Communication

Grand Unified Origin of Gauge Interactions and Families Replication in the Standard Model

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Abstract: The tremendous phenomenological success of the Standard Model (SM) suggests that its flavor structure and gauge interactions may not be arbitrary but should have a fundamental first-principle explanation. In this work, we explore how the basic distinctive properties of the SM dynamically emerge from a unified New Physics framework tying together both flavor physics and Grand Unified Theory (GUT) concepts. This framework is suggested by a novel anomaly-free supersymmetric chiral $E_6 \times SU(2)_F \times U(1)_F$ GUT containing the SM. Among the most appealing emergent properties of this theory is the Higgs-matter unification with a highly-constrained massless chiral sector featuring two universal Yukawa couplings close to the GUT scale. At the electroweak scale, the minimal SM-like effective field theory limit of this GUT represents a specific flavored three-Higgs doublet model consistent with the observed large hierarchies in the quark mass spectra and mixing already at tree level.

1. Introduction

With a handful of physical parameters such as fermion masses and gauge couplings, the Standard Model (SM) explains a huge variety of collider and low energy data spanning over several orders of magnitude for the corresponding energy scales. Its success builds strongly on the gauge principle. However, it is fundamentally incomplete as it leaves the cosmological Dark Matter and baryon asymmetry of the universe unexplained. Moreover, it neither contains mechanisms for generating the tiny neutrino masses nor explains the structure of the SM fermion families. This suggests that the SM is not the ultimate theory but an excellent effective field theory (EFT) of the subatomic world.

Since the birth of the SM in mid-1970, there have been numerous attempts to come up with a consistent first-principle explanation of the well-measured but yet totally arbitrary and rather odd properties of the SM. Among these are the remarkable proton stability, the specific structure of gauge and Yukawa interactions and the properties of the Higgs and Yukawa sectors which are intimately connected to the rather peculiar observed patterns in the neutrino and charged fermion mass spectra and generation mixings (the so-called flavor problem).

It is fairly easy to achieve unification of the gauge couplings at higher energy scales by postulating the existence of additional scalars and/or fermions belonging to incomplete representations of SU(5) [1]. This is for example realized in supersymmetric (SUSY) extensions of the SM [2]. This unification is a necessary requirement to embed the SM into a larger gauge group such as as SU(5), SO(10), or E_6 [3–13], so-called grand unified theories

(GUTs). For a recent thorough discussion of theoretical features and most important phenomenological implications of the E_6 GUTs, see e.g., Refs. [14–21]. Remarkably, in Ref. [22] it has been demonstrated that the well-known hierarchy and doublet-triplet splitting problems appear to be naturally resolved in the framework of SUSY SU(6) GUT (see also Refs. [23,24]). This model also provides means for the explanation of the origin of the fermion mass hierarchy, i.e., why the third fermion family in the SM is heavier than first two [25]. Other promising scenarios designed to address the flavor problem invoke new "horizontal" symmetries at high energies; for recent studies, see e.g., Refs. [26–32]. For original works, where it was suggested, in particular in the context of $SU(3)_F$, that the observed fermion spectrum with its hierarchies of masses and mixing angles are due to horizontal symmetry breaking hierarchy, i.e., by the flavon VEV hierarchy breaking it, see Refs. [33–36].

2. Gauge and Yukawa Structure

In general, it is rather difficult to combine both gauge symmetries and horizontal symmetries without a proliferation of unknown parameters in the Yukawa sector. Our main goal here is to find a consistent GUT framework in four spacetime dimensions, with both types of unification realized dynamically in the gauge and Yukawa sectors. For this purpose, we would like to study high-scale SUSY-based framework binding together both observed fermion families' replication in the SM and grand unification. For this purpose, let us consider the $\mathcal{N} = 1$ SUSY $E_6 \times SU(2)_F \times U(1)_F$ GUT in four dimensions where the gauge symmetries of the SM originate from E_6 whereas additional group factors $SU(2)_F \times U(1)_F$ conveniently represent a "horizontal" gauge symmetry distinguishing the fermion families, i.e., the family symmetry[1].

Note that a further constrained scenario based on $SU(3)_F$ has been previously studied by some of the authors in Refs. [39,40]. Indeed, promoting $SU(2)_F \times U(1)_F$ to $SU(3)_F$, the model seems to become more compact and natural. However, it was found that the top and charm quark tree-level masses are degenerate and a strong fine-tuning in the soft SUSY breaking sector is necessary in order to induce a realistic mass splitting at one-loop level. Such fine-tuning implies a rather strong hierarchy between different soft SUSY breaking parameters that have prompted the search for a less constrained fully-gauged family symmetry such as the one considered in this work.

The subsequent symmetry breaking steps can be realized by means of the Higgs mechanism as follows:

$$E_6 \times SU(2)_F \times U(1)_F \xrightarrow{M_6} [SU(3)]^3 \times SU(2)_F \times U(1)_F \tag{1}$$

$$\xrightarrow{M_3} SU(3)_C \times [SU(2) \times U(1)]^2 \tag{2}$$

$$\times SU(2)_F \times U(1)_F \xrightarrow{M_S} \ldots . \tag{3}$$

where $[SU(3)]^3 \equiv SU(3)_C \times SU(3)_L \times SU(3)_R$ is the trinification group and $[SU(2) \times U(1)]^2 \equiv SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$. We adopt at this stage that $E_6 \times SU(2)_F \times U(1)_F$ theory originates from a certain large gauge group $\mathcal{G}$ at the upper-most GUT-scale M_{GUT} (with a single universal gauge coupling) by means of some unknown dynamics and formulate the basic criteria for phenomenological consistency of such a scenario.

The mass scales of the rank-preserving symmetry breaking steps in Equations (1) and (2) are given by the sizes of the superpotential quadratic terms implying a nearly-compressed scale hierarchy $M_{GUT} \gtrsim M_6 \gtrsim M_3$. The ... in Equation (3) represent the subsequent low-scale breaking steps down to the SM gauge group triggered by soft-SUSY breaking interactions at the soft scale M_S. The latter can be decoupled from the trinification breaking scale, i.e., $M_S \ll M_3$, in consistency with the low-scale electroweak spontaneous symmetry breaking (EW-SSB) in the SM.

In this work, our main goal is to briefly discuss the main features of the SUSY $E_6 \times SU(2)_F \times U(1)_F$ GUT with the symmetry breaking pattern given in Equations (1)–(3) and

with a particular anomaly-free superfield content summarised in Table 1. In order to build a minimal working GUT scenario in this framework, one simple way to proceed is to adopt a scheme enabling to give large masses to the majority of supermultiplets listed in Table 1 apart from those that we need for phenomenological explorations. For simplicity, we introduce an extra $\mathbb{Z}_2$ parity that plays a similar role to *R*-parity of the MSSM yielding the superpotential of the $\mathbb{Z}_2$-odd superfields analogical to that of the MSSM leptonic sector with right-handed neutrinos,

$$W_{\mathcal{HN}} = \mu_{\mathcal{H}} \mathcal{H}_{\mathcal{U}} \mathcal{H}_{\mathcal{D}} + y_{\mathcal{L}} \mathcal{H}_{\mathcal{U}} \mathcal{L} \mathcal{E} + y_{\mathcal{N}} \mathcal{H}_{\mathcal{D}} \mathcal{L} \mathcal{N} + \mu_{\mathcal{N}} \mathcal{N} \mathcal{N} . \tag{4}$$

Hence, an analogue of right-handed neutrino, $\mathcal{N}$, receives Majorana mass at some large scale $\mu_{\mathcal{N}}$, while other additional superfields $\mathcal{H}_{\mathcal{U,D}}$, $\mathcal{L}$ and $\mathcal{E}$ acquire their masses upon breaking of $SU(2)_F \times U(1)_F$ symmetry through VEVs in scalar components of $SU(2)_F$ $\mathcal{H}_{\mathcal{U,D}}$ doublets. This resembles the Higgsino, (s)neutrinos' and (s)leptons' mass generation via the EW symmetry breaking mechanism in the conventional MSSM framework, but for the family $SU(2)_F \times U(1)_F$ SSB and at a larger scale, $\mu_{\mathcal{H}} \gtrsim M_S \gg M_{EW}$.

Table 1. Fundamental superfield content of the anomaly-free 4-dimensional SUSY $E_6 \times SU(2)_F \times U(1)_F$ GUT. Here, $i = 1,2$ and $k = 1,2,3$.

$\mathbb{Z}_2$-**Even**	$\mathbb{Z}_2$-**Odd**
$\psi^{\mu i} = (\mathbf{27}, \mathbf{2})_{(1)}, \quad \psi^{\mu 3} = (\mathbf{27}, \mathbf{1})_{(-2)}$	
$\mathcal{H}_{\mathcal{U}} = (\mathbf{1}, \mathbf{2})_{(-1)}, \quad \mathcal{H}_{\mathcal{D}} = (\mathbf{1}, \mathbf{2})_{(+1)}$	$\mathcal{L}_k = (\mathbf{1}, \mathbf{2})_{(-1)}$
$\mathcal{A} = (\mathbf{78}, \mathbf{1})_{(0)}$	$\mathcal{E}_k = (\mathbf{1}, \mathbf{1})_{(+2)}$
$\Sigma, \Sigma' = (\mathbf{650}, \mathbf{1})_{(0)}$	$\mathcal{N}_k = (\mathbf{1}, \mathbf{1})_{(0)}$
$\Psi = (\mathbf{2430}, \mathbf{1})_{(0)}$	

Provided that $\mathcal{H}_{\mathcal{U,D}}$ are $\mathbb{Z}_2$-even, their VEVs are not affecting $\mathbb{Z}_2$ symmetry which is therefore preserved in this model and survives down to low scales. Similarly to *R*-parity, this symmetry provides a possible way to stabilize the lightest state among the $\mathbb{Z}_2$-odd components of $\mathcal{L}$, $\mathcal{E}$ and $\mathcal{N}$ superfields. Whether or not such a state can play a role of a Dark Matter candidate remains one of the interesting topics for further studies in this model. Besides, neutrino-like states may receive a relatively small mass scale due to a seesaw-type mixing with the Majorana fermion from $\mathcal{N}$. Communication of such $\mathbb{Z}_2$-odd sector with the SM sectors would be suppressed due to a large mass scale of family $SU(2)_F \times U(1)_F$ gauge bosons at tree level and due to a small loop-generated coupling to the SM Higgs boson. Apart from potentially light and decoupled $\mathbb{Z}_2$-odd states, all the other fields in $W_{\mathcal{HN}}$ are well above the EW scale and their impact on the low-scale phenomenology is expected to be strongly suppressed. Thus, they can be safely integrated out below M_S scale.

It is worth noticing here that the particle content of the considered $E_6 \times SU(2)_F \times U(1)_F \times \mathbb{Z}_2$ GUT and charge assignments in Table 1 enable the superpotential mass terms to all the fundamental $\mathbb{Z}_2$-even superfields except for $\psi^{\mu i}$ and $\psi^{\mu 3}$, providing a novel GUT framework manifestly free of the gauge and Witten anomalies.

The large E_6 representations Σ, Σ' and Ψ trigger (through their scalar VEVs) the spontaneous rank- and SUSY-preserving breaking of E_6 symmetry at M_6 scale in Equation (1), while the components of the $\mathbb{Z}_2$-even E_6-adjoint representation $\mathcal{A}$ play a critical role in triggering the subsequent trinification SSB in Equation (2). All these fields conveniently received large masses and can be integrated out below either M_6 (large E_6 reps) or M_3 (adjoint E_6 rep) scale in the considered SUSY GUT. Taking the trinification breaking at M_3 scale as an example, the $SU(3)_{L,R}$-adjoint superfields $\Delta^a_{L,R} \subset (\mathbf{78}, \mathbf{1})$ (see Table 2) originating from $\mathcal{A}$ superfield have a universal mass term $\mu_{78} \sim M_{GUT}$ in the superpotential, which together with a cubic term triggers a rank- and SUSY-preserving VEV in one of its scalar components $\langle \tilde{\Delta}^{a=8}_{L,R} \rangle \equiv M_3$ [40]. The SUSY-preserving breaking of a gauge symmetry implies that the *D*- and *F*-terms have to vanish separately. This means that the scalar

potential has zero value in both the $SU(3)_{L,R}$-symmetric and $SU(3)_{L,R}$-broken vacua in the exact SUSY case. Thus, the presence of even a tiny soft-SUSY breaking effect already at the M_{GUT} scale is needed to make these vacua non-equivalent [41], hence, enabling the trinification SSB in Equation (2). As a result, all the components of $\Delta_{L,R}^a$ acquire a universal mass, $M_{\Delta_{L,R}} \sim M_3$ due to D-terms and thus are integrated out below the $M_{\Delta_{L,R}} \sim M_3$ scale.

The remaining massless superfields $\psi^{\mu i}$ and $\psi^{\mu 3}$ neatly unite the SM Higgs and matter (neutrino, charged lepton and quark) sectors hence imposing very specific constraints on the structure of the resulting low-energy EFT below M_S. How does such Higgs-matter unification comply with observations? Notably, the considered $E_6 \times SU(2)_F \times U(1)_F$ SUSY GUT enables for a phenomenologically consistent splitting between the second- and third-generation quark masses already at tree level with only two distinct quark Yukawa couplings below M_6. Let us explore this interesting phenomenon in more detail.

Table 2. Upper part: fundamental chiral superfields in the $[SU(3)]^3 \times SU(2)_F \times U(1)_F$ theory–components of the massless $\psi^{\mu i}$ and $\psi^{\mu 3}$ superfields of $E_6 \times SU(2)_F \times U(1)_F$ [42]. Lower part: the corresponding components of the massive superfield $(\mathbf{78}, \mathbf{1})_0$. Accidental symmetries' charges are shown in last two columns.

	$SU(3)_L$	$SU(3)_R$	$SU(3)_C$	$SU(2)_F$	$U(1)_F$	$U(1)_W$	$U(1)_B$
L^i	**3**	$\bar{\mathbf{3}}$	**1**	**2**	1	1	0
L^3	**3**	$\bar{\mathbf{3}}$	**1**	**1**	-2	1	0
Q_L^i	$\bar{\mathbf{3}}$	**1**	**3**	**2**	1	$-1/2$	$1/3$
Q_L^3	$\bar{\mathbf{3}}$	**1**	**3**	**1**	-2	$-1/2$	$1/3$
Q_R^i	**1**	**3**	$\bar{\mathbf{3}}$	**2**	1	$-1/2$	$-1/3$
Q_R^3	**1**	**3**	$\bar{\mathbf{3}}$	**1**	-2	$-1/2$	$-1/3$
Δ_L	**8**	**1**	**1**	**1**	0	0	0
Δ_R	**1**	**8**	**1**	**1**	0	0	0
Δ_C	**1**	**1**	**8**	**1**	0	0	0
Ξ	**3**	$\bar{\mathbf{3}}$	**3**	**1**	0	0	0
Ξ'	$\bar{\mathbf{3}}$	**3**	$\bar{\mathbf{3}}$	**1**	0	0	0

First of all, the SUSY $E_6 \times SU(2)_F \times U(1)_F$ theory below M_{GUT} features a vanishing dim-3 superpotential $d_{\mu\nu\lambda}\varepsilon_{ij}\psi^{\mu i}\psi^{\nu j}\psi^{\lambda 3} = 0$, caused by anti-symmetry of family index contractions, where $d_{\mu\nu\lambda}$ is a completely symmetric E_6 tensor [43,44], and ε_{ij} is the totally anti-symmetric $SU(2)$ Levi-Civita pseudotensor. Since the renormalisable E_6 interactions cannot generate a non-trivial Yukawa structure in this theory, the effects of high-dimensional operators become important. In particular, the relevant part of the superpotential below $M_{GUT} \gtrsim M_6$ scale that contains the SM matter and Higgs sectors reads

$$W_\psi = \frac{\varepsilon_{ij}\psi^{\mu i}\psi^{\nu j}\psi^{\lambda 3}}{2M_{GUT}} \left[\tilde{\lambda}_1 \Sigma_\mu^\alpha d_{\alpha\nu\lambda} + \tilde{\lambda}_2 \Sigma_\nu^\alpha d_{\alpha\mu\lambda} + \tilde{\lambda}_4 \Sigma'^\alpha_\mu d_{\alpha\nu\lambda} + \tilde{\lambda}_5 \Sigma'^\alpha_\nu d_{\alpha\mu\lambda} \right], \quad (5)$$

where the minimal superfield content necessary to generate two distinct Yukawa couplings below M_6 requires the presence of two different bi-fundamental **650**-superfields of E_6 [45], Σ_ν^μ and Σ'^μ_ν. Their VEVs, $\langle \Sigma \rangle \propto k_\Sigma M_6$ and $\langle \Sigma' \rangle \propto k_{\Sigma'} M_6$, trigger subsequent breaking of E_6 [46] down to the trinification symmetry (1). As a result, an EFT superpotential of massless fields below M_6 reads

$$W_{eff} = \varepsilon_{ij}(\mathcal{Y}_1 L^i \cdot Q_L^3 \cdot Q_R^j - \mathcal{Y}_2 L^i \cdot Q_L^j \cdot Q_R^3 + \mathcal{Y}_2 L^3 \cdot Q_L^i \cdot Q_R^j) \quad (6)$$

in terms of the massless trinification leptonic $L^{i,3}$ and quark $Q^{i,3}_{L,R}$ superfields – components of the original massless $\psi^{\mu i}$ and $\psi^{\mu 3}$ superfields of $E_6 \times SU(2)_F \times U(1)_F$ described in Table 2. Next, it is convenient to perform the following decomposition

$$
(L^{i,3})^l{}_r = \begin{pmatrix} \chi^{\bar{l}}_{\bar{r}} & \ell^{\bar{l}}_L \\ \ell_{R\bar{r}} & \phi \end{pmatrix}^{i,3}, \quad (Q^{i,3}_L)^x{}_l = \begin{pmatrix} q^x_{L\bar{l}} & D^x_L \end{pmatrix}^{i,3},
$$
$$
(Q^{i,3}_R)^r{}_x = \begin{pmatrix} q^{\bar{r}}_{Rx} & D_{Rx} \end{pmatrix}^{\top\, i,3},
$$
(7)

such that, upon further splitting into $SU(2)_{L,R}$ representations, one has

$$
\chi^{\bar{l}}_{\bar{r}} = \begin{pmatrix} H^{\bar{l}}_u & H^{\bar{l}}_d \end{pmatrix}^{i,3} \quad \ell_{R\bar{r}} = \begin{pmatrix} e_R & \nu_R \end{pmatrix}^{i,3} \quad q^{\bar{r}}_{Rx} = \begin{pmatrix} u_{Rx} & d_{Rx} \end{pmatrix}^{\top\, i,3}.
$$
(8)

Above, l, r and x represent $SU(3)_L$, $SU(3)_R$ and $SU(3)_C$ triplet indices, $\bar{l}$ and $\bar{r}$ denote $SU(2)_L$ and $SU(2)_R$ doublet indices, respectively, i is the $SU(2)_F$ index, while the labels L (R) should not be identified with left (right) chiralities at this stage (all fermionic components are L-handed Weyl spinors). In Equation (8) one can see that the model offers three up specific $H^{i,3}_u$ and three down specific $H^{i,3}_d$ Higgs doublet candidates. While by no means unique, and in fact not a preferred scenario according to the discussion in [47], a MSSM-like Higgs sector is a possible low-scale limit of the model. However, due to a non-trivial mixing structure, such doublets result from a linear combination of those in Equation (8) and cannot be promptly identified at the level of unbroken trinification. The Higgs-matter unification here implies that the Higgs doublet superfields of the EW theory are unified together with the lepton and quark $SU(3)_{L,R}$-superfields in the $\psi^{\mu i}$ and $\psi^{\mu 3}$. Such a unification is thus enforced by the gauge symmetry of the high-scale theory and that cannot be consistently realized in the MSSM.

The effective trinification superpotential (6) contains two universal Yukawa couplings

$$
\mathcal{Y}_1 = \zeta \frac{k_{\Sigma'}}{\sqrt{6}} \tilde{\lambda}_{45}, \quad \mathcal{Y}_2 = \zeta \frac{k_{\Sigma}}{2\sqrt{2}} (\tilde{\lambda}_{21} - \tilde{\lambda}_{45}),
$$
(9)

where $\tilde{\lambda}_{ij} \equiv \tilde{\lambda}_i - \tilde{\lambda}_j$ and $\zeta \simeq M_6/M_{GUT}$. As we will demonstrate below, due to a very steep Renormalisation Group (RG) evolution of the gauge couplings in the $E_6 \times SU(2)_F \times U(1)_F$ theory at high scales and the required matching of the SM gauge couplings to their measured values at the electroweak (EW) scale, one has $\zeta \sim 1$ and $k_{\Sigma} \simeq -k_{\Sigma'}$. On another hand, a possible common origin of the dim-4 operators from yet unknown M_{GUT}-scale dynamics in the superpotential (5) and a compressed hierarchy $M_{GUT} \gtrsim M_6$ imply that $\tilde{\lambda}_{21} \simeq \tilde{\lambda}_{45}$ suggesting the following hierarchy $\mathcal{Y}_2 \ll \mathcal{Y}_1 \sim 1$. It turns out that such an emergent hierarchy is consistent with the existence of an order-one top-quark Yukawa coupling given by $\mathcal{Y}_1$. Besides, it leads to the observed top-charm and bottom-strange quark mass hierarchies in the SM as well as to the down-type vector-like quark mass hierarchy already at tree level, namely,

$$
\frac{\mathcal{Y}_1}{\mathcal{Y}_2} = \frac{m_t}{m_c} \approx \frac{m_b}{m_s} \approx \frac{m_B}{m_{D,S}} \sim \mathcal{O}(100),
$$
(10)

implying also a possibility for two light vector-like D, S-quark species potentially within the reach of the LHC or future collider measurements.

The superpotential (6) possesses an accidental Abelian $U(1)_W \times U(1)_B$ symmetry whose charges, W and B, are summarised in Table 2. Furthermore, the theory has an extra $\mathbb{Z}_2$ parity denoted as $\mathbb{P}_B$-parity defined as

$$
\mathbb{P}_B = (-1)^{2W+2S} = (-1)^{3B+2S},
$$
(11)

where S is the spin. In the considered GUT theory, the $\mathbb{P}_B$-parity replaces the conventional R-parity and forbids triple-squark or quark-quark-squark trilinear interactions in the

soft-SUSY breaking sector capable of destabilising the proton at the soft scale. Together with the baryon-number $U(1)_B$-symmetric Yukawa sector, this ensures that only E_6 gauge interactions can trigger the proton decay, highly suppressed by a large M_6 close to M_{GUT}.

The dim-3 superpotential of Ξ, Ξ' and $\Delta_{L,R,C}$ superfields–components of the massive chiral $\mathcal{A}$ superfield (see Table 2, also Ref. [42])–reads

$$W_{78} = \sum_{A=L,R,C} \left[\frac{1}{2}\mu_{78}\text{Tr}\Delta_A^2 + \frac{1}{3!}\mathcal{Y}_{78}\text{Tr}\Delta_A^3 \right] + \mu_{78}\text{Tr}(\Xi\Xi') + \sum_{A=L,R,C} \mathcal{Y}_{78}\text{Tr}(\Xi\Xi'\Delta_A), \quad (12)$$

with the universal $\mu_{78} \simeq M_{GUT}$. As was mentioned above, the last rank/SUSY-preserving breaking step in Equation (2) represents the trinification breaking by means of degenerate VEVs at $M_3 \lesssim M_6$ in the $SU(3)_L$, $SU(3)_R$ octet superfields Δ_L, Δ_R, respectively [39,40]. In this case, all the $\Delta_{L,R,C}$ components acquire large masses $M_{\Delta_{L,R,C}} \sim M_3$ and hence are integrated out leaving no heavy fields in the resulting left-right (LR) symmetric SUSY EFT [47]

$$\begin{aligned} W_{LR} = \mathcal{Y}_1\varepsilon_{ij}&\left[\chi^i \cdot q_L^3 \cdot q_R^j + \ell_R^i \cdot D_L^3 \cdot q_R^j + \ell_L^i \cdot q_L^3 \cdot D_R^j + \phi^i \cdot D_L^3 \cdot D_R^j \right] \\ &- \mathcal{Y}_2\varepsilon_{ij}[j \leftrightarrow 3] + \mathcal{Y}_2\varepsilon_{ij}[i \leftrightarrow 3] \end{aligned} \quad (13)$$

written in terms of the massless components of trinification bi-triplets introduced in Equation (7). The further symmetry breaking steps down to the SM and hence the masses/mixings of the $L^{i,3}$ and $Q_{L,R}^{i,3}$ components are controlled by the structure of the soft-SUSY breaking mass terms and tri-linear interactions as well as by the tree-level Yukawa hierarchy (10).

In the LR symmetric SUSY theory, the largest amount of free parameters comes from the soft-SUSY breaking sector, namely, 17 trilinear couplings (5 involving sleptons and 12–squarks), 16 soft $\tilde{L}\tilde{L}$- and $\tilde{Q}\tilde{Q}$-type mass terms, 2 high-scale gaugino mass parameters (in E_6 and gauge-family sectors). In addition, there are 4 gauge couplings in the gauge sector whereas all the low-scale Yukawa couplings are matched to two universal high-scale ones defining the strongest hierarchies (10) already at tree level. The loop corrections to the Yukawa sector are controlled by the soft-SUSY breaking parameters and gauge couplings, whose number is sufficient to accommodate the measured values of the SM fermion masses and mixing angles.

3. The Unification Picture

Let us now investigate how strong the hierarchy between the soft and trinification breaking scales, $M_S \ll M_3$, can be–the question of primary importance for a realistic low-energy theory. Provided the compressed hierarchy $M_{GUT} \gtrsim M_6$, the unknown M_{GUT}-scale effects may induce significant threshold corrections to the trinification gauge couplings at M_6 scale. Indeed, the relevant gauge-kinetic dim-5 operators [46]

$$\mathcal{L}_{5D} = -\frac{\tilde{\xi}}{M_{GUT}} \left[\frac{1}{4C}\text{Tr}(F_{\mu\nu} \cdot \tilde{\Phi}_{E_6} \cdot F^{\mu\nu}) \right] \quad (14)$$

where C is the charge normalization, $F_{\mu\nu}$ is the E_6 field strength tensor, $\tilde{\xi} \sim 1$ is a non-renormalisable coupling constant, and $\tilde{\Phi}_{E_6}$ is a linear combination of the scalar fields originating from the symmetric product of two E_6 adjoint representations $\tilde{\Phi}_{E_6} \in (\mathbf{78} \otimes \mathbf{78})_{\text{sym}} = \mathbf{1} \oplus \mathbf{650} \oplus \mathbf{2430}$, with two $\mathbf{650}$-reps Σ_ν^μ and Σ'^μ_ν already utilised above. The E_6-

breaking VEVs in these fields modify the gauge coupling unification condition at M_6 scale via dim-5 threshold corrections from Equation (14) [46]

$$\alpha_{3C}^{-1}(1+\zeta\delta_C)^{-1} = \alpha_{3L}^{-1}(1+\zeta\delta_L)^{-1} = \alpha_{3R}^{-1}(1+\zeta\delta_R)^{-1},$$

$$\alpha_{3A}^{-1} = \frac{4\pi}{g_A^2}, \qquad \delta_C = -\frac{1}{\sqrt{2}}k_\Sigma - \frac{1}{\sqrt{26}}k_\Psi, \tag{15}$$

$$\delta_{L,R} = \frac{1}{2\sqrt{2}}k_\Sigma \pm \frac{3}{2\sqrt{2}}k_{\Sigma'} - \frac{1}{\sqrt{26}}k_\Psi, \quad k_\Psi \propto \frac{\langle \mathbf{2430} \rangle}{M_6},$$

where α_{3A}^{-1}, $A = L, R, C$, are the inverse trinification structure constants, $k_{\Sigma,\Sigma'}$ and $\zeta \sim 1$ were defined above. Provided that the family $U(1)_T$ and the hypercharge $U(1)_Y$ gauge groups remain unbroken above the EW scale, their the T- and Y-charges are related to the high-scale ones as $T_T = 6T_R^3 - 4T_F^3 + \frac{2}{\sqrt{3}}(T_L^8 - T_R^8 - 2T_F^8)$ and $T_Y = 2T_R^3 + \frac{2}{\sqrt{3}}(T_L^8 + T_R^8)$, respectively. The corresponding inverse structure constants are matched (at tree level) to the high-scale ones below M_3-scale as follows: $\alpha_T^{-1} = \frac{4}{9}(\alpha_{2F}^{-1} + \frac{1}{12}[\alpha_L^{-1} + \alpha_R^{-1} + 4\alpha_F^{-1}]) + \alpha_{2R}^{-1}$ and $\alpha_Y^{-1} = \frac{1}{3}(\alpha_L^{-1} + \alpha_R^{-1}) + \alpha_{2R}^{-1}$, respectively. (Here, α_{2A} and α_A are the structure constants for $SU(2)_A$ and $U(1)_A$, respectively.)

We have performed a sophisticated numerical analysis of the one-loop RG flow of gauge couplings between M_{GUT} and M_{EW} scales accounting for tree-level matching at intermediate scales as well as the matching to their measured counterparts at M_{EW}. We have demonstrated that the presence of threshold corrections δ_A to the gauge couplings at M_6 enables a perturbative universal coupling with $M_{GUT} = 10^{16}$–10^{18} GeV as well as low-scale soft-SUSY breaking down to as low as $M_S \lesssim 10^3$ TeV, in overall consistency with the SM phenomenology. For all valid points, we have found a compressed $M_6 - M_{GUT}$ hierarchy with $\zeta \simeq 1$ as well as $k_\Sigma \simeq -k_{\Sigma'}$. One particular example for such RG flow for a valid parameter space point is shown in Figure 1. The E_6 gauge coupling evolves very fast as indicated by a steep line stretched between M_{GUT} and M_6 scales. The threshold corrections are quite sizable in this example. We give the corresponding parameters in Table 3. Note, in an unrealistic case of a strong $M_6 - M_{GUT}$ hierarchy $\zeta \ll 1$, one could recover an approximate unification $\alpha_{3C}^{-1} \simeq \alpha_{3L}^{-1} \simeq \alpha_{3R}^{-1}$ corresponding to $\mathbb{Z}_3$-permutation symmetry in the trinification gauge sector, originally imposed in Ref. [48]. However, a small $\zeta \ll 1$ implies unacceptably small Yukawa couplings (see Equation (9)).
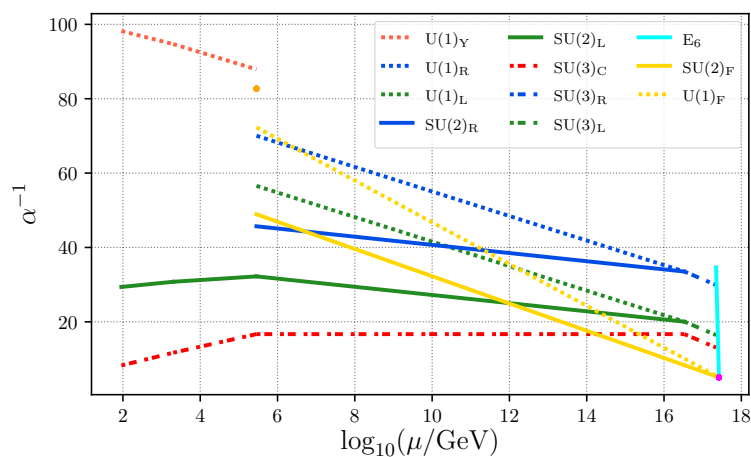

Figure 1. An illustration of RG evolution of the gauge couplings in the considered model for a parameter space point with a low-scale soft-SUSY breaking consistent with the SM. Here, all the soft-induced symmetry breaking scales (except $M_{EW} \ll M_S$) are assumed to be compressed for simplicity and are fixed to a universal value M_S where the SM gauge symmetry emerges. The purple dot represents the universal $\alpha_U^{-1}(M_{GUT})$, while the orange dot shows the $\alpha_T^{-1}(M_S)$ value. The purple dot represents an example of a unified gauge couplings where non-perturbative effects can start playing role.

Table 3. Benchmark points used for the running of the gauge couplings in Figure 1. The top line corresponds to a parameter space point where δ_i differ considerably whereas in the bottom line their absolute values are of the same order. Here, $t_i = \log_{10} \frac{M_i}{\text{GeV}}$.

t_8	t_3	t_S	ζ	$\alpha_8^{-1}(M_8)$	$\alpha_T^{-1}(M_S)$	δ_L	δ_R	δ_C	k_Ψ	k_Σ	k_Σ'	k_σ
17.42	16.53	5.455	0.844	5.067	82.70	-0.622	-0.161	-0.737	0.862	0.326	-0.377	0.0998

4. Quark Masses and Mixing

A simple configuration of soft-scale induced VEVs breaking the symmetry in Equation (3) down to $\text{SU}(3)_C \times \text{U}(1)_{\text{E.M.}}$ reads

$$\left\langle \tilde{L}^k \right\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u_k & 0 & 0 \\ 0 & d_k & 0 \\ 0 & \sim M_S & \sim M_S \end{pmatrix}, \qquad k = 1, 2, 3, \tag{16}$$

where u_k and d_k represent the EW-breaking Higgs up-type and down-type VEVs in the low-energy SM-like EFT, respectively, while all other VEVs are considered to be of order M_S, for simplicity. With such a VEV setting, we can have up to six Higgs doublets in the low-energy limit of the theory. However, for the sake of simplicity, here we consider that only five Higgs doublets acquire VEVs. In this case we found that there is only one particular configuration of these VEVs with $d_1 = 0$ that provides the Cabibbo–Kobayashi–Maskawa (CKM) matrix and the quark mass spectrum compatible with those in the SM, i.e.,

$$V_{\text{CKM}} = \begin{pmatrix} \frac{d_2 u_2 \mathcal{Y}_1^2 + d_3 u_3 \mathcal{Y}_2^2}{\sqrt{\mathcal{AB}}} & -\frac{u_1 \mathcal{Y}_1}{\sqrt{\mathcal{A}}} & \frac{(d_2 u_3 - d_3 u_2)\mathcal{Y}_1 \mathcal{Y}_2}{\sqrt{\mathcal{AB}}} \\ -\frac{d_2 u_1 \mathcal{Y}_1}{\sqrt{\mathcal{BC}}} & -\frac{u_2}{\sqrt{\mathcal{C}}} & \frac{d_3 u_1 \mathcal{Y}_2}{\sqrt{\mathcal{BC}}} \\ \frac{(\mathcal{C}d_3 - d_2 u_2 u_3)\mathcal{Y}_1 \mathcal{Y}_2}{\sqrt{\mathcal{ABC}}} & \frac{u_1 u_3 \mathcal{Y}_2}{\sqrt{\mathcal{AC}}} & \frac{\mathcal{C}d_2 \mathcal{Y}_1^2 + d_3 u_2 u_3 \mathcal{Y}_2^2}{\sqrt{\mathcal{ABC}}} \end{pmatrix} \tag{17}$$

$$\mathcal{A} = \mathcal{C}\mathcal{Y}_1^2 + u_3^2 \mathcal{Y}_2^2, \quad \mathcal{B} = d_2^2 \mathcal{Y}_1^2 + d_3^2 \mathcal{Y}_2^2, \quad \mathcal{C} = u_1^2 + u_2^2,$$

$$m_c^2 = \tfrac{1}{2}\mathcal{Y}_2^2(u_1^2 + u_2^2 + u_3^2) \quad m_t^2 = \tfrac{1}{2}[\mathcal{Y}_1^2(u_1^2 + u_2^2) + \mathcal{Y}_2^2 u_3^2],$$

$$m_s^2 = \tfrac{1}{6}(d_3 - d_2)^2 \mathcal{Y}_2^2, \quad m_b^2 = \tfrac{1}{2}(d_2^2 \mathcal{Y}_1^2 + d_3^2 \mathcal{Y}_2^2),$$

while the *u*- and *d*-quarks as well as charged leptons and light neutrinos do not acquire masses at tree level. In the limit of small $\mathcal{Y}_2/\mathcal{Y}_1 \ll 1$ suggested by the second- and third-generation mass hierarchies (10), an approximate Cabibbo mixing arises, with the angle $\theta_C \approx \arctan(u_1/u_2)$. Indeed, in this limit the top-bottom mixing element $V_{\text{tb}} \simeq 1 - (\mathcal{Y}_2/\mathcal{Y}_1)^2 \sim \mathcal{O}(1)$ is well under control. Moreover, the same ratio (10) provides a strong suppression for V_{td}, V_{ts}, V_{bu} and V_{bc} CKM elements, in agreement with measurements. In addition, small tree-level contributions to the masses and CKM from a seesaw-type mixing with the heavy vector-like quarks are present. This is similar to the model presented in Ref. [47] which we refer to for further details. Furthermore, loop contributions generate additional (small) terms to the CKM mixing entries and fermion masses. Note, the minimal scenario that is compatible with the CKM quark mixing and mass spectrum corresponds to a Three Higgs Doublet Model with $d_{1,3} = u_3 = 0$; a suitable benchmark scenario for detailed phenomenological explorations.

Note, as an interesting possibility for future studies, the fields Σ, Σ' can break E_6 not only to the trinification group but also to $\text{SU}(6) \times \text{SU}(2)$. The two scenarios have an intersection and the presence of adjoint $\mathcal{A}$ which breaks trinification at a lower scale M_3 makes a difference in the associated symmetry breaking pattern and in the corresponding low-energy SM-like EFT limit. Further analysis would be necessary to conclude on which of these two E_6 breaking schemes is favored by the vacuum structure of the theory, as well as by phenomenology. Such studies should include the full RG flow analysis and the matching to the SM Higgs and flavor sectors in both scenarios.

5. Conclusions

In summary, the suggested flavored SUSY-GUT framework exhibits two-fold unification in the gauge and Yukawa sectors as a consequence of the Higgs-matter and the SM gauge couplings' Grand Unification under E_6. While higher-dimensional E_6 operators in the $E_6 \times SU(2)_F \times U(1)_F$ GUT theory generate the necessary splittings in the Yukawa and gauge sectors, the gauge couplings' RG flow suggests a strongly-decoupled energy scale for the soft-SUSY breaking sector, giving rise to a consistent low-scale SM-like EFT. The latter exhibits the minimum of three light Higgs doublets for the model to be generically compatible with SM quark masses and CKM mixing. The main features of the SM fermion spectra such as the observed top-charm and bottom-strange mass hierarchies as well as a Cabibbo-type mixing in the quark sector are generated already at tree level. Other parameters of the light fermion spectra such as the small CKM mixing elements, u, d-quark and charged lepton masses, neutrino masses and mixing should be established at higher-loop orders via a mixture of different-type seesaw mechanisms which is planned for further studies. However, it is already clear that vast phenomenological prospects offered in the proposed framework by a rich scalar, neutrino and heavy vector-like fermion sectors as well as by the gauge family interactions can be expected in the reach of future collider experiments. One of the remaining key theoretical goals for further studies is to explore the potential of the ultimate Left-Right-Color-Family gauge and Yukawa couplings' unification through a dynamical origin of $E_6 \times SU(2)_F \times U(1)_F$ from yet unknown large gauge group $\mathcal{G}$ at the M_{GUT} scale.

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Note

1　For a previous discussion of implications of non-abelian family symmetries in supersymmetric GUT model building, see e.g., Refs. [37,38]

References

1. Calibbi, L.; Ferretti, L.; Romanino, A.; Ziegler, R. Gauge coupling unification, the GUT scale, and magic fields. *Phys. Lett. B* **2009**, *672*, 152–157. [CrossRef]
2. Martin, S.P. A Supersymmetry primer. *Adv. Ser. Direct. High Energy Phys.* **1998**, *18*, 1–98. [CrossRef]
3. Georgi, H.; Glashow, S.L. Unity of All Elementary Particle Forces. *Phys. Rev. Lett.* **1974**, *32*, 438–441. [CrossRef]

4.   Fritzsch, H.; Minkowski, P. Unified Interactions of Leptons and Hadrons. *Ann. Phys.* **1975**, *93*, 193–266. [CrossRef]
5.   Chanowitz, M.S.; Ellis, J.R.; Gaillard, M.K. The Price of Natural Flavor Conservation in Neutral Weak Interactions. *Nucl. Phys. B* **1977**, *128*, 506–536. [CrossRef]
6.   Georgi, H.; Nanopoulos, D.V. T Quark Mass in a Superunified Theory. *Phys. Lett. B* **1979**, *82*, 392–394. [CrossRef]
7.   Georgi, H.; Nanopoulos, D.V. Ordinary Predictions from Grand Principles: T Quark Mass in O(10). *Nucl. Phys. B* **1979**, *155*, 52–74. [CrossRef]
8.   Georgi, H.; Nanopoulos, D.V. Masses and Mixing in Unified Theories. *Nucl. Phys. B* **1979**, *159*, 16–28. [CrossRef]
9.   Georgi, H. Lie algebras in particle physics. From isospin to unified theories. *Front. Phys.* **1982**, *54*, 1–255.
10.   Gursey, F.; Ramond, P.; Sikivie, P. A Universal Gauge Theory Model Based on E6. *Phys. Lett. B* **1976**, *60*, 177–180. [CrossRef]
11.   Gursey, F.; Serdaroglu, M. E6 gauge field theory model revisited. *Nuovo Cim. A* **1981**, *65*, 337. [CrossRef]
12.   Achiman, Y.; Stech, B. Quark Lepton Symmetry and Mass Scales in an E6 Unified Gauge Model. *Phys. Lett. B* **1978**, *77*, 389–393. [CrossRef]
13.   Pati, J.C.; Salam, A. Lepton Number as the Fourth Color. *Phys. Rev. D* **1974**, *10*, 275–289; Erratum in **1975**, *11*, 703. [CrossRef]
14.   Athron, P.; King, S.F.; Miller, D.J.; Moretti, S.; Nevzorov, R.; King, S.F.; Miller, D.J.; Moretti, S.; Nevzorov, R. The Constrained E$_6$SSM. 2008. Available online: https://arxiv.org/abs/0810.0617 (accessed on 22 November 2021).
15.   Hall, J.P.; King, S.F.; Nevzorov, R.; Pakvasa, S.; Sher, M.; Nevzorov, R.; Pakvasa, S.; Sher, M. Novel Higgs Decays and Dark Matter in the E(6)SSM. *Phys. Rev. D* **2011**, *83*, 075013. [CrossRef]
16.   Rizzo, T.G. Gauge Kinetic Mixing in the E_6SSM. *Phys. Rev. D* **2012**, *85*, 055010. [CrossRef]
17.   Nevzorov, R. E_6 inspired supersymmetric models with exact custodial symmetry. *Phys. Rev. D* **2013**, *87*, 015029. [CrossRef]
18.   Nevzorov, R.; Pakvasa, S. Exotic Higgs decays in the E_6 inspired SUSY models. *Phys. Lett. B* **2014**, *728*, 210–215. [CrossRef]
19.   Nevzorov, R.; Thomas, A.W. E_6 inspired composite Higgs model. *Phys. Rev. D* **2015**, *92*, 075007. [CrossRef]
20.   Athron, P.; Harries, D.; Nevzorov, R.; Williams, A.G. E_6 Inspired SUSY Benchmarks, Dark Matter Relic Density and a 125 GeV Higgs. *Phys. Lett. B* **2016**, *760*, 19–25. [CrossRef]
21.   Ishihara, T.; Maekawa, N.; Takegawa, M.; Yamanaka, M. Leptogenesis in $E_6 \times U(1)_A$ SUSY GUT model. *JHEP* **2016**, *2*, 108. [CrossRef]
22.   Berezhiani, Z.G.; Dvali, G.R. Possible solution of the hierarchy problem in supersymmetrical grand unification theories. *Bull. Lebedev Phys. Inst.* **1989**, *5*, 55–59.
23.   Berezhiani, Z.; Csaki, C.; Randall, L. Could the supersymmetric Higgs particles naturally be pseudoGoldstone bosons? *Nucl. Phys. B* **1995**, *444*, 61–91. [CrossRef]
24.   Dvali, G.R.; Pokorski, S. The Role of the anomalous U(1)-A for the solution of the doublet - triplet splitting problem. *Phys. Rev. Lett.* **1997**, *78*, 807–810. [CrossRef]
25.   Barbieri, R.; Dvali, G.R.; Strumia, A.; Berezhiani, Z.; Hall, L.J. Flavor in supersymmetric grand unification: A Democratic approach. *Nucl. Phys. B* **1994**, *432*, 49–67. [CrossRef]
26.   Ordell, A.; Pasechnik, R.; Serôdio, H.; Nottensteiner, F. Classification of anomaly-free 2HDMs with a gauged U(1)′ symmetry. *Phys. Rev. D* **2019**, *100*, 115038. [CrossRef]
27.   Cárcamo Hernández, A.E.; Kovalenko, S.; Pasechnik, R.; Schmidt, I. Sequentially loop-generated quark and lepton mass hierarchies in an extended Inert Higgs Doublet model. *JHEP* **2019**, *6*, 056. [CrossRef]
28.   Vien, V.V.; Long, H.N.; Cárcamo Hernández, A.E. Fermion Mass and Mixing in a Low-Scale Seesaw Model based on the S4 Flavor Symmetry. *Prog. Theor. Exp. Phys.* **2019**, *11*, 113. [CrossRef]
29.   Cárcamo Hernández, A.E.; Hidalgo Velásquez, Y.; Pérez-Julve, N.A. A 3-3-1 model with low scale seesaw mechanisms. *Eur. Phys. J. C* **2019**, *79*, 828. [CrossRef]
30.   Cárcamo Hernández, A.E.; Long, H.N.; Vien, V.V. The first $\Delta(27)$ flavor 3-3-1 model with low scale seesaw mechanism. *Eur. Phys. J. C* **2018**, *78*, 804. [CrossRef]
31.   Björkeroth, F.; de Medeiros Varzielas, I.; López-Ibáñez, M.L.; Melis, A.; Vives, O. Leptogenesis in $\Delta(27)$ with a Universal Texture Zero. *JHEP* **2019**, *09*, 050. [CrossRef]
32.   Ding, G.J.; King, S.F.; Liu, X.G.; Lu, J.N. Modular S_4 and A_4 Symmetries and Their Fixed Points: New Predictive Examples of Lepton Mixing. *JHEP* **2019**, *12*, 030. [CrossRef]
33.   Berezhiani, Z.G.; Chkareuli, J.L. Quark-leptonic families in a model with su(5) × su(3) symmetry. *Sov. J. Nucl. Phys.* **1983**, *37*, 618–626. (In Russian)
34.   Berezhiani, Z.G. Horizontal Symmetry and Quark-Lepton Mass Spectrum: The SU(5) × SU(3)-h Model. *Phys. Lett. B* **1985**, *150*, 177–181. [CrossRef]
35.   Berezhiani, Z.G.; Khlopov, M.Y. The Theory of broken gauge symmetry of families. *Sov. J. Nucl. Phys.* **1990**, *51*, 739–746. (In Russian)
36.   Berezhiani, Z. Unified picture of the particle and sparticle masses in SUSY GUT. *Phys. Lett. B* **1998**, *417*, 287–296. [CrossRef]
37.   Ross, G.G.; Vives, O. Yukawa structure, flavor and CP violation in supergravity. *Phys. Rev. D* **2003**, *67*, 095013. [CrossRef]
38.   Ross, G.G.; Velasco-Sevilla, L.; Vives, O. Spontaneous CP violation and nonAbelian family symmetry in SUSY. *Nucl. Phys. B* **2004**, *692*, 50–82. [CrossRef]
39.   Camargo-Molina, J.E.; Morais, A.P.; Ordell, A.; Pasechnik, R.; Sampaio, M.O.; Wessén, J. Reviving trinification models through an E6 -extended supersymmetric GUT. *Phys. Rev. D* **2017**, *95*, 075031. [CrossRef]

40. Camargo-Molina, J.E.; Morais, A.P.; Ordell, A.; Pasechnik, R.; Wessén, J. Scale hierarchies, symmetry breaking and particle spectra in SU(3)-family extended SUSY trinification. *Phys. Rev. D* **2019**, *99*, 035041. [CrossRef]
41. Witten, E. Dynamical Breaking of Supersymmetry. *Nucl. Phys. B* **1981**, *188*, 513. [CrossRef]
42. Slansky, R. Group Theory for Unified Model Building. *Phys. Rep.* **1981**, *79*, 1–128. [CrossRef]
43. Kephart, T.W.; Vaughn, M.T. Tensor Methods for the Exceptional Group E6. *Ann. Phys.* **1983**, *145*, 162. [CrossRef]
44. Deppisch, T. E6Tensors: A Mathematica Package for E6 Tensors. *Comput. Phys. Commun.* **2017**, *213*, 130–135. [CrossRef]
45. Gursey, F. Symmetry Breaking Patterns in E6. In *First Workshop on Grand Unification*; Birkhäuser Boston: Boston, MA, USA, 1980; p. 0039.
46. Chakrabortty, J.; Raychaudhuri, A. A Note on dimension-5 operators in GUTs and their impact. *Phys. Lett. B* **2009**, *673*, 57–62. [CrossRef]
47. Morais, A.P.; Pasechnik, R.; Porod, W. Prospects for new physics from gauge left-right-colour-family grand unification hypothesis. *Eur. Phys. J. C* **2020**, *80*, 1162. [CrossRef]
48. Glashow, S. *Fifth Workshop on Grand Unification*; Kang, K., Fried, H., Frampton, F., Eds.; World Scientific, Singapore, 1984.