one, the Erlang type distribution simplifies to the exponential distribution described above.[113]

Mixture of K exponentials (H_K)

H_K describes the use of a hyper-exponential distribution with a parameter, i.e. a mixture of K exponential functions.[114]

Phase Type (PH)

PH describes the use of a distribution that differentiates into different types of service in a queueing system. A PH distribution can be applied, for example, if a customer needs different types of services to be completed before leaving the system as in a public office where the service is offered at different counters.[115]

General (G)

G describes the use of a general probability distribution without further specifying the corresponding properties. A G/G/1 queueing system has for example been used by BANDI, BERTSIMAS AND YOUSSEF in a robust Queueing Theory approach.[116]

2.2.1.1 Poisson Processes

The Poisson process is often used to model arrivals into a queueing system. The process can be understood as describing events that occur 'randomly' in time. A stochastic process $\{N(t), t \geq 0\}$ describes the collection of random variables that are indexed by the parameter t. A counting process is a stochastic process in which N(t) has non-negative and integer values and can be described as the cumulative number of events that have

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[112] Cf. SHORTLE, J. F. ET AL., 2018, p. 8
[113] Cf. SHORTLE, J. F. ET AL., 2018, p. 159
[114] Cf. BHAT, U. N., 2015, p. 299
[115] Cf. JACKSON, R. R. P., 1954, p. 109
[116] Cf. BANDI, C.; BERTSIMAS, D.; YOUSSEF, N., 2015, p. 677

occurred by time t.[117]

Definition III: A Poisson Process can be described as a counting process N(t) with a rate of $\lambda > 0$ that exhibits the following properties:[118]

1. N(0) = 0.
2. Pr{1 event between t and t + Δt} = λΔt + o(Δt).
3. Pr{2 events between t and t + Δt} = o(Δt).
4. The number of events in non-overlapping intervals are statistically independent.

o(Δt) is defined as a quantity that becomes a negligible factor for small values of t and can be assumed to be zero. The second property states that the probability to observe exactly one event over a shorter time period is approximately proportional to the length of the time interval. The third property requires the probability of observing two events in a relatively small time period to be essentially zero. The fourth property states that events in different, non-overlapping intervals are assumed to happen independently of each other. In practical terms this means that the knowledge about how many events are likely to occur in one time period provides no information on how many events are expected to occur in another, disjoint time period.

Definition III provides some useful characterizations of the Poisson Process, but does not specify any possible quantification for the stochastic process N(t). Therefore, a Poisson Random Variable has to be defined.

Definition IV: A Poisson Random Variable is a random variable with the following probability mass function:[119]

$$(2.2.1.4)\quad p_n = e^{-A} \frac{A^n}{n!} \quad (n=1, 2, \dots)$$

A is a constant greater than zero.

[117] Cf. SHORTLE, J. F. ET AL., 2018, p. 39
[118] Cf. SHORTLE, J. F. ET AL., 2018, p. 39; CHING, W.-K. ET AL., 2013, pp. 19–20
[119] Cf. SHORTLE, J. F. ET AL., 2018, p. 40; STEWART, W. J., 2009, p. 141; CHING, W.-K. ET AL., 2013, p. 20

The following theorem links the Poisson Process and its random variable.[120]

Theorem II: N(t) is a Poisson Process with a rate of $\lambda > 0$. The number of events occurring up to time t are given by the Poisson Variable with the mean of λt which can be written as:[121]

$$(2.2.1.5)\ p_n(t) = \frac{(\lambda t)}{n!} e^{-\lambda t}$$

$p_n(t)$ describes the probability that n events have occurred at time t, $Pr\{N(t) = n\}$.

Theorem II quantifies the distribution of events in a Poisson Process with respect to the length of the considered time interval t. Theorem III below links the Poisson Process with the exponential distribution.

Theorem III: N(t) is a Poisson Process with a rate of $\lambda > 0$. The times between successive events are independent and exponentially distributed with rate λ and a mean of $1/\lambda$.[122] [123]

Therefore, it can be summarized that the Poisson Process is interlinked with the memoryless property of the exponential distribution which is relevant for the Markov Chains detailed in chapter 2.5.1.2.

2.2.1.2 Markov Chains

Markov Chains describe a class of models in which the analyzed system transitions among a discrete set of states. The main property of Markov models states that, if the present state of the system (X_n) is known, the future state (X_{n+1}) is independent of the past.[124]

There are several additional properties that define Markov Chains:[125]

1. State j is accessible from state i such that p_{ij} is greater than zero.[126] This means

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[120] For an example regarding the calculation of different values of n, the reader is referred to SHORTLE ET AL., p. 40 and 41.
[121] Cf. SHORTLE, J. F. ET AL., 2018, p. 40; CHING, W.-K. ET AL., 2013, p. 20
[122] For the detailed proof of theorem III, the reader is referred to SHORTLE ET AL. (2018), p. 43.
[123] Cf. SHORTLE, J. F. ET AL., 2018, p. 42; STEWART, W. J., 2009, p. 297; CHING, W.-K. ET AL., 2013, p. 21
[124] Cf. CHING, W.-K. ET AL., 2013, p. 3
[125] Cf. SHORTLE, J. F. ET AL., 2018, pp. 51–52
[126] Note that there is also a differing notation in the literature for which a number of steps n between states i and j is introduced such that p_{ij}^n is greater than zero.

that there is a path between states i and j with a non-zero probability.

2. Two states i and j communicate with each other if i is accessible from j and vice versa. This property partitions the set of states into subclasses that are able to communicate within each other but not with states that do not belong to the respective subclass. A Markov Chain is called irreducible if all states can communicate with each other and reducible if not.

3. A state j can be called recurrent if, starting in state j, the probability to return to state j is 1. Otherwise it is called transient.

Markov Chains can either be of discrete or continuous time. While the first group assumes that state transitions can only happen at discrete points in time, the second group also assumes state transitions, but the time spent at each state is an exponential distributed random variable in continuous time.[127]

Discrete time

As mentioned above, the key property of the Markov Chain is its independence of past events which is also called the Markov property and can be written as:[128]

(2.2.1.6) $Pr\ \{X_{n+1}=j\ |X_0 = i_0, X_1 = i_1,\ldots, X_n = i_n\} = Pr\{X_{n+1} = j|X_n = i\}$

The Markov property states that knowing the current state of the system leads to the same results as knowing the current state as well as the entire history of state transitions. Therefore, it can be considered as ‘memoryless’.

The conditional probabilities of the transitions between two states, i and j, can then be calculated as follows:[129]

(2.2.1.7) $p_{ij} = Pr\{X_{n+1} = j|X_n = i)$

The conditional probabilities between the states then result in a transition matrix unique to the respective system, as illustrated by a simple example for a three state system below:

[127] Cf. SHORTLE, J. F. ET AL., 2018, pp. 49–62; STEWART, W. J., 2009, p. 194
[128] Cf. SHORTLE, J. F. ET AL., 2018, p. 49; STEWART, W. J., 2009, p. 194
[129] Cf. SHORTLE, J. F. ET AL., 2018, p. 49; STEWART, W. J., 2009, p. 194

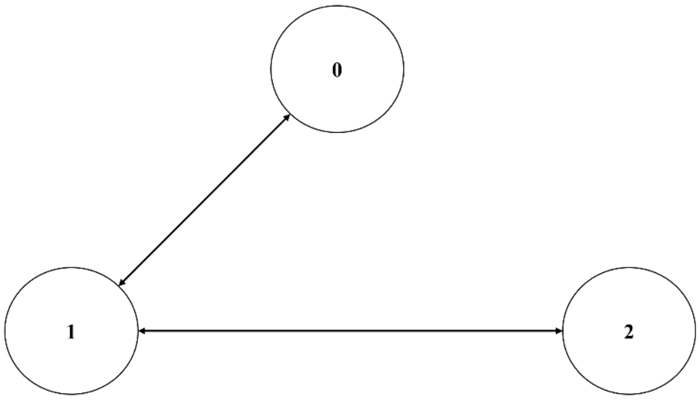

Figure 3: Simple Markov example

State ‘0’ can transition to ‘1’ and vice versa. ‘1’ can transition into ‘2’ and vice versa. ‘0’ cannot transition into ‘2’ or backwards. The individual transition rates then result in the following transition matrix:

$$\mathbf{P} = \begin{Bmatrix} 0 & p_{01} & 0 \\ p_{10} & 0 & p_{12} \\ 0 & p_{21} & 0 \end{Bmatrix}$$

<u>*Continuous time*</u>

A (time-homogenous) continuous time Markov Chain is a stochastic process $\{X(t), t \geq 0\}$ with a countable state space that exhibits the following properties:[130]

1. Each time the system enters state i, it remains in state i for a period of time that is exponentially distributed by rate υ_i. The time spent at time i is independent of the past.

2. Whenever the system departs from state i, it enters state j with the probability p_{ij}. This process happens independently of the past.

The continuous time Markov Chain transitions between states as for the discrete time, but the time spent at each state is now an exponentially distributed random variable. Therefore, the Markov property has to be rewritten as:[131]

(2.2.1.8) $\Pr\{X(t+s) = j \mid X(t) = i, X(u), 0 \leq u < t\} = \Pr\{X(t+s) = j \mid X(t) = i\}$

[130] Cf. SHORTLE, J. F. ET AL., 2018, p. 62
[131] Cf. SHORTLE, J. F. ET AL., 2018, p. 62; BHAT, U. N., 2015, p. 27

In simple terms, this means that, given the current state i, the transition into a future state j is independent of the systems history X(u). The continuous time Markov Chain has the Markov property since the time spent at each state is an exponentially distributed random variable which also has the 'memoryless' property of the exponential distribution.

2.2.1.3 Simple Markovian Queueing Models

In the following a simple M/M/1 Markovian Queueing model is presented. The simple models are characterized by a 'birth-death' process which consists of a set of states {0; 1; 2; …} that is called a population. As detailed in chapter 2.5.1.2, the system transitions up or down from the current state. If the system is in state $n \geq 0$, the time until the next arrival (also called 'birth') is an exponentially distributed random variable with the rate of λ. Whenever an arrival occurs, the system transitions from state n to state n+1. If the system is in state $n \geq 1$, the time until the next departure (also called 'death') can be expressed through the exponentially distributed random variable μ. The system transitions accordingly from state n to state n-1. The states denote the number of customers in the system.[132] The state transitions for a three-state system are illustrated in figure 4 below.

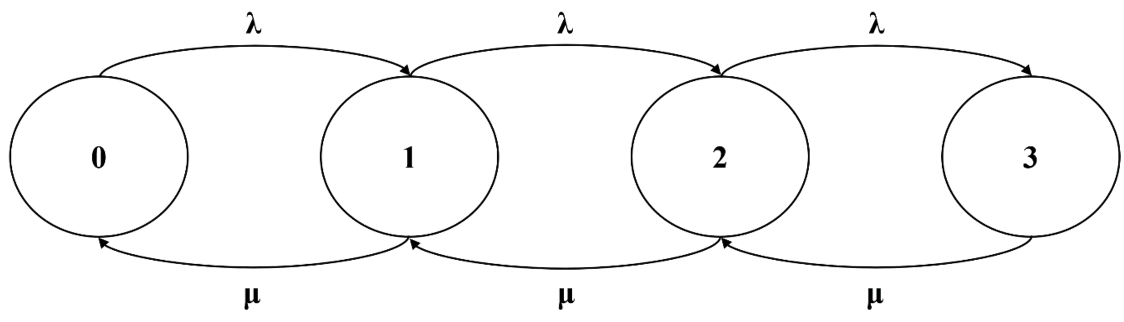

Figure 4: M/M/1 queue

Generally, in a queueing system, a birth or arrival to the system can be considered as demand, while a death or departure can be considered as service. As mentioned before, in the case of an M/M/1 queue the interarrival and service times are exponentially distributed random variables with the functions of:[133]

(2.2.1.9) **Interarrival times:** $a(t) = \lambda e^{-\lambda t}$

(2.2.1.10) **Service times:** $b(t) = \mu e^{-\mu t}$

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[132] Cf. SHORTLE, J. F. ET AL., 2018, p. 74; STEWART, W. J., 2009, p. 209; CHING, W.-K. ET AL., 2013, p. 21
[133] Cf. SHORTLE, J. F. ET AL., 2018, p. 77

In order to derive the state probabilities, balance-flow equations have to be formulated and can then be rewritten as:[134]

(2.2.1.11) $\lambda p_0 = \mu p_1 \rightarrow p_1 = \frac{\lambda}{\mu} p_0$

(2.2.1.12) $(\lambda + \mu) p_n = \lambda p_{n-1} + \mu p_{n+1} \rightarrow p_{n+1} = \frac{\lambda + \mu}{\mu} p_n + \frac{\lambda}{\mu} p_{n-1}$

The basic concept of the balance-flow equations requires all flows into one state to be equal to all flows out of the respective state. If, and only if, this holds true, it can be considered as a steady state. There are several solution methods for the steady-state probabilities described above which all simplify to the two basic equations presented below:[135]

(2.2.1.13) $p_0 = \left(1 - \frac{\lambda}{\mu}\right) \quad \left(\frac{\lambda}{\mu} < 1\right)$

(2.2.1.14) $p_n = p_0 \left(\frac{\lambda}{\mu}\right)^n \quad \left(\frac{\lambda}{\mu} < 1\right), n \geq 1$

A steady state can only be reached if the condition of $\frac{\lambda}{\mu} < 1$ holds true. For $\frac{\lambda}{\mu} > 1$, or in other words, if demand always exceeds service, the system would fall back further and further behind.

The steady-state probabilities can then be used to calculate key performance metrics of the system like the expected number of customers in the system. The number of customers in the system at steady state 'L' and the number of customers in queue 'L_q' can be calculated as follows:[136]

(2.2.1.15) $L = (1 - \rho) \sum_{n=0}^{\infty} n p_n \rightarrow L = \frac{\lambda}{\mu - \lambda}$

(2.2.1.16) $L_q = \sum_{n=0}^{\infty} n p_n - \sum_{n=0}^{\infty} p_n \rightarrow L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$

The number of customers in the system can then be used to calculate the average waiting

[134] Cf. SHORTLE, J. F. ET AL., 2018, p. 78; SMITH, D. K., 2002, p. 45
[135] Cf. SHORTLE, J. F. ET AL., 2018, p. 79
[136] Cf. SHORTLE, J. F. ET AL., 2018, p. 84; STEWART, W. J., 2009, p. 399

time 'W' and time delay 'W_q' by using Little´s Law:[137]

$$(2.2.1.17)\ W = \frac{L}{\lambda} = \frac{1}{\mu - \lambda}$$

$$(2.2.1.18)\ W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)}$$

2.2.2 Basic Hypercube Queueing Model

The HQM is based on M/M/N Queueing Theory principles and was introduced by Larson to describe and analyze medical and other emergency systems with a server-to-customer relationship to greater detail.[138] The basic idea is to expand the state descriptions of a queueing system to represent each server individually as noted by CHIYOSHI, IANNONI AND MORABITO.[139] As a result, each server is represented individually in the state description, which causes the basic HQM to not incorporate a queue by default. Thus, it can only consider the demands that can be immediately served into its analysis.

Most HQM models consider emergency systems at least to some degree. Therefore, they incorporate the serving entities, like ambulances, fire trucks or other medical and emergency vehicles, as servers. The use of the HQM allows for the modelling of system-specific details like dispatch policies, levels of backup introduced as well as spatial and temporal variations. The main advantage of the HQM is the option to derive server-specific probabilities that indicate whether the server is available or unavailable for incoming demands as well as other related performance metrics. Moreover, it allows for dependence between the individual servers. The obvious downside of the incorporation of the HQM is explained by the increased state space which can be calculated by $2^{\text{Number of Servers}}$. Note that the modelling and subsequent computational efforts to derive and solve the linear equation system depend significantly on the number of servers that are considered. For example, a three-server system has 8 equations while a five- or ten-server system results in 32 and 1,024 equations respectively. The implied efforts that remain relevant to date, limit the use of the HQM to study areas with a moderate size and have triggered the development and use of approximation techniques as initially debuted by LARSON.[140]

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137 Cf. LITTLE, J. D. C., 1961, p. 387
138 Cf. LARSON, R. C., 1974
139 Cf. CHIYOSHI, F. Y.; IANNONI, A. P.; MORABITO, R., 2011, p. 272
140 Cf. LARSON, R. C., 1975

Those approximation techniques have some disadvantages like the compromised incorporation of system-specific assumptions and potentially less accurate performance metrics.

The name 'hypercube' derives from the state space describing the system. In the basic HQM, each server has two states: available which is denoted by a '0' and unavailable which is denoted by a '1' in the corresponding state description. Therefore, a three-server system in which the first and third servers are currently unavailable while the second server is available can be described as {1, 0, 1}. If the system provides more than three servers, it is called a hypercube.[141] In the basic HQM, only single step transitions are allowed which means that the state description can only be changed at one spot at once. Applied to the EMS systems theory, this means that only one emergency unit (server) can be dispatched at once. The flows within a three-server system are illustrated in figure 5 below:

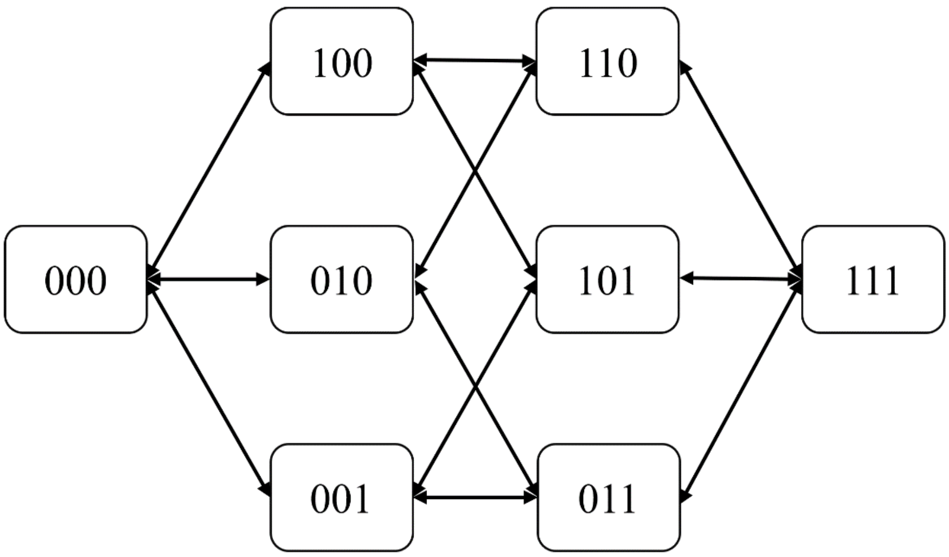

Figure 5: State flows within a three-server HQM

In order to derive the steady-state probabilities, one equation for each state of the HQM has to be formulated that balances all flows into and out of the respective state. This procedure can be generalized by the following two equations:[142]

[141] Cf. LARSON, R. C., 1974, p. 69
[142] Cf. LARSON, R. C., 1974, p. 75

$$(2.2.2.1)\quad P\{B_b\}\left[\sum_{\substack{a \\ \{B_a \in C_N : d^-_{ab}=1\}}} \lambda_{ab} + \sum_{\substack{a \\ \{B_a \in C_N : d^+_{ab}=1\}}} \mu_{ab}\right] = \sum_{\substack{a \\ \{B_a \in C_N : d^-_{ab}=1\}}} P\{B_a\}\mu_{ab} + \sum_{\substack{a \\ \{B_a \in C_N : d^+_{ab}=1\}}} P\{B_a\}\lambda_{ab} \;\forall\, b = 0, 1, \ldots, 2^N\text{-}1$$

$$(2.2.2.2)\quad \sum_{a=0}^{2^N-1} P\{B_a\} = 1$$

Equation 2.2.2.1 above describes the construction of the steady-state balancing equations. Equation 2.2.2.2 requires the sum of all steady-state probabilities to be equal to one, d^+_{ab} and d^-_{ab} denote the upward and downward Hamming-distances between states a and b and control whether one-step or multiple-step transitions between states are allowed. For example, the difference (d^+_{ab}) between state a which is described by {1, 0, 0} and state b which is described by {1, 0, 1} is one. As mentioned above, the steady-state balancing equations then result in a linear equation system with the size of $2^{\text{Number of Servers}}$.

A simple numerical example is provided below to demonstrate the character of the HQM. For the sake of simplicity, only three servers and three demand points are used. Moreover, it is assumed that each server can respond to any incident[143], which results in the dispatch (preference) list provided in table 3 for each demand point. For other basic numerical examples, which include multiple dispatches, partial backup as well as different classes of demand, the reader is referred to CHIYOSHI, IANNONI AND MORABITO.[144]

	Server Preferences		
Demand Point	**1st**	**2nd**	**3rd**
1	**1**	**2**	**3**
2	**2**	**3**	**1**
3	**3**	**1**	**2**

Table 3: Preference list numerical example HQM

The following demand and service rates are used:

[143] Also called total backup.
[144] Cf. CHIYOSHI, F. Y.; IANNONI, A. P.; MORABITO, R., 2011

	λ_1	λ_2	λ_3	λ $(=\lambda_1+\lambda_2+\lambda_3)$	μ_1	μ_2	μ_3	μ $(=\mu_1+\mu_2+\mu_3)$
Example	0,5	0,5	0,5	1,5	1	1	1	3

Table 4: Parameters numerical example HQM

The steady-state equations can then be formulated as follows:

(2.2.2.I) $\lambda P\{B_{000}\} = \mu_1 P\{B_{100}\} + \mu_2 P\{B_{010}\} + \mu_3 P\{B_{001}\}$

(2.2.2.II) $(\lambda + \mu_1)P\{B_{100}\} = \lambda_1 P\{B_{000}\} + \mu_2 P\{B_{110}\} + \mu_3 P\{B_{101}\}$

(2.2.2.III) $(\lambda + \mu_2)P\{B_{010}\} = \lambda_2 P\{B_{000}\} + \mu_1 P\{B_{110}\} + \mu_3 P\{B_{011}\}$

(2.2.2.IV) $(\lambda + \mu_3)P\{B_{001}\} = \lambda_3 P\{B_{000}\} + \mu_1 P\{B_{101}\} + \mu_2 P\{B_{011}\}$

(2.2.2.V) $(\lambda + \mu_1 + \mu_2)P\{B_{110}\} = (\lambda_1 + \lambda_2) P\{B_{100}\} + \lambda_1 P\{B_{010}\} + \mu_3 P\{B_{111}\}$

(2.2.2.VI) $(\lambda + \mu_1 + \mu_3)P\{B_{101}\} = \lambda_3 \{B_{100}\} + (\lambda_1 + \lambda_3) P\{B_{001}\} + \mu_2 P\{B_{111}\}$

(2.2.2.VII) $(\lambda + \mu_2 + \mu_3) P\{B_{011}\} = (\lambda_2 + \lambda_3) P\{B_{010}\} + \lambda_2 P\{B_{001}\} + \mu_1 P\{B_{111}\}$

(2.2.2.VIII) $\mu P\{B_{111}\} = \lambda P\{B_{110}\} + \lambda P\{B_{101}\} + \lambda P\{B_{011}\}$

(2.2.2.IX) $P\{B_{000}\} + P\{B_{100}\} + P\{B_{010}\} + P\{B_{001}\}$

$$+ P\{B_{110}\} + P\{B_{101}\} + P\{B_{011}\} + P\{B_{111}\} = 1$$

For example, equation (2.2.2.V) can be explained as follows: The left-hand side describes all flows of the state B_{110} into all other possible states. Due to the full backup, this can happen for all incoming demands of all demand points (λ) just as if one of the two unavailable servers completed its service and was available again ($\mu_1+\mu_2$). The right-hand side describes all flows from state B_{100} into state B_{110} that can be accomplished with incoming demands from demand points 1 and 2 ($\lambda_1 + \lambda_2$), since the primary server (server 1) is unavailable in the respective states and server 2 is the preferential backup server. State B_{010} can transform into B_{110} by a demand from demand point 1 (λ_1). State B_{111} can transform into B_{110} by a downward transition, i.e. server 3 completes its service, with the service rate of μ_3.

The linear equation system is then solved with MatLabR2020 software and results in the following steady state probabilities for the example:

State	Steady-State-Probability
$P\{B_{000}\}$	0,2388
$P\{B_{100}\}$	0,1194
$P\{B_{010}\}$	0,1194
$P\{B_{001}\}$	0,1194
$P\{B_{110}\}$	0,0896
$P\{B_{101}\}$	0,0896
$P\{B_{011}\}$	0,0896
$P\{B_{111}\}$	0,1343

Table 5: Steady-state probabilities numerical example HQM

As can be from table 5 above, a service rate that is twice the rate of the incoming demand leads to a rather uncongested system in which all servers are unavailable with a probability of 13,43 percent. The equal probabilities for the states in which the same number of servers is unavailable, e.g. $P\{B_{110}\}$, $P\{B_{101}\}$ and $P\{B_{011}\}$, can be explained by to the equal amount of demand points all servers are responsible for.

As initially noted by LARSON , a variety of system performance metrics can be calculated with the help of the steady-state probabilities which can then be used to evaluate the performance of the overall system as well as of specific regions or servers. The calculations of some of the key performance metrics that can be derived with the use of the HQM are briefly described and illustrated in the following.[145]

Average Expected Loss Probabilities[146]

The average expected loss probabilities (AELP) describe the probability to which an incoming demand is assumed to be lost. Thereby, they sum up the probabilities of each state in which no server is able to respond to an incoming demand. Note that the calculation of loss probabilities has to be done with respect to districting and the dispatching policy

[145] Cf. LARSON, R. C., 1974, pp. 86–87
[146] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008, p. 211

applied. If a total backup and no queue is utilized,, as for the presented example, the loss probability is the probability of the state in which all servers are busy (here $P\{B_{111}\}$). For cases where not every server can respond to any incident, the preference lists for each demand point have to be taken into account.

The loss probability per demand point is calculated by the sum of the probabilities of all states corresponding to the set of states in which no server is available to respond to a demand, N_j. The system-wide loss probability can be expressed as the weighted sum of the loss probabilities of the demand points with the respective fraction, f_j, of demands as the weight.

$$(2.2.2.3)\ PLoss_j = \sum_{B_a \in N_j}^{2^N-1} P\{B_a\}\ \forall\ j \in J$$

$$(2.2.2.4)\ AELP = \sum_{j=1}^{J} f_j PLoss_j$$

The calculation of the (system-wide) loss probabilities is straightforward due to the above mentioned full-backup and the equal demand rates of all demand points in the presented numerical example. Therefore, the loss probabilities for every demand point ($PLoss_j$) as well as the system-wide loss probabilities (PLoss) are 13,43 percent for the example. Note that ‘0,1343’ in table 5 refers to 13,43 percent. This remains true for the rest of this work.

Fraction of dispatches[147]

The fraction of dispatches is usually formulated in respect to a server and can, for example, express the fraction of dispatches one server sends to one demand point (or region). Thereby, it sums up the probability of all states in which the respective server is responsible[148] to be dispatched to a demand from the associated demand point.

The fraction of dispatches p_{nj} server n sends to demand point j is therefore calculated by the sum of all states a in which server n is responsible and will be dispatched ($B_a \in E_{nj}$) in the case of a demand in demand point j.

[147] Cf. LARSON, R. C., 1974, p. 87
[148] Please note that this formulation does not require the definition of a dispatch policy.

$$(2.2.2.5)\ p_{nj} = \frac{\sum_{B_a \in E_{nj}} P\{B_a\}}{1 - PLoss_j}\ \forall\, j \in J, n \in N$$

Since the loss probabilities usually are greater than zero, the denominator of the equation above requires the sum of the dispatches all servers n send to demand point j to be equal to one. Note that the formulation used here differs from other common formulations as in LARSON[149], GEROLIMINIS, KARLAFTIS AND SKABARDONIS[150] or ENAYATI ET AL. [151] These would use the expression (1- $P\{B_{111}\}$) in the case of a three-server system. That, however, is only valid for a full backup and would lead to an underestimation of the resulting response time. The formulation developed here can be used for partial as well as for full backups alike. Additionally, the fraction of demands of demand point j is not used in the calculation of the fraction of dispatches, but for the response time. The results for the numerical example are given in table 6 below:

	Server		
Demand Point	**1**	**2**	**3**
1	0,6551	0,2414	0,1035
2	0,1035	0,6551	0,2414
3	0,2414	0,1035	0,6551

Table 6: Fraction of dispatches numerical example HQM

p_{11} is calculated by $\frac{P\{B_{000}\}+ P\{B_{010}\}+ P\{B_{001}\}+ P\{B_{011}\}}{(1- P\{B_{111}\})} = \frac{0,2388+0,1194+0,1194+0,0897}{1-0,1343} = 0,6551$. Obviously, the preferential server for each demand point is dispatched in the majority of cases. In the case of a partial backup, some servers would not be part of the preference list for a demand point and therefore would have the probability of zero percent to be dispatched.

[149] Cf. LARSON, R. C., 1974, p. 87
[150] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009, p. 801
[151] Cf. ENAYATI, S. ET AL., 2019, p. 422

Average Expected Response Time (AERT) [152]

The calculation of the fraction of dispatches is mostly used for the estimation of the average expected response time (AERT) of the overall system or for certain demand points (regions). Therefore, the fraction of dispatches of server n to demand point j is multiplied with the corresponding expected travel time, t_{nj}, and weighted with the fraction of demand point j on the overall demand, f_j.

$$(2.2.2.6)\quad AERT = \sum_{n=1}^{N} \sum_{j=1}^{j} f_j \, p_{nj} \, t_{nj}$$

For the numerical example, the following travel times (in minutes) are assumed:

	Server		
Demand Point	**1**	**2**	**3**
1	**6**	**8**	**10**
2	**10**	**6**	**8**
3	**8**	**10**	**6**

Table 7: Travel times numerical example HQM

For the example, the system-wide AERT as well as the demand point specific response time, $AERT_j$, is 6,897 minutes.

There are several other performance metrics that make use of the response time, like the shortest and longest response times to a demand point or for a region of the overall study area.

Workloads[153]

The steady-state probabilities can also be used to express the extent to which a given server is expected to be unavailable for incoming demands, which is called a workload. Therefore, the probabilities of all states in which a specific server is unavailable, $B_a \in N_B^n$, are summed up.

[152] Cf. LARSON, R. C., 1974, p. 87
[153] Cf. LARSON, R. C., 1974, p. 86

$$(2.2.2.7)\ w_n = \sum_{B_a \in N_B^n}^{2^N-1} P\{B_a\}$$

For the numerical examples, the following server workloads can be calculated:

	Server 1	Server 2	Server 3
Workload	**0,4329**	**0,4329**	**0,4329**

Table 8: Server workloads numerical example HQM

The workload of the respective examples of servers are identical due to the full backup and identical demand and service rates between the demand points. The workload of server 1 is calculated as follows:

$P\{B_{100}\}+ P\{B_{110}\}+ P\{B_{101}\}+ P\{B_{111}\}= 0{,}1194 + 0{,}0896 + 0{,}0896 + 0{,}1343 = 0{,}4329$.

2.3 Hypercube Queueing Models and Hypercube Queueing Location Models

The following chapter contains a review of the literature on Hypercube Queueing Models and Hypercube Queueing Location Models. Since some of the Hypercube Queueing Location Models also offer theoretical contributions to the general field of Hypercube Queueing Models, chapter 2.3.1 includes types of models. In Chapter 2.3.2 an overview about Hypercube Queueing Location Models will be given, focusing on aspects that are relevant for the location decision.

The literature reviews of chapters 2.3 to 2.6 are all performed with a procedure built on WEBSTER AND WATSON[154], SAUNDER, LEWIS AND THORNHILL[155] and FARAHANI ET AL.[156] which is described below. The aim of the literature review is to survey the existing body of academic literature with regard to EMS system facility location and analysis.

Sources

SCOPUS and Google Scholar are used as sources for the literature review. The latter is applied especially to double check on papers that are either not listed in SCOPUS at all or not yet listed.

[154] Cf. WEBSTER, J.; WATSON, R. T., 2002
[155] Cf. SAUNDER, M.; LEWIS, P.; THORNHILL, A., 2012, p. 127
[156] Cf. FARAHANI, R. Z. ET AL., 2019, p. 3

Keywords

The literature review was performed with the sole use or the combination of the following keywords: “Queueing Theory”, “Hypercube Queueing”, “Hypercube Queueing Model”, “Emergency Medical”, “Emergency Medical System”, “Emergency Service”, “Emergency Service System”, “Facility Location”, “Robust Optimization”, “Goal Programming”, “Multi-Objective”, “Uncertainty”, “Coverage Model”, “Maximum Expected Coverage”.

Moreover, all papers have been screened for additional literature relevant to this work. If any single author or a combination of authors are found to be of particular relevance, their respective profiles on platforms like ResearchGate were also searched for additional literature. All literature reviews that have been found were also checked for additional literature.

Inclusion and exclusion criteria

The literature review focuses on peer-reviewed journal papers but also includes book chapters, conference proceedings, monographs and presumably non-peer-reviewed journals.

Any paper that does not explicitly focus on operations research techniques that are relevant for the design and operation of EMS systems is excluded from further analysis. As a result, the emphasis, especially for chapters 2.4, 2.5 and 2.6, of the review is on papers that discuss the facility location problem. However, some papers do also consider other neighboring problem sets like the vehicle routing problem. The papers listed in chapter 2.3.1 do focus on the broader field of, exact or approximate, HQM models. Note that not all papers analyzed in this chapter include a location decision.

Time Horizon

The research is not limited to any time period. However, the analysis shows that the research field has gained attention since the early 1970s and the majority of papers has been published since 1990.

2.3.1 Hypercube Queueing Models

The original HQM was introduced by LARSON as a combination of facility location and analysis.[157] By using queueing-theory based arguments, its original purpose was to explore the operational behavior of response systems, like police patrols, fire stations or vehicles of emergency medical systems. The model allows for the decision maker to analyze several system-wide performance metrics that can be calculated by the 'server busy' probabilities. LARSON additionally states the calculations for some key performance metrics such as the system response time, the server workloads and several specific metrics for individual areas of the study area.

The use of the exact HQM can still be computationally prohibitive to date for the analysis of large-scale study areas in which more than 15 servers have to be located in order to derive sufficient performance metrics. Therefore, LARSON developed an approximate-HQM (A-HQM) to deal with the computational burdens.[158] The original HQM results in $2^{\text{Number of Servers}}$ states of the system. For each state one linear equation is formulated which forms an equation system that needs to be solved in order to derive the 'server busy' probabilities. While the location of ten servers results in 1,024 equations, the location of fifteen servers requires 32,768 equations. It may well be possible that the solving time for the linear equation system will be exceeded by the time that is necessary to derive its coefficients. The procedure by LARSON reduces the state-space from $2^{\text{Number of Servers}}$ to N by using the theory of M/M/N queues in which servers are randomly selected until an available server is found. The exact HQM is approximated by so-called 'correction factors'. A convergence algorithm is stated which allows for the approximation of up to one-hundred servers. The A-HQM results in server workloads that are up to two percent less accurate than those obtained by the exact method.

Up to the date of this work, a fixed value for the service provided by the system is used by a large amount of studies. Especially in large-scale study areas, the service provided can be highly dependent on the location of the incoming demand call for emergency help due to long travel times from the server location to the demand incident. HALPERN therefore formulate an approach that uses demand point specific service rates and analyze the consequences for a two-server-system. They also provide an approximation method for

157 Cf. LARSON, R. C., 1974
158 Cf. LARSON, R. C., 1975

simple systems.[159]

LARSON AND FRANCK analyze the dispatching consequences of an automatic emergency vehicle location. Therefore, they assume that the nearest-available server is dispatched to all incidents and evaluate the consequences of the fixed dispatch policy. Additionally, they state computation and storage minimizing techniques which build on the theory of M/M/N queues as for the A-HQM.[160]

BERMAN, LARSON AND ODONI extend the Q-median problem for congestion of the system by applying a HQM. Thereby, they use probabilistic factors in the usually deterministic nature of the Q-median problem which then allows for the possibility that a requested server is not available at the time of an incident. [161] Their study has later been extended by BERMAN AND LARSON for the consideration of backup coverages.[162]

All HQM models described above use the assumption that only one server can be dispatched to an incident at once. Since large-scale emergencies may require the dispatch of multiple units to account for the larger number of affected persons, CHELST AND BARLACH incorporate the multiple dispatch into the HQM. Therefore, they apply a fixed preference list of servers for each demand point considered and, in the case of a multiple dispatch, dispatch the two highest ranked and available servers to the incident. For simplification, they assume identical service rates for single and multiple dispatches of emergency units. To lower computational burdens, they use the A-HQM.[163]

JARVIS develops an approximation procedure for the HQM (J-HQM) that explicitly considers multiple types of customers and distinguishable servers. As in LARSON[164], the approximation method results in a N-sized linear equation system with N as the number of servers that need to be located. The resulting linear equation system is then solved by a convergence algorithm with an error percentage of up to five percent.[165]

BRANDEAU AND LARSON use the HQM to deploy ambulances in Boston. They also introduce an option for varying service times, a mean service time calibration and a travel time

[159] Cf. HALPERN, J., 1977
[160] Cf. LARSON, R. C.; FRANCK, E. A., 1978
[161] Cf. BERMAN, O.; LARSON, R. C.; ODONI, A. R., 1981
[162] Cf. BERMAN, O.; LARSON, R. C., 1982
[163] Cf. CHELST, K. R.; BARLACH, Z., 1981
[164] Cf. LARSON, R. C., 1975
[165] Cf. JARVIS, J. P., 1985

estimation algorithm which adds increased realism to the model.[166]

LARSON AND RICH apply the HQM for New York City Police Patrol cars to analyze new patrol and response strategies which results in larger areas of responsibility for each server.[167]

BATTA, DOLAN AND KRISHNAMURTHY combine the HQM and the Maximum Expected Coverage Location Problem (MEXCLP). They use the HQM to relax the assumptions that the servers operate independently and have the same busy probabilities even without considering their location. Therefore, as in BERMAN, LARSON AND ODONI[168] and BERMAN AND LARSON[169], the HQM is applied to model the potential congestion of the system. To reduce computational burden, BATTA, DOLAN AND KRISHNAMURTHY propose to use the locations obtained by the MEXCLP in the HQM for a more thorough analysis. Additionally, they state an approximate HQM (B-HQM) with the help of 'correction factors' as for the A-HQM.[170] CHIYOSHI ET AL. issue a note on the study to underline the existing differences between the MEXCLP and the HQM which cannot be easily compared because of the difference in their objective functions and the need to clearly state the service times used in the HQM due to its major influence on the system behavior.[171]

GOLDBERG AND PAZ develop an approximate HQM that uses the demand location for the calculation of the service rates. The approximation approach is based on the J-HQM but extended for call-specific service rates and multiple objectives. Additionally, they compare different approximation techniques.[172]

BURWELL, MCKNEW AND JARVIS apply the HQM to a real-world case by using historical data. The authors conclude that the HQM can provide a realistic approximation of the behavior of the analyzed system if the input factors are defined correctly and to enough detail.[173] BURWELL, MCKNEW AND JARVIS use the same case study and extend the model to locate more than one server at a server location. Co-located servers can result in equal preferences since the servers are equally preferable in the case of an incoming demand

[166] Cf. BRANDEAU, M. L.; LARSON, R. C., 1986
[167] Cf. LARSON, R. C.; RICH, T. F., 1987
[168] Cf. BERMAN, O.; LARSON, R. C.; ODONI, A. R., 1981
[169] Cf. BERMAN, O.; LARSON, R. C., 1982
[170] Cf. BATTA, R.; DOLAN, J. M.; KRISHNAMURTHY, N. N., 1989
[171] Cf. CHIYOSHI, F. Y.; GALVÃO, R. D.; MORABITO, R., 2003
[172] Cf. GOLDBERG, J.; PAZ, L., 1991
[173] Cf. BURWELL, T. H.; MCKNEW, M. A.; JARVIS, J. P., 1992

call if for example the nearest-available dispatch policy is used. They show that co-located servers are equally beneficial to the key performance metrics of the system and can offer improvements in parts of the study area in which there is a high density of incoming demand calls. In addition, the authors state a model that randomly selects a responsible server in the case of equal preferences.[174]

ZHU AND MCKNEW exert the HQM in a goal-programming EMS system location study. They intend to minimize the imbalances between the workloads of the servers and therefore formulate a workload balance model. The HQM is then used to reproduce the real-world behavior of the current system and to validate the findings.[175]

MENDONÇA AND MORABITO apply the HQM to analyze the performance metrics in a real-world case study and propose a technique of varying the areas of responsibilities to reduce inequalities among server workloads.[176]

AYTUG AND SAYDAM state a Genetic Algorithm (GA) that combines the HQM with an expected coverage approach to solve the MEXCLP with greater realism and accuracy. The J-HQM is employed to compute the objective function and the congestion for each solution candidate.[177]

GEROLIMINIS, KARLAFTIS AND STATHOPOULOS embed the HQM in the MCLP for districting and dispatching decisions.[178]

GALVÃO, CHIYOSHI AND MORABITO present a study that compares the MEXCLP to the Maximum Availability Location Problem (MALP) to identify similarities and existing differences. The J-HQM is used to relax the server independence and workload equality assumption for greater realism.[179]

ATKINSON ET AL. develop two heuristics that approximate the loss probabilities of incoming demand calls (AT-HQM). The first heuristic builds on the calculation of the fraction of last calls for each server location individually while the second heuristic groups the server locations.[180]

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[174] Cf. BURWELL, T. H.; JARVIS, J. P.; MCKNEW, M. A., 1993
[175] Cf. ZHU, Z.; MCKNEW, M. A., 1993
[176] Cf. MENDONÇA, F. C.; MORABITO, R., 2001
[177] Cf. AYTUG, H.; SAYDAM, C., 2002
[178] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; STATHOPOULOS, A., 2004
[179] Cf. GALVÃO, R. D.; CHIYOSHI, F. Y.; MORABITO, R., 2005
[180] Cf. ATKINSON, J. B. ET AL., 2006

IANNONI AND MORABITO propose a HQM for the analysis of EMS systems on highways that incorporates dispatches of multiple servers to one incident and a partial backup. Additionally, they introduce different types of servers that can then be dispatched to different types of incidents. They also allow for demand to be located directly at the server location which could be created by patients coming directly to this location.[181]

TAKEDA, WIDMER AND MORABITO analyze the consequences of server decentralization for a real-world case study. Instead of locating all servers at one place, they decentralize them at different locations and introduce additional servers. The authors show that, if a large number of servers is located over the study area and not centralized, key performance metrics like the response time, fractions of calls served by backups and server workloads improve.[182]

ATKINSON ET AL. extend the earlier study by ATKINSON ET AL.[183] for customer dependent service rates and different demand classes. The resulting HQM is then extended for a third state and the state descriptions which consist of the respective server either being free or serving one of the two demand classes. Since the state space then increases to $3^{\text{Number of Servers}}$, the authors state two different heuristics (AT-HQM2) and a simulation method to estimate the loss probabilities of the system. Their approximation results in an error percentage of up to two percent.[184]

ERKUT, INGOLFSSON AND ERDOGAN state a maximum survival location model (MSLM) that builds on an approximate HQM. The objective function calculates the expected number of patients that survive after a cardiac incident. The J-HQM is used to estimate the system behavior. Additionally, the results of the MSLM are compared to the MEXCLP and MCLP. [185]

GALVÃO AND MORABITO use the HQM to solve common probabilistic location models like the MEXCLP and MALP with an increased realism. They argue that some of the simplifying assumptions do not reconstruct real-world conditions to enough detail and employ the HQM to model the congestion and the portion of incoming demand calls that

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181 Cf. IANNONI, A. P.; MORABITO, R., 2007
182 Cf. TAKEDA, R. A.; WIDMER, J. A.; MORABITO, R., 2007
183 Cf. ATKINSON, J. B. ET AL., 2006
184 Cf. ATKINSON, J. B. ET AL., 2008
185 Cf. ERKUT, E.; INGOLFSSON, A.; ERDOGAN, G., 2008

cannot be served by the first server on the preference list.[186]

IANNONI, MORABITO AND SAYDAM combine the HQM with a genetic algorithm to derive the server locations and the system configurations. Additionally, they vary the size of the areas of responsibility for each server in order to find sufficient dispatch strategies for the derived server locations.[187]

INGOLFSSON, BUDGE AND ERKUT state an approximate HQM (I-HQM) and incorporate uncertainty in the delay as well as the travel time into their analysis., They claim that their model estimates the provided service correctly by incorporating the uncertainty.[188]

MORABITO, CHIYOSHI AND GALVÃO study the difference in the use of homogeneous and non-homogenous servers in the application of the HQM. Therefore, they state three basic HQM models. The first model uses a fixed-dispatch policy with a full backup, the second model a fixed dispatch policy with a partial backup and the third model a random dispatch since all servers are located at the same base and hence the preference lists do not differ. The authors compare the basic operational characteristics of a system with homogenous servers to a system with non-homogenous servers. They conclude that using homogenous servers leads to less predictability than a system with non-homogenous servers.[189]

RAJAGOPALAN, SAYAM AND XIAO extend the Queueing Probabilistic Location Set Covering Model proposed by MARIANOV AND REVELLE[190] by the use of the J-HQM to calculate 'server busy' probabilities and the expected coverage that is provided by the system.[191]

BUDGE, INGOLFSSON AND ERKUT develop an approximate HQM that allows for multiple servers to be stationed at one location. They use the A-HQM and generalize its procedure by computing location-specific (in contrast to server-specific) busy and dispatch probabilities (BIE-HQM). The error percentage produced by this approximation averages to 1,4 percent. If only sections of the study area are considered, the error percentage regarding the coverage values could be up to 8 percent.[192]

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[186] Cf. GALVÃO, R. D.; MORABITO, R., 2008
[187] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008
[188] Cf. INGOLFSSON, A.; BUDGE, S.; ERKUT, E., 2008
[189] Cf. MORABITO, R.; CHIYOSHI, F. Y.; GALVÃO, R. D., 2008
[190] Cf. MARIANOV, V.; REVELLE, C., 1994
[191] Cf. RAJAGOPALAN, H. K.; SAYDAM, C.; XIAO, J., 2008
[192] Cf. BUDGE, S.; INGOLFSSON, A.; ERKUT, E., 2009

GEROLIMINIS, KARLAFTIS AND SKABARDONIS extend their earlier conference paper[193] presenting an HQM-based EMS system location study. Their model inherently designs the first- and lower-tier areas of responsibility for each server and hence provides an approach that determines location and dispatching decisions jointly.[194]

IANNONI, MORABITO AND SAYDAM expand their earlier study[195] by the continuous location of emergency vehicles on highways and discuss measures to deal with trade-offs between key performance metrics of the system such as the response time of the system and the workload imbalance between the individual servers.[196]

MCLAY AND MAYORGA use the A-HQM in an EMS system location study to discuss the influence on response time thresholds on the overall expected survivability of patients.[197]

CHANTA ET AL. employ the A-HQM to derive 'server busy' probabilities in order to calculate equity between demand points.[198]

GEROLIMINIS, KEPAPTSOGLOU AND KARLAFTIS extend the study of GEROLIMINIS, KARLAFTIS AND SKABARDONIS by the incorporation of a larger amount of servers into the location decision. Therefore, they reduce the state-space of the underlying equation system from $2^{\text{Number of Servers}}$ to 'Number of Servers' by assuming symmetrically located servers and, as a result, identical workloads among servers. Thereby, they formulate one equation for each server which is made possible by assuming an equal distribution of the demand points to the servers.[199]

IANNONI, MORABITO AND SAYDAM discuss the EMS system location problem for large-scale emergencies on highways using the AT-HQM. They jointly determine the server locations and the areas of responsibility for each server. Additionally, they compare the results of the approximate and exact HQM and state an error percentage of 0,36 percent.[200]

BAPTISTA AND OLIVEIRA use the J-HQM to compute new system performance metrics such as demand served by an individual unit, demand served subject to a time constraint

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193 Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2006
194 Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009
195 Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008
196 Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2009
197 Cf. MCLAY, L. A.; MAYORGA, M. E., 2010
198 Cf. CHANTA, S. ET AL., 2011
199 Cf. GEROLIMINIS, N.; KEPAPTSOGLOU, K.; KARLAFTIS, M. G., 2011
200 Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2011

and average unit workloads. They validate their findings with a simulation study.[201]

CHANTA, MAYORGA AND MCLAY deploy Tabu-Search in an exact HQM study and compare the nearest-available dispatch policy as well as the random swap of responsibilities after the optimization. They conclude that the nearest-available dispatch policy does not always yield optimal results.[202]

SAYDAM ET AL. apply the J-HQM to minimize the number of servers that is required to cover the study area while meeting minimum coverage requirements. Additionally, they minimize the necessary number of server redeployments to ensure the coverage level.[203]

TORO-DIAZ ET AL. use an exact HQM in an EMS system location study while calculating the expected coverage provided by the system as an additional performance metric. They implement their model in three case studies. The dispatch policy is modelled as a decision variable.[204]

DAVOUDPOUR, MORTAZ AND HOSSEINIJOU employ the HQM to extend the MEXCLP.[205]

GRANNAN, BASTIAN AND MCLAY extend the MEXCLP by using the J-HQM. They use distinct types of servers in a location and dispatching study of military medical evacuation.[206]

SUDTACHAT, MAYORGA AND MCLAY discuss strategies to dispatch emergency vehicles to optimize the likelihood of patient survival. The authors introduce multiple dispatches as well as multiple classes of demand. They make recommendations for dispatch policies based on a simulation study.[207]

BOYACI AND GEROLIMINIS extend their conference paper[208] and solve the HQM with stochastic demand and travel time. The uncertainty in both factors is modelled through a scenario approach. The HQM is extended for a third state in such way that the three states capture whether the server is available, busy at his location or busy responding to a demand call further away. This extends the state space to $3^{\text{Number of Servers}}$. BOYACI AND

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201 Cf. BAPTISTA, S.; OLIVEIRA, R. C., 2012
202 Cf. CHANTA, S.; MAYORGA, M.; MCLAY, L. A., 2012
203 Cf. SAYDAM, C. ET AL., 2013
204 Cf. TORO-DÍAZ, H. ET AL., 2013
205 Cf. DAVOUDPOUR, H.; MORTAZ, E.; HOSSEINIJOU, S. A., 2014
206 Cf. GRANNAN, B. C.; BASTIAN, N. D.; MCLAY, L. A., 2014
207 Cf. SUDTACHAT, K.; MAYORGA, M. E.; MCLAY, L. A., 2014
208 Cf. BOYACI, B.; GEROLIMINIS, N., 2012

GEROLIMINIS tackle this problem by aggregating the servers into groups with an error percentage of less than two percent.[209]

DE SOUZA ET AL. extend the exact HQM by different demand classes and a priority in queue. In the exact HQM, the length of the queue and the priorities of the demand classes represent individual states of the model. Thereby, even relatively small systems and queues extend the state space to a rather large degree. To lower the computational burden, the authors consider a full backup dispatch policy in which an incoming demand call can only be queued if all servers are unavailable. The authors use a simulation study to validate their findings.[210]

In a related study, IANNONI, CHIYOSHI AND MORABITO analyze the behavior of server reservation policies for EMS systems with an HQM. The authors propose an extension for finite-sized queues and a cut-off policy to deal with the different demand classes. The server reservation policy leads to low priority calls not being served even if there are servers available in order to maintain capacities for high priority demands. Therefore, three different demand classes are modelled. The server reservation policy leads to improvements in the key performance metrics for the two higher priority demand classes.[211]

ZAKI AND GHANI use the J-HQM in a location optimization study with multiple server types that allows for redeployment.[212]

TORO-DÍAZ ET AL. apply the AT-HQM2 and BIE-HQM in an EMS system location study. They state several additional performance criteria like the expected coverage provided by the system, the expected average response time, the server workloads and the Gini coefficient as an equity metric. The latter shows the increase or decrease in the response time that the individual demand points are served with if compared to the average. Additionally, they compute the Gini coefficient to measure the differences between the server workloads.[213]

ANSARI, MCLAY AND MAYORGA study the maximum expected covering problem for district design with the help of an approximate HQM. They use the same approximation

[209] Cf. BOYACI, B.; GEROLIMINIS, N., 2015
[210] Cf. DE SOUZA, R. M. ET AL., 2015
[211] Cf. IANNONI, A. P.; CHIYOSHI, F. Y.; MORABITO, R., 2015
[212] Cf. ZAKI, H.; GHANI, N. A., 2015
[213] Cf. TORO-DÍAZ, H. ET AL., 2015

techniques as TORO-DIAZ ET AL. to calculate 'server busy' probabilities and server workloads. Their model maximizes the expected coverage and the proportion of high priority calls that can be served subject to a pre-defined time constraint while maintaining a balanced server workload. The model determines the dispatch policy and the server location and results in an iterative update to the 'server busy' probabilities. The error percentage of the approximate HQM is up to two percent.[214]

ANSARI, YOON AND ALBERT state an approximate HQM (AYA-HQM) that co-locates servers, accounts for the multiple dispatch of servers and does not assume a particular dispatch policy. They determine an approximation technique and an error percentage of up to 5,6 percent but show that their model is, on average, more accurate than other approximation techniques for their case study.[215]

RODRIGUES ET AL. propose an extension to the study of DE SOUZA ET AL. for partial backup and priority in queue with regard to different classes of demand. Additionally, they extend the finite queue for the exact tracking of the location of the queued demand and use a three-server system. Since the assumption of partial backup and the exact tracking lead to an excessive amount of states for larger-scale systems, the authors state an approximation technique that does not include the demand location in the state definition. They show an error percentage of 0,5 percent.[216]

SABEGH ET AL. use the HQM in a multi-objective EMS system location study that minimizes the expected average response time, the fixed costs as well as the environmental impact of the system.[217]

VAN BARNEFELD, VAN DER MEI AND BHULAI apply the J-HQM in an EMS system relocation study that allows for multiple types of vehicles and demands.[218]

AKDOĞAN, BAYINDIR AND IYIGUN state a service-rate formulation extension for the HQM. They explicitly use the demand origin in their service-rate calculation. They validate their findings with a simulation study for different levels of backup. Their model assumes a fixed dispatch policy but inherently defines the areas of responsibility for each server.[219]

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[214] Cf. ANSARI, S.; MCLAY, L. A.; MAYORGA, M. E., 2017
[215] Cf. ANSARI, S.; YOON, S.; ALBERT, L. A., 2017
[216] Cf. RODRIGUES, L. F. ET AL., 2017
[217] Cf. SABEGH, M. H. Z. ET AL., 2017
[218] Cf. VAN BARNEVELD, T. C.; VAN DER MEI, R. D.; BHULAI, S., 2017
[219] Cf. AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018

KARIMI, GENDREAU AND VERTER determine an approximate HQM (K-HQM) that builds on the original A-HQM. Their model extends the existing body of literature with the assumption that each individual server is responsible for an arbitrary subset of the study area. They also include the condition that the service time is dependent on the location of the incoming demand. The stated error percentage is up to 5,5 percent.[220]

RODRIGUES ET AL. extend the study of RODRIGUES ET AL.[221] for a six server system while also using a simulation study to validate the findings.[222]

ENAYATI ET AL. use the HQM in an EMS system location study with different demand classes. They calculate several performance metrics and use a multi-objective approach. Therefore, they calculate pareto-optimal solutions between two criteria, like loss probabilities and average response time.[223]

BEOJONE, DE SOUZA AND IANNONI employ an exact HQM with distinct servers where one group of servers can only respond to immediately life-threatening calls. In order to deal with the computational efforts, an aggregation model that groups servers at the same station is proposed.[224]

BLANK states an aggregation technique that builds on facilitating the computational effort on deriving the linear equation system for large-scale systems.[225]

JENKINS, LUNDSAY AND ROBBINS use a B-HQM to locate assets for the military evacuation in a robust optimization approach.[226]

RODRIGUEZ, DE LA FUENTE AND AGUAYO combine the Facility Location Equipment Emplacement Technique (FLEET) and the J-HQM. They apply the latter to calculate the server busy probabilities while also accounting for multiple dispatches and multiple types of demands.[227]

BLANK uses the exact HQM in a location optimization study for refugee camps. After

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[220] Cf. KARIMI, A.; GENDREAU, M.; VERTER, V., 2018
[221] Cf. RODRIGUES, L. F. ET AL., 2017
[222] Cf. RODRIGUES, L. F. ET AL., 2018
[223] Cf. ENAYATI, S. ET AL., 2019
[224] Cf. BEOJONE, C. V.; DE SOUZA, R. M.; IANNONI, A. P., 2020
[225] Cf. BLANK, F., 2020
[226] Cf. JENKINS, P. R.; LUNDAY, B. J.; ROBBINS, M. J., 2020
[227] Cf. RODRIGUEZ, S. A.; DE LA FUENTE, R. A.; AGUAYO, M. M., 2020

completing the location decision, outbreaks of COVID-19 are simulated to test the resilience of the system. Additionally, two servers are located that solely treat virus patients.[228]

LIU ET AL. use the exact HQM in multi-dispatch and cooperative service study.[229]

To provide a comprehensive overview of the existing HQM literature, all mentioned contributions are listed in table 11 and categorized according to the following categories:

- Single or multiple dispatch
- Total or partial Backup
- Homogenous or non-homogenous server
- Demand Classes
- Queue
- Exact or approximate HQM
- Location Optimization
- Main contribution

[228] Cf. BLANK, F., 2021
[229] Cf. LIU, H. ET AL., 2021

	Single Dispatch	**Multiple Dispatch**	**Total Backup**	**Partial Backup**	**Ho-moge-nous Server**	**Non-Homo-genous Server**	**Demand Classes**	**Queue**	**Exact HQM**	**Approx. HQM**	**Location Optimiza-tion**	**Main Contribution**
Larson (1974)	X		X		X				X			Initial HQM
Larson (1975)	X		X		X			X		X		Approxima-tion
Halpern (1977)	X		X		X		X		X	X		Service rates and perfor-mance criteria
Larson and Franck (1978)	X		X		X				X			Evaluation of dispatch consequences
Berman, Larson and Odoni (1981)	X		X		X					X	X	Extensions of network prob-lems for congestion
Chelst and Barlach (1981)		X	X		X		X		X	X		Extension for multiple dispatch
Berman and Larson (1982)	X		X		X		X	X		X	X	P-Median with conges-tion
Larson and McKnew (1982)	X		X			X				X		Extension for third state
Jarvis (1985)	X		X			X	X			X		Approxima-tion

	Single Dispatch	Multiple Dispatch	Total Backup	Partial Backup	Homogenous Server	Non-Homogenous Server	Demand Classes	Queue	Exact HQM	Approx. HQM	Location Optimization	Main Contribution
Brandeau and Larson (1986)	X		X			X			X			Case-Study
Larson and Rich (1987)	X		X		X				X			Travel time analysis
Batta, Dolan and Krishmamurthy (1989)	X		X		X			X		X	X	Extension of MEXCLP
Goldberg and Paz (1991)	X		X		X		X			X	X	Stochastic travel times
Burwell, McKnew and Jarvis (1992)	X		X		X		X			X		Case-Study
Burwell, McKnew and Jarvis (1993)	X		X			X	X			X		Extension for co-located and dispatch preferences
Zhu and McKnew (1993)	X		X			X				X	X	Goal Programming approach

	Single Dispatch	Multiple Dispatch	Total Backup	Partial Backup	Homogenous Server	Non-Homogenous Server	Demand Classes	Queue	Exact HQM	Approx. HQM	Location Optimization	Main Contribution
Mendonça and Morabito (2001)	X			X	X				X			Case-Study
Chiosyhi, Galvão and Morabito (2003)	X		X		X		X		X		X	Note on Batta, Dolan and Krishamurty (1989)
Aytug and Saydam (2003)	X		X		X			X		X	X	Extension of MEXCLP
Geroliminis, Karlaftis and Stathopoulos (2004)	X			X	X				x		X	MCLP + Hypercube
Galvão, Chiyoshi and Morabito (2005)	X		X		X					X	X	Extensions of MEXCLP and MALP
Atkinson et al. (2006)	X			X	X		X			X		Heuristics for approximating server busy probabilities

	Single Dispatch	Multiple Dispatch	Total Backup	Partial Backup	Homogenous Server	Non-Homogenous Server	Demand Classes	Queue	Exact HQM	Approx. HQM	Location Optimization	Main Contribution
Geroliminis, Karlaftis and Skabardonis (2006)	X			X	X				X		X	Conference prequel to Geroliminis, Karlaftis and Skabardonis (2009)
Iannoni and Morabito (2007)		X		X		X	X		X			Multiple dispatch with partial backups in a case study
Takeda, Widmer and Morabito (2007)	X		X		X		X	X	X			Case study
Atkinson et al. (2008)	X			X	X		X			X		Customer-dependent service rates
Erkut, Ingolfsson and Erdogan (2008)	X		X			X				X		Maximization of survival rates

	Single Dispatch	Multiple Dispatch	Total Backup	Partial Backup	Homogenous Server	Non-Homogenous Server	Demand Classes	Queue	Exact HQM	Approx. HQM	Location Optimization	Main Contribution
Galvão and Morabito (2008)	X		X		X				X		X	Extension of MEXCLP and MALP
Iannoni, Morabito and Saydam (2008)		X		X		X	X		X		X	Combination of HQM and Genetic Algorithm
Ingolfsson, Budge and Erkut (2008)	X		X			X	X			X	X	Ambulance location under random delays
Morabito, Chiyoshi and Galvão (2008)	X		X			X			X			Case-Study
Rajagopalan, Sayam and Xiao (2008)	X		X		X					X	X	Multi-period Set Covering Model for ambulance redeployment
Budge, Ingolfsson and Erkut (2009)	X		X			X	X			X		Approximation of server busy probabilities

	Single Dispatch	**Multiple Dispatch**	**Total Backup**	**Partial Backup**	**Homogenous Server**	**Non-Homogenous Server**	**Demand Classes**	**Queue**	**Exact HQM**	**Approx. HQM**	**Location Optimization**	**Main Contribution**
Geroliminis, Karlaftis and Skabardonis (2009)	X			X	X				X		X	Combination of HQM, districting and dispatching decisions
Iannoni, Morabito and Saydam (2009)		X		X		X	X		X		X	Extension of Iannoni, Morabito and Saydam (2008) for districting decisions
Mclay and Mayorga (2010)	X		X		X		X	X		X		Evaluation of performance of time threshold criteria
Chanta et al. (2011)	X			X	X					X	X	Minimization of envy between demand points
Geroliminis et al. (2011)	X			X	X					X	X	State reduction

	Single Dispatch	Multiple Dispatch	Total Backup	Partial Backup	Homogenous Server	Non-Homogenous Server	Demand Classes	Queue	Exact HQM	Approx. HQM	Location Optimization	Main Contribution
Iannoni, Morabito and Saydam (2011)	X			X	X					X	X	Use of the approx. HQM for large-scale problems
Baptista and Oliveira (2012)	X		X			X	X	X		X		Computation of performance metrics
Boyaci and Geroliminis (2012)	X			X		X	x			X	X	Introduction of additional states
Chanta, Mayorga and McLay (2012)	X			X	X				x		X	Use of Tabu-Search
Saydam et al. (2013)	X		X		X					X	X	Minimization of number of ambulances and redeployments
Toro-Díaz et al. (2013)	X		X	X	X				X		X	Performance criteria in an exact HQM location study

	Single Dispatch	**Multiple Dispatch**	**Total Backup**	**Partial Backup**	**Homogenous Server**	**Non-Homogenous Server**	**Demand Classes**	**Queue**	**Exact HQM**	**Approx. HQM**	**Location Optimization**	**Main Contribution**
Davoudpour, Mortaz and Hosseinijou (2014)		X	X			X	X		X		X	Combination of MEXCLP and HQM
Grannan, Bastian and McLay (2014)	X		X			X	X			X	x	Extensions of MEXCLP
Sudtachat, Mayorga and McLay (2014)		X		X	X		X			X		Recommendations for dispatch policies
Boyaci and Geroliminis (2015)	X		X			X	X			X	X	Scenario-based uncertainty in three-state HQM
De Souza et al. (2015)	X		X		X		X	X	X			Combination of demand classes and queues
Iannoni, Chiyoshi and Morabito (2015)	X		X			X	X	X	X			Server reservation and cut-off HQM

	Single Dispatch	**Multiple Dispatch**	**Total Backup**	**Partial Backup**	**Homogenous Server**	**Non-Homogenous Server**	**Demand Classes**	**Queue**	**Exact HQM**	**Approx. HQM**	**Location Optimization**	**Main Contribution**
Toro-Díaz et al. (2015)	X		X			X	X			X	X	Large-scale and fairness criteria
Zaki and Ghani (2015)	X			X		X	X			X	X	Redeployment + two server types
Ansari, McLay and Mayorga (2017)	X		X		X		X			X	X	HQM approximation for district design
Ansari, Yoon and Albert (2017)		X	X			X	X			X		Approximation
Rodrigues et al. (2017)	X			X	X		X	X	X	X		Partial backup in queue
Sabegh et al. (2017)	X			X	X				X		X	Multi-objective optimization with HQM
Van Barnefeld, van der Mei and Bhulai (2017)		X		X		X	X			X	X	Relocation study

	Single Dispatch	Multiple Dispatch	Total Backup	Partial Backup	Homogenous Server	Non-Homogenous Server	Demand Classes	Queue	Exact HQM	Approx. HQM	Location Optimization	Main Contribution
Akdoğan, Bayindir and Iyigun (2018)	X		X		X					X	X	Service-rate formulation
Karimi, Gendrau and Verter (2018)	X		X		X			X		X		Approximation technique
Rodrigues et al. (2018)	X			X	X		X	X	X			Extension of Rodrigues et al. (2017) for more Servers
Enayati et al. (2019)	X			X	X		X		X		X	Multi-objective optimization + equity criteria
Beojone et al. (2020)	X			X		X	X		X			Exact HQM and distinguishable servers
Blank (2020)	X			X	X				X		X	Aggregation technique
Jenkins, Lundsay and Robbins (2020)	X			X	X					X	X	Robust optimization and multiple objectives

	Single Dispatch	Multiple Dispatch	Total Backup	Partial Backup	Homogenous Server	Non-Homogenous Server	Demand Classes	Queue	Exact HQM	Approx. HQM	Location Optimization	Main Contribution
RODRIGUEZ, DE LA FUENTE AND AGUAYO (2020)		X	X			X	X			X	X	Combination of FLEET and HQM
BLANK (2021)	X			X	X		X		X		X	Optimization and simulation study
LIU ET AL. (2021)		X		X		X	X		X			Multi-dispatch study

Table 9: HQM literature overview

As will become obvious from table 9, the majority of the HQM related literature only considers the single dispatch of one server to an incident. From a modelling and computational performance view, this can be a benefit since the necessary calculations for the coefficients have to be done only for the single dispatch which may lead to approximately of up to twice the computational time.

The very early HQM studies mostly use the full-backup assumption. More recent studies use partial backup. The decision to which degree backup servers are available for incoming demands should be considered as a strategic decision. As mentioned, for example, by GEROLIMINIS, KARLAFTIS AND SKABARDONIS[230] and ENAYATI ET AL.[231] a high level of backup is not necessarily beneficial for all performance metrics. Usually the expected response time of the system will increase if more distant servers are dispatched to an incident. Obviously, other performance metrics may decrease like the loss probabilities.

To this day a majority of the HQM related literature uses homogenous servers in such way that every server is able to respond to every incident. This can be seen as a rather significant simplification since there can be demand for emergency help, like cardiac incidents, that require the response of a trained emergency medic.

The concept of differentiating incoming demand classes is also referred to as 'layering' in the literature. The layering concept has been used from the very first HQM studies on and can contribute towards more realistic assumptions in HQM and EMS system models in general. Considering the fraction of studies that use layering, there is no obvious trend towards a more or less frequent use. Some of the assumptions, like multiple dispatch, server reservation policies or priority queues, require the incorporation of different demand classes.

The queueing of incoming demand calls that cannot be immediately served due to a congested system or server reservation policies has been a focus in recent research. For approximate HQMs, this procedure has been employed more extensively in past studies. This is due to the use of traditional Queueing Theory based arguments to approximate the HQM. For the exact HQM, there has been a shift towards the incorporation only recently. Interestingly, there has been no location study using the exact HQM that models a real-world waiting line even for smaller systems.

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[230] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009, p. 802
[231] Cf. ENAYATI, S. ET AL., 2019, p. 12

Due to the computational efforts of solving the HQM exactly, approximation techniques have been one string of research since the very beginning. The approximate HQM was used more often than the exact version up to the early 2010s. Since computational power began to increased significantly, recent HQM studies have been using the exact hypercube more often. This can also be observed in HQM location optimization studies.

2.3.2 Hypercube Queueing Location Models

As mentioned in chapter 2.2.2, a comprehensive overview will be given over HQM related location optimization studies for EMS systems which concentrate on the factors relevant for the location optimization decision. Note that only studies that use the approximate or exact HQM for the necessary calculations in the location decision will be discussed. Therefore, studies like ZHU AND MCKNEW[232] and GALVÃO AND MORABITO[233] are excluded since they use the HQM only for further analysis of their findings. The studies are presented separately for the use of a) the approximate HQM and b) the exact HQM.

2.3.2.1 Hypercube Queueing Location Models with approximate HQM

As noted in chapter 2.2.1, the early approximation techniques for the HQM are used for the analysis of larger scale systems but not in the location decision. The very early HQM related EMS system location studies mostly applied the HQM for the extension of well-known coverage or median problems. BERMAN, LARSON AND ODONI[234] and BATTA, DOLAN AND KRISHNAMURTHY include the HQM in the p-median model and the MEXCLP to model the system congestion.

GOLDBERG AND PAZ state an approximate HQM based on the J-HQM and use their model in an EMS system location study that intends to maximize the number of calls that are served with respect to a pre-defined time constraint.[235]

The studies of AYTUG AND SAYDAM and GALVÃO, CHIYOSHI AND MORABITO both use the J-HQM to extend the MEXCLP and the MALP. Both studies employ the HQM to relax the simplifications of the base models and to account for the system congestion.[236]

ERKUT, INGOLFSSON AND ERDOGAN incorporate the J-HQM in an EMS system location

[232] Cf. ZHU, Z.; MCKNEW, M. A., 1993
[233] Cf. GALVÃO, R. D.; MORABITO, R., 2008
[234] Cf. BERMAN, O.; LARSON, R. C.; ODONI, A. R., 1981
[235] Cf. GOLDBERG, J.; PAZ, L., 1991
[236] Cf. AYTUG, H.; SAYDAM, C., 2002; GALVÃO, R. D.; CHIYOSHI, F. Y.; MORABITO, R., 2005

study that intends to locate servers to maximize the patient survivability.[237]

INGOLFSSON, BUDGE AND ERKUT intend to maximize the expected coverage that is provided by the system by modelling the server busy probabilities through the use of a newly proposed approximate HQM.[238]

RAJAGOPALAN. SAYDAM AND XIAO minimize the number of servers that are necessary to serve the study area. They use the J-HQM to calculate basic coverage metrics of the system and the 'server busy' probabilities.[239]

CHANTA ET AL. use an approximate HQM to derive 'server busy' probabilities in a location model that considers equity between demand points.[240]

GEROLIMINIS, KEPAPTSOGLOU AND KARLAFTIS propose the approximate HQM counterpart to GEROLIMINIS, KARLAFTIS AND SKABARDONIS[241]. Their objective function intends to minimize the response time of the system.[242]

IANNONI, MORABITO AND SAYDAM use the AT-HQM in a large-scale emergency study for EMS systems on highways. They state three separate performance metrics that are used for the minimization of the response time of the system.[243]

SAYDAM AND AYTUG calculate the 'server busy' probabilities by the J-HQM. Their objective function minimizes the number of servers as well as their redeployments during the study period.[244]

GRANNAN, BASTIAN AND MCLAY apply the J-HQM to calculate 'server busy' probabilities in a military evacuation study that maximizes the expected coverage provided by the system.[245]

BOYACI AND GEROLIMINIS[246] and BOYACI AND GEROLIMINIS[247] minimize the demand that is covered by multiple groups of servers in order to maintain a balanced workload.

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[237] Cf. ERKUT, E.; INGOLFSSON, A.; ERDOGAN, G., 2008
[238] Cf. INGOLFSSON, A.; BUDGE, S.; ERKUT, E., 2008
[239] Cf. RAJAGOPALAN, H. K.; SAYDAM, C.; XIAO, J., 2008
[240] Cf. CHANTA, S. ET AL., 2011
[241] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009
[242] Cf. GEROLIMINIS, N.; KEPAPTSOGLOU, K.; KARLAFTIS, M. G., 2011
[243] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2011
[244] Cf. SAYDAM, C. ET AL., 2013
[245] Cf. GRANNAN, B. C.; BASTIAN, N. D.; MCLAY, L. A., 2014
[246] Cf. BOYACI, B.; GEROLIMINIS, N., 2012
[247] Cf. BOYACI, B.; GEROLIMINIS, N., 2015

TORO-DÍAZ ET AL. employ the AT-HQM2 to calculate 'server busy' probabilities in an EMS system location study that intends to minimize disparities between the considered demand classes. Their model intends to minimize the response time of the system.[248]

ZAKI AND GHANI use the J-HQM in a location optimization study that intends to minimize the number of servers that are necessary to provide a pre-defined coverage level. Their study does not include computational experiments. Therefore, no remarks are made regarding the solution technique or the study area or the period.[249]

ANSARI, MCLAY AND MAYORGA maximize the expected coverage provided by the system and calculate the server 'busy probabilities' through a newly stated approximate HQM.[250]

JENKINS, LUNDAY AND ROBBINS state a robust optimization approach that uses the B-HQM to calculate the 'server busy' probabilities.[251]

RODRIGUEZ, DE LA FUENTE AND AGUAYO calculate the 'server busy' probabilities with the help of the J-HQM in a FLEET model based location study.[252]

To provide a comprehensive overview over existing EMS system location studies that use an approximate HQM, a table is provided in the following to categorize the studies according to the following categories:

- Objective function
- Use of additional performance metrics in the analysis
- Number of servers located
- Solution technique
- Decision variable(s)
- Time period
- Dispatch policy

[248] Cf. TORO-DÍAZ, H. ET AL., 2015
[249] Cf. ZAKI, H.; GHANI, N. A., 2015
[250] Cf. ANSARI, S.; MCLAY, L. A.; MAYORGA, M. E., 2017
[251] Cf. JENKINS, P. R.; LUNDAY, B. J.; ROBBINS, M. J., 2020
[252] Cf. RODRIGUEZ, S. A.; DE LA FUENTE, R. A.; AGUAYO, M. M., 2020

EMS System Location Studies with approximate HQM							
	Objective Function	**Performance Metrics**	**Number of Servers located**	**Solution Technique**	**Decision Variable(s)**	**Time Period**	**Dispatch Policy**
Berman, Larson and Odoni (1981)	Min. distance	Yes	2	Proprietary	Location	One	Nearest-Available
Batta, Dolan and Krishnamurthy (1989)	Max. expected coverage	No	3	Proprietary	Location	One	Nearest - Available
Goldberg and Paz (1991)	Max. calls served	Yes	Up to 10	Proprietary	Location	One	Most-Preferred
Aytug and Saydam (2003)	Max. expected coverage	Yes	Up to 6	Genetic Algorithm	Location	One	Most-Preferred
Galvão, Chiyoshi and Morabito (2005)	Max. expected coverage	No	Up to 22	Simulated Annealing	Location	One	Nearest - Available
Erkut, Ingolfsson and Erdogan (2008)	Max. survival	Yes	Up to 25	Branch-and-Cut	Location	One	Most-Preferred
Ingolfsson, Budge and Erkut (2008)	Max. expected coverage	Yes	Up to 21	Branch-and-Bound	Location	One	Nearest-Available
Rajagopalan. Saydam and Xiao (2008)	Min. servers	Yes	Up to 30	Tabu-Search	Location + Number of servers	Varying	Nearest-Available

	Objective Function	Performance Metrics	Number of Servers located	Solution Technique	Decision Variable(s)	Time Period	Dispatch Policy
Chanta et al. (2011)	Min. envy	No	Up to 10	Tabu-Search	Location + Districting	One	Nearest-Available
Geroliminis, Kepaptsoglou and Karlaftis (2011)	Min. response time	No	Up to 25	Genetic Algorithm	Location	One	Nearest-Available
Geroliminis, Kepaptsoglou and Karlaftis (2011)	Min. response time	No	Up to 50	Proprietary	Location + Districting	One	Nearest-Available
Boyaci and Geroliminis (2012)	Min. uncovered demand	No	Up to 54	Branch-and-Cut and Matlab	Location + Districting	One	Nearest-Available
Saydam and Aytug (2013)	Min. servers + redeployments	Yes	Up to 18	Proprietary	Location + Number of servers	One	Nearest-Available
Grannan, Bastian and McLay (2014)	Max. expected coverage	Yes	Up to 14	Branch-and-Bound	Location + Dispatch policy + Districting	One	Decision Variable
Boyaci and Geroliminis (2015)	Min. of demand covered for each partion of the study area	No	12	Branch-and-Cut and Matlab	Location + Districting	One	Nearest-Available
Toro-Díaz et al. (2015)	Min. response time	Yes	Up to 18	Tabu-Search	Location + Dispatch policy	One	Decision Variable

	Objective Function	Performance Metrics	Number of Servers located	Solution Technique	Decision Variable(s)	Time Period	Dispatch Policy
Zaki and Ghani (2015)	Min. servers	No	/	/	Location + Dispatch policy + Districting	/	Decision Variable
Ansari, McLay and Mayorg (2017)	Max. expected coverage	Yes	Up to 8	Proprietary	Location + Districting	One	Varying
Jenkins, Lunday and Robbins (2020)	Max. expected coverage + Min. servers + redeployments	No	Up to 11	Branch-and-Cut	Location + Districting + Redeployment	One	Nearest-Available
Rodriguez, De la Fuente and Aguayo (2020)	Max. expected coverage	No	Up to 13	Proprietary	Location + Districting	One	Nearest-Available

Table 10: EMS system location studies with approximate HQM

The existing body of studies on approximate HQM location uses a variety of objective functions. The J-HQM is used most often. Since the approximate HQM has often been applied to relax existing simplifications of common coverage, the objective function consequently maximizes the expected coverage that can be provided by the system. Due to the significantly lower computational efforts implied, the number of servers used can be easily varied and included in the analysis. RAJAGOPALAN, SAYDAM AND XIAO[253] as well as SAYDAM ET AL.[254] employ the number of servers located in their objective function and in the decision variable.

One of the key benefits of the, exact or approximate, HQM is the possibility of further analysis of the solution found by other performance metrics that can be calculated (mostly) through the 'server busy' probabilities. Approximately half of the studies mentioned use additional performance metrics.

As mentioned above, the number of servers can be easily varied in approximate HQM location studies. Note that only a minority of studies actually utilizes the number of servers as a decision variable. Most of the studies vary the number of servers to actually calculate the objective function for the respective number of servers as well as to analyze the computational performance of the approximation with regard to the error percentage or the computational time.

Since usually the presented objective functions have no closed-form expression, a solution technique is required to derive an optimal or near-optimal solution. The analyzed body of literature therefore employs a variety of techniques. Some of the studies use commercial solvers, like CPLEX, and Branch-and-Cut or Branch-and-Bound procedures. Other studies develop own techniques suited for their individual problems which are categorized in table 12 as 'proprietary'. The remaining studies apply modern heuristics like the Genetic Algorithm, Tabu-Search or Simulated Annealing.

Most of the studies determine the location of the servers as their sole decision variable. IANNONI, MORABITO AND SAYDAM[255] and ANSARI, MCLAY AND MAYORGA[256] both

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[253] Cf. RAJAGOPALAN, H. K.; SAYDAM, C.; XIAO, J., 2008
[254] Cf. SAYDAM, C. ET AL., 2013
[255] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2011
[256] Cf. ANSARI, S.; MCLAY, L. A.; MAYORGA, M. E., 2017

choose the portioning of the study area as an additional decision variable while RAJAGOPALAN, SAYDAM AND XIAO[257] and SAYDAM ET AL.[258] also inherently use the actual number of ambulances. Interestingly, only TORO-DÍAZ ET AL. use the dispatch policy as a decision variable.[259] Note that, especially for large-scale study areas, this can lead to an excess amount of solutions that need to be calculated. Moreover, if the dispatch policy is applied as a decision variable, it can potentially vary between neighboring demand points which can then lead to legal and ethical issues.

Only RAJAGOPALAN, SAYDAM AND XIAO include more than one time period in their analysis.[260]

As mentioned above, the dispatch policy is mostly pre-determined in the model and resorts mostly to the 'nearest' or 'first' available assumption. Some studies use the 'most-preferred' dispatch policy which may also lead to the closest available server being dispatched. ANSARI, MCLAY AND MAYORGA include the multiple dispatch of servers in their study and vary the dispatch policy accordingly.[261]

2.3.2.2 Hypercube Queueing Location Models with exact HQM

GEROLIMINIS, KARLAFTIS AND STATHOPOULOS extend the MCLP with an exact HQM.[262]

GEROLIMINIS, KARLAFTIS AND SKABARDONIS build on their conference[263] , formulating an optimization model based on an exact HQM that simultaneously determines server locations and their areas of responsibility by assuming the nearest-available dispatch policy for all incoming demands. Their objective function minimizes the system response time.[264]

IANNONI, MORABITO AND SAYDAM use the exact HQM in two studies on the location optimization decision for EMS systems on highways.[265] The second study is based on the first and extends it by the possibility that servers can be located anywhere in the study area. They state three, response time-based, objective functions that are employed for the minimization of the system response time. The same functions were also used in IANNONI,

[257] Cf. RAJAGOPALAN, H. K.; SAYDAM, C.; XIAO, J., 2008
[258] Cf. SAYDAM, C. ET AL., 2013
[259] Cf. TORO-DÍAZ, H. ET AL., 2015
[260] Cf. RAJAGOPALAN, H. K.; SAYDAM, C.; XIAO, J., 2008
[261] Cf. ANSARI, S.; MCLAY, L. A.; MAYORGA, M. E., 2017
[262] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; STATHOPOULOS, A., 2004
[263] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2006
[264] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009
[265] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008; 2009

MORABITO AND SAYDAM.[266] The studies employ a districting algorithm that optimizes the areas of responsibility for each located server.

CHANTA, MAYORGA AND MCLAY use the exact HQM in an EMS system location study that implements Tabu-Search as solution technique.[267]

TORO-DÍAZ ET AL. employ the exact HQM in an EMS system location study that discusses three cases – one of small, one of medium and one of largesize. They model the dispatch policy as a decision variable for the small case and assume the nearest-available dispatch policy for the two other cases. Their objective function minimizes the system response time.[268]

DAVOUDPOUR, MORTAZ AND HOSSEINIJOU maximize the expected coverage provided by an EMS system by using an exact HQM.[269]

SABEGH ET AL. propose a multi-objective HQM location study with an exact HQM. Their objective functions minimize the fixed costs of the system, its response time and its environmental impact. They calculate pareto-optimal solutions with the use of simulated annealing.[270]

AKDOĞAN, BAYINDIR AND IYIGUN use the exact HQM for the proposal of demand-specific service rates. Subsequently, they embed their service-rate formulations into an exact HQM optimization model with an artificial study area. Their objective function minimizes the system response time.[271]

ENAYATI ET AL. apply the exact HQM for the incorporation of equity concerns between demand classes in an EMS system location study. They propose several performance metrics and use them in a multi-criteria optimization study. Therefore, they choose pairwise performance metrics and calculate a pareto-optimal solution between the criteria. A GA is employed as solution technique.[272]

BLANK proposes a technique to dynamically aggregate demand points into super demand areas to mitigate computational performance concerns. The model minimizes the system

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[266] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2011
[267] Cf. CHANTA, S.; MAYORGA, M.; MCLAY, L. A., 2012
[268] Cf. TORO-DÍAZ, H. ET AL., 2013
[269] Cf. DAVOUDPOUR, H.; MORTAZ, E.; HOSSEINIJOU, S. A., 2014
[270] Cf. SABEGH, M. H. Z. ET AL., 2017
[271] Cf. AKDOGAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018
[272] Cf. ENAYATI, S. ET AL., 2019

response time and uses the genetic algorithm.[273]

BLANK uses the exact HQM in a location optimization study to derive location for emergency vehicles in a refugee camp. The genetic algorithm is applied as solution technique.[274]

As for the HQM location studies that provide an approximate HQM, table 11 with the same categories is provided in the following.

[273] Cf. BLANK, F., 2020
[274] Cf. BLANK, F., 2021

EMS System Location Studies with exact HQM							
	Objective Function	**Performance Metrics**	**Number of Servers located**	**Solution Technique**	**Decision Variable(s)**	**Time Period**	**Dispatch Policy**
Geroliminis, Karlaftis and Stathopoulos (2004)	Max. expected coverage	No	5	Proprietary	Location	One	Nearest-Available
Geroliminis, Karlaftis and Skabardonis (2006)	Min. response time	No	5	Proprietary	Location	One	Nearest-Available
Iannoni, Morabito and Saydam (2008)	Min. response time	Yes	5	Genetic Algorithm	Location	One	Nearest-Available
Geroliminis, Karlaftis and Skabardonis (2009)	Min. response time	No	5	Proprietary	Location	One	Nearest-Available
Iannoni, Morabito and Saydam (2009)	Min. response time	Yes	5	Genetic Algorithm	Location	One	Nearest-Available
Chanta, Mayorga and McLay (2012)	Min. response time	Yes	5	Tabu-Search	Location + Dispatch policy	One	Decision Variable
Toro-Díaz et al. (2013)	Min. response time	Yes	5	Genetic Algorithm	Location + Dispatch policy	One	Decision Variable

	Objective Function	Performance Metrics	Number of Servers lo-cated	Solution Technique	Decision Variable(s)	Time Period	Dispatch Policy
Davoudpour, Mortaz and Hosseinijou (2014)	Max. expected coverage	No	4	Branch-and-Bround and MatLab	Location	One	Nearest-Available
Sabegh et al. (2017)	(Pareto) Multi-Criteria	No	5	Simulated Anneal-ing	Location	One	Nearest-Available
Akdoğan, Bayindir and Iyigun (2018)	Min. response time	No	up to 7	Genetic Algorithm	Location	One	Nearest-Available
Enayati et al. (2019)	(Pareto) Multi-Criteria	Yes	5	Genetic Algorithm (NSGA-II)	Location + Dispatch policy	One	Decision Variable
Blank (2020)	Min. response time	No	up to 10	Genetic Algorithm	Location	One	Nearest-Available
Blank (2021)	Min. response time	Yes	5	Genetic Algorithm	Location	One	Nearest-Available

Table 11: EMS system location studies with exact HQM

In comparison with the approximate HQM location studies, it seems noteworthy that most of the exact studies use the minimization of the response time in the objective function. Only DAVOUDPOUR, MORTAZ AND HOSSEINIJOU maximize the expected coverage.[275] There are two studies that employ more than one criterion in the objective function, both calculate pareto-optimal solutions between the criteria used.

Approximately half of the studies apply additional performance metrics for their analysis. Since the exact HQM is believed to be more accurate, the calculation of performance metrics for either the objective function or for further analysis should help on the way towards a more thorough analysis of the solution found.

The computational effort required for solving the exact HQM for a vast amount of candidate solutions has been prohibitive in a way that only one existing study has considered double-digit servers. Most studies use five or less servers. Therefore, the exact HQM has yet to be applied for the EMS system location optimization for large-scale study areas.

Most of the existing studies employ the genetic algorithm. DAVOUDPOUR, MORTAZ AND HOSSEINIJOU use MatLab to solve the linear equation system and export the ‘server busy’ probabilities into LINGO which then applies a Branch-and-Bound algorithm.[276] SABEGH ET AL. provide the simulated-annealing based ‘Pareto archived simulated annealing algorithm’.[277] ENAYATI ET AL. apply a similar procedure for the NSGA-II which is a heuristic procedure that is based on a genetic algorithm and pareto-optimality.[278]

The computational hurdles already mentioned have limited the ability to implement a larger number of decision variables in the optimization process. Therefore, most existing studies only use the location as a decision variable. TORO-DÍAZ ET AL.[279] and ENAYATI ET AL.[280] also employ the dispatch policy. Hence, these two studies are the only ones that differ from the nearest-available assumption for the dispatch policy.

All existing exact HQM location studies use only one time period.

[275] Cf. DAVOUDPOUR, H.; MORTAZ, E.; HOSSEINIJOU, S. A., 2014, p. 1161
[276] Cf. DAVOUDPOUR, H.; MORTAZ, E.; HOSSEINIJOU, S. A., 2014, p. 1164
[277] Cf. SABEGH, M. H. Z. ET AL., 2017, p. 413
[278] Cf. ENAYATI, S. ET AL., 2019, pp. 11–12
[279] Cf. TORO-DÍAZ, H. ET AL., 2013, p. 920
[280] Cf. ENAYATI, S. ET AL., 2019, p. 6

2.4 Location models for Emergency Medical Service Systems

The following chapter contains the description of basic models in the location optimization of EMS systems. Some recent applications and, if existent, extensions are given for each model. Because of the excessive amount of literature on the field, an overview of the existing literature is given according to the respective focus areas.

The very early EMS system location models were generally limited by the computational power available. In order to simplify the necessary calculations, they mostly ignored the inherent complexities and uncertainties of such systems. For example, it was assumed that demands do not change and emergency vehicles are always available.

Set Covering Location Problem (SCLP)

One of the earliest contributions to this field is the Set Covering Location Problem (SCLP) originally introduced by TOREGAS ET AL.[281] The general aim is to minimize the number of servers necessary to provide coverage of the underlying study area. The model has several shortcomings as, for example, the assumption that some demand points are uncovered once an emergency vehicle is dispatched. ALY AND WHITE extend the SCLP for stochastic response times.[282] DASKIN AND STERN use the SCLP in a location study that intends to minimize the number of facilities while ensuring full coverage within a pre-defined distance and then maximizing the number of demand points that are covered more than once.[283] HOGAN AND REVELLE incorporate backup coverage into the SCLP.[284] MARIANOV AND REVELLE use Queueing Theory to relax the assumption that the emergency vehicles operate independently.[285] DASKIN AND OWEN state the partial set covering location problem (PSCLP) which only requires a pre-defined portion of demand to be covered.[286] Due to the mentioned shortcomings, the SCLP has not been used extensively in recent research. HWANG uses the SCLP in a facility location study for deteriorating items while incorporating stochasticity.[287] ALIZADEH AND NISHI combine the SCLP and the Maximal Covering Location Problem (see below) to combine the benefits of both methods.[288] CORDEAU, FURINI AND LJUBIC extend the PSCLP by the assumption that the

281 Cf. TOREGAS, C. ET AL., 1971
282 Cf. ALY, A. A.; WHITE, J. A., 1978
283 Cf. DASKIN, M. S.; STERN, E. H., 1981
284 Cf. HOGAN, K.; REVELLE, C., 1986
285 Cf. MARIANOV, V.; REVELLE, C., 1994
286 Cf. DASKIN, M. S.; OWEN, S. H., 1999
287 Cf. HWANG, H. S., 2004
288 Cf. ALIZADEH, R.; NISHI, T., 2019

number of customers exceeds the number of available facilities to a large extent.[289]

Maximal Covering Location Problem (MCLP)

Some of the mentioned shortcomings are overcome by the Maximal Covering Location Problem (MCLP) formulated by CHURCH AND REVELLE.[290] It intends to maximize the coverage that is provided for the underlying study area subject to limited availability of the emergency vehicles. Therefore, the MCLP maximizes the value provided due to constrained resources. As for the SCLP, HOGAN AND REVELLE introduce a backup coverage criterion into the MCLP.[291] MARIANOV AND SERRA combine Queueing Theory and the MCLP for a location-allocation problem.[292] As mentioned by MURRAY, the MCLP remains highly relevant to date and has since been widely used for the location decision in a variety of fields like fire stations, outlets and airports.[293] ALSALLOUM AND RAND combine the MCLP and the Goal Programming approach.[294] In a more recent extension of the MCLP, DAVARI, FAZEL ZARANDI AND HEMMATI incorporate fuzzy travel times from the location of the emergency unit to the demand point.[295] FARAHANI ET AL. extend the possibility of locations failing in their formulation of the MCLP and use a bee-colony-algorithm to efficiently derive a solution.[296] ZHANG, PENG AND LI model the MCLP in an uncertain environment by using different modelling techniques.[297]

Tandem equipment allocation model (TEAM) and facility location equipment emplacement technique (FLEET)

The early location optimization models for EMS systems assume all emergency vehicles to be of equal type. SCHILLING ET AL. relax this assumption by stating the tandem equipment allocation model (TEAM) and facility location equipment emplacement technique (FLEET).[298] Both models extend the MCLP by defining a hierarchy between the types of vehicles considered. The TEAM model assumes that specialized emergency vehicles can only be located in tandem with basic emergency vehicles. The FLEET model relaxes this assumption and allows for specialized vehicles to be located on their own or in a tandem.

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289 Cf. CORDEAU, J. F.; FURINI, F.; LJUBIC, I., 2019
290 Cf. CHURCH, R. L.; REVELLE, C., 1974
291 Cf. HOGAN, K.; REVELLE, C., 1986
292 Cf. MARIANOV, V.; SERRA, D., 1994
293 Cf. MURRAY, A. T., 2016, p. 6
294 Cf. ALSALLOUM, O. I.; RAND, G. K., 2006
295 Cf. DAVARI, S.; FAZEL ZARANDI, M. H.; HEMMATI, A., 2011
296 Cf. FARAHANI, R. Z. ET AL., 2014
297 Cf. ZHANG, B.; PENG, J.; LI, S., 2017
298 Cf. SCHILLING, D. ET AL., 1979

BIANCHI AND CHURCH combine the Maximum Expected Covering Location Problem and the FLEET model.[299] Their model minimizes the probability that an emergency vehicle is unavailable at the time of an incoming demand while allowing for backup coverage provided by other vehicles. TAVAKOLI AND LIGHTNER use this model in a location study that intends to maximize the number of demands that can be served subject to a time threshold.[300] Recently, RODRIGUEZ, DE LA FUENTE AND AGUAYO use the FLEET model and combine it with an approximate HQM to model the system congestion.[301]

Maximum Expected Covering Location Problem (MEXCLP)

The Maximum Expected Covering Location Problem (MEXCLP) is proposed by DASKIN to extend the MCLP by the possibility that when a demand enters the system not all servers are available.[302] The, at that time, still existing computational burdens are mitigated by the assumptions of independently operating servers and equally busy probabilities between servers that are invariant with respect to the server location. These assumptions are lifted by BATTA, DOLAN AND KRISHNAMURTHY who combine the MEXCLP with an approximate HQM to model the system congestion and state the adjusted MEXCLP.[303] REPEDE AND BERNARDO incorporate demands that vary during the study period.[304] In a recent application of the MEXCLP, SAYDAM AND AYTUG use the MEXCLP to test the performance of the genetic algorithm against other solution techniques.[305] MCLAY extends the MEXCLP for two different types of servers[306] while SORENSON AND CHURCH incorporate the local reliability of individual servers[307].

Maximum Availability Location Problem (MALP)

The Maximum Availability Location Problem (MALP), by REVELLE AND HOGAN, locates a number of servers in such a way that any incoming demand will be served by an available server within a pre-defined time threshold and a level of reliability.[308] The MALP has later been extended by MARIANOV AND REVELLE with the help of Queueing

[299] Cf. BIANCHI, G.; CHURCH, R. L., 1988
[300] Cf. TAVAKOLI, A.; LIGHTNER, C., 2004
[301] Cf. RODRIGUEZ, S. A.; DE LA FUENTE, R. A.; AGUAYO, M. M., 2020
[302] Cf. DASKIN, M. S., 1983
[303] Cf. BATTA, R.; DOLAN, J. M.; KRISHNAMURTHY, N. N., 1989
[304] Cf. REPEDE, J. F.; BERNARDO, J. J., 1994
[305] Cf. AYTUG, H.; SAYDAM, C., 2002
[306] Cf. MCLAY, L. A., 2009
[307] Cf. SORENSEN, P.; CHURCH, R., 2010
[308] Cf. REVELLE, C.; HOGAN, K., 1989

Theory to incorporate congestion concerns.[309] HAREWOOD uses the MALP in a multi-objective location optimization problem to dispatch emergency vehicles.[310] KARASAKAL AND KARASAKAL relax the assumption that all incoming demand needs to be covered and introduce the concept of partial coverage for the MALP.[311] Thereby, their model gradually decreases the coverage provided for longer distances to the respective demand points. HUANG AND FAN embed the MALP in a scenario approach to model the uncertainty in the emergency services resource allocation.[312]

Double Standard Model (DSM)

The Double Standard Model (DSM) is another model that introduces backup coverage for a more realistic reproduction of the real-world. Originally proposed by GENDREAU, LAPORTE AND SEMET, it operates under the definition of two pre-defined time standards.[313] It intends to maximize the coverage under the first, usually more strictly defined, time standard while ensuring that all demands are covered under the second standard. DOERNER ET AL. include a penalty function for unmet demands and uneven workloads into the DSM.[314] SCHMID AND DOERNER use the DSM in an ambulance location and relocation study with time-dependent travel times. They extend it for capacity concerns.[315] KEPAPTSOGLOU, KARLAFTIS AND MINTSIS apply the DSM in a location study that intends to locate emergency vehicles for road safety.[316] LIU ET AL. extend the DSM to guarantee service reliability to maximize the provided coverage. [317]

Dynamic allocation and reallocation models

Most basic EMS system location models assume static locations as home bases for the emergency vehicles during the study period. One possible strategy to face changing demands over the study period would be to station emergency vehicles in areas of the study area for which a higher demand is to be expected. GENDREAU, LAPORTE AND SEMET extend the DSM and state the Dynamic Double Standard Model.[318] Their model still intends to maximize the coverage subject to two time constraints minus the expected penalty that

[309] Cf. MARIANOV, V.; REVELLE, C., 1996
[310] Cf. HAREWOOD, S. I., 2002
[311] Cf. KARASAKAL, O.; KARASAKAL, E. K., 2004
[312] Cf. HUANG, Y.; FAN, Y., 2011
[313] Cf. GENDREAU, M.; LAPORTE, G.; SEMET, F., 1997
[314] Cf. DOERNER, K. F. ET AL., 2005
[315] Cf. SCHMID, V.; DOERNER, K. F., 2010
[316] Cf. KEPAPTSOGLOU, K.; KARLAFTIS, M. G.; MINTSIS, G., 2012
[317] Cf. LIU, Y. ET AL., 2016
[318] Cf. GENDREAU, M.; LAPORTE, G.; SEMET, F., 2001

is implied by the relocation of the emergency vehicles. Other approaches use the locations obtained in advance[319] or incorporate the randomness of the system into the model, for example, by making decisions under particular system configurations[320].

Due to the high amount of research that has been conducted in the field of EMS system location models, several literature reviews have summarized the existing contributions. A selection of relevant reviews is provided in the following with the respective focus of the review.

- OWEN AND DASKIN (1998): Strategic facility location in a general context[321]
- MARIANOV AND SERRA (2002): Coverage and network problems as well as their applications and extensions[322]
- BROTCORNE, LAPORTE AND SEMET (2003): Coverage problems[323]
- SNYDER (2006): Facility location under uncertainty[324]
- SIMPSON AND HANCOCK (2009): Relevant operations research articles for emergency response[325]
- BASAR, CATAY AND ÜNLÜYURT (2012): Taxonomy based on emergency service station location models[326]
- CAUNHYE, NIE AND POKHAREI (2012): Optimization models for emergency response[327]
- ABOUELJINANE AND JEMAI (2013): Simulation models[328]
- ARINGHIERI ET AL. (2017): Inclusion of various emergency service systems[329]

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[319] Cf. GENDREAU, M.; LAPORTE, G.; SEMET, F., 2006, p. 23
[320] Cf. ANDERSSON, T.; VÄRBRAND, P., 2007, p. 197
[321] Cf. OWEN, S. H.; DASKIN, M. S., 1998
[322] Cf. MARIANOV, V.; SERRA, D., 2002
[323] Cf. BROTCORNE, L.; LAPORTE, G.; SEMET, F., 2003
[324] Cf. SNYDER, L. V., 2006
[325] Cf. SIMPSON, N. C.; HANCOCK, P. G., 2009
[326] Cf. BAŞAR, A.; ÇATAY, B.; ÜNLÜYURT, T., 2012
[327] Cf. CAUNHYE, A. M.; NIE, X.; POKHAREL, S., 2012
[328] Cf. ABOUELJINANE, L.; SAHIN, E.; JEMAI, Z., 2013
[329] Cf. ARINGHIERI, R. ET AL., 2017

- BÉLANGER, RUIZ AND SORIANO ET AL. (2019): Recent trends in EMS system location models[330]

- FARAHANI ET AL. (2019): Operations research models for public facilities[331]

2.5 Multi-objective optimization for Emergency Medical Service Systems

The optimization of multiple objectives is used in private and public facility location decision problems to integrate conflicting goals. Recent applications in the private supply chain design sector include, for example, the combination of cost and shortage minimization[332], efficiency maximization and risk minimization[333], cost, environmental and social impact minimization[334], cost and carbon emission minimization[335], the maximization of created jobs, resource utilization and cost minimization[336] as well as cost and environmental impact minimization and capacity maximization[337].

The strategic location of EMS system facilities also includes the consideration of multiple objectives. One example is the trade-off between loss probabilities of incoming demands for emergency help and the average expected response time of the system. Other things being equal, lower loss probabilities usually correspond with a higher expected response time and vice versa. This effect can be explained by the dispatch of more distant servers, when the primary units are unavailable, and the resulting longer travel times from the location to the scene of the incident. Since both criteria are key external performance metrics, as described in IANNONI, MORABITO AND SAYDAM[338], the degree to which worse values in one criterion are accepted to benefit the other is one of the key strategic decisions during the design of EMS systems. Note that not all factors influencing the response and loss behavior should be included in the consideration of multiple objectives for the location optimization decision. So, for example, the preference lists between demand areas should be kept as equal as possible in order to mitigate potential ethical and legal issues.

The consideration of multiple objectives during the location decision for EMS systems

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[330] Cf. BÉLANGER, V.; RUIZ, A.; SORIANO, P., 2019
[331] Cf. FARAHANI, R. Z. ET AL., 2019
[332] Cf. MIRZAPOUR AL-E-HASHEM, S. M. J.; MALEKLY, H.; ARYANEZHAD, M. B., 2011
[333] Cf. HUANG, E.; GOETSCHALCKX, M., 2014
[334] Cf. RAMOS, T. R. P.; GOMES, M. I.; BARBOSA-PÓVOA, A. P., 2014
[335] Cf. NURJANNI, K. P.; CARVALHO, M. S.; COSTA, L., 2017
[336] Cf. ALLAOUI, H. ET AL., 2018
[337] Cf. MOHEBALIZADEHGASHTI, F.; ZOLFAGHARINIA, H.; AMIN, S. H., 2020
[338] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008, pp. 219–220

has been a major research field. The relevant literature is chronologically listed and briefly described in the following.

CHARNES AND STORBECK use Goal Programming in a EMS system location study to maximize coverage for two distinct demand groups.[339]

DASKIN, HOGAN AND REVELLE extend SCLP and MCLP by the consideration of multiple objectives seeking to minimize the number of necessary facilities while maximizing the coverage provided. They do not provide computational results for their extensions.[340]

BAKER, CLAYTON AND TAYLOR use Goal Programming to incorporate various criteria into their optimization problem. They include the number of available emergency units, monetary concerns, the response time as well as the capacity in their goal formulations.[341]

ZHU AND MCKNEW state an EMS system location study that intends to balance the workloads between the emergency units. They only employ the workload balance as their singular goal and estimate the system congestion with an approximate HQM.[342]

MANDELL AND BECKER maximize the expected survival of the patients of the EMS system while minimizing occurring inequalities between patient groups. They use the ε-constraint method.[343]

BADRI, MORTAGY AND ALI ALSAYED state a Goal Programming approach for the location of fire stations that includes monetary, political and service related goals, like the response time, in its optimization decision.[344]

HAREWOOD develops two objective functions – the first maximizes the coverage provided while the second minimizes the implied costs. However, there is no clear declaration which method was used to solve the bi-objective optimization problem.[345]

ALSALLOUM AND RAND combine the MCLP with a Goal Programming approach. They maximize the fraction of covered demands while minimizing the number of spare capacities in order to determine an efficient solution.[346]

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[339] Cf. CHARNES, A.; STORBECK, J., 1980
[340] Cf. DASKIN, M. S.; HOGAN, K.; REVELLE, C., 1988
[341] Cf. BAKER, J. R.; CLAYTON, E. R.; TAYLOR, B. W., 1989
[342] Cf. ZHU, Z.; MCKNEW, M. A., 1993
[343] Cf. MANDELL, M. B.; BECKER, L. R., 1996
[344] Cf. BADRI, M. A.; MORTAGY, A. K.; ALI ALSAYED, C., 1998
[345] Cf. HAREWOOD, S. I., 2002
[346] Cf. ALSALLOUM, O. I.; RAND, G. K., 2006

ARAZ, SELIM AND OZKARAHAN apply Fuzzy Goal Programming in an EMS location optimization problem. Their objective functions involve the maximization of (backup) coverage provided as well as the minimization of the travel time to ensure a pre-defined service level. They use Fuzzy Goal Programming as well as Lexicographic Multi-Objective Linear Programming to solve the proposed model.[347]

BASTIAN employs the Goal Programming approach in a robust optimization study to maximize the provided coverage as well as the available spare capacities of the system while minimizing the vulnerability of their EMS system in a military context.[348]

KANOUN, CHABCHOUB AND AOUNI use a weighted Goal Programming approach in their EMS system location optimization study.[349] Additionally, they consider the decision makers' preference by using satisfaction functions as originally proposed by MARTEL AND AOUNI[350]. Their objectives consist of coverage, cost and response time goals.

ZHAN AND LIU utilize stochastic Goal Programming considering uncertainty in demand and in the routes for the emergency units. They implement the response time of the system as well as the number of unserved demands as goals.[351]

CHANTA, MAYORGA AND MCLAY intend to minimize the inequality between urban and rural demand areas. Therefore, they state a bi-objective model that minimizes the distance to unserved demands and the unserved demands themselves while using the ε-constraint method.[352]

ZHANG AND JIANG employ a robust bi-objective optimization model that simultaneously minimizes unserved demands. They include unserved demands as a penalty function in their cost related objective function and calculate pareto-optimal solutions.[353]

STEINER ET AL. apply a multi-objective optimization approach to partition the health care system of their study. Their objectives include the maximization of equity between demand areas with respect to their size, the maximization of served demands as well as the

[347] Cf. ARAZ, C.; SELIM, H.; OZKARAHAN, I., 2007
[348] Cf. BASTIAN, N. D., 2010
[349] Cf. KANOUN, I.; CHABCHOUB, H.; AOUNI, B., 2010
[350] Cf. MARTEL, J. M.; AOUNI, B., 1990
[351] Cf. ZHAN, S. L.; LIU, N., 2011
[352] Cf. CHANTA, S.; MAYORGA, M. E.; MCLAY, L. A., 2014
[353] Cf. ZHANG, Z. H.; JIANG, H., 2014

minimization of the system response time. They calculate pareto-optimal solutions between the objectives.[354]

KHODARAPARASTI ET AL. maximize the efficiency of the system while minimizing occurring inequalities by using the ε-constraint method.[355]

PAUL, LUNDAY AND NURRE extend the MCLP by multiple objectives and apply the maximization of the expected coverage and the minimization of modifications to the existing structure of the system in their formulation. They employ the ε-constraint method to determine a set of non-inferior solutions. [356]

SABEGH ET AL. state three different objective functions that represent the minimization of the response time, the costs and the environmental impact of the system. They use the HQM to derive the necessary performance metrics while calculating pareto-optimal solutions between the objective functions.[357]

BERMAN ET AL. develop two objective functions that maximize the coverage provided by the system while minimizing the amount of changes to the existing structure of the system. They use the ε-constraint method.[358]

HE ET AL. use the weighted sum approach to minimize the response time to covered and to uncovered demands with the intention to reduce inequalities between urban and rural areas.[359]

ENAYATI ET AL. incorporate the HQM to derive the inputs for their combination of different objective functions and then calculate pareto-optimal solutions between the components. Their objective functions include the response time of the system, the loss probabilities and several equity measures that are represented by a Gini coefficient approach.[360]

JENKINS, LUNDAY AND ROBBINS state a robust multi-objective optimization model for the location of military evacuation assets. Their objective functions maximize the expected coverage while minimizing the number of emergency units and the number of necessary

[354] Cf. STEINER, M. T. A. ET AL., 2015
[355] Cf. KHODAPARASTI, S. ET AL., 2016
[356] Cf. PAUL, N. R.; LUNDAY, B. J.; NURRE, S. G., 2017
[357] Cf. SABEGH, M. H. Z. ET AL., 2017
[358] Cf. BERMAN, O. ET AL., 2018
[359] Cf. HE, Z. ET AL., 2018
[360] Cf. ENAYATI, S. ET AL., 2019

redeployments. They use the ε-constraint method.[361]

The aforementioned contributions are categorized with respect to the below listed aspects in table 12:

- Objective Function
- Method
- Decision variables
- Main Contribution

[361] Cf. JENKINS, P. R.; LUNDAY, B. J.; ROBBINS, M. J., 2020

	Objectives	Method	Decision Variable(s)	Main Contribution
Charnes and Storbeck (1980)	Coverage for two distinct demand groups	Goal Programming	Location	Introduction of demand groups
Daskin, Hogan and ReVelle (1988)	Number of facilities + Coverage	/	Location	Extension of coverage models for multiple objectives
Baker, Clayton and Taylor (1989)	Available emergency units + Cost + Response time + Capacity utilization	Goal Programming	Districting	Combination of budget and standard EMS system goals
Zhu and McKnew (1993)	Workload balance	Goal Programming	Location	Combination of HQM and Goal Programming
Mandell and Becker (1996)	Survival + Equity	ε-constraint	Location + Districting	Balance between survival and equity
Badri et al. (1998)	Cost + Demand + Distance + Number of facilities + Service overlap + Political + Available resources	Goal Programming	Location	Goal Programming and definition of various goals for emergency services
Harewood (2002)	Cost + Coverage	/	Location + Districting	Extension of MALP

	Objectives	Method	Decision Variable(s)	Main Contribution
Alsalloum and Rand (2006)	Coverage + Efficiency	Goal Programming	Location + Number of emergency units + Districting	Extension of MCLP
Araz, Selim and Ozkarahan (2007)	Coverage + Travel time	Fuzzy Goal Programming and Lexicographic Linear Programming	Location + Districting	Incorporation of fuzzy parameters
Bastian (2010)	Coverage + Spare capacities + Vulnerability	Goal Programming	Location + Districting	Robust Goal Programming
Kanoun, Chabchoub and Aouni (2010)	Coverage + Number of incidents + Infrastructure + Response time	Goal Programming	Location	Use of satisfaction functions
Zhan and Liu (2011)	Response time + Unserved demands	Goal Programming	Location + Districting	Stochastic Goal Programming
Chanta, Mayorga and McLay (2014)	Distance to unserved demands + Unserved demands	ε-constraint	Location + Number of emergency units + Districting	Equity between urban and rural areas

	Objectives	Method	Decision Variable(s)	Main Contribution
Zhang and Jiang (2014)	Costs + Unserved demands	Pareto	Location + Number of emergency units + Districting	Bi-Objective Robust Optimization
Steiner et al. (2015)	Districting equity + Served demands + Response time	Pareto	Location + Districting	Multi-Objective partitioning of the health service
Berman et al. (2017)	Coverage + Modifications to existing structure	ε-constraint	Location	Uncertainty in travel time and multi-objective
Khodaparasti et al. (2016)	Efficiency + Equity	ε-constraint	Location + Districting	Balance between efficiency and equity
Paul, Lunday and Nurre (2017)	Coverage + Modifications to existing structure	ε-constraint	(Re)Location + Districting	Multi-Objective Extension of MCLP
Sabegh et al. (2017)	Response time + Cost + CO_2	Pareto	Location	Multi-Objective HQM location study
He et al. (2018)	Response time to covered + Uncovered demands	Weighted Sum	Location	Equity between urban and rural areas

	Objectives	Method	Decision Variable(s)	Main Contribution
Enayati et al. (2019)	Response time + Unserved demands + Equity + Workload	Pareto	Location + Districting	Multi-Objective HQM location study
Jenkins, Lunday and Robbins (2020)	Coverage + Number of emergency units + Redeployments	ε-constraint	Location + Districting + Redeployments	Multi-Objective Robust Optimization

Table 12: Multi-objective EMS system location studies

2.6 Uncertainty in Emergency Medical Service systems location models

The behavior of any EMS system regarding performance metrics like the average response time, the loss probability of incoming demands or the coverage provided is highly influenced by key (input) variables. Among those is the demand and its spatial distribution or the expected travel time from the server location to the scene of the incident. Since, by nature, the occurrence of incoming demand for emergency help and its geographical origin cannot be known during the planning phase of the EMS system, it has to be considered as uncertain to at least some degree. In order to mitigate the implied theoretical, modelling and also computational burdens, a common approach in deterministic models is to assume fixed, unchangeable values for the respective parameters.

It was only recently that FARAHANI ET AL. criticized that most EMS system location models use deterministic and static inputs and thereby ignore potential changes.[362] This, of course, leads to a simplification of the respective model and, as a result, solutions can be derived more easily. Thus, the existing body of literature with a real consideration of uncertainty in the location decision of EMS systems is rather small. In this chapter, as in chapter 2.6, the relevant contributions to this field are briefly described. Note that this chapter only considers literature in which (input) parameters are assumed to be uncertain. Thus, common probabilistic models like the probabilistic SCLP of CHAPMAN AND WHITE[363] are excluded from further analysis here.

One of the earliest works in this field was done by SCHILING. His model extends the SCLP towards the incorporation of demand scenarios for the location optimization decision.[364]

GOLDBERG AND PAZ use the HQM to derive the system congestion and introduce stochastic travel times.[365]

BERALDI AND RUSZCZYNSKI combine Stochastic Programming and Probability Constraints to model uncertain demands.[366]

BERALDI, BRUNI AND CONFORTI employ Stochastic Programming to include the randomness of the system into the location optimization decision. Therefore, they assume randomly generated demands for emergency services and determine locations as well as the

[362] Cf. FARAHANI, R. Z. ET AL., 2019, p. 12
[363] Cf. CHAPMAN, S.; WHITE, J., 1974
[364] Cf. SCHILLING, D. A., 1982
[365] Cf. GOLDBERG, J.; PAZ, L., 1991
[366] Cf. BERALDI, P.; RUSZCZYSKI, A., 2002

number of emergency units stationed.[367]

ARAZ, SELIM AND OZKARAHAN use Fuzzy Goal Programming that defines the coverage provided by the system as well as the travel time as fuzzy goals.[368]

JIA, ORDOÑEZ AND DESSOUSKY state a general model that can be used as an extension to p-median and coverage models. They implement a scenario approach to model the uncertainty in demand.[369]

INGOLFSSON, BUDGE AND ERKUT apply the HQM to model the congestion of the EMS system in a Stochastic Programming location optimization study. They consider uncertainty in travel times as well as random delays.[370]

BERALDI AND BRUNI extend the work of BERALDI, BRUNI AND CONFORTI and formulate a two-stage stochastic programming approach. In the first stage the location and capacity decisions are taken while the second stage contains the allocation of emergency units to the respective areas of responsibility.[371]

NOYAN also builds on BERALDI, BRUNI AND CONFORTI and relaxes the assumption of independent operating emergency units and incoming demands.[372]

HUANG AND FAN extend the MCLP with a Stochastic Programming approach that considers uncertainty in the service provided by the EMS system and in the accessibility to the service.[373]

BERMAN, HAJIZADEH AND KRASS combine the MCLP with a Robust Optimization approach that considers uncertainty in the travel times from the location to the scene of the incident.[374]

ZHANG AND JIANG state a bi-objective Robust Optimization model that simultaneously minimizes costs and unserved demands. They consider uncertainty in demand as well as in the number of available emergency units.[375]

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[367] Cf. BERALDI, P.; BRUNI, M. E.; CONFORTI, D., 2004
[368] Cf. ARAZ, C.; SELIM, H.; OZKARAHAN, I., 2007
[369] Cf. JIA, H.; ORDOÑEZ, F.; DESSOUKY, M., 2007
[370] Cf. INGOLFSSON, A.; BUDGE, S.; ERKUT, E., 2008
[371] Cf. BERALDI, P.; BRUNI, M. E., 2009
[372] Cf. NOYAN, N., 2010
[373] Cf. HUANG, Y.; FAN, Y., 2011
[374] Cf. BERMAN, O.; HAJIZADEH, I.; KRASS, D., 2013
[375] Cf. ZHANG, Z. H.; JIANG, H., 2014

AN ET AL. use a scenario approach within the context of Stochastic Programming and consider network disruptions. Additionally, they include a M/M/1 queueing system in their model.[376]

SHISHEBORI AND BABADI apply the Robust Optimization approach to incorporate potential system disruptions in the location optimization decision. As a result, they include the repair and improvement decision of the respective links in the system in their set of decision variables. Additionally, they consider uncertainty in the travel costs and the demand.[377]

NICKEL, REUTER-OPPERMANN, SALDANHA-DA-GAMA employ Stochastic Programming to consider uncertain demands. They demonstrate the computational limitations opposing an excessive amount of scenarios but conclude that the use of only one sample of scenarios can lead to sub-optimal decisions. Therefore, they introduce 'stochastic sampling' for which they solve their model with various samples of scenarios.[378]

BERMAN ET AL. extend BERMAN, HAJIZADEH AND KRASS by a second objective function that minimizes the changes to the existing network structure.[379]

BOUJEMAA ET AL. include a Stochastic Programming approach to consider uncertainty in demands in a two-tier EMS system.[380]

SUNG AND LEE use a two-stage Stochastic Programming approach. During the first stage the location decision is taken while the second stage maximizes the coverage provided.[381]

BERTSIMAS AND NG state a Stochastic Programming and Robust Optimization approach to extend the MEXCLP and MALP in an EMS system location optimization study. They construct their uncertain-demand sets using historical data. Additionally, they compare Stochastic Programming and Robust Optimization and find only minor differences.[382]

BOUJEMAA ET AL. extend BOUJEMAA ET AL.[383] and use Stochastic Programming in a multi-period location study for a two-tier EMS system. They consider uncertainty in two

[376] Cf. AN, S. ET AL., 2015
[377] Cf. SHISHEBORI, D.; YOUSEFI BABADI, A., 2015
[378] Cf. NICKEL, S.; REUTER-OPPERMANN, M.; SALDANHA-DA-GAMA, F., 2016
[379] Cf. BERMAN, O. ET AL., 2018
[380] Cf. BOUJEMAA, R. ET AL., 2018
[381] Cf. SUNG, I.; LEE, T., 2018
[382] Cf. BERTSIMAS, D.; NG, Y., 2019
[383] Cf. BOUJEMAA, R. ET AL., 2018

demand classes.[384]

JENKINS, LUNDAY AND ROBBINS formulate a Robust Optimization Approach that incorporates uncertainty in demand and availability of the emergency units through a scenario approach.[385]

The body of relevant literature is categorized in table 3 according to the following aspects:

- Considered Uncertainty
- Method
- Decision Variable(s)
- Main Contribution

[384] Cf. BOUJEMAA, R. ET AL., 2020
[385] Cf. JENKINS, P. R.; LUNDAY, B. J.; ROBBINS, M. J., 2020

	Considered Uncertainty	Method	Decision Variable(s)	Main Contribution
Schiling (1982)	Demand	Scenario Approach	Location	Use of scenario approach in SCLP
Goldberg and Paz (1991)	Travel time	Stochastic Programming	Location	Combination of HQM and Stochastic Programming
Beraldi and Ruszczynski (2002)	Demand	Stochastic Programming	Location	Combination of Stochastic Programming and probability constraints
Beraldi, Bruni and Conforti (2004)	Demand	Stochastic Programming	Location + Number of emergency units	Incorporation of randomness of the system through uncertain demand
Araz, Selim and Ozkarahan (2007)	Coverage + Travel time	Fuzzy Programming	Location + Districting	Use of Fuzzy Goal Programming
Jia, Ordoñez and Dessousky (2007)	Demand	Scenario Approach	Location + Districting	Extension of coverage and p-median model through scenario development
Ingolfsson, Budge and Erkut (2008)	Travel time + Random delays	Stochastic Programming	Location + Number of emergency units	Combination of HQM and Stochastic Programming

	Considered Uncertainty	Method	Decision Variable(s)	Main Contribution
Beraldi and Bruni (2009)	Demand	Stochastic Programming	Location + Number of emergency units + Districting	Formulation of a two-stage Stochastic Programming Approach
Noyan (2010)	Demand	Stochastic Programming	Location + Number of emergency units	Extension of Beraldi et al. (2004) for dependence between emergency units and demands
Huang and Fan (2011)	Availability + Accessibility	Stochastic Programming	Location	Extension of MCLP for uncertain availability and accessibility
Zhan and Liu (2011)	Demand + Service + Accessibility	Stochastic Programming	Location + Districting	Combination of Goal and Stochastic Programming
Berman, Hajizadeh and Krass (2013)	Travel time	Robust Optimization	Location + Districting	Extension of MCLP for uncertain travel time
Zhang and Jiang (2014)	Demand + Number of emergency units	Robust Optimization	Location + Number of emergency units + Districting	Bi-Objective Robust Optimization
An et al. (2015)	Availability	Stochastic Programming	Location + Districting	Incorporation of network disruptions and queueing

	Considered Uncertainty	Method	Decision Variable(s)	Main Contribution
Shishebori and Babadi (2015)	Demand + Travel costs + Availability	Robust Optimization	Location + Repair/Improvement decision	Incorporation of network disruptions
Nickel, Reuter-Oppermann, Saldanha-da-Gama (2016)	Demand	Stochastic Programming	Location	Stochastic Sampling
Berman et al. (2017)	Travel time	Robust Optimization	Location	Extension of Berman, Hajizadeh and Krass (2013) for maintaining existing system structure
Boujemaa et al. (2018)	Demand	Stochastic Programming	Location + Districting + Number of emergency units	Stochastic Programming for two-tiered EMS system
Sung and Lee (2018)	Demand	Stochastic Programming	Location	Two-Stage Stochastic Programming + Assumption of a dispatch policy
Bertsimas and Ng (2019)	Demand	Robust Optimization + Stochastic Programming	Location + Districting	Data-Driven construction of uncertainty sets
Jenkins, Lunday and Robbins (2020)	Demand + Availability	Robust Optimization	Location + Districting + Redeployments	Multi-Objective Robust Optimization

Table 13: Uncertainty in EMS system location optimization studies

2.7 Research Gap and Research Contribution

The literature review given in chapter 2.6 has identified certain research gaps this work intends to close. Each gap and the contribution(s) of this work are briefly described hereafter.

Application of the exact HQM for a larger size real-world case

The literature review indicates that the exact HQM has not been used for a larger case study due to the still existing modelling and computational burdens. However, existing approximation techniques may lead to rather significant approximation errors. Additionally, the validity of the performance metrics as well as the number of performance metrics that can be calculated by using an approximate HQM may be compromised. This work intends to close this gap and will use the exact HQM in a larger size real-world case that locates up to double-digit servers. As a result, the linear equation system will have over 1,000 equations that have to be solved simultaneously.

Therefore, a real-world case is modelled on a virtual grid which also takes the spatial varieties of demand for EMS into account. Additionally, this work aims to contribute by offering a greenfield location optimization approach as well as by making suggestions how the existing EMS system could be improved.

Calculation of (new) performance metrics for the optimization and analysis for an exact HQM location optimization study

The literature review has also shown that approximately half of the existing exact HQM location optimization studies do not use additional performance metrics for their analysis. Accordingly, most of the studies only use a singular criterion in their objective function(s). The most common formulation of the objective function is the minimization of the average response time of the system. Usually some trade-offs have to be accepted in the design of an EMS system, like the mitigation of differences in the service between urban and rural areas. As a result, more than one criterion should be used in the optimization and subsequent analysis. This study intends to fulfill this gap by using a variety of criteria in the location optimization study as well as by providing further insights into the behavior of the EMS system by a thorough analysis through additional metrics that, for example, analyze the response behavior in certain sub-areas of the study area.

Proposal of a Goal Programming approach with an embedded exact HQM

As already mentioned, existing exact HQM location optimization studies chiefly use a singular criterion. The existing multi-objective studies calculate pareto-fronts between two criteria. However, the literature review of chapter 2.5 has also shown that in the broader field of multi-objective optimization studies for EMS systems, the Goal Programming approach has been used successfully. As mentioned by BADRI, MORTAGY AND AL-SAYED, Goal Programming has several benefits and is often the only applicable technique when it comes to formulating and measuring objectives and goals for any decision maker.[386] This work intends to make use of the possibility of calculating valid performance metrics and to then employ them in a Goal Programming approach. In order to mitigate some of the existing shortcomings of Goal Programming, the weighted Goal Programming is combined with the satisfaction function approach of MARTEL AND AOUNI.[387]

Incorporation of different day times as scenarios in a Robust Optimization approach

The analysis of the existing literature has shown that most of the location optimization studies only use a singular time period for their optimization study. Frequently, the demand for EMS, which may fluctuate significantly over the day, is aggregated into one reference period. Thereby, fluctuations are smoothed out and differences in the spatial distribution of demand are ignored. This work plans to formulate different day times as scenarios and includes them into a Robust Optimization approach. Note that the definition of scenarios in future studies does not have to be limited to the mentioned day time approach but could also include travel time variations or other inputs that can be assumed to be uncertain.

As a result, the study area is modelled for different day times and the actual demand for each period varies with respect to the respective time of the day. In order to make use of the exact HQM, a thorough analysis with respect to each time period is conducted to gain further insight into the behavior of the EMS if the quantity and location of the demand for EMS changes during the course of the day.

[386] Cf. BADRI, M. A.; MORTAGY, A. K.; ALI ALSAYED, C., 1998, p. 246
[387] Cf. MARTEL, J. M.; AOUNI, B., 1990

Joint Robust Goal Programming approach

The Goal Programming and Robust Optimization Approach are then integrated in order to contribute to a multi-objective robust optimization for an EMS system. In contrast to the common Robust Optimization, Robust Goal Programming makes use of the variety of calculated performance metrics and tries to offer a thorough approach introducing different day times and the possibility of capturing existing trade-offs.

Proposal of the use of a different metaheuristic and comparison to the state of the art

The literature review of chapter 2.6.2 has also shown that the majority of studies use the GA or other GA-based techniques. Therefore, a rather significant research gap exists as to whether the use of other metaheuristics may contribute to a better solution or at least an adequate solution in less time. The latter is especially important for an exact HQM with double-digit servers since the computational time still is one of the strongest limiting factors. In order to close that gap the Ant-Colony-Optimization is adapted to the HQM location optimization problem. For any new optimization technique proposed, it should be ensured that it will produce at least similarly valid results as the current state of the art. Since metaheuristics usually do not always return the same result, an experimental setting is proposed here. Therefore, one hundred repetitions of the experiment are performed for each parameter setting in order to ensure that any difference perceived due to parameter settings and optimization techniques has actually been caused by these. As already mentioned, the computation times of a double-digit server exact HQM are one of the major limiting factors. In conjunction with the highly necessary repetitions of the experiment, it is not feasible to conduct the metaheuristic experiments with a double-digit server exact HQM. Therefore, an artificial study area with five servers is proposed to validate the results of the ACO against the GA.

Dynamic Caching Strategy

In order to mitigate the problem of computational burden, AKDOĞAN, BAYINDIR AND IYIGUN have proposed a caching strategy. They memorize already calculated combinations of decision variables and the corresponding value of the objective function. If a such a combination has already been calculated, the value of the objective function is retrieved from memory and the actual calculation is skipped.[388] This work intends to extend this

[388] Cf. AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018

approach by not saving the value of the objective function but of the corresponding performance metric(s). In this way, already calculated solutions can not only be used in a single optimization run, but also between different optimizations that may differ in their weights on objective or even the mentioned satisfaction functions. Consider that the Dynamic Caching Strategy is used throughout this work, but the cache of the model proposed in chapter 4.1 cannot be used for the models in chapter 4.2 and 4.3. This can be explained due to the different model inputs.

3. Hypercube Queueing Model

3.1 Hypercube Queueing Model and districting process

This chapter describes the HQM as well as the underlying districting process. The relevant assumptions for the HQM are briefly described in the following.

1. Network of J individual regions

The HQM is used to optimize and subsequently analyze a network of J individual regions. A singular demand point represents the demand for emergency medical services of the entire region.[389] The set of all demand points represents the relevant study area from a demand viewpoint.

2. Set of N distinctive and individual servers

In order to provide service to any incoming demand occurring at the demand points of the regions, a set of N distinctive and individual servers has to be located within the study area. It is assumed that their location is not limited to the demand points. All servers are assumed to be equal and to provide equal services. Only one server can be located at one location.

3. Construction of districts and level of districting D

Each server is assumed to have primary areas of responsibility for which it is dispatched even if all servers are available. Additionally, it is presupposed that each server has lower-tier areas of responsibility for which it is only dispatched if the primary servers are unavailable. Therefore, a level of districting D has to be determined which describes the degree to which lower-tier servers can be dispatched if the primary servers are unavailable. If D is smaller than the number of servers to be located, a partial backup is considered while a full backup is considered for an equal value.

4. Dispatch policy

All servers are assumed to be dispatched according to the nearest-available dispatch policy. Therefore, for each region/demand point a set of the nearest-available servers is constructed which then represents the corresponding preference list. As a result, if the nearest-available server is not available for an arriving demand, the second-nearest server is

[389] Note that the terms ‘atom’ and ‘service region’ are also used in the academic literature.

dispatched and so forth.

5. One-step transitions

It is assumed that only one-step transitions between states are allowed. Applied to the field of EMS systems, this implies that only one server can be dispatched in the case of an incident. Therefore, for a three-server system, a state transition from state {0; 0; 0} to state {0; 1; 1} is not allowed, but a transition to state {0; 1; 0} would be allowed.

6. State space

The sum of all states forms the state space. Moreover, it is presupposed that each server can only have the state of '0' as available and '1' as unavailable. The number of states depends solely on the number of servers that are considered and can be calculated by $2^{\text{Number of Servers}}$. The states describe the vertices of the N-dimensional hypercube.

7. Relation to queueing theory principles

The HQM is based on M/M/N queueing theory principles. Therefore, it is assumed that knowing the current state of the system provides information of equal value as knowing the entire history of the system's state transitions which constitutes the Markov process. The HQM, as a result, is meant to be 'memoryless' and can be classified as a finite-state Markov process.[390] Demand is required to occur solely at the center of each demand point and to follow a time-homogeneous Poisson distribution. The service provided is assumed to follow an exponential distribution with a mean of $\frac{1}{\mu}$.

8. Travel and service times

The travel times from the server locations to the demand points as well as the service time are assumed to be deterministic and known. The service time refers to the time period the server spends at the demand point to provide the emergency medical services.

9. Service procedure

After a request for service is received, the nearest-available server is immediately dispatched and travels to the demand point. The server then spends a certain amount of time at the demand point to provide the emergency medical service and then returns to his base

[390] Note that the HQM can theoretically have infinite states for an infinite number of state descriptions for each server or for an infinite number of servers.

location. After return, he is assumed to be available again for service requests.

10. Queueing

If an incoming demand call cannot be immediately served by an available server, it is assumed to be lost. Therefore, no dedicated waiting line is considered as in DE SOUZA ET AL.[391] or RODRIGUES ET AL.[392]. Note that in real-life the call would be served later or by a neighboring EMS system. This simplification is chosen to reduce the computational and modelling efforts that would be implied by modelling specific state descriptions that allow for the queueing of demand calls.

Based on the assumptions detailed above, the districting process and the subsequent formulation of the flow balancing equations for the 2^N-size linear equation system are performed. The former is the pre-requisite for the calculation of the upward as well as downward transition rates. For the determination of districts, a vector T_j, with the length of the maximum level of districting D, is created for each demand point $j \in J$. T_j is an ordered vector that lists, from nearest to furthest, all servers that are eligible to respond in the case of a service request. It is then used to determine the areas of responsibility D_{lk}^d for each server with respect to the state of the HQM. D_{lk}^d is formulated in conjunction with the corresponding state of the HQM with w_l as the nearest and w_k the d-nearest server for demand point j.

$$(3.1.1)\ D_{lk}^d = \begin{cases} j \in J : T_j(1) = w_l\ ,\ T_j(d) = w_k, & \text{if } k \neq l,\ 1 < d \leq D \\ j \in J : T_j(1) = w_l, & \text{if } k = l,\ d = 1 \end{cases}$$

D_{lk}^d is the sub-area of consolidated demand points for which server k is the nearest-available server for districting level d. This formulation explicitly considers the unavailable server l in the districting level d for the construction of the sub-areas. The area-based formulation leads to the definition of areas of responsibility with regard to the level of districting and the respective server. λ_{lk}^d is the corresponding demand to the D_{lk}^d sub-area. The overall incoming demand needs to be fully covered for every level of districting.

$$(3.1.2) \sum_{l=1}^{L} \sum_{k=1}^{K} \lambda_{lk}^d = \lambda \ \ \forall\ d = 1, \ldots, D$$

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[391] Cf. DE SOUZA, R. M. ET AL., 2015
[392] Cf. RODRIGUES, L. F. ET AL., 2017

The upward transition rate λ_{ab} between state a and state b of the N-dimensional hypercube can then be formulated as follows:

$$(3.1.3)\ \lambda_{ab} = \lambda^{1}_{kk} + \sum_{l_1 \in N:\, b_{l_1}=1} \lambda^{2}_{l_1 k} + \sum_{d=2}^{D} \sum_{l_1, \ldots, l_{d-1} \in N:\, \prod_{i=1}^{d-1} b_{l_i} = 1} \lambda^{d}_{l_1 k} \cap \lambda^{d-1}_{l_1 l_{d-1}} \cap \lambda^{d-2}_{l_1 l_{d-2}} \cap \ldots \cap \lambda^{2}_{l_1 l_2}$$

The equation above denotes that, given the state of the system a, server k responds to any demand of its primary area of responsibility D^{1}_{kk}, with λ^{1}_{kk} as the corresponding demand, as well as any other downstream demand from subarea D^{d}_{lk}, with λ^{d}_{lk} as demand, if server l is unavailable in state a. The intersections in the equation above require the higher-tier servers to be unavailable for $d > 2$ which ensures that the d-nearest server is only dispatched if nearer servers are not available.

The construction of sub-areas is illustrated in table 14 below for a five-server system, twenty demand points as well as a partial-backup with a districting level of 3. The reader is referred to AKDOĞAN, BAYINDIR AND IYIGUN for an example of a full backup.[393]

Districting Level		
d = 1	**d = 2**	**d = 3**
$D^{1}_{1,1}$ = {1, 2, 3, 4}	$D^{2}_{5,1}$ = {17, 18, 19, 20}	$D^{3}_{4,1}$ = {13, 14, 15, 16}
$D^{1}_{2,2}$ = {5, 6, 7, 8}	$D^{2}_{1,2}$ = {1, 2, 3, 4}	$D^{3}_{5,2}$ = {17, 18, 19, 20}
$D^{1}_{3,3}$ = {9, 10, 11, 12}	$D^{2}_{2,3}$ = {5, 6, 7, 8}	$D^{3}_{1,3}$ = {1, 2, 3, 4}
$D^{1}_{4,4}$ = {13, 14, 15, 16}	$D^{2}_{3,4}$ = {9, 10, 11, 12}	$D^{3}_{2,4}$ = {5, 6, 7, 8}
$D^{1}_{5,5}$ = {17, 18, 19, 20}	$D^{2}_{4,5}$ = {13, 14, 15, 16}	$D^{3}_{3,5}$ = {9, 10, 11, 12}

Table 14: Districting example

Note that for illustration purposes, the backup procedure is heavily simplified. In a real-world setting, it is highly unlikely that each server would serve an equal amount of demand points at each districting level. Server 1 is the nearest-available for demand points one to four. If server 1 is unavailable, server 2 is then the nearest-available one and will be assigned to demand points one to four at districting level two. For level three, it is assumed that server 3 is the nearest-available. At each districting level, all demand points have to be assigned to exactly one server. The difference in districting between a full- and

[393] Cf. AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018, p. 140

partial-backup is obvious from table 14 in such a way that a full-backup would require that each demand point is assigned to a given server over the sum of all districting levels which then would equal the number of servers considered.

The construction of upward transition rates from state a which is denoted by the state description of {0; 1; 1; 1; 1} to state b which is denoted by {1; 1; 1; 1;1} can be expressed, with respect to the example provided above, as follows:

$$\lambda_{ab} = \lambda^1_{1,1} + \lambda^2_{5,1} + \lambda^2_{4,5} \cap \lambda^3_{4,1}$$

Server 1 becomes unavailable for the described state transition. Therefore, the upward transition rates include its primary area of responsibility, $D^1_{1,1}$ with $\lambda^1_{1,1}$ as the corresponding demand. Additionally, server 1 also responds to the demand regions 17 through 20 since server 5 is unavailable in state a which then includes $D^2_{5,1}/\lambda^2_{5,1}$ in the transition rate. Server 5 is the primary backup for the demand points of server 4 {13; 14; 15; 16}, but both servers are not available due to the current state of the system. Therefore, the third-nearest server has to be dispatched which is server 1. The transition rate then also contains the corresponding sum of demand points which is expressed through the intersection between $D^2_{4,5}$ and $D^3_{4,1}$. It is obvious from the districting formulation, that incoming demands for demand points 13 through 16 would be lost if servers 1, 4 and 5 are unavailable. This illustrates the need for defining an adequate level of backup which should also take into account the overall demand as well as the spatial complexities of the study area. From a modelling and computational standpoint, the introduction of a higher level of backup leads to higher efforts in both categories, especially for larger-scale study areas which include a higher number of demand points and the need to locate a high number of servers to serve them sufficiently.

The formulation of the downward transition rates builds mainly on the concept of districting introduced above. There are several alternatives proposed in the literature to calculate the downward transition rates which are also named service rate:

Fixed value

The fixed-value approach assumes a pre-defined value which includes the travel time from the server location to the demand point as well as the time spent at the scene. This value is then used for all servers as well as all demand points and therefore does not register the spatial complexities of the relation between server and demand point. A fixed value for the service rates has for example been used by TORO-DÍAZ ET AL.[394] and ENAYATI ET AL.[395].

Weighted average

The weighted average approach calculates a weighted average between varying service rates of different servers. For a one-step transition, it simplifies to calculating one service rate for one transition regardless of the spatial relationship between server and demand points. It can then be formulated as follows:

(3.1.4)

$$\mu_{ab} = \frac{\lambda_{ab}}{\frac{\lambda^1_{kk}}{\mu^1_{kk}} + \frac{\sum_{l_1 \in N:\, b_{l1}=1} \lambda^2_{l_1 k}}{\sum_{l_1 \in N:\, b_{l1}=1} \mu^2_{l_1 k}} + \frac{\sum_{d=2}^{D} \sum_{l_1, \dots, l_d \in N \,:\, \prod_{i=1}^{d-1} b_{l_i} = 1} \lambda^d_{l_1 k} \cap \lambda^{d-1}_{l_1 l_{d-1}} \cap \lambda^{d-2}_{l_1 l_{d-2}} \cap \dots \cap \lambda^2_{l_1 l_2}}{\sum_{d=2}^{D} \sum_{l_1, \dots, l_d \in N \,:\, \prod_{i=1}^{d-1} b_{l_i} = 1} \mu^d_{l_1 k} \cap \mu^{d-1}_{l_1 l_{d-1}} \cap \mu^{d-2}_{l_1 l_{d-2}} \cap \dots \cap \mu^2_{l_1 l_2}}}$$

λ_p and μ_p denote one term of the upward- and downward-transition rate formulation. The expression can then be simplified as:

(3.1.5) $$\mu_{ab} = \left(\frac{\sum_{p=1}^{P} \left(\frac{\lambda_p}{\mu_p} \right)}{\lambda_{ab}} \right)^{-1}$$

The weighted average approach has been introduced by GEROLIMINIS, KARLAFTIS AND SKABARDONIS[396] and later used by GEROLIMINIS, KEPAPTSOGLOU AND KARLAFTIS[397].

Travel time based

Since, as mentioned above, the travel time from the server location to the demand point can have a significant influence on the overall service provided, AKDOĞAN, BAYINDIR AND IYIGUN include it in their service rate formulation.[398] They propose various alternatives and prove their validity in a simulation study. This study will use the weighted average variant that uses the demand fraction of the demand point as weight. The service provided for one individual demand point can be formulated as follows:

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[394] Cf. TORO-DÍAZ, H. ET AL., 2013, p. 925
[395] Cf. ENAYATI, S. ET AL., 2019, p. 427
[396] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009, p. 803
[397] Cf. GEROLIMINIS, N.; KEPAPTSOGLOU, K.; KARLAFTIS, M. G., 2011
[398] Cf. AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018, p. 142

$$(3.1.6)\ \phi'_{nj} = \frac{T}{\phi + 2\, t_{nj}}$$

The formulation calculates the number of possible deployments, ϕ'_{nj}, of a server to the respective demand point. T is the time period, ϕ is the time spent at the scene of the incident while t_{nj} denotes the travel time from the server location n to the demand point j. Since the server has to travel back to its location, t_{nj} is used twice.

The calculation of the downward transition rate between state a and state b, that uses the weighted average of all demand points that can cause the transition, can be formulated as follows:

$$(3.1.7)\ \mu_{ab} = \sum_{j \in L_{ab}} \left(\frac{f_j}{\sum_{j \in L_{ab}} f_j} \right) \phi'_{nj}$$

L_{ab} is the set of all demand points that can lead to the downward transition from state a to state b. f_j describes the demand fraction of demand point j.

3.2 Model Formulation

The following model formulation is used as the underlying hypercube queueing model for the Goal Programming, Robust Optimization[399] as well as for the Robust Goal Programming approaches detailed in chapter 4.

$$(3.2.1)\ \sum_{j=1}^{J} f_j y_j^{cov} \geq C_{cov}$$

$$(3.2.2)\ \sum_{i \in W_j}^{I} x_i \geq y_j \quad \forall\, j \in J$$

$$(3.2.3)\ \sum_{i=1}^{I} x_i = N$$

$$(3.2.4)\ x_i, y_j, y_j^{cov} \in [0;1] \quad \forall\, i \in I, j \in J$$

---

399 The scenario-dependent formulations are given in chapter 4.2.

$$(3.2.5)\quad P\{B_b\}\left[\sum_{\substack{a \\ \{B_a \in C_N : d^-_{ab}=1\}}} \lambda_{ab} + \sum_{\substack{a \\ \{B_a \in C_N : d^+_{ab}=1\}}} \mu_{ab}\right] =$$

$$\sum_{\substack{a \\ \{B_a \in C_N : d^-_{ab}=1\}}} P\{B_a\}\,\mu_{ab} + \sum_{\substack{a \\ \{B_a \in C_N : d^+_{ab}=1\}}} P\{B_a\}\,\lambda_{ab} \quad \forall\, b = 0, 1, \ldots, 2^N-1$$

$$(3.2.6)\quad \sum_{a=0}^{2^N-1} P\{B_a\} = 1$$

Constraint (3.2.1) requires a certain amount of the study area to be coverable within a pre-determined time span. The overall goal of the model is to provide coverage of the entire study area. Therefore, a more or less evenly spread location of the server locations over the study area is assumed to be profitable. The constraint largely avoids solution candidates that are geographically skewed towards one side of the study area. They can also be considered as beneficial in terms of computational times since the existing solution space is decreased due to the time requirement. y_j^{cov} shows whether demand point j can be covered subject to a pre-defined time constraint. Note that the constraint is calibrated over C_{cov} and it is potentially possible that good solutions are eliminated by the constraint. Therefore, the value of C_{cov} should be chosen rather loosely and in such a way that it only avoids extreme solution candidates. Constraint (3.2.2) controls the variable y_j and only takes the value 'one' if there is at least one server covering demand point j. Constraint (3.2.3) guarantees that only the pre-defined number of servers is located while (3.2.4) defines variables x_i, y_j and y_j^{cov} as binary variables.

The constraints (3.2.5) and (3.2.5) are formulated similarly to the ones used for the HQM numerical example of chapter 2.5.2.

4. Extensions for Hypercube Queueing Location Models

4.1 Goal Programming with an embedded HQM

The Goal Programming approach has initially been introduced by CHARNES AND COOPER.[400] It is one of the most used approaches in the multi-objective optimization literature and allows for the consideration of multiple and, if relevant, conflicting goals in the optimization approach. In order to utilize it to its fullest potential, measurable and relevant objectives for the corresponding optimization problem have to be identified. The goals may or may not be of the same unit scale.[401] One of the benefits of the Goal Programming approach lies in its practicability. While it may be impossible to determine the utility function of a decision maker, it is often possible to name certain aspiration levels which can then be used in a Goal Programming approach. In order to formulate a meaningful Goal Programming model, some key terms are briefly characterized below:[402]

Objective

The objective is usually defined by a mathematical function that captures the key wishes of the decision maker.

Aspiration level

The aspiration level is a (realistic) target value for the corresponding objective. It measures the achievement or non-achievement of an objective.

Goal

The combination of an objective and an aspiration level constitutes the corresponding goal. Usually, goals can be formulated with b as the defined aspiration level.

(4.1.1) satisfy $f(x) \geq b$

(4.1.2) or, satisfy $f(x) \leq b$

(4.1.3) or, satisfy $f(x) = b$

In order to evaluate the quality of an obtained solution, usually the deviation between the desired aspiration level and the actual achievement (or non-achievement) of each goal is

[400] Cf. CHARNES, A.; COOPER, W., 1961
[401] Cf. BADRI, M. A.; MORTAGY, A. K.; ALI ALSAYED, C., 1998, p. 246
[402] Cf. IGNIZIO, J. P., 1983, pp. 279–282

measured and can be characterized as follows:

(4.1.4) $d_g = b_g - f_g(x)$

Since the deviations can be positive or negative, d_g can be written as:

(4.1.5) $d_g = \delta_g^+ + \delta_g^-$

δ_g^+ and δ_g^- show the positive and negative deviation from the desired aspiration level and can be used to measure the achievement or non-achievement. As noted by IGNIZIO, there are several ways to compare obtained solution candidates to each other in order to derive the most appropriate solution to the optimization problem. Examples include the minimization of some linear form of goal deviation variables, to minimize the maximum goal deviation, the lexicographic minimum and the development of a set of non-dominated solutions that are close to one of the solutions obtained by one of the methods named above.[403] One of the most used approaches is the minimization of the sum of the weighted goal deviation which is also called weighted goal programming. The optimization problem can then be formulated as follows[404]:

(4.1.6) $\textbf{Min } Z = \sum_{g=1}^{G} \left(W_g^+ \delta_g^+ + W_g^- \delta_g^- \right)$

subject to

(4.1.7) $f_g(x) - \delta_g^+ + \delta_g^- = b_g \ \forall \ g \in G$

(4.1.8) $x \in F$

(4.1.9) $\delta_g^+, \delta_g^- \geq 0 \ \forall \ g \in G$

Usually the individual goals are expressed in different unit scales, for example monetary units, the amounts of a good produced or, in the case of EMS, the number of emergency calls that can be covered by the system. The differing unit scales complicate the interpretation of the optimization process result and should be normalized. There is a variety of literature that treats this problem and, for example, an Euclidean distance approach has been proposed to normalize the positive and negative goal deviations. A related problem

[403] Cf. IGNIZIO, J. P., 1983, p. 283
[404] Cf. KANOUN, I.; CHABCHOUB, H.; AOUNI, B., 2010, p. 148; IGNIZIO, J. P., 1983, p. 283

arising from the weighted Goal Programming approach is the insufficient incorporation of the decision maker's preferences.[405] In order to deal with the mentioned shortcomings, MARTEL AND AOUNI propose an extension to the weighted Goal Programming which incorporates so called satisfaction functions.[406] These functions can be used to precisely model the preferences of a decision maker or to reward and penalize certain achievements or non-achievements of an aspiration level. The concept of satisfaction functions is, for example, applied by KANOUN, CHABCHOUB AND AOUNI[407] in the location optimization decision for EMS systems and by JAYARAMAN ET AL.[408] in the broader context of sustainable economic development.

The satisfaction functions build on the Prométhée method and use a generalized criterion. Their application leads to direct and comparable differences for actions on one criterion and allows for an easy and simple incorporation of the decision makers preferences into the optimization process. MARTEL AND AOUNI propose six types of satisfaction functions that build on the Prométhée method, but emphasize that their list is "neither exhaustive nor restrictive"[409] and can be easily extended for different optimization problems or preferences. For each goal a corresponding satisfaction function is chosen which returns a function value for the achievement or non-achievement of the goal by the solution candidate. The six types of satisfaction functions are briefly described in the following:

[405] Cf. MARTEL, J. M.; AOUNI, B., 1990, p. 1124
[406] Cf. MARTEL, J. M.; AOUNI, B., 1990
[407] Cf. KANOUN, I.; CHABCHOUB, H.; AOUNI, B., 2010
[408] Cf. JAYARAMAN, R. ET AL., 2017
[409] Cf. MARTEL, J. M.; AOUNI, B., 1990, p. 1126

Usual criterion (C1)

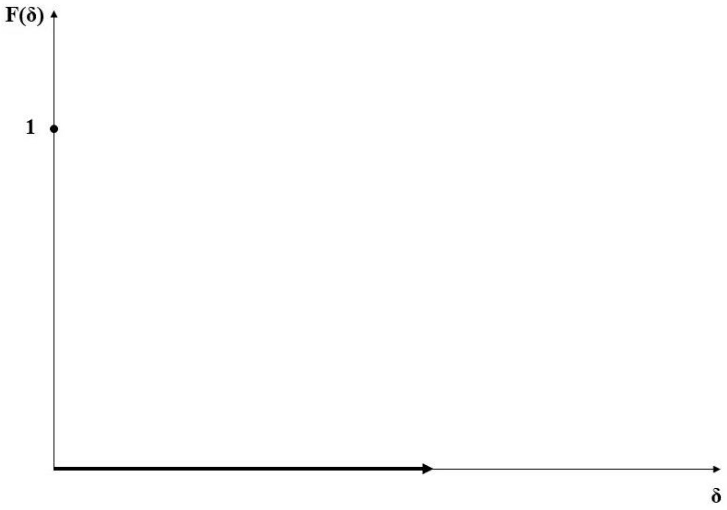

Figure 6: Criterion C1

The Usual criterion (C1) penalizes any deviation from the defined aspiration level and returns the function value of zero. Mathematically, this can be formulated as follows:

(4.1.10) $F(\delta) = \begin{cases} 1, \delta = 0 \\ 0, \delta > 0 \end{cases}$

Quasi-criterion (C2)

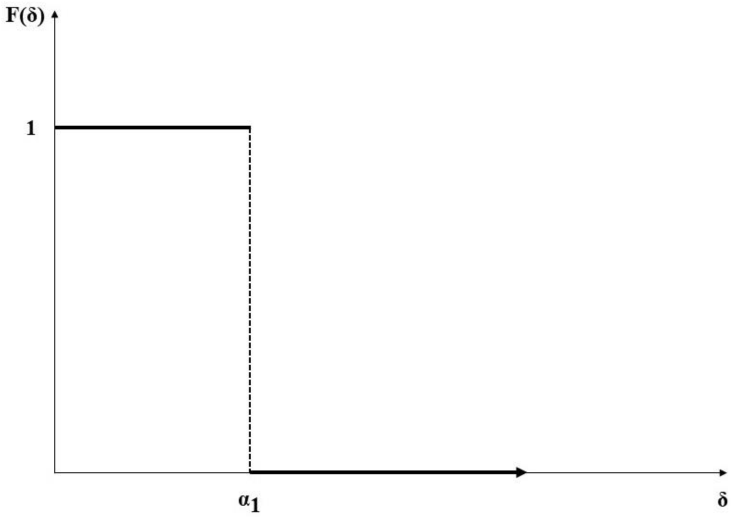

Figure 7: Criterion C2

The Quasi-criterion allows for a higher deviation from the defined aspiration level than the Usual criterion. After the threshold of α_1, the function value of zero is returned. Mathematically, this can be formulated as follows:

(4.1.11) $F(\delta) = \begin{cases} 1, \delta \leq \alpha_1 \\ 0, \delta > \alpha_1 \end{cases}$

Criterion with linear preference (C3)

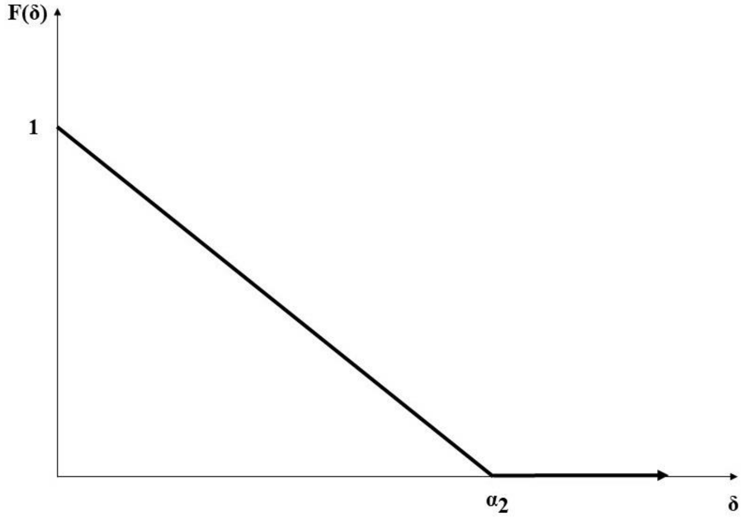

Figure 8: Criterion C3

The criterion with linear preference (C3) penalizes deviations linearly until a threshold value of α_2. Afterwards the function returns a value of zero. Mathematically, this can be formulated as follows:

(4.1.12) $F(\delta) = \begin{cases} 1 - \left(\frac{\delta}{\alpha_2}\right), \delta \leq \alpha_2 \\ 0, \delta > \alpha_2 \end{cases}$

Level criterion (C4)

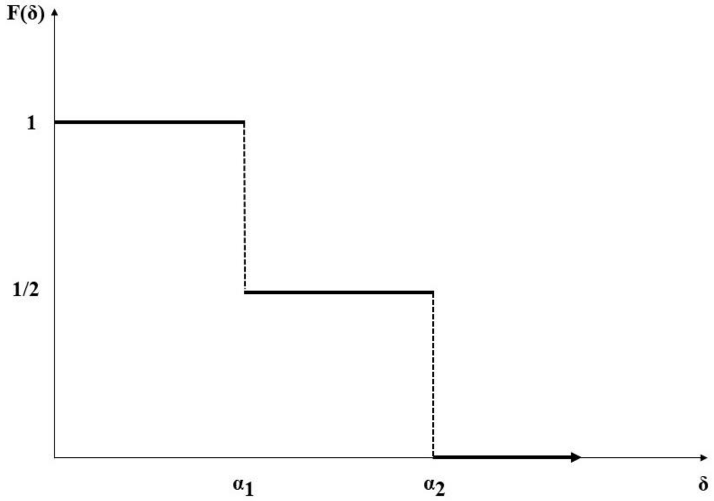

Figure 9: Criterion C4

The Level criterion is an extension of the Quasi-criterion (C2) and allows for a more precise penalization of the achievement or non-achievement of the defined aspiration level. It considers the incorporation of one or more levels that represent different function values for the respective deviations from the defined aspiration level.[410] Mathematically, this can be formulated as follows:

$$(4.1.13)\ F(\delta) = \begin{cases} 1, & \delta \leq \alpha_1 \\ \frac{1}{2}, & \alpha_1 < \delta \leq \alpha_2 \\ 0, & \delta \geq \alpha_2 \end{cases}$$

[410] Note that the '1/2' in figure 9 represents a suggestion. It can be easily adjusted according to the decision makers preferences.

Criterion with linear preference and indifference area (C5)

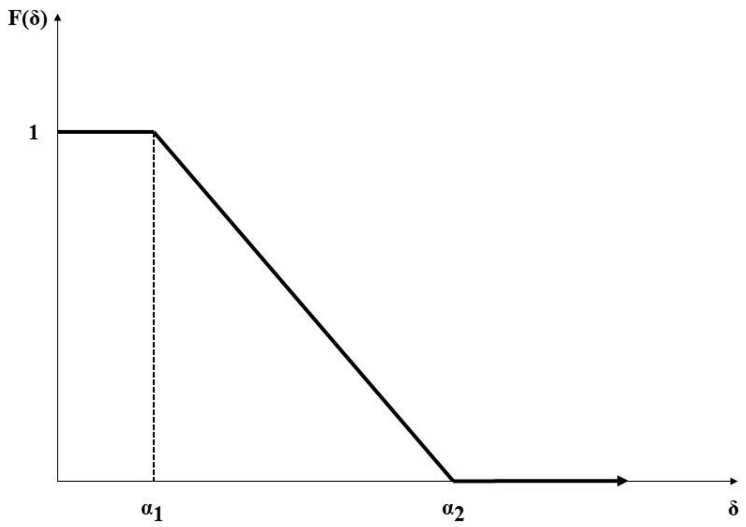

Figure 10: Criterion C5

The criterion with linear preference and an indifference area (C5) penalizes deviations from the defined aspiration level only after a certain threshold value α_1. Afterwards, the function value decreases linearly until α_2 is exceeded. Mathematically, this can be formulated as follows:

$$(4.1.14)\ F(\delta) = \begin{cases} 1, \delta \leq \alpha_1 \\ \frac{\alpha_2 - \delta}{\alpha_2 - \alpha_1}, \alpha_1 < \delta \leq \alpha_2 \\ 0, \delta \geq \alpha_2 \end{cases}$$

Gaussian criterion (C6)

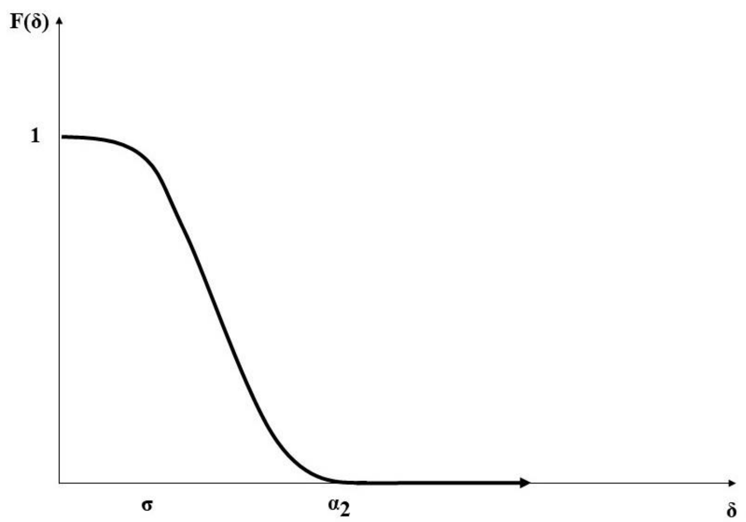

Figure 11: Criterion C6

The Gaussian criterion (C6) penalizes values after a certain threshold, σ, more strongly if compared to values before that threshold. After α_2 is exceeded, the function returns the value zero. Mathematically, this can be formulated as follows:

$$(4.1.15)\ F(\delta) = \begin{cases} \exp\left(\frac{-\delta^2}{2\sigma^2}\right), & \delta \leq \alpha_2 \\ 0, & \delta > \alpha_2 \end{cases}$$

The returned function values of the satisfaction functions are used in the Goal Programming formulation instead of the deviations. The Goal Programming model can then be described/calculated as follows:

$$(4.1.16)\ \textbf{Max}\ Z = \sum_{g=1}^{G} \left(W_g^+ F_g^+\left(\delta_g^+\right) + W_g^- F_g^-\left(\delta_g^-\right)\right)$$

s.t.

$$(4.1.17)\ f_g(x) - \delta_g^+ + \delta_g^- = g_g \ \forall\ g \in G$$

$$(4.1.18)\ x \in F$$

$$(4.1.19)\ 0 \leq \delta_g^+ \leq \alpha_{gv}^+ \ \ \forall\ g \in G$$

$$(4.1.20)\ 0 \leq \delta_g^- \leq \alpha_{gv}^- \ \ \forall\ g \in G$$

The concept of satisfaction functions rewards less deviation from the defined aspiration level with a higher function value. Therefore, a higher function value can be considered as superior. The optimization problem is then formulated as a maximization problem. Note that it is possible to put varying weights on the positive and negative deviation from an aspiration level. The selection of the type of the satisfaction function should be done with respect to the corresponding goal and the preferences of the decision maker.

The linear equation system defined by the model presented in chapter 3 returns probabilities that, on their own, cannot be used in an optimization process. Therefore, they have to be employed in the calculation of performance metrics that can be analyzed in order to evaluate the performance of the corresponding solution candidate. As is obvious from the literature review in chapter 2.3.2.2, most exact HQM studies only use the AERT metric in their objective functions. With the exception of SABEGH ET AL.[411] and ENAYATI ET

[411] Cf. SABEGH, M. H. Z. ET AL., 2017, p. 411

AL.[412], all studies apply only one criterion. The approach presented here wants to make use of the variety of performance metrics that can be derived by the use of the HQM. Those can then be considered for a precise evaluation of the deviations from the aspiration levels that are defined in conjunction. The defined goals and their respective aspiration levels were formulated to fulfill most common goals for the design of an EMS system. The performance metrics as well as the corresponding satisfaction functions for the Goal Programming approach are described hereafter. The relevant aspiration levels and threshold values are calibrated in chapter 7.2.

G1 Average Expected Loss Probability (AELP) (criterion C6)

Any incoming demand for emergency medical help requires an immediate response. Additionally, EMS systems usually have to respond to any demand for emergency help due to legal but also ethical concerns. The loss probabilities for any M/M/N queueing system that does not explicitly consider an infinitely sized queue have to be expected to be greater than zero. This can be explained by the probability that a request for service occurs in the case of all servers being unavailable. Such cases should be understood as demands not being immediately served. In reality, the demand can be served later or by a neighboring EMS system.

The loss probabilities can be calculated for every demand point or as a system-wide performance metric. If calculated per demand point, $PLoss_j$ is calculated by the sum of the probabilities of all states $B_a \in N_j$. N_j is the set of states in which an incoming demand call of demand point j cannot be handled because all servers on the preference list are unavailable.

$$(4.1.21)\ PLoss_j = \sum_{B_a \in N_j}^{2^N-1} P\{B_a\}\ \forall\ j \in J$$

For a system-wide calculation, $PLoss_j$ is then summed up over all demand points and weighted by the fraction of the demand by the individual demand point of the overall demand. This can be expressed as follows:

412 Cf. ENAYATI, S. ET AL., 2019, p. 428

$$(4.1.22)\ \text{AELP} = \sum_{j=1}^{J} f_j \text{PLoss}_j$$

In order to penalize solutions that result in a higher AELP, a Gaussian criterion (C6) is introduced.

$$(4.1.23)\ F(\delta_1^+) = \begin{cases} \exp\left(\frac{-\delta_1^{+2}}{2\sigma^2}\right), & \delta_1^+ < \alpha_2^1 \\ 0, & \delta_1^+ \geq \alpha_2^1 \end{cases}$$

<u>G2 Average Expected Response Time (AERT) (criterion C3)</u>

The AERT metric is one of the most applied performance metrics for the evaluation and analysis of EMS systems and is frequently used for external communication purposes. It has also legal implications since most EMS systems are benchmarked against certain AERT thresholds as described in chapter 2. As already detailed for the numerical example of chapter 2, AERT is calculated by the sum, from all demand points and servers, of the fractions of dispatches multiplied by the corresponding travel times. Since the fraction of dispatches is normalized to account for the loss probabilities, the fraction of dispatches should be less sensitive to an increase in demand if compared to the AELP metric.

As mentioned above, the fraction of dispatches server n sends to demand point j, p_{nj}, has to be normalized to account for demand that cannot be immediately served, which can be calculated as follows:

$$(4.1.24)\ p_{nj} = \frac{\sum_{B_a \in E_{nj}} P\{B_a\}}{1 - \text{PLoss}_j}\ \ \forall\ j \in J, n \in N$$

$$(4.1.25)\ \sum_{n=1}^{N} p_{nj} = 1,\ \ \forall\ j \in J$$

The sum of the fraction of all dispatches for all servers has to be ‘one’ for every demand point (4.1.25). The fraction of dispatches can then be used to calculate the system-wide response time by the weighted sum of all demand points and the corresponding fraction of dispatches, p_{nj}, multiplied by the travel time, t_{nj}. The fractions of demand of the individual demand points are employed as weights.

$$(4.1.26)\ \text{AERT} = \sum_{n=1}^{N}\sum_{j=1}^{J} f_j\, p_{nj}\, t_{nj}$$

Due to the less sensitive reaction of the AERT metric to an increase in demand, a linear preference criterion (C3) is introduced here as satisfaction function.

$$(4.1.27)\ F\left(\delta_2^+\right) = \begin{cases} -\frac{1}{\alpha_2^2}\delta_2^+ + 1, & \delta_2^+ < \alpha_2^2 \\ 0, & \delta_2^+ \geq \alpha_2^2 \end{cases}$$

G3 Percentage of demand points covered under 9 minutes (RTCov) (criterion C3)

All demands for emergency help require a more or less instantaneous response. The very early EMS systems frequently used coverage requirements in their objective function which are still relevant to date. The coverage metric can offer insight into the coverage provided to the more distant areas of the underlying service area. Note that it is not identical to the coverage constraint of chapter 3. The latter is applied before the calculation of the equation system and is used to avoid extremely skewed solution candidates. It also does not take account of system congestion. In contrast, the percentage of demand points that can be covered under 9 minutes should theoretically be lower due to the congestion of the system and the resulting fraction of dispatches served by backup servers. The threshold of 9 minutes is chosen due to the extra time that is allowed for slightly less serious emergencies.[413] $ytime_j$ is a binary variable that takes the value of 'one' if demand point j is part of the set of demand points, N^T, that can be covered within the pre-defined time constraint. It is then divided by the number of demand points.

$$(4.1.28)\ \text{RTCov} = \frac{\sum_{j \in N^T}^{J} ytime_j}{|J|}$$

$$(4.1.29)\ ytime_j \in [0;1]\ \forall\, j \in J$$

The response time of the system is calculated by a weighted average. In this way, the analysis of the percentage of demand points that can be covered under a pre-determined time constraint offers additional insight. A linear preference criterion (C3) is used to capture the deviations.

[413] Cf. REUTER-OPPERMANN, M.; VAN DEN BERG, P. L.; VILE, J. L., 2017, p. 188

$$(4.1.30)\ F(\delta_3^-)=\begin{cases}-\frac{1}{\alpha_2^3}\delta_3^-+1, & \delta_3^-<\alpha_2^3\\ 0, & \delta_3^-\geq\alpha_2^3\end{cases}$$

G4 Longest response time to a demand point (LRT) (criterion C3)

As described for G3, any demand requires a response by the EMS system as soon as possible. Since EMS systems always have a responsibility to provide medical services to more distant parts of their underlying service area, limiting the longest response time is a desirable goal in the optimization process. Additionally, minimizing the longest response time can also contribute to the goal of providing equity among the different parts of the service area. The longest response time to a priority demand is calculated by the maximal response time for all demand points.

$$(4.1.31)\ LRT \geq \sum_{n=1}^{N} p_{nj}\, t_{nj}\quad \forall\, j \in J$$

In order to capture the potential non-achievement of G4, a linear preference criterion (C3) is introduced.

$$(4.1.32)\ F(\delta_4^+)=\begin{cases}-\frac{1}{\alpha_2^4}*\delta_4^++1, & \delta_4^+<\alpha_2^4\\ 0, & \delta_4^+\geq\alpha_2^4\end{cases}$$

G5 Workload-Balance-Metric (criterion C3)

Usually, not all servers serve the same number of demand points due to their spatial distribution over the study area. As a result, the workloads between individual servers may vary to a rather large degree, which could then lead to some servers having significantly higher workloads. The unequal distribution of dispatches may also influence the ability to respond to incoming demands. In contrast, in most cases, servers have to be located unevenly over the study area, as there are regions from which more demand can be expected. The workload of a server is denoted by w_n and is calculated by the sum of all states, $B_a \in N_B^n$, in which server n is unavailable.

$$(4.1.33)\ w_n = \sum_{B_a \in N_B^n}^{2^N-1} P\{B_a\}\ \forall\, n \in N$$

Balanced server workloads may or may not be beneficial for other performance metrics like AERT or AELP, but can be helpful for possible future spikes in demand calls. A similar approach is used by Iannoni, MORABITO AND SAYDAM[414]. The WBM is calculated by the absolute deviation of the workload of server n, w_n, from the average server workloads of all servers.

$$(4.1.34)\ \mathrm{WBM} = \sum_{n=1}^{N} \left| \frac{w_n}{\sum_{n=1}^{N} \frac{w_n}{N}} - 1 \right| \frac{1}{N}$$

A linear preference criterion (C3) is utilized here.

$$(4.1.35) F\left(\delta_5^+\right) = \begin{cases} -\frac{1}{\alpha_2^5} * \delta_5^+ + 1, & \delta_5^+ < \alpha_2^5 \\ 0, & \delta_5^+ \geq \alpha_2^5 \end{cases}$$

The performance metrics used as well as their corresponding satisfaction functions, aspiration levels and threshold values are summarized in table 15 below:

	Goal	**Criterion type**
G1	AELP	C6
G2	AERT	C3
G3	Percentage of demand points covered under 9 minutes (RTCov)	C3
G4	Longest response time (LRT)	C3
G5	Workload-Balance-Metric (WBM)	C3

Table 15: Goal definitions

The linear preference criterion (C3) is used as the satisfaction function for goals two to five (G2-G5). This is assumed for the following reasons:

- The Usual-, Quasi- and Level-criteria (C1, C2 and C4) seem to be inappropriate

[414] Cf. IANNONI, A. P.; MORABITO, R.; SAYDAM, C., 2008, p. 215

since they cannot penalize deviations adequately for the current optimization problem due to their discrete function types.

- The linear preference criterion with an indifference area (C5) is not used since deviations from the aspiration levels shall be penalized from the outset.
- The Gaussian criterion (C6) is only employed for the AELP metric, which puts additional emphasis on lowering the expected loss probabilities of the system if they exceed a certain threshold.

The performance metrics used in the Goal Programming approach only reflect metrics that are directly and solely connected to the performance of the EMS system. Budget constraints are not considered since it is assumed that they are of secondary importance.[415] In order to illustrate the existing trade-offs, the weights of the goals are varied in chapter 7.

4.2 Robust Optimization with an embedded HQM

Since its proposal, the robust optimization framework proposed by MULVEY, VANDERBEI AND ZENIOS has been widely used in the incorporation of uncertainty factors into optimization decisions.[416] For the integration of the robustness terms, the objective function is modified and incorporates penalty functions for the solution and, in some modifications, the model robustness, which are both weighted by individual parameters. The solution robustness refers to the solution being nearly optimal in every scenario while the model robustness refers to the solution being nearly feasible in every scenario. The definition of the modifier 'nearly' is up to the decision maker or modeler. The relationship between the parameters should reflect the decision maker's preferences for solution or model robustness. For further details on robust optimization principles the reader is referred to MULVEY, VANDERBEI AND ZENIOS, BEN-TAL AND NEMIROVSKI[417] and BEN-TAL, EL GHAOUI AND NEMIROVSKI[418].

The used Robust Optimization approach follows the principles described by MULVEY, VANDERBEI AND ZENIOS, also kown as 'discrete Robust Optimization'[419]. The derivation

[415] A budget constraint is introduced in chapter seven as an addition that penalizes newly opened server locations.
[416] Cf. MULVEY, J. M.; VANDERBEI, R. J.; ZENIOS, S. A., 1995
[417] Cf. BEN-TAL, A.; NEMIROVSKI, A., 2002
[418] Cf. BEN-TAL, A.; EL GHAOUI, L.; NEMIROVSKI, A., 2009
[419] Cf. HAFEZALKOTOB, A.; HAJI-SAMI, E.; OMRANI, H., 2015, p. 199

of the robust optimization problem is described in the following section.

In a general context, the linear programming model can be formulated as follows [420]:

(4.2.1) **min** $f(x,y)= c^T x+ d^T y$

Subject to:

(4.2.2) $Ax = b$

(4.2.3) $Bx + Cy = e$

(4.2.4) $x, y \geq 0$

In this formulation, x is a vector of the decision variables, while y is the vector of the control variables. The matrices of the parameter are described by A, B, and C. The vectors of the parameters are denoted by b and e.

For the introduction of Robust Optimization approaches, it is assumed that A and b are deterministically known while B, C and e are uncertain. For the definition of the robust optimization problem, a set of scenarios $\Omega = \{1, 2, \ldots, S\}$ is introduced. With each scenario, a set of realizations $\{d_s, B_s, C_s, e_s\}$ as well as a scenario probability ($\sum_{s=1}^{S} p_s = 1$) are presupposed. The control variable y is also defined with respect to the scenarios.

If the linear optimization problem is solved, with respect to the different scenarios, realizations and control variables, it is highly unlikely that a solution is both optimal and feasible for all scenarios. Therefore, a robust solution has to be found that is adequate for all scenarios and their different parameter realizations. Therefore, an error vector δ_s is introduced to capture the infeasibility of the model under the corresponding scenarios. In the case of the model being feasible, δ_s will be zero. Based on these assumptions, the linear programming model can be reformulated as [421]:

(4.2.5) $$\mathbf{min}\ Z = \sum_{s=1}^{S} p_s \psi_s + \vartheta \sum_{s=1}^{S} p_s \left(\psi_s - \sum_{s'=1}^{S} p_{s'} \psi_{s'} \right)^2$$

[420] Cf. MULVEY, J. M.; VANDERBEI, R. J.; ZENIOS, S. A., 1995, p. 265

[421] Cf. LEUNG, S. C. H. ET AL., 2007, p. 227

Subject to:

(4.2.6) $Ax = b$

(4.2.7) $B_s x + C_s y_s + \delta_s = e_s \; \forall \; s \in S$

(4.2.8) $x, y_s, \delta_s \geq 0$

ϑ is the weighting parameter for the variance while ψ_s represents the scenario-dependent value of the function $f(x, y_s)$, which can be a cost-minimizing function, or, in the case of an EMS system, the minimization of the overall response time or the loss probabilities of incoming demands. For this work, the minimization of the overall response time is chosen as the objective function.

The quadratic expression in (4.2.5) implies high computational efforts that can be reduced by applying the approach by YU AND LIN[422]. In their approach the quadratic term is replaced by an absolute deviation which can be written as follows:

$$\text{(4.2.9)} \quad \mathbf{min}\, Z = \sum_{s=1}^{S} p_s \psi_s + \vartheta \sum_{s=1}^{S} p_s \left| \psi_s - \sum_{s'=1}^{S} p_{s'} \psi_{s'} \right|$$

Furthermore, they propose a way to minimize the objective function effectively. The optimization problem can then be described as follows:

$$\text{(4.2.10)} \quad \mathbf{min}\, Z = \sum_{s=1}^{S} p_s \psi_s + \vartheta \sum_{s=1}^{S} p_s \left(\psi_s - \sum_{s'=1}^{S} p_{s'} \psi_{s'} + 2\theta_s \right)$$

$$\text{(4.2.11)} \quad \psi_s - \sum_{s=1}^{S} p_s \psi_s + \theta_s \geq 0 \; \forall \; s \in S$$

(4.2.12) $\theta_s \geq 0 \; \forall \; s \in S$

θ_s is zero in the case of $\psi_s - \sum_{s=1}^{S} p_s \psi_s > 0$. If $\psi_s - \sum_{s=1}^{S} p_s \psi_s < 0$, θ_s takes the value of $\sum_{s=1}^{S} p_s \psi_s - \psi_s$. As a result, it can be stated that the result of (4.2.10) is equal to (4.2.9).

The second robustness term in the objective function, the model robustness, is a penalty function that penalizes the infeasibility of the model in some or all of the scenarios used

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[422] Cf. YU, C. S.; LI, H. L., 2000, pp. 387–391

[423]. The resulting objective function can then be expressed as follows:

$$(4.2.13)\ \mathbf{min}\, Z = \sum_{s=1}^{S} p_s\psi_s + \vartheta \sum_{s=1}^{S} p_s \left(\psi_s - \sum_{s'=1}^{S} p_{s'}\psi_{s'} + 2\theta_s \right) + \omega \sum_{s=1}^{S} p_s\delta_s$$

The Robust Optimization approach in this paper considers different day times as the individual scenarios and therefore considers uncertainty in demand location and quantity within a scenario-based approach. One of the assumptions of an M/M/N queueing system, constituting the theoretical foundation of both the approximate and the exact HQM, are constant interarrival and service rates as explained in chapter 2.2.1. Considering these as permanent allows the formation of a steady state. Therefore, varying demand or service quantities cannot be expressed adequately through any single M/M/N queueing model. BANDI, BERSTSIMAS AND YOUSSEF state a robust queueing model which incorporates uncertainty through a probability distribution and a G/G/N queueing model. [424] This approach shall not be used here. Instead, the HQM is formulated and solved for each scenario individually. JENKINS, LUNDAY AND ROBBINS formulate a very similar approach and employ an approximate HQM in their optimization study. The approximate HQM uses the steady-state formulation of a common M/M/N queueing model and introduces so-called correction factors to calculate the server-specific fraction of dispatches. The main difference between the approximate and exact HQM lies in the explicit consideration of individual servers, which is assumed in the exact model from the outset. JENKINS, LUNDAY AND ROBBINS study a military evacuation asset location problem in which several and distinctive deployment phases in a military operation are introduced as scenarios. The quantity of expected demand as well as the demand locations vary among the scenarios. Their objective function maximizes the demand fulfillment over all scenarios. Thus, the approach of JENKINS, LUNDAY AND ROBBINS may be considered comparable to the approach used in this work.[425] A scenario-based queueing-theory approach can also be found in AN ET AL. for an emergency service facility location study. [426]

BETTINELLI ET AL. use different day times in a M/M/N queueing-theory-based simulation study. They discuss the problem of homogenous time periods and time-homogenous interarrival and service rates and conclude that the individual time periods considered in

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[423] Cf. MIRZAPOUR AL-E-HASHEM, S. M. J.; MALEKLY, H.; ARYANEZHAD, M. B., 2011, p. 31
[424] Cf. BANDI, C.; BERTSIMAS, D.; YOUSSEF, N., 2015
[425] Cf. JENKINS, P. R.; LUNDAY, B. J.; ROBBINS, M. J., 2020
[426] Cf. AN, S. ET AL., 2015

the model have to be of adequate length. If the time span is too short, the model is unlikely to reach a true steady state. If it is too long, it is possible that significant changes in the demand patterns and locations occur in reality but are represented inadequately. Consequently, BETTINELLI ET AL. recommend the application of at least four different time periods with a minimal length of two hours each. In their analysis, they use five different periods with a length of at least three hours. [427]

The approach taken in this study employs the following assumptions to incorporate different day times into the optimization problem:

- The HQM is formulated and solved S times. S denotes the number of scenarios that are used to model the different day times. The corresponding terms of the model are then varied with respect to the scenario $s \in S$.
- The interarrival and service rates within each scenario remain constant. Therefore, changing demand conditions within one scenario are not considered.
- The S different model results in S different sets of steady-state probabilities.
- The S different sets of steady-state probabilities are then used to calculate S sets of performance metrics and the scenario-related part of the objective function, which is independent of other scenarios.
- The S sets of performance metrics are then also employed to calculate the other relevant terms of the model, for example the relevant restrictions.

The different demand locations and quantities for each scenario result in different formulations for the districting process and, subsequently, for the upward- and downward transition rates, which will be described below. The different locations of the demand points for the individual scenarios may result in scenario-dependent preference lists. Note that only a change in the quantity of the demand would not have an impact on the preference lists. Therefore, if the location of demand points is identical in different scenarios, the same preference lists can be used.

It should be considered that a location decision is taken once for all scenarios and that, as a result, the actual locations of the servers are assumed to be invariable. There exist differing formulations in the broader field of EMS location optimization literature which,

[427] Cf. BETTINELLI, A. ET AL., 2014

for example, intend to minimize the number of relocations necessary to solve EMS demands adequately. Since a restocking and replenishment is often necessary, it is considered beneficial for the server to return to its base location after completing the service. Moreover, the use of a fixed location also benefits more distant regions of the study area since basic coverage can always be assured. Additionally, it represents the current conditions as well as the operating characteristics of the applied case study best.

A set of scenario-specific demand points, J^s, is introduced. For every demand point $j \in J^s$ and every scenario s an ordered vector T_j^s (from nearest to furthest) of candidate server locations is constructed, with D as the maximum level of districting allowed. Every vector is defined with respect to the corresponding hypercube state with w_l^s being the nearest and w_k^s the d-nearest server for demand point j. Mathematically, the scenario-dependent formulation is expressed as follows:

$$(4.2.14)\ D_{lk}^{d,s} = \begin{cases} j \in J^s : T_j^s(1) = w_l^s,\ T_j^s(d) = w_k^s, & \text{if } k \neq l,\ 1 < d \leq D \\ j \in J^s : T_j^s(1) = w_l^s, & \text{if } k = l,\ d = 1 \end{cases}$$

$D_{lk}^{d,s}$ denotes the sub-area of consolidated demand points in which server k is the nearest-available server for districting level d and scenario s in the event server l is not available in the case of a demand. $\lambda_{lk}^{d,s}$ is the corresponding demand of the $D_{lk}^{d,s}$ sub-area for emergency services offered by the system. Furthermore, the incoming demand for every scenario has to be fully covered for every level of districting d.

$$(4.2.15) \sum_{l=1}^{L} \sum_{k=1}^{K} \lambda_{lk}^{d,s} = \lambda^s \quad \forall\ d=1,\ldots,D,\ s \in S$$

$$(4.2.16)\ \lambda_{ab}^{s} = \lambda_{kk}^{1,s} + \sum_{l_1 \in N:\ b_{l_1}=1} \lambda_{l_1 k}^{2,s} + \sum_{d=2}^{D} \sum_{l_{1,\ldots,}l_{d-1} \in N\ :\ \prod_{i=1}^{d-1} b_{l_i} = 1} \lambda_{l_1 k}^{d,s} \cap \lambda_{l_1 l_{d-1}}^{d-1,s} \cap \lambda_{l_1 l_{d-2}}^{d-2,s} \cap \ldots \cap \lambda_{l_1 l_2}^{2,s}$$

The downward transition rates also have to be formulated with respect to the individual scenarios. It is assumed here that the time spend at the scene of the incident, ϕ, is not subject to changes in different scenarios. Since the daytimes are formulated as scenarios, they have different time lengths. Therefore, the calculation now denotes how much service can be provided per time period T^s of scenario s. t_{nj}^s is also varied due to the different demand locations and resulting travel times from server n to demand point j.

$$(4.2.17)\ \phi_{nj}^{\prime,s} = \frac{T^s}{\phi + 2\, t_{nj}^s}$$

The calculation of the downward transition rate between state a and state b in scenario s expresses again the weighted average of all demand points that can cause that transition.

$$(4.2.18)\ \mu_{ab}^s = \sum_{j \in L_{ab}^s} \left(\frac{f_j^s}{\sum_{j \in L_{ab}^s} f_j^s} \right) \phi_{nj}^{\prime,s}$$

L_{ab}^s is the set of all demand points that can lead to the downward transition from state a to state b in scenario s. f_j^s describes the demand fraction of demand point j in scenario s.

The scenario-based upward and downward transition rates are used to formulate the Robust Optimization approach with the embedded HQM. Note that the objective function only considers the response time of the system but does not include multiple objectives as in chapter 4.3.

$$(4.2.19)\ \textbf{min T} = \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{j=1}^{J^s} p_s\, f_j^s\, p_{nj}^s\, t_{nj}^s + \vartheta \sum_{s=1}^{S} p_s \left(\psi_s - \sum_{s'=1}^{S} p_{s'} \psi_{s'} + 2\theta_s \right) + \omega \sum_{s=1}^{S} p_s \delta_s$$

Subject to:

$$(4.2.20)\ \sum_{s=1}^{S} \sum_{j=1}^{J} p_s f_j^s y_j^{cov,s} \geq C_{cov}$$

$$(4.2.21)\ \sum_{i \in W_j^s}^{I} x_i \geq y_j^s \quad \forall\, j \in J^s$$

$$(4.2.22)\ \sum_{i=1}^{I} x_i = N$$

$$(4.2.23)\ x_i, y_j^s, y_j^{cov,s} \in [0;1] \quad \forall\, i \in I, j \in J^s$$

$$(4.2.24)\quad P^s\{B_b\}\left[\sum_{\substack{a\\ \{B_a\in C_N:d^-_{ab}=1\}}} \lambda^s_{ab} + \sum_{\substack{a\\ \{B_a\in C_N:d^+_{ab}=1\}}} \mu^s_{ab}\right] =$$

$$\sum_{\substack{a\\ \{B_a\in C_N:d^-_{ab}=1\}}} P^s\{B_a\}\,\mu^s_{ab} + \sum_{\substack{a\\ \{B_a\in C_N:d^+_{ab}=1\}}} P^s\{B_a\}\,\lambda^s_{ab}\ \forall\, b = 0, 1, \ldots, 2^N\text{-}1,\ s \in S$$

$$(4.2.25)\quad \sum_{a=0}^{2^N-1} P^s\{B_a\} = 1\ \forall\, s \in S$$

$$(4.2.26)\quad p^s_{nj} = \frac{\sum_{B_a\in E^s_{nj}} P^s\{B_a\}}{1\text{-}\ PLoss^s_j}\ \forall\, j \in J^s,\ n \in N,\ s \in S$$

$$(4.2.27)\quad PLoss^s_j = \sum_{B_a\in N^s_j}^{2^N-1} P^s\{B_a\},\ \forall\, j \in J^s,\ s \in S$$

$$(4.2.28)\quad \sum_{n=1}^{N} p^s_{nj} = 1,\ \forall\ j \in J^s,\ s \in S$$

$$(4.2.29)\quad \psi_s - \textstyle\sum_{s=1}^{S} p_s\psi_s + \theta_s \geq 0\ \forall\, s \in S$$

$$(4.2.30)\quad \sum_{j=1}^{J^s} f^s_j\ PLoss^s_j = \delta_s\ \forall\, s \in S$$

$$(4.2.31)\quad \delta_s \geq 0\ \forall\, s \in S$$

For simplicity, the model is formulated only in robust terms. ψ_s describes the realization of the objective function under scenario s, i.e. $\sum_{n=1}^{N}\sum_{j=1}^{J^s} p_s\ f^s_j\ p^s_{nj}\ t^s_{nj}\ \forall\, s \in S$. Since most of the equations formulated above are identical to the plain HQM not recognizing scenarios, the reader is referred to chapter 3.2 for a more detailed presentation. Only those equations that have changed significantly or are newly introduced are discussed to a greater extent:

Constraint (4.2.20) is applied before the actual optimization to avoid solutions that are heavily skewed towards one side of the study area. Since it is not assumed that servers can change their base locations between scenarios, the decision variable x_i is formulated scenario-independent. Therefore, (4.2.20) calculates a probability-weighted average of the number of demand points that can be covered with respect to a pre-defined time constraint. Due to the change in demand locations, $y_j^{cov,s}$ is formulated scenario-dependent. Constraint (4.2.21) controls the decision variable and only takes the value of ‘one’ if demand point j is covered at least once in scenario s. Constraints (4.2.22) and (4.2.23) remain to the same as in the model of chapter 3.2 but are, if relevant, related to the respective scenarios.

Constraint (4.2.24) defines the linear equation system with regard to a scenario-dependent formulation. Constraints (4.2.25), (4.2.26), (4.2.27) and (4.2.28) are expressed scenario-dependent and use the steady-state probabilities of the corresponding scenario.

Constraints (4.2.30) and (4.2.31) refer to the calculation of the robustness parameters. Constraint (4.2.29) uses the linearization approach introduced by YU AND LIN[428] to calculate the deviations for each scenario (θ_s). The parameter for the penalty function, δ_s, is calculated in constraint (4.2.30). The AELP is used as the reference point for the calculation of the model robustness parameter.

4.3 Robust Goal Programming with an embedded HQM

The Robust Goal Programming approach was initially developed by KUCHTA.[429] It is a relatively recent optimization technique integrating the two widely used techniques Goal Programming and Robust Optimization.[430] The use of the Robust Goal Programming extends the usually deterministic Goal Programming. Robust Goal Programming has recently been applied for the multi-objective portfolio selection problem[431], a capital budgeting problem[432], a production problem[433], in a fuzzy multi-objective facility problem[434], transportation research[435] and for the sustainable development problem[436]. As can be seen by the studies mentioned above, Robust Goal Programming is a relatively new extension

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[428] Cf. YU, C. S.; LI, H. L., 2000
[429] Cf. KUCHTA, D., 2004
[430] Cf. HANKS, R. W.; LUNDAY, B. J.; WEIR, J. D., 2020
[431] Cf. GHAHTARANI, A.; NAJAFI, A. A., 2013
[432] Cf. GHASEMI BOJD, F.; KOOSHA, H., 2018
[433] Cf. HANKS, R. W.; WEIR, J. D.; LUNDAY, B. J., 2017
[434] Cf. SHISHEBORI, D.; BABADI, A. Y.; NOORMOHAMMADZADEH, Z., 2018
[435] Cf. HANKS, R. W.; LUNDAY, B. J.; WEIR, J. D., 2020
[436] Cf. JIA, R.; LIU, Y.; BAI, X., 2020

to the Robust Optimization technique and has not been extensively researched.[437]

The Robust Optimization approach by MULVEY, VANDERBEI AND ZENIOS (chapter 4.2) incorporates uncertainty over a scenario method. In order to use this approach in the Goal Programming of chapter 4.1, the objective function and the performance metrics have to be formulated scenario-dependent. The satisfaction functions are not changed here for simplification purposes. If not marked otherwise, the restrictions of chapter 4.2 remain relevant for this chapter. Therefore, the formulation of upward and downward transition rates as well as for the basic restrictions of the HQM are not repeated here.[438]

Average expected loss probability

The AELP may vary due to the changing demands between individual scenarios.

$$(4.3.1)\ AELP^s = \sum_{j=1}^{J^s} f_j^s \, PLoss_j^s \ \forall\, s \in S$$

The formulation is identical to the calculation of δ_s in chapter 4.2 and similar to chapter 4.1 but considers the individual scenarios.

Average expected response time

The AERT may vary between the scenarios due to changing demand locations and quantities and can be calculated as follows.

$$(4.3.2)\ AERT^s = \sum_{n=1}^{N} \sum_{j=1}^{J^s} f_j^s \, p_{nj}^s \, t_{nj}^s \ \forall\, s \in S$$

The formulation is identical to the first part of the objective function of the Robust Optimization and does only differ from the formulation in chapter 4.1 for the consideration of the respective scenarios.

Percentage of demand points covered under 9 minutes

The coverage of demand points depends on their locations as well as on the overall congestion of the system. Thus, the coverage provided by the system may vary with respect

[437] Cf. HANKS, R. W.; LUNDAY, B. J.; WEIR, J. D., 2020
[438] This is relevant for all restrictions, (1)-(12). The objective function is reformulated with respect to the Goal Programming approach.

to the different demand considered in the individual scenarios.

$$(4.3.3)\ RTCov^s = \frac{\sum_{j \in N^{Ts}}^{J^s} ytime_j^s}{|J^s|} \quad \forall\, s \in S$$

$$(4.3.4)\ ytime_j^s \in [0;1] \quad \forall\, j \in J^s,\ s \in S$$

$ytime_j^s$ denotes whether demand point j can be covered subject to a time constraint pre-defined for the base model and whether it could be varied with regard to the respective scenario. This may lead to ethical and legal issues since the EMS system has to ensure basic coverage at any time of the day.

Longest response time

The longest response time may vary with the individual scenarios for the same reasons as the percentage of demand points covered under 9 minutes.

$$(4.3.5)\ LRT^s \geq \sum_{n=1}^{N} p_{nj}^s t_{nj}^s \quad \forall\, j \in J^s,\ s \in S$$

The formulation is similar to chapter 4.1 but scenario-dependent.

Workload-Balance-Metric

Since the model is calculated S times, S different sets of probabilities can be used to calculate scenario-dependent workloads for the servers as well as the WBM.

$$(4.3.6)\ p_n^s = \sum_{B_a \in N_B^n}^{2^N-1} P^s\{B_a\} \quad \forall\, n \in N,\ s \in S$$

As for the scenario-independent formulation of the server workloads, the server workloads are calculated by the sum of all probabilities $P^s\{B_a\}$ in which server n is unavailable ($B_a \in N_B^n$). N_B^n denotes the corresponding set of states. Since the basic structure of the HQM does not change with respect to the scenario approach, the set of (un)available states remains identical.

The calculation of the WBM then uses the scenario-dependent workloads.

$$(4.3.7)\ WBM^s = \sum_{n=1}^{N} \left| \frac{p_n^s}{\sum_{n=1}^{N} \frac{p_n^s}{N}} - 1 \right| \frac{1}{N}$$

The scenario-dependent performance metrics are applied in the formulation of the objective function for the Robust Goal Programming.

(4.3.8) **Max** Z =

$$\sum_{g=1}^{G} \left(\sum_{s=1}^{S} p_s \left(W_{gs}^{+} F_g^{+}\left(\delta_{gs}^{+}\right) + W_{gs}^{-} F_g^{-}\left(\delta_{gs}^{-}\right) \right) - \vartheta \sum_{s=1}^{S} p_s \left(\psi_s^g - \sum_{s'=1}^{S} p_{s'} \psi_{s'}^g + 2\theta_s^g \right) - \omega \sum_{s=1}^{S} p_s \delta_s \right)$$

Subject to:

$$(4.3.9)\ f_g(x^s) - \delta_{gs}^{+} + \delta_{gs}^{-} = g_{gs}\ \forall\, g \in G,\ s \in S$$

$$(4.3.10)\ x^s \in F\ \forall\, s \in S$$

$$(4.3.11)\ 0 \le \delta_{gs}^{+} \le \alpha_{gvs}^{+}\ \ \forall\, g \in G,\ s \in S$$

$$(4.3.12)\ 0 \le \delta_{gs}^{-} \le \alpha_{gvs}^{-}\ \ \forall\, g \in G,\ s \in S$$

$$(4.3.13)\ \psi_s - \sum_{s=1}^{S} p_s \psi_s + \theta_s \ge 0\ \forall\, s \in S$$

$$(4.3.14) \sum_{j=1}^{J^s} f_j^s\, PLoss_j^s = \delta_s \forall\, s \in S$$

$$(4.3.15)\ \delta_s \ge 0\ \forall\, s \in S$$

The objective function for the Goal Programming with satisfaction functions has a maximizing character. Hence, the addition of solution and model robustness terms would lead to an additional score for presumably inferior solutions. Therefore, solution and model robustness are incorporated with their respective negative values. The same approach is employed by SAFAEI, ROOZBEH AND PAYDAR in a Robust Optimization supply chain design study in which the objective function maximizes the profit.[439] ψ_s describes the realization of the objective function, i.e. $W_{gs}^{+} F_g^{+}\left(\delta_{gs}^{+}\right) + W_{gs}^{-} F_g^{-}\left(\delta_{gs}^{-}\right)\ \forall\, s \in S,\ g \in G$. The calculation of the model robustness term (equation 4.3.14) is identical to chapter 4.2.

[439] Cf. SAFAEI, A. S.; ROOZBEH, A.; PAYDAR, M. M., 2017

5. Case Study and Study Area

As detailed in the literature review in chapter 2.6.2, the exact HQM has not been extensively used for larger-size case studies. Recent advances in computational power and efficient metaheuristics allow for the incorporation of the exact HQM into location optimization studies for larger study areas, such as a larger city. Therefore, one aim of this study is to design and implement a large-area real-world case study for which a location optimization study with an exact HQM is performed. Due to the availability of open source data, the city of Munich is chosen. The respective data as well as the location optimization decision refer to the system of first responders. The number of emergency vehicles also includes the location that solely serves children. Therefore, no demand differentiation in an adult and infant population is considered.

Case description	
Inhabitants	1,561,720 (2020)
Geographical size	310,71 sq. km
City districts	25
Yearly (daily) [hourly] emergency medical dispatches	~ 43,000 (117) [4,875] (2019)
Emergency vehicles	12

Table 16: Case description

To account for different demands for emergency medical services in different areas of the city, the population density of Munich was modelled on a 200x200 grid with respect to the different densities of the (sub-)districts. A total of seven different population densities are considered. For the reconstruction of actual demand, a total of three-hundred demand points is located in the study area. The locations are positioned randomly but according to the individual population density of each area. It is assumed that areas with a higher population density have a higher demand for emergency medical services.[440] The demand is then uniformly distributed over all demand points. As a result, different, e.g. higher, demand in certain areas of the study area is not reflected in different demands for the demand points, but through the number of demand points in that area. The amount of

[440] Note that this assumption is relaxed for two of the three demand point maps that are used for the robust model.

yearly, daily and hourly emergency medical dispatches is provided by the Integrated Control Center of Munich.[441] The design and mapping of the demand points for the base study area is illustrated in figure 12 below.

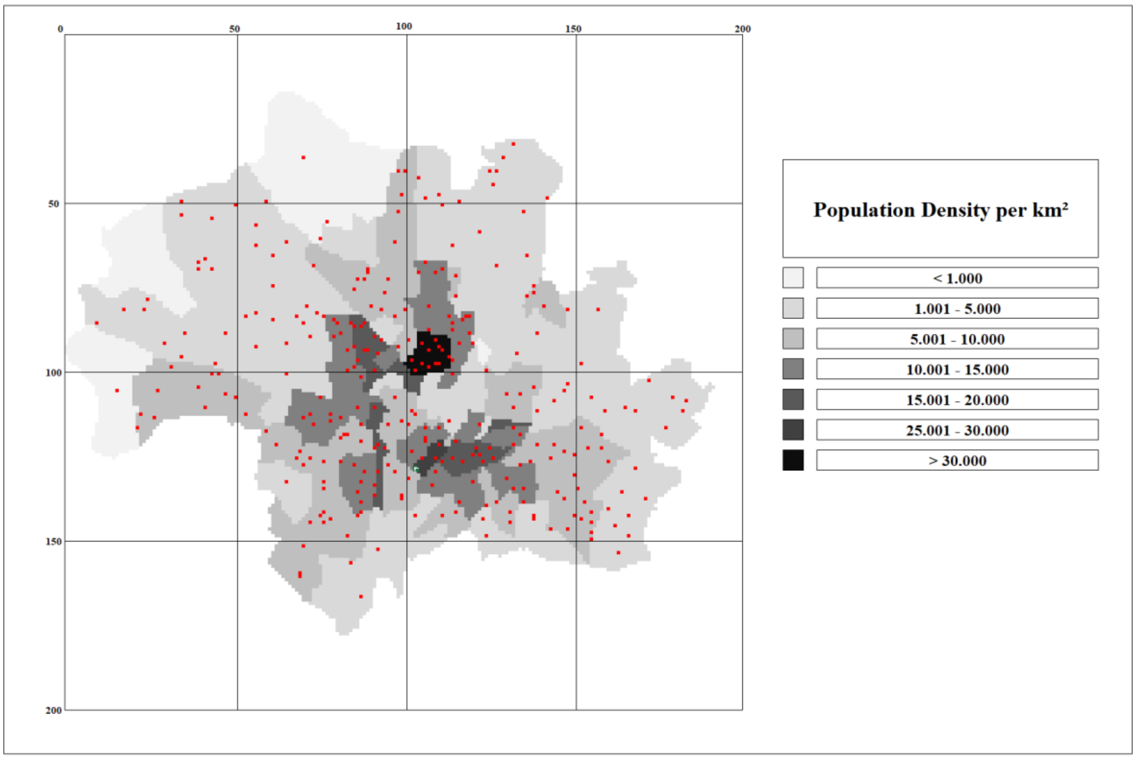

Figure 12: Base study area

As described in the literature review of chapter 2.3.2, all existing HQM models use only one time period. One of the aims of this work is to demonstrate the varying influence of different daytime periods on an EMS system. Therefore, the construction of additional study areas with respect to different times of the day is required. Since the model has to be solved separately for each defined scenario, a higher amount of scenarios would lead to excess computational times. BETTINELLI ET AL. recommend a minimum length of two hours per period for the model to reach a true steady state. They employ a total of four different periods in their analysis.[442]

Usually, EMS systems are confronted with changes in spatial distribution and frequency of incoming demands over the course of one day. Applied to urban contexts, this implies that events like the commute of the working population and varying locations, like home and work sites, lead to different origins of demand. Furthermore, varying activities, the

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[441] Cf. INTEGRIERTE LEITSTELLE MÜNCHEN, 2019, p. 13
[442] Cf. BETTINELLI, A. ET AL., 2014, p. 4

resulting variation of risks over the day, and different risk groups of potential patients lead to a non-uniform distribution of incoming demands over the day. Applied to this work, this means that each time period or scenario has to be assigned a certain amount of demand. As mentioned by CANTWELL ET AL., the incoming demand starts relatively low at the beginning of the day and reaches its peak at about noon time. Afterwards it remains relatively stable into the evening hours and declines towards the end of the day.[443] BETTINELLI ET AL. stress the impact of rush hours, especially for large urban agglomerations like the city of Milan which is used in their case study.[444]

In order to capture the different variations of demand and not over-stress the still existing computational limits, a total of five different time periods for one day is considered here. As already mentioned, due to commutes and different centers of activity over the course of the day, it is also assumed that the demand points shift between the scenarios. Therefore, three different demand point mappings are considered. The base demand point mapping (B) is identical to the base study area. The rush-hour demand point mapping (R) is introduced to capture especially the commutes to and from the workplace. The work demand point mapping (W) is used to represent the amount of the day the working population spend at the worksite. Moreover, it is assumed that not all potential demand callers do change their location over the day, one third of the demand points remains identical in the different demand point mappings. This is to account for the part of the population that does not commute to the workplace or does not significantly change its location over the day. The R and the W demand point mapping are designed with the help of traffic intensity maps[445] as well as maps that show the distribution of worksites over the city area[446]. Both maps are provided by the City of Munich. Note that the different study areas as well as the varying amounts of demand between the scenarios are only used for the models of chapter 4.2 and 4.3. The Goal Programming model considers solely the study area presented in figure 12. The different demand point mappings for the individual scenarios are presented in figure 13.

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[443] Cf. CANTWELL, K. ET AL., 2015, p. 3
[444] Cf. BETTINELLI, A. ET AL., 2014, p. 4
[445] Cf. LANDESHAUPTSTADT MÜNCHEN, 2019
[446] Cf. LANDESHAUPTSTADT MÜNCHEN - REFERAT FÜR STADTPLANUNG UND BAUORDNUNG, 2017, p. 14–15

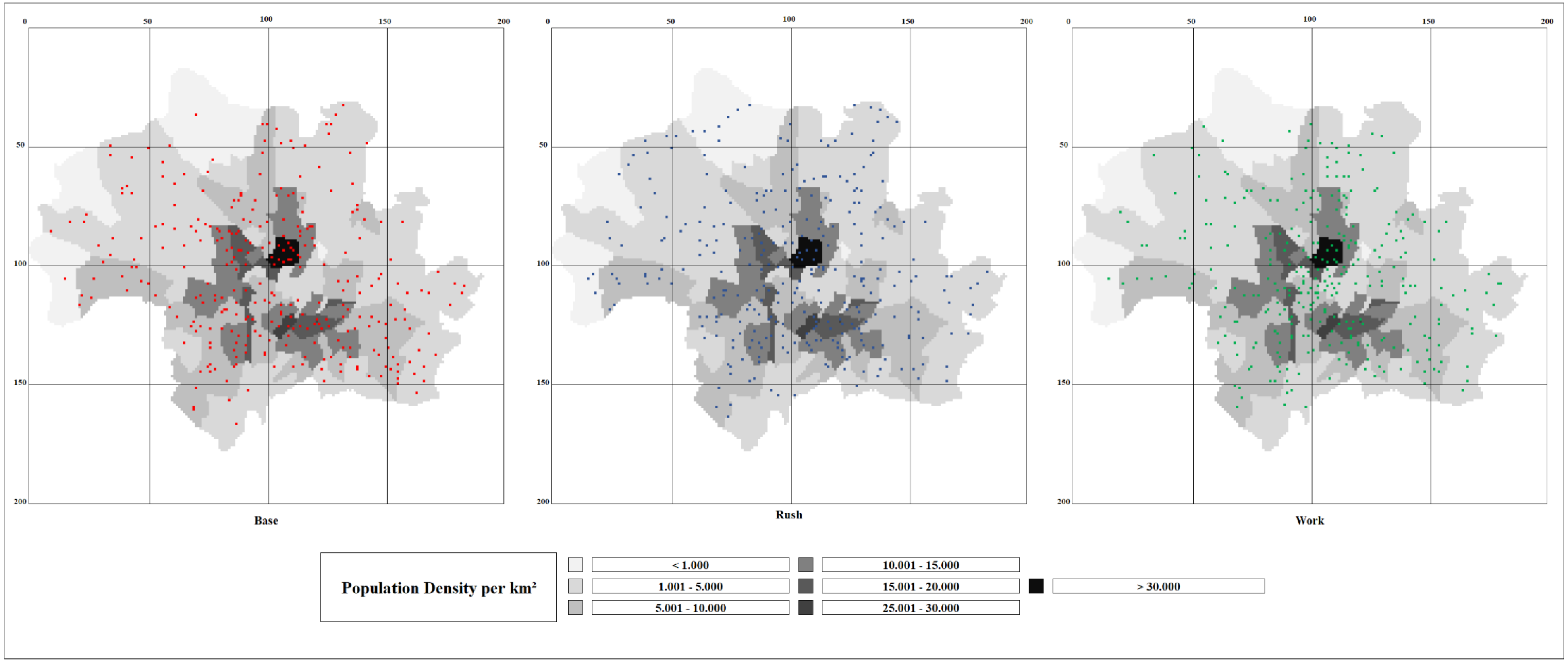

Figure 13: Demand point mappings

The distribution of demand in the respective scenarios is presented in table 17 below. The scenario probabilities reflect the fraction of the corresponding daytime on the overall day.

Scenario	Daytime	Fraction of total daily demand as percent factor	Demand point mapping	Scenario probability
1	2am-6am	0,13	B	0,208
2	7am-8am	0,1	R	0,0835
3	9am-4pm	0,38	W	0,333
4	5pm-6pm	0,12	R	0,0835
5	7pm-1am	0,27	B	0,292

Table 17: Scenario generation

In order to perform an adequate location optimization decision, a set of candidate server locations has to be located in the study area. Since this study wants to showcase the potential improvements for a greenfield location decision, all twelve existing first responder locations are reconstructed in the study area. Additionally, twenty-eight locations are randomly generated. This leads to a set of forty candidate locations which are illustrated in figure 14. The green dots represent existing server locations while the red dots represent the randomly generated locations.

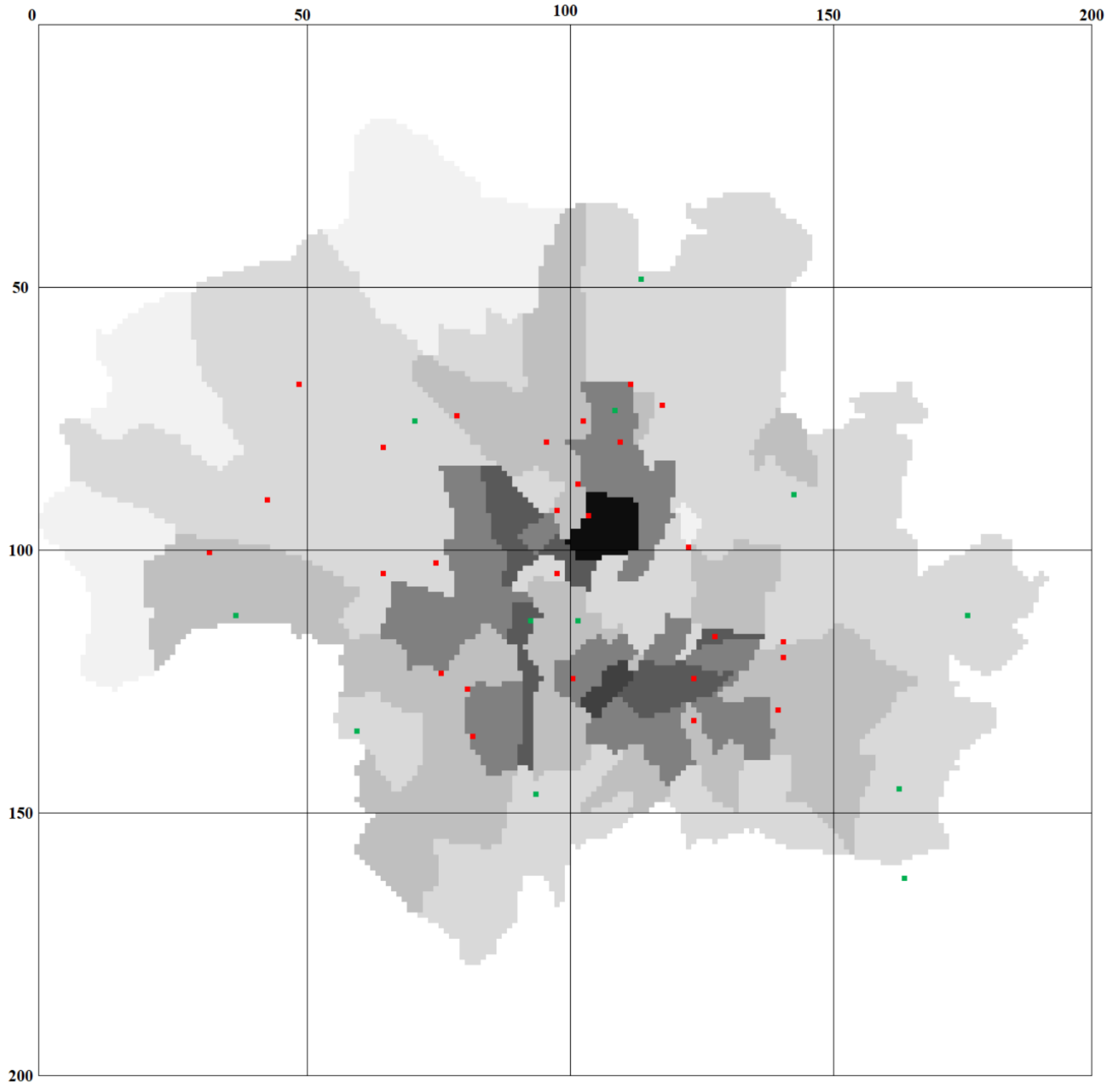

Figure 14: Server Locations

6. Metaheuristic

6.1 Genetic Algorithm

The Genetic Algorithm (GA), initially proposed by HOLLAND [447], is a population-based metaheuristic that intends to mimic the real-life evolutionary process by gradually eliminating individuals with a lower fitness value. Compared to single-solution approaches, like Simulated Annealing, population-based techniques are supposed to be superior in terms of avoiding local optima.[448] Additionally, the GA is easy to understand due to its use of principles of genetic evolution which are common knowledge. As a result, the GA is relatively easy to adapt to the respective optimization problem.

For a review of recent literature, the reader is referred to MIRJALILI ET AL.[449] The GA uses a chromosome structure to represent the different decision variables of the model. In order to evolve towards better solutions, a fitness function, in this case the value of the objective function, is used to compare the different individuals of each population generation. The main components which are relevant for the current work are described in the following[450]:

Initial population

At the start of the GA, a pre-defined number of individuals, the population size, is randomly initialized with N (the number of decision variables) chromosomes.

Calculation of reproduction probabilities

After each individual of the current generation ($s_n \in S$) has completed the model, the fitness values of each individual are compared and assigned a probability. For a maximizing objective function, as for the models presented in sections 4.1 and 4.3, this is done by the following equation:

$$(6.1.1)\quad p_s = \frac{fit_s}{\sum_{s \in S} fit_s} \quad \forall\, s \in S$$

fit_s represents the value of the objective function.

For a minimizing objective function as for the model presented in section 4.2, the probability is assigned by the following equation:

[447] Cf. HOLLAND, J. H., 1975
[448] Cf. KATOCH, S.; CHAUHAN, S. S.; KUMAR, V., 2020
[449] Cf. MIRJALILI, S. ET AL., 2020
[450] Cf. AKDOGAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018, pp. 147–149

$$(6.1.2)\ p_s = \frac{\frac{1}{fit_s}}{\sum_{s \in S} \frac{1}{fit_s}} \ \forall\ s \in S$$

Due to the minimizing character of the objective function, smaller fitness values are considered superior.

After that, individuals are selected in pairs according to their probability to create the next generation.

Crossing over

Crossing over is used to transfer genes from the parents to the offspring. Here, it is assumed that two parents produce two children. A random crossover point x^{cross} between 1 and N is employed. If a pre-defined crossover probability is met, the first x^{cross} genes from parent 1 are transferred to the first child while the genes after the crossover point are transferred from parent 2. For the second child, the inverse procedure is applied. If the crossover probability is not met, the children are produced by the same chromosomes as their parents. The crossover mechanism, for the case that the crossover probability is met and all other restrictions are fulfilled, is illustrated below.

Parent 1 (1 2 3 ⁝ 4 5 6)   Parent 2 (7 8 9 ⁝ 10 11 12)

Child 1(1 2 3 ⁝ 10 11 12)   Child 2 (7 8 9 ⁝ 4 5 6)

Afterwards, the offspring is added to the solution pool.

Mutation procedures

After the crossover procedures, each chromosome of each individual is subject to mutation to create larger genetic diversity and to avoid local minima. A rule of thumb for the mutation probability is to choose values between 1/(number of chromosomes) and 1/(population size). [451]

Solution pool and reproduction conditions

After the crossover and mutation procedures, the model is performed for each individual candidate solution. After that, the solutions are compared to each other and only the superior individuals are selected to start the reproduction process for the next generation.

[451] Cf. GEROLIMINIS, N.; KEPAPTSOGLOU, K.; KARLAFTIS, M. G., 2011, p. 294

Termination criterion

As for other metaheuristics, a termination criterion is applied. It is possible to either use a fixed criterion which terminates the GA after a pre-defined number of iterations or to use a dynamic criterion that leads to a termination if the best found solution has not improved after a pre-defined number of iterations.

The procedure of the GA is illustrated in the pseudo-code provided below:

```
start
if g = 0
  for int s_n, ..., S
    for int c_i, ..., N
        choose solution components c_i randomly
        endfor
    if (restrictions are fulfilled)
      return solution s_n from model
  calculate p_s for s_n ∈ S
  for int x = 1, ..., (S/2)
  select two s_n with p_s
  if (p_c is met)
  determine random crossover point
    for int c_i, ..., N
      transfer c_i according to crossover point
    endfor
  endif
  if else
  copy s_n to s_n+x
  for int c_i, ..., N
      if (p_m is met)
        mutate c_i
    endfor
   create two s_n+x and add to S^New
   endfor
if g > 0
```

for int $s_n, ..., S^{New}$
if (restrictions are fulfilled)
return solution s_n from model
endfor
select S best solutions
kill (S^{New} – S) worst solutions
repeat reproduction procedure from above
until termination criterion is satisfied
return best solution found

Table 18: Pseudo-code GA

As mentioned above, the GA has several parameters that can be adjusted for the respective optimization problem:

- Size of the population
- Crossover probability
- Mutation probability
- Selection process[452]

6.2 Ant-Colony-Optimization

The Ant-Colony-Optimization (ACO) is a population-based metaheuristic that intends to mimic the real-life behavior of ants in the search of new food sources. Originally proposed by DORIGO as a metaheuristic, it is nowadays often used to solve optimization problems of different facets.[453] For a review of recent literature, the reader is referred to MIRJALILI, SONG DONG AND LEWIS.[454] In real life, ants swarm out of the anthill to search food sources. If they are successful, they return to the anthill and deploy a pheromone trail on their way back. The closer the food source is located, the stronger the pheromone trail is. Newly swarmed out ants then choose, if more than one option is available, the food source with the stronger pheromone trail with a higher likelihood. Usually, the food source is being used up over time and therefore the pheromone trail evaporates. This leads to less

[452] The procedure described here refers to the roulette selection process. A tournament selection process would create a competition between two randomly selected individuals in which the inferior would be eliminated from the pool.
[453] Cf. DORIGO, M., 1992
[454] Cf. MIRJALILI, S.; SONG DONG, J.; LEWIS, A., 2020

and less ants choosing the corresponding food source.

The process of finding new food sources and the evaporation of pheromone trails is applied in the ACO metaheuristic. The individual steps that are relevant to the modelling and execution of the ACO are described below[455]:

Modelling of the decision variables

For a discrete and binary optimization problem as the one studied in this work, the entire space of decision variables is modelled as potential food sources. In order to guarantee equality among them, they are all given an equal probability at the start. Applied to the real-life behavior of ants, this would be comparable to a completely random swarm-out at the initial food search.

Construction of the population

After the initial modelling period is completed, an initial population has to be created. The number of ants, which are the solution candidates, is one of the key parameters that can be adjusted for the ACO. For each ant, the pre-defined number of solution components is chosen with respect to the probability distribution over the solution space. If there are relevant restrictions, like the coverage constraint and the requirement that only one server can be located at a location, they have to be met accordingly.

Completion of the model and choice of solutions

After the population is constructed, the model is performed for each candidate solution and the value of the objective function is registered. After all ants have completed the model, the solution candidates are ordered with respect to the value of their objective functions. One or more of the best solutions is then selected to update the probabilities.

Probability update

The selected solutions are used to update the probabilities and the evaporation process is applied. Thus, for each element of the solution space the following equation is applied:

$$(6.2.1)\ \tau_i = (1 - \rho)\ \tau_i + \rho \sum_{c_i \in s}^{S_{upd}} \omega_s\ F(s)\ \forall\ i \in I$$

[455] Cf. DORIGO, M.; BLUM, C., 2005, p. 253; BLUM, C., 2005, p. 359–361

τ_i is the probability of server location i. The evaporation proportion is denoted by ρ. Solution s has different solution components described by c_i. The S_{upd} best solutions are chosen to update the pheromone trails. ω_s is the weight of solution s. F(s) is a quality function, that ensures that the condition of $F(s) \geq F(s')$ is fulfilled. This guarantees that more pheromones and therefore higher probability are given to the solution components that have performed superior.

After the probabilities are updated, the respective iteration of the ACO is completed and the next population is constructed under the new probability distribution of the solution components.

Termination criterion

The termination criterion is identical to the GA.

The procedure of the ACO is illustrated in the pseudo-code provided below:

start

for int i, …, I

set τ_i to $\frac{1}{I}$

endfor

repeat

for int s_a*, …, S*

for int c_i*, …, N*

choose N solution components c_i *with* τ_i

endfor

if (restrictions are fulfilled)

return solution s_a from model

endfor

select S_{upd} best solutions

for int i, …, I

$\tau_i = (1 - \rho)\,\tau_i$

if ($c_i \in s_a \in S_{upd}$)

$\tau_i = \tau_i + \rho \sum_{c_i \in \mathbf{s_a}}^{S_{upd}} \omega_s F(s)$

endif

endfor

until termination criterion is satisfied

return best solution found

Table 19: Pseudo-code ACO

As already mentioned above, the ACO has several parameters that can be adjusted to the specific optimization problem:

- Size of the population
- Number of solutions used to update the probabilities
- Weights of S_{upd}
- Evaporation proportion

Due to the probability distribution based approach, it is possible to lose well performing solution components over the run-time. Therefore, several approaches have been stated in the literature which can be summarized as 'Daemon-strategies'. Those strategies, for example, maintain the component of the solutions that have performed best, or apply local search procedures[456].

6.3 Dynamic Caching Strategy

The use of the exact HQM in a larger-scale optimization study is expected to be computationally intensive. As noted by Galvão and Morabito[457] and Toro-Díaz et al.[458], the computational times of the HQM are mainly dependent on the number of servers considered and the effort to construct the 2^N linear equations necessary. The latter is influenced by the study area and the corresponding number of demand points or regions but also by the districting level that is considered. As already mentioned in chapters 2.3.1 and 2.3.2, there are numerous approaches that intend to reduce the computational time of the HQM with the downside of reducing the expressiveness of the HQM to at least some degree.

The approach presented here keeps the full expressiveness of the HQM but intends to lower the number of solutions that are actually calculated. The overall number of individ-

[456] Cf. Stützle, T. et al., 2013, p. 193
[457] Cf. Galvão, R. D.; Morabito, R., 2008, p. 535
[458] Cf. Galvão, R. D.; Morabito, R., 2008, p. 535; Toro-Díaz, H. et al., 2013, p. 921

ual solutions computed highly depends on the parameters chosen for the respective metaheuristic which are discussed in chapter 7.1.

The use of a so-called 'Dynamic Caching Strategy' is inspired by AKDOĞAN, BAYINDIR AND IYIGUN[459]. They cache the composition of the decision variables and the corresponding value of the objective function. Before any new solution is calculated, the cache is checked whether the solution has already been calculated. If so, the calculation process is skipped and the value of the objective function is retrieved from the cache. This procedure is applied and extended here. The Goal Programming (chapter 4.1), the Robust Optimization (chapter 4.2) as well as the Robust Goal Programming (chapter 4.3) all rely on the calculation of performance metrics for their objective function and, if relevant, for their restrictions. Since one goal of this study is to determine the trade-offs between the individual performance metrics for EMS systems, multiple analysis is necessary. Thus, the weighting parameters are varied to capture the trade-offs and relationships between the performance metrics. All models are controlled over decision variable x_i which results in the same performance metrics for different weighting parameters if all x_i are identical. This is used in the proposed 'dynamic caching strategy' which saves the values of the applied individual performance metrics. After the metaheuristic has determined the composition of decision variables, the existing memory is checked. If, and only if, the decision variables are identical to an existing composition, even for different weighting parameters and computational runs of the optimization, the construction and calculation of the linear equation system is skipped and the values are retrieved from memory. Subsequently, the value of the objective function is calculated accordingly. If the composition of the decision variables is unique, i.e. it cannot be found in the memory created up to this point, it can be considered as new. It is then newly calculated and subsequently added to the memory and the values of the performance metrics are saved.

The 'dynamic caching strategy' for a single solution candidate is described in the pseudocode provided below:

[459] Cf. AKDOĞAN, M. A.; BAYINDIR, Z. P.; IYIGUN, C., 2018, p. 148

Start

determine I decision variables x_i for solution s^{new}
for int s, …, S^{Mem}
if all $x_i = x_i^s$
for int p,…, P^s
retrieve pm_p^s from PM_p^s
endfor
calculate objective function with pm_p^s
set x^{skip} to one
skip calculation process
endfor
if $x^{skip} = 0$
calculate model
for int 1, …, I
add x_i to $X_{s^{new}}^{Mem}$
endfor
for int p, …, P^s
add $pm_p^{s^{new}}$ to $PM_{s^{new}}^{Mem}$
endfor
endif

End

Table 20: Pseudo-code Dynamic Caching Strategy

The actual performance gains by the Dynamic Caching Strategy are dependent on the overall computational effort necessary to solve the proposed model as well as the calibration and nature of the metaheuristic. These are analyzed in chapter 7.

7. Computational Results

7.1 Calibration of metaheuristic and experiments

The Genetic Algorithm as well as the Ant-Colony-Optimization require the adjustment of several parameters to fit the respective optimization problem. Although some of the parameters can be set rather easily, others need to be analyzed thoroughly. Therefore, this chapter aims at the basic calibration of both metaheuristics in a simplified study area. The Goal Programming model from chapter 4.1 is used as the underlying methodological approach. The rather large case study of chapter 5 does not represent an appropriate experimental environment due to the high number of servers that need to be located in order to reach satisfying results and the corresponding high computational efforts. Therefore, in the following an artificial study area will be presented which resembles a small to medium sized town. 50 demand points are modelled on a 50x50 grid, which is equivalent to 15 square kilometers. C_{Cov} represents the degree of demand points that can be covered under 8 minutes; it is set to 0,94. The study period is set to 1 hour and the time spent at each incident, on average, to 45 minutes. D is set to 4. The demand λ is set to 2. In order to serve incoming demands, 5 servers are located within the study area. The study area provides an agglomeration zone towards the middle of the study area which resembles a city center. In order to reconstruct some of the responsibilities of EMS systems for more distant regions of their region to be served, some demand points are located further away from the center as in the lower left corner (see figure 15). A set of 40 candidate server locations is randomly located in the study area.

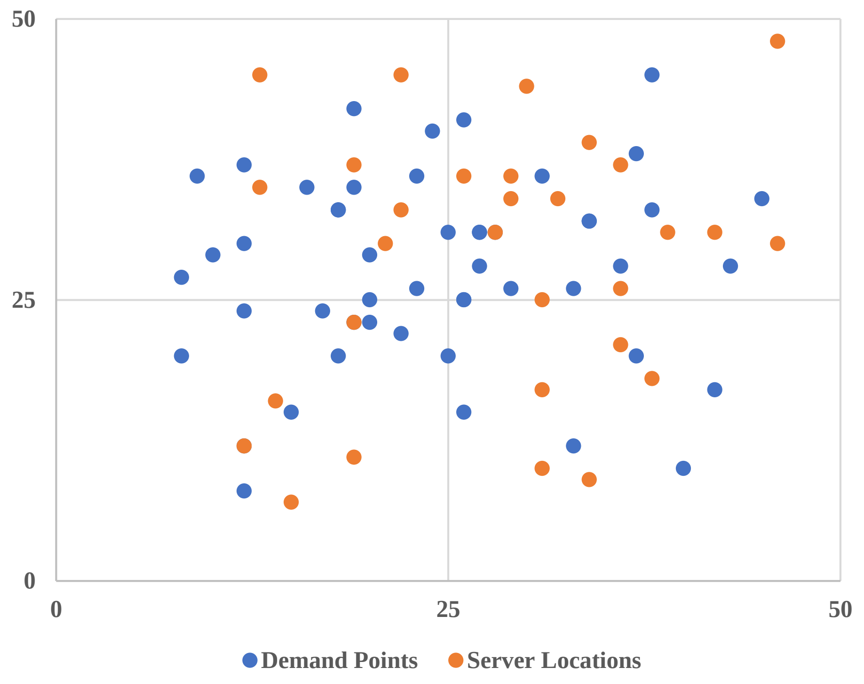

Figure 15: Artificial study area

All computational results were calculated on an Intel i7-10875H CPU running at 2.3 GHz and 16 GB RAM. The models are coded in C++ in Codeblocks. The linear equation system is solved with the use of the Eigen library.

Before the actual optimization, the aspiration levels and the threshold values of the Goal Programming model for chapter 4.1 have to be calibrated. Therefore, for each goal an optimization run is performed that solely tries to minimize or maximize, depending on the character of the goal, the performance metric corresponding to the respective goal definition. After all five optimization runs have been performed, the best values of the performance metric for each goal are retrieved from the created cache in order to mitigate any potentially existing optimality gap between the value obtained from the optimizations and the true optimal value. This is done to ensure that each aspiration level can theoretically be fully reached. Table 20 describes the aspiration levels and threshold values used for the calibration and experiments of the metaheuristic.

	Aspiration Levels	Threshold Values	α_2^i
AELP	0,0487805	0,0787805	0,03
AERT	6,38014	10	3,61986
RTCov	0,98	0,8	0,18
Longest RT	9,14059	14	4,85941
WBM	0,0226952	0,5	0,4773048

Table 21: Goal calibration – metaheuristic experiments

The parameter for the Gaussian criterion, σ, is set to 0,01. Each goal is given the weight of 0,2.

Both the Genetic Algorithm as well as the Ant-Colony-Optimization return heuristic results. Therefore, a rerun of the optimization procedure with the same parameter setting does not necessarily return the same results. In order to mitigate this influence and to gain reliable insights into different parameter combinations of both metaheuristics, the optimization procedures are repeated one hundred times for each parameter combination. The basic conditions for the metaheuristic experiments are listed below:

- One hundred repetitions for each optimization run for each parameter combination
- Varying sets of population sizes and parameter combinations for both metaheuristics
- Fixed termination criterion of one-hundred iterations (generations) for both metaheuristics

The Genetic Algorithm, as shown in chapter 2.6.2., has been the main solution method applied for (exact) HQM location optimization studies. Note that the metaheuristic experiments therefore should be understood as a feasibility proof of the Ant-Colony-Optimization for HQM location optimization studies rather than the proof of optimality of the solution returned under every circumstance.

The two metaheuristics have some comparable and some differing adjustable parameters. The calibration of the parameters for the metaheuristic should always take into account both the quality of the solution and the computational time required to derive the solution. Generally, a slower convergence should theoretically lead to a better solution being found with a higher likelihood.

The parameters that can be calculated for both metaheuristics are listed below:

Genetic Algorithm

- Population size
- Crossover probability
- Mutation probability

Note that the crossing over point at the chromosome strings can also be varied for the GA. With respect to the homogenous nature of the decision variables in this study, the crossing over point is not further analyzed and randomly set.

Ant-Colony-Optimization

- Population size
- Evaporation proportion
- Number of solutions used to update the probabilities

The weights of the solutions within a given number of solutions that are used to update the probabilities are not varied here.

Several performance metrics are calculated for each parameter setting which are then applied to analyze the behavior of the metaheuristic. One particular solution has resurfaced over all parameter settings and is considered to be 'pseudo-optimal' here. As already noted, both metaheuristics do not guarantee an optimal solution. Therefore, an optimality gap, as described by PAN AND NAGI[460], cannot be neglected here. In the case of the presented HQM model(s), only a complete enumeration would return the undoubtedly optimal value. This approach is considered to be computationally prohibitive. Still, it is highly likely that the described objective function score of 0,934266 does represent the value of the objective function for the optimal solution to the optimization problem. The calculated performance metrics are briefly described below.

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[460] Cf. PAN, F.; NAGI, R., 2010, p. 675

Average deviation to 'pseudo-optimal' solution (ADPOS)

The value of the objective function for the best solution found for each repetition of the experiment is compared to the 'pseudo-optimal' solution. After that, an average deviation is calculated and returned as a percentage value.

Average iteration best solution found (AIBSF)

The iteration (generation) in which the best solution of the repetition is found is calculated as an average over all repetitions of the experiment.

Average calculations per experiment (ACPE)

The calculations per experiment are calculated by the sum of all newly calculated solutions per experiment. It is then computed as the average over all experiments.

Additionally, for each parameter combination a graphical illustration is provided that shows how the number of newly calculated solutions in each iteration (generation) changes.

7.1.1 Results for the Genetic Algorithm

For enhanced clarity of the presented results, the results are given in schemes of 'ADPOS; AIBSF; ACPE' per cell of the respective tables. Please note that the results for the Genetic Algorithm are presented in individual tables for each population size due to better visibility.

		Crossover Probability		
		0,4	**0,6**	**0,8**
Mutation Probability	**0,01**	4,261; 47,53; 66,6	3,735; 39,48; 72,82	3,736; 35,22; 78,71
	0,05	2,036; 49,65; 224,03	2,023; 42,32; 228	1,933; 43,2; 232,27
	0,1	1,528; 44,74; 372,84	1,693; 42,5; 374,48	1,466; 43,29; 384
	0,15	1,559; 40,87; 488,75	1,439; 41,14; 492,93	1,482; 39,31; 493,25
	0,2	1,388; 37,76; 580,83	1,37; 39,69; 582,28	1,515; 31,87; 586,45
	0,25	1,12; 40,55; 651,14	1,135; 39,32; 656,85	1,184; 41,88; 659,9
	0,3	1,215; 39,92; 708,06	1,224; 44,33; 711,73	1,186; 35,24; 717,95

Table 22: GA - population size 20

The diversity in the number of solutions calculated and in the solution components is highly dependent on the overall population size in the case of the GA. If no mutation would be considered, it would converge, in the best case, towards the best solution that is possible to achieve by the optimal mixture of the already existing solution components of the initial generation. This pool of solutions can be altered by mutation procedures. Therefore, the lowest mutation percentage (0,01) results in the highest deviation from the best solution found. A lower deviation can be observed if higher mutation percentages are considered. For low mutation percentages, a higher crossover probability leads to a higher genetic diversity. Interestingly, a higher crossover probability does not always result in significantly better solutions, but in a slightly higher average number of calculations per experiments (ACPE). The ACPE-metric does also increase significantly for a higher mutation probability. This can be explained by the genetic diversity introduced through a higher mutation likelihood. As a result, even in later generations, a relatively high number of solutions has to be newly calculated. The AIBSF-metric reacts relatively stable against parameter changes. Overall, a relatively high average deviation from the 'pseudo-optimal' solution can be observed.

		Crossover Probability		
		0,4	**0,6**	**0,8**
Mutation Probability	**0,01**	2,69; 40,32; 131,75	2,591; 36,12; 146,88	2,313; 38,98; 163,42
	0,05	1,583; 40,27; 415,35	1,572; 33,93; 428,4	1,365; 35,75; 444,84
	0,1	1,171; 36,95; 684,49	1,191; 37,78; 693,65	1,226; 28,2; 7 03,87
	0,15	0,958; 38,2; 880,74	1,066; 32,81; 883,72	0,924; 35,11; 898,74
	0,2	0,859; 36,17; 1030,57	0,757; 32,84; 1042,06	0,806; 30,28; 1048
	0,25	0,862; 38,74; 1150,27	0,83; 38,4; 1152,16	0,873; 38,16; 1164
	0,3	0,731; 35,44; 1235,31	0,951; 36,91; 1241,77	0,881; 34,26; 1250,05

Table 23: GA - population size 40

Due to the non-satisfying solutions for a population size of 20, the population size is then increased to 40.[461] Over all parameter combinations, the ADPOS- and AIBSF-metric decrease while the ACPE-metric increases. Therefore, on average, a better solution is found earlier during the runtime, but more overall calculations are necessary. The higher number of calculations can be explained by the higher population size and the higher number of overall DNA strings that are subsequently subject to mutation. The main relationships, as described for a population size of 20, remain also valid for a population size of 40.

[461] Please note that the population size of 40 does represent the whole population (parents + children) in the respective generation. Only the children as well as the parents for which a chromosome has mutated are newly calculated.

		Crossover Probability		
		0,4	**0,6**	**0,8**
Mutation Probability	**0,01**	2,351; 32,75; 198,32	1,772; 32,5; 218,95	1,766; 29,96; 244,52
	0,05	1,333; 28,86; 595,27	1,131; 31,15; 613,25	1,158; 25,48; 636,75
	0,1	1,042; 32,77; 948,9	1,023; 28,08; 963,51	0,84; 30,52; 980,83
	0,15	0,783; 32,29; 1206,61	0,77; 31,52; 1218,94	0,803; 27,32; 1230,33
	0,2	0,645; 27,22; 1395,11	0,589; 28,39; 1403,79	0,547; 26,26; 1411,24
	0,25	0,646; 34,11; 1532,18	0,506; 34,84; 1541,37	0,513; 31; 1551,23
	0,3	0,53; 32,55; 1632,04	0,435; 29,82; 1644,74	0,369; 35,69; 1651,83

Table 24: GA - population size 60

In order to gain additional insights into the functioning of the Genetic Algorithm, the population size is then increased to 60. The change in results (see table 21) is mainly equivalent to the change from a population size of 20 to 40: On average, a better solution is found earlier during the runtime, but more calculations are necessary. The main relationships, as described for a population size of 20, remain also valid for the population size of 60.

Altogether, it can be stated that a higher population size is very useful for the quality of the returned solution but takes significantly more calculations. A higher mutation probability does lead to better results on average. As already discussed, it also leads to a higher number of solutions that are calculated over the runtime. If the mutation percentage is varied from 0,01 to 0,3, the number of calculated solutions increases to a more than ten times higher number for a population size of 20. This relationship gets somewhat less relevant for a higher population size due to the higher overall number of solutions calculated. A higher crossover probability is observed to have the highest impact for lower mutation percentages and lower population sizes. This can be explained by the higher genetic diversity which can then be provided. Generally, a crossover probability of 0,8 leads to the best results with the downside of a higher number of necessary calculations. The latter becomes less relevant for higher mutation probabilities since a higher number of solutions needs to be calculated anyway.

The average calculations per iteration over all experiments are presented, separated into the different population sizes, in figure 13 below to allow insight into the behavior during the runtime and into the entire convergence behavior of the metaheuristic. For better comparability, all results for the parameter combinations are given in one figure. The orange (green) [blue] values represent the population size of 20 (40) [60]. A darker line corresponds to a lower mutation percentage with the darkest line representing a mutation probability of 0,01 and the brightest line representing 0,3. Since there is no significant difference between the individual crossover probabilities, their lines are not varied.

The average number of calculations per generation depends mostly on the whole population size. The mutation probability has a significant influence on the number of solutions that are newly calculated in each generation. If a low mutation percentage is considered, an exponential decrease in calculated solutions can be observed. This results in only a relatively small number of solutions to be newly calculated after 10 generations. For a higher mutation probability the decrease is more linear and there is only a slight decrease in calculations between generations. The resulting high number of calculations can, at least to some degree, explain why a better solution can be obtained if a higher mutation probability is considered.

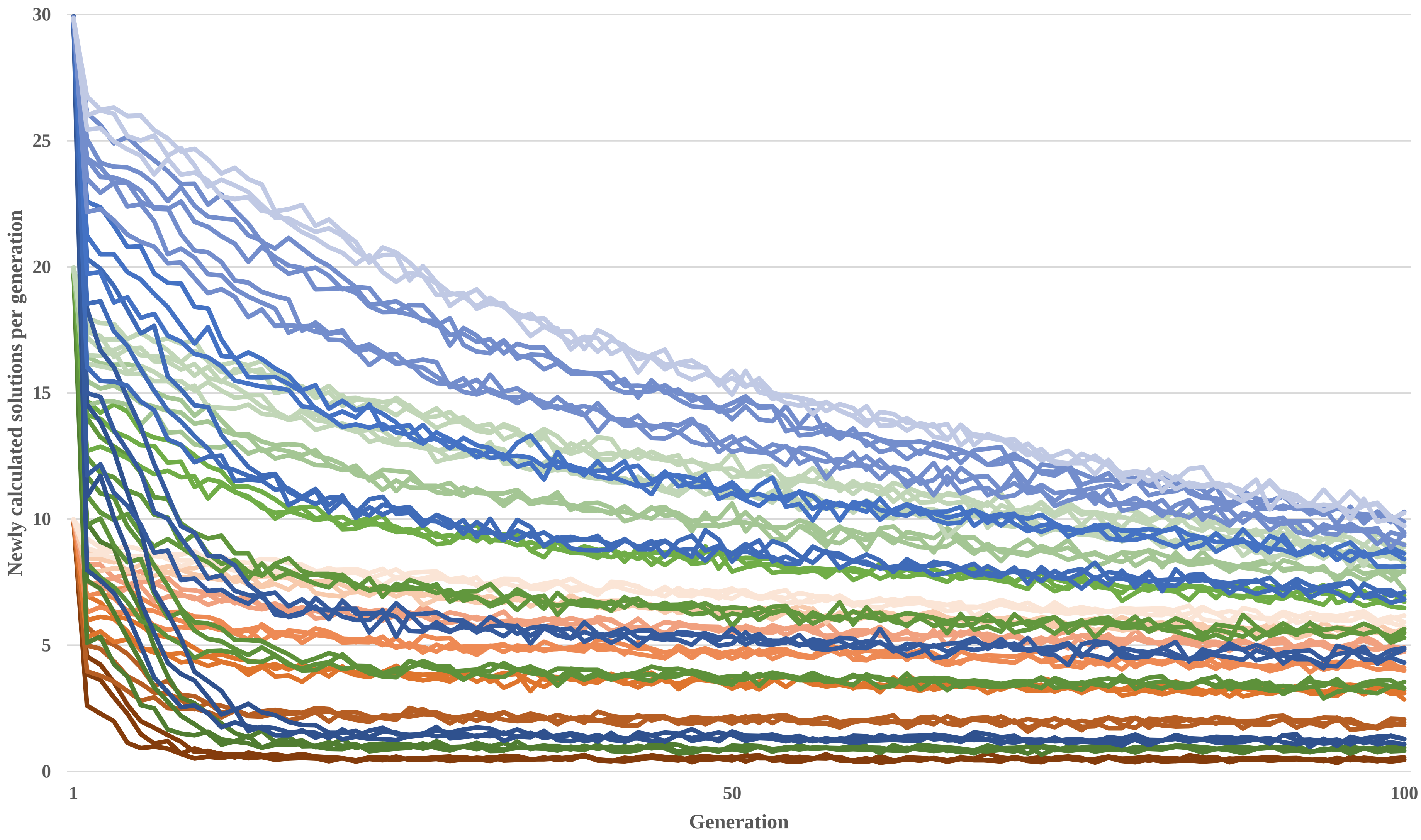

Figure 16: Average calculations per generation - Genetic Algorithm

7.1.2 Results for the Ant-Colony-Optimization

For the sake of clarity, the results are given in the scheme of 'ADPOS; AIBSF; ACPE' per cell of the respective tables. Note that the weights within a given S_{upd} are not varied for reasons of simplicity.

	Evaporation Proportion					
Number of solutions (S_{upd}) used to update the probabilities	**0,02**	**0,05**	**0,1**	**0,2**	**0,3**	**0,5**
1	0,039; 29,4; 749,71	0,407; 15,53; 413,99	0,654; 10,26; 241,8	1,464; 7,34; 133,07	2,161; 5,27; 94,52	3,128; 3,47; 58,94
2 w_s= {0,7; 0,3)	0,037; 27,46; 791,48	0,198; 18,63; 451,54	0,57; 11,63; 268,2	1,146; 7,22; 156,81	1,891; 5,93; 110,99	2,755; 4,7; 67,4
3 w_s= {0,5; 0,3; 0,2}	0,019; 27,57; 830,77	0,095; 19,35; 477,46	0,398; 12,45; 291,64	0,81; 8,63; 170,66	1,591; 6,51; 121,07	2,243; 5,17; 75,55
4 w_s= {0,4; 0,3; 0,2; 0,1}	0,019; 31,96; 857,18	0,039; 20,81; 499,25	0,461; 12,6; 304,21	0,914; 8,48; 178,71	1,209; 6,6; 130,38	2,315; 5,15; 84,6
5 w_s= {0,3; 0,25; 0,2; 0,15; 0,1}	0; 29,34; 895,9	0,02; 23,56; 539,48	0,318; 15,05; 336,52	1,033; 9,12; 200,13	1,461; 6,8; 140,08	2,224; 5,27; 91,85
6 w_s= {0,25; 0,2; 0,175; 0,15; 0,125; 0,1}	0; 30,6; 919,26	0,039; 25,27; 575,97	0,242; 15,58; 345,56	0,74; 11,71; 211,98	1,459; 7,7; 152,01	2,163; 4,77; 97,33
7 w_s= {0,25; 0,2; 0,175; 0,15; 0,1; 0,075; 0,05}	0; 31,78; 922,74	0,019; 24,76; 572,02	0,177; 15,31; 352,59	0,822; 10,77; 214,24	1,657; 9,42; 150,94	2,249; 5,44; 100,54

Table 25: ACO - population size 20

The results in table 22 refer to a population size of twenty ants. A higher number of S_{upd} leads, for a given evaporation proportion, to a higher score in the ADPOS-metric. This can be explained by the more even spread of pheromones on well performing solution components. As a result, the ACO converges more slowly, which can also be seen in the higher numbers for AIBSF and ACPE. This relationship can be observed for all evaporation proportions when a higher number of S_{upd} is considered. For an evaporation proportion of 0,02 a very slow convergence towards the best solution can be seen. For $S_{upd} > 4$ and $\rho = 0{,}02$ the optimal solution is found in every one of the one hundred repetitions of the experiment. An evaporation proportion of 0,05 leads to very comparable results. All in all, a lower score of the objective function and the corresponding lower ADPOS-metric is seen for higher evaporation proportions. In this case well performing solution components are forgotten too quickly so that only an inferior solution can be achieved. As a result, the ACO converges quickly which also explains that the best solution is, on average, found earlier during the runtime if compared to lower evaporation proportions (AISBF) as well as the relatively low number of average calculations (ACPE). The evaporation proportions of 0,02 and 0,05 do lead to a relatively high number of solutions that need to be computed.

The average calculations per iteration are presented below. A low number of solutions used to update the probabilities is represented by a dark line, while a high number is represented by a more bright line. The values for $\rho = 0{,}02$ are given in black, for 0,05 in orange, for 0,1 in green, for 0,2 in yellow, for 0,3 in red and for 0,5 in blue.

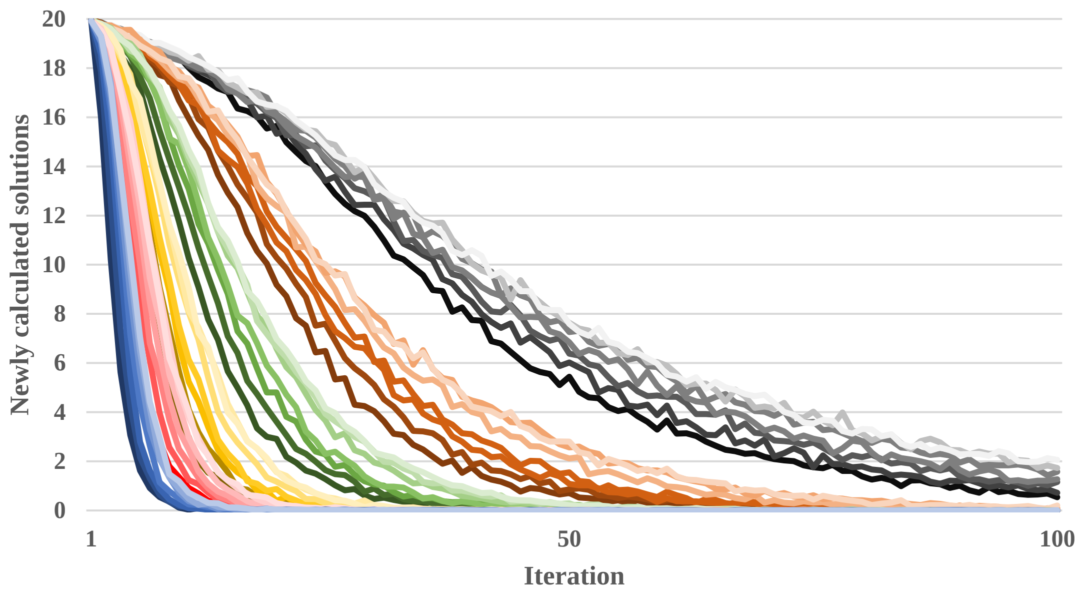

Figure 17: Average calculations per iteration – Ant-Colony-Optimization

Figure 17 shows that a lower evaporation proportion does lead to significantly higher numbers of solutions calculated in each iteration. A higher number of S_{upd} leads to more solutions calculated even though the difference is not as significant as for a variation of the evaporation proportion.

Since the proposed HQM models intend to locate server numbers in the double-digit range, the computational times are expected to be one of the main limiting factors. Therefore, a trade-off has to be made between the quality of the solution and the computational time. Table 25 above shows that an evaporation proportion of 0,1 performs reasonably well while also leading to a relatively low number of solutions to be calculated over the runtime. As a result, the parameter combinations of $S_{upd} \geq 5$ and an evaporation proportion of 0,1 are analyzed with respect to different population sizes.

		Population Size				
		10	**15**	**20**	**25**	**30**
Number of solutions (S_{upd}) used to update the probabil-ities	**5**	0,839; 22,35; 259,63	0,481; 18,2; 292,8	0,318; 15,05; 336,52	0,236; 12,2; 369,47	0,16; 11,75; 407,94
	6	1,315; 25,22; 286,07	0,4; 20,87; 322,53	0,242; 15,58; 345,56	0,121; 13,31; 394,80	0,058; 12,63; 418,25
	7	1,433; 22,81; 284,59	0,515; 21,73; 327,91	0,177; 15,31; 352,59	0,157; 14,93; 390,37	0,141; 10,87; 419,89

Table 26: ACO - different population sizes

If relatively low population sizes (10 or 15) are considered, lower values of S_{upd} perform better, which can be explained by the rather small population size. A higher value would presumably benefit from inferior solutions. If rather high population sizes (25 or 30) are considered, a higher value for S_{upd} does benefit from the entire optimization process. It should be noted that an increase in population size does not lead to a directly proportional increase in calculated solutions due to the overall convergence of the ACO. It does lead to a better score in the AIBSF-metric due to a higher number of already calculated solutions earlier in the runtime.

Altogether, it can be stated that for medium to larger sized populations (20-30) the deviations from the ‘pseudo-optimal’ value are relatively minor. Among the combinations listed in table 26, S_{upd} = 6 or 7 performs best over all experiments. It should be noticed that the weights have been chosen very similarly between S_{upd} = 6 and 7, which may lead to easily comparable results. Therefore, it is highly likely that both values would lead to, more or less, similar results. If population sizes are considered, there is a relatively small increase in performance if the population size is chosen higher than twenty.

As already mentioned, the primary goal of this section was to compare the GA, as one of the established solution techniques in the field of HQM location optimization studies, to the ACO. It should be noted, before any further remarks on their respective performance are made, that any comparison between the GA and the ACO may yield different results for different settings. If the performance metrics are compared, the ACO performs better than the GA in the proposed experimental setting. For the GA, there has been no parameter combination that leads to the 'pseudo-optimal' solution in every one of the one hundred repetitions of the experiment, even for a population size of 60.[462] The differences to the 'pseudo-optimal' solution are only minor if extreme parameter settings, like a mutation probability of under ten percent or a population size of twenty, are excluded. The ACO reacts even more robust to a change in the parameters. Only for cases like $S_{upd} \leq 3$ and $\rho \geq 0{,}3$, large deviations to the 'pseudo-optimal' solution can be observed. If a modest convergence speed is chosen, as for $S_{upd} \geq 5$ and $\rho = 0{,}02$, 0,05 or 0,1, the deviations, if any, are very small. The analysis for different population sizes has also shown very modest deviations if smaller populations are utilized. NEUMANN, SUDHOLT AND WITT have shown that a population size set to a value as small as two can yield sufficient results for the ACO if an adequate evaporation proportion is chosen.[463] Since any comparison between two metaheuristics is always subject to the corresponding research environment, some related studies comparing the GA to the ACO are listed below with respect to their area of application as well as their results:

- TEWOLDE AND SHENG: Tool path planning; ACO performs better[464]
- LI ET AL.: Traveling Salesman Problem; ACO performs better[465]
- SARIFF AND BUNIYAMIN: Robot path planning; ACO performs better[466]
- SANTANA ET AL: Protein classes and image recreation; ACO performs better for small instances, GA for large instances[467]
- ALHANJOURI AND BELAL: Traveling Salesman Problem; GA performs better[468]

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[462] If population sizes are compared, the actual newly calculated solutions should be the main factor. Therefore, for an entire population size of 60, a maximum of thirty solutions are newly calculated in each generation of the GA.
[463] Cf. NEUMANN, F.; SUDHOLT, D.; WITT, C., 2010, p. 70
[464] Cf. TEWOLDE, G. S.; SHENG, W., 2008, p. 287
[465] Cf. LI, K. ET AL., 2008, p. 75
[466] Cf. SARIFF, N. B.; BUNIYAMIN, N., 2009, p. 137
[467] Cf. SANTANA, L. E. A. ET AL., 2010
[468] Cf. ALHANJOURI, M.; BELAL, A., 2011, p. 578

- PUTHA, QUADRIFOGLIO AND ZECHMAN: Traffic Signal Coordination; ACO performs better[469]

Based on the results of the metaheuristic experiments as well as the presented research studies, the ACO can be viewed as an adequate solution technique. If solely the results of the metaheuristic experiments are compared, the ACO should perform at least similar to the GA in HQM location optimization studies. For the remainder of this work, the ACO is chosen as the solution technique. In order to limit computational times while still reaching sufficient solutions, the parameter combination of $S_{upd} = 6$ and $\rho = 0{,}1$ as well as a population size of twenty ants are utilized. Additionally, a fixed termination criterion of fifty iterations per optimization is set.

7.2 Initial configuration

In order to replicate the current conditions, a set of twelve servers is located within the study area. The optimization is performed with D = 4. The values of C_{cov}, T and $\emptyset_{rk}$ are set to 0,94, 60, and 45, respectively. The time threshold relevant to C_{cov} is set to 8 minutes. The aspiration levels and the threshold values for each goal can be obtained from table 27 below. The parameter for the Gaussian criterion, σ, is set to 0,01.

The calibration process for the aspiration levels and threshold values of chapter 7.1 is repeated here. Table 27 indicates the relevant aspiration levels and the applied threshold values.

	Aspiration levels	**Threshold values**	$\boldsymbol{\alpha_2^i}$
AELP	0,0481702	0,0781702	0,03
AERT	6,9907	10	3,0093
RTCov	0,966667	0,8	0,166667
LRT	11,4943	14	2,5057
WBM	0,152802	0,5	0,347198

Table 27: Goal calibration

Additionally, the chosen level of backup, D = 4, should be validated. The obtained solution while solely optimizing for the AELP metric is therefore simulated with varying levels of backup. The aim of the initial configuration is to choose a level of backup that

[469] Cf. PUTHA, R.; QUADRIFOGLIO, L.; ZECHMAN, E., 2012, p. 27

allows for a sufficient service behavior of the system while not exceeding certain thresholds of computational time. The analysis is performed for backup levels of one to five.

	D1	D2	D3	D4	D5
AELP	0,29341	0,14325	0,080787	0,0481702	0,026257
AERT	5,653	6,3057	6,7267	7,0267	7,1892
RTCov	0,96333	0,95666	0,94667	0,933333	0,92333
LRT	12,1431	12,2344	12,3141	12,4603	12,6261
WBM	0,17147	0,16551	0,159516	0,157126	0,157201
Computational time in seconds	17	18	20	45	607

Table 28: Numerical results - initial configuration

The performance metrics change significantly for different levels of backups that are considered. The AELP metric decreases clearly for a higher level of backup since only then calls that cannot immediately be served can be transferred to lower-tier servers. If no backup is considered, a server cannot be dispatched in about thirty percent of the cases. Most other response-time related metrics, like the AERT, the percentage of demand points covered in less than 10 minutes and the longest time to respond to a demand point, do decrease. This can be explained by the normalized calculations for the response time which lead to a lower increase in the response time. A higher level of backup leads to a more even spread of workloads among servers since they are able to respond to other incidents as well. The actual decrease in some performance metrics should not lead to the impression that a lower level of backup is beneficial. Instead, the relatively high levels of calls that cannot be served immediately (AELP metric) play a significant role in the mentioned increase. The computational times rise significantly for a higher level of backup.

This can be explained by much more complex districting formulations as well as a higher computational effort to construct the linear equation system. Additionally, the time needed to solve the resulting equation system may be higher. The computational times do remain relatively stable up to a third-level backup.

As already mentioned in existing (exact) HQM location optimization studies as by GEROLIMINIS, KARLAFTIS AND SKABARDONIS[470] and ENAYATI ET AL.[471], the modelling and computational efforts, the decrease in some performance metrics should be considered during the decision which level of backup is incorporated into the model. From a practical perspective EMS system administrators may preferably request help from a third party or a neighboring EMS system only if a distant server is available to respond.[472] For less congested systems or lower priority demands, it may also be a reasonable strategy to wait until a nearby server becomes available again to preserve resources for other incoming demand calls. The incorporation of D = 5 would lead to very high computational times. In view of the points mentioned above, D = 4 is chosen for the remainder of the current work.

7.3 Goal Programming

The initial configurations of chapter 7.2 are performed with the use of the Goal Programming model of chapter 4.1. Therefore, the derived solutions are already cached by the Dynamic Caching Strategy and can be used for further analysis. The analysis showed that there are some significant trade-offs between the individual performance metrics that need to be further analyzed. Moreover, a holistic system configuration has to be found that considers all analyzed performance metrics as well as the mentioned trade-offs between them. Table 29 shows the weights for the relevant optimizations, the corresponding values of the performance metrics, the number of unique solutions per optimization (‘USO’) as well as the newly calculated solutions per optimization due to the caching strategy (‘NCSO’). Note that the column ‘Performance gain’ refers solely to the difference in unique and newly calculated solutions per optimization.

[470] Cf. GEROLIMINIS, N.; KARLAFTIS, M. G.; SKABARDONIS, A., 2009, p. 802
[471] Cf. ENAYATI, S. ET AL., 2019, p. 427
[472] Cf. ENAYATI, S. ET AL., 2019, p. 427

Number	Code	Weights	OF value	AELP	AERT	RTCov	LRT	WBM	USO	NCSO	Performance gain
1	G1	{1; 0; 0; 0; 0}	1	0,04817	7,0267	0,933333	12,4603	0,157126	710	710	0 %
2	G2	{0; 1; 0; 0; 0}	1	0,050881	6,9907	0,92	13,1133	0,165664	729	599	17,83 %
3	G3	{0; 0; 1; 0; 0}	1	0,05406	7,4494	0,966667	11,537	0,196848	667	501	24,89 %
4	G4	{0; 0; 0; 1; 0}	1	0,053705	7,3854	0,95	11,4943	0,199461	758	550	27,44 %
5	G5	{0; 0; 0; 0; 1}	1	0,049195	7,2495	0,933333	12,5319	0,152802	766	468	38,9 %
6	A1	{0,5; 0,5; 0; 0; 0}	0,9966	0,048916	7,0028	0,926667	12,4551	0,171068	805	466	42,11 %
7	A2	{0,75; 0,25; 0; 0; 0}	0,997	0,04817	7,0267	0,933333	12,4603	0,157126	723	458	36,65 %
8	A3	{0,25; 0,75; 0; 0; 0}	0,9963	0,048916	7,0028	0,926667	12,4551	0,171068	765	312	59,22 %
9	A4	{0; 0,5; 0,5; 0; 0}	0,9469	0,050589	7,1295	0,956667	11,6305	0,172541	690	286	58,55 %
10	A5	{0; 0,75; 0,25; 0; 0}	0,9504	0,050589	7,1295	0,956667	11,6305	0,172541	781	288	63,12 %
11	A6	{0; 0,25; 0,75; 0; 0}	0,9619	0,05406	7,4494	0,96667	11,537	0,196848	603	394	34,66 %
12	A7	{0; 0,5; 0; 0,5; 0}	0,9633	0,049486	7,1111	0,926667	11,5781	0,175447	745	277	62,82 %
13	A8	{0; 0,75; 0; 0,25; 0}	0,9594	0,050578	7,1081	0,923333	11,6076	0,182186	776	303	60,95 %
14	A9	{0; 0,25; 0; 0,75; 0}	0,9698	0,053984	7,3426	0,94	11,4974	0,208786	742	236	68,19 %
15	A10	{0,5; 0; 0,5; 0; 0}	0,9645	0,051407	7,4844	0,963333	11,5843	0,191341	645	283	56,12 %

Number	Code	Weights	OF value	AELP	AERT	RTCov	LRT	WBM	USO	NCSO	Performance gain
16	A11	{0,75; 0; 0,25; 0; 0}	0,9694	0,05022	7,3221	0,956667	11,6282	0,159122	689	294	57,33 %
17	A12	{0,25; 0; 0,75; 0; 0}	0,9701	0,053217	7,4603	0,966667	11,5376	0,196341	659	270	59,03 %
18	A13	{0,5; 0; 0; 0,5; 0}	0,9792	0,049535	7,1958	0,916667	11,5755	0,188568	644	265	58,85 %
19	A14	{0,75; 0; 0; 0,25; 0}	0,9842	0,049374	7,1294	0,926667	11,5984	0,171608	694	228	67,15 %
20	A15	{0,25; 0; 0; 0,75; 0}	0,976	0,051477	7,3247	0,956667	11,5302	0,182873	676	265	60,8 %
21	A16	{0; 0; 0,5; 0,5; 0}	0,9915	0,05406	7,4494	0,966667	11,537	0,196848	637	228	64,21 %
22	A17	{0; 0; 0,75; 0,25; 0}	0,9957	0,05406	7,4494	0,966667	11,537	0,196848	671	196	70,79 %
23	A18	{0; 0; 0,25; 0,75; 0}	0,9872	0,05406	7,4494	0,966667	11,537	0,196848	671	207	69,15 %
24	A19	{0; 0; 0; 0,5; 0,5}	0,9726	0,051046	7,388	0,953333	11,5756	0,160543	685	222	67,59 %
25	A20	{0; 0; 0; 0,75; 0,25}	0,9677	0,05076	7,3777	0,953333	11,5643	0,168503	643	225	65,01 %
26	A21	{0; 0; 0; 0,25; 0,75}	0,9844	0,050417	7,3283	0,95	11,5974	0,15524	756	198	73,81 %
27	A22	{0,2; 0,2; 0,2; 0,2; 0,2}	0,9529	0,050724	7,1396	0,956667	11,5794	0,173659	660	178	73,03 %
28	A23	{0,35; 0,35; 0,1; 0,1; 0,1}	0,9560	0,050724	7,1396	0,956667	11,5794	0,173659	719	177	75,38 %

Table 29: Results of Goal Programming

The analysis shows that the USO metric varies between 603 and 805 unique solutions per optimization. The variation can be explained by the heuristic nature of the ACO. The difference in value compared to the ACPE metric of chapter 7.1 is due to the broader solution space of the twelve-server system. However, the Dynamic Caching Strategy yields rather high benefits especially for the first fifteen optimizations. Afterwards, only slight gains in performance can be observed. The NCSO-metric as well as the gains in performance with respect to the optimizations are graphically illustrated in figure 18 below.

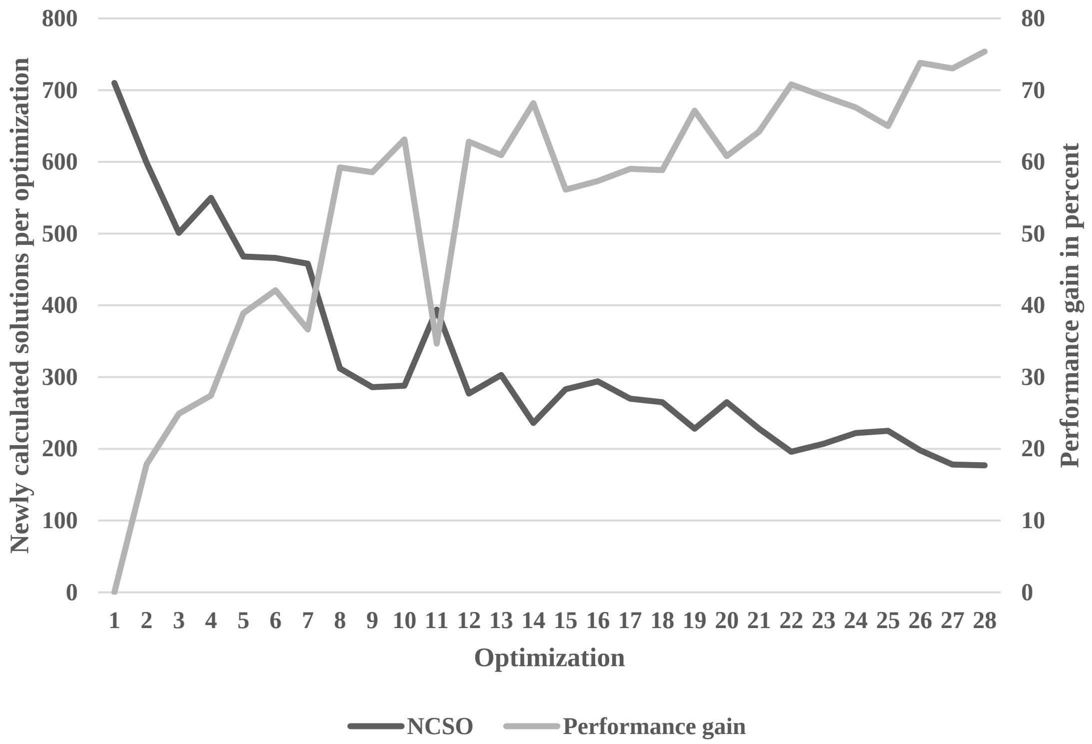

Figure 18: NCSO and performance gain - Goal Programming

As already mentioned, the performance gains decrease for a higher number of optimizations. Therefore, the behavior of the ACO with respect to newly calculated solutions per iteration and per optimization should be analyzed to gain further insight. Figure 19 below shows the newly calculated solutions per iteration. Please note that the darker lines refer to earlier optimizations while the brighter lines refer to later optimizations.

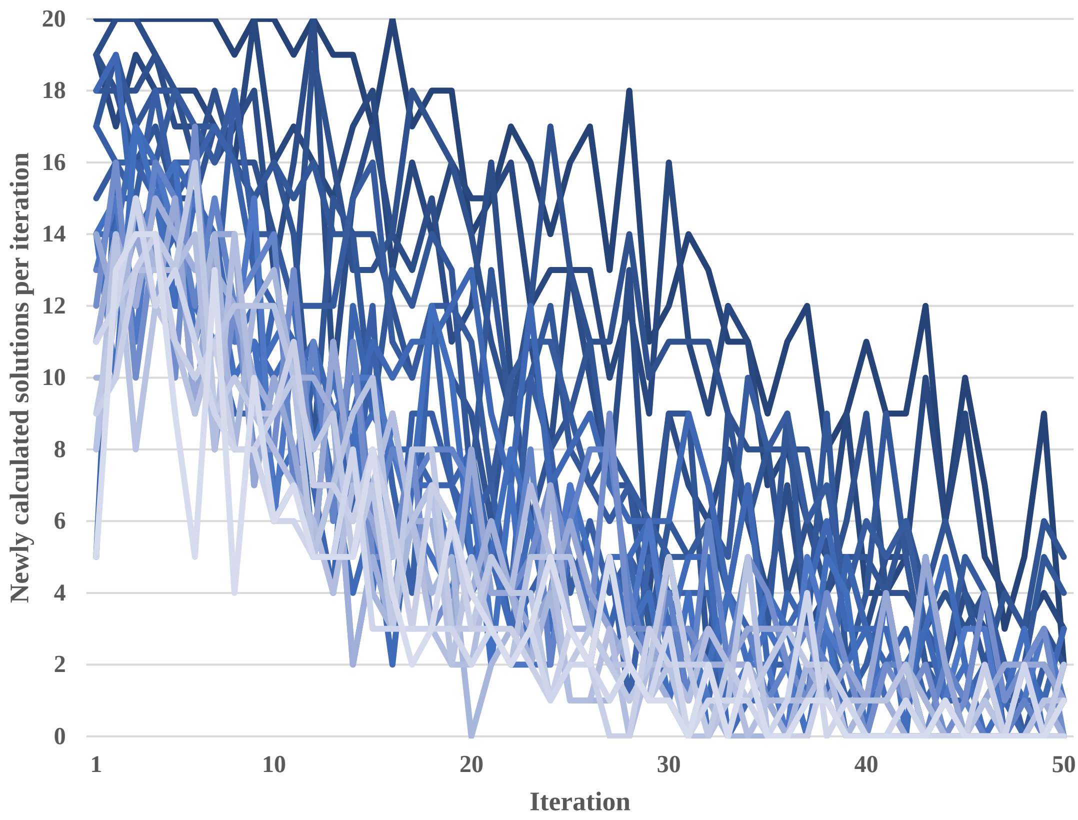

Figure 19: Calculations per Iteration – Goal Programming

Based on figure 19, the decreasing performance gains can be explained by the non-decreasing number of calculations after approximately five optimizations. This holds especially true for the first ten iterations for each optimization. As for other metaheuristics, the ACO starts at different points in the search space and converges towards the solution derived over the runtime. If more solutions are saved in the cache, it becomes more likely that the first solutions of the respective optimization have already been calculated. Presumably, a high number of optimizations would be necessary to further limit the calculations during the first few iterations by a significant degree. The number of newly calculated solutions over the whole runtime will most likely further decrease for a higher number of optimizations but with even more decreasing marginal performance gains. Therefore, it can be concluded that the Dynamic Caching Strategy is most beneficial during the first few optimizations but can also benefit the computational times for later optimizations with decreasing returns.

As already mentioned, the five optimizations (G1-G5) for each of the performance metrics have revealed some trade-offs between the metrics that need to be further analyzed.

- The minimization of the AELP metric results in a relatively low value for the AERT metric. Furthermore, it also leads to good values for all other computed metrics. Therefore, it can be stated that the AELP and AERT metric are conflicting objectives during the design decision of EMS systems. This can be explained due to the following reasoning: The AELP metric intends to maximize the service that can be provided for all demand points. As a result, it also includes the distant demand points. However, the AERT metric aims at minimizing the response time to all demand points. Therefore, it can be beneficiary if the service to the distant demand points is neglected to at least some degree. This then results in a decrease for the response time if the AELP metric is to be maximized.
- The minimization of the AERT metric leads to a relatively high increase for the AELP metric if the only modest decrease from G1 to G2 for the AERT metric is considered. Additionally, all other performance metrics decrease also. The minimization of the AERT metric leads to rather low values for the portion of demand that can be covered under 9 minutes. Since especially the AERT metric intends to minimize the overall response time, it may be beneficial to neglect a fast response for a higher portion of demand due to the calculation of an average value for the metric. This results in a localization of the servers towards more demand intensive regions while simultaneously accepting higher response times to more distant parts. A similar relationship can be observed between the AERT and LRT metric. The increase from G1 to G2 for the LRT metric is especially significant. Altogether, it can be summarized that lowering the AERT metric beyond a certain value, which may be represented by the AERT value from G1, leads to rather significant losses in all other metrics.

  The minimization of the AELP/AERT metric also leads to a relatively even distribution among the server workloads (WBM). A low value for the first two mentioned metrics implies a rather high availability of a) any server on the preference list and b), on average, a relatively close server for any given demand point. If all demand points are considered, this should lead to a rather equal distribution between the workload of the individual servers.
- The maximization of the portion of demand points that can be covered under 9 minutes (RTCov) leads to rather unfavorable values for the AELP/AERT metric. This can be explained by the reverse relation of the one detailed in the paragraph

above. Additionally, a rather high value for the WBM can be observed. The maximization of demand points that can be covered subject to any time constraint always includes the very distant demand points. Therefore, the servers should in theory be located in a way that also ensures the coverage for those demand points, which also explains the rather low longest response time value. Since more demand is expected towards more central parts of the study area, the servers that primarily serve the more distant parts of the study area are ranked lower in the preference lists of the demand points in the central parts. This results in more uneven server workloads.

- Very similar relationships to the RTCov metric can be observed for the minimization of the longest response time for the AELP/AERT metric as well as for the WBM. It also leads to a rather high value for the portion of demand points that can be covered under 9 minutes (RTCov).
- The minimization of the WBM leads to slightly higher values for the AELP metric and to moderately higher for the AERT metric in comparison to the optimization for the latter two metrics. The difference in value between the WBM for G1/G2 and G5 optimizations is very small. The higher AELP/AERT metrics can then be explained due to the most even distribution between server workloads. This may result in servers being located less favorably in terms of loss probabilities and response time. A similar relationship can be observed for the RTCov metric and the longest response time.

Altogether, it can be stated that the minimization of the AELP/AERT metric leads to good results in all performance metrics with the exception of RTCov and the LRT. If the latter two are optimized, the loss probabilities and the average response time of the system will increase. The aim of even workloads does not benefit the other analyzed performance metrics and uneven workloads should therefore be accepted to a certain degree.

In order to gain additional insight into the Goal Programming model, the weights between the individual goals are varied to better capture the existing trade-offs as well as the sensitivity of the model to different weights. The respective weights can be found in table 29.

AELP and AERT (G1, G2, A1-A3)

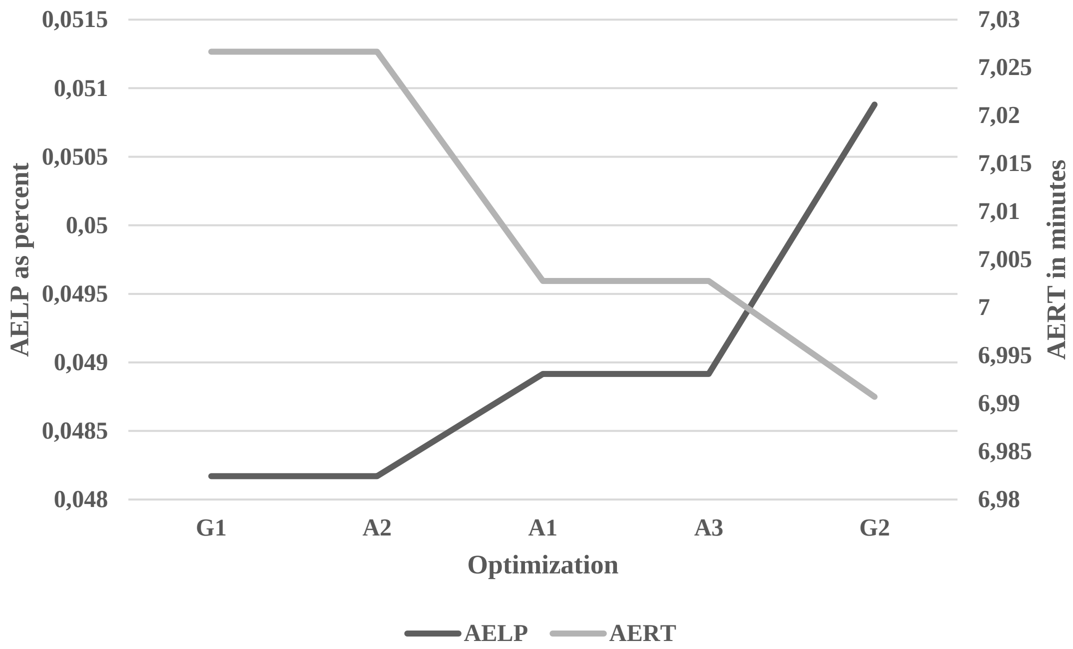

Figure 20: AELP and AERT - Goal Programming

Figure 20 shows the different results of the AELP and AERT metric with respect to optimizations. The AELP value reacts relatively insensitive to changes in its weight up until solely the AERT metric is to be minimized. In contrast, the AERT metric seems to decline more steadily. Especially between A2 and A1, a rather high decrease for the AERT metric (7,0267 vs. 7,0028) can be observed while the AELP metric only increases moderately (0,04817 vs. 0,048916).

AERT and RTCov (G2, G3, A4-A6)

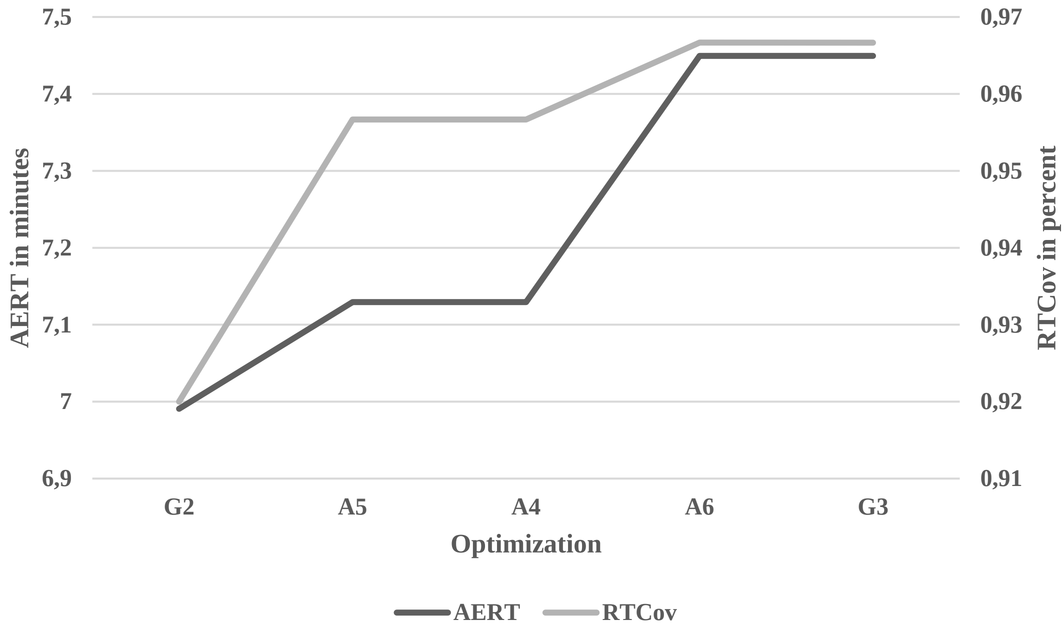

Figure 21: AERT and RTCov - Goal Programming

Figure 21 shows the different realizations of the AERT and RTCov metric with respect to the individual weights. If RTCov is to be maximized, both metrics increase for higher weights on RTCov. For the minimization of AERT, both metrics decrease. For a weight greater than 0,5 on RTCov, as for A6 and G3, the AERT metric increases significantly in value. The increase of RTCov from 0,956667 (A4/A5) to 0,966667 (A6/G3) leads to an increase for AERT from 7,1295 to 7,4494. Therefore, it can be concluded that benefitting a higher portion of demand points with a response time of up to 9 minutes leads to a significant increase for the average response time.

AERT and Longest Response Time (G2, G4, A7-A9)

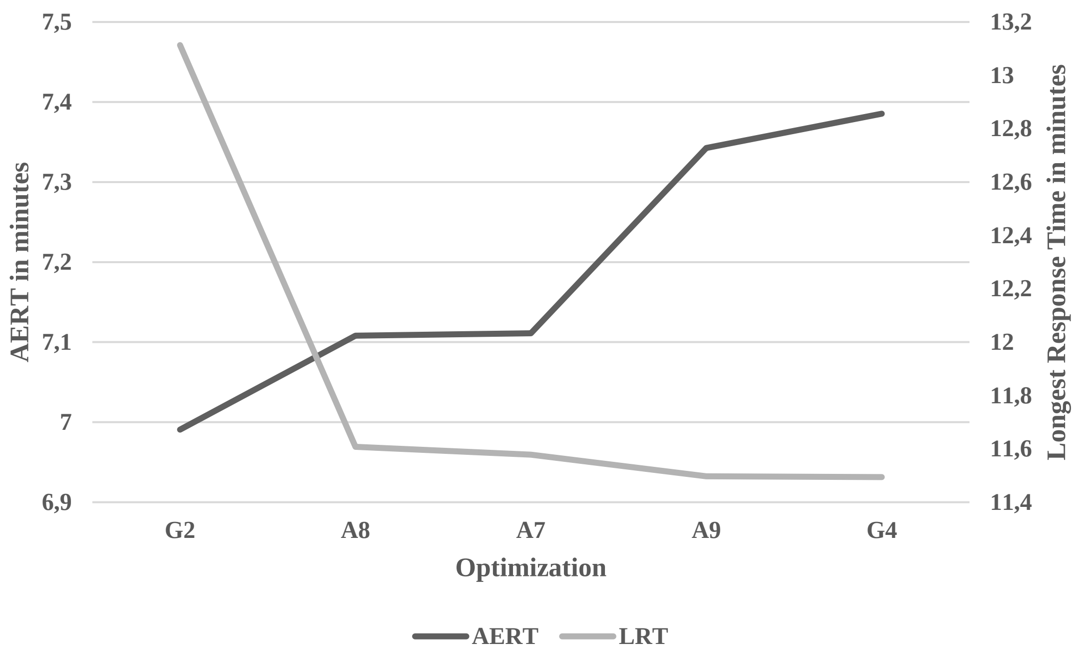

Figure 22: AERT and Longest Response Time - Goal Programming

Figure 22 shows the different realizations of the AERT- and LRT- metric with respect to the individual weights. With the G2 optimization, a rather high LRT can be observed. If a moderate weight (A7/A8) is put on minimizing the LRT, its value drops significantly while the increase for the AERT remains modest. For a relatively high value on the LRT metric, the AERT increases significantly. For the A9 and G4 optimizations only a slight improvement of the LRT can be stated. If compared to A7, the AERT increases from 7,1111 to 7,3426 (7,3854) for A9 (G4) while the LRT decreases from 11,5781 to 11,4974 (11,4943). Comparable to the relationship between the AERT and RTCov metrics, it can be said that limiting the response time to the most distant demand point beyond a certain degree heavily affects the average response time to all other demand points.

<u>*AELP and RTCov (G1, G3, A10-A12)*</u>

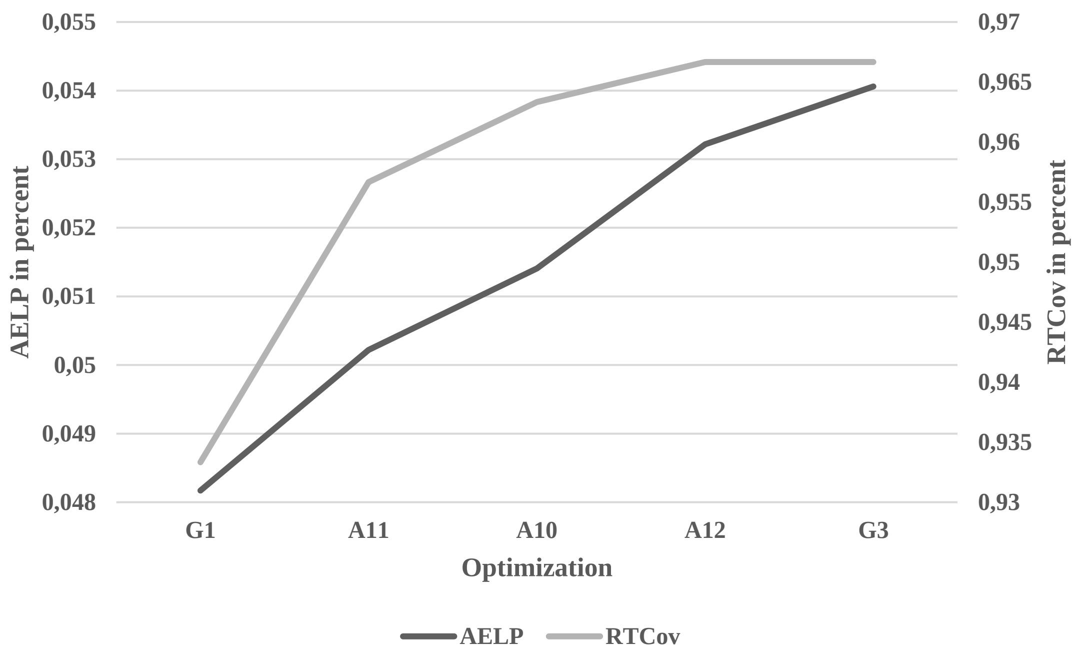

Figure 23: AELP and RTCov - Goal Programming

Figure 23 shows the different realizations of the AELP and RTCov metrics with respect to the individual weights. The general relationship between AERT and RTCov, as described for figure 20, remains also valid for the AELP metric. The most significant difference can be observed for the higher increase of the AELP if a moderate weight is put on maximizing RTCov. Consequently, if the decision maker intends to maximize RTCov, even smaller weights result in noticeable gains for the AELP metric. Therefore, the benefit of shorter response times for a certain, more distant, group of demand points leads to higher average loss probabilities for incoming demands in the overall system.

AELP and Longest Response Time (G1, G4, A13-A15)

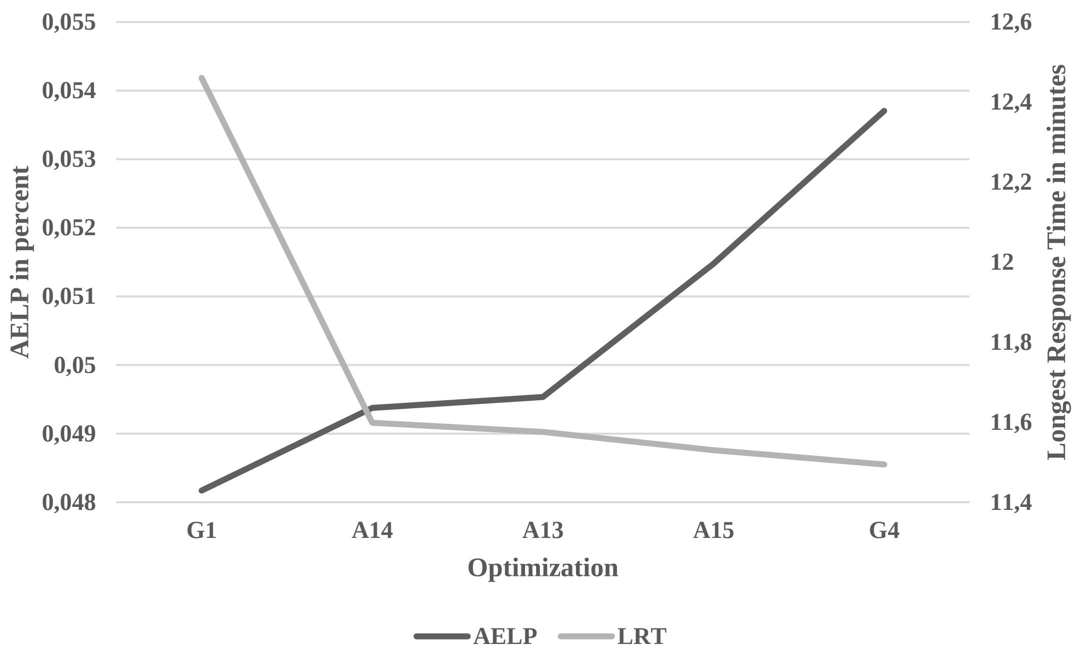

Figure 24: AELP and Longest Response Time - Goal Programming

Figure 24 shows the different realizations of the AELP and LRT metrics with respect to the individual weights. The general relationship between both metrics remains largely identical to the one described for figure 22. Thus, decreasing the longest response time to any demand point beyond a certain degree has rather high negative implications for the AELP of the overall system. This for example can be seem if A13 is compared to G4. The LRT decreases from 11,5755 (A13) to 11,4943 (G4) while the AELP increases from 0,049535 to 0,053705.

RTCov and Longest Response Time (G3, G4, A16-A18)

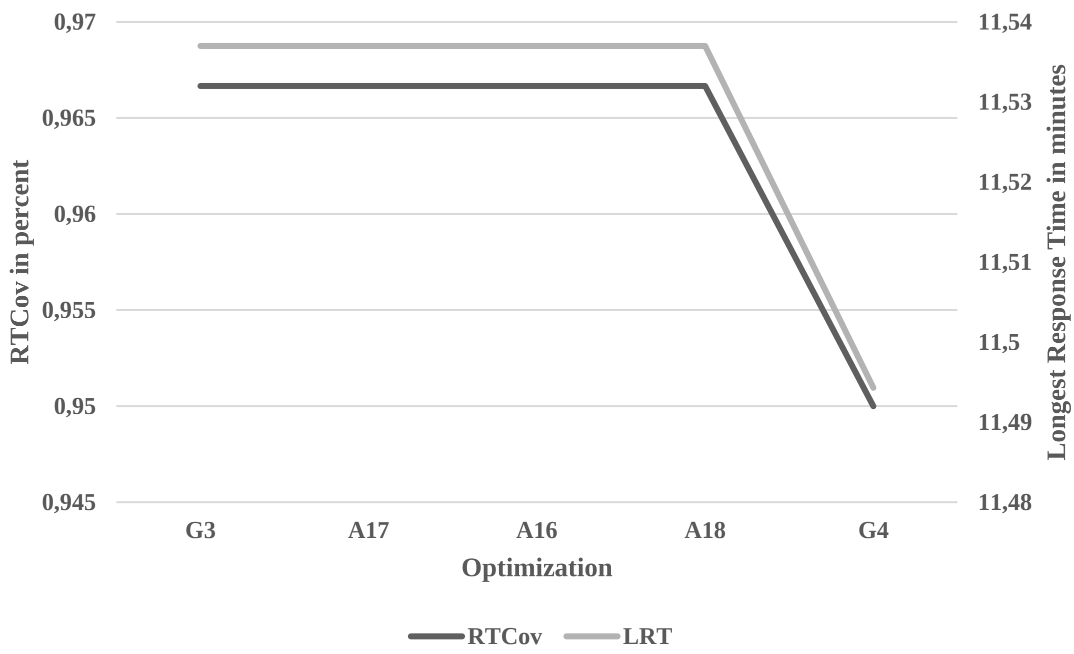

Figure 25: RTCov and Longest Response Time - Goal Programming

Figure 25 shows the different realizations of the RTCov and LRT metrics with respect to the individual weights. The solution variables remain identical between the solutions obtained except for the case that all weight is put on the minimization of the LRT. For the G4 optimization a modest drop in the RTCov metric can be observed. Therefore, only for the extreme case of G4, minimizing the LRT conflicts with the goal of maximizing RTCov.

Longest Response Time and WBM (G4, G5, A19-A21)

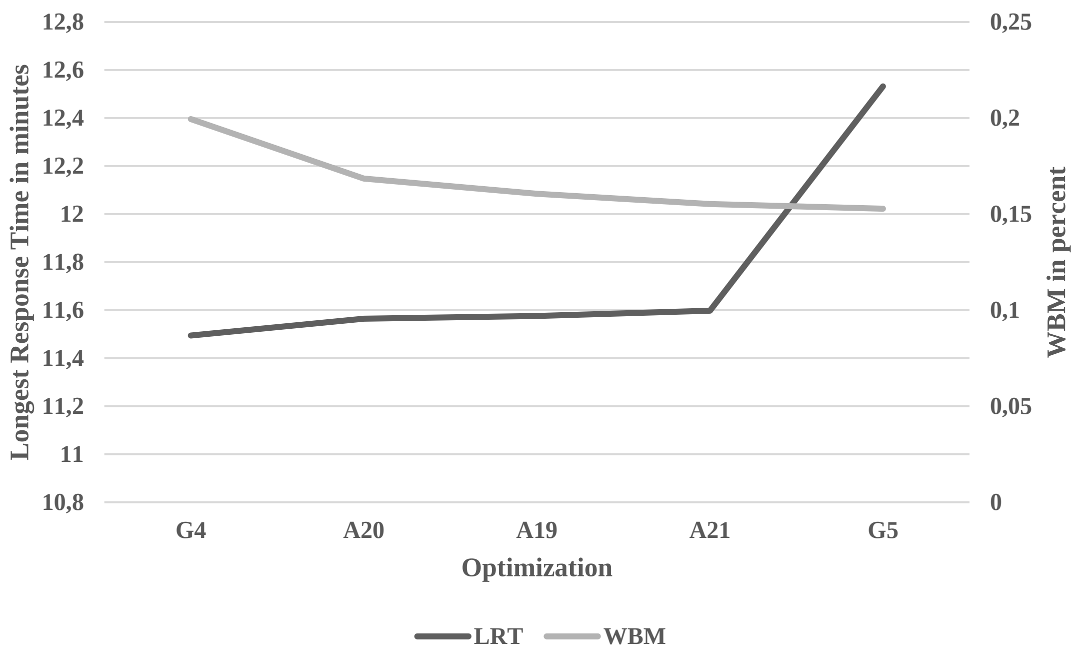

Figure 26: Longest Response Time and WBM - Goal Programming

Figure 26 shows the different realizations of the LRT metric and the WBM with respect to the individual weights. All in all, the LRT reacts rather insensitive to changes in its weight, which can be seen from the only minor increases for G4, A20, A19 and A21. The small increases do lead to relatively large decreases for the WBM. While the LRT increases from 11,4943 (G4) to 11,5974 (A21), the WBM decreases from 0,199461 to 0,15524. Only if the entire weight is put on minimizing the WBM, the LRT increases significantly to 12,5319. As already mentioned, this is explained by a localization of servers that tries to equalize server workloads as much as possible. This neglects the response time for more distant areas of the study area more seriously and therefore increases the LRT.

In order to analyze the behavior of the model with respect to the variation in more than two weights, two additional optimizations are performed. The first (A22) puts the identical weight on each goal while the second (A23) intends to take the different performance metrics into account and proposes a holistic solution. Interestingly, the same solution is derived for both weight combinations. The results for both optimizations can be found in table 29 above.

The weights for A23 were chosen in such a way as to benefit the most crucial performance metrics, like the AELP or AERT. Simultaneously, adequate values in other performance metrics were ensured. As a result, all performance metrics decrease from their best value. Table 30 contains a comparison between the respective optimal value and the value realized in A23 for each performance metric.

	AELP	AERT	RTCov	LRT	WBM
Best value (from G1-G5)	0,04817	6,9907	0,966667	11,4943	0,152802
A23	0,050724	7,1396	0,956667	11,5794	0,173659
Relative Difference to G1-G5	+ 5,30 %	+ 2,13 %	- 1,05 %	+ 0,74 %	+ 13,65 %

Table 30: Comparison of A23 - Goal Programming

The differences between the best value and A23 are relatively minor for all performance metrics with the exception of the WBM. The values for RTCov and LRT increase the least. With regard to the performance of A23 in all computed performance metrics and in comparison to G1-G5, it can be stated that the weight combination of A23 offers a good proposal how all performance metrics can be considered without sacrificing too much in a particular metric. Therefore, A23 is chosen for further analysis in the following.

Since most of the performance metrics represent average or aggregated values, further insights into the behavior of the model can be gained through a more detailed statistical analysis of the loss probabilities and response times.

Average	**Min**	**Max**	**Portion of demand points with loss probability under … in %**				
			4 %	**5 %**	**6 %**	**7 %**	**7,5 %**
0,050724	0,025111	0,071366	38,33	57,33	64	77,67	1

Table 31: AELP Statistics - Goal Programming

The loss probabilities between the individual demand points vary significantly, encompassing values from 2,5 % to 7,1366 %. The majority of demand points has a loss probability below 5 %. About 23 % of the demand points range between 7 % and the maximum value of 7,1366 %. The loss probabilities depend highly on the number of demand points that share the identical preference lists as well as the number of demand points a particular server is responsible for. If a demand point is served by a server (or server combination) that is responsible for an above-average number of demand points, above-average loss probabilities can be expected.[473] The demand points with the highest loss probabilities are located in the lower middle part of the study area and are mostly served by the four servers in close proximity, which can be seen in figure 27 below.

Average	**Min**	**Max**	**Portion covered under … minutes in %**				
			6	**8**	**9**	**10**	**12**
7,1396	4,7105	11,5794	15,67	81,67	95,67	97,67	100

Table 32: AERT Statistics - Goal Programming

The response time for the demand points varies between 4,71049 and 11,5794 minutes. As is obvious from table 32, only a rather small extent of demand points are covered under 6 minutes while only a minority of demand points cannot be reached within 8 minutes. Thus, the response time is between 6 and 8 minutes for the majority of the demand points.

The server locations for A23 are illustrated in figure 26 below. (Server locations are indicated by the black ambulance symbol.)

[473] The values for the loss probabilities and the response times for each demand point can be found in appendix A.

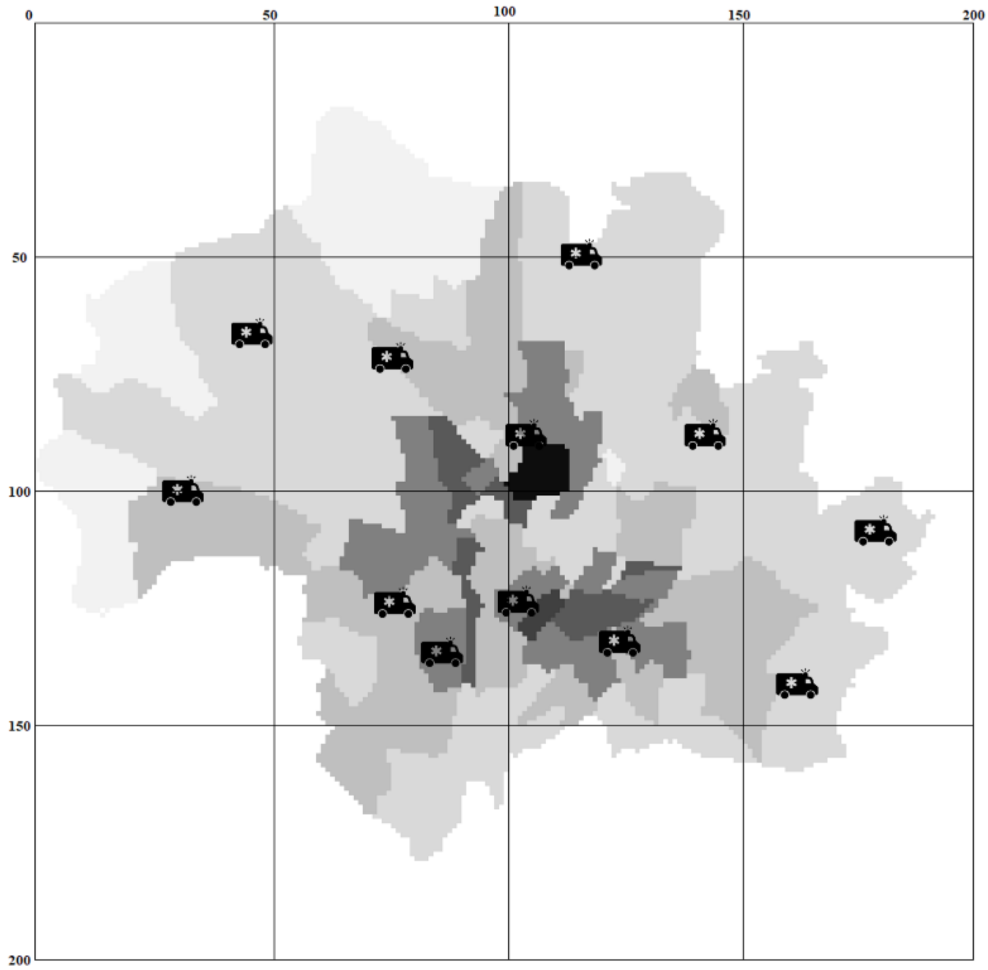

Figure 27: Server locations - Goal Programming

7.4 Robust Optimization

The results for the Robust Optimization are performed using the model detailed in chapter 4.2. Since the basic foundation for the model is identical to the Robust Goal Programming model, all relevant performance metrics of chapter 4.3 are calculated and subsequently cached for all candidate solutions. In this way, the already existing number of cached solutions are used, which were found as the computational results of the Robust Goal Programming in chapter 4.5. The parameter settings for the time spend on the scene of the incident as well as for C_{cov}, as described for chapter 7.3, are kept identical for chapters 7.4 and 7.5. The time periods and demand values are varied with respect to the scenarios. The coding following the scheme of 'SRX-MRY' denotes the parameter setting for the Robust Optimization and shows the values for the solution robustness (SR) and the model robustness (MR). Therefore, for example, the coding SR1-MR10 represents $\vartheta = 1$ and $\omega = 10$.

The results for the Robust Optimization can be found in table 33 below. As for the presentation of the results of Goal Programming, ‘USO’ denotes the number of unique solutions per optimization while ‘NCSO’ represents the number of newly calculated solutions with respect to the already existing cache. The Robust Optimization shows that there are significant changes in the value of the AELP and AERT metrics with respect to the individual scenarios analyzed further down in this section. If the scenario-weighted values are considered, nearly the exact same value to the G2 value of the Goal Programming for the AERT metric can be observed. The AELP metric increases rather significantly from 0,04817 to 0,056837 (G1 of chapter 7.3 to SR0-MR0). The values for the individual scenarios can be found in appendix B.

Number	Code	OF value	AERT	AELP	USO	NCSO	Performance gain
1	SR0-MR0	7,0481	7,0481	0,056837	653	653	0 %
2	SR05-MR0	7,9374	7,0481	0,056837	764	513	32,85 %
3	SR1-MR0	8,8267	7,0481	0,056837	653	494	24,35 %
4	SR3-MR0	12,3838	7,0481	0,056837	696	428	38,51 %
5	SR1-MR10	9,395	7,0481	0,056837	670	350	47,76 %
6	SR1-MR100	14,5104	7,0481	0,056837	659	295	55,24 %
7	SR1-MR300	25,8898	7,1321	0,056531	679	301	55,67 %
8	SR1-MR500	37,1951	7,1407	0,056508	553	263	52,44 %
9	SR1-MR1000	65,4489	7,1407	0,056508	527	362	31,31 %
10	SR1-MR10000	572,961	7,2747	0,056385	479	343	28,39 %

Table 33: Results of Robust Optimization

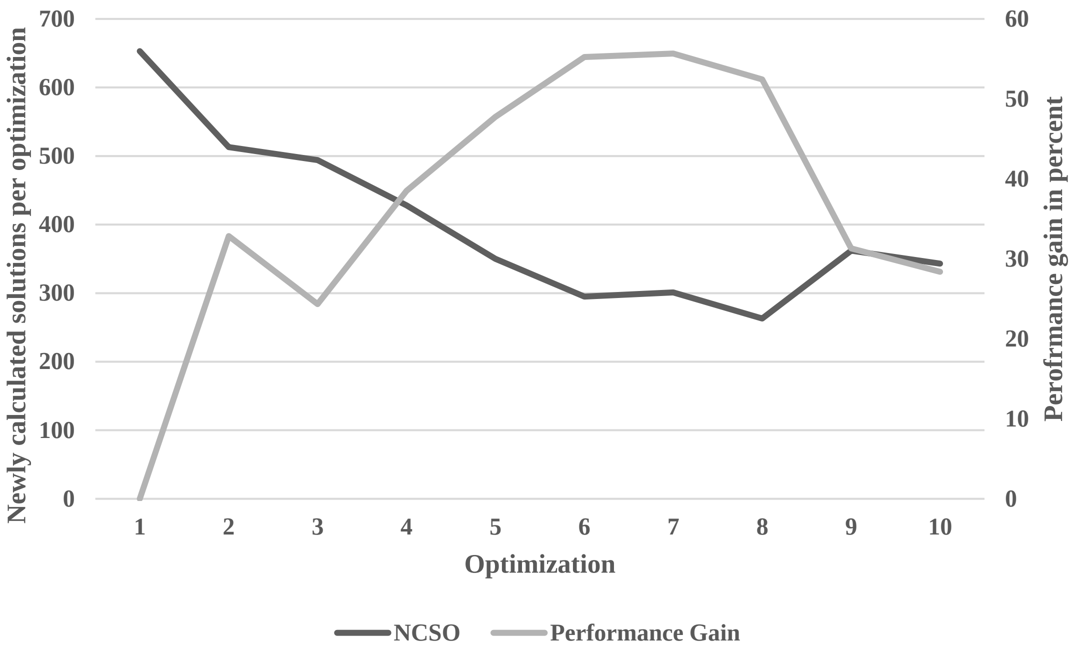

Figure 28: NCSO and performance gain - Robust Optimization

Similar to the results of the Dynamic Caching Strategy for the Goal Programming model, the NCSO metric steadily declines for an increasing number of optimizations. Note that the number of optimizations for the Robust Optimization approach is significantly lower than for the Goal Programming. Therefore, the increases in newly calculated solutions for the optimizations 9 and 10 are assumed to be of only temporary nature and not to represent a significant shift in the pattern of decreasing values for the NCSO metric.[474] Combined with the decrease in the 'USO' metric, the curve of the performance gain has a negative slope for optimizations 9 and 10. The former may be caused by the rather high parameter settings for ω which may limit the relevant search space for the ACO. If optimizations 9 and 10 are excluded from further analysis, a pattern comparable to the Dynamic Caching Strategy for the Goal Programming model can be observed. The 'NCSO' metric decreases, however with diminishing marginal returns for a higher number of optimizations. The number of newly calculated solutions per iteration and for each optimization can be found in figure 29 below. Note that the darker lines refer to earlier optimizations while the brighter lines refer to later optimizations.

[474] Since the cache of chapter 7.4 is also used for 7.5, this relationship is further analyzed with more data in chapter 7.5.

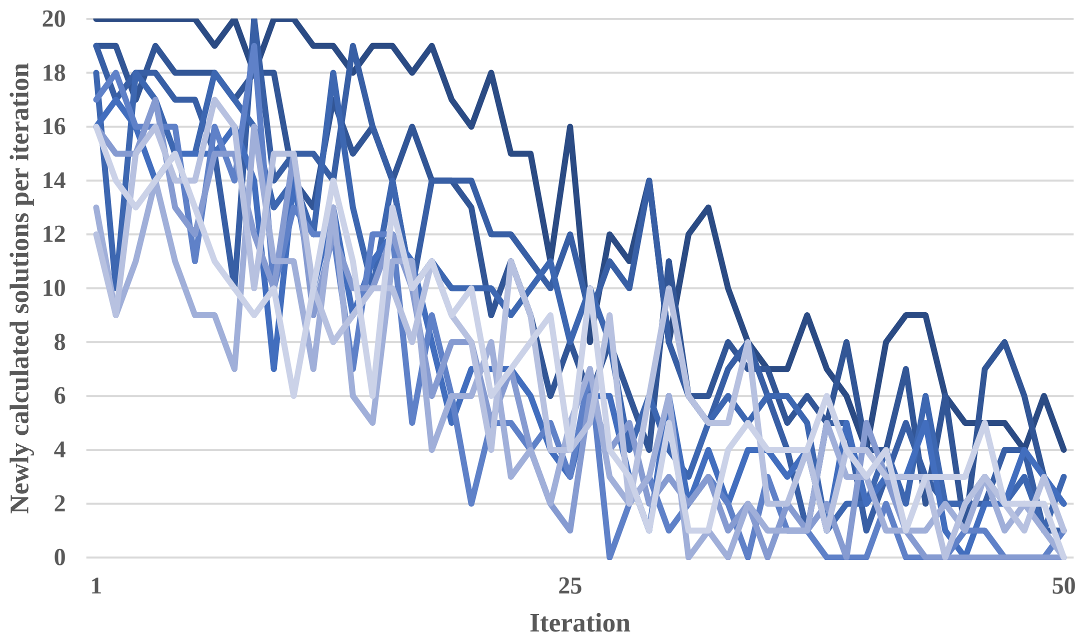

Figure 29: Calculations per iteration - Robust Optimization

If the results presented in figure 29 are compared to the calculations per iteration of the Goal Programming (see figure 19), a very similar pattern emerges. The benefits of the Dynamic Caching Strategy seem to be most relevant during the early iterations of each optimization while the effect becomes somewhat less significant during the later iterations. As a result, exactly as with Goal Programming, it can be stated that the ACO starts at different points in the search space and converges towards its solution. Moreover, the calculations during the early iterations could most likely be reduced for a significantly higher number of optimizations and a higher number of cached solutions. The actual relative benefits of the Dynamic Caching Strategy seem to be the most beneficial during the first few optimizations.

The parameter settings for the solution and model robustness require further explanation and analysis. In order to mitigate differences in the solutions for individual scenarios, the solution robustness parameter is varied to capture potential trade-offs. However, no difference in the solutions derived with varying parameters for the solution robustness could be observed. Therefore, it can be stated that the solution that leads to the lowest AERT minimizes the variance of the AERT values between the individual scenarios. As already mentioned, the ACO returns heuristic solutions. Hence, it cannot be fully guaranteed that a solution exists that leads to a lower solution robustness with the costs of an increased AERT. Additionally, a positive weight for the solution robustness may lead to beneficial results during the optimization process since solutions with a higher variance are penalized more heavily. Therefore, the solution robustness parameter, ϑ, is set to one for the remainder of this chapter.[475]

The AELP metric is used to capture the model robustness. For higher weights of ω, more emphasis is put on lowering the loss probabilities of the overall system. As for the Goal Programming model, minimizing the system response time also leads to a relatively low AELP value. If more weight is put on minimizing the AELP, the AERT increases rather significantly. The relationship for varying values of ω can be seen in figure 29.

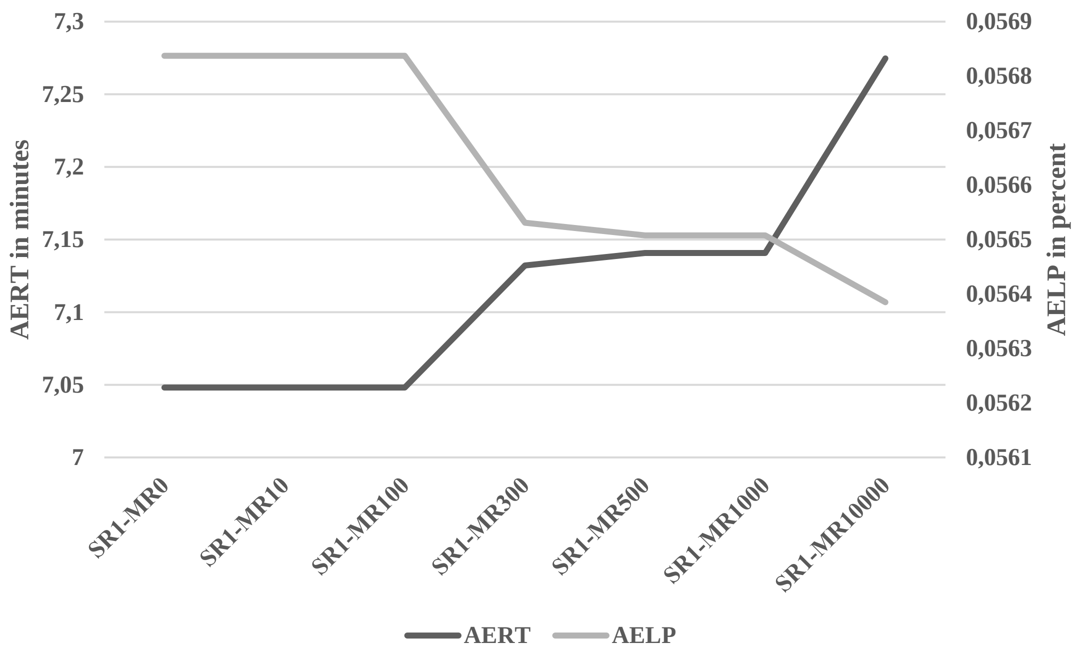

Figure 30: AERT and AELP - Robust Optimization

Note that the actual differences between the AELPs are very small. Therefore, rather high values of ω are necessary to capture the existing trade-offs between both metrics. The Goal Programming model shows that there is a slight difference in the solution for the minimization of the AELP or the AERT. However, in conjunction with the results presented in figure 30, it can be said that the minimization of the AERT leads to very good results for the AELP even if multiple scenarios and demand point mappings are considered.

The solution derived with the parameter setting of ϑ = 1 and ω = 300 ('SR1-MR300') is chosen for further analysis here. Table 34 and 35 contain the different characterizations for the AERT and AELP with respect to the individual scenarios as well as additional

[475] The calibration of the solution robustness parameter is repeated for chapter 7.5.

statistical values.

		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Min		4,3631	5,0290	4,9797	5,3881	5,0209
Max		12,0134	11,8531	11,4139	12,1155	12,3875
Average		6,6691	7,5109	7,1924	7,7568	7,1062
Median		6,5242	7,2534	7,0839	7,4752	6,9284
% covered under … minutes	6	0,31333	0,09	0,086667	0,03	0,17
	7	0,876667	0,73	0,826667	0,686667	0,84
	8	0,933333	0,87	0,95	0,856667	0,92
	10	0,98	0,92	0,973333	0,906667	0,963333
	12	0,996667	1	1	0,996667	0,996667

Table 34: AERT Statistics - Robust Optimization

The AERT metric shows a modest increase for scenarios in which a higher demand is to be considered. Nonetheless, increases in the AERT statistics can be observed, especially for scenarios 2 and 4. The rise is significantly higher for the AELP. While during the early hours of the day (scenario 1) almost all demand is served immediately, approximately 12,5 % of the demand cannot be answered in the second rush hour period (scenario 4). This can also be seen by the sharp decrease in the number of demand points that are below the defined thresholds in table 35 for scenarios with higher demand proportions, for example scenario 2, 3 and 4.

Note that the model shows a higher than one-to-one increase for the AELP for higher demand values. The system becomes significantly more congested during the peak hours. Since the AERT increases for scenarios with higher demand, more distant servers on the preference lists of the demand points are dispatched due to the congestion of the system. Due to their higher distance and longer response times, the overall service behavior of the system declines, which may in turn lead to additional congestion. Another reason can be found in the overall loss behavior of the system which causes incoming demands to be lost with greater likelihood when the overall congestion is high. This also explains the deviation between the standard Goal Programming and the Robust Optimization model for the AELP metric.

		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Max		0,016865	0,109943	0,12031	0,16529	0,055913
Min		0,003176	0,040981	0,024275	0,068244	0,014226
Average		0,011359	0,081332	0,076095	0,125298	0,03964
Median		0,009995	0,083331	0,078119	0,129894	0,037077
% of demand points with loss probability under …	**4%**	1	0	0,1	0	0,58
	5%	1	0,14	0,29	0	0,76
	6%	1	0,15	0,3	0	1
	7%	1	0,33	0,46	0,01	1
	7,50%	1	0,38	0,49	0,03	1
	10%	1	0,78	0,71	0,16	1
	15%	1	1	1	0,78	1

Table 35: AELP Statistics - Robust Optimization

The AERT and AELP values of tables 34 and 35 represent a weighted average or aggregated values. To gain insight into the behavior of the system with regard to the individual demand points, some additional statistical analysis needs to be performed. Therefore, the loss probabilities and response times of every demand point are computed and visualized via a boxplot. The boxplots have been created with RStudio and can be found in figure 30. The values for both metrics can be found in appendix C.

The boxplots of figure 31 show that, in contrast to the AELP plot, the boxes and whiskers of the plots of the AERT plot do not significantly increase. The difference can be explained by the computation of the AERT which normalizes the sum of the fraction of dispatches for all servers to one.[476] This is done because the loss probabilities for any demand point, with a positive overall demand and any queueing system without an infinite queue, are expected to be greater than zero. As a result, the response time is only measured if there actually is a dispatch. Altogether, the AERT reacts significantly less sensitive to a change in demand than the AELP. Therefore, any sensitivity analysis, especially for changing demand patterns, should always include the analysis of the AELP metric and not be limited to changes in response times.

[476] See chapter 2.2.2 for detailed calculations.

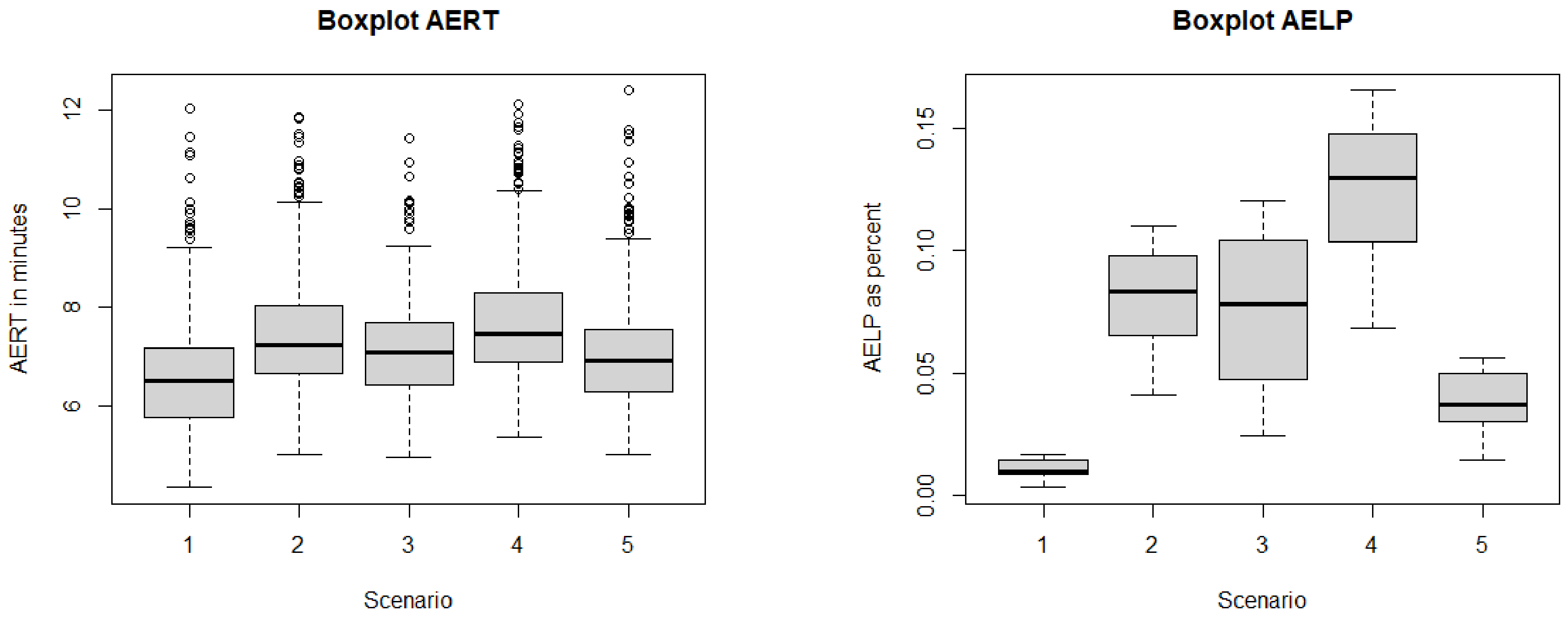

Figure 31: Boxplots Robust Optimization

The solutions generated by the robust model should be verified against their deterministic counterparts. Therefore, five separate optimizations, one for each of the scenarios, are performed with the sole focus on minimizing the response time. The resulting solutions are then used in the robust model to compute the AERT and AELP. Subsequently, the corresponding values are compared against the robust solution which represents the base-line. The results are illustrated in figure 32 (AERT) and 33 (AELP) below.

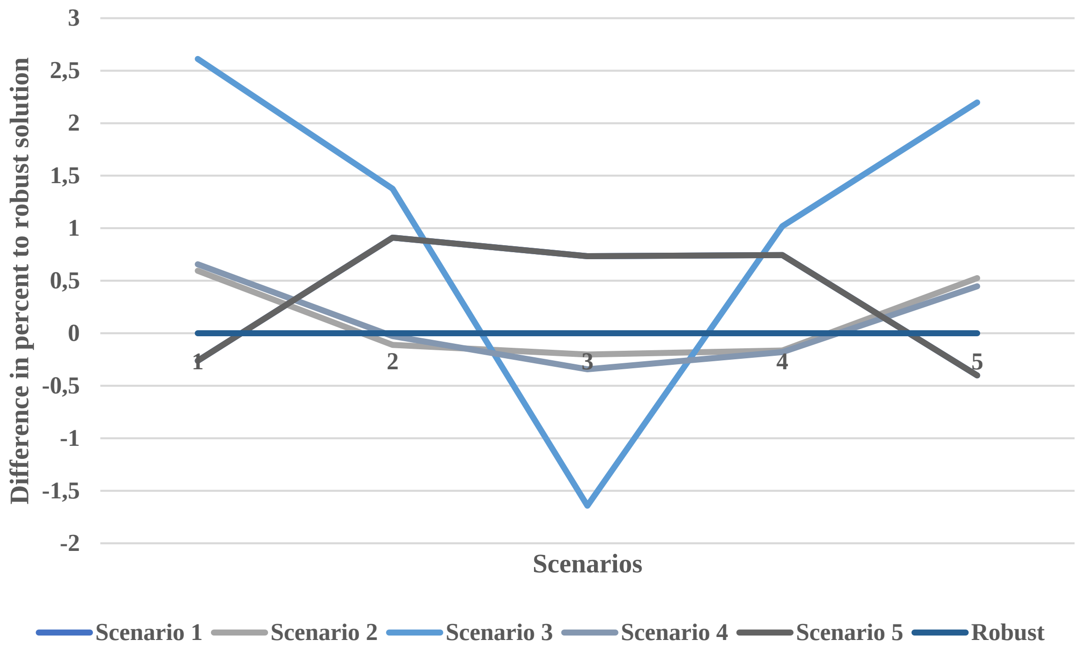

Figure 32: Solution Comparison Robust Optimization – AERT

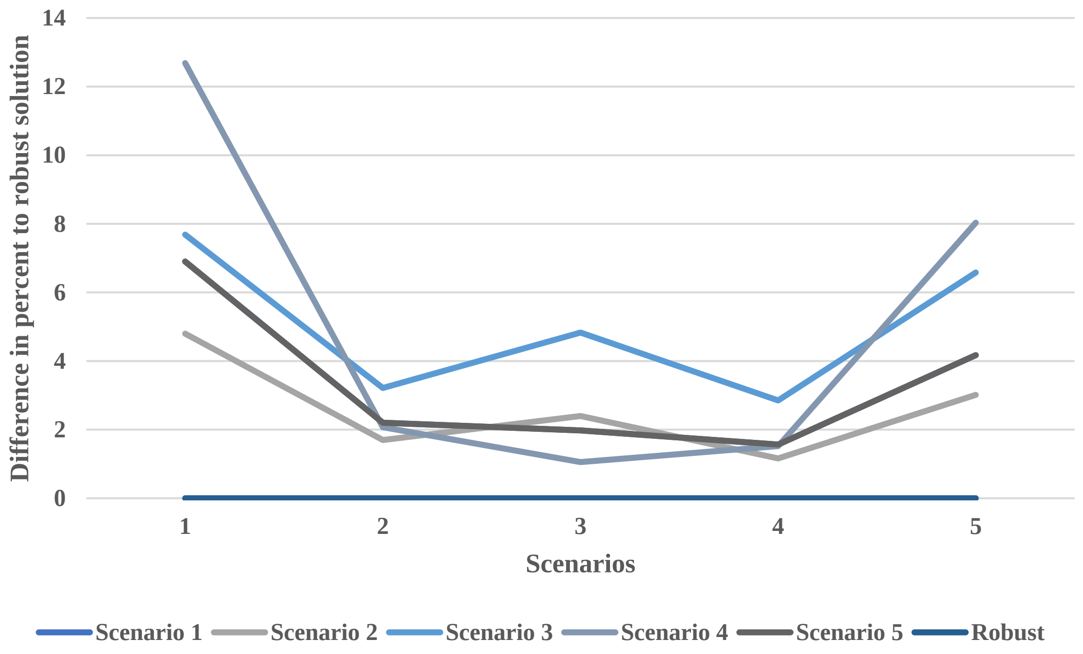

Figure 33: Solution Comparison Robust Optimization – AELP

Note that the solutions obtained for scenarios 1 and 5 are identical. In contrast, the solutions for scenarios 2 and 4 are not identical even though they share the same demand point mapping. This can be explained by the higher portion of demand for scenario 4 which may lead to an alternate location decision. Yet, the differences are very small. Insofar as the AERT is concerned, all scenario optimized solutions perform better for their respective scenario. The solutions for the two rush hour scenarios, 2 and 4, also perform well for scenario 3 which can be explained by the commute to the workplace and the higher spatial proximity. The solution for scenario 3 leads to a very low AERT but to relatively high values for all other scenarios. The chosen robust solution, 'SR1-MR300', puts some weight on lowering the AELP. Therefore, in terms of AELP, the robust solution, compared to the scenario optimized solutions, performs better in all scenarios. The main relationships described above remain valid for the AELP comparison.

The server locations for 'SR1-MR300' are illustrated in figure 34 below. (Server locations are indicated by the black ambulance symbol .)

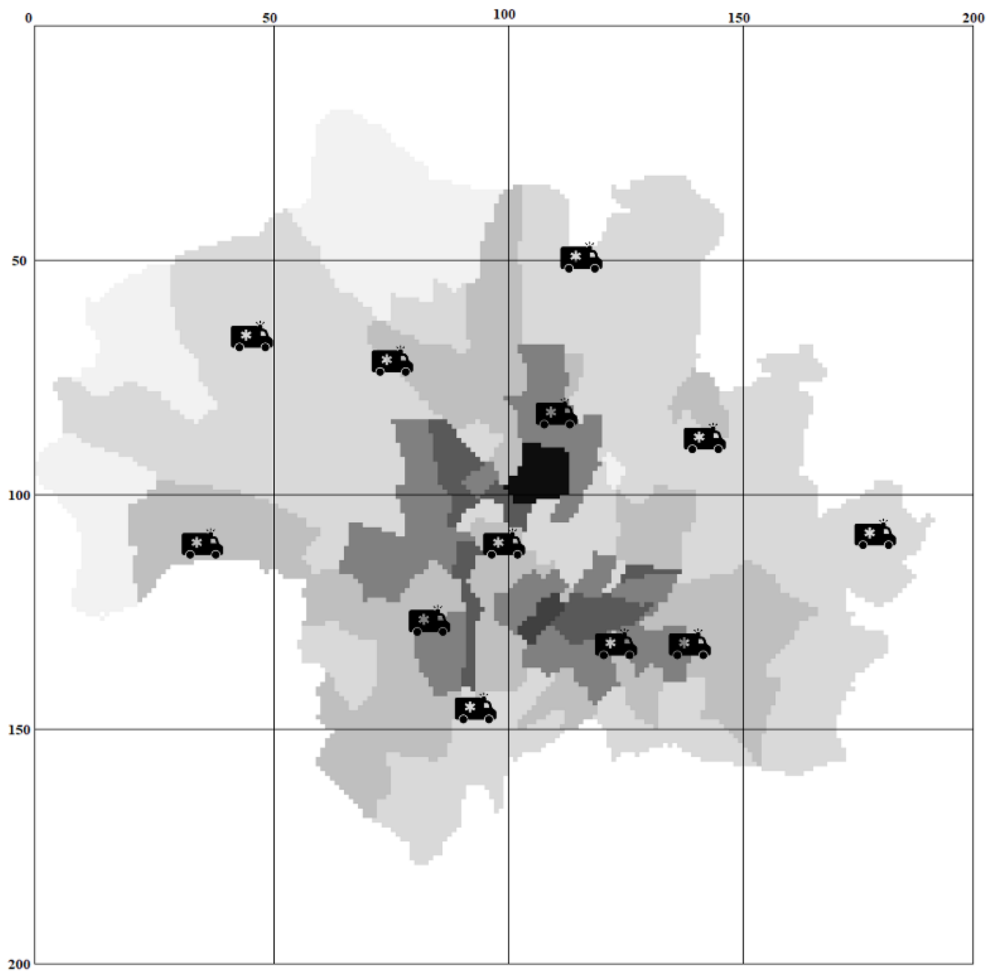

Figure 34: Server locations - Robust Optimization

7.5 Robust Goal Programming

As already mentioned, the Robust Goal Programming uses the existing cache of the Robust Optimization. As for the metaheuristic calibration and experiments as well as the Goal Programming approach, the aspiration levels and the corresponding threshold values need to be calibrated. Therefore, the calibration process of chapters 7.1 and 7.3 is repeated here. The weighting parameters for both the solution and the model robustness are set to zero. Additionally, the created cache is searched for the respective best value for each of the five performance metrics and for each of the five scenarios.

The same parameter values for the time spend at the scene of the incident as well as for C_{cov} as in chapter 7.3 and 7.4 are used here. In order to ensure comparability to the Goal Programming model of chapter 7.1, the threshold values for the goals corresponding to the AERT, RTCov, LRT and WBM metrics are not changed here. Since the analysis of chapter 7.4 has shown that the value of the AELP metric is most sensitive to changes in the demand inputs, 0,03 is added to the respective best value for each scenario as threshold value. The parameter for the Gaussian criterion, σ, is set to 0,01. The derived aspiration levels as well as the threshold values can be found in table 36 below. The weights (W^{+}_{gs}, W^{-}_{gs}) described in table 37 are relevant for all scenarios.

As for the Robust Optimization approach, the values derived for the performance metrics refer to the weighted average over all scenarios; see table 37. The values for each scenario can be found in appendix D. Note that the coding for the individual optimization is formulated as follows: ‘G’ refers to the preliminary optimization runs for each goal, ‘SRX-MRY’ to the calibration of the solution and model robustness parameter, ‘A’ to the variation in the weighting parameters, ‘B’ to the budget goal of chapter 7.5.1 and ‘S’ to the additional scenario optimizations of chapter 7.5.2. The corresponding results are analyzed in the respective chapters.

		Aspiration level	Threshold value	α_2^{is}
AELP	Scenario 1	0,0110509	0,0410509	0,03
	Scenario 2	0,08029	0,11029	0,03
	Scenario 3	0,07596	0,10596	0,03
	Scenario 4	0,12396	0,15396	0,03
	Scenario 5	0,0389675	0,0689675	0,03
AERT	Scenario 1	6,53419	10	3,46581
	Scenario 2	7,42039	10	2,57961
	Scenario 3	7,04826	10	2,95174
	Scenario 4	7,65594	10	2,34406
	Scenario 5	6,95288	10	3,04712
RTCov	Scenario 1	0,98	0,8	0,18
	Scenario 2	0,91	0,8	0,11
	Scenario 3	0,986667	0,8	0,186667
	Scenario 4	0,883333	0,8	0,083333
	Scenario 5	0,97	0,8	0,17
LRT	Scenario 1	11,3088	14	2,6912
	Scenario 2	11,6856	14	2,3144
	Scenario 3	9,83286	14	4,16714
	Scenario 4	11,7826	14	2,2174
	Scenario 5	11,3555	14	2,6445
WBM	Scenario 1	0,184242	0,5	0,315758
	Scenario 2	0,133789	0,5	0,366211
	Scenario 3	0,192646	0,5	0,307354
	Scenario 4	0,123154	0,5	0,376846
	Scenario 5	0,160271	0,5	0,339729

Table 36: Goal calibration - Robust Goal Programming

Number	Code	Weight	OF value	AELP	AERT	RTCov	LRT	WBM	USO	NCSO	Performance gain
1	G1	{1; 0; 0; 0; 0}	0,9976	0,056385	7,2747	0,929542	11,6703	0,197303	603	255	57,71 %
2	G2	{0; 1; 0; 0; 0}	0,9822	0,056837	7,0481	0,928713	11,607	0,182961	705	295	58,16 %
3	G3	{0; 0; 1; 0; 0}	0,9596	0,063925	7,449	0,960935	11,2501	0,23023	635	405	36,22 %
4	G4	{0; 0; 0; 1; 0}	0,9474	0,062636	7,4151	0,945245	11,06	0,226881	579	308	46,8 %
5	G5	{0; 0; 0; 0; 1}	0,9711	0,05848	7,1354	0,907185	11,993	0,179877	607	335	44,81 %
6	SR0-MR0	{0,2; 0,2; 0,2; 0,2; 0,2}	0,9039	0,057203	7,1301	0,93316	11,5649	0,187538	639	277	56,65 %
7	SR0,5-MR0	{0,2; 0,2; 0,2; 0,2; 0,2}	0,7900	0,057203	7,1301	0,93316	11,5649	0,187538	635	240	62,2 %
8	SR1-MR0	{0,2; 0,2; 0,2; 0,2; 0,2}	0,6761	0,057203	7,1301	0,93316	11,5649	0,187538	613	270	55,95 %
9	SR2-MR0	{0,2; 0,2; 0,2; 0,2; 0,2}	0,4484	0,057203	7,1301	0,93316	11,5649	0,187538	563	444	21,14 %
10	SR3-MR0	{0,2; 0,2; 0,2; 0,2; 0,2}	0,2207	0,057203	7,1301	0,93316	11,5649	0,187538	685	423	38,25 %
12	SR1-MR0,5	{0,2; 0,2; 0,2; 0,2; 0,2}	0,6475	0,057203	7,1301	0,93316	11,5649	0,187538	612	223	63,56 %
13	SR1-MR1	{0,2; 0,2; 0,2; 0,2; 0,2}	0,6189	0,057203	7,1301	0,93316	11,5649	0,187538	589	193	67,23 %

Number	Code	Weight	OF value	AELP	AERT	RTCov	LRT	WBM	USO	NCSO	Perfor-mance gain
14	SR1-MR2	{0,2; 0,2; 0,2; 0,2; 0,2}	0,5617	0,057203	7,1301	0,93316	11,5649	0,187538	670	255	61,94 %
15	SR1-MR3	{0,2; 0,2; 0,2; 0,2; 0,2}	0,5041	0,056929	7,1028	0,93191	11,5694	0,187159	630	202	67,94 %
16	SR1-MR10	{0,2; 0,2; 0,2; 0,2; 0,2}	0,1056	0,056929	7,1028	0,93191	11,5694	0,187159	552	168	69,57 %
17	A1	{0,5; 0,5; 0; 0; 0}	0,7382	0,056837	7,0481	0,928713	11,607	0,182961	603	189	68,66 %
18	A2	{0,75; 0,25; 0; 0; 0}	0,74	0,056837	7,0481	0,928713	11,607	0,182961	649	201	69,03 %
19	A3	{0,25; 0,75; 0; 0; 0}	0,7363	0,056837	7,0481	0,928713	11,607	0,182961	672	230	65,77 %
20	A4	{0; 0,5; 0,5; 0; 0}	0,6641	0,061504	7,0697	0,936497	11,8709	0,231407	715	447	37,48 %
21	A5	{0; 0,75; 0,25; 0; 0}	0,6955	0,061504	7,0697	0,936497	11,8709	0,231407	737	294	60,11 %
22	A6	{0; 0,25; 0,75; 0; 0}	0,7042	0,065104	7,2933	0,958163	11,436	0,236358	590	272	53,9 %
23	A7	{0; 0,5; 0; 0,5; 0}	0,6862	0,062404	7,2224	0,931767	11,1561	0,221869	699	300	57,08 %
24	A8	{0; 0,75; 0; 0,25; 0}	0,7004	0,058911	7,0564	0,922187	11,5915	0,202653	641	173	73,01 %
25	A9	{0; 0,25; 0; 0,75; 0}	0,6905	0,062636	7,4151	0,945245	11,06	0,226881	581	191	67,13 %
26	A10	{0,5; 0; 0,5; 0; 0}	0,6719	0,057956	7,1704	0,936494	11,6581	0,199479	596	207	65,27 %

Number	Code	Weight	OF value	AELP	AERT	RTCov	LRT	WBM	USO	NCSO	Perfor-mance gain
27	A11	{0,75; 0; 0,25; 0; 0}	0,7055	0,057203	7,1301	0,93316	11,5441	0,187538	645	223	65,43 %
28	A12	{0,25; 0; 0,75; 0; 0}	0,6728	0,062162	7,4304	0,95733	11,1799	0,226223	551	230	58,26 %
29	A13	{0,5; 0; 0; 0,5; 0}	0,6804	0,057235	7,1258	0,928022	11,5611	0,193229	562	173	69,22 %
30	A14	{0,75; 0; 0; 0,25; 0}	0,7103	0,057235	7,1258	0,928022	11,5611	0,193229	584	184	68,49 %
31	A15	{0,25; 0; 0; 0,75; 0}	0,679	0,061501	7,4368	0,951357	11,0693	0,220199	559	230	58,86 %
32	A16	{0; 0; 0,5; 0,5; 0}	0,6807	0,061501	7,4368	0,951357	11,0693	0,220199	616	262	57,47 %
33	A17	{0; 0; 0,75; 0,25; 0}	0,6685	0,061501	7,4368	0,951357	11,0693	0,220199	611	223	63,5 %
34	A18	{0; 0; 0,25; 0,75; 0}	0,6929	0,061501	7,4368	0,951357	11,0693	0,220199	559	205	63,33 %
35	A19	{0; 0; 0; 0,5; 0,5}	0,672	0,0622	7,416	0,951073	11,0681	0,218224	600	173	71,17 %
36	A20	{0; 0; 0; 0,75; 0,25}	0,6888	0,0622	7,416	0,951073	11,0681	0,218224	625	203	67,52 %
37	A21	{0; 0; 0; 0,25; 0,75}	0,6892	0,056996	7,1037	0,927467	11,5659	0,185814	618	155	74,92 %
38	A22	{0,35; 0,35; 0,1; 0,1; 0,1}	0,704	0,057314	7,0697	0,929548	11,5593	0,189204	527	198	62,43 %
39	B1	{0; 0; 0; 0; 0; 1}	0,8	0,06115	7,5182	0,944552	11,5954	0,2229	410	212	48,29 %
40	B2	{0,25; 0,25; 0,1; 0,1; 0,1; 0,2}	0,6165	0,059204	7,2225	0,905798	12,3285	0,19176	538	198	63,2 %

Number	Code	Weight	OF value	AELP	AERT	RTCov	LRT	WBM	USO	NCSO	Performance gain
41	B3	{0,175; 0,175; 0,05; 0,05; 0,05; 0,5}	0,6045	0,06115	7,5182	0,944552	11,5954	0,2229	455	191	58,02 %
42	B4	{0,05; 0,05; 0,05; 0,05; 0,05; 0,75}	0,5997	0,06115	7,5182	0,944552	11,5954	0,2229	414	160	61,35 %
43	S1	{1; 0; 0; 0; 0}	1	0,056679	7,2264	0,923162	11,926	0,200326	592	154	73,99 %
44	S2	{0; 1; 0; 0; 0}	0,9999	0,059732	7,1021	0,919967	12,2225	0,22725	660	169	74,39 %
45	S3	{0; 0; 1; 0; 0}	0,9849	0,064056	7,2968	0,957052	11,4162	0,22866	627	276	55,98 %
46	S4	{0; 0; 0; 1; 0}	0,9898	0,059346	7,1288	0,928162	11,5214	0,202307	603	379	37,15 %
47	S5	{0; 0; 0; 0; 1}	0,9972	0,056837	7,0481	0,928713	11,607	0,182961	540	121	77,59 %
48	S6	{1; 0; 0; 0; 0}	1	0,056385	7,2747	0,929542	11,6703	0,197303	565	169	70,09 %
49	S7	{0; 1; 0; 0; 0}	1	0,059532	7,1044	0,904125	11,9816	0,18814	673	159	76,37 %
50	S8	{0; 0; 1; 0; 0}	1	0,063925	7,449	0,960935	11,2501	0,23023	486	194	60,08 %
51	S9	{0; 0; 0; 1; 0}	1	0,062627	7,482	0,957883	11,1984	0,229818	576	221	61,63 %
52	S10	{0; 0; 0; 0; 1}	1	0,0565	7,1407	0,919413	11,9125	0,187413	671	200	70,19 %

Table 37: Results of Robust Goal Programming

Much in the same way as with the Robust Optimization, a rather high variation for the USO-metric can be observed when compared to the Goal Programming model. Moreover, the relatively high variation in the NCSO-metric leads to volatile results for the performance gain yielded through the use of the Dynamic Caching Strategy. So, for example, the optimization of S4 leads to 379 newly calculated solutions, which is a relatively high number considering the corresponding optimization number (46). Additionally, a modest amount of unique solutions, 603, is necessary to derive the solution. Similar observations can be made for other outliers like A4 and S3. Altogether, there is the impression that the corresponding search space for these optimizations has not been explored sufficiently, which then results in the high NCSO number. Figure 35 shows the course of the NCSO and of the performance gain with respect to the optimizations.

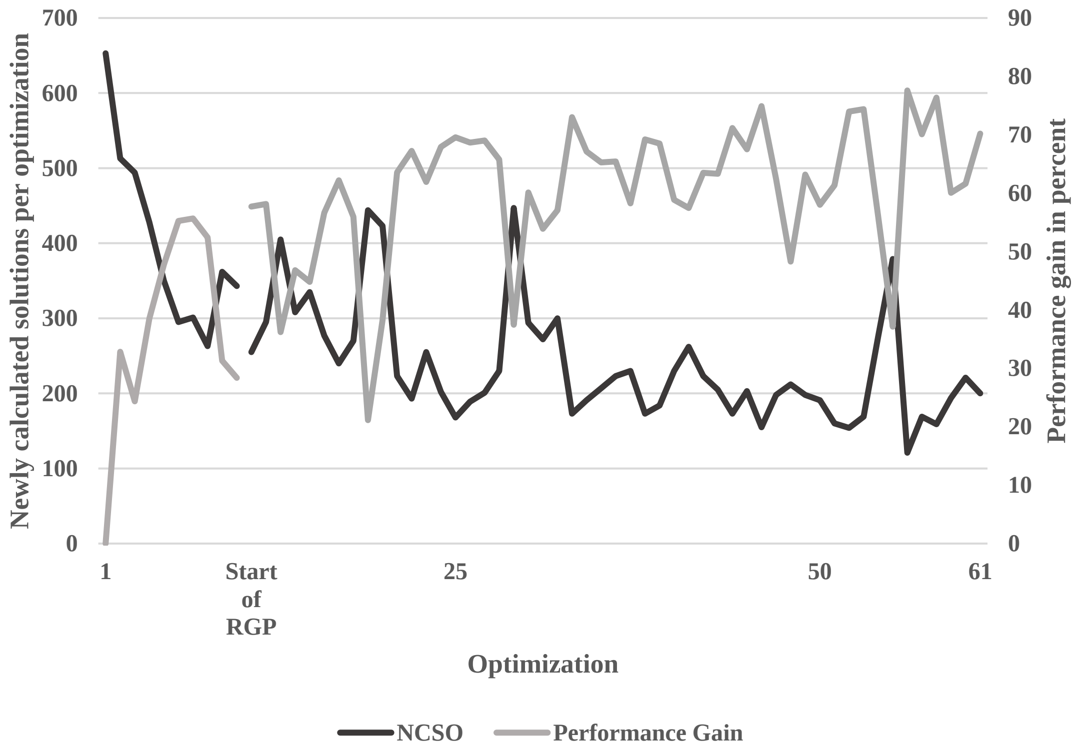

Figure 35: NCSO and performance gain - Robust Goal Programming

For reasons already mentioned, there is a rather high fluctuation in both graphs. As for the Goal Programming, it can be stated that the Dynamic Caching Strategy leads to the highest relative performance gains during the first few optimizations. Afterwards, only minor gains can be observed. As already mentioned in chapter 7.3, this could be explained by the different starting points of the ACO, which then converge towards already explored end points. This argument is further supported by the number of solutions that are newly calculated during the individual iterations of the ACO, illustrated in figure 36 below. Note that the darker lines refer to earlier optimizations while the brighter ones refer to the following optimizations.

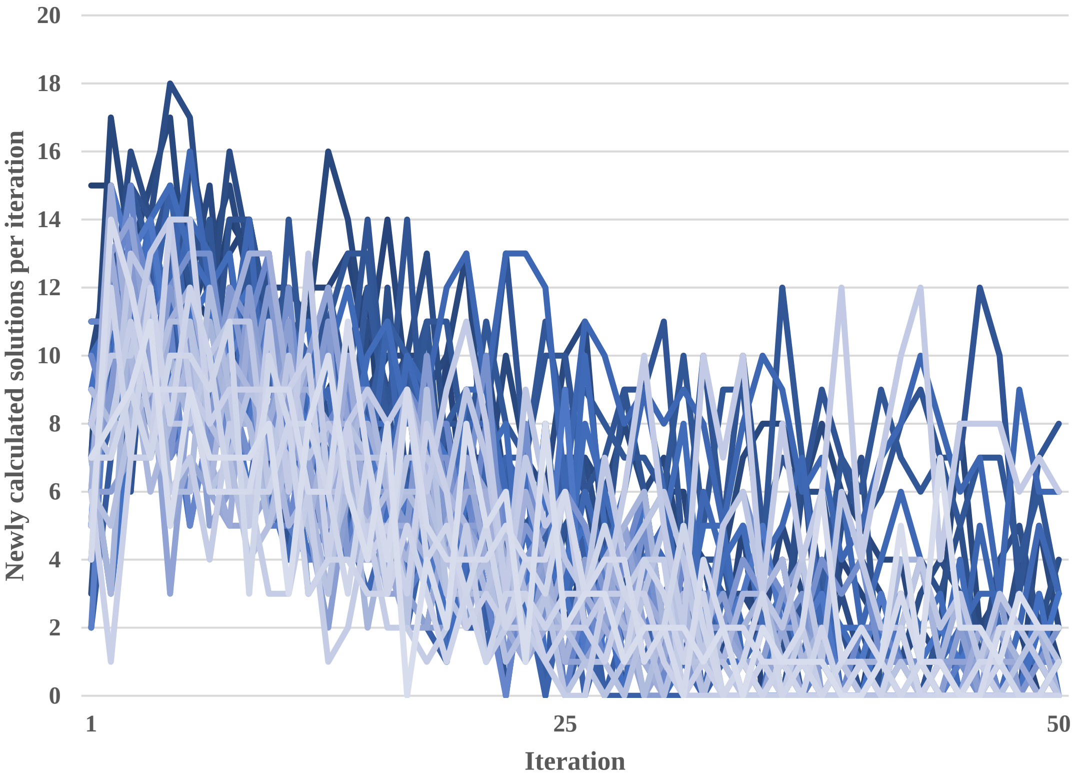

Figure 36: Calculations per iteration - Robust Goal Programming

As with the Goal Programming model, the optimization for each of the five goals (G1-G5) has revealed some trade-offs that require further analysis and discussion. Note that the analysis below only refers to the significant differences that have not already been mentioned for the Goal Programming. For a thorough analysis and description of the mechanisms between the performance metrics, the reader is referred to chapter 7.3. The values of the performance metrics corresponding to each scenario can be found in appendix D.

- The minimization of the AELP metric (G1) leads to relatively good values in all other performance metrics. As already discussed for the Robust Optimization, the AELP metric reacts most sensitive to the changes in demand quantity and location and leads to a steep increase for the AERT metric. Additionally, lowering the AELP metric beyond a certain value leads to unfavorable results for the overall response time. This becomes especially clear if the only minor decrease for the AELP metric between G1 and G2 is considered. There is no significant difference between the derived solutions for the RTCov and LRT metrics between the solutions for G1 and G2. This also holds true if the individual scenarios are considered. The only real significant differences can be observed for the WBM in scenario 1 (0,240713 - G1 vs. 0,196339 - G2) which is then smoothened out over the remaining four scenarios.

- The minimization of the AERT metric (G2) leads to the most balanced results in comparison to G1 and G3-G5. Therefore, none of the computed performance metrics is obstructed to a relevant degree. The highest decrease in any performance metric can be observed for RTCov. As already mentioned for the Goal Programming model, this is due to the calculation of an average of the AERT, which in consequence neglects the coverage of the highest portion of demand points within 9 minutes that would theoretically be possible.

- The maximization of the RTCov metric (G3) leads to relatively good values for the LRT metric. All other performance metrics suffer a rather high decrease from their observed best value. The RTCov metric also fluctuates between the scenarios due to the different demand quantities and locations. Interestingly, especially poor values for the AELP and AERT metric can be observed for scenarios in which the relative demand is high (scenarios 2 and 4). This can be explained by the rather large congestion of the system in those scenarios which makes it even more difficult to maintain a high coverage value.

- The minimization of the LRT metric (G4) results in a rather high RTCov value. As for G3, all other performance metrics decrease rather significantly. However, slightly better values for the AELP, AERT metric as well as the WBM can be observed in comparison to G3. The low values for the AELP and AERT metric for scenarios 2 and 4 can also be noticed for G4. Note that the lowest LRT can be observed for scenario 3 which can be explained by the ‘work’ demand point mapping (see chapter 5). Therefore, the locations of the demand points are located more towards the city center so that it can be reached faster.

- The minimization of the WBM (G5) leads to relatively good values for the AELP and AERT metrics but to rather poor values for the RTCov and LRT metrics. The WBM achieves lower values for scenarios with higher demands, especially for scenarios 2 and 4. This can be explained by the congestion of the system which in turn causes lower-tier servers to be dispatched more often. As a result, the workload becomes more evenly distributed among all servers and the metric decreases in value.

Altogether, it can be stated that the incorporation of different day times and the resulting changes in quantity of demand and location lead to changes in the value of the performance metrics. The values for the AELP and AERT metric behave identical to the Robust Optimization approach. Similarly to the AERT, the RTCov metric shows only minor increases due to the lower sensitivity for increases in demand. The LRT metric even decreases. It has to be noted that the LRT metric represents an average value over all scenarios and the decrease is caused by the relatively low value during the third, ‘work’-related scenario (see also appendix D). The changes in the quantity of demand also explain the increase for the WBM if compared to the Goal Programming model. If compared to the Goal Programming model, it can be summarized that the trade-offs between the metrics are less severe. This can be explained due to the several scenarios which then smoothen the existing deviations for one scenario. This is especially valid for the values of the ‘work’-related scenario.

As with the Robust Optimization approach, the parameters for the solution and model robustness need to be calibrated. For the purpose of simplification, all goals are given the same weight of 0,2. As in the Robust Optimization, the solution robustness is introduced to penalize solutions that show a variation between the scenarios and should lead to a solution that is nearly optimal in every scenario. In order to mitigate any potentially existing trade-offs between solution and model robustness, the parameter for the model robustness is set to 0. However, as in chapter 7.4, no effect on the solution robustness can be observed for higher weighting factors. As a result, it can be concluded that the best solution derived under equal weights for all performance metrics leads to the lowest variability between the scenarios. Since this may only be true for the best solution(s) of each optimization, the solution robustness is assumed to be beneficial towards the convergence of the search process. Therefore, its weight, ϑ, is set to 1 for the remainder of this chapter.

Additionally, the parameter for the model robustness requires further calibration. Since, for reasons of simplification and comparability to the Robust Optimization model, the model robustness penalizes solutions with higher AELPs, additional emphasis would be put on lowering the AELP for a positive weight. However, only for high weighting parameters of $\omega = 3$ or 10, a very small difference between the derived solutions can be observed. In order to avoid any obstruction of the model with respect to an additional emphasis on lowering the AELPs, ω is set to zero for the remainder of this chapter.

As with the Goal Programming model, the existing trade-offs between the metrics need to be further analyzed. Therefore, the weights between two performance metrics are varied to illustrate the mechanism of the Robust Goal Programming model. In order to gain additional value through the scenario approach and the corresponding variance in the value of the performance metrics, the scenario-dependent values for the relevant performance metrics are given in the respective analysis. The values are presented in the format of (performance metric A; performance metric B) per scenario.

AELP and AERT (G1, G2, A1-A3)

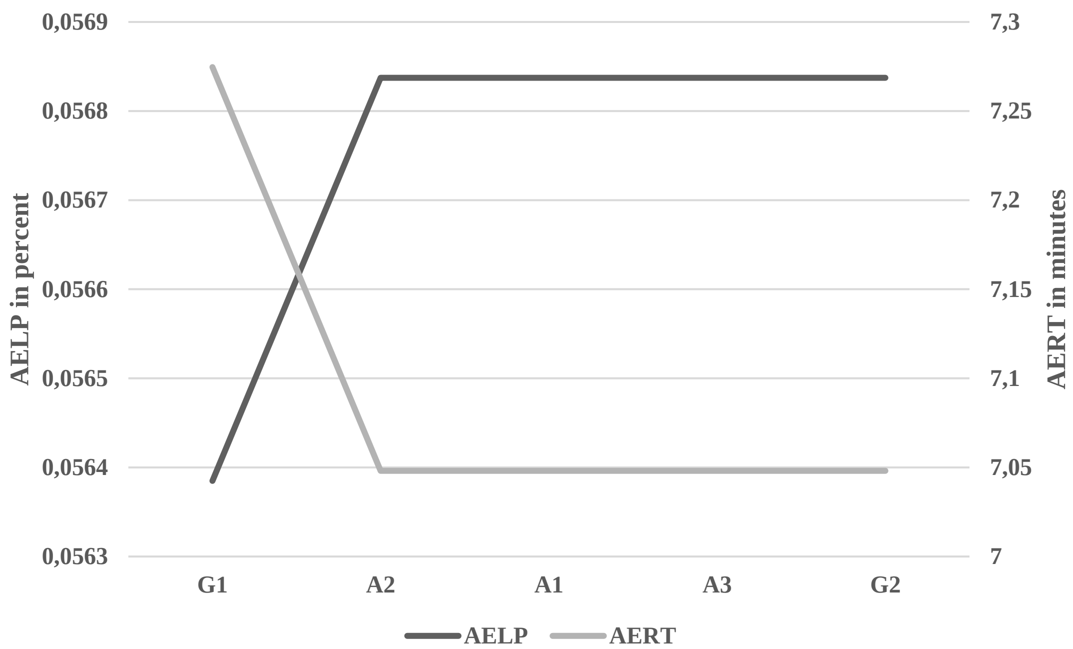

Figure 37: AELP and AERT - Robust Goal Programming

As with the Robust Optimization, the optimized solution for the AERT metric leads to good results in the AELP metric as well. Only if the complete weight is put on lowering the AELP (G1), a slight increase can be observed which then corresponds to a rather large increase for the AERT. Interestingly, the difference in AELP values between G1 and the other solutions can be fully explained by a better value for the third scenario, as can be seen in table 38 below. The difference in the AERT metric between the solutions is the highest for scenario 3 while a more or less identical difference in value can be observed for all other scenarios.

	Scenarios					
(AELP; AERT)	**1**	**2**	**3**	**4**	**5**	**Average**
G1	(0,01124; 6,7394)	(0,08192; 7,6413)	(0,07596; 7,4397)	(0,12571; 7,8877)	(0,03909; 7,1875)	(0,056385; 7,2747)
G2, A1-A3	(0,01105; 6,5514)	(0,08110; 7,4287)	(0,07785; 7,1659)	(0,12506; 7,6698)	(0,03903; 6,9809)	(0,056837; 7,0481)

Table 38: Scenario values AELP and AERT - Robust Goal Programming

AERT and RTCov (G2, G3, A4-A6)

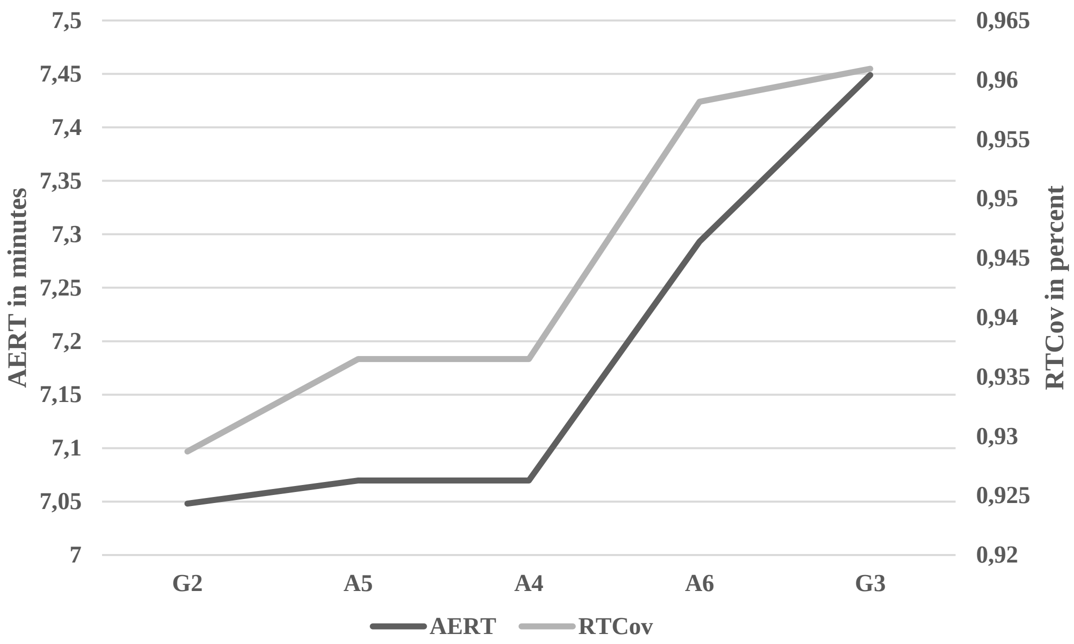

Figure 38: AERT and RTCov - Robust Goal Programming

As detailed for the Goal Programming, the optimizations of the AERT and RTCov metrics lead to contrary results. For higher weights on lowering the AERT, the overall value of the RTCov metric decreases. If RTCov is to be maximized, higher weights on RTCov lead to higher values of the AERT metric. Both metrics react relatively insensitive until a weight of 0,75 or higher is put on increasing the RTCov metric. If G2 and G3 are compared, G2 (G3) has better values for the AERT (RTCov) in all scenarios as is obvious from table 39 below. For the third scenario, a difference in A4 and A5 can be observed in comparison to G2. Therefore, A4 and A5 have a lower AERT and RTCov even though a higher emphasis is put on maximizing the latter. A similar relationship can also be observed for A6 and G3 in the second and fourth scenario. Even though higher coverage rates can be observed, the AERT is lower for A6 in comparison to G3. However, the overall RTCov is lower since the proposed solution on the whole performs worse due to the relatively low probabilities for the second and fourth scenario. The overall value for the RTCov metric only increases slightly between A6 and G3 but a relatively high difference can be seen for the AERT metric. Therefore, it can be stated that improving the provided coverage under 9 minutes for as many demand points as possible leads to a rather high decrease in the overall response time for the majority of other demand points.

	Scenario					
(AERT; RTCov)	**1**	**2**	**3**	**4**	**5**	**Average**
G2	(6,5514; 0,95)	(7,4287; 0,873333)	(7,1659; 0,95)	(7,6698; 0,853333)	(6,9809 0,926667)	(7,0481; 0,928713)
A4, A5	(6,562; 0,97)	(7,5272; 0,876667)	(7,1435; 0,94)	(7,7725; 0,863333)	(7,0155; 0,946667)	(7,0697; 0,936497)
A6	(6,7689; 0,973333)	(7,7058; 0,906667)	(7,4042; 0,976667)	(7,9622; 0,88)	(7,2309; 0,963333)	(7,2933; 0,958163)
G3	(6,8819; 0,973333)	(7,8898; 0,893333)	(7,5973; 0,986667)	(8,1524; 0,863333)	(7,3566; 0,97)	(7,449; 0,960935)

Table 39: Scenario values AERT and RTCov - Robust Goal Programming

AERT and LRT (G2, G4, A7-A9)

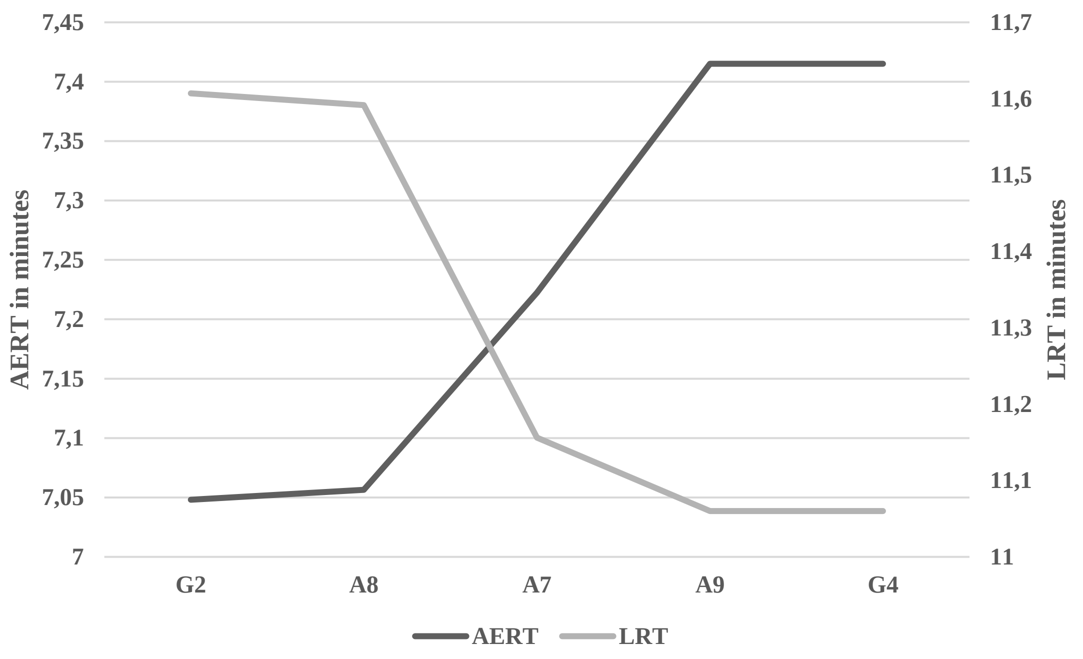

Figure 39: AERT and LRT - Robust Goal Programming

As already described for the Goal Programming, the optimization of both the AERT and the LRT metrics are conflicting objectives. A high emphasis on lowering the AERT leads to a relatively high score for the LRT. Both metrics react rather insensitive to minor changes in their weights (G2 to A8; A9 to G4). The only significant difference in scenario values can be observed for the LRT metric and the second and fourth scenario when A8 is compared to G2 and A7; see table 40 below. The relatively low values in these scenarios are compensated for by high values in all other scenarios. As with the trade-offs between the AERT and RTCov metric, decreasing the LRT metric from A7 to A9/G4 leads to a rather high increase of the AERT.

	Scenarios					
(AERT; LRT)	**1**	**2**	**3**	**4**	**5**	**Average**
G2	(6,5514; 11,4088)	(7,4287; 12,2041)	(7,1659; 11,4217)	(7,6698; 12,496)	(6,9809; 11,5345)	(7,0481; 11,607)
A8	(6,5855; 11,4162)	(7,4288; 11,8278)	(7,1453; 11,512)	(7,6654; 12,0951)	(7,0128; 11,5955)	(7,0564; 11,5915)
A7	(6,7118; 11,4324)	(7,6789; 11,8706)	(7,3039; 10,1936)	(7,9232; 12,1299)	(7,1624; 11,5743)	(7,2224; 11,1561)
A9, G4	(6,8683; 11,4142)	(7,8553; 11,7544)	(7,5376; 10,0175)	(8,1127; 11,9687)	(7,3395; 11,5383)	(7,4151; 11,06)

Table 40: Scenario values AERT and LRT - Robust Goal Programming

AELP and RTCov (G1, G3, A10-A12)

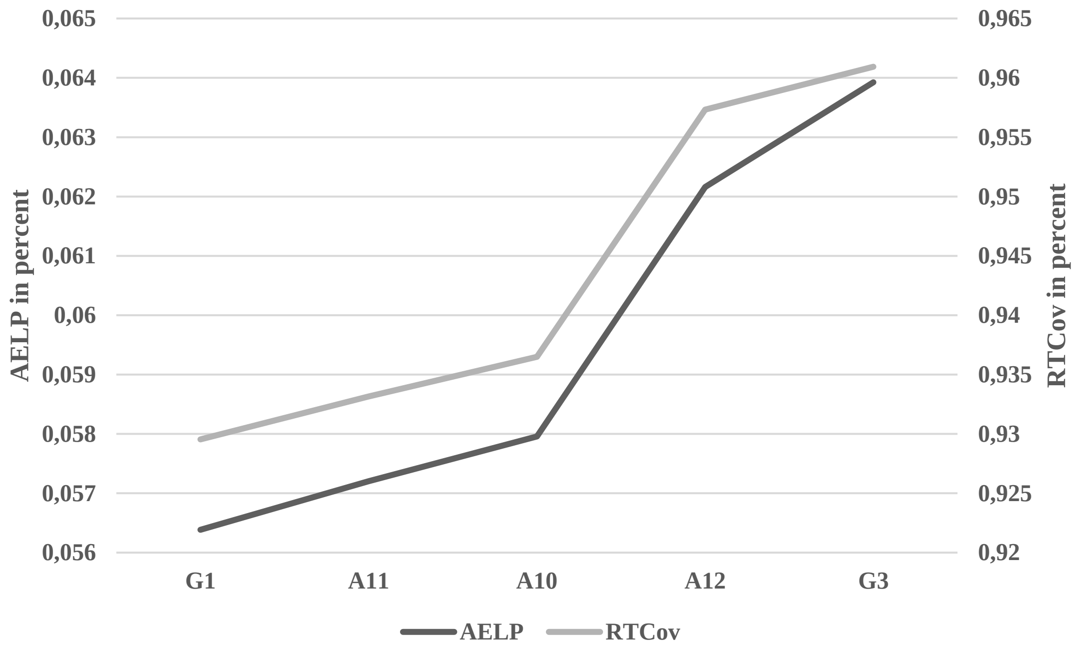

Figure 40: AELP and RTCov - Robust Goal Programming

A relationship that is similar to the one between AERT and RTCov metrics can be observed between the AELP and RTCov metrics. Lowering the AELP leads to relatively low values for the RTCov metric. Comparing both metrics, the RTCov reacts more sensitive to changes in its weight. When the scenario values are considered, an increase in coverage and a decrease in loss probabilities for the second and fourth scenario can be observed for the comparison of A12 to G3 (see table 41). The minor difference is offset by the values of the other scenarios due to already mentioned low probabilities of scenarios 2 and 4. Thus, A12 provides better values in both metrics for the rush hours but performs worse in all other scenarios.

			Scenario			
(AELP; RTCov)	**1**	**2**	**3**	**4**	**5**	**Average**
G1	(0,011245; 0,95)	(0,081917; 0,873333)	(0,07596; 0,956667)	(0,12571; 0,836667)	(0,03909; 0,926667)	(0,056385; 0,929542)
A11	(0,011457; 0,95)	(0,081473; 0,88333)	(0,077784; 0,956667)	(0,12538; 0,87)	(0,039882; 0,926667)	(0,057203; 0,93316)
A10	(0,011613; 0,95)	(0,081791; 0,886667)	(0,079439; 0,96)	(0,125772; 0,87)	(0,040261; 0,93333)	(0,057957; 0,936494)
A12	(0,012645; 0,97)	(0,088456; 0,896667)	(0,085637; 0,976667)	(0,133704; 0,876667)	(0,042685; 0,966667)	(0,062162; 0,95733)
G3	(0,013123; 0,973333)	(0,090618; 0,893333)	(0,088415; 0,986667)	(0,136264; 0,863333)	(0,043866; 0,97)	(0,063925; 0,960935)

Table 41: Scenario values AELP and RTCov - Robust Goal Programming

AELP and LRT (G1, G4, A13-A15)

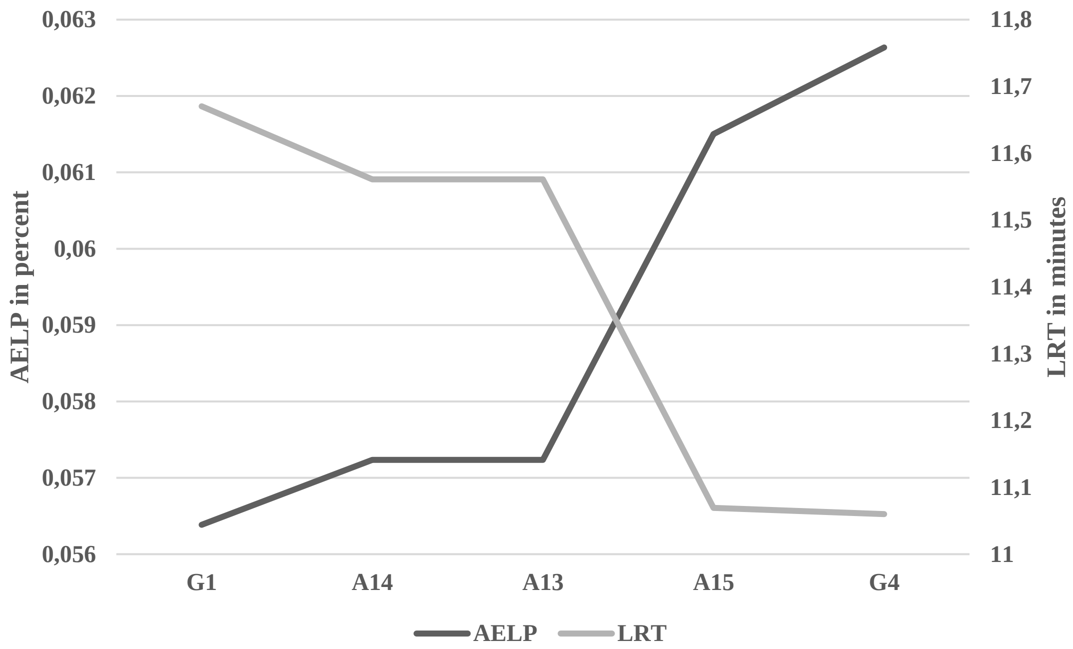

Figure 41: AELP and LRT - Robust Goal Programming

Comparably to the AERT, trade-offs between the AELP and LRT metric can be noticed. Therefore, an AELP focused optimization leads to relatively high values for the LRT. The metrics react rather stable to changes in their weights until a weight of 0,75 or higher is put on lowering the LRT. No real significant difference can be observed if the scenario values of the solutions are compared to each other which can be seen in table 42 below.

			Scenarios			
(AELP; LRT)	**1**	**2**	**3**	**4**	**5**	**Average**
G1	(0,011245; 11,4481)	(0,08191; 12,3448)	(0,07596; 11,4525)	(0,12571; 12,6384)	(0,03909; 11,6071)	(0,056385; 11,6703)
A14, A13	(0,01146; 11,4247)	(0,08164; 11,8678)	(0,07779; 11,4285)	(0,12557; 12,1253)	(0,03987; 11,5605)	(0,05723; 11,5611)
A15	(0,01261; 11,4159)	(0,08788; 11,7602)	(0,08405; 10,0339)	(0,13303; 11,9888)	(0,04262; 11,5426)	(0,06150; 11,0693)
G4	(0,01301; 11,4142)	(0,08919; 11,7544)	(0,08560; 10,0175)	(0,13453; 11,9687)	(0,04363; 11,5383)	(0,06263; 11,06)

Table 42: Scenario values AELP and LRT - Robust Goal Programming

RTCov and LRT (G3, G4, A16-A18)

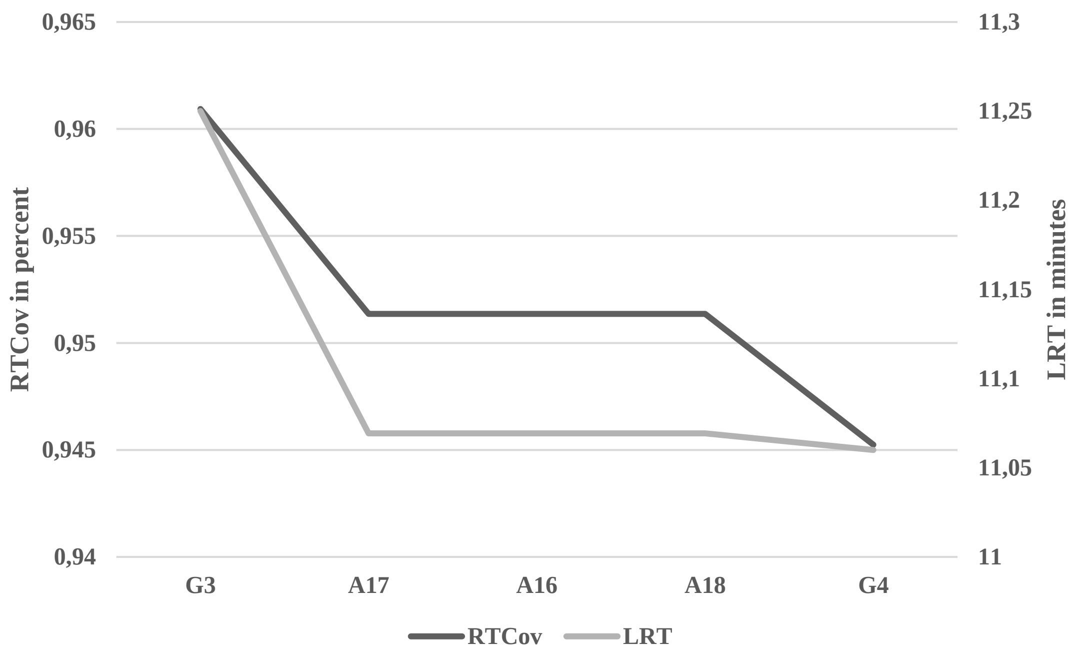

Figure 42: RTCov and LRT - Robust Goal Programming

Like the results of the Goal Programming model, optimization of both the RTCov and LRT metric results in conflicting goals due to the mechanism mentioned in chapter 7.3. The Robust Goal Programming model reacts more sensitive to a change in weighting parameters than the Goal Programming model. This may be explained by the scenario values for the third scenario which show a high RTCov and low LRT value for all three solutions derived (see table 43). Note that the changes between A16-A18 and G4 are very small for the LRT metric but slightly more significant for the RTCov metric.

	Scenario					
(RTCov; LRT)	**1**	**2**	**3**	**4**	**5**	**Average**
G3	(0,97333; 11,5196)	(0,89333; 12,4843)	(0,986667; 9,9682)	(0,86333; 12,924)	(0,97; 11,6886)	(0,96093; 11,2501)
A16-A18	(0,97; 11,4159)	(0,89; 11,7602)	(0,966667; 10,0339)	(0,86333; 11,9888)	(0,96333; 11,5426)	(0,95136; 11,0693)
G4	(0,97; 11,4142)	(0,88; 11,7544)	(0,956667; 10,0175)	(0,86333; 11,9687)	(0,956667; 11,5383)	(0,94525; 11,06)

Table 43: Scenario values RTCov and LRT - Robust Goal Programming

LRT and WBM (G4, G5, A19-A21)

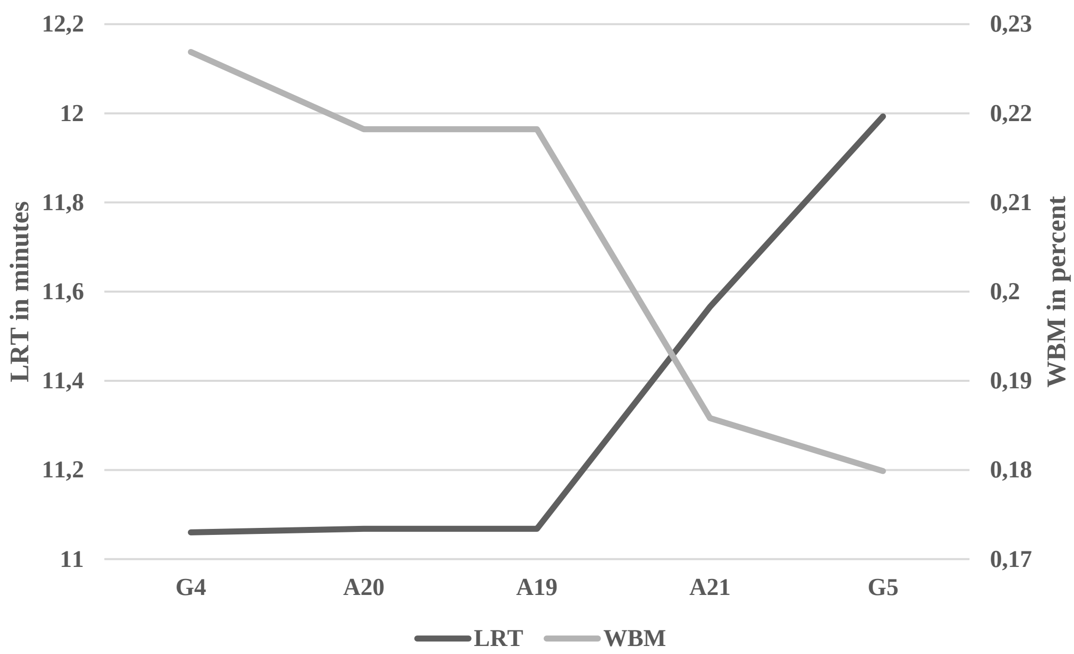

Figure 43: LRT and WBM - Robust Goal Programming

Optimization of the LRT metric and the WBM generates conflicting goals as can be seen in figure 43 above. One explanation might be that servers are located so they can serve even the most remote demands relatively fast, which then may result in more uneven workloads for the entire system. Especially the LRT metric reacts rather insensitive to changes in its weight until a weight of 0,75 is put on minimizing the WBM. The value of the LRT metric increases significantly for A21 and G5. A similar relationship can be observed for the decrease of the WBM. If the individual scenarios are considered, A21 performs better for the WBM of scenarios 2 to 4 if compared to G5. The values for scenarios 1 and 5 offset the difference.

	Scenario					
(LRT; WBM)	**1**	**2**	**3**	**4**	**5**	**Average**
G4	(11,4142; 0,248159)	(11,7544; 0,198583)	(10,0175; 0,248607)	(11,9687; 0,17586)	(11,5383; 0,20963)	(11,06; 0,226881)
A20, A19	(11,4174; 0,230626)	(11,7574; 0,196227)	(10,0318; 0,245064)	(11,9748; 0,173932)	(11,5448; 0,197736)	(11,0681; 0,218224)
A21	(11,4233; 0,203165)	(11,8736; 0,136563)	(11,443; 0,208658)	(12,1335; 0,125975)	(11,5574; 0,178597)	(11,5659; 0,185814)
G5	(11,6299; 0,184242)	(11,8556; 0,137569)	(12,0629; 0,213148)	(12,1206; 0,127914)	(12,1747; 0,165781)	(11,993; 0,179877)

Table 44: Scenario values LRT and WBM - Robust Goal Programming

As with the Goal Programming model, an appropriate configuration of the weighting parameters should be found to ensure adequate values in all five performance metrics. Therefore, in the same way as with the Goal Programming model, the weights are set to {0,35; 0,35; 0,1, 0,1; 0,1} for A22. If the computed performance metrics of A22 are compared to G1-G5, values very similar to the G1 and G2 solutions can be observed. A22 performs better for RTCov and LRT while slightly worse for AELP, AERT and WBM if compared to G1 and G2. The differences of A22 to the (global) best values from G1-G5 can be found in table 45 below.

	AELP	AERT	RTCov	LRT	WBM
Best values from G1-G5	0,056385	7,0481	0,960935	11,06	0,179877
A22	0,057314	7,0697	0,929548	11,5593	0,189204
Difference in Percent to G1-G5	+ 1,65 %	+ 0,31 %	- 3,38 %	+ 4,51 %	+ 5,19 %

Table 45: Comparison of A22 to G1-G5

Just like the Robust Optimization model, the Robust Goal Programming model is then optimized separately for each scenario. After that, for each of the five derived solutions, the value of the objective function is calculated for each scenario with the parameter setting described above. The values of the objective function are then set in relation to the robust optimization as the baseline. The relative values can be found in table 46 below.

Note that the solutions for scenario 1 and 5 are identical.

		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
	Robust	0 %	0 %	0 %	0 %	0 %
Optimized for	Scenario 1	-1,73 %	13,74 %	5,58 %	21,69 %	-1,44 %
	Scenario 2	0,06 %	-0,09 %	0,23 %	0,18 %	-0,18 %
	Scenario 3	4,68 %	1,86 %	-1,37 %	1,69 %	5,13 %
	Scenario 4	0,05 %	0,13 %	0,38 %	-0,17 %	0,28 %
	Scenario 5	-1,73 %	13,74 %	5,58 %	21,69 %	-1,44 %

Table 46: Relative values solution comparison - Robust Goal Programming

Unsurprisingly, each derived solution performs best in the scenario for which it was optimized. Since they are identical, the solutions for scenario 1 (5) perform superior in scenario 5 (1). The solutions derived for scenarios 2 to 4 work relatively well if used in the robust model. In contrast, the solutions derived for scenarios 1 and 5 perform especially poor in scenarios 2 and 4 but are also inferior in comparison to all others except for scenario 3.

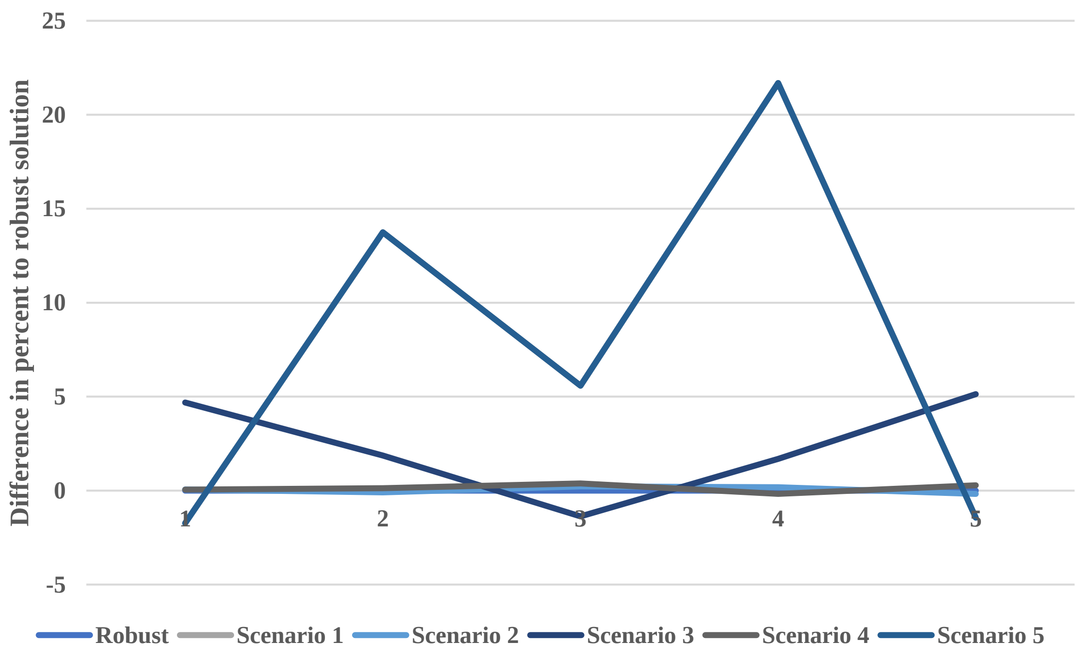

Figure 44: Solution comparison Robust Goal Programming

For simplification and due to the comparable results to the Robust Optimization, a detailed analysis of the AELP and AERT with regard to individual scenario values as in tables 34 and 35 is not repeated here. The server locations for A22 are illustrated in figure

45 below.

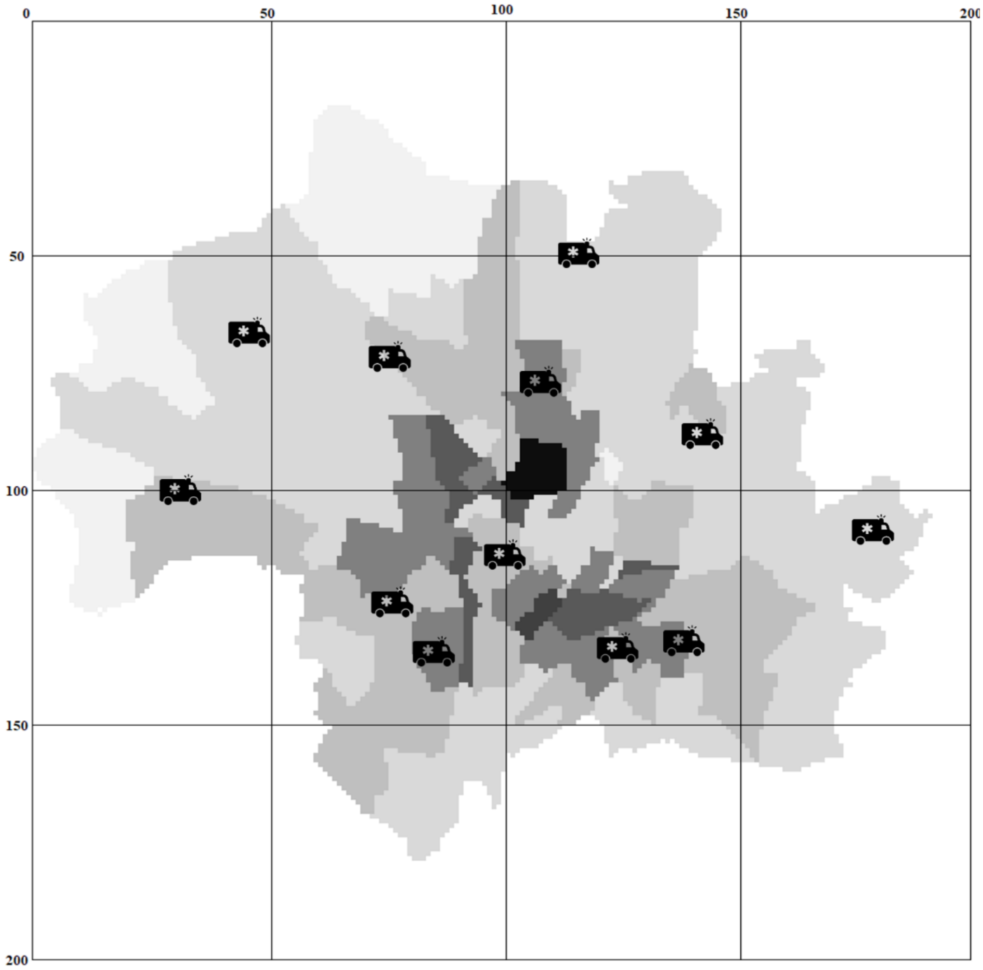

Figure 45: Server locations - Robust Goal Programming

The existing system configuration is then compared to A22 in order to demonstrate the gains achieved in the overall system performance. The Robust Goal Programming model is run with the set of the current server location as inputs. For the sake of simplicity, only the weighted average values of each metric are discussed. The scenario-dependent values can be found in appendix D.

	AELP	AERT	RTCov	LRT	WBM
Existing system configuration	0,071021	7,7643	0,81536	14,9629	0,243828
A22	0,057314	7,0697	0,929548	11,5593	0,189204
Difference in percent to existing configuration	- 19,3 %	- 8,95 %	+ 14 %	- 22,75 %	- 22,4 %

Table 47: Comparison to existing system configuration

Table 47 shows improvements in the performance metrics of up to 22,75 %. Note that the coverage constraint for C_{cov} needs to be relaxed if the existing system configuration can provide an eligible solution. The reader is referred to chapter 8 for a brief discussion on the parameter setting of C_{cov}.

7.5.1 Budget goal

Up to this point, no constraint has been placed on the number of new server locations that can be opened. As discussed in the literature review of chapter 2, budget constraints or cost minimizing objective functions are not among the most discussed objectives for EMS system location optimization studies. However, a new goal is introduced here that penalizes newly opened server locations that differ from the set of locations that is already in place. Note that the budget goal could be expressed either as a linear preference or a level criterion due to its discrete value. The linear preference criterion is chosen here. It should also be considered that the server locations are assumed to be immobile and therefore cannot change their location between scenarios. As a result, the budget goal is defined identically for all scenarios. The satisfaction function can then be expressed as follows:

$$(7.5.1.1)\ F\left(\delta_6^+\right) = \begin{cases} -\frac{1}{\alpha_2^6}\delta_6^+ + 1, & \delta_6^+ < \alpha_2^6 \\ 0, & \delta_6^+ \geq \alpha_2^6 \end{cases}$$

The parameters for the calibration of the budget goal can be found in table 48 below:

	Aspiration levels	Threshold value	α_2^6
Budget goal	12	2	10

Table 48: Goal calibration budget goal

The optimization for B1 was performed with the solution robustness parameter set to zero while B2-B4 with a solution robustness parameter set to one. The optimization B1 shows that, under the current setting of C_{cov}, a maximum of ten from the twelve existing servers can be selected. If a weight greater than 0,2 is put on the budget goal, the same solution is derived for B1, B3 and B4. The solution derived for the optimizations of B1/B3/B4 and B2 are compared to A22 in the following in order to demonstrate the impact of the budget goal on the performance metrics. As in the comparison to the existing system configuration, only the weighted average values of each metric are discussed. The scenario-dependent values can be found in appendix D.

B1, B3 and B4 each choose ten of the twelve existing server locations while B2 chooses nine. Interestingly, B2 does not perform better in all performance metrics and leads to inferior values for RTCov and LRT. Altogether, a positive weight on the budget goal plays a rather significant role in the location optimization decision and leads to notable differences in at least some of the performance metrics. If the WBM is excluded, the differences are up to 6,75 % (see table 49). B2 performs better than B1, B3 and B4 in comparison to A22 for the AELP and AERT metric.

	AELP	**AERT**	**RTCov**	**LRT**	**WBM**
B1, B3, B4	0,06115	7,5182	0,944552	11,5954	0,2229
B2	0,059204	7,2225	0,905798	12,3285	0,19176
A22	0,057314	7,0697	0,929548	11,5593	0,189204
Difference in percent to A22 (B1, B3, B4)	+ 6,75 %	+ 6,34 %	- 1,61 %	+ 0,31 %	+ 17,81 %
Difference in percent to A22 (B2)	+ 3,3 %	+ 2,16 %	+ 2,62 %	+ 6,65 %	+ 1,35 %

Table 49: Comparison budget goal

7.5.2 Additional scenario optimization

The Robust Goal Programming model can be used to put additional weights on the optimization of a certain performance metric for one of the scenarios. Since the scenarios differ in the location of demand, only the ‘base’ demand point mapping has already been analyzed in chapter 7.3. The ‘rush’ and ‘work’ demand point mappings are further discussed here. The present chapter analyzes the influence of the decision to solely optimize one performance metric for a certain scenario. To that aim, ten optimizations, S1-S10, are performed. S1-S5 focus on the ‘rush’ demand point mapping while S6-S10 focus on the ‘work’ demand point mapping. In order to focus these scenarios, the scenario probabilities have to be changed from their original proportion of the day time. For S1-S5 scenarios 2 and 4 are both given the equal probability of 0,5, while S6-S10 put the probability of 1 on scenario 3. All other scenario probabilities are set to zero. The solution robustness parameter is also set to zero. For the analysis of scenarios 2 and 4 it should be noted that the corresponding optimizations do not always result to the theoretically best value possible (see table 36) for both scenarios. This can be explained by the higher demand and the congestion for the fourth scenario which may then lead to different optimal solutions for the scenarios. However, the observed differences are only minor. The performance metrics are described for the respective optimized scenarios as well as for their average value considering all scenarios (table 50). The remaining scenario-dependent values can be found in appendix D.

	S1			S2		
	Scenario 2	**Scenario 4**	**Average of all scenarios**	**Scenario 2**	**Scenario 4**	**Average of all scenarios**
AELP	0,08029	0,12396	0,056679	0,0836556	0,127834	0,059732
AERT	7,5961	7,849	7,2264	7,4204	7,6566	7,1021
RTCov	0,87	0,86	0,923162	0,876667	0,856667	0,919967
LRT	12,413	12,8319	11,926	12,5539	12,8356	12,2225
WBM	0,142452	0,126634	0,200326	0,163485	0,148974	0,22725
	S3			**S4**		
S3	**Scenario 2**	**Scenario 4**	**Average of all scenarios**	**Scenario 2**	**Scenario 4**	**Average of all scenarios**
AELP	0,090296	0,135595	0,064056	0,083052	0,127189	0,059346
AERT	7,7004	7,9565	7,2968	7,4345	7,698	7,1288
RTCov	0,906667	0,883333	0,957052	0,886667	0,86	0,928162
LRT	11,9316	12,3651	11,4162	11,7257	11,7892	11,5214
WBM	0,199407	0,179501	0,22866	0,153954	0,139387	0,202307

	S5		
	Scenario 2	**Scenario 4**	**Average of all scenarios**
AELP	0,081101	0,12506	0,056837
AERT	7,4287	7,6698	7,0481
RTCov	0,873333	0,853333	0,928713
LRT	12,2041	12,496	11,607
WBM	0,133789	0,125277	0,182961

Table 50: Scenario optimization for scenarios 2 and 4

- S1 optimizes scenarios 2 and 4 for the AELP metric. The lowest value for the AELP metric in scenario 2 corresponds to the lowest value for scenario 4. The overall value of the AELP metric, if all scenarios are considered, is very good compared to the best global solution found, G1. All other metrics suffer notably, this also holds true if only the values for scenarios 2 and 4 are considered.

- S2 focuses on the AERT metric. None of the desired aspiration levels for either scenario 2 or scenario 4 are fully reached but there are only minor deviations. Compared to S1, all other performance metrics decrease even more significantly from their overall best value. This can be observed for the scenario-dependent values as well as the averages.

- S3 maximizes the RTCov metric. The defined aspiration level for scenario 4 is fully reached while the deviation from the best value for scenario 2 is very small. Similarly to the trade-offs described in chapter 7.5, the maximization of RTCov for scenarios 2 and 4 leads to comparably good values of the LRT metric but inferior values for all other performance metrics. The difference to the best-observed global value, G3, is modest.

- S4 minimizes the LRT metric. None of the defined aspiration levels for scenarios 2 and 4 are reached fully. However, as for the AERT metric, the deviations are only minor. The other performance metrics do not suffer in the same way as for S3. Interestingly, the average value differs rather significantly from the best global value, G4. This can be explained by the location of the demand points which are located according to the commutes into and out of the city center. Therefore, the objective to minimize the LRT for the rush hour scenarios contradicts rather heavily the entire goal of minimizing the LRT for all scenarios.

- S5 minimizes the deviation in workloads between the servers (WBM). Only the desired aspiration level for scenario 2 is fully reached. Note that the derived solution is identical to the G2 solution. The deviations for all other performance metrics are thus minor (AELP, WBM) or modest (RTCov, LRT).

	S6		S7		S8	
	Scenario 3	**Average of all scenarios**	**Scenario 3**	**Average of all scenarios**	**Scenario 3**	**Average of all scenarios**
AELP	0,07596	0,056385	0,081618	0,059532	0,088415	0,063925
AERT	7,4397	7,2747	7,0483	7,1044	7,5973	7,449
RTCov	0,956667	0,929542	0,93	0,904125	0,986667	0,960935
LRT	11,4525	11,6703	12,0494	11,9816	9,9682	11,2501
WBM	0,20017	0,197303	0,224126	0,18814	0,26064	0,23023

	S9		S10	
	Scenario 3	**Average of all scenarios**	**Scenario 3**	**Average of all scenarios**
AELP	0,086677	0,062627	0,076074	0,056508
AERT	7,6573	7,482	7,1963	7,1407
RTCov	0,976667	0,957883	0,94	0,919413
LRT	9,8329	11,1984	11,39	11,9125
WBM	0,259101	0,229818	0,192646	0,187413

Table 51: Scenario optimization for scenario 3

Table 51 contains the results of the optimization solely for scenario 3. Since only a singular scenario is concerned, the defined aspiration level is reached for all optimizations. It is also notable that there are fewer deviations between the relationships of the scenario-dependent and the average values for the performance metrics. Therefore, the relationships between the metrics in scenario 3 remain mostly valid for all other scenarios.

- S6 minimizes the AELP metric for scenario 3. Note that S6 is identical to G1. Therefore, minimizing the AELP for the 'work' scenario also leads to the lowest global value for the AELP metric that is observed. Therefore, the relationships to all other performance metrics, as described for G1, remain identical.

- S7 minimizes the AERT metric for scenario 3. If the individual scenario values are considered, S7 leads to significant decreases (increases) for the RTCov (LRT) metric. A similar relationship can be observed for the WBM. Since the demand points of the 'work' demand point mapping are located more towards the center of the study area, the minimization of the AERT for scenario 3 neglects a fast coverage for the more distant demand points for the other four scenarios.

- S8 focuses on the maximization of the RTCov metric. Note that S8 is identical to G3. The relationships to all other performance metrics, as analyzed for G3, remain valid.

- S9 minimizes the LRT metric for scenario 3. It can be seen that S9 does lead to a relatively high increase in the overall LRT metric. This can also be explained by the more centralized demand point mapping which subsequently leads to the neglection of providing a relatively fast response to all remote demand points for all other scenarios.

- S10 balances the workloads among all servers for scenario 3. As for other optimizations, for example G5, this leads to relatively good values for the AELP and AERT metrics but comparably poor values for the RTCov and LRT metrics.

Altogether, satisfying one aspiration level of one scenario does not necessarily lead to the fulfillment in the other scenarios. This means that even if the spatial demand distribution remains identical, further analysis regarding different demand intensities is necessary. This is further accentuated by the described system congestion for higher proportion on the overall demands, as for scenario 4. The scenario optimization has shown that the derived system configuration may or may not differ from the best-observed global value. For some optimizations, like S6 and S8, it leads to the same results. Therefore, an optimization that only considers a singular spatial distribution of demand should always be verified against other demand point distributions.

7.6 Location of an additional server

In order to demonstrate the behavior of the model in the case that an additional server is located, the model is performed with 13 servers to be located. To allow additional insight, the robust model is utilized. Due to the larger equation system that needs to be constructed and subsequently solved, the computational times per solution increase approximately by a factor of three. Therefore, the ACO is terminated after 5 iterations to reduce computational time. The following analysis should be understood as a general demonstration of how the model performs with an additional server rather than the proof of an optimal location of 13 servers. Since the analysis up to this point has shown that optimizing for the AERT metric provides fairly good values for all other performance metrics, an AERT optimization is performed. Afterwards, the derived performance metrics are compared to G2 of chapter 7.5.

	Scenario				
(13 Servers; 12 Servers)	**1**	**2**	**3**	**4**	**5**
AELP	(0,00874; 0,011051)	(0,064728; 0,081101)	(0,063729; 0,077857)	(0,1026; 0,12506)	(0,031365; 0,039034)
AERT	(6,39617; 6,55134	(7,1159; 7,4287)	(6,75495; 7,1659)	(7,3406; 7,6698)	(6,77812; 6,9809)
RTCov	(0,956667; 0,95)	(0,896667; 0,873333)	(0,956667; 0,95)	(0,883333; 0,853333)	(0,93; 0,926667)
LRT	(11,3961; 11,4088)	(11,7658; 12,2041)	(11,3639; 11,4217)	(11,8277; 12,496)	(11,5105; 11,5345)
WBM	(0,242723; 0,196339)	(0,146253; 0,133789)	(0,235647; 0,208721)	(0,133828; 0,125277)	(0,207456; 0,174611)

Table 52: Performance metrics with 12 and 13 servers

Table 52 demonstrates how the performance metrics change from 12 to 13 servers. It can be seen that not all performance metrics react similarly. The AELP metric, for example, does react rather sensitive to an additional server. This holds especially true for the high demand scenarios, like scenarios 2, 3 and 4. The decrease can be explained by the additional service that can be provided through the extra server. The AERT metric decreases due to the presumably shorter average travel times from the server location to the demand points. The RTCov and LRT metric do not decrease in a similar fashion. This can be explained by the complicated coverage of remote demand points, which is not facilitated through the use of an additional server. The WBM increases. Since this metric is not necessarily dependent on the number of servers, the increase can be explained by the uneven workloads that may even be elevated through the location of an additional server that serves specific areas of the study area. The weighted average for each performance metric for the 12- and 13-server-case as well as the relative difference in percent are given in table 53 below.

	AELP	**AERT**	**RTCov**	**LRT**	**WBM**
13 Servers	0,04617	6,7661	0,937747	11,4857	0,212921
12 Servers	0,056837	7,0481	0,928713	11,607	0,182961
Difference in percent to 12 servers	- 18,77 %	- 4 %	+ 0,97 %	- 1,05 %	+ 16,38 %

Table 53: Comparison with 12 and 13 servers

The server locations are illustrated in figure 46 below:

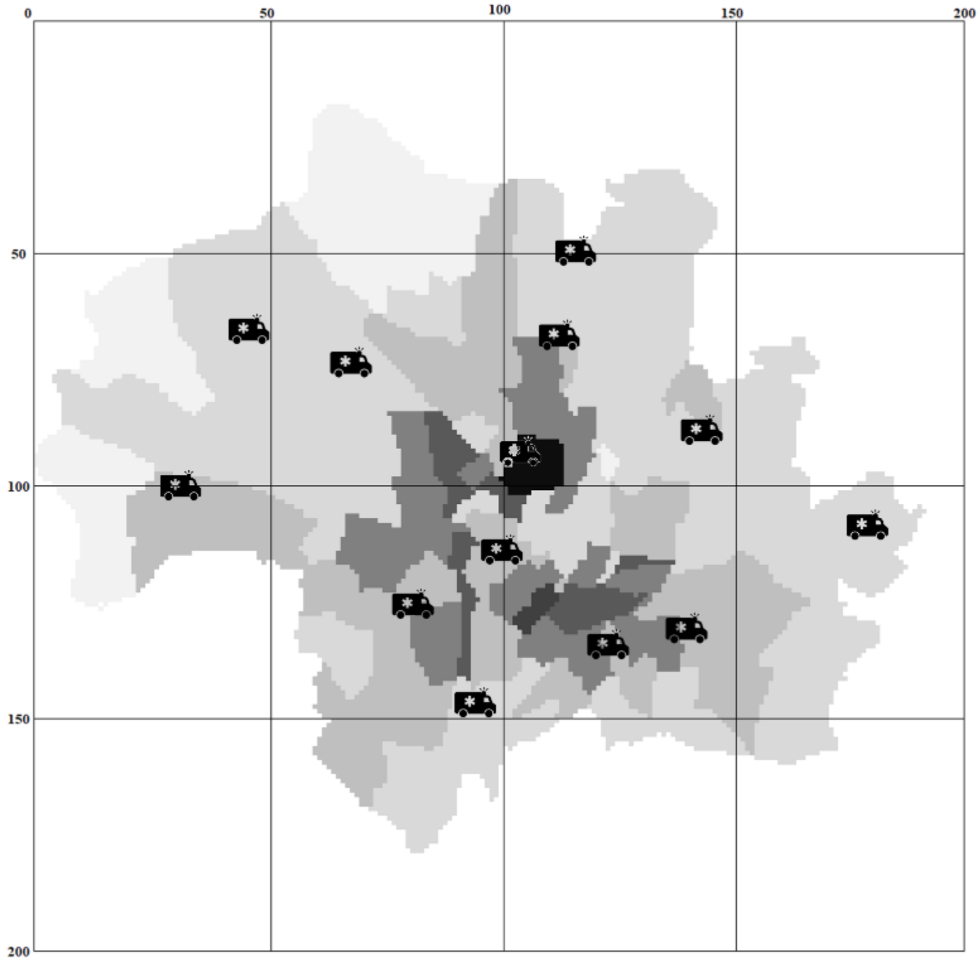

Figure 46: Server locations - 13 server

Figure 46 illustrates the fact that the additional server is located towards the middle of the study area which explains the decrease in the AELP and AERT metrics as well as, if relevant, the only minor decreases for the other performance metrics.

8. Conclusion

This study intends to contribute towards the use of the exact HQM for larger-scale EMS system location optimization studies. In order to make use of the potential of the HQM, several performance metrics are defined and calculated and subsequently utilized during the optimization process through the application of Goal Programming. To account for different spatial demand distributions and quantities, a Robust Optimization approach is formulated and then extended for multiple objectives through the proposal of a Robust Goal Programming technique. A larger-sized real-world case is constructed and used to demonstrate the usability of the proposed models. The still existing computational burdens are mitigated by the application of a new metaheuristic (ACO) in combination with a caching strategy. The research questions raised in chapter one are discussed in the following.

RQ1: Which criteria and objectives are relevant for the design of an EMS system and which influence do they exercise on the resulting decision?

Most existing HQM-based EMS system location optimization studies use the AERT as their objective function, as has been detailed in chapter 2.3. The analysis of chapter 7 has shown that the use of the AERT leads to a balanced system configuration that does not neglect any of the five computed performance metrics to a greater extent. However, depending on the preferences of the decision maker, the application of other performance metrics in the objective function yields some benefits. For example, from an ethical viewpoint, it may be desirable to limit the LRT more strictly than through the use of the AERT as the sole measure. Additionally, it is shown that providing fast and reliable coverage to more distant areas of the study area, as described by the RTCov metric, counteracts the fulfilment of response-time requirements of the other demand points. It should be noted that the influence of the objectives on the overall system configuration is highly dependent on the overall spatial distribution of demand over the service area. For more compact systems, trade-offs between the AERT and RTCov or LRT may be less severe than for systems that have large responsibilities for rural and more distant areas.

RQ2: How can a holistic approach be formulated and which additional insights can it offer?

The Robust Optimization and Robust Goal Programming approaches try to introduce a more holistic view through the use of different day times as scenarios. The analysis has shown that the computed performance metrics are highly subjective to the overall amount of demand that has to be served during the reference period and its spatial distribution. However, the performance metrics do not all display similar results. The AELP metric is most affected while the other, response-time-based, metrics do not show similar influences. Therefore, any decision maker should include different spatial distributions and quantities of emergency demand into the analysis whenever a location or districting decision is taken. The Robust Goal Programming further accentuates this finding and demonstrates the use of the different performance metrics. Even though the described trade-offs for the Goal Programming mostly stay relevant for the Robust Goal Programming, some additional insight can be gained from the scenario optimization. This approach further intensifies the need to take a more holistic view in designing EMS systems.

RQ3: Which measures can be taken in order to significantly reduce the computational burdens of the HQM in a location optimization study?

The computational times of the HQM still remain one of the most limiting factors during the optimization process. For the Goal Programming approach, one solution takes approximately 45 seconds to compute and increases by a factor of five for the Robust Optimization and Robust Goal Programming. Note that the computational times increase approximately by a factor of three for any additional server. The ACO is proven to be an adequate solution technique that yields better results in presumably less computational time if compared to the GA. Note that the computational time does not significantly differ between the ACO and the GA if the same amount of solutions needs to be calculated. The difference between both metaheuristics lies in the quality of the solution derived. Based on the results of the metaheuristic experiments, the use of the GA may have resulted in more generations and a higher population which would have then increased the computational time significantly. Altogether, it can be said that the use of the ACO has contributed towards reducing the computational burdens. Additionally, a Dynamic Caching Strategy is proposed that intends to make use of already calculated solutions. Even tough the relative performance gains are most significant during the first few optimizations, all optimizations benefit from the already created cache.

Nonetheless, this study has some drawbacks that should be mentioned and discussed. Therefore, some particular research areas should be tackled in the future.

- The assumption of fourth-level districting contributes towards the relatively high AELP metric to at least some degree and leads to comparably short preference lists. As a result, only servers in relative proximity can be dispatched. Despite the mentioned points, it may be beneficial to limit the ability of downstream servers to respond to distant demands in a real-life setting. Altogether, this assumption should be validated in future studies. It is made solely due to the computational limitations.

- The value of C_{cov} results in the current configuration of the system to be an ineligible solution. However, the existing server configuration performs poorly for the proposed model. Therefore, C_{cov} rejects presumably inferior solutions. It cannot be ruled out that it may as well reject a well-performing solution. Future studies could therefore include a more thorough validation.

- The scenario approach used for the Robust Optimization and Robust Goal Programming model leads to hard boundaries between the scenarios. Applied to the day-time approach, this means that there is no smooth transition between the demand point mappings and the resulting spatial distribution of demand. Therefore, the results of the models can only be assumed to be scenario-robust. Future studies could introduce more scenarios which model the transitions between the demand point mappings. Moreover, some complexities like changing demands for different days of the week or for public events are completely neglected. Additional value could therefore be provided through a simulation study that simulates demand-intense events at certain areas of the study area and the subsequent analysis of their influence on the overall system behavior.

- The metaheuristic experiments are limited to a rather small study area and a small number of servers to be located. Therefore, any increasing optimality gap for a higher amount of servers cannot be neglected. However, it should be noted that the current state of the art metaheuristic, the GA, is also not validated for the combination of an exact HQM and a larger number of servers. Future studies could focus on a more thorough validation of the results for varying system sizes.

- The models also do not incorporate either heterogenous servers, multiple dispatches or a finite- or infinite-sized queue. Note that all of these extensions already exist for the descriptive use of the exact HQM. The incorporation of any of these would lead to a further increase in modelling and computational efforts. However, future studies could at least include some of these aspects in medium-sized case studies.

- From a practical standpoint, the use of the proposed exact HQM may be limited to skilled professionals with background in coding, queueing and optimization theory. Additionally, the practical use may also be limited to a maximum of up to fifteen servers due to the still existing computational burdens.

Appendix A: Goal Programming A23

	Response Time	Loss Probability
1	10,2559	0,0463507
2	9,19333	0,0463507
3	8,25777	0,0396751
4	8,01271	0,0396751
5	8,02088	0,0463507
6	8,31186	0,0463507
7	7,22685	0,0463505
8	7,68284	0,0463507
9	7,52054	0,0396751
10	5,86906	0,0463506
11	6,43474	0,0463505
12	10,2346	0,0463507
13	9,95906	0,0399344
14	8,7209	0,0396757
15	5,67189	0,0463506
16	8,33036	0,0399345
17	5,76946	0,0463506
18	7,55102	0,0396751
19	8,84212	0,0463507
20	9,24875	0,0399344
21	11,5794	0,0396757
22	7,91582	0,0399345
23	8,07905	0,0396755
24	7,21936	0,0399345
25	7,14996	0,0463507
26	7,24158	0,0396755
27	7,40957	0,0399345
28	7,99791	0,0396754
29	6,22554	0,0399345
30	7,10277	0,0463506
31	6,57025	0,0399345
32	8,856	0,046351
33	6,39396	0,0399344
34	6,68381	0,0399344
35	7,66472	0,046351
36	6,24834	0,0396755
37	8,59737	0,0463507
38	6,68007	0,0341526
39	5,94676	0,0399344
40	6,66882	0,0396756
41	7,72705	0,046351
42	6,61439	0,0396756

43	7,52602	0,046351
44	7,60169	0,046351
45	7,93761	0,046351
46	6,24492	0,0396755
47	6,44771	0,0396756
48	7,17371	0,039676
49	6,63034	0,0399345
50	7,9586	0,0251113
51	6,11781	0,0396755
52	6,92338	0,039676
53	7,72638	0,0251111
54	7,90014	0,046351
55	7,64839	0,0251111
56	8,77252	0,034152
57	6,60759	0,0483053
58	6,979	0,0396756
59	7,03618	0,0463514
60	7,16232	0,0251111
61	9,39897	0,034152
62	8,4844	0,034152
63	6,74228	0,0513706
64	6,46058	0,0513706
65	7,26978	0,0251111
66	8,16421	0,0251111
67	7,18508	0,0341526
68	6,68809	0,0483053
69	7,19955	0,0341526
70	7,21917	0,0399343
71	6,69058	0,0483053
72	6,47892	0,0513706
73	7,57529	0,046351
74	7,83787	0,046351
75	7,89234	0,046351
76	7,79285	0,0341526
77	6,74638	0,0483053
78	7,78537	0,046351
79	10,5357	0,034152
80	7,32298	0,0483053
81	6,89227	0,0483053
82	6,97082	0,0483053
83	6,96469	0,067983
84	7,62335	0,046351
85	7,00895	0,0679826
86	7,0521	0,067983
87	6,90339	0,0590719

88	7,58287	0,0590714
89	6,58042	0,034152
90	7,3932	0,034152
91	7,27787	0,0483053
92	7,95042	0,0452472
93	6,35181	0,0356719
94	7,8206	0,0483053
95	7,43369	0,0483053
96	6,83421	0,0679831
97	6,74239	0,0679831
98	6,48635	0,0679831
99	7,35116	0,0590717
100	6,47792	0,034152
101	8,52218	0,0341524
102	6,93864	0,0590718
103	7,731	0,0656651
104	7,92138	0,0656651
105	8,54567	0,034152
106	7,47996	0,067983
107	6,69067	0,0679831
108	7,47416	0,0590717
109	7,73459	0,067983
110	7,30628	0,067983
111	7,23644	0,067983
112	7,25189	0,0590715
113	7,58833	0,0590717
114	7,12034	0,0679831
115	7,0181	0,0656649
116	5,72252	0,034152
117	7,76779	0,0656654
118	7,70573	0,067983
119	7,10199	0,0679828
120	7,86426	0,0656654
121	6,77217	0,034152
122	7,34776	0,0679828
123	7,57931	0,0656654
124	7,6257	0,0656654
125	7,00668	0,033291
126	5,4275	0,034152
127	7,96634	0,067983
128	7,55829	0,0656653
129	8,13845	0,067983
130	7,59058	0,0679829
131	7,3616	0,0679828
132	8,15188	0,0656649

133	6,72245	0,034152
134	7,01071	0,034152
135	8,9136	0,0341523
136	7,9839	0,0656654
137	7,88937	0,0679831
138	6,21539	0,033291
139	7,4522	0,033291
140	6,57844	0,034152
141	7,81611	0,0356721
142	9,19351	0,034152
143	6,91825	0,034152
144	7,70287	0,033291
145	7,66983	0,0295291
146	8,20192	0,065665
147	8,02661	0,065665
148	8,06926	0,0295291
149	7,25242	0,0704337
150	7,16794	0,0704341
151	7,84666	0,033291
152	5,5905	0,0332911
153	8,14255	0,0332912
154	6,16787	0,0332911
155	7,53961	0,0295291
156	6,74787	0,0704337
157	6,88362	0,0704338
158	8,28533	0,0656649
159	6,23108	0,033291
160	6,82878	0,071366
161	8,54474	0,035971
162	7,17654	0,033291
163	5,76079	0,0332911
164	5,76906	0,0332911
165	8,85001	0,0295291
166	8,68266	0,0295291
167	6,57906	0,0553088
168	6,15356	0,0704337
169	6,76061	0,0713659
170	8,4996	0,0295291
171	6,70587	0,041743
172	6,02107	0,0704337
173	5,76078	0,0704337
174	6,309	0,0704338
175	7,11632	0,0696906
176	5,83708	0,0704337
177	6,11438	0,0704337

178	6,08476	0,0704338
179	6,26708	0,0703154
180	7,44668	0,0656649
181	9,64518	0,0295291
182	6,33828	0,0696905
183	6,62155	0,0696906
184	7,99947	0,0472359
185	8,51326	0,033291
186	5,33452	0,0332911
187	7,83765	0,041743
188	5,28302	0,0704337
189	5,38611	0,0704337
190	7,99148	0,0472359
191	7,53636	0,0332911
192	5,05724	0,0704337
193	5,8872	0,0703157
194	5,68597	0,0704337
195	5,76323	0,0703157
196	6,81627	0,0696902
197	7,1856	0,041743
198	5,58648	0,0703154
199	6,13386	0,0703157
200	7,57554	0,047236
201	8,28838	0,0472359
202	8,6205	0,033291
203	5,60156	0,0703154
204	5,37629	0,0703154
205	6,69583	0,0656649
206	6,79354	0,0656649
207	8,28545	0,0332916
208	7,76415	0,0332911
209	5,84812	0,041743
210	5,03054	0,0703154
211	8,61857	0,0332911
212	6,40525	0,0696902
213	6,56612	0,0523636
214	8,44735	0,0332916
215	6,06517	0,041743
216	5,37644	0,041743
217	5,40618	0,0703157
218	5,85663	0,0703157
219	6,34175	0,0703154
220	6,65546	0,047236
221	8,25309	0,0332911
222	4,71049	0,0704337

223	4,81334	0,0703154
224	6,09544	0,0703157
225	6,21154	0,0703154
226	6,37244	0,0523636
227	7,71988	0,0472361
228	7,35308	0,0332916
229	5,83473	0,041743
230	4,97569	0,0703161
231	5,26198	0,0703154
232	7,38764	0,0472361
233	7,0674	0,0332916
234	5,09788	0,0703161
235	5,48078	0,0703154
236	5,36329	0,0703154
237	6,00362	0,0703157
238	7,78448	0,0332916
239	5,70984	0,0703154
240	6,7222	0,0472361
241	7,11789	0,041743
242	5,06381	0,0703161
243	4,8261	0,0703161
244	6,10726	0,0503296
245	5,36787	0,0703158
246	6,30601	0,0703157
247	5,12494	0,0703161
248	7,12284	0,0472361
249	7,35733	0,0472361
250	7,35143	0,0332916
251	4,76862	0,0703161
252	7,9624	0,0378401
253	6,27039	0,0332916
254	5,54063	0,0703158
255	6,30974	0,0703154
256	6,46131	0,0703154
257	7,63618	0,0378401
258	6,6187	0,0332916
259	5,28395	0,0703161
260	6,71789	0,0703154
261	6,95523	0,0503297
262	7,62381	0,0378395
263	6,89492	0,0332916
264	6,96964	0,0503297
265	5,96127	0,0332916
266	6,03721	0,0703161
267	5,79774	0,0703161

268	7,17533	0,0703154
269	7,63122	0,0503296
270	6,55896	0,0332916
271	6,34594	0,0703161
272	5,91224	0,0703161
273	7,4913	0,0703154
274	7,43828	0,0703154
275	8,0976	0,0378395
276	7,18699	0,0378401
277	5,80654	0,0332916
278	6,21032	0,0703161
279	7,61995	0,0503297
280	8,19629	0,0378395
281	6,93615	0,0378401
282	7,11184	0,0703161
283	6,60272	0,0703161
284	8,00226	0,0503296
285	6,60392	0,0332916
286	5,66756	0,0332916
287	8,329	0,0378401
288	7,62865	0,0378401
289	6,78025	0,0378401
290	7,10449	0,0703161
291	8,5015	0,0503297
292	6,41247	0,0332916
293	7,01481	0,0378401
294	8,65425	0,0703161
295	8,28494	0,0703158
296	7,38897	0,0332916
297	8,88916	0,0703161
298	10,3531	0,0703161
299	10,5581	0,0703161
300	11,4056	0,0703158

Appendix B: Robust Optimization

SR0-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837

SR0,5-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837

SR1-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837

SR3-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837

SR1-MR10	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837

SR1-MR100	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837

SR1-MR300	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,6691	7,5109	7,1924	7,7568	7,1062	7,1321
AELP	0,01136	0,081332	0,076095	0,125298	0,03964	0,056531

SR1-MR500	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,6819	7,5194	7,1964	7,7631	7,1179	7,1407
AELP	0,01132	0,081456	0,076074	0,125395	0,039549	0,056508

SR1-MR1000	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,6819	7,5194	7,1964	7,7631	7,1179	7,1407
AELP	0,01132	0,081456	0,076074	0,125395	0,039549	0,056508

SR1-MR10000	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AERT	6,7394	7,6413	7,4397	7,8877	7,1875	7,2747
AELP	0,011245	0,081917	0,07596	0,125711	0,03909	0,056385

Appendix C: Boxplots Robust Optimization

(Loss probability; Response time)	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
1	(0,00714177; 9,16214)	(0,0743538; 11,5077)	(0,0452104; 8,24623)	(0,114371; 11,6607)	(0,0273156; 9,72264)
2	(0,00714177; 8,04681)	(0,0827344; 10,1363)	(0,0452109; 10,1681)	(0,127116; 10,5118)	(0,0273156; 8,61962)
3	(0,00756544; 7,47349)	(0,0827344; 10,881)	(0,0452104; 9,01068)	(0,127116; 11,2194)	(0,0272105; 7,92517)
4	(0,00756544; 7,15119)	(0,0743538; 11,8267)	(0,0616038; 7,69117)	(0,114371; 11,9091)	(0,0272105; 7,64658)
5	(0,00714177; 6,78412)	(0,0827344; 11,3454)	(0,0616038; 8,30821)	(0,127116; 11,6596)	(0,0273156; 7,38195)
6	(0,00714177; 7,13802)	(0,0743527; 11,5181)	(0,0452109; 9,18735)	(0,114372; 11,6092)	(0,0273156; 7,71139)
7	(0,00714177; 6,22949)	(0,0827343; 11,4397)	(0,0452104; 7,70931)	(0,127115; 11,7269)	(0,0273156; 6,79689)
8	(0,00714177; 6,52163)	(0,0827344; 10,5136)	(0,0616039; 6,79658)	(0,127116; 10,814)	(0,0273156; 7,07544)
9	(0,00756544; 6,78741)	(0,0827343; 11,8531)	(0,0616039; 6,02648)	(0,127115; 12,1155)	(0,0272105; 7,20007)
10	(0,00714177; 4,65997)	(0,0743538; 8,59396)	(0,0616039; 5,70841)	(0,114371; 8,94711)	(0,0273156; 5,27991)
11	(0,00714177; 5,4085)	(0,0743527; 10,8365)	(0,024275; 8,67111)	(0,114372; 10,9576)	(0,0273156; 5,94454)
12	(0,00714153; 9,61167)	(0,0743539; 10,2414)	(0,0452104; 7,71352)	(0,114371; 10,5138)	(0,0273155; 9,918)
13	(0,00477099; 9,67397)	(0,0743539; 10,2932)	(0,0616039; 5,79313)	(0,114371; 10,5017)	(0,0184164; 10,512)
14	(0,00756544; 8,27637)	(0,0827344; 8,09699)	(0,0616039; 5,95227)	(0,127116; 8,44786)	(0,0272107; 8,82048)
15	(0,00714177; 4,40706)	(0,0516342; 10,4493)	(0,0616038; 8,91236)	(0,0831513; 10,9075)	(0,0273156; 5,03283)
16	(0,00477099; 7,96898)	(0,0743539; 10,0421)	(0,0361115; 9,59954)	(0,114371; 10,4059)	(0,0184164; 8,76915)
17	(0,00714177; 4,60861)	(0,0743538; 8,38484)	(0,024275; 8,03022)	(0,114371; 8,65422)	(0,0273156; 5,18593)
18	(0,00756544; 6,96647)	(0,0516342; 10,7905)	(0,0616038; 6,57728)	(0,0831513; 11,2715)	(0,0272105; 7,28502)
19	(0,00714177; 8,18317)	(0,0743527; 9,76123)	(0,0616039; 6,43567)	(0,114372; 9,92301)	(0,0273156; 8,50812)
20	(0,00823534; 8,98602)	(0,0743538; 7,98902)	(0,0616039; 6,302)	(0,114371; 8,29789)	(0,0294176; 9,7513)
21	(0,00756544; 11,4506)	(0,0827345; 5,9883)	(0,0616039; 6,7232)	(0,127116; 6,41527)	(0,0272107; 11,5957)
22	(0,00823534; 7,51936)	(0,0827344; 6,6936)	(0,0616038; 7,09385)	(0,127116; 7,08431)	(0,0294176; 8,31058)
23	(0,00756389; 7,66523)	(0,0827344; 9,38738)	(0,045211; 7,40628)	(0,127116; 9,65385)	(0,0272104; 8,09294)
24	(0,00823534;	(0,0827345;	(0,045211;	(0,127116;	(0,0294176;

	6,84289)	6,16595)	7,4145)	6,56117)	7,54071)
25	(0,00714177; 6,2571)	(0,0516342; 9,35791)	(0,0781188; 7,78883)	(0,0831513; 9,83218)	(0,0273156; 6,59515)
26	(0,00756395; 6,76119)	(0,0516342; 9,76201)	(0,0361115; 6,17084)	(0,0831513; 10,2604)	(0,0272104; 7,28776)
27	(0,0075655; 7,08933)	(0,0827344; 9,13724)	(0,0616037; 7,06291)	(0,127116; 9,3623)	(0,0272106; 7,53763)
28	(0,0109929; 7,6118)	(0,0623254; 10,3398)	(0,0616039; 6,92788)	(0,0986227; 10,7746)	(0,0383364; 7,71223)
29	(0,00823534; 5,84137)	(0,0743539; 8,69937)	(0,0616039; 6,93823)	(0,114371; 8,95686)	(0,0294176; 6,54589)
30	(0,00714177; 6,34172)	(0,0827344; 8,93263)	(0,0616038; 6,99416)	(0,127116; 9,15454)	(0,0273156; 6,6012)
31	(0,00823534; 6,26003)	(0,0623254; 10,1249)	(0,0452107; 7,44078)	(0,0986227; 10,5472)	(0,0294175; 6,82607)
32	(0,0075866; 8,38475)	(0,0827345; 7,27959)	(0,0452108; 6,29452)	(0,127116; 7,51045)	(0,0290923; 8,56909)
33	(0,00823534; 6,13452)	(0,0743527; 8,05931)	(0,0616037; 8,18111)	(0,114372; 8,31369)	(0,0294176; 6,91702)
34	(0,009009; 6,5313)	(0,0827344; 8,42076)	(0,0452108; 6,27052)	(0,127116; 8,6071)	(0,0314327; 7,26781)
35	(0,0109928; 5,99032)	(0,0975122; 7,44499)	(0,0452108; 6,1463)	(0,147536; 7,58566)	(0,0383368; 6,3479)
36	(0,00756395; 5,6517)	(0,0637114; 10,4155)	(0,0781188; 7,06248)	(0,10014; 10,8065)	(0,0272104; 6,31798)
37	(0,00714082; 7,50598)	(0,0743539; 7,83318)	(0,0781188; 6,96976)	(0,114371; 8,10445)	(0,0273142; 7,71984)
38	(0,009009; 6,52709)	(0,0975119; 7,90332)	(0,0616037; 6,72922)	(0,147536; 7,99242)	(0,0314327; 7,23358)
39	(0,00900954; 5,67812)	(0,0827344; 7,94878)	(0,0616037; 6,96489)	(0,127116; 8,11878)	(0,0322791; 6,41826)
40	(0,0109923; 6,01974)	(0,0827345; 7,05156)	(0,0616037; 8,03009)	(0,127116; 7,24357)	(0,0383377; 6,51708)
41	(0,00714076; 5,62341)	(0,0827345; 7,07989)	(0,0381255; 6,27036)	(0,127116; 7,27167)	(0,0273142; 6,10687)
42	(0,0109923; 5,95525)	(0,0827344; 7,37468)	(0,0781184; 6,75864)	(0,127116; 7,55509)	(0,0383377; 6,46292)
43	(0,0109928; 5,85225)	(0,0623254; 7,77423)	(0,0616037; 6,62434)	(0,0986227; 8,22738)	(0,0383368; 6,28307)
44	(0,00714076; 5,51066)	(0,0827342; 7,02767)	(0,0616037; 6,73757)	(0,127116; 7,22113)	(0,0273142; 6,03511)
45	(0,00714082; 5,63535)	(0,0827342; 7,07576)	(0,0361115; 6,98998)	(0,127116; 7,28255)	(0,0273142; 6,14896)
46	(0,00756389; 5,55178)	(0,0827344; 7,78591)	(0,0652491; 5,99628)	(0,127116; 7,92785)	(0,02721; 6,23961)
47	(0,0109923; 5,76356)	(0,0409811; 9,45826)	(0,0781188; 6,79695)	(0,0682436; 9,76988)	(0,0383377; 6,32985)
48	(0,0109923; 6,57989)	(0,0975119; 6,8752)	(0,0616037; 6,75653)	(0,147536; 7,06407)	(0,0383377; 6,85157)
49	(0,00823534; 6,42049)	(0,0827342; 8,17446)	(0,0361115; 6,12088)	(0,127116; 8,30653)	(0,0294176; 6,86696)
50	(0,00758672; 6,69755)	(0,0743527; 6,76281)	(0,065249; 5,87103)	(0,114372; 7,13354)	(0,0290921; 7,14993)

51	(0,0127446; 5,38598)	(0,0975108; 6,99992)	(0,0452107; 6,39721)	(0,147537; 7,25057)	(0,0433216; 6,02268)
52	(0,0109923; 6,47722)	(0,0975108; 7,08468)	(0,0616037; 6,57943)	(0,147537; 7,29824)	(0,0383376; 6,79229)
53	(0,00758672; 6,34853)	(0,0975119; 7,12798)	(0,0652491; 6,37948)	(0,147536; 7,2879)	(0,0290921; 6,85305)
54	(0,00758624; 5,3207)	(0,0827342; 8,00774)	(0,0781188; 6,86233)	(0,127116; 8,15292)	(0,0290912; 5,98931)
55	(0,00758672; 6,28637)	(0,0849603; 8,31667)	(0,0616037; 6,71806)	(0,130898; 8,4326)	(0,0290921; 6,76081)
56	(0,00900871; 9,58315)	(0,0654108; 6,99998)	(0,0781186; 6,94011)	(0,103541; 7,40984)	(0,0314326; 9,97751)
57	(0,0127447; 6,14731)	(0,0743524; 6,84983)	(0,0781188; 6,52766)	(0,114372; 7,212)	(0,0433218; 6,66369)
58	(0,0109923; 6,2672)	(0,0975108; 6,84322)	(0,0616037; 6,4166)	(0,147537; 7,1225)	(0,0383376; 6,70201)
59	(0,0109929; 5,00794)	(0,0827342; 6,5628)	(0,0352582; 10,6396)	(0,127116; 6,84595)	(0,0383363; 5,69914)
60	(0,00317591; 5,60702)	(0,0827342; 6,67142)	(0,0744339; 7,29596)	(0,127116; 6,94246)	(0,014226; 6,35785)
61	(0,00900871; 11,0723)	(0,0743525; 6,74564)	(0,0744331; 6,70687)	(0,114372; 7,13314)	(0,0314326; 11,3751)
62	(0,00900871; 9,90296)	(0,0975119; 6,58727)	(0,0342019; 7,56478)	(0,147536; 6,82141)	(0,0314326; 10,2315)
63	(0,0109923; 6,64218)	(0,0827342; 6,78444)	(0,0342019; 7,24779)	(0,127116; 7,04837)	(0,0383376; 6,95655)
64	(0,0109929; 5,87927)	(0,0874667; 6,23957)	(0,0352582; 10,1462)	(0,134192; 6,60221)	(0,0383363; 6,35204)
65	(0,00317591; 5,85692)	(0,0849603; 7,62967)	(0,0781188; 6,80753)	(0,130898; 7,79572)	(0,014226; 6,64031)
66	(0,00581801; 7,10628)	(0,0849603; 7,81092)	(0,0342019; 7,49528)	(0,130898; 7,95996)	(0,0232096; 7,66408)
67	(0,00900954; 7,00236)	(0,0637114; 8,10967)	(0,0445657; 8,25144)	(0,10014; 8,43348)	(0,0322791; 7,3371)
68	(0,0127447; 6,20802)	(0,0623242; 7,43853)	(0,0381255; 7,26983)	(0,0986235; 7,7192)	(0,0433218; 6,74372)
69	(0,009009; 7,06908)	(0,0849608; 7,58643)	(0,0381255; 7,37419)	(0,130898; 7,87959)	(0,0314327; 7,41217)
70	(0,00900805; 6,87049)	(0,0975108; 7,03593)	(0,0744339; 7,02623)	(0,147537; 7,22958)	(0,0322787; 7,32505)
71	(0,0127447; 6,30638)	(0,0975108; 6,80372)	(0,0381255; 7,33287)	(0,147537; 7,08761)	(0,0433218; 6,84553)
72	(0,0109929; 6,38508)	(0,0849603; 7,28)	(0,0867574; 6,66315)	(0,130898; 7,48128)	(0,0383363; 6,76652)
73	(0,0112704; 5,41009)	(0,0849608; 7,33408)	(0,074433; 7,01733)	(0,130898; 7,62612)	(0,041421; 6,04276)
74	(0,00758612; 5,85856)	(0,0637113; 7,29656)	(0,104164; 7,18167)	(0,10014; 7,58045)	(0,0290909; 6,39751)
75	(0,00758612; 5,96063)	(0,0975119; 6,26584)	(0,0352582; 7,604)	(0,147536; 6,59204)	(0,0290909; 6,46932)
76	(0,00900954; 7,664)	(0,0409809; 7,3057)	(0,0523178; 6,71768)	(0,0682436; 7,80535)	(0,0322791; 7,88938)
77	(0,0127447; 6,35866)	(0,0637114; 10,5016)	(0,0445657; 7,47291)	(0,10014; 10,7183)	(0,043322; 6,88095)

78	(0,00758612; 5,85556)	(0,0654096; 7,81986)	(0,0652491; 7,18593)	(0,103542; 8,01094)	(0,0290909; 6,40838)
79	(0,00900823; 12,0134)	(0,0975119; 6,72301)	(0,104164; 7,08101)	(0,147536; 6,97365)	(0,0314312; 12,3875)
80	(0,0127447; 6,96874)	(0,0849604; 6,54341)	(0,0867574; 6,95979)	(0,130898; 6,88186)	(0,0433218; 7,39679)
81	(0,0127447; 6,47586)	(0,040981; 7,55347)	(0,0744338; 7,01288)	(0,0682436; 8,03025)	(0,043322; 6,96833)
82	(0,0127446; 6,51184)	(0,0445555; 8,33296)	(0,104164; 7,36736)	(0,0737178; 8,68803)	(0,0433216; 6,96018)
83	(0,0167522; 6,64872)	(0,0654108; 7,61988)	(0,0867574; 7,11352)	(0,103541; 7,82257)	(0,0557914; 6,96881)
84	(0,0112704; 5,72482)	(0,0874666; 7,2322)	(0,0352582; 8,00403)	(0,134192; 7,50153)	(0,041421; 6,28596)
85	(0,0167523; 6,64804)	(0,0849608; 7,40776)	(0,0352582; 7,96446)	(0,130898; 7,60213)	(0,0557918; 7,00725)
86	(0,0167522; 6,73674)	(0,0654096; 7,70735)	(0,104164; 7,24031)	(0,103542; 7,98042)	(0,0557914; 7,0509)
87	(0,0112703; 5,80624)	(0,0975119; 7,05057)	(0,0445659; 6,45762)	(0,147536; 7,27161)	(0,0414209; 6,33873)
88	(0,0112704; 5,96202)	(0,0849604; 6,87487)	(0,0867574; 7,26908)	(0,130898; 7,15389)	(0,041421; 6,47008)
89	(0,00900823; 8,26664)	(0,0445551; 6,9259)	(0,0867573; 7,41845)	(0,0737179; 7,44355)	(0,0314312; 8,53201)
90	(0,00900871; 7,80081)	(0,109942; 7,16334)	(0,0816222; 6,51922)	(0,16529; 7,37299)	(0,0314326; 8,01952)
91	(0,0167523; 6,86827)	(0,0952908; 6,62207)	(0,104164; 7,53948)	(0,146912; 6,91982)	(0,0557918; 7,24145)
92	(0,00758612; 6,46063)	(0,0637113; 7,95389)	(0,104164; 7,22669)	(0,10014; 8,12521)	(0,0290909; 6,89369)
93	(0,00581801; 4,73996)	(0,109942; 7,30735)	(0,0867574; 7,2776)	(0,16529; 7,49219)	(0,0232096; 5,51752)
94	(0,012073; 7,52806)	(0,0849609; 7,13231)	(0,0980112; 7,46227)	(0,130898; 7,41213)	(0,0421982; 7,90921)
95	(0,0167523; 7,06979)	(0,0952908; 6,84487)	(0,0352578; 8,53228)	(0,146912; 7,11444)	(0,0557918; 7,44225)
96	(0,0167523; 7,32546)	(0,0952908; 6,9379)	(0,0352578; 8,3342)	(0,146912; 7,19439)	(0,0557918; 7,46258)
97	(0,0167529; 7,1444)	(0,0637103; 9,88435)	(0,0381254; 8,56111)	(0,10014; 10,1187)	(0,0557913; 7,32548)
98	(0,0167528; 6,49987)	(0,0637114; 8,19927)	(0,0867574; 7,36339)	(0,10014; 8,34119)	(0,0557904; 6,82195)
99	(0,0112704; 6,19909)	(0,109942; 7,52626)	(0,0980113; 7,47504)	(0,16529; 7,73296)	(0,041421; 6,66884)
100	(0,00900823; 8,21321)	(0,109942; 7,43559)	(0,0816221; 7,13605)	(0,16529; 7,61889)	(0,0314312; 8,60612)
101	(0,00900805; 8,32795)	(0,0445551; 6,55245)	(0,0352582; 8,90342)	(0,0737178; 7,06861)	(0,0322788; 8,56248)
102	(0,0112703; 6,39757)	(0,0637114; 8,31032)	(0,0980113; 7,53703)	(0,10014; 8,43884)	(0,0414209; 6,81226)
103	(0,00998676; 6,6382)	(0,0654096; 8,6012)	(0,104164; 7,76932)	(0,103542; 8,71604)	(0,0389473; 7,02179)
104	(0,00998652; 6,89069)	(0,0841478; 8,30524)	(0,104164; 7,50308)	(0,130057; 8,48971)	(0,038947; 7,18501)

105	(0,00900871; 8,69453)	(0,0952908; 6,99017)	(0,104164; 7,26277)	(0,146912; 7,24699)	(0,0314326; 8,77908)
106	(0,0167523; 7,40079)	(0,109942; 7,57083)	(0,0867574; 7,5331)	(0,16529; 7,68925)	(0,0557918; 7,57949)
107	(0,0167529; 6,66786)	(0,109942; 7,48466)	(0,0867574; 7,55227)	(0,16529; 7,60322)	(0,0557913; 6,94776)
108	(0,0112704; 6,47878)	(0,109943; 7,25535)	(0,098012; 7,41875)	(0,16529; 7,41498)	(0,041421; 6,89166)
109	(0,0167523; 7,53223)	(0,109943; 7,13056)	(0,0445659; 6,71355)	(0,16529; 7,3308)	(0,0557918; 7,71782)
110	(0,0167531; 7,13731)	(0,0637103; 8,90394)	(0,104164; 7,17429)	(0,10014; 9,15433)	(0,0557915; 7,36586)
111	(0,0167531; 7,06266)	(0,0654096; 8,75505)	(0,104164; 7,2714)	(0,103542; 8,89093)	(0,0557915; 7,305)
112	(0,0112703; 6,6244)	(0,092128; 7,39144)	(0,0816221; 7,48225)	(0,142887; 7,60395)	(0,0414209; 7,01116)
113	(0,0112704; 6,63748)	(0,0798143; 7,17532)	(0,0737818; 6,82798)	(0,125687; 7,43286)	(0,041421; 7,01948)
114	(0,0167529; 6,77108)	(0,109942; 8,04392)	(0,104164; 7,82883)	(0,16529; 8,18537)	(0,0557913; 7,08671)
115	(0,0101351; 6,02738)	(0,109942; 7,48504)	(0,104164; 7,46206)	(0,165289; 7,62003)	(0,0378993; 6,47603)
116	(0,00900823; 7,17144)	(0,0952908; 7,34174)	(0,0980113; 7,71919)	(0,146912; 7,56589)	(0,0314312; 7,6251)
117	(0,00998676; 6,99171)	(0,0952908; 7,36338)	(0,0980113; 8,07296)	(0,146912; 7,58125)	(0,0389473; 7,30117)
118	(0,0167531; 7,10031)	(0,0841478; 9,05163)	(0,0473213; 7,04017)	(0,130057; 9,17152)	(0,0557916; 7,39235)
119	(0,0167529; 6,27177)	(0,109942; 7,25156)	(0,0980113; 7,65965)	(0,16529; 7,46692)	(0,0557913; 6,70941)
120	(0,00998676; 7,17869)	(0,0637102; 9,30028)	(0,0980113; 7,74248)	(0,100141; 9,34924)	(0,0389473; 7,44303)
121	(0,00900823; 6,92072)	(0,109942; 7,47483)	(0,104164; 7,83954)	(0,165289; 7,62516)	(0,0314312; 7,30138)
122	(0,0167529; 6,38356)	(0,109942; 7,11594)	(0,104164; 7,35533)	(0,16529; 7,35553)	(0,0557913; 6,84583)
123	(0,00998688; 6,78108)	(0,109942; 7,22632)	(0,0980113; 8,12232)	(0,16529; 7,47945)	(0,0389482; 7,19869)
124	(0,00998688; 6,87573)	(0,092127; 7,56856)	(0,104164; 7,03109)	(0,142886; 7,78686)	(0,0389482; 7,25675)
125	(0,00852633; 6,18059)	(0,0494679; 7,09386)	(0,0980113; 7,89291)	(0,0798029; 7,35817)	(0,0301389; 6,58728)
126	(0,00900823; 6,95272)	(0,0798143; 7,60756)	(0,073782; 7,2967)	(0,125687; 7,7898)	(0,0314312; 7,50316)
127	(0,0167531; 7,07407)	(0,109942; 7,76445)	(0,0352579; 8,58013)	(0,165289; 7,91254)	(0,0557916; 7,39393)
128	(0,00998688; 6,54227)	(0,109942; 7,15876)	(0,12031; 7,1179)	(0,165289; 7,37119)	(0,0389482; 7,06231)
129	(0,016753; 7,26744)	(0,109942; 6,81495)	(0,12031; 7,40145)	(0,16529; 7,07599)	(0,0557913; 7,56551)
130	(0,016753; 6,37676)	(0,109942; 6,81572)	(0,104164; 6,94708)	(0,16529; 7,1055)	(0,0557914; 6,8524)
131	(0,0167529; 6,03669)	(0,092127; 7,64623)	(0,104164; 6,91787)	(0,142886; 7,84865)	(0,0557914; 6,58128)

132	(0,00998718; 7,82829)	(0,092127; 7,77519)	(0,0980113; 7,98241)	(0,142886; 7,94016)	(0,0389484; 7,9494)
133	(0,00900823; 6,43136)	(0,0637103; 7,34837)	(0,073782; 7,97347)	(0,10014; 7,61045)	(0,0314312; 6,90625)
134	(0,00900823; 6,61121)	(0,0637103; 8,34905)	(0,0352578; 7,47811)	(0,10014; 8,50112)	(0,0314312; 7,05037)
135	(0,0100089; 9,49168)	(0,109942; 7,16239)	(0,0980113; 7,84248)	(0,165289; 7,37796)	(0,0370764; 9,51728)
136	(0,00998688; 7,23247)	(0,0921271; 7,43604)	(0,047321; 6,07116)	(0,142886; 7,70099)	(0,0389482; 7,54105)
137	(0,016753; 6,526)	(0,0921271; 7,78624)	(0,0352578; 6,18019)	(0,142886; 7,95628)	(0,0557913; 6,95714)
138	(0,00852585; 5,61525)	(0,092127; 8,19838)	(0,104164; 6,71821)	(0,142886; 8,27588)	(0,0301383; 6,07773)
139	(0,00852627; 6,72818)	(0,0763652; 7,24976)	(0,12031; 7,37483)	(0,119827; 7,47091)	(0,030139; 7,02769)
140	(0,00900829; 5,5956)	(0,075232; 8,93455)	(0,098012; 8,50146)	(0,118926; 9,03406)	(0,0314312; 6,2355)
141	(0,0095607; 6,88149)	(0,0494677; 7,28831)	(0,0352578; 9,80271)	(0,079803; 7,52055)	(0,0356563; 7,16497)
142	(0,00900829; 9,21065)	(0,0494667; 7,03812)	(0,0352578; 7,36152)	(0,0798039; 7,38033)	(0,0314312; 9,87603)
143	(0,00900829; 6,72576)	(0,0637103; 9,96396)	(0,0352578; 6,37601)	(0,10014; 10,3518)	(0,0314312; 7,4297)
144	(0,00852627; 6,98296)	(0,0637103; 8,38979)	(0,104164; 7,64379)	(0,10014; 8,78671)	(0,030139; 7,22371)
145	(0,00900841; 6,28906)	(0,0637103; 6,79101)	(0,12031; 6,81674)	(0,10014; 7,15434)	(0,0314312; 6,8083)
146	(0,0112661; 7,85218)	(0,105667; 7,32021)	(0,0473216; 7,53431)	(0,159908; 7,5748)	(0,041435; 7,94653)
147	(0,0112661; 7,44122)	(0,0637103; 6,67412)	(0,12031; 6,41999)	(0,10014; 7,05199)	(0,041435; 7,59122)
148	(0,00900841; 6,67342)	(0,0833326; 8,25501)	(0,12031; 7,17657)	(0,129894; 8,32)	(0,0314312; 7,13287)
149	(0,010009; 7,53584)	(0,0494679; 7,56224)	(0,079692; 8,02659)	(0,0798029; 7,68151)	(0,0370765; 7,90104)
150	(0,0141706; 4,70244)	(0,0494667; 6,42692)	(0,0352578; 8,50744)	(0,0798039; 6,63641)	(0,050571; 5,52306)
151	(0,00852585; 7,59231)	(0,0494667; 6,22313)	(0,035258; 7,13187)	(0,0798039; 6,45943)	(0,0301383; 7,67196)
152	(0,00852585; 4,87596)	(0,0637103; 10,3425)	(0,104164; 7,30579)	(0,10014; 10,7414)	(0,0301383; 5,51626)
153	(0,00852627; 7,37844)	(0,0637103; 7,91706)	(0,12031; 6,29818)	(0,10014; 8,3355)	(0,030139; 7,53531)
154	(0,00852585; 5,52799)	(0,0637103; 6,97629)	(0,12031; 7,42988)	(0,10014; 7,29765)	(0,0301383; 6,18484)
155	(0,00900841; 5,10372)	(0,0833327; 8,00647)	(0,097436; 7,76309)	(0,129895; 8,08081)	(0,0314312; 5,83435)
156	(0,0136537; 6,12292)	(0,0833327; 7,91596)	(0,097436; 8,2241)	(0,129895; 7,99939)	(0,0479504; 6,61802)
157	(0,0138779; 5,4248)	(0,0763652; 7,56152)	(0,047321; 5,71877)	(0,119827; 7,70774)	(0,0483351; 6,07045)
158	(0,0112653; 7,54431)	(0,105667; 6,75805)	(0,047321; 5,88468)	(0,159908; 7,06421)	(0,0414354; 7,65908)

159	(0,00852585; 5,76305)	(0,0833327; 8,04913)	(0,12031; 6,7889)	(0,129895; 8,10568)	(0,0301383; 6,12528)
160	(0,0141706; 5,28492)	(0,0494679; 7,73115)	(0,12031; 7,2309)	(0,0798029; 7,83527)	(0,050571; 5,97794)
161	(0,00852555; 7,17448)	(0,0637103; 7,65221)	(0,0796909; 8,23172)	(0,10014; 8,09078)	(0,0301394; 7,39329)
162	(0,00852573; 6,82242)	(0,0637103; 6,39399)	(0,0796914; 8,07232)	(0,10014; 6,80383)	(0,0301383; 7,01966)
163	(0,00852573; 5,22371)	(0,0637102; 7,44775)	(0,0473216; 7,73323)	(0,10014; 7,7225)	(0,0301383; 5,67329)
164	(0,00852585; 5,20261)	(0,105667; 6,05563)	(0,0473216; 7,66612)	(0,159908; 6,44261)	(0,0301383; 5,87595)
165	(0,00900829; 7,68209)	(0,0494667; 5,69859)	(0,0473216; 7,67174)	(0,0798039; 6,05693)	(0,0314312; 8,44645)
166	(0,00860441; 7,05609)	(0,109942; 6,73773)	(0,035258; 6,9376)	(0,165289; 7,03509)	(0,0315191; 7,4802)
167	(0,0100091; 7,16656)	(0,0833312; 8,01912)	(0,0538451; 8,42297)	(0,129894; 8,05528)	(0,0370766; 7,63027)
168	(0,0136548; 6,52394)	(0,0494677; 7,70514)	(0,0538448; 8,56244)	(0,079803; 7,82987)	(0,0479508; 7,01816)
169	(0,0168637; 5,48819)	(0,105667; 6,54078)	(0,0828125; 7,03658)	(0,159908; 6,85962)	(0,0559117; 6,12003)
170	(0,00900829; 6,86663)	(0,0494667; 5,02903)	(0,107828; 6,4426)	(0,0798039; 5,38806)	(0,0314312; 7,65946)
171	(0,0099948; 7,2961)	(0,0752319; 8,12767)	(0,12031; 6,38572)	(0,118926; 8,34606)	(0,0359094; 7,73599)
172	(0,0136548; 6,15217)	(0,0752319; 8,06374)	(0,12031; 7,43246)	(0,118926; 8,2959)	(0,0479508; 6,6472)
173	(0,0136548; 6,20356)	(0,0918148; 6,99397)	(0,047321; 7,6969)	(0,142368; 7,26697)	(0,0479508; 6,74172)
174	(0,0168637; 5,23959)	(0,0958831; 6,52253)	(0,0828125; 7,20073)	(0,146749; 6,8558)	(0,0559117; 5,83528)
175	(0,0165366; 6,81452)	(0,0935329; 7,50158)	(0,107828; 6,68877)	(0,143101; 7,73362)	(0,0543896; 7,13554)
176	(0,0099948; 6,66974)	(0,0494667; 6,22405)	(0,12031; 7,06951)	(0,0798039; 6,42737)	(0,0359094; 7,21612)
177	(0,0168647; 5,80488)	(0,0637103; 9,77469)	(0,047321; 6,23524)	(0,10014; 10,2128)	(0,055912; 6,21322)
178	(0,0168636; 5,30441)	(0,104456; 6,16694)	(0,111048; 6,36549)	(0,156532; 6,52242)	(0,0559117; 5,82335)
179	(0,0168637; 5,50461)	(0,105667; 6,42195)	(0,111048; 6,39841)	(0,159908; 6,7447)	(0,0559117; 6,0419)
180	(0,0112662; 6,8163)	(0,0833315; 7,48799)	(0,12031; 6,62123)	(0,129894; 7,57442)	(0,0414356; 7,10083)
181	(0,00900829; 8,14689)	(0,0752319; 7,95675)	(0,12031; 6,73441)	(0,118926; 8,20215)	(0,0314312; 8,92866)
182	(0,0168638; 6,11894)	(0,0918151; 7,38017)	(0,12031; 7,47132)	(0,142369; 7,64478)	(0,0559119; 6,54885)
183	(0,0165366; 6,53326)	(0,069586; 8,24531)	(0,047321; 7,46079)	(0,110113; 8,45364)	(0,0543896; 6,89458)
184	(0,0112653; 6,52439)	(0,104456; 6,79171)	(0,047321; 4,97968)	(0,156532; 7,06499)	(0,0414354; 6,78979)
185	(0,00852537; 7,19043)	(0,104456; 6,56169)	(0,0538448; 8,08026)	(0,156532; 6,81055)	(0,0301395; 7,40017)

186	(0,00852573; 4,8719)	(0,0494666; 7,16307)	(0,0606729; 7,82205)	(0,0798026; 7,3779)	(0,0301383; 5,49389)
187	(0,00999403; 8,0939)	(0,104456; 6,195)	(0,082812; 7,53218)	(0,156532; 6,46705)	(0,0359081; 8,3049)
188	(0,0168647; 5,29444)	(0,104456; 6,41867)	(0,111048; 6,57151)	(0,156532; 6,68408)	(0,055912; 5,8534)
189	(0,0168647; 5,27979)	(0,0950434; 7,38196)	(0,12031; 7,26923)	(0,144357; 7,55894)	(0,055912; 5,81842)
190	(0,0112653; 6,08268)	(0,0833311; 7,23881)	(0,047321; 6,92765)	(0,129895; 7,38041)	(0,0414354; 6,42867)
191	(0,00852573; 7,23506)	(0,0494666; 6,05651)	(0,111048; 6,76887)	(0,0798038; 6,28256)	(0,0301383; 7,39079)
192	(0,0168647; 5,17188)	(0,0637103; 8,47461)	(0,111047; 6,62648)	(0,10014; 8,94873)	(0,055912; 5,76286)
193	(0,0168638; 6,36665)	(0,0918151; 6,55074)	(0,111048; 6,35887)	(0,142369; 6,84687)	(0,0559119; 6,68396)
194	(0,0168647; 5,20766)	(0,0494666; 5,63547)	(0,111048; 6,59999)	(0,0798038; 5,97426)	(0,055912; 5,65903)
195	(0,0168638; 6,46292)	(0,0833311; 6,84463)	(0,111048; 6,55844)	(0,129895; 6,99415)	(0,0559119; 6,74254)
196	(0,0144917; 6,40022)	(0,0494666; 7,31622)	(0,0473208; 7,52461)	(0,0798025; 7,45576)	(0,0494959; 6,71361)
197	(0,00999475; 7,71535)	(0,0637103; 9,08587)	(0,047321; 5,65792)	(0,10014; 9,55724)	(0,035909; 8,03573)
198	(0,0168647; 5,70612)	(0,104456; 6,24016)	(0,0606732; 8,15551)	(0,156532; 6,5432)	(0,055912; 6,00057)
199	(0,016864; 6,78661)	(0,104456; 6,15237)	(0,111048; 6,4289)	(0,156532; 6,42481)	(0,0559126; 6,99182)
200	(0,0112653; 5,78029)	(0,0833311; 6,77818)	(0,111047; 6,31328)	(0,129895; 6,9607)	(0,0414354; 6,16705)
201	(0,00852561; 5,48617)	(0,104456; 6,92127)	(0,111047; 6,31322)	(0,156532; 7,08543)	(0,0301394; 6,01285)
202	(0,00852561; 5,80684)	(0,0833311; 6,58109)	(0,111048; 6,62691)	(0,129895; 6,75263)	(0,0301394; 6,30401)
203	(0,0168647; 5,54186)	(0,0494667; 7,32366)	(0,111048; 7,00529)	(0,0798025; 7,44914)	(0,055912; 5,86787)
204	(0,0168647; 5,95659)	(0,069586; 7,4667)	(0,047321; 6,44479)	(0,110114; 7,75374)	(0,055912; 6,18742)
205	(0,0144917; 5,60383)	(0,104456; 5,97456)	(0,111047; 6,23115)	(0,156532; 6,22336)	(0,0494959; 6,03542)
206	(0,0112662; 5,48013)	(0,0833315; 6,21493)	(0,111048; 7,01771)	(0,129894; 6,43227)	(0,0414356; 5,96032)
207	(0,00852543; 6,89873)	(0,0494666; 6,30047)	(0,111047; 6,04046)	(0,0798038; 6,60176)	(0,0301394; 7,18902)
208	(0,00852543; 7,32622)	(0,0695862; 8,26955)	(0,0606732; 8,19289)	(0,110113; 8,45031)	(0,0301394; 7,51164)
209	(0,00999469; 6,78535)	(0,0695862; 8,04952)	(0,111047; 6,2779)	(0,110113; 8,26119)	(0,0359092; 7,32875)
210	(0,0168637; 6,59484)	(0,069586; 7,71564)	(0,111048; 7,20508)	(0,110113; 7,97451)	(0,0559117; 6,79367)
211	(0,00852555; 5,9187)	(0,104456; 6,21591)	(0,0473209; 6,34992)	(0,156532; 6,38829)	(0,0301393; 6,40658)
212	(0,0144917; 5,36499)	(0,104456; 7,17991)	(0,111047; 6,15141)	(0,156533; 7,30769)	(0,0494959; 5,82299)

213	(0,0144917; 5,19148)	(0,0494666; 6,44666)	(0,111048; 6,78016)	(0,0798026; 6,73256)	(0,0494959; 5,67324)
214	(0,00852555; 6,31821)	(0,104456; 6,09806)	(0,086114; 6,27729)	(0,156532; 6,28201)	(0,0301393; 6,7522)
215	(0,00999492; 6,93446)	(0,0889438; 6,34112)	(0,0473208; 7,17646)	(0,13634; 6,56008)	(0,0359092; 7,45908)
216	(0,00999486; 6,11109)	(0,0494667; 7,2119)	(0,0606732; 7,86728)	(0,0798025; 7,36998)	(0,0359092; 6,72092)
217	(0,0168636; 6,97985)	(0,069586; 7,68966)	(0,0606729; 7,73869)	(0,110114; 7,95247)	(0,0559118; 7,07921)
218	(0,016864; 6,64479)	(0,069586; 6,10713)	(0,111048; 7,09127)	(0,110114; 6,52214)	(0,0559126; 6,84196)
219	(0,0144917; 6,03595)	(0,104456; 6,33957)	(0,0861142; 6,70767)	(0,156532; 6,47301)	(0,0494959; 6,39627)
220	(0,0144917; 4,94646)	(0,0889438; 7,05179)	(0,086114; 6,41485)	(0,13634; 7,20769)	(0,0494959; 5,45949)
221	(0,00852555; 5,12038)	(0,0889439; 6,65606)	(0,111047; 6,17881)	(0,13634; 6,84242)	(0,0301393; 5,74642)
222	(0,00999486; 5,28704)	(0,0833315; 6,10797)	(0,0861141; 6,92971)	(0,129894; 6,32537)	(0,0359092; 5,98158)
223	(0,0168649; 4,36306)	(0,104456; 6,65063)	(0,086114; 5,9217)	(0,156532; 6,73445)	(0,055912; 5,02086)
224	(0,0144917; 6,36316)	(0,0889438; 6,00152)	(0,0473209; 5,85022)	(0,13634; 6,2426)	(0,0494959; 6,65209)
225	(0,0144917; 5,51932)	(0,0889438; 5,90186)	(0,0473208; 7,14344)	(0,13634; 6,15516)	(0,0494959; 5,95597)
226	(0,0144917; 4,79101)	(0,0695861; 8,24791)	(0,086114; 6,37591)	(0,110114; 8,44546)	(0,0494959; 5,32514)
227	(0,00852561; 4,79011)	(0,0889438; 6,94818)	(0,086114; 6,23327)	(0,13634; 7,10326)	(0,0301394; 5,46235)
228	(0,00852543; 7,55167)	(0,104456; 6,93061)	(0,047321; 7,4271)	(0,156533; 7,05754)	(0,0301394; 7,75402)
229	(0,00999486; 6,50887)	(0,0889438; 6,29523)	(0,111047; 5,60813)	(0,13634; 6,50365)	(0,0359092; 7,07342)
230	(0,0168649; 4,75045)	(0,0889438; 5,66451)	(0,111048; 6,93542)	(0,13634; 5,93277)	(0,055912; 5,25751)
231	(0,0168649; 6,15254)	(0,0889438; 5,77092)	(0,111047; 5,67662)	(0,13634; 6,03164)	(0,055912; 6,32175)
232	(0,0112653; 4,9888)	(0,104456; 5,58302)	(0,047321; 7,76058)	(0,156532; 5,84487)	(0,0414354; 5,55068)
233	(0,00852579; 7,40243)	(0,104456; 5,83993)	(0,0606731; 8,35732)	(0,156532; 6,15962)	(0,0301383; 7,7191)
234	(0,0168649; 5,17834)	(0,104456; 7,06184)	(0,111047; 6,10722)	(0,156533; 7,13007)	(0,055912; 5,54876)
235	(0,0168649; 5,69631)	(0,0889438; 5,51479)	(0,111049; 6,60738)	(0,13634; 5,79004)	(0,055912; 5,93603)
236	(0,0168647; 6,43394)	(0,0889438; 5,68495)	(0,111049; 6,73381)	(0,13634; 5,94985)	(0,0559125; 6,53295)
237	(0,0168641; 6,5071)	(0,0494666; 7,75355)	(0,0861139; 6,05727)	(0,0798038; 7,92713)	(0,0559125; 6,71859)
238	(0,00852555; 6,17858)	(0,0494666; 7,76824)	(0,0473209; 6,36997)	(0,0798038; 8,00755)	(0,0301393; 6,7013)
239	(0,0168646; 6,45606)	(0,069586; 7,6514)	(0,047321; 8,1309)	(0,110113; 7,92567)	(0,0559125; 6,59667)

240	(0,0144919; 4,82339)	(0,104456; 6,0887)	(0,111047; 6,10567)	(0,156532; 6,22416)	(0,0494959; 5,35384)
241	(0,00999492; 7,65134)	(0,104456; 6,59529)	(0,0861139; 6,02144)	(0,156532; 6,65822)	(0,0359092; 8,0824)
242	(0,00999486; 5,73087)	(0,0494665; 6,76714)	(0,086114; 5,58769)	(0,0798026; 7,05353)	(0,0359092; 6,34415)
243	(0,0168649; 5,29439)	(0,0695861; 8,41963)	(0,0473209; 6,7168)	(0,110114; 8,6249)	(0,055912; 5,63768)
244	(0,0144919; 4,68985)	(0,104456; 5,93154)	(0,0861139; 5,48478)	(0,156532; 6,09367)	(0,0494959; 5,23946)
245	(0,0168647; 5,68301)	(0,0494665; 6,80516)	(0,0606731; 8,80907)	(0,0798026; 7,09847)	(0,0559125; 5,90185)
246	(0,0168641; 6,53936)	(0,0494667; 7,57127)	(0,0606729; 7,98518)	(0,0798025; 7,77027)	(0,0559125; 6,70427)
247	(0,00999486; 5,97887)	(0,069586; 6,69054)	(0,0606729; 6,79292)	(0,110113; 7,05075)	(0,0359092; 6,55595)
248	(0,0144919; 5,26129)	(0,104456; 5,76046)	(0,111047; 5,87452)	(0,156532; 6,0041)	(0,0494959; 5,74472)
249	(0,0144909; 5,18301)	(0,088944; 6,74187)	(0,086114; 6,51477)	(0,13634; 6,89911)	(0,0494958; 5,747)
250	(0,00852555; 6,61636)	(0,088944; 6,5812)	(0,0861139; 5,53968)	(0,13634; 6,7566)	(0,0301393; 7,14057)
251	(0,0168649; 5,56397)	(0,0889438; 5,92697)	(0,111047; 6,81094)	(0,13634; 6,16208)	(0,055912; 5,86664)
252	(0,00852555; 5,75548)	(0,0889438; 5,61367)	(0,0861139; 5,58956)	(0,13634; 5,8855)	(0,0301393; 6,41493)
253	(0,00852543; 8,64809)	(0,104456; 5,85254)	(0,0861139; 5,1539)	(0,156532; 6,04623)	(0,0301394; 8,87723)
254	(0,0168647; 5,24793)	(0,104456; 6,65767)	(0,0473209; 7,52906)	(0,156532; 6,74158)	(0,0559125; 5,56835)
255	(0,0168646; 5,5197)	(0,104456; 6,90878)	(0,0606729; 7,08679)	(0,156533; 6,98583)	(0,0559125; 5,8191)
256	(0,0168646; 5,38319)	(0,0889438; 5,63504)	(0,0861145; 5,79114)	(0,13634; 5,89703)	(0,0559125; 5,71203)
257	(0,00852555; 6,33094)	(0,104456; 6,84215)	(0,111049; 6,27832)	(0,156533; 6,93897)	(0,0301393; 6,94536)
258	(0,00852579; 9,09963)	(0,0889438; 5,78227)	(0,0473209; 6,20879)	(0,13634; 6,04511)	(0,0301383; 9,37827)
259	(0,0168647; 5,36694)	(0,0494665; 5,98825)	(0,0473209; 6,98958)	(0,0798026; 6,43907)	(0,0559125; 5,7197)
260	(0,0144917; 5,84259)	(0,069586; 6,96205)	(0,111049; 6,1538)	(0,110113; 7,30507)	(0,0494958; 6,22779)
261	(0,0144919; 5,24022)	(0,104456; 6,04935)	(0,111049; 5,99993)	(0,156532; 6,27487)	(0,0494959; 5,74604)
262	(0,0144909; 5,78315)	(0,104456; 5,99218)	(0,086114; 6,33763)	(0,156532; 6,15553)	(0,0494958; 6,29083)
263	(0,00852555; 7,39713)	(0,104456; 6,0441)	(0,0606729; 8,24119)	(0,156532; 6,18536)	(0,0301393; 7,88964)
264	(0,0144919; 5,36277)	(0,0889438; 5,75876)	(0,111049; 5,94993)	(0,13634; 6,01205)	(0,0494959; 5,85165)
265	(0,00852543; 8,695)	(0,0889441; 5,93634)	(0,086114; 6,81186)	(0,13634; 6,22942)	(0,0301394; 9,05155)
266	(0,0168649; 6,95535)	(0,104456; 6,80638)	(0,0861139; 6,07154)	(0,156533; 6,92284)	(0,055912; 7,31996)

267	(0,0168647; 5,14939)	(0,0889438; 5,6581)	(0,0473209; 7,66321)	(0,13634; 5,92478)	(0,0559125; 5,5893)
268	(0,0144917; 6,27414)	(0,104456; 5,98904)	(0,043296; 7,36876)	(0,156532; 6,15179)	(0,0494958; 6,59501)
269	(0,0144919; 6,08188)	(0,104456; 5,92749)	(0,0473209; 8,44635)	(0,156532; 6,13475)	(0,0494959; 6,52244)
270	(0,00852555; 8,11657)	(0,0889438; 6,08105)	(0,086114; 6,81492)	(0,13634; 6,32176)	(0,0301393; 8,57649)
271	(0,00999486; 7,26897)	(0,0889438; 6,00232)	(0,0606729; 8,14278)	(0,13634; 6,25607)	(0,0359092; 7,70747)
272	(0,0168647; 5,28021)	(0,104456; 6,82211)	(0,111049; 6,01559)	(0,156532; 7,02849)	(0,0559125; 5,73118)
273	(0,0168645; 5,49111)	(0,0889438; 6,16962)	(0,0861139; 6,66917)	(0,13634; 6,41199)	(0,055913; 5,87484)
274	(0,0133364; 6,72267)	(0,044372; 7,07519)	(0,0473208; 10,0042)	(0,0732182; 7,49019)	(0,0450197; 6,93975)
275	(0,00767231; 6,58063)	(0,104456; 5,85944)	(0,0606729; 8,37699)	(0,156532; 6,09614)	(0,0281348; 7,19165)
276	(0,00852555; 7,53324)	(0,069586; 7,98955)	(0,111049; 5,44522)	(0,110113; 8,27938)	(0,0301393; 8,08368)
277	(0,00852543; 9,74803)	(0,088944; 6,85696)	(0,0861139; 6,77591)	(0,13634; 7,02935)	(0,0301394; 10,0039)
278	(0,0168647; 6,79756)	(0,0494665; 8,78544)	(0,0432959; 7,56143)	(0,0798026; 9,0314)	(0,0559125; 7,16581)
279	(0,0144919; 6,16299)	(0,069586; 8,02046)	(0,0473209; 8,52597)	(0,110113; 8,312)	(0,0494959; 6,57783)
280	(0,00767231; 6,77413)	(0,088944; 6,93473)	(0,111049; 5,91915)	(0,13634; 7,10855)	(0,0281348; 7,37008)
281	(0,00852555; 7,96605)	(0,0494667; 10,0184)	(0,0861139; 7,12953)	(0,0798025; 10,1631)	(0,0301393; 8,48199)
282	(0,00999486; 7,93567)	(0,0889438; 6,85812)	(0,043296; 8,32037)	(0,13634; 7,06505)	(0,0359092; 8,33808)
283	(0,0168647; 7,22869)	(0,044372; 7,69591)	(0,0473209; 8,91952)	(0,0732182; 8,08139)	(0,0559125; 7,59397)
284	(0,0144919; 6,6247)	(0,044372; 8,17368)	(0,111049; 5,02411)	(0,0732182; 8,5286)	(0,0494959; 7,03387)
285	(0,00852555; 8,55453)	(0,0494665; 8,61226)	(0,0861139; 7,32088)	(0,0798026; 8,90984)	(0,0301393; 9,01985)
286	(0,00852543; 9,67326)	(0,0889438; 7,23367)	(0,0758193; 7,10154)	(0,13634; 7,44952)	(0,0301394; 10,0199)
287	(0,00767225; 7,62314)	(0,104456; 8,06458)	(0,111049; 6,90958)	(0,156532; 8,28836)	(0,0281347; 8,21099)
288	(0,00767225; 8,01834)	(0,044372; 8,21102)	(0,111049; 6,58648)	(0,0732182; 8,5752)	(0,0281347; 8,57575)
289	(0,00852555; 9,03938)	(0,104456; 6,45099)	(0,111049; 5,83796)	(0,156532; 6,74497)	(0,0301393; 9,50732)
290	(0,0168647; 6,08308)	(0,0494665; 9,5041)	(0,0473208; 10,9296)	(0,0798026; 9,78255)	(0,0559125; 6,59332)
291	(0,0144919; 7,16511)	(0,104456; 6,88515)	(0,043296; 9,23412)	(0,156532; 7,17197)	(0,0494959; 7,51617)
292	(0,00852543; 10,6363)	(0,0494667; 10,9545)	(0,0606744; 9,90694)	(0,0798025; 11,1208)	(0,0301394; 10,9323)
293	(0,00767225; 9,3856)	(0,0889438; 8,19257)	(0,0606744; 9,73858)	(0,13634; 8,37366)	(0,0281347; 9,85739)

294	(0,00999469; 8,9009)	(0,0695862; 9,63941)	(0,111049; 6,42889)	(0,110113; 9,91946)	(0,0359097; 9,34247)
295	(0,0168646; 5,44527)	(0,104456; 6,40766)	(0,0473208; 11,4139)	(0,156532; 6,74188)	(0,0559125; 6,05852)
296	(0,00852543; 11,1491)	(0,104456; 6,80041)	(0,111049; 6,76606)	(0,156532; 7,12544)	(0,0301394; 11,5113)
297	(0,0168646; 7,05627)	(0,0785928; 7,36071)	(0,0606744; 10,0981)	(0,121932; 7,70072)	(0,0559125; 7,62181)
298	(0,00999475; 9,98388)	(0,104456; 9,5671)	(0,111049; 7,87905)	(0,156532; 9,85136)	(0,0359097; 10,494)
299	(0,00999475; 10,1215)	(0,0695862; 10,8292)	(0,0606744; 10,932)	(0,110113; 11,1438)	(0,0359097; 10,6405)
300	(0,0168646; 9,03271)	(0,104456; 10,5403)	(0,111049; 7,99621)	(0,156532; 10,8304)	(0,0559125; 9,60132)

Appendix D: Robust Goal Programming

G1	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011245	0,081917	0,07596	0,125711	0,03909	0,056385
AERT	6,7394	7,6413	7,4397	7,8877	7,1875	7,2747
RTCov	0,95	0,873333	0,956667	0,836667	0,926667	0,929542
LRT	11,4481	12,3448	11,4525	12,6384	11,6071	11,6703
WBM	0,240713	0,158647	0,20017	0,137225	0,191344	0,197303

G2	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
RTCov	0,95	0,873333	0,95	0,853333	0,926667	0,928713
LRT	11,4088	12,2041	11,4217	12,496	11,5345	11,607
WBM	0,196339	0,133789	0,208721	0,125277	0,174611	0,182961

G3	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,013123	0,090618	0,088415	0,136264	0,043866	0,063925
AERT	6,8819	7,8898	7,5973	8,1524	7,3566	7,449
RTCov	0,973333	0,893333	0,986667	0,863333	0,97	0,960935
LRT	11,5196	12,4843	9,9682	12,924	11,6886	11,2501
WBM	0,247474	0,193935	0,26064	0,173197	0,209953	0,23023

G4	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,013017	0,089195	0,085602	0,134531	0,043635	0,062636
AERT	6,8683	7,8553	7,5376	8,1127	7,3395	7,4151
RTCov	0,97	0,88	0,956667	0,863333	0,956667	0,945245
LRT	11,4142	11,7544	10,0175	11,9687	11,5383	11,06
WBM	0,248159	0,198583	0,248607	0,17586	0,20963	0,226881

G5	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011427	0,082752	0,080295	0,127313	0,040496	0,05848
AERT	6,7597	7,5493	7,0819	7,7675	7,1648	7,1354
RTCov	0,93	0,863333	0,93	0,84	0,896667	0,907185
LRT	11,6299	11,8556	12,0629	12,1206	12,1747	11,993
WBM	0,184242	0,137569	0,213148	0,127914	0,165781	0,179877

SR0-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR0,5-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average

AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR1-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR2-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR3-MR0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR1-MR0,5	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR1-MR1	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR1-MR2	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114574	0,0814727	0,0777842	0,12538	0,0398821	0,057203
AERT	6,63734	7,50225	7,23966	7,74695	7,07327	7,1301
RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316

LRT	11,4212	11,8675	11,4475	12,1253	11,5542	11,5649
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

SR1-MR3	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	0,056929
AELP	0,0113377	0,0807825	0,0777261	0,12452	0,0395385	7,1028
AERT	6,60464	7,46242	7,23209	7,70567	7,03495	0,93191
RTCov	0,95	0,88	0,953333	0,86	0,93	11,5694
LRT	11,42	11,8739	11,4606	12,1342	11,5513	0,187159
WBM	0,204437	0,137366	0,210186	0,127075	0,18001	0,056929

SR1-MR10	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0113377	0,0807825	0,0777261	0,12452	0,0395385	0,056929
AERT	6,60464	7,46242	7,23209	7,70567	7,03495	7,1028
RTCov	0,95	0,88	0,953333	0,86	0,93	0,93191
LRT	11,42	11,8739	11,4606	12,1342	11,5513	11,5694
WBM	0,204437	0,137366	0,210186	0,127075	0,18001	0,187159

A1	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
RTCov	0,95	0,873333	0,95	0,853333	0,926667	0,928713
LRT	11,4088	12,2041	11,4217	12,496	11,5345	11,607
WBM	0,196339	0,133789	0,208721	0,125277	0,174611	0,182961

A2	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
RTCov	0,95	0,873333	0,95	0,853333	0,926667	0,928713
LRT	11,4088	12,2041	11,4217	12,496	11,5345	11,607
WBM	0,196339	0,133789	0,208721	0,125277	0,174611	0,182961

A3	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
RTCov	0,95	0,873333	0,95	0,853333	0,926667	0,928713
LRT	11,4088	12,2041	11,4217	12,496	11,5345	11,607
WBM	0,196339	0,133789	0,208721	0,125277	0,174611	0,182961

A4	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012672	0,087598	0,083803	0,133064	0,042934	0,061504
AERT	6,562	7,5272	7,1435	7,7725	7,0155	7,0697
RTCov	0,97	0,876667	0,94	0,863333	0,946667	0,936497
LRT	11,7179	12,1969	11,7028	12,3145	11,9515	11,8709
WBM	0,240593	0,195709	0,257701	0,17835	0,220257	0,231407

A5	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012672	0,087598	0,083803	0,133064	0,042934	0,061504
AERT	6,562	7,5272	7,1435	7,7725	7,0155	7,0697
RTCov	0,97	0,876667	0,94	0,863333	0,946667	0,936497
LRT	11,7179	12,1969	11,7028	12,3145	11,9515	11,8709
WBM	0,240593	0,195709	0,257701	0,17835	0,220257	0,231407

A6	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,013853	0,090833	0,090421	0,135257	0,045321	0,065104
AERT	6,7689	7,7058	7,4042	7,9622	7,2309	7,2933
RTCov	0,973333	0,906667	0,976667	0,88	0,963333	0,958163
LRT	11,4024	11,887	11,0519	12,3023	11,5213	11,436
WBM	0,243127	0,202037	0,27286	0,181773	0,215332	0,236358

A7	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012945	0,089476	0,085065	0,134826	0,043341	0,062404
AERT	6,7118	7,6789	7,3039	7,9232	7,1624	7,2224
RTCov	0,97	0,86	0,94	0,846667	0,94	0,931767
LRT	11,4324	11,8706	10,1936	12,1299	11,5743	11,1561
WBM	0,252144	0,184973	0,235789	0,166687	0,210758	0,221869

A8	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011996	0,082762	0,080781	0,127012	0,041096	0,058911
AERT	6,5855	7,4288	7,1426	7,6654	7,0128	7,0564
RTCov	0,946667	0,873333	0,94	0,846667	0,92	0,922187
LRT	11,4162	11,8278	11,512	12,0951	11,5955	11,5915
WBM	0,222048	0,150242	0,226233	0,136431	0,19587	0,202653

A9	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,013017	0,089195	0,085602	0,134531	0,043635	0,062636
AERT	6,8683	7,8553	7,5376	8,1127	7,3395	7,4151
RTCov	0,97	0,88	0,956667	0,863333	0,956667	0,945245
LRT	11,4142	11,7544	10,0175	11,9687	11,5383	11,06
WBM	0,248159	0,198583	0,248607	0,17586	0,20963	0,226881

A10	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011613	0,081791	0,079439	0,125772	0,040261	0,057956
AERT	6,6714	7,5375	7,2914	7,7848	7,1072	7,1704
RTCov	0,95	0,886667	0,96	0,87	0,93333	0,936494
LRT	11,4085	12,3836	11,4506	12,8101	11,5355	11,6581
WBM	0,221303	0,141304	0,224386	0,129308	0,192229	0,199479

A11	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011457	0,081473	0,077784	0,12538	0,039882	0,057203
AERT	6,6373	7,5023	7,2397	7,747	7,0733	7,1301

RTCov	0,95	0,88333	0,956667	0,87	0,926667	0,93316
LRT	11,3212	11,8675	11,4475	12,1253	11,5542	11,5441
WBM	0,207264	0,139088	0,206473	0,127302	0,182971	0,187538

A12	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012645	0,088456	0,085637	0,133704	0,042685	0,062162
AERT	6,8501	7,8866	7,5823	8,1527	7,3335	7,4304
RTCov	0,97	0,896667	0,976667	0,876667	0,966667	0,95733
LRT	11,4138	12,4711	9,9539	12,9112	11,5472	11,1799
WBM	0,245895	0,194023	0,253121	0,171353	0,206433	0,226223

A13	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011457	0,081635	0,077798	0,12557	0,039876	0,057235
AERT	6,6456	7,5033	7,2176	7,746	7,0778	7,1258
RTCov	0,95	0,883333	0,946667	0,86	0,923333	0,928022
LRT	11,4247	11,8678	11,4285	12,1253	11,5605	11,5611
WBM	0,223994	0,143662	0,204359	0,129601	0,190989	0,193229

A14	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011457	0,081635	0,077798	0,12557	0,039876	0,057235
AERT	6,6456	7,5033	7,2176	7,746	7,0778	7,1258
RTCov	0,95	0,883333	0,946667	0,86	0,923333	0,928022
LRT	11,4247	11,8678	11,4285	12,1253	11,5605	11,5611
WBM	0,223994	0,143662	0,204359	0,129601	0,190989	0,193229

A15	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012611	0,08788	0,084048	0,133025	0,042617	0,061501
AERT	6,8719	7,8756	7,5805	8,1386	7,349	7,4368
RTCov	0,97	0,89	0,966667	0,863333	0,963333	0,951357
LRT	11,4159	11,7602	10,0339	11,9888	11,5426	11,0693
WBM	0,235598	0,196782	0,246981	0,173221	0,198816	0,220199

A16	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012611	0,08788	0,084048	0,133025	0,042617	0,061501
AERT	6,8719	7,8756	7,5805	8,1386	7,349	7,4368
RTCov	0,97	0,89	0,966667	0,863333	0,963333	0,951357
LRT	11,4159	11,7602	10,0339	11,9888	11,5426	11,0693
WBM	0,235598	0,196782	0,246981	0,173221	0,198816	0,220199

A17	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012611	0,08788	0,084048	0,133025	0,042617	0,061501
AERT	6,8719	7,8756	7,5805	8,1386	7,349	7,4368
RTCov	0,97	0,89	0,966667	0,863333	0,963333	0,951357
LRT	11,4159	11,7602	10,0339	11,9888	11,5426	11,0693
WBM	0,235598	0,196782	0,246981	0,173221	0,198816	0,220199

A18	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012611	0,08788	0,084048	0,133025	0,042617	0,061501
AERT	6,8719	7,8756	7,5805	8,1386	7,349	7,4368
RTCov	0,97	0,89	0,966667	0,863333	0,963333	0,951357
LRT	11,4159	11,7602	10,0339	11,9888	11,5426	11,0693
WBM	0,235598	0,196782	0,246981	0,173221	0,198816	0,220199

A19	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012904	0,08886	0,084816	0,134228	0,043302	0,0622
AERT	6,8619	7,8582	7,5396	8,1199	7,342	7,416
RTCov	0,973333	0,886667	0,97	0,853333	0,96	0,951073
LRT	11,4174	11,7574	10,0318	11,9748	11,5448	11,0681
WBM	0,230626	0,196227	0,245064	0,173932	0,197736	0,218224

A20	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012904	0,08886	0,084816	0,134228	0,043302	0,0622
AERT	6,8619	7,8582	7,5396	8,1199	7,342	7,416
RTCov	0,973333	0,886667	0,97	0,853333	0,96	0,951073
LRT	11,4174	11,7574	10,0318	11,9748	11,5448	11,0681
WBM	0,230626	0,196227	0,245064	0,173932	0,197736	0,218224

A21	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011344	0,080972	0,077773	0,12477	0,039584	0,056996
AERT	6,6216	7,4711	7,2123	7,7107	7,0447	7,1037
RTCov	0,946667	0,88	0,946667	0,853333	0,926667	0,927467
LRT	11,4233	11,8736	11,443	12,1335	11,5574	11,5659
WBM	0,203165	0,136563	0,208658	0,125975	0,178597	0,185814

A22	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0114	0,081606	0,078226	0,125499	0,039728	0,057314
AERT	6,5842	7,4302	7,179	7,6722	7,0155	7,0697
RTCov	0,95	0,876667	0,95	0,86	0,926667	0,929548
LRT	11,4256	11,8527	11,4294	12,1157	11,5595	11,5593
WBM	0,208109	0,139425	0,209415	0,129144	0,184097	0,189204

B1	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0124616	0,087406	0,083768	0,132407	0,042146	0,06115
AERT	6,9565	7,9427	7,6616	8,2101	7,4357	7,5182
RTCov	0,963333	0,886667	0,96	0,863333	0,953333	0,944552
LRT	11,8738	12,2997	10,4549	12,7332	12,171	11,5954
WBM	0,2459	0,192846	0,247421	0,168308	0,202757	0,2229
Existing servers						10

B2	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average

AELP	0,012	0,083277	0,080832	0,127751	0,041678	0,059204
AERT	6,8615	7,6029	7,1678	7,8259	7,2608	7,2225
RTCov	0,92	0,863333	0,933333	0,846667	0,893333	0,905798
LRT	12,9102	12,085	12,085	12,4808	12,2178	12,3285
WBM	0,201671	0,142878	0,225694	0,130517	0,177493	0,19176
Existing servers						9

B3	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0124616	0,087406	0,083768	0,132407	0,042146	0,06115
AERT	6,9565	7,9427	7,6616	8,2101	7,4357	7,5182
RTCov	0,963333	0,886667	0,96	0,863333	0,953333	0,944552
LRT	11,8738	12,2997	10,4549	12,7332	12,171	11,5954
WBM	0,2459	0,192846	0,247421	0,168308	0,202757	0,2229
Existing servers						10

B4	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0124616	0,087406	0,083768	0,132407	0,042146	0,06115
AERT	6,9565	7,9427	7,6616	8,2101	7,4357	7,5182
RTCov	0,963333	0,886667	0,96	0,863333	0,953333	0,944552
LRT	11,8738	12,2997	10,4549	12,7332	12,171	11,5954
WBM	0,2459	0,192846	0,247421	0,168308	0,202757	0,2229
Existing servers						10

S1	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011258	0,08029	0,077492	0,12396	0,039307	0,056679
AERT	6,739	7,5961	7,3268	7,849	7,1755	7,2264
RTCov	0,936667	0,87	0,946667	0,86	0,92	0,923162
LRT	11,8592	12,413	11,4269	12,8319	12,1446	11,926
WBM	0,23925	0,142452	0,210427	0,126634	0,198704	0,200326

S2	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,013067	0,083656	0,080075	0,127834	0,043459	0,059732
AERT	6,6538	7,4204	7,1838	7,6566	7,0786	7,1021
RTCov	0,94	0,876667	0,94	0,856667	0,913333	0,919967
LRT	11,7452	12,5539	12,2373	12,8356	12,2755	12,2225
WBM	0,269576	0,163485	0,229406	0,148974	0,235258	0,22725

S3	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,013451	0,090296	0,088342	0,135595	0,044447	0,064056
AERT	6,7896	7,7004	7,3928	7,9565	7,2447	7,2968
RTCov	0,976667	0,906667	0,973333	0,883333	0,96	0,957052
LRT	11,4311	11,9316	10,8981	12,3651	11,5776	11,4162
WBM	0,240622	0,199407	0,255503	0,179501	0,211951	0,22866

S4	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average

AELP	0,012273	0,083052	0,081343	0,127189	0,041614	0,059346
AERT	6,6628	7,4345	7,2316	7,698	7,0934	7,1288
RTCov	0,95	0,886667	0,94333	0,86	0,926667	0,928162
LRT	11,4097	11,7257	11,4635	11,7892	11,5321	11,5214
WBM	0,221558	0,153954	0,226621	0,139387	0,192687	0,202307

S5	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011051	0,081101	0,077857	0,12506	0,039034	0,056837
AERT	6,5514	7,4287	7,1659	7,6698	6,9809	7,0481
RTCov	0,95	0,873333	0,95	0,853333	0,926667	0,928713
LRT	11,4088	12,2041	11,4217	12,496	11,5345	11,607
WBM	0,196339	0,133789	0,208721	0,125277	0,174611	0,182961

S6	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,011245	0,081917	0,07596	0,125711	0,03909	0,056385
AERT	6,7394	7,6413	7,4397	7,8877	7,1875	7,2747
RTCov	0,95	0,873333	0,956667	0,836667	0,926667	0,929542
LRT	11,4481	12,3448	11,4525	12,6384	11,6071	11,6703
WBM	0,240713	0,158647	0,20017	0,137225	0,191344	0,197303

S7	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,0119	0,083709	0,081618	0,128625	0,041603	0,059532
AERT	6,7225	7,5309	7,0483	7,7479	7,1343	7,1044
RTCov	0,93	0,856667	0,93	0,833333	0,89	0,904125
LRT	11,6305	11,8168	12,0494	12,0763	12,1743	11,9816
WBM	0,188913	0,145555	0,224126	0,135778	0,173701	0,18814

S8	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,013123	0,090618	0,088415	0,136264	0,043866	0,063925
AERT	6,8819	7,8898	7,5973	8,1524	7,3566	7,449
RTCov	0,973333	0,893333	0,986667	0,863333	0,97	0,960935
LRT	11,5196	12,4843	9,9682	12,924	11,6886	11,2501
WBM	0,247474	0,193935	0,26064	0,173197	0,209953	0,23023

S9	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,012771	0,088511	0,086677	0,133763	0,042972	0,062627
AERT	6,8905	7,9291	7,6573	8,1978	7,371	7,482
RTCov	0,973333	0,893333	0,976667	0,866667	0,97	0,957883
LRT	11,5193	12,4889	9,8329	12,9087	11,669	11,1984
WBM	0,25071	0,192448	0,259101	0,170301	0,209247	0,229818

S10	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,01132	0,081456	0,076074	0,125395	0,039549	0,056508
AERT	6,6819	7,5194	7,1963	7,7631	7,1179	7,1407
RTCov	0,933333	0,866667	0,94	0,853333	0,92	0,919413

LRT	12,0159	11,8576	11,39	12,119	12,3913	11,9125
WBM	0,219047	0,145963	0,192646	0,129941	0,187199	0,187413

Existing set	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Average
AELP	0,015607	0,105346	0,094131	0,154539	0,050441	0,071021
AERT	7,3422	8,3312	7,6848	8,5626	7,7653	7,7643
RTCov	0,846667	0,736667	0,85	0,696667	0,81	0,81536
LRT	14,6538	15,3516	14,5702	15,6733	15,3166	14,9629
WBM	0,29256	0,21372	0,232031	0,191418	0,246164	0,243828

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Eidesstattliche Erklärung

Hiermit erkläre ich gemäß § 7 Abs. 2 Nr. 2 der Promotionsordnung der wirtschaftswissenschaftlichen Fakultät der Universität Würzburg vom 06.12.2016, dass ich diese Dissertation eigenständig, d.h. insbesondere selbständig und ohne Hilfe eines kommerziellen Promotionsberaters angefertigt habe, und dass ich außer den im Schrifttumsverzeichnis angegebenen Hilfsmitteln keine weiteren benutzt habe und alle Stellen, die aus dem Schrifttum ganz oder annähernd entnommen sind, als solche kenntlich gemacht und einzeln nach ihrer Herkunft unter Bezeichnung der Ausgabe (Auflage und Jahr des Erscheinens), des Bandes und ggf. der Seite des benutzten Werkes nachgewiesen habe.

Würzburg, März 2021