Methodological Advances in Composite-based Structural Equation Modeling

Tamara Schamberger

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For my family.

Zusammenfassung

Diese Arbeit beschäftigt sich mit kompositen-basierter Strukturgleichungmodellierung. Strukturgleichungsmodellierung kann genutzt werden um sowohl theoretische Konzepte als auch deren Beziehungen untereinander zu modellieren. In der traditionellen faktor-basierten Strukturgleichungsmodellierung werden diese theoretischen Konzepte als "common factor", d.h. als latente Variablen, die die Kovarianzstruktur ihrer beobachteten Variablen erklären, modelliert. Im Gegensatz dazu, können in kompositen-basierter Strukturgleichungsmodellierung die theoretischen Konzepte sowohl als "common factor" als auch als Komposite, also als Linearkombinationen beobachteter Variablen, die die gesamte Information zwischen ihren beobachteten Variablen und allen anderen Variablen im Modell übertragen, modelliert werden. Diese Arbeit stellt einige methodische Weiterentwicklungen im Bereich der kompositen-basierten Strukturgleichungsmodellierung vor. Sie besteht aus insgesamt 7 Kapiteln.

Kapitel 1 gibt zunächst einen Überblick über das zugrundeliegende Modell sowie über die Definition des Begriffs der kompositen-basierten Strukturgleichungsmodellierung.

In Kapitel 2 wird anschließend eine Anleitung dafür gegeben, wie Monte Carlo Simulationen in der Statistik Software R mittels des Pakets "cSEM" für verschiedene Schätzer, die der kompositen-basierten Strukturgleichungsmodellierung zugeordnet werden, durchgeführt werden können. Diese Anleitung wird anhand einer beispielhaften Simulationsstudie veranschaulicht, die das Verhalten von Partial Least Squares Path Modeling (PLS-PM) und consistent Partial Least Squares (PLSc) Schätzungen

in endlichen Stichproben untersucht, insbesondere im Hinblick auf die Auswirkungen von Stichprobenkorrelationen zwischen Messfehlern auf statistische Inferenz.

Im dritten Kapitel werden Schätzer der kompositen-basierten Strukturgleichungsmodellierung vorgestellt, die robust gegenüber Ausreißern sind. Dafür werden Schätzer der kompositen-basierten Strukturgleichungsmodellierung, PLS-PM und PLSc, angepasst. Im Gegensatz zu den ursprünglichen Schätzern, können mit diesen Anpassungen Verzerrungen, die durch zufällig entstandene Ausreißer in Stichproben entstehen können, vermieden werden, was anhand einer Simulationsstudie gezeigt wird.

In Kapitel 4 wird eine Methode zur Durchführung von Vorhersagen auf Basis von Modellen vorgestellt, die mit ordinal Partial Least Squares und ordinal consistent Partial Least Squares geschätzt wurden. Die beobachteten Variablen sind dabei ordinal kategorial skaliert, was sowohl bei der Schätzung als auch der Vorhersage explizit berücksichtigt wird. Die Vorhersagegüte wird mittels einer Simulationsstudie untersucht. Zusätzlich wird eine Anleitung, wie solche Vorhersagen mittels des R Pakets “cSEM” durchgeführt werden können, gegeben. Diese wird anhand eines empirischen Beispiels demonstriert.

In Kapitel 5 wird die konfirmatorische Kompositenanalyse für Forschung im Bereich von “Human Development” vorgestellt. Mittels konfirmatorischer Kompositenanalyse können Kompositenmodelle geschätzt und auch evaluiert werden. In diesem Kapitel wird die Henseler-Ogasawara Spezifikation für Kompositenmodelle verwendet, wodurch beispielsweise die Maximum Likelihood Methode zur Parameterschätzung verwendet werden kann.

Da der auf der Henseler-Ogasawara Spezifikation basierende Maximum Likelihood Schätzer Nachteile aufweist, wird in Kapitel 6 eine andere Spezifikation des Kompositmodells vorgestellt, mit der Kompositenmodelle mit der Maximum Likelihood Methode geschätzt werden können. Die Ergebnisse dieses Maximum Likelihood Schätzers werden mit denen von PLS-PM verglichen und somit gezeigt, dass dieser Maximum Likelihood Schätzer auch in endlichen Stichproben valide Ergebnisse liefert.

Das letzte Kapitel, Kapitel 7, gibt einen Überblick über die Entwicklung und die verschiedenen Stränge der kompositen-basierten Strukturgleichungsmodellierung. Darüber hinaus wird hier der Beitrag, den die vorangegangenen Kapitel zur weiteren Verbreitung kompositen-basierter Strukturgleichungsmodellierung leisten, aufgezeigt.

Summary

This thesis is about composite-based structural equation modeling. Structural equation modeling in general can be used to model both theoretical concepts and their relations to one another. In traditional factor-based structural equation modeling, these theoretical concepts are modeled as common factors, i.e., as latent variables which explain the covariance structure of their observed variables. In contrast, in composite-based structural equation modeling, the theoretical concepts can be modeled both as common factors and as composites, i.e., as linear combinations of observed variables that convey all the information between their observed variables and all other variables in the model. This thesis presents some methodological advancements in the field of composite-based structural equation modeling. In all, this thesis is made up of seven chapters.

Chapter 1 provides an overview of the underlying model, as well as explicating the meaning of the term composite-based structural equation modeling.

Chapter 2 gives guidelines on how to perform Monte Carlo simulations in the statistic software R using the package “cSEM” with various estimators in the context of composite-based structural equation modeling. These guidelines are illustrated by an example simulation study that investigates the finite sample behavior of partial least squares path modeling (PLS-PM) and consistent partial least squares (PLSc) estimates, particularly regarding the consequences of sample correlations between measurement errors on statistical inference.

The third Chapter present estimators of composite-based structural equation mod-

eling that are robust in responding to outlier distortion. For this purpose, estimators of composite-based structural equation modeling, PLS-PM and PLSc, are adapted. Unlike the original estimators, these adjustments can avoid distortion that could arise from random outliers in samples, as is demonstrated through a simulation study.

Chapter 4 presents an approach to performing predictions based on models estimated with ordinal partial least squares and ordinal consistent partial least squares. Here, the observed variables lie on an ordinal categorical scale which is explicitly taken into account in both estimation and prediction. The prediction performance is evaluated by means of a simulation study. In addition, the chapter gives guidelines on how to perform such predictions using the R package "cSEM". This is demonstrated by means of an empirical example.

Chapter 5 introduces confirmatory composite analysis (CCA) for research in "Human Development". Using CCA, composite models can be estimated and assessed. This chapter uses the Henseler-Ogasawara specification for composite models, allowing, for example, the maximum likelihood method to be used for parameter estimation.

Since the maximum likelihood estimator based on the Henseler-Ogasawara specification has limitations, Chapter 6 presents another specification of the composite model by means of which composite models can be estimated with the maximum likelihood method. The results of this maximum likelihood estimator are compared with those of PLS-PM, thus showing that this maximum likelihood estimator gives valid results even in finite samples.

The last chapter, Chapter 7, gives an overview of the development and different strands of composite-based structural equation modeling. Additionally, here I examine the contribution the previous chapters make to the wider distribution of composite-based structural equation modeling.

Samenvatting

Deze dissertatie behandelt op composiet gebaseerde structurele vergelijking modellering. Structurele vergelijkingsmodellen kunnen worden gebruikt om zowel theoretische concepten als hun relaties te modelleren. In traditionele factorge-baseerde structurele vergelijkingsmodellering worden deze theoretische concepten gemodelleerd als “common factors”, oftewel als latente variabelen die de covariantiestructuur van hun geobserveerde variabelen verklaren. Bij op composiet gebaseerde structurele vergelijkingsmodellering daarentegen kunnen de theoretische concepten gemodelleerd worden, zowel als “common factors” en als composieten, als lineaire combinaties van geobserveerde variabelen die alle informatie overbrengen tussen hun geobserveerde variabelen en alle andere variabelen in het model. Deze disseratie presenteert enkele methodologische vorderingen op het gebied van op composiet gebaseerde structurele vergelijkingsmodellering. Deze dissertatie beslaat zeven hoofdstukken.

Hoofdstuk 1 geeft een overzicht van het onderliggende model, alsmede een uitleg van de betekenis van de term op composiet gebaseerde structurele vergelijkingsmodellering.

Hoofdstuk 2 geeft richtlijnen voor het uitvoeren van Monte-Carlosimulaties in de statistische software R met behulp van het pakket “cSEM” met verschillende schatters in de context van “composite-based structural equation modeling“. Deze richtlijnen worden geïllustreerd door een voorbeeld simulatiestudie die het eindigesteekproefgedrag van “Partial Least Squares Path Modeling” (PLS-PM) en “consistent partial least squares” (PLSc) wordt onderzocht, met name wat betreft de gevolgen

van steekproefcorrelaties tussen meetfouten voor de statistische gevolgtrekking.

In het derde hoofdstuk worden schatters van op composiet gebaseerde structurele vergelijkingsmodellering gepresenteerd die robuust zijn om te reageren op uitbijtervervorming. Daartoe worden schatters van op composiet gebaseerde structurele vergelijkingsmodellering, PLS-PM en PLSc, aangepast. In tegenstelling tot de oorspronkelijke schatters kunnen deze aanpassingen vertekening als gevolg van willekeurige uitbijters in steekproeven voorkomen, zoals wordt aangetoond met een simulatiestudie.

Hoofdstuk 4 presenteert een aanpak voor het doen van voorspellingen op basis van modellen geschat met "ordinal partial least squares" en "ordinal consistent partial least squares". Hierbij liggen de waargenomen variabelen op een ordinale categorische schaal, waarmee zowel bij de schatting als bij de voorspelling expliciet rekening wordt gehouden. De voorspellingsprestaties worden geëvalueerd door middel van een simulatiestudie. Daarnaast geeft het hoofdstuk richtlijnen over hoe dergelijke voorspellingen kunnen worden uitgevoerd met behulp van het R-pakket "cSEM". Dit wordt gedemonstreerd aan de hand van een empirisch voorbeeld.

Hoofdstuk 5 introduceert "confirmatory composite analysis" (CCA) voor onderzoek in "Human Development". Met behulp van CCA kunnen composietmodellen worden geschat en geëvalueerd. Dit hoofdstuk maakt gebruik van de Henseler-Ogasawara specificatie voor composietmodellen, waardoor bijvoorbeeld de maximum likelihoodmethode kan worden gebruikt voor de parameterschatting.

Aangezien de maximum likelihood-schatter op basis van de Henseler-Ogasawara specificatie beperkingen heeft, wordt in hoofdstuk 6 een andere specificatie van het composietmodel gepresenteerd waarmee composietmodellen kunnen worden geschat met de maximum likelihood-methode. De resultaten van deze maximum likelihoodschatter worden vergeleken met die van PLS-PM, waarmee wordt aangetoond dat deze maximum likelihoodschatter ook valide resultaten oplevert bij eindige steekproeven.

Het laatste hoofdstuk, hoofdstuk 7, geeft een overzicht van de ontwikkeling en de verschillende onderdelen van op composiet gebaseerde structurele vergelijkingsmodellering. Daarnaast ga ik hier in op de bijdrage van de voorgaande hoofdstukken aan de bredere verspreiding van op composiet gebaseerde structurele vergelijkingsmodellering.

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Chapter 1

Introduction

Structural equation modeling (SEM) is a popular method in social and behavioral sciences. Its ability to model theoretical concepts, to take into account various forms of measurement errors, and to model relations between theoretical concepts ensure that this method is frequently and widely applied (Bagozzi and Phillips, 1982; Bollen, 1989). Traditionally, in SEM theoretical concepts are modeled as common factors which are latent variables that explain the variance-covariance structure of their related observed variables (Jöreskog, 1970a). Consequently, this type of SEM is often referred to as 'factor-based SEM'. Factor-based SEM is well established, with various estimators such as a generalized least squares (GLS, Jöreskog and Goldberger, 1972) estimator or a maximum likelihood (ML, Jöreskog, 1969) estimator proposed to obtain the parameter estimates. Also, it is often applied in various disciplines such as business management (Mak and Sockel, 2001) and psychology (MacCallum and Austin, 2000).

Besides SEM with common factors, SEM with composites – labeled as composite-based SEM – has recently been established as a second type of SEM. While traditionally, composites were regarded only as the outcome of dimension reduction procedures, nowadays we regard the composite model as a statistical model (Cho et al., in press; Dijkstra, 2015, 2017; Henseler et al., 2014) and thus, it is used to represent theoretical concepts (Henseler, 2021). Moreover, with the introduction of confirmatory composite analysis (CCA, Henseler et al., 2014; Schuberth et al., 2018a), a statistical tool to assess composites is available. Consequently, composite-based SEM has gained attraction in various research fields, such as marketing research (Cheah et al., 2018), psychology (Pant et al., 2018), and business management (Liem and Hien, 2020; Henseler and Schuberth, 2020).

The most dominant approach in estimating composite models is partial least squares path modeling (PLS-PM, Wold, 1975). Although it was introduced as an approach to estimate path models with latent variables (Wold, 1982), in its traditional form, PLS-PM is only able to consistently estimate composite models. However, if a correction for attenuation is applied, i.e., if consistent PLS (PLSc) is applied, it can also be used to obtain consistent parameter estimates for common factor models and models incorporating both composites and common factors (Dijkstra and Henseler, 2015a,b). Besides enhancing PLS-PM to obtain consistent parameter estimates for composite and common factor models, i.e., PLSc, various improvements and extensions, such as an overall model fit test (Dijkstra and Henseler, 2015a), a possibility to take measurement errors into account (Rademaker et al., 2019), robust versions which are not as vulnerable to outlier distortion as the original approaches, specifically robust PLS and robust PLSc (Schamberger et al., 2020), and approaches used to obtain out-of-sample predictions (Danks et al., 2019; Shmueli, 2010; Shmueli et al., 2016, 2019) have been proposed.

Besides PLS-PM, the literature proposes various other approaches for estimating composite models. While generalized structured component analysis (GSCA, Hwang and Takane, 2004) can be used to estimate composite models, GSCA with uniqueness terms for accommodating measurement error (GSCAm, Hwang et al., 2017) can be used to consistently estimate models with common factors. Further, integrated GSCA (IGSCA, Hwang et al., 2021) can deal with models containing both composites and common factors. Moreover, a specification of the composite model in terms of composite loadings, emergent variables, and excrescent variables has been introduced (Henseler, 2021; Schuberth, forthcoming). Using this specification, structural equation models relating composites can be estimated with the common estimators of factor-based SEM. Consequently, a ML estimator or a GLS estimator, for example, can be applied to estimate composite models.

This thesis contributes to the existing literature by providing several enhancements for composite-based SEM. This chapter provides a brief introduction to composite-based SEM. First, Section 1.1 describes the underlying statistical model. Section 1.2 describes what can be understood as composite-based SEM. Finally, the introduction closes in Section 1.3 with a short overview of the following chapters and their contributions to the composite-based SEM literature.

1.1 Structural equation models

Structural equation models are composed of two parts. The first part comprises the relations between theoretical concepts that are captured in the structural model. In the second part, the theoretical concepts are modeled, thus it depicts the relations between constructs – which can be either common factors or composites – and their related observed variables. In the process a block of observed variables belonging to the j-th construct η_j is composed in the $(K_j \times 1)$ vector $\boldsymbol{x}_j$, which is called the j-th block of observed variables. In total, the model contains J constructs and K observed variables.

1.1.1 Structural model

The structural model comprises the relations between constructs, which are not dependent on the type of construct. In general, modeling relations between constructs can be understood as restricting the covariances between them. To model these relations, the endogenous constructs which are explained through at least one other construct are stored in the $(J_{\text{en}} \times 1)$ vector $\boldsymbol{\eta}_{\text{en}}$ and the exogenous constructs which are not explained through other constructs are stored in the $(J_{\text{ex}} \times 1)$ vector $\boldsymbol{\eta}_{\text{ex}}$:

$$\boldsymbol{\eta}_{\text{en}} = \boldsymbol{B}\boldsymbol{\eta}_{\text{en}} + \boldsymbol{\Gamma}\boldsymbol{\eta}_{\text{ex}} + \boldsymbol{\zeta} \tag{1.1}$$

$$\Leftrightarrow \qquad \boldsymbol{\eta}_{\text{en}} = \boldsymbol{\Pi}\boldsymbol{\eta}_{\text{ex}} + (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{\zeta} \tag{1.2}$$

where $\boldsymbol{\Pi} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{\Gamma}$. The $(J_{\text{en}} \times J_{\text{en}})$ matrix $\boldsymbol{B}$ represents the relations between the endogenous constructs and the $(J_{\text{en}} \times J_{\text{ex}})$ matrix $\boldsymbol{\Gamma}$ represents the relations between the exogenous and the endogenous constructs. The vector $\boldsymbol{\zeta}$ contains the J_{en} structural error terms with an assumed mean vector of zeros. In my thesis, I restrict my attention to recursive structural models with uncorrelated structural error terms.

The exogenous constructs $\boldsymbol{\eta}_{\text{ex}}$ are assumed to be uncorrelated with the structural error terms $\boldsymbol{\zeta}$. Figure 1.1 depicts an example structural model with one exogenous and two endogenous constructs.

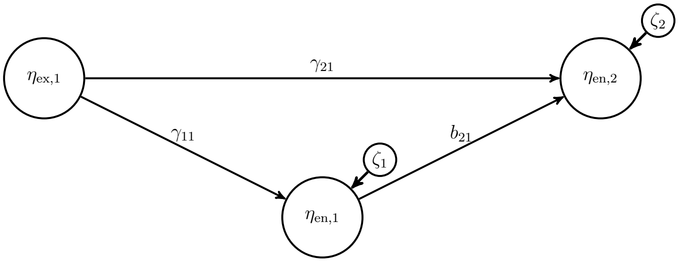

Figure 1.1: Structural model

1.1.2 Reflective measurement model

Traditionally, theoretical concepts are modeled as common factors which are latent variables that explain the covariance structure of their related observed variables:

$$x_{ji} = \lambda_{ji} \cdot \eta_j + \varepsilon_{ji} \qquad i = 1, \ldots, K_j; \qquad j = 1, \ldots, J \tag{1.3}$$

$$\boldsymbol{x}_j = \boldsymbol{\lambda}_j \cdot \eta_j + \boldsymbol{\varepsilon}_j \qquad j = 1, \ldots J \tag{1.4}$$

where $\boldsymbol{\lambda}_j$ is a vector containing the K_j factor loadings, where the loading λ_{ji} represents the influence of the j-th common factor η_j on its i-th observed variable x_{ji}. The vector $\boldsymbol{\varepsilon}_j$ contains the K_j random measurement errors. In my thesis, I restrict my attention to measurement errors with a zero mean, which are not correlated with one another, nor with the constructs. The model with a common factor as articulated in Equation 1.4 is also known as reflective measurement model. Figure 1.2 depicts a reflective measurement model representing the relation between one block of observed variables and its related common factor.

1.1.3 Composite model

Besides modeling theoretical concepts as common factors, such concepts can be modeled as composites, i.e., as linear combinations of observed variables:

$$\eta_j = \boldsymbol{w}_j' \boldsymbol{x}_j \qquad j = 1, \ldots, J \tag{1.5}$$

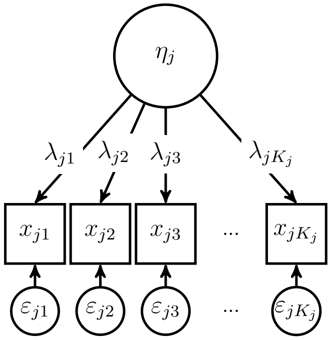

Figure 1.2: Common factor model

where the vector $\boldsymbol{w}_j$ contains the K_j weights of the j-th block of observed variables. The composites are assumed to convey the information between their observed variables and all other variables in the model. Figure 1.3 depicts the relations between one composite and its related observed variables.

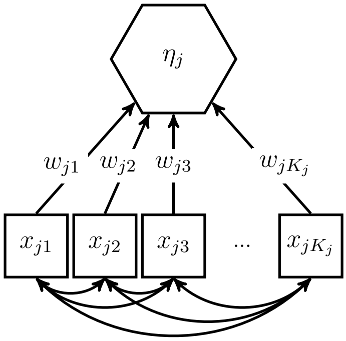

Figure 1.3: Composite model

1.1.4 Model-implied observed variables' variance-covariance matrix

With the model definition given above, the $(K \times K)$ model-implied observed variables' variance-covariance matrix $\boldsymbol{\Sigma}(\boldsymbol{\theta})$ can be specified for use in model estimation if ML is employed, and for data generation in Monte Carlo simulations.

Using the reduced form of the structural model as articulated in Equation 1.2, we derive the variance-covariance matrix of the constructs as follows:

$$\mathrm{V}(\boldsymbol{\eta}) = \begin{pmatrix} \boldsymbol{\Phi} & \boldsymbol{\Phi}\boldsymbol{\Pi}' \\ \boldsymbol{\Pi}\boldsymbol{\Phi} & \boldsymbol{\Pi}\boldsymbol{\Phi}\boldsymbol{\Pi}' + (\boldsymbol{I}-\boldsymbol{B})^{-1}\boldsymbol{\Psi}(\boldsymbol{I}-\boldsymbol{B})'^{-1} \end{pmatrix} \tag{1.6}$$

where $\boldsymbol{\eta}$ is a vector of both the exogenous and the endogenous constructs, the matrix $\boldsymbol{\Phi}$ contains the variances and covariances of the exogenous constructs $\boldsymbol{\eta}_{\text{ex}}$, and the matrix $\boldsymbol{\Psi}$ comprises the variance-covariance matrix of the structural error terms $\boldsymbol{\zeta}$.

Using the relations between the observed variables and their related constructs we can derive the inter-block covariances, i.e., the covariances between observed variables of different blocks, as follows:

$$\mathrm{Cov}(x_{ji}, x_{lk}) = \lambda_{ji} \cdot \lambda_{lk} \cdot \mathrm{cov}(\eta_j, \eta_l) \tag{1.7}$$

where λ_{ji} and λ_{lk} are the factor loadings as shown in Equation 1.4 if the corresponding constructs are common factors and composite loadings if the constructs are modeled as composites. Composite loadings are derived from the corresponding weights and the intra-block variances and covariances captured in the intra-block variance-covariance matrix $\boldsymbol{\Sigma}_{jj}$:

$$\boldsymbol{\lambda}_j = \boldsymbol{\Sigma}_{jj} \cdot \boldsymbol{w}_j \tag{1.8}$$

In contrast, the variance-covariance matrix of a block of observed variables, i.e., the intra-block variance-covariance matrix depends on the type of constructs. For composite models, the intra-block variances and covariances are unrestricted:

$$\boldsymbol{\Sigma}_{jj} = \begin{pmatrix} \sigma_{11}^{(j)} & \sigma_{12}^{(j)} & \cdots & \sigma_{1K_j}^{(j)} \\ \sigma_{21}^{(j)} & \sigma_{22}^{(j)} & \cdots & \sigma_{2K_j}^{(j)} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{K_j1}^{(j)} & \sigma_{K_j2}^{(j)} & \cdots & \sigma_{K_jK_j}^{(j)} \end{pmatrix} \tag{1.9}$$

where $\sigma_{kl}^{(j)}$ represents the covariance between x_{jk} and x_{jl}. In contrast, the common factor model usually imposes constraints on the intra-block variance-covariance matrix. Specifically, the intra-block variance-covariance matrix depends on the factor loadings, the variance of the common factor, and the variances of the random measurement errors:

$$\boldsymbol{\Sigma}_{jj} = \boldsymbol{\lambda}_j \boldsymbol{\lambda}_j' \cdot \mathrm{var}(\eta_j) + \boldsymbol{\Theta}_{\boldsymbol{\varepsilon}_j} \tag{1.10}$$

where $\boldsymbol{\Theta}_{\boldsymbol{\varepsilon}_j}$ is the variance-covariance matrix of the random measurement errors of block j.

1.2 Composite-based structural equation modeling

In general, two types of SEM can be distinguished – factor-based and composite-based SEM. While factor-based SEM is often understood as SEM with theoretical concepts modeled as common factors, composite-based SEM is often understood as SEM with theoretical concepts modeled as composites. Yet, this distinction does not allow us to account for the different natures the various estimators have. Therefore, to clarify, I will describe my understanding of composite-based SEM in the following section. This is in line with the definition put forward by Henseler (2021) and Yu et al. (2021). Obviously, this distinction is not unique and depends on the researcher's view. Consequently, others could argue in a different way.

I understand composite-based SEM as a variety of approaches that build on composites for parameter estimation (Henseler, 2021). Consequently, two types of composite-based SEM can be distinguished: First, if the theoretical concept of interest is modeled as a composite, and thus represented as in Equation 1.5, this should be classified as composite-based SEM, for the simple reason that the common estimators of factor-based SEM, namely ML and GLS, cannot be applied to estimate composite models if they are specified in terms of weights. Consequently, PLS-PM, GSCA, and other variance-based estimators with which weight estimates are directly obtained illustrate estimators in composite-based SEM. Additionally, if the theoretical concept of interest is modeled as a composite, but specified in terms of composite loadings, emergent, and excrescent variables – i.e., if the Henseler-Ogasawara specification (Henseler, 2021; Schuberth, forthcoming) is used – this type of SEM should also be classified as composite-based SEM. Although in this case, the composites are specified in terms of composite loadings and the model is estimated with the common estimators of factor-based SEM, the composite nature must explicitly be taken into account to obtain the model parameters. Specifically, the composite loading estimates must be inverted to obtain weight estimates. Consequently, although ML and GLS are used to estimate the model parameters, due to the composite nature of the constructs, this should be classified as composite-based SEM.

The second type of composite-based SEM considers theoretical concepts which are modeled as common factors. Traditionally, factor-based SEM estimators such as ML or GLS are used to estimate common factor models. Nevertheless, composite-based SEM estimators such as PLSc and GSCAm can also be used to obtain the parameter estimates for this type of construct. PLSc and GSCAm are enhancements of traditional composite-based estimators, namely PLS-PM and GSCA, respectively. Both these approaches build composites to obtain the model parameters and should therefore be classified as composite-based SEM.

To summarize, if the theoretical concepts are modeled as composites, composite-based SEM has to be applied to obtain the model parameters. If the theoretical concepts are modeled as common factors, both composite-based SEM and factor-based SEM can be applied to obtain the model parameters. Therefore, if a model contains both theoretical concepts modeled as composites and theoretical concepts modeled as common factors, composite-based estimators such as PLSc, IGSCA, or the Henseler-Ogasawara specification of the composite model need to be applied to obtain the model parameters.

1.3 Contribution to composite-based structural equation modeling

This thesis contributes to the literature by presenting various improvements in the context of composite-based SEM. Specifically, Chapter 2 provides guidelines for performing Monte Carlo simulations for SEM with PLS-PM and other variance-based estimators in R using the package *cSEM* (Rademaker and Schuberth, 2020), which includes most of the traditional estimators for composited-based SEM such as PLS-PM and GSCA. In addition, these guidelines are illustrated using an example Monte Carlo simulation with ready-to-use R code. Chapter 3 introduces robust versions of PLS-PM and PLSc, namely robust PLS and robust PLSc (Schamberger et al., 2020), respectively. The chapter shows that the traditional versions of PLS-PM and PLSc lead to distorted estimates if the considered sample contains outliers. The chapter demonstrates the performance of robust PLS and robust PLSc using a Monte Carlo simulation comparing their results to those of their traditional counterparts. Robust PLS and robust PLSc provide researchers with composite-based SEM estimators that are less vulnerable to outlier distortion than their traditional counterparts. Chapter 4 presents a procedure to perform out-of-sample predictions using the composite-based SEM estimators ordinal partial least squares (OrdPLS, Cantaluppi and Boari, 2016) and ordinal consistent partial least squares (OrdPLSc, Schuberth and Cantaluppi, 2017; Schuberth et al., 2018b). OrdPLS and OrdPLSc should be used if the observed variables of the constructs are on an ordinal categorical scale. Moreover, performing out-of-sample predictions using model estimates by PLS-PM and PLSc has increasingly gained attention in composite-based SEM literature (Shmueli et al., 2016). Still, existing approaches to performing out-of-sample predictions for model estimates with PLS-PM and PLSc do not allow us to account for the ordinal categorical nature of the observed variables. As a remedy, Chapter 4 provides guidelines on how to perform out-of-sample predictions using OrdPLS and OrdPLSc. This is illustrated by an example using the R package *cSEM*. Chapter 5 contributes to the literature by introducing confirmatory composite analysis (CCA, Henseler and Schuberth, 2021b; Henseler, 2021; Schuberth et al., 2018a) for human development research based on the Henseler-Ogasawara specification (Schuberth, forthcoming) for composite models, which enables the specification of composite models in terms of composite loadings, as well as emergent and excrescent variables. Thus, these kinds of models can be estimated using the common tools of factor-based SEM, such as Mplus (Muthén and Muthén, 1998-2017), and the R package *lavaan* (Rosseel, 2012). Consequently, the parameter estimates can be obtained by ML if the Henseler-Ogasawara specification is used. Finally, Chapter 6 contributes to the composite-based SEM literature by providing a specification of the composite model in terms of weights and a corresponding ML estimator. This ML approach's performance is compared to the corresponding results obtained by PLS-PM in a Monte Carlo simulation. This thesis closes with an epilogue in Chapter 7 providing an overview of the different strands of composite-based SEM and how the previous chapter contributed to the wider spread of composite-based SEM in the literature.

Chapter 2

Conducting Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models: A tutorial using the R package cSEM

2.1 Introduction

Structural equation modeling (SEM) is a popular method in social and behavioral sciences. It allows for modeling relationships between theoretical concepts and between theoretical concepts and observed variables (Bagozzi and Phillips, 1982). Moreover, it is able to take into account various forms of measurement errors (Bollen, 1989). As a consequence, SEM is widely applied in various disciplines such as marketing research (Steenkamp and Baumgartner, 2000), psychology (Fassinger, 1987; MacCallum and Austin, 2000; Higgins, 2002), business management (Hult et al., 2006; Mak and Sockel, 2001), and information systems research (Urbach et al., 2010).

Traditionally, SEM uses covariance-based estimators such as a maximum likelihood (ML, Jöreskog, 1970b) estimator or a generalized least squares (GLS, Jöreskog and Goldberger, 1972) estimator to obtain parameter estimates. Covariance-based estimators obtain parameter estimates by minimizing the discrepancy between the observed variables' model implied and sample variance-covariance matrix. Traditionally, covariance-based estimators were used to estimate structural equation models in which theoretical concepts are modeled as common factors which explain the variance-covariance structure of their related observed variables (Jöreskog, 1970a). However, a recently introduced specification of the composite model in terms of composite loadings (Henseler, 2021; Schuberth, forthcoming) allows us to employ covariance-based approaches to obtain the model parameters for structural equation models with theoretical concepts modeled as composites which are linear combinations of observed variables (Benitez et al., 2020; Henseler, 2017; Hubona et al., 2021).

Besides covariance-based estimators, variance-based estimators which build proxies for the theoretical concepts first and estimate the model parameters based on these proxies afterwards, have been introduced to estimate structural equation models. Traditionally, variance-based estimators such as partial least squares path modeling (PLS-PM, Wold, 1975) or generalized structured component analysis (GSCA, Hwang and Takane, 2004) are used to estimate composite models. Nevertheless, enhancements of these approaches such as consistent partial least squares (PLSc, Dijkstra and Henseler, 2015a) or generalized structured component analysis with uniqueness terms for accommodating measurement error (GSCAm, Hwang et al., 2017) can be used to estimate common factor models.

SEM in general and variance-based estimators for structural equation models in particular, are constantly being refined and new methods are constantly being introduced, such as a new criterion to assess discriminant validity (Roemer et al., 2021), a combination of ML and PLS-PM to obtain parameter estimates (Ghasemy et al., 2021), or an approach to estimate second-order constructs (Schuberth et al., 2020). These need methodological and theoretical justification, e.g., statistical properties such as the bias or the standard errors need to be evaluated. Instead of formally proving statistical properties – which is often difficult or even impossible – a common practice is to provide evidence for these by using Monte Carlo simulations. For this purpose, a series of samples is drawn from a given population and analyzed using the method of interest. To evaluate the estimates, they can be compared to their known population counterparts. The Monte Carlo method was introduced in 1949 by Metropolis and Ulam in physics. With increasing computational power, the Monte Carlo method gained increasing attention and Monte Carlo simulations are now frequently applied in various research fields, including physics (Landau and Binder, 2021), econometrics (Hendry, 1984), biology (Manly, 2018), and medicine (Mode, 2011).

In general, Monte Carlo simulations can be performed in any statistical environment. Nevertheless, R (R Core Team, 2020) is popular among researchers because of its open-source nature. The most popular R package to estimate structural equation models with covariance-based estimators such as ML and GLS is arguably *lavaan* (Rosseel, 2012) for which several tutorials are available (Lee, 2015; Rosseel, 2014). Nevertheless, *lavaan* cannot be used to estimate structural equation models with variance-based estimators such as PLS-PM and GSCA. For that purpose, the R package *cSEM* (Rademaker and Schuberth, 2020) can be used. *cSEM* provides researchers with a tool to estimate structural equation models with PLS-PM, PLSc, GSCA, and other variance-based estimators. Moreover, *cSEM* is accompanied by the R package *cSEM.DGP* to simulate data for predetermined structural equation models (Rademaker and Schamberger, 2020). Thus, it provides users with all the necessary tools to perform Monte Carlo simulations for SEM with variance-based estimators in R. However, as yet, no tutorial is available on how to conduct Monte Carlo simulations for SEM using *cSEM*.

To address this gap, this chapter provides guidelines for performing Monte Carlo simulations for SEM with PLS-PM and other variance-based estimators using the open-source software *cSEM*. To demonstrate the guidelines, I conduct an exemplary Monte Carlo simulation to investigate PLS-PM's and PLSc's finite sample behavior, particularly regarding the consequences of sample correlations among measurement errors on statistical inference. In more detail, this paper evaluates whether a t-test can hold the significance level for a path coefficient estimate of which the population value is zero, and which is estimated by PLS-PM or PLSc if one of the two constructs is assumed to function according to a common factor.

The remainder of the chapter is structured as follows. Section 2.2 gives an overview of Monte Carlo simulations for SEM and existing software tools to perform Monte Carlo simulations for SEM with PLS-PM and other variance-based estimators demonstrating the need for guidelines on Monte Carlo simulations for SEM using *cSEM*. Section 2.3 gives step-by-step guidelines on how to conduct Monte Carlo simulations for SEM using the R package *cSEM*. Section 2.4 illustrates the guidelines by an exemplary Monte Carlo simulation. The chapter closes with a conclusion in Section 2.5.

2.2 The need for further guidelines on Monte Carlo simulations for SEM

"Monte Carlo is the confluence of deterministic, stochastic, and computational methods with computer generated random numbers an important component."(Hurd, 1985) Although it is assumed that the first application of Monte Carlo methods was to estimate the number π, the first published Monte Carlo method was as an approach to answer questions in physics (Metropolis and Ulam, 1949). Originally, the method was introduced as an approach to obtain probabilities that could not be obtained analytically. It is based on the idea of the law of large numbers and other asymptotic theorems of statistics (Metropolis and Ulam, 1949). Monte Carlo simulations are often applied to estimate parameters of interest by using a large number of simulated samples. These are often generated using inverse sampling (Johansen, 2010). Inverse sampling proceeds to generate values according to the quantile function of the considered distribution, such that these follow the considered distribution. For each of the simulated samples, the parameters of interest are estimated. Naturally, "the estimate will never be confined within given limits with certainty, but only - if the number of trials is great - with great probability" (Metropolis and Ulam, 1949). Consequently, the parameter estimates can be evaluated, e.g., in terms of bias, consistency, and efficiency. With increasing computational power, Monte Carlo simulations have gained increasing attention in various research fields – also in the context of SEM.

Monte Carlo simulations for SEM are used for different purposes. First, they are used to demonstrate the performance of new approaches to obtain parameter estimates such as PLSc (Dijkstra and Henseler, 2015b), GSCAm (Hwang et al., 2017), PLSe1 (Huang, 2013), and PLSe2 (Ghasemy et al., 2021) where the bias and standard errors in small samples are evaluated, or of enhancements for model assessment such as goodness of fit indices, tests for the overall model fit (Moshagen, 2012), or bootstrap-based techniques for inference (Jung et al., 2019). These kinds of Monte Carlo simulation are often conducted to demonstrate the performance of an enhancement in the specific situation for which it was developed. Second, Monte Carlo simulations for SEM are often conducted to demonstrate the limitations of SEM approaches in specific situations (Rönkkö and Evermann, 2013). Third, Monte Carlo simulations for SEM are used to compare the performance of different estimators (Reinartz et al., 2009; Hwang et al., 2017).

Since Monte Carlo simulations for SEM are frequently applied, various tools have been proposed to perform such simulations. For example, the commercial software Mplus (Muthén and Muthén, 1998-2017) or LISREL (Jöreskog and Sörbom, 1993) provide tools for data generation and model estimation using covariance-based approaches such as ML or GLS. Besides commercial software, open source software such as the R packages *lavaan* (Rosseel, 2012) and *simsem* (Pornprasertmanit et al., 2021) are available for Monte Carlo simulations for SEM with covariance-based estimators and there are various guidelines on how to perform such Monte Carlo simulations. The R package *lavaan* comprises most of the available approaches for covariance-based estimators, interacts with the R package *simsem* to simulate data, and thus provides users with all the necessary tools to perform Monte Carlo simulations for SEM if covariance-based estimators are used to obtain the model parameters. Nevertheless, these tools cannot be used to perform Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models.

Variance-based estimators for structural equation models such as PLS-PM or GSCA gained traction over the last two decades, thus, Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models, increasingly gained attention. Nevertheless, compared to covariance-based estimators, existing software tools for this type of SEM are still limited. Consequently, one rarely comes across guidelines for Monte Carlo simulations for PLS-PM and other variance-based estimators for structural equation models in the literature. While available commercial software such as ADANCO (Henseler, 2019) or SmartPLS (Ringle et al., 2015) do not provide tools for generating data and thus cannot directly be applied to conduct a Monte Carlo simulation, the R package *matrixpls* (Rönkkö, 2021) can be used to obtain parameter estimates with variance-based estimators such as PLS-PM, PLSc and GSCA. The package interacts with the R package *simsem* to simulate data and can thus also be used for Monte Carlo simulations for SEM. Nevertheless, *matrixpls* does not provide users with the possibility of obtaining model parameters with approaches such as Kettenring's (1971) approaches for generalized canonical correlation analysis (GCCA). Additionally, higher-order constructs (Schuberth et al., 2020) or non-linear structural equation models containing higher-order terms (Dijkstra and Schermelleh-Engel, 2014) cannot be estimated using *matrixpls*. While most available software for SEM uses the data as input to obtain parameter estimates, *matrixpls* uses a variance-covariance matrix as input to obtain parameter estimates, which makes it less flexible in terms of evaluating a model's prediction performance.

As an alternative, the R package *cSEM* was introduced. This package comprises a majority of variance-based estimators for SEM and is well established (Rademaker and Schuberth, 2020). To elaborate, researchers using *cSEM* are provided with several possibilities to determine standard errors of the parameter estimates, such as jackknife (Tukey, 1958) or bootstrap (Efron, 1979). Moreover, nonlinear structural equation models and models containing higher-order constructs can be estimated. Besides model estimation, *cSEM* provides users with the possibility of identifying inadmissible solutions, performing out-of-sample predictions, and assessing the models. Finally, it is accompanied by the R package *cSEM.DGP* (Rademaker and Schamberger, 2020) which was designed to simulate data for predefined structural equation models. Consequently, *cSEM* provides users with all the necessary tools to perform Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models in R, which is why I provide guidelines for applying these tools in the following section.

2.3 Monte Carlo simulation for SEM using the R package cSEM

To support researchers in conducting Monte Carlo simulations for SEM with variance-based estimators using the R package *cSEM*, I present guidelines in this section by explaining the different steps that need to be performed, namely determining the simulation study's objective, determining the underlying population, determining other simulation parameters, determining the expectations related to the simulation results, generating samples according to the simulation study's design, estimating the model parameters based on the generated samples, and evaluating the results of the Monte Carlo simulation as displayed in Figure 2.1.

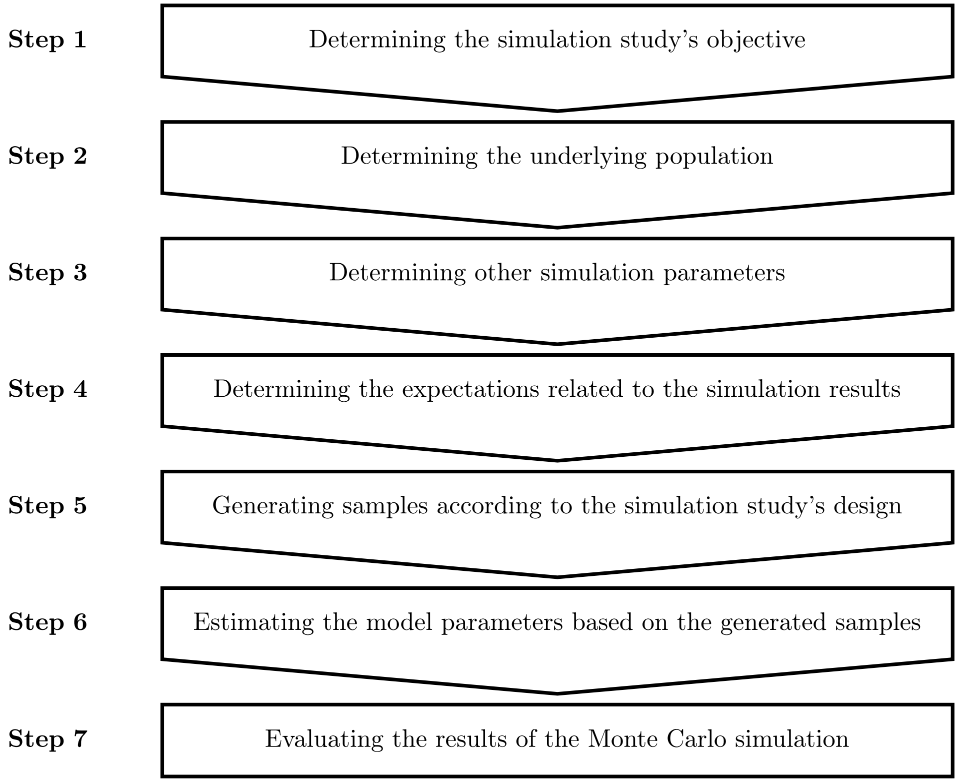

Figure 2.1: Steps to perform a Monte Carlo simulation with variance-based estimators using the R package *cSEM*

2.3.1 Determining the objective of the Monte Carlo simulation

The starting point of a Monte Carlo simulation is to determine its objective. Considering PLS-PM and other variance-based estimators, most Monte Carlo simulations have one of the following objectives: (i) to demonstrate the performance of a new methodology (e.g., Dijkstra and Henseler, 2015a; Henseler et al., 2012; Henseler and Sarstedt, 2013; Henseler et al., 2015, 2016b; Klesel et al., 2019); (ii) to compare the results of different estimators (e.g., Dijkstra and Henseler, 2015a; Henseler and Chin, 2010; Henseler and Sarstedt, 2013; Reinartz et al., 2009); (iii) to evaluate the performance of an existing approach in a specific situation (e.g., Henseler, 2010; Henseler et al., 2012, 2015; Rönkkö and Evermann, 2013).

2.3.2 Determining the underlying population

Structural equation models consist of two parts: (i) equations describing the relations between the constructs, and (ii) equations describing the relations between the constructs and their related observed variables. The relations between the constructs can be written as follows:

$$\boldsymbol{\eta} = \boldsymbol{B}\boldsymbol{\eta} + \boldsymbol{\zeta} \tag{2.1}$$

where $\boldsymbol{\eta}$ is a vector of the constructs η_j, $\boldsymbol{B}$ contains the corresponding path coefficients, and $\boldsymbol{\zeta}$ is a vector of structural error terms.

Besides the relations between the constructs, two different types of relations between the constructs and their related observed variables can be distinguished. First, if the theoretical concepts are modeled as common factors which explain the variance-covariance structure of their related observed variables, the relations between the construct η_j and its observed variables $\boldsymbol{x}_j$ are represented in terms of loadings:

$$\boldsymbol{x}_j = \boldsymbol{\lambda}_j \eta_j + \boldsymbol{\varepsilon}_j \tag{2.2}$$

The observed variables related to the $j-$th construct are stored in a block of observed variables $\boldsymbol{x}_j$, with $\boldsymbol{\lambda}_j$ as a vector of loadings and $\boldsymbol{\varepsilon}_j$ as a vector of measurement errors. Second, the theoretical concept can be modeled as a composite which emerges from its observed variables and conveys all information between its observed variables and the other variables in the model. Composites are linear combinations of observed variables:

$$\eta_j = \boldsymbol{w}_j' \boldsymbol{x}_j \tag{2.3}$$

The vector $\boldsymbol{x}_j$ represents the block of observed variables related to the composite η_j, and $\boldsymbol{w}_j$ is a vector of the corresponding weights.

The concrete population is directly influenced by the objective of the Monte Carlo simulation, and all the population model's parameters should be determined in accordance with the Monte Carlo simulation's objective. In more detail, Figure 2.2 shows the relevant steps to determine the underlying population. To evaluate the influence of the population model on the simulation results, researchers can consider different population models for their simulation study.

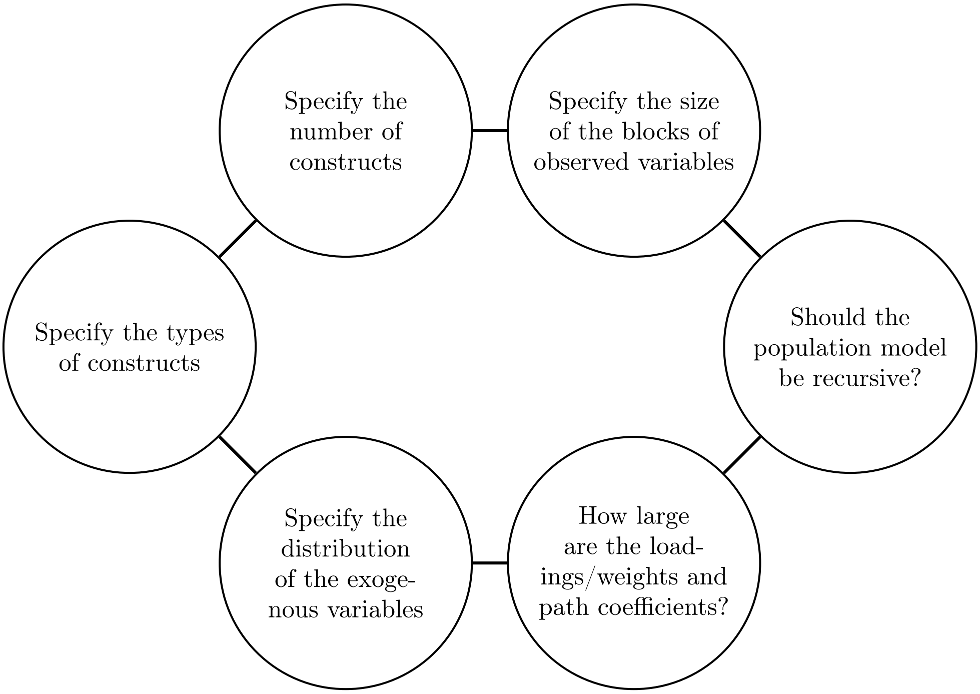

Figure 2.2: How to specify a population

Once the model has been determined, it needs to be specified in lavaan syntax for the later estimation. To specify a model in lavaan syntax, it is specified as a string in which the structural relations are determined using $\sim$, correlations are determined using $\sim\sim$, relations between common factors and their observed variables are determined using $=\sim$, and relations between composites and their observed variables are determined using $<\sim$.

As Figure 2.2 shows, the distribution of the exogenous variables needs to be determined. Using this distribution and the population relations between the variables, the distribution of the observed variables can be determined. For simplicity, it is customary to assume a multivariate standard normal distribution for the exogenous variables (e.g., Jannoo et al., 2014; Jung et al., 2019; Rademaker et al., 2019), therefore, samples can be generated by using the population variance-covariance matrix.

As an alternative to calculating the population variance-covariance matrix by hand, the population correlation matrix can be obtained using the R package *cSEM.DGP* (Rademaker and Schamberger, 2020). In *cSEM.DGP*, several assumptions about the error terms are imposed. First, the structural error terms ζ_k are assumed to be uncorrelated with those constructs η_j that solely occur in an exogenous position and thus are not explained through other constructs. Second, the measurement errors $\boldsymbol{\varepsilon}_j$ are assumed to be uncorrelated among one another, with the measurement errors of other constructs and the structural error terms. To obtain the population correlation matrix, in a first step the model has to be specified in lavaan syntax including the population parameters. This population model has to be used as input for the function `generateData()` to obtain the population correlation matrix. Further, the argument .return_type of the `generateData()` function needs to be set to "cor" and the argument .empirical needs to be set to "TRUE" to obtain the considered matrix.

2.3.3 Determining other simulation parameters

Besides determining the population for the Monte Carlo simulation, other simulation parameters have to be specified. The simulation parameters that need to be considered depend on the simulation study's objective. Nevertheless, some parameters need to be considered for all simulation studies:

- Sample size:

  For a Monte Carlo simulation, samples need to be drawn from the above described population. To provide empirical evidence for statistical properties such as consistency or asymptotic efficiency, large samples should be considered. Moreover, Monte Carlo simulations are often applied to evaluate the estimator's finite sample behavior. Thus, sample sizes of 200 and 500 observations are commonly used for Monte Carlo simulations with PLS-PM and other variance-based estimators. (Dijkstra and Henseler, 2015b; Jung et al., 2019). To evaluate the effect of the sample size on the performance of an approach, different sample sizes should be considered.

- Number of draws:

  As the Monte Carlo principle relies on the law of large numbers, several samples need to be drawn and used to estimate the model parameters. A high number of draws implies estimates that are more precise (Metropolis and Ulam, 1949). However, a high number of draws implies a longer computational time. For simulation studies with variance-based estimators for structural equation models, it is common to set the number of draws to 500 (Hwang et al., 2010; Goodhue et al., 2012; Jung et al., 2019) or 1000 (Aguirre-Urreta and Rönkkö, 2015; Dijkstra and Henseler, 2015b).

2.3.4 Determining the expectations related to the simulation results

The fourth step of a Monte Carlo simulation with variance-based estimators for structural equation models is to determine the expectations regarding the simulation results. The expectations are closely related to the Monte Carlo simulation's objective. Nevertheless, the expectations should be determined when the population is specified to be able to determine expectations about all parameters of the Monte Carlo simulation. In general, formulating the expectations helps to check whether the simulation design, i.e., the chosen population and other simulation parameters, fits the simulation study's objective or whether the design needs to be adjusted.

2.3.5 Generating samples according to the simulation study's design

Once the design of the Monte Carlo simulation has been specified, samples need to be generated according to the population and the other simulation parameters. Using *cSEM* the easiest way to simulate data according to the simulation design, i.e., the population model and other simulation parameters, is to use the `generateData()` function of the package *cSEM.DGP*. In doing so, samples from a multivariate normal distribution and also from a distribution with predefined values for the skewness and kurtosis can be drawn:

`generateData`(.model, .N, .skewness, .kurtosis, .return_type)

To obtain samples from a distribution with predefined values for skewness and kurtosis and thus nonnormally distributed samples, the Fleishman-Vale-Maurelli procedure (Fleishman, 1978; Vale and Maurelli, 1983) is used. The argument .model contains the population model in lavaan syntax including the population parameters as described above. The argument .N equals the corresponding sample size and .skewness and .kurtosis can be used if a distribution of the observed variables different to the multivariate normal distribution is considered. As default, .skewness and .kurtosis are set to the values of the normal distribution, i.e. to a skweness of 0 and a kurtosis of 3. The argument .return_type determines the requested output format. As default it is set to „data.frame", thus, it does not need to be adjusted to obtain a simulated sample. Note that *cSEM.DGP* is still limited in terms of model complexity, i.e., considering the number of concepts that can be taken into account. If the defined population model is not supported by *cSEM.DGP*, samples according to the simulation design can be drawn by using other well developed R packages such as *simsem*. For more flexibility considering data generation, the R package *covsim* (Grønneberg and Foldnes, 2017), for example, could be used. As an alternative, samples can be drawn by using the corresponding quantile function of the considered distribution.

2.3.6 Estimating the model parameters based on the generated samples

Once the samples have been drawn according to the simulation design, the model parameters need to be estimated. Using the R package *cSEM*, model estimation is done using the function `csem()`, which has a variety of possible arguments. In its simplest form, `csem()` only requires a sample (.data) and a model in lavaan syntax (.model):

`csem`(.data, .model)

Added to this, `csem()` has a variety of optional arguments. I will focus on the optional arguments that are relevant for the exemplary Monte Carlo simulation that I will show later. The optional arguments all have default values and thus only need further definition if different options should be used. For a detailed explanation of all optional arguments and their corresponding default values, please refer to the manual of the *cSEM* package (Rademaker and Schuberth, 2020).

- *.approach_weights*: defines the approach that is used to obtain the weights of the composites. The default is set to "PLS-PM".
- *.disattenuate*: defines whether the construct correlations should be disattenuated in order to get consistent estimates for the loadings and path coefficients in case of concepts modeled as common factors. The default is set to "TRUE".
- *.PLS_weight_scheme_inner*: defines the inner weighting scheme that is used during the PLS-PM algorithm. The default is set to the path weighting scheme ("path").
- *.PLS_modes*: defines the PLS-PM mode that should be used for estimating the weights for each construct. The default is set to "NULL", and consequently, for common factors PLS-PM Mode A is used, and for composites PLS-PM Mode B is used.
- *.resample_method*: defines the resample method that is used. The default is set to "NULL", and consequently, no resampling is done and no standard errors are provided.
- *.R*: defines the number of resampling replications. The default is set to 499.
- *.seed*: defines the seed that is used for the resampling. If no seed is provided, a random seed is produced.
- *.handle_inadmissibles*: defines how inadmissible results that appear during the resampling should be treated. Inadmissible results are results which did not converge, where at least one standardized loading is larger than 1, where the construct correlation matrix is not positive semi-definite, at least one reliable estimate is larger than 1, or where the observed variables' population variance-covariance matrix is not positive definite. The default is set to "drop". Consequently, if inadmissible solutions occur during the resampling, the final results can be based on less than 499 resamples.

The `csem()` result is a list with the parameter estimates and further information about the estimation. Moreover, several post estimation functions like `assess()`, `infer()`, `predict()`, `summarize()` and `verify()` can be applied. The post-estimation function `verify()` checks for inadmissible results. An inadmissible result indicates a problem with the model for the available sample. Consequently, depending on the simulation study's objective, inadmissible results could be removed from the simulation and replaced with admissible counterparts.[1] The post-estimation function `summarize()` gives a summary of the estimation results, including the parameter estimates and their standard errors – if resampling was applied. For an explanation of the other post-estimation functions, please refer to Rademaker and Schuberth (2020).

2.3.7 Evaluating the results of the Monte Carlo simulation

As a last step of a Monte Carlo simulation with PLS-PM and other variance-based estimators, the results need to be evaluated. The type of evaluation depends highly on the Monte Carlo simulation's objective, therefore, general guidelines cannot be provided. Nevertheless, some possibilities are given in the following exposition.

If an approach to obtain parameter estimates is evaluated, statistical properties, such as the bias, consistency, and efficiency should often be evaluated. The bias can be estimated by comparing the average parameter estimates to their population values: $\frac{1}{n_{draws}} \sum_{i=1}^{n_{draws}} \hat{\theta}_i - \theta_i$. To provide evidence for the consistency of parameter estimates, one often evaluates whether the bias decreases for increasing sample sizes and whether estimations based on very large samples – for example, 100'000 observations – show any bias. Further, the efficiency of parameter estimates obtained by different approaches is evaluated by comparing the standard errors for different sample sizes. To evaluate both the bias and the variance jointly, measures such as the root mean squared error (RMSE) or the mean absolute error (MAE) are applied. Additionally, visualizing the results by using barplots, density plots, or similar can improve the comprehensibility of the results.

Besides evaluating the results in terms of numbers and statistical properties, the results should also be compared to the expectations about the results that were formulated at the beginning of the simulation study. Moreover, expectations that were not met should be justified, and where other expectations were met, those reasons should also be given.

[1] Note that in terms of good research practice, the share of inadmissible results generated through the simulation study should be reported since a huge share of inadmissible results indicates that the corresponding estimator is problematic in the considered research situation.

2.4 Example: The consequences of sample correlations among measurement errors on statistical inference concerning the PLS-PM results

To illustrate the guidelines to conducting Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models using the R package *cSEM*, I conducted an exemplary Monte Carlo simulation.[2]

2.4.1 Determining the objective

The exemplary Monte Carlo simulation investigates PLS-PM's and PLSc's finite sample behavior, particularly regarding the consequences of sample correlations among measurement errors on statistical inference. Since correlations of zero are hardly ever found in empirical research, PLS-PM's assumption of uncorrelated error terms can yield a bimodal distribution of the parameter estimates in finite samples. This effect has previously been discussed in the literature (Rönkkö, 2014). The illustrative Monte Carlo simulation presented here, evaluates the effect of this assumption in empirical research on an extreme case with uncorrelated constructs. Specifically, the study evaluates whether a t-test regarding a path coefficient estimate which is zero in the population is able to hold the level of significance if the parameter estimates are obtained with PLS-PM and PLSc if one construct is modeled as a composite and one construct is modeled as a common factor. Additionally, we evaluate the guidelines that propose that PLS-PM and PLSc need a nomological net to provide valuable estimates, and thus that the constructs should not be isolated (Henseler et al., 2014). Consequently, we evaluate whether the bimodal distribution of the parameter estimates vanishes and whether the t-test is able to hold its significance level if the model is embedded in more contexts.

The illustrative Monte Carlo simulation's objective can be classified using the classes described above. Since no enhancement is introduced, but two approaches are compared and their performance is evaluated in a specific situation, the exemplary Monte Carlo simulation can be classified in the second and third group of Monte Carlo simulations for SEM.

[2] Note that the results of this section are obtained using the *cSEM* package version 0.2.0.

2.4.2 Underlying Population

Some parts of the underlying population are directly motivated by the Monte Carlo simulation's objective. Since we consider an extreme case of two uncorrelated constructs in which one construct is modeled as a common factor and another is modeled as a composite, the types of constructs are already specified. Further, the exogenous construct is modeled as a common factor and the endogenous construct is modeled as a composite. The population model should be non-recursive, and exactly two constructs are included. Also, the two constructs are related to three observed variables. Consequently, the following structural relation is considered:

$$\eta_2 = 0.0 \cdot \eta_1 + \zeta_2 \tag{2.4}$$

We use the following relations between the observed variables and their associated constructs:

$$x_{11} = 0.9 \cdot \eta_1 + \varepsilon_{11} \tag{2.5}$$

$$x_{12} = 0.8 \cdot \eta_1 + \varepsilon_{12} \tag{2.6}$$

$$x_{13} = 0.7 \cdot \eta_1 + \varepsilon_{13} \tag{2.7}$$

$$\eta_2 = 0.6 \cdot x_{21} + 0.4 \cdot x_{22} + 0.2 \cdot x_{23} \tag{2.8}$$

The random measurement errors are assumed to be uncorrelated and ζ_2 is assumed to be uncorrelated with both η_1 and the measurement errors. The variances of the measurement errors of the first construct η_1 are set to 0.19, 0.36, and 0.51, respectively. The correlations between the observed variables of the composite are set to 0.5 each, such that the composite η_2 has unit variance. The observed variables are assumed to be multivariate normally distributed. Figure 2.3 displays the population model.

Once the population model has been determined, the model has to be specified in lavaan syntax as input for the later simulation:

```
model <- '
# Relations between the constructs and the observed variables
eta1 =~ x11 + x12 + x13
eta2 <~ x21 + x22 + x23

# Structural relations
eta2 ~ eta1'
```

The observed variables' population variance-covariance matrix depends on the loadings, the intra-block correlations of the composite's observed variables, the mea-

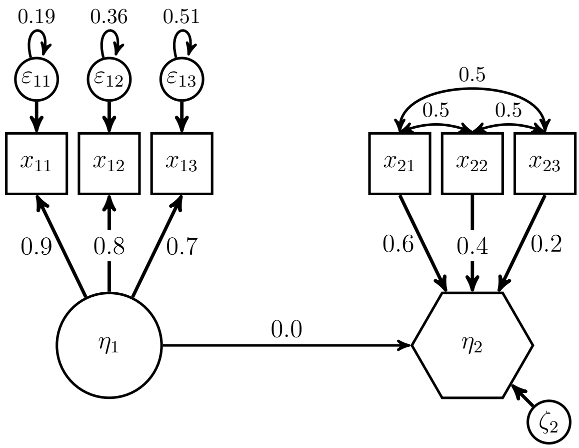

Figure 2.3: Population model with two constructs

surement errors' variances, and the structural relations. Note that *cSEM* as well as *cSEM.DGP* rely on correlations and not on variances and covariances. Due to the zero path between the constructs η_1 and η_2, the correlations between the observed variables associated with η_1 and η_2 are zero. The correlations between the observed variables of a common factor can be obtained as follows:

$$\text{Cov}(x_{ji}, x_{jk}) = \lambda_{ji} \cdot \lambda_{jk}$$

The correlations between the observed variables of a composite are given in the model definition. Consequently, the observed variables' population correlation matrix is given as follows:

$$\boldsymbol{\Sigma} = \begin{array}{c} \begin{array}{cccccc} \underline{x_{11}} & \underline{x_{12}} & \underline{x_{13}} & \underline{x_{21}} & \underline{x_{22}} & \underline{x_{23}} \end{array} \\ \begin{pmatrix} 1.00 & & & & & \\ 0.72 & 1.00 & & & & \\ 0.63 & 0.56 & 1.00 & & & \\ 0.00 & 0.00 & 0.00 & 1.00 & & \\ 0.00 & 0.00 & 0.00 & 0.50 & 1.00 & \\ 0.00 & 0.00 & 0.00 & 0.50 & 0.50 & 1.00 \end{pmatrix} \end{array} \tag{2.9}$$

As explained above, the observed variables' population correlation matrix can be obtained using the R package *cSEM.DGP* by using the model including the population parameters as input for the `generateData()` function:

```r
library(cSEM.DGP)
model_dgp <- '
# Relations between the constructs and the observed variables
eta1 =~ 0.9*x11 + 0.8*x12 + 0.7*x13
eta2 <~ 0.6*x21 + 0.4*x22 + 0.2*x23

# Structural relations
eta2 ~ 0.0*eta1

# Intra-block correlations
x21 ~~ 0.5*x22 + 0.5*x23
x22 ~~ 0.5*x23'

Sigma <- generateData(.model = model_dgp, .return_type = "cor", .empirical = TRUE)
```

To evaluate the influence of the population model on the simulation results, as well as the guidelines that state PLS-PM and PLSc need the constructs to be embedded in a nomological net and not to be isolated, we considered a second population model. The relation between η_1 and η_2, as well as how they relate to their observed variables remain the same as for the first population model. In addition, both constructs are connected to one composite each with a non zero path. The following relations between the constructs η_1 and η_2 and the new composites η_3 and η_4 are considered:

$$\eta_2 = 0.0 \cdot \eta_1 + 0.3 \cdot \eta_3 \tag{2.10}$$

$$\eta_1 = 0.3 \cdot \eta_4 \tag{2.11}$$

Both composites η_3 and η_4 are assumed to be composed of three indicators each. The population model is displayed in Figure 2.4. The observed variables are assumed to be multivariate normally distributed. Note that the population correlation matrix, as well as the R code for simulating the model with four constructs, are given in the Appendix.

2.4.3 Determining other simulation parameters

Considering the rest of the simulation design, the following parameters are chosen: First, two sample sizes, namely 200 and 500 observations, are considered to evaluate the estimators' finite sample behavior. Second, the number of draws is set to 500. Consequently, in total two estimators, two population models with two sample sizes each, and one number of draws, i.e., eight conditions, are considered.

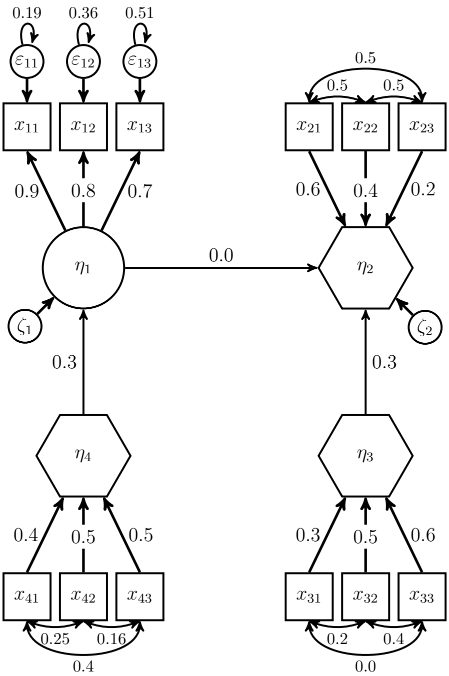

Figure 2.4: Population model with four constructs

2.4.4 Expectations regarding the simulation results

Considering the situation outlined above, we expected that the t-test would not hold the significance level for the path coefficient in the case of the population model with two constructs. Further, we expected that the parameter estimates would show a bimodal distribution in this case, which would be in line with previous findings in the literature (Rönkkö, 2014). For the population model with four constructs, we expected that the bimodal form of the distribution would vanish for the path coefficient between η_1 and η_2. Further, we assumed that the t-test is able to hold its significance level in this case, which would also be in line with previous findings in the literature that showed that PLS-PM and PLSc need the constructs to be embedded in a nomological net (Henseler et al., 2014). Considering the two sample sizes, namely 200 and 500 observations, we expected that the results would become more accurate in the sense that the bias as well as the standard errors would decrease.

2.4.5 Data generation

Using the *cSEM.DGP* package, we generated samples with 200 observations by first determining the population model including the model parameters in lavaan syntax, and afterwards using the `generateData()` function:

```
library(cSEM.DGP)
model_dgp <- '
# Relations between the constructs and the observed variables
eta1 =~ 0.9*x11 + 0.8*x12 + 0.7*x13
eta2 <~ 0.6*x21 + 0.4*x22 + 0.2*x23

# Structural relations
eta2 ~ 0.0*eta1

# Intra-block correlations
x21 ~~ 0.5*x22 + 0.5*x23
x22 ~~ 0.5*x23'

data <- generateData(.model = model_dgp, .N = 200)
```

Since the observed variables are assumed to be multivariate normally distributed, the other arguments of `generateData()` do not have to be adjusted from their defaults.

Note that according to the simulation design, several samples need to be simulated. These can either all be simulated first and estimated afterwards or each sample can be estimated before simulating a new sample. The samples with 500 observations are simulated similarly by setting the argument .N to 500.

2.4.6 Model estimation

To investigate PLS-PM's and PLSc's finite sample behavior, particularly regarding the consequences of sample correlations among measurement errors on statistical inference, the model described above in lavaan syntax and the simulated samples need to be used as input for the `csem()` function. Additionally, the standard errors of the path coefficient estimates need to be obtained. Consequently, the argument .resample_method needs to be adjusted from its default. For example, the standard errors can be obtained using a bootstrap approach:

`csem`(.data, .model, .resample_method = “bootstrap”)

To ensure that all results rely on the same number of bootstrap samples and that all bootstrap results taken into account are in fact admissible, the argument .handle_inadmissibles needs to be set to "replace". Also, `csem()` uses a disattenuation for common factors as default. To obtain PLS-PM estimates instead of PLSc estimates in the case of common factors, the argument .disattenuate needs to be set to "FALSE". The argument .seed is set to "123" to ensure that the bootstrap results can be reproduced. The other optional arguments are not changed from their defaults. Consequently, the following two commands provided the requested estimates:

```
# Estimation of the model based on the simulated sample
res_PLSc <- csem(.model = model, .data = data, .resample_method = "bootstrap",
                 .handle_inadmissibles = "replace", .seed = 123)

# Estimation of the model based on the simulated sample without disattenuation
res_PLS <- csem(.model = model, .data = data, .resample_method = "bootstrap",
                .disattenuate = FALSE, .handle_inadmissibles = "replace",
                .seed = 123)
```

In the illustrative Monte Carlo simulation, for each sample size, 500 samples needed to be simulated and estimated using PLSc and PLS-PM using the simulation design. To do so, we applied the following code:

```
library(cSEM)
library(cSEM.DGP)
# Define model for the data generation
model_dgp <- '
# Relations between the constructs and the observed variables
eta1 =~ 0.9*x11 + 0.8*x12 + 0.7*x13
eta2 <~ 0.6*x21 + 0.4*x22 + 0.2*x23

# Structural relations
eta2 ~ 0.0*eta1

# Intra-block correlations
x21 ~~ 0.5*x22 + 0.5*x23
x22 ~~ 0.5*x23'

# Define model for the parameter estimation
model <- '
# Relations between the constructs and the observed variables
eta1 =~ x11 + x12 + x13
eta2 <~ x21 + x22 + x23

# Structural relations
eta2 ~ eta1'

# Define lists to store the simulation results
res_PLS <- list()
res_PLSc <- list()
set.seed(123)
while(i < 501){
  data <- generateData(.model = model_dgp, .N = 200)
  res_PLSc_temp <- csem(.model = model, .data = data)
  res_PLS_temp <- csem(.model = model, .data = data, .disattenuate = FALSE)

  if(sum(verify(res_PLSc_temp)) == 0 && sum(verify(res_PLS_temp)) == 0){
    res_PLSc[[i]] <- csem(.model = model, .data = data, .resample_method = "bootstrap",
                          .handle_inadmissibles = "replace", .seed = 123)
    res_PLS[[i]] <- csem(.model = model, .data = data, .resample_method = "bootstrap",
                         .disattenuate = FALSE, .handle_inadmissibles = "replace",
                         .seed = 123)
    i <- i+1
}
```

Note that we included only admissible results in the results. Thus, if estimating one of the generated samples yields an inadmissible result, this is replaced by an admissible one. Consequently, all results are based on 500 admissible solutions with 499 admissible bootstrap runs each. To obtain the simulation results for the population model with two constructs and a sample size of 500, the argument .N has to be set to 500. I give the code that needs to be run to obtain the results for the population model with four constructs in the Appendix.

2.4.7 Evaluating the simulation results

To evaluate the results, in a first step, the share of significant path coefficient estimates has to be determined for the various sample sizes and population models. Note that for the larger model with four constructs, only the path coefficient between the common factor η_1 and the composite η_2 is considered to be able to evaluate the population model's effect on the test's performance regarding significance for this path coefficient. The other parameters are not considered here.

Using bootstrap, standard errors $\hat{\sigma}_{\hat{b}_{21}}$ were obtained for all path coefficient estimates. These estimated standard errors can be used to test the null hypothesis: $H_0 : b_{21} = 0$ for each estimation, achieved by using the t-test statistic

$$t = \frac{\hat{b}_{21}}{\hat{\sigma}_{\hat{b}_{21}}} \tag{2.12}$$

which is also provided in the output of the `csem()` function. The test statistic is asymptotically standard normally distributed. The significance level α is set to 0.01, 0.05, and 0.1, respectively. To obtain the p-value, the asymptotic distribution of the test statistic was used. The p-value of the test statistic is compared to the level of significance to decide whether the structural coefficient is significant at the given level of significance. If the share of significant coefficients is smaller or equal to the level of significance, the test was able to hold the level of significance. Besides evaluating the share of significant path coefficients in terms of a t-test, we could also obtain the share of significant path coefficients using bootstrap confidence intervals. Since the share of significant path coefficients was similar if confidence intervals were used, we do not report the results here. Nevertheless, I give a table with the corresponding results in the Appendix.

The results of the illustrative Monte Carlo simulations have been stored in lists where each list element contains the results for one simulated sample. As explained earlier, every result of the `csem()` function is a list, and several post-estimation functions can be applied. The post-estimation function `summarize()` yields an overview of the estimation, and the parameter estimates including their estimated standard errors, t-statistics, and p-values:

```
summarize(res_PLS[[1]])
```

```
## --------------------------------------------------------------------------------
## ----------------------------------- Overview -----------------------------------
##
##  General information:
##  ------------------------
##  Estimation status                  = Ok
##  Number of observations             = 200
##  Weight estimator                   = PLS-PM
##  Inner weighting scheme             = "path"
##  Type of indicator correlation      = Pearson
##  Path model estimator               = OLS
##  Second-order approach              = NA
##  Type of path model                 = Linear
##  Disattenuated                      = No
##
##  Resample information:
##  ---------------------
##  Resample method                    = "bootstrap"
##  Number of resamples                = 499
##  Number of admissible results       = 499
##  Approach to handle inadmissibles = "replace"
##  Sign change option                 = "none"
##  Random seed                        = 123
##
##  Construct details:
##  ------------------
##  Name  Modeled as     Order         Mode
##
##  eta1  Common factor  First order   "modeA"
##  eta2  Composite      First order   "modeB"
##
## ----------------------------------- Estimates ----------------------------------
##
## Estimated path coefficients:
## ============================
##                                                              CI_percentile
##   Path           Estimate  Std. error   t-stat.   p-value         95%
##   eta2 ~ eta1      0.1064      0.1850    0.5754    0.5650 [-0.2610; 0.2700 ]
```

```
## Estimated loadings:
## ===================
##                                                               CI_percentile
##   Loading        Estimate  Std. error   t-stat.   p-value         95%
##   eta1 =~ x11     -0.8789      0.4158   -2.1134    0.0346 [-0.8822; 0.9642 ]
##   eta1 =~ x12     -0.8187      0.3950   -2.0726    0.0382 [-0.8250; 0.9389 ]
##   eta1 =~ x13     -0.2443      0.3767   -0.6484    0.5167 [-0.6913; 0.9751 ]
##   eta2 =~ x21     -0.8635      0.4904   -1.7609    0.0783 [-0.6115; 0.9882 ]
##   eta2 =~ x22      0.0251      0.4233    0.0592    0.9528 [-0.4657; 0.9735 ]
##   eta2 =~ x23     -0.4103      0.3718   -1.1033    0.2699 [-0.3613; 0.9440 ]
##
## Estimated weights:
## ==================
##                                                               CI_percentile
##   Weight         Estimate  Std. error   t-stat.   p-value         95%
##   eta1 <~ x11     -0.7687      0.3447   -2.2302    0.0257 [-0.7026; 0.8455 ]
##   eta1 <~ x12     -0.5355      0.2619   -2.0442    0.0409 [-0.4618; 0.6801 ]
##   eta1 <~ x13      0.4664      0.4096    1.1388    0.2548 [-0.8339; 1.1183 ]
##   eta2 <~ x21     -1.0299      0.6980   -1.4754    0.1401 [-1.0375; 1.1659 ]
##   eta2 <~ x22      0.6202      0.6930    0.8950    0.3708 [-1.0602; 1.2290 ]
##   eta2 <~ x23     -0.2319      0.6089   -0.3808    0.7034 [-0.9968; 1.2552 ]
##
## Estimated indicator correlations:
## =================================
##                                                               CI_percentile
##   Correlation    Estimate  Std. error   t-stat.   p-value         95%
##   x21 ~~ x22       0.4554      0.0530    8.6003    0.0000 [ 0.3523; 0.5586 ]
##   x21 ~~ x23       0.5008      0.0535    9.3572    0.0000 [ 0.3885; 0.5926 ]
##   x22 ~~ x23       0.5440      0.0521   10.4400    0.0000 [ 0.4334; 0.6405 ]
##
## ----------------------------------- Effects ------------------------------------
##
## Estimated total effects:
## ========================
##                                                                CI_percentile
##   Total effect     Estimate  Std. error   t-stat.   p-value         95%
##   eta2 ~ eta1        0.1064      0.1850    0.5754    0.5650 [-0.2610; 0.2700 ]
## ________________________________________________________________________________
```

Consequently, the p-value corresponding to the t-statistic of Equation 2.12 can be extracted by using "summarize(res_PLS[[i]])$Estimates$path_estimates$p_value". The complete results are displayed in Figure 2.5 using a bar plot. Also, the concrete results in terms of numbers, are given in the Appendix. Further, besides the results in terms of test statistics, we give the results using confidence intervals to detect significant path coefficients in the Appendix.

The results show that for the model with two constructs, the t-test for the path coefficient is not able to hold the level of significance. For both sample sizes and all levels of significance we detected higher shares of significant path coefficients for models estimated with PLS-PM and PLSc. Consequently, significant relations between the two constructs are detected too often, which can lead to false conclusions. If the

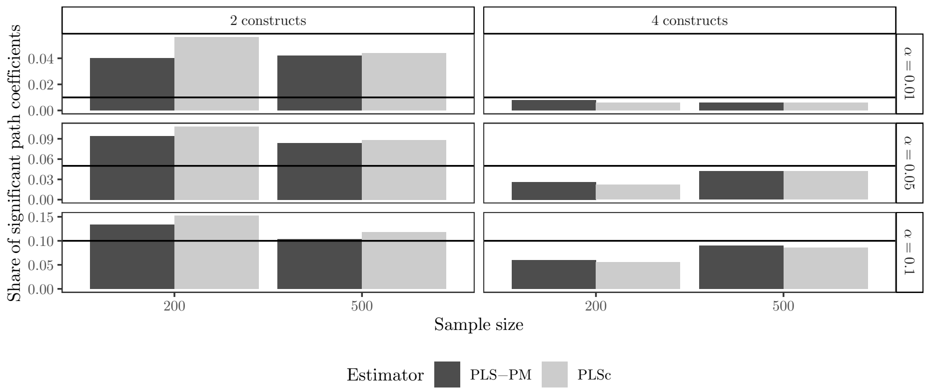

Figure 2.5: Share of significant path coefficients between η_1 and η_2

model with four constructs, i.e., with a higher context, is considered, the significance test for the path coefficient is able to hold the level of significance for all values of α. The results for models estimated with PLS-PM and PLSc only, hardly differ at all.

Eventually, Figure 2.6 shows the density for the path coefficient between η_1 and η_2 for the different simulation conditions. The results demonstrate that for the model with two constructs, the path coefficients show the bimodal distribution as expected. In contrast, for the model with four constructs, the results do not have the bimodal distribution. This is in line with previous findings in the literature which state that PLS-PM and PLSc need models with more context to yield valuable results (Henseler et al., 2014).

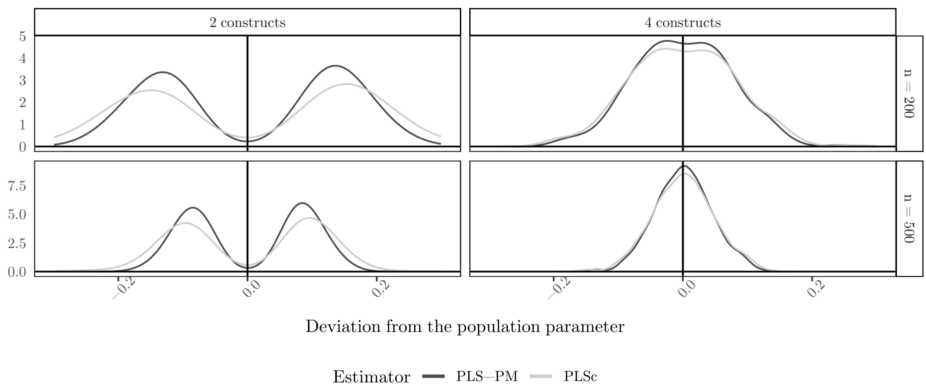

Figure 2.6: Density for the path coefficients

As stated before, we ensured that all simulation results only consider admissible results. Therefore, in the analysis we removed all inadmissible results from the results we used. Nevertheless, the shares of inadmissible solutions are displayed in Table 2.1[3]. This shows that for the model with two constructs which are connected through a zero path, more than 50% of the simulated samples result in an inadmissible solution. Moreover, for the model with four constructs, the share of inadmissible solutions is much lower and also decreases with increasing sample size. High shares of inadmissible results indicate a problem with the considered model and thus, we find support for the hypothesis that PLS-PM and PLSc need the constructs to be embedded in a nomological net. The results of the Monte Carlo simulation are completely in line with the expectations that were described above, and thus are also in line with previous findings in the literature.

Table 2.1: Share of inadmissible solutions

sample size	number of constructs	share of inadmissible solutions
200	2	0.5292
500	2	0.5136
200	4	0.1379
500	4	0.0512

2.5 Conclusion

This chapter has presented guidelines for performing Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models using the R package *cSEM*. First, the need for further guidelines on Monte Carlo simulations for SEM is explained. Second, step-by-step guidelines for how to conduct a Monte-Carlo simulation with PLS-PM and other variance-based estimators for structural equation models are given. The guidelines are accompanied by an illustrative example and the corresponding ready-to-use R code. Moreover, suggestions for evaluating the results and the general design for this type of Monte Carlo simulation, are provided. The latter are based on commonly used design patterns for Monte Carlo simulations with variance-based estimators given in the literature.

[3]Note that these only refer to the original estimation and do not consider the inadmissible solutions generated during the resampling to obtain standard errors.

Besides providing guidelines for Monte Carlo simulations using the R package *cSEM*, data storage and availability of the data is a relevant issue. In terms of good research practice, the exact procedure researchers followed for their Monte Carlo simulation had to be explained in detail. Further, all functions and packages researchers used have to be reported to ensure that the results can be reproduced. The code researchers used have to be provided by the authors – according to good practice, at least upon request. Considering the data used for the simulation study, it should either be made publicly available, or researchers need to ensure that the data could be reproduced by using a seed during the simulation study. Using a seed ensures that although the data is randomly drawn, if the code is run again, the same random samples are drawn. If a seed was to be used during the simulation, this should be mentioned in the simulation design.

Future research should provide guidelines for Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models using the R package *cSEM* for more specific situations, e.g. in using the post-estimation function `predict()`. Moreover, future research should provide guidelines for larger Monte Carlo simulations for which using high performance clusters could be helpful. Nevertheless, the interested reader is referred to Rademaker and Schuberth (2020) for a more detailed explanation of the R package *cSEM* and the optional arguments.

Chapter 3

Robust partial least squares path modeling

3.1 Introduction [1]

Structural equation modeling (SEM) is a popular psychometric method in social and behavioral sciences. Its ability to operationalize abstract concepts, estimate their relationships and take into account measurement errors make it a frequently applied tool for answering various types of research questions (Bollen, 1989).

Generally, two kinds of SEM estimators can be distinguished. On the one hand, covariance-based estimators, such as the maximum likelihood (Jöreskog, 1970a) and the generalized least squares estimator (Browne, 1974), minimize the discrepancy between the empirical and the model-implied observed variable variance-covariance matrix to obtain the model parameter estimates. On the other hand, variance-based

[1] This chapter is based on joint work with Florian Schuberth, Jörg Henseler and Theo K. Dijkstra. It was published as a peer-reviewed article:
Schamberger, T., Schuberth, F., Henseler, J., Dijkstra, T.K., 2020. Robust partial least squares path modeling. Behaviormetrika 47, 307 - 334.
 Compared to the published article, I adjusted the notation such that it is consistent with the other chapters.

(VB) estimators, such as generalized structured component analysis (Hwang and Takane, 2004) and generalized canonical correlation analysis (Kettenring, 1971), first build proxies for the constructs as linear combinations of the observed variables and, subsequently, estimate the model parameters.

Among VB estimators, partial least squares path modeling (PLS-PM, Wold, 1975) is one of the most often applied and thoroughly studied estimators. Its performance has been investigated for various population models, for normally and non-normally distributed data and in comparison to other estimators (Dijkstra and Henseler, 2015a; Hair et al., 2017c; Sarstedt et al., 2016; Takane and Hwang, 2018). Moreover, in empirical research, PLS-PM has been used across a variety of fields, such as Marketing (Hair et al., 2012), Information Systems (Marcoulides and Saunders, 2006), Finance (Avkiran et al., 2018), Family Business (Sarstedt et al., 2014), Human Resources (Ringle et al., 2020) and Tourism (Müller et al., 2018).

Over the last several years, PLS-PM has undergone numerous enhancements. In its current form, known as consistent partial least squares (PLSc), it consistently estimates linear and non-linear structural models containing both composites and common factors (Dijkstra and Schermelleh-Engel, 2014; Dijkstra and Henseler, 2015b). Moreover, it can estimate models containing hierarchical constructs (Becker et al., 2012; Fassott et al., 2016; van Riel et al., 2017), deal with ordinal categorical observed variables (Schuberth et al., 2018b) and correlated measurement errors within a block of observed variables (Rademaker et al., 2019), and can be employed as an estimator in confirmatory composite analysis (Schuberth et al., 2018a). In addition to model estimation, PLS-PM can be used in multigroup comparisons (Klesel et al., 2019; Sarstedt et al., 2011) and to reveal unobserved heterogeneity (Becker et al., 2013; Ringle et al., 2014). Furthermore, the fit of models estimated by PLS-PM can be assessed in two ways: first, by measures of fit, such as the standardized root mean square residual (SRMR, Henseler et al., 2014), and second by bootstrap-based tests of the overall model fit (Dijkstra and Henseler, 2015a). An overview of the methodological research on PLS-PM is provided by Khan et al. (2019).

Despite the numerous enhancements of PLS-PM and suggested guidelines (e.g., Benitez et al., 2020; Henseler et al., 2016a; Rigdon, 2016), handling outliers in the context of PLS-PM has been widely neglected, although outliers are often encountered in empirical research (Filzmoser, 2005). This is not without problems, since PLS-PM and many of its enhancements such as PLSc use the Pearson correlation, which is known to be very sensitive to outliers (e.g., Boudt et al., 2012). Therefore, ignoring outliers is very likely to lead to distorted results and thus to questionable conclusions.

Outliers are observations that differ significantly from the rest of the data (Grubbs, 1969). Two types of outliers can be distinguished (Niven and Deutsch, 2012).[2] First, outliers can arise completely unsystematic and therefore not follow any structure. Second, outliers can arise systematically, e.g., from a different population than the rest of the observations.

To deal with outliers in empirical research, two approaches are commonly used. The first encompasses using robust estimators that are not or are only to a lesser extent distorted by outliers. The second entails identifying and manually removing outliers before the final estimation. The latter is often regarded as the inferior approach. First, it cannot be guaranteed that outliers are identified as such because outliers can affect the results in a way that they may not be identified by visualization or statistics such as the Mahalanobis distance (Hubert et al., 2008). Second, even if outliers can be identified, removing them implies a loss of useful information they contain (Gideon and Hollister, 1987); additionally, for small datasets, reducing the effective number of observations reduces statistical power.

In light of this situation, this chapter contributes to the existing SEM literature by presenting robust versions of PLS-PM. Specifically, we introduce robust partial least squares path modeling (robust PLS) and robust consistent partial least squares (robust PLSc), which combine the robust covariance estimator minimum covariance determinant (MCD) with PLS-PM and PLSc, respectively. Consequently, if robust PLS/PLSc are used to estimate structural equation models, outliers do not have to be removed manually.

The remainder of the chapter is structured as follows. Section 3.2 develops robust PLS/PLSc as a combination of PLS-PM/PLSc with the robust minimum covariance determinant (MCD) estimator of covariances. Section 3.3 and Section 3.4 present the setup of our Monte Carlo simulation to assess the efficacy of robust PLS/PLSc and the corresponding results. Section 3.5 demonstrates the performance of robust PLS/PLSc by two empirical examples. Finally, the chapter is concluded by Section 3.6 with a

[2]It is noted that extant literature provides various taxonomy descriptions of outliers (e.g., Sarstedt and Mooi, 2014).

discussion of findings, conclusions and an outlook on future research.

3.2 Developing robust partial least squares path modeling

Originally, PLS-PM was developed by Wold (1975) to analyze high-dimensional data in a low-structure environment. PLS-PM is capable of emulating several of Kettenring's (1971) approaches to generalized canonical correlation analysis (Tenenhaus et al., 2005). However, while traditionally it only consistently estimates structural models containing composites (Dijkstra, 2017)[3], in its current form, known as PLSc, it is capable of consistently estimating structural models containing both composites and common factors (Dijkstra and Henseler, 2015a). The following section presents PLS-PM and its consistent version expressed in terms of the correlation matrix of the observed variables.

3.2.1 Partial least squares path modeling

Consider K standardized observed variables for J constructs, each of which belongs to one construct only. The n observations of the standardized observed variables belonging to the j-th construct are stored in the data matrix $\boldsymbol{X}_j$ of dimension $(n \times K_j)$ such that $\sum_{j=1}^{J} K_j = K$. The empirical correlation matrix of these observed variables is denoted by $\boldsymbol{S}_{jj}$. To ensure the identification of the weights, they need to be normalized. This normalization is typically done by fixing the variance of each proxy to one, i.e., $\hat{\boldsymbol{w}}_j^{(0)'} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(0)} = 1$. Typically, unit weights are used as starting weights for the iterative PLS-PM algorithm. To obtain the weights to build the proxies, the iterative PLS-PM algorithm performs the following three steps in each iteration (l).

In the first step, outer proxies for the construct are built by the observed variables as follows:

$$\hat{\boldsymbol{\eta}}_j^{(l)} = \boldsymbol{X}_j \hat{\boldsymbol{w}}_j^{(l)} \tag{3.1}$$

The weights are scaled in each iteration by $\left(\hat{\boldsymbol{w}}_j^{(l)'} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(l)}\right)^{-\frac{1}{2}}$. Consequently, the proxy $\hat{\boldsymbol{\eta}}_j$ has zero mean and unit variance.

[3]It is noted that in the context of PLS-PM only mode B consistently estimates composite models (Dijkstra, 2017).

In the second step, inner proxies for the constructs are built by the outer proxies of the previous step:

$$\tilde{\boldsymbol{\eta}}_j^{(l)} = \sum_{j'=1}^{J} e_{jj'}^{(l)} \hat{\boldsymbol{\eta}}_{j'}^{(l)} \tag{3.2}$$

There are three different ways of calculating the inner weight $e_{jj'}^{(l)}$, all of which yield similar results (Noonan and Wold, 1982): centroid, factorial and the path weighting scheme. The factorial scheme calculates the inner weight $e_{jj'}^{(l)}$ as follows:[4]

$$e_{jj'}^{(l)} = \begin{cases} \mathrm{cov}(\hat{\boldsymbol{\eta}}_j^{(l)}, \hat{\boldsymbol{\eta}}_{j'}^{(l)}), & \text{if } \eta_j \text{ and } \eta_{j'} \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

The resulting inner proxy $\tilde{\boldsymbol{\eta}}_j$ is scaled to have unit variance again.

In the third step of each iteration, new outer weights $\hat{\boldsymbol{w}}_j^{(l+1)}$ are calculated.[5] Using mode A, the new outer weights (correlation weights) are calculated as the scaled correlations of the inner proxy $\tilde{\boldsymbol{\eta}}_j^{(l)}$ and its corresponding observed variables $\boldsymbol{X}_j$:

$$\hat{\boldsymbol{w}}_j^{(l+1)} \propto \sum_{j'=1}^{J} e_{jj'} \boldsymbol{S}_{jj'} \hat{\boldsymbol{w}}_{j'}^{(l)} \qquad \text{with} \qquad \hat{\boldsymbol{w}}_j^{(l+1)'} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(l+1)} = 1 \tag{3.4}$$

Using mode B, the new outer weights (regression weights) are the scaled estimates from a multiple regression of the inner proxy $\tilde{\boldsymbol{\eta}}_j^{(l)}$ on its corresponding observed variables:

$$\hat{\boldsymbol{w}}_j^{(l+1)} \propto \boldsymbol{S}_{jj}^{-1} \sum_{j'=1}^{J} e_{jj'}^{(l)} \boldsymbol{S}_{jj'} \hat{\boldsymbol{w}}_{j'}^{(l)} \qquad \text{with} \qquad \hat{\boldsymbol{w}}_j^{(l+1)'} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(l+1)} = 1 \tag{3.5}$$

The final weights are obtained when the new weights $\hat{\boldsymbol{w}}_j^{(l+1)}$ and the previous weights $\hat{\boldsymbol{w}}_j^{(l)}$ do not change significantly; otherwise, the algorithm starts again from Step 1, building outer proxies with the new weights. Subsequently, the final weight estimates $\hat{\boldsymbol{w}}_j$ are used to build the final proxies $\hat{\boldsymbol{\eta}}_j$:

$$\hat{\boldsymbol{\eta}}_j = \boldsymbol{X}_j \hat{\boldsymbol{w}}_j \tag{3.6}$$

Loading estimates are obtained as correlations of the final proxies and their observed variables. The path coefficients are estimated by ordinary least squares (OLS) according to the structural model.

[4]For more details on the other weighting schemes, see, e.g., Tenenhaus et al. (2005).

[5]In the following, we only consider mode A and mode B. mode C, which is a combination of Modes A and B, is not considered here (Dijkstra, 1985).

3.2.2 Consistent partial least squares

For models containing common factors, it is well known that the estimates are only *consistent at large*, i.e., the parameter estimates converge in probability only to their population values when the number of observations *and* the number of observed variables tend to infinity (Wold, 1982).

To overcome this shortcoming, Dijkstra and Henseler (2015b) developed PLSc. PLSc applies a correction for attenuation to consistently estimate factor loadings and path coefficients among common factors. The consistent factor loading estimates of observed variables' block j can be obtained as follows:

$$\hat{\boldsymbol{\lambda}}_j = \hat{c}_j \hat{\boldsymbol{w}}_j \tag{3.7}$$

The correction factor $\hat{c}_j$ is obtained from the following equation:

$$\hat{c}_j = \sqrt{\frac{\hat{\boldsymbol{w}}_j' \left(\boldsymbol{S}_{jj} - \mathrm{diag}(\boldsymbol{S}_{jj})\right) \hat{\boldsymbol{w}}_j}{\hat{\boldsymbol{w}}_j' \left(\hat{\boldsymbol{w}}_j \hat{\boldsymbol{w}}_j' - \mathrm{diag}\left(\hat{\boldsymbol{w}}_j \hat{\boldsymbol{w}}_j'\right)\right) \hat{\boldsymbol{w}}_j}} \tag{3.8}$$

To obtain consistent path coefficient estimates, the correlation estimates among the proxies need to be corrected for attenuation to consistently estimate the construct correlations:

$$\widehat{\mathrm{cor}}(\eta_j, \eta_{j'}) = \frac{\hat{\boldsymbol{w}}_j' \boldsymbol{S}_{jj'} \hat{\boldsymbol{w}}_{j'}}{\hat{Q}_j \hat{Q}_{j'}} \tag{3.9}$$

Here, $\hat{Q}_j = \hat{c}_j \sqrt{\hat{\boldsymbol{w}}_j{}' \hat{\boldsymbol{w}}_j}$ is the reliability estimate. In case of composites, typically, no correction for attenuation is applied, i.e., $\hat{Q}_j$ is set to 1 if the j-th construct is modeled as a composite. Finally, based on the consistently estimated construct correlations, the path coefficients are estimated by OLS for recursive structural models and by two-stage least squares for non-recursive structural models.

3.2.3 Selecting a robust correlation

As illustrated, PLS-PM and PLSc can both be expressed in terms of the correlation matrix of observed variables. For this purpose, typically, the Pearson correlation estimates are used. However, it is well known in the literature that the Pearson correlation is highly sensitive to outliers (e.g., Abdullah, 1990). Hence, a single outlier can cause distorted correlation estimates and therefore distorted PLS-PM/PLSc results. To overcome this shortcoming, we propose to replace the Pearson correlation by robust correlation estimates. Similar was already proposed for covariance-based estimators (e.g., Yuan and Bentler, 1998a,b).

The existing literature provides a variety of correlation estimators that are robust against unsystematic outlier. Table 3.1 presents an overview of several robust correlation estimators and their asymptotic Breakdown Points. The Breakdown Point (BP) of an estimator is used to judge its robustness against unsystematic outliers and thus indicates the minimum share of outliers in a dataset that yields a breakdown of the estimate, i.e., a distortion of the estimate caused by random/unsystematic outliers (Donoho and Huber, 1983). Formally, the BP of an estimator T can be described as follows:

$$\mathrm{BP} = \inf\{\varepsilon : \sup|T(X^*) - T(X)| = \infty\},$$

with $T(X)$ being the estimate based on the sample X which is not contaminated by outliers and $T(X^*)$ being the estimate based on the sample X^* which is contaminated by outliers of share ε. Usually, an estimator with a higher asymptotic BP is preferred, as it is more robust, i.e., less prone to outlier's distortion, than an estimator with a lower asymptotic BP. In addition to ranking various estimators by their asymptotic BPs, the estimators can be distinguished by their approach to obtaining the correlation estimate: using robust estimates in the Pearson correlation, using non-parametric correlation estimates, using regression-based correlations and performing an iterative procedure that estimates the correlation matrix by using the correlation of a subsample that satisfies a predefined condition.

To protect the Pearson correlation from being distorted by outliers, robust moment estimates can be used for the calculation of correlation. For instance, the mean and standard deviation can be replaced by, respectively, the median and the median absolute deviation (Falk, 1998), or variances and covariances can be estimated based on a winsorized or trimmed dataset (Gnanadesikan and Kettenring, 1972). In addition to the Pearson correlation, robust non-parametric estimators such as Spearman's, Kendall's or the Gaussian rank correlation can be used (Boudt et al., 2012). Regression weights that indicate if an observation is regarded as an outlier can also be applied to weight the variances and covariances in the Pearson correlation (Abdullah, 1990). Finally, iterative algorithms, such as the Minimum Covariance Determinant (MCD) and the Minimum Volume Ellipsoid (MVE) estimator, can be used to select a

Table 3.1: Review of various robust correlation coefficients

Approach	Estimator	Source	Description	BP
Use of robust estimates in the Pearson correlation	**Correlation Median**	Falk (1998)	The median absolute deviation is used instead of the standard deviation, and the comedian is used instead of the covariance.	
	Trimmed dataset	Gnanadesikan and Kettenring (1972)	The variances and covariance is calculated for a trimmed dataset.	
	Winsorized dataset	Gnanadesikan and Kettenring (1972)	The variances and covariance are calculated for a winsorized dataset.	
Non-parametric correlation estimators	**Spearman's rank correlation**	Boudt et al. (2012)	The correlation is calculated for ranked observations.	20.6%
	Kendall's rank correlation	Boudt et al. (2012)	The correlation is calculated based on the similarity of ranked observations.	29,3%
	Gaussian rank correlation	Boudt et al. (2012)	The correlation is calculated for the Gaussian scores of the ranked observations.	12,4%
Regression-based correlation	**Weighted Least Squares**	Abdullah (1990)	Weights used in the weighted least squares approach that deemphasize outliers less are applied in/to the estimation of the variances and covariance used in the Pearson correlation.	$\leq$50%
Use the subsample that satisfies a predefined condition to estimate the correlation	**Minimum Volume Ellipsoid**	Rousseeuw (1985)	The correlation matrix of a subsample that yields the ellipsoid with the smallest volume ellipsoid is used.	50%
	Minimum Covariance Determinant	Rousseeuw (1985)	The correlation matrix of a subsample that yields to the smallest correlation determinant is used.	50%

representative subsample unaffected by outliers for the calculation of the covariance and the standard deviations.

Among all considered approaches, the MCD estimator is a promising candidate for developing a robust version of PLS-PM and PLSc. Although both MVE and MCD estimators have an asymptotic BP of 50%, which is the highest BP an estimator can have and is much larger than the asymptotic BP of 0% of the Pearson correlation, in contrast to the MVE estimator, the MCD estimator is asymptotically normally distributed. Moreover, robust estimates based on the MCD are more precise (Butler et al., 1993; Gnanadesikan and Kettenring, 1972) and a closed-form expression of the standard error exists (Rousseeuw, 1985).

The MCD estimator estimates the variance-covariance matrix of a sample $\boldsymbol{X}$ of dimension $n \times K$ as the variance-covariance matrix of a subsample of dimension $h \times K$ with the smallest positive determinant. To identify this subsample, theoretically, the variance-covariance matrices of $\binom{n}{h}$ different subsamples have to be estimated. The choice of h also determines the asymptotic BP of the MCD estimator. A maximum asymptotic BP of 50% is reached if $h = (n + K + 1)/2$; otherwise, it will be smaller (Rousseeuw, 1985).

The rationale behind the MCD estimator can be given by considering two random variables, each with n observations. While the Pearson correlation is based on all n observations to estimate the correlation, the MCD estimator calculates the variances and the covariance based on a subsample containing only h observations. The h observations are determined by the ellipse with the smallest area containing the h observations. Similarly is done in case of more than two variables, the subsample is determined by the ellipsoid with smallest volume containing the h observations. Generally, the MCD estimator finds the confidence ellipsoid to a certain confidence level with the minimum volume to determine the variances and covariances.

To reduce the computational effort of calculating the MCD estimator, a fast algorithm has been developed that considers only a fraction of all potential subsamples (Rousseeuw and Driessen, 1999). The fast MCD algorithm is an iterative procedure. In each iteration (l), the algorithm applies the following three steps to a subsample $\boldsymbol{H}^{(l)}$ of sample $\boldsymbol{X}$ consisting of h observations and K variables:

- Calculate the Mahalanobis distance $d_i^{(l)}$ for every observation $\boldsymbol{x}_i$ of $\boldsymbol{X}$:

$$d_i^{(l)} = \sqrt{\left(\boldsymbol{x}_i - \bar{\boldsymbol{x}}^{(l)}\right)' \boldsymbol{S}^{(l)^{-1}} \left(\boldsymbol{x}_i - \bar{\boldsymbol{x}}^{(l)}\right)} \qquad i = 1, \ldots, n, \tag{3.10}$$

  where $\bar{\boldsymbol{x}}^{(l)}$ is the sample mean, and $\boldsymbol{S}^{(l)}$ is the variance-covariance matrix of the subsample $\boldsymbol{H}^{(l)}$.

- Create the new subsample $\boldsymbol{H}^{(l+1)}$ consisting of the observations corresponding to the h smallest distances.

- Return $\boldsymbol{S}^{(l)}$ if the determinant of $\boldsymbol{S}^{(l+1)}$ equals the determinant of $\boldsymbol{S}^{(l)}$ or zero; otherwise, start from the beginning.

Since $\det(\boldsymbol{S}^{(l)}) \geq \det(\boldsymbol{S}^{(l+1)})$ and $\det(\boldsymbol{S}^{(1)}) \geq \det(\boldsymbol{S}^{(2)}) \geq \det(\boldsymbol{S}^{(3)}) \ldots$ is a non-negative sequence, the convergence of this procedure is guaranteed (Rousseeuw and Driessen, 1999). Once the iterative procedure has converged, the procedure is repeated several times for different initial subsamples $\boldsymbol{H}^{(1)}$.

The initial subsample $\boldsymbol{H}^{(1)}$ is chosen as follows: First, a random subsample $\boldsymbol{H}^{(0)}$ of size $K+1$ of $\boldsymbol{X}$ is drawn. If $\det(\boldsymbol{S}^{(0)}) = 0$, which is not desirable, further observations of $\boldsymbol{X}$ are added to $\boldsymbol{H}^{(0)}$ until $\det(\boldsymbol{S}^{(0)}) > 0$, where $\boldsymbol{S}^{(0)} = \mathrm{cov}(\boldsymbol{H}^{(0)})$. Second, the initial distances $d_i^{(0)}$ are calculated based on the mean vector and the variance-covariance matrix of $\boldsymbol{H}^{(0)}$, see Equation 3.10. Finally, the initial subsample $\boldsymbol{H}^{(1)}$ consists of the observations belonging to the h smallest distances $d_i^{(0)}$.

Figure 3.1 illustrates the difference between the Pearson correlation and the MCD estimator for a normally distributed dataset with 300 observations, where 20 percent of observations are replaced by randomly generated outliers. The population correlation is set to 0.5. As shown, the estimate of the Pearson correlation is strongly distorted by the outliers, while the MCD correlation estimate is robust against outliers, and thus very close to the population correlation.

3.2.4 Robust partial least squares path modeling and robust consistent partial least squares

To deal with outliers in samples without manually removing them before the estimation, we propose modifications of PLS-PM and PLSc called robust PLS and robust PLSc, respectively. In contrast to traditional PLS-PM and its consistent version using the Pearson correlation, the proposed robust counterparts use the MCD correlation estimate as input to the PLS-PM algorithm. As a consequence, the steps of the PLS-PM algorithm and the principle of PLSc of correcting for attenuation remain unaffected. Figure 3.2 contrasts robust PLS and PLSc with their traditional counterparts.

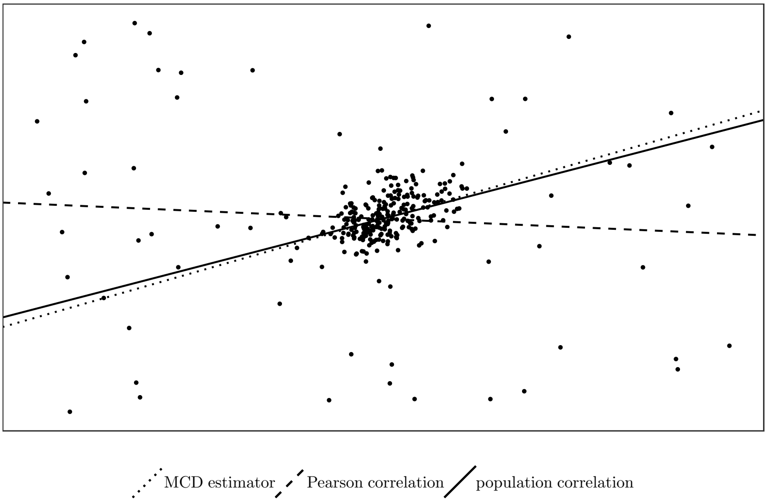

Figure 3.1: Difference between the MCD and Pearson correlation

As shown in Figure 3.2, the only difference between robust PLS/PLSc and its traditional counterparts is the input of the estimation. The subsequent steps remain unaffected, and thus, robust PLS/PLSc can be easily implemented in most common software packages. However, due to the iterative algorithm, robust PLS/PLSc are more computationally intensive than their traditional counterparts that are based on the Pearson correlation.

3.3 Computational experiments using Monte Carlo simulation

The purpose of our Monte Carlo simulation is twofold: First, we examined the behavior of PLS-PM and PLSc in case of unsystematic outlier. Although the reliance of traditional PLS-PM/PLSc on Pearson correlation implies that outliers would be an issue, there is no empirical evidence so far of whether the results of traditional PLS-PM/PLSc are affected by outliers and, if so, how strong the effect is. Second, we were interested in the efficacy of robust PLS/PLSc. More concretely, we investigated the convergence behavior, bias and efficiency of robust PLS/PLSc and compared them to their traditional counterparts.

The experimental design was full factorial and we varied the following experimental

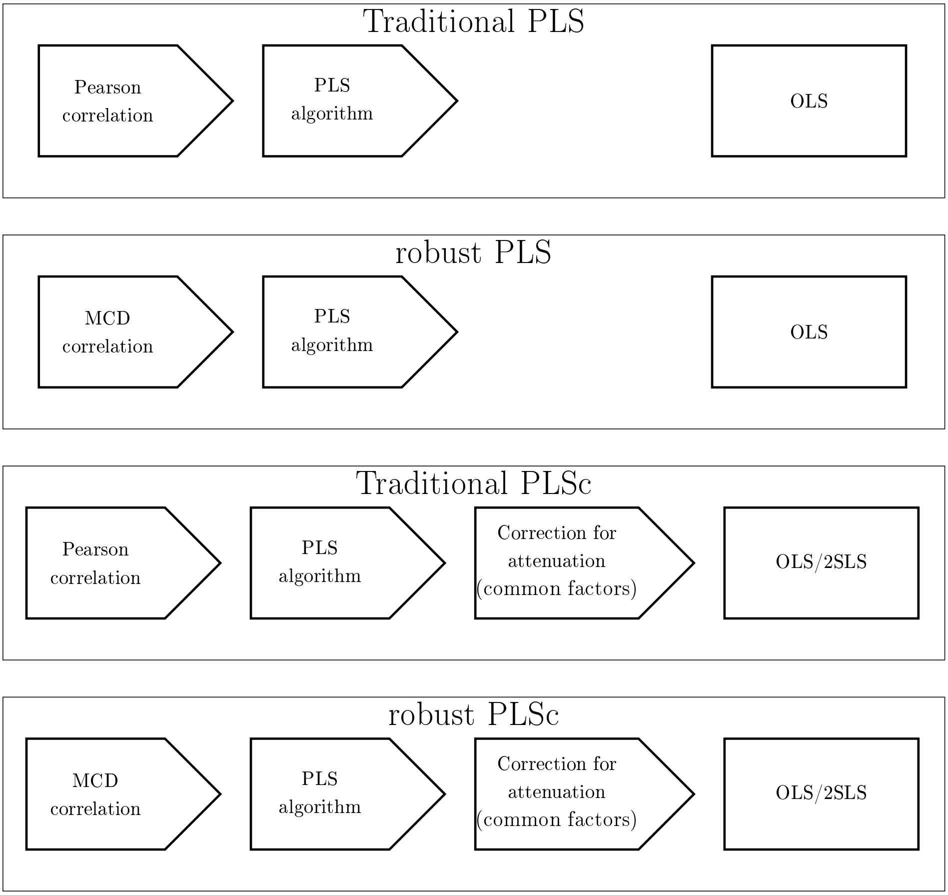

Figure 3.2: Conceptual differences of traditional and robust PLS/PLSc

conditions: [6]

- concept operationalization (all constructs are specified either as composites or common factors),

[6]Additionally, three other conditions were examined. First, it was examined whether the model complexity had an influence on the results by including a model containing 5 constructs and 20 observed variables. In doing so, all constructs were either specified as composites or as common factors. Second, we investigated the estimators' performance in case that only a fraction of the observed variables, i.e., two observed variables, are contaminated by outliers. Third, we examined the estimators' performance in case of systematic outliers. In doing so, the outliers were drawn from the univariate continuous uniform distribution with lower bound 2 and upper bound 5 representing a situation where respondents always score high. Since the results are very similar to the results presented, these conditions are not be explained in detail in this chapter. For the results as well as further information on both conditions, we refer to the Appendix.

- sample size ($n = 100$, 300 and 500) and
- share of outliers (0%, 5%, 10%, 20%, 40% and 50%).

3.3.1 Population models

To assess whether the type of construct, i.e., composite or common factor, affects the estimators' performance, we considered two different population models.

Population model with three common factors

The first population model consists of three common factors and has the following structural model:

$$\eta_2 = \gamma_{21}\eta_1 + \zeta_2 \tag{3.11}$$

$$\eta_3 = \gamma_{31}\eta_1 + \gamma_{32}\eta_2 + \zeta_3, \tag{3.12}$$

where $\gamma_{21} = 0.5$, $\gamma_{31} = 0.3$ and $\gamma_{32} = 0.0$. As Figure 3.3 depicts, each block of three observed variables loads on one common factor with the following population loadings: 0.9, 0.8 and 0.7 for η_1, 0.7, 0.7 and 0.7 for η_2, and 0.8, 0.8 and 0.7 for η_3.

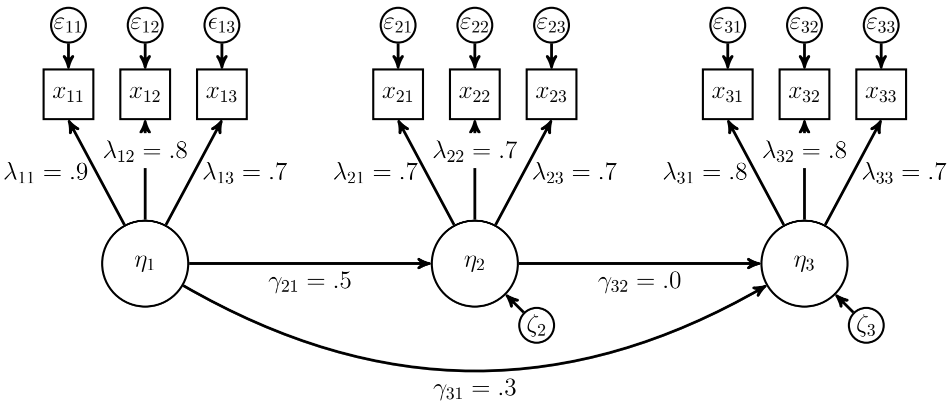

Figure 3.3: Population model containing three common factors

All structural and measurement errors are mutually independent and the common factors are assumed to be independent of measurement errors. The observed variables' population correlation matrix is given by the following:

$$
\boldsymbol{\Sigma} = \begin{array}{c} \begin{array}{ccccccccc} \underline{x_{11}} & \underline{x_{12}} & \underline{x_{13}} & \underline{x_{21}} & \underline{x_{22}} & \underline{x_{23}} & \underline{x_{31}} & \underline{x_{32}} & \underline{x_{33}} \end{array} \\ \begin{pmatrix}
1.000 & & & & & & & & \\
0.720 & 1.000 & & & & & & & \\
0.630 & 0.560 & 1.000 & & & & & & \\
0.315 & 0.280 & 0.245 & 1.000 & & & & & \\
0.315 & 0.280 & 0.245 & 0.490 & 1.000 & & & & \\
0.315 & 0.280 & 0.245 & 0.490 & 0.490 & 1.000 & & & \\
0.216 & 0.192 & 0.168 & 0.084 & 0.084 & 0.084 & 1.000 & & \\
0.216 & 0.192 & 0.168 & 0.084 & 0.084 & 0.084 & 0.640 & 1.000 & \\
0.189 & 0.168 & 0.147 & 0.074 & 0.074 & 0.074 & 0.560 & 0.560 & 1.000
\end{pmatrix} \end{array} \tag{3.13}
$$

Population model with three composites

The second population model illustrated in Figure 3.4 is similar to the first, but all common factors are replaced by composites. The composites are built as follows: $\eta_1 = \boldsymbol{x}_1'\boldsymbol{w}_1$ with $\boldsymbol{w}_1' = (0.6, 0.4, 0.2)$; $\eta_2 = \boldsymbol{x}_2'\boldsymbol{w}_2$ with $\boldsymbol{w}_2' = (0.3, 0.5, 0.6)$; and $\eta_3 = \boldsymbol{x}_3'\boldsymbol{w}_3$ with $\boldsymbol{w}_3' = (0.4, 0.5, 0.5)$.

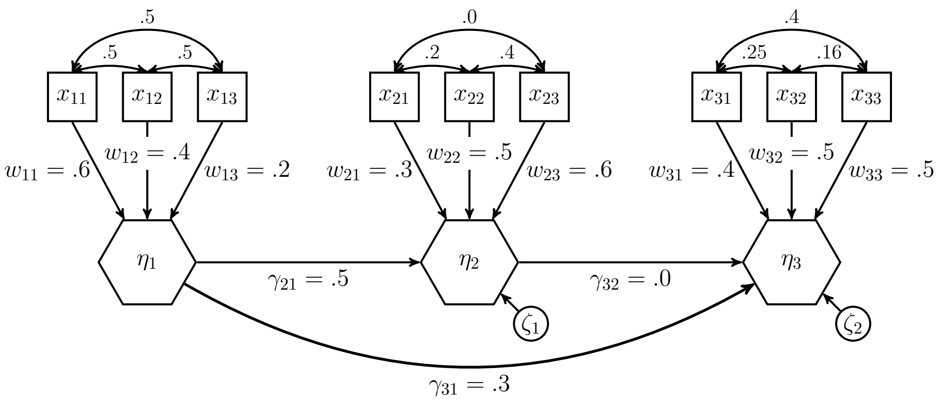

Figure 3.4: Population model containing three composites

The observed variables' population correlation matrix has the following form:[7]

$$
\boldsymbol{\Sigma} = \begin{array}{c}
\begin{array}{ccccccccc}
\underline{x_{11}} & \underline{x_{12}} & \underline{x_{13}} & \underline{x_{21}} & \underline{x_{22}} & \underline{x_{23}} & \underline{x_{31}} & \underline{x_{32}} & \underline{x_{33}}
\end{array} \\
\begin{pmatrix}
1.000 & & & & & & & & \\
0.500 & 1.000 & & & & & & & \\
0.500 & 0.500 & 1.000 & & & & & & \\
0.180 & 0.160 & 0.140 & 1.000 & & & & & \\
0.360 & 0.320 & 0.280 & 0.200 & 1.000 & & & & \\
0.360 & 0.320 & 0.280 & 0.000 & 0.400 & 1.000 & & & \\
0.196 & 0.174 & 0.152 & 0.044 & 0.087 & 0.087 & 1.000 & & \\
0.184 & 0.163 & 0.143 & 0.041 & 0.082 & 0.082 & 0.250 & 1.000 & \\
0.200 & 0.178 & 0.155 & 0.044 & 0.089 & 0.089 & 0.400 & 0.160 & 1.000
\end{pmatrix}
\end{array} \tag{3.14}
$$

3.3.2 Sample size

Although the asymptotic BP of MCD equals 50% (Rousseeuw, 1985), its finite sample behavior in the context of the PLS-PM algorithm needs to be examined. Therefore, we varied the sample size from 100 to 300 and 500 observations. For an increasing sample size, we expect almost no effect on the behavior of robust PLS and PLSc except that standard errors of their estimates decrease, i.e., the estimates become more accurate.

3.3.3 Outlier share in the datasets

To assess the robustness of our proposed estimator and to investigate the performance of PLS-PM and PLSc in case of randomly distributed outliers, we varied the outlier share in the datasets from 0% to 50% with the intermediate levels of 5%, 10%, 20% and 40%. We deliberately included a share of 0% to investigate whether robust PLS and PLSc perform comparably to their non-robust counterparts if outliers are absent. In this case, we would expect the traditional versions of PLS-PM/PLSc to outperform our proposed modifications as they are based on the Pearson correlation which is known to be asymptotically efficient under normality (Anderson and Olkin, 1985). As the share of outliers increases, we expect an increasing distortion in PLS-PM and

[7] All correlations were rounded to three decimal places.

PLSc estimates. In contrast, due to the MCD estimator's asymptotic BP of 50%, we expect robust PLS/PLSc to be hardly affected by outliers unless the asymptotic BP is reached.

3.3.4 Data generation and analysis

The simulation was carried out in the statistical programming environment R (R Core Team, 2017). The datasets without outliers were drawn from the multivariate normal distribution using the function `mvrnorm()` from the *MASS* package (Venables and Ripley, 2002). The outliers were randomly drawn from the univariate continuous uniform distribution with the lower bound of -10 and the upper bound of 10 using the function `runif()` from the *stats* package (R Core Team, 2017). To contaminate the datasets with outliers, the last observations of each dataset were replaced by those. The MCD correlation estimates were calculated by the `cov.rob()` function from the *MASS* package (Venables and Ripley, 2002). PLS-PM and PLSc as well as the estimates of our proposed robust versions were obtained using the function `csem()` from the *cSEM* (version 0.0.0.9000, Rademaker and Schuberth, 2020) package.[8] The inner weights were obtained by the factorial scheme and the observed variable weights in case common factors were calculated by mode A and in case of composites by mode B.

Although we considered the number of inadmissible solutions, the presented final results are based on 1,000 admissible estimations per estimator for each condition, i.e., the inadmissible solutions were excluded and replaced by proper ones. To assess the performance of the different estimators, we consider the empirical smoothed density of the deviations of a parameter estimate from its population value. The range of the density represents the accuracy of an estimator, i.e., the fatter the tails are the less precise the estimator. A narrow symmetric density with a mode at zero is desired, as it indicates an unbiased estimator with a small standard error.

[8] Since the cSEM package is currently under development, the results for PLS-PM and PLSc were validated by ADANCO (Henseler, 2019) and SmartPLS (Ringle et al., 2015).

3.4 Results

This section presents the results of our Monte Carlo simulation. Due to the large number of results, we report only a representative part of the results. In doing so, we only consider some of the model parameters since the results are very similar for all parameters. The complete results are given in the Appendix.

3.4.1 Population model with three common factors

This section shows the result for the population model consisting of three common factors. Figure 3.5 shows the performance of robust PLSc for various sample sizes and outlier shares when all observed variables are affected by unsystematic outliers. For clarity, only the two path coefficients γ_{21} and γ_{32} and the factor loading λ_{13} are considered. The results for the other parameters are similar.

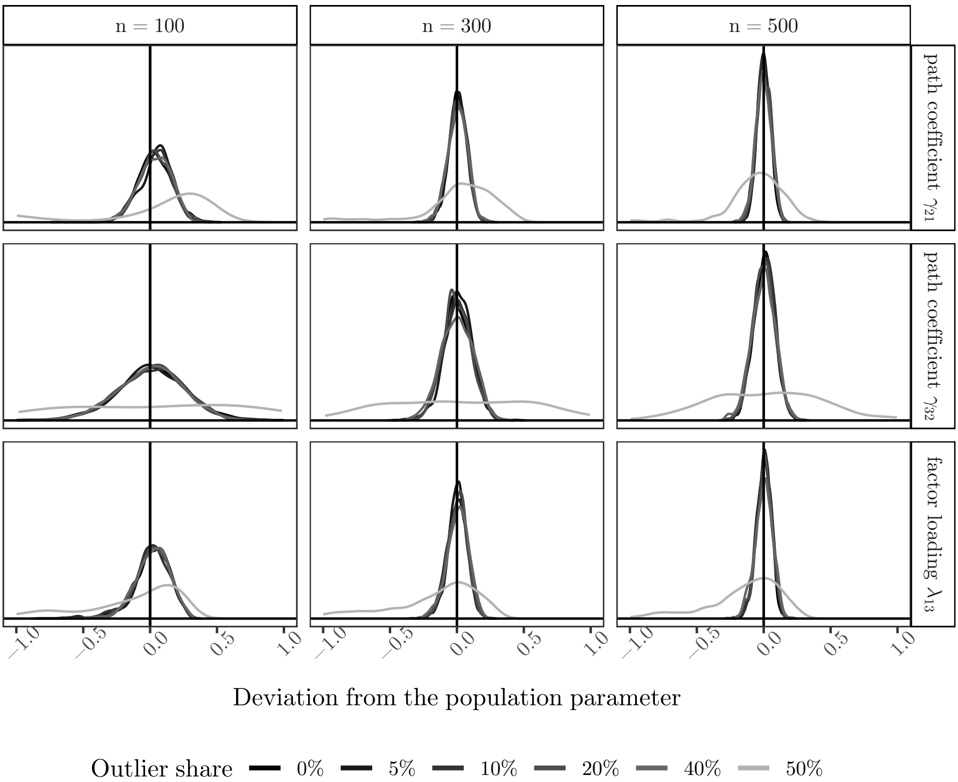

Figure 3.5: Performance of robust PLSc

As illustrated, the outlier share does not affect the parameter estimates of robust PLSc. On average, all estimates are close to their population value. Only when the proportion of outliers reaches 50% will the estimates be clearly distorted. Moreover, the results are similar for larger sample sizes, except that the estimates become more accurate.

Figure 3.6 compares the estimates of robust and traditional PLSc. Since their results are very similar across various sample sizes, the results are only shown for a sample size of 300 observations. Moreover, as the results for robust PLSc are almost unaffected by the share of outliers, only the results for outlier shares of 0%, 5%, 40% and 50% are considered.

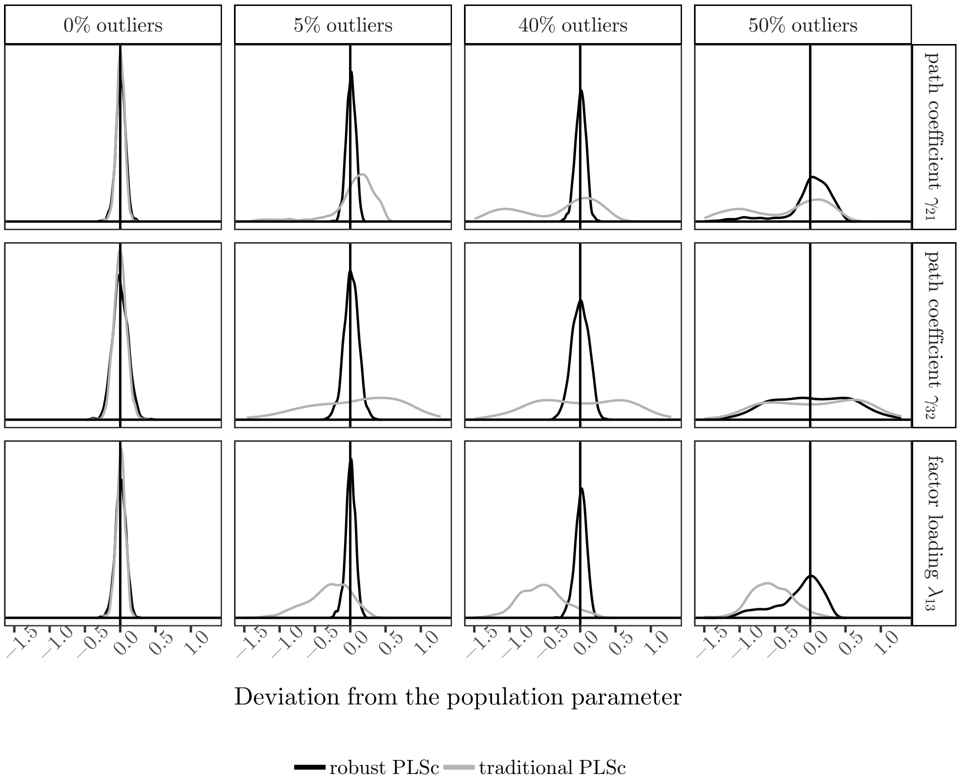

Figure 3.6: Comparison of robust and traditional PLSc for $n = 300$

For samples without outliers, both approaches yield similar estimates, but PLSc produces slightly smaller standard errors. However, while robust PLSc estimates show almost no distortion until the asymptotic BP of 50% of the MCD estimator is reached, traditional PLSc estimates are already distorted for a small outlier share. This distortion increases if the outlier share is increased.

3.4.2 Population model with three composites

In the following, the results for the population model consisting of three composites are shown. To preserve clarity, only the results for the two path coefficients γ_{21} and γ_{32} and the weight w_{13} are reported. However, the results for the other parameters are similar. Figure 3.7 illustrates the performance of robust PLS.

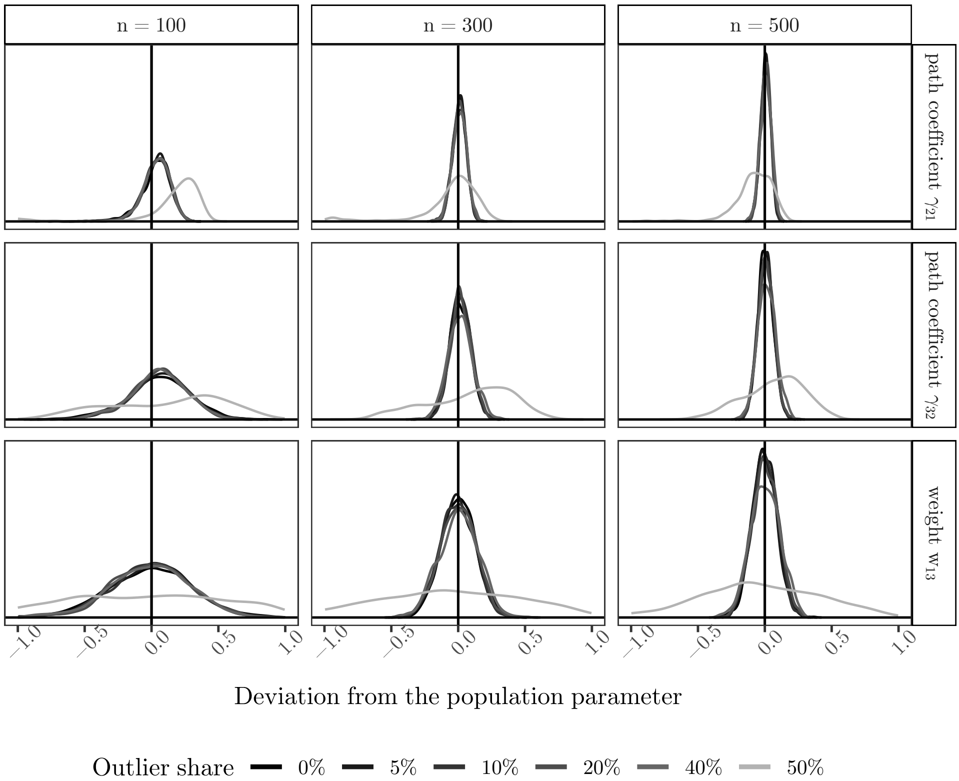

Figure 3.7: Performance of robust PLS

Similar to the model with three common factors, the outlier share has almost no effect on the performance of robust PLS. Only when the share of outliers reaches 50% are the estimates significantly distorted. On average, the robust estimates are very close to their population value; for an increasing sample size, the estimates becomes more precise.

Figure 3.8 compares the performance of robust PLS and that of its original version for various shares of outliers. Since the results are very similar across the considered sample sizes, the results for 300 observations are representative of the results for other sample sizes. Moreover, robust PLS behaves similarly for different outlier shares; therefore, only the results for outlier shares of 0%, 5%, 40% and 50% are shown.

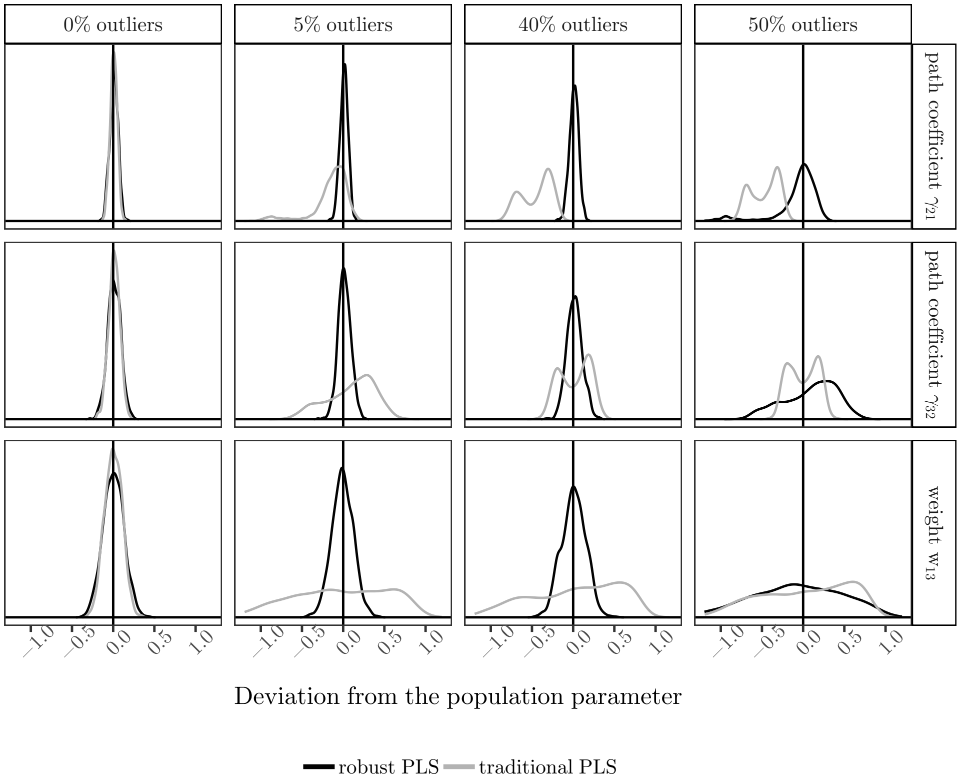

Figure 3.8: Comparison of robust and traditional PLS for $n = 300$

In the case of no outliers, the two estimators yield similar estimates, but PLS-PM results in slightly smaller standard errors. While robust PLS shows a distortion only at the share of 50% of outliers, traditional PLS-PM estimates are already distorted at the outlier share of 5%. As the outlier share increases, the PLS-PM estimates become increasingly distorted, and in case of outlier share of 10% or above, the estimates even show a bimodal distribution.

3.4.3 Inadmissible solutions

Figures 3.9 and 3.10 illustrate the share of inadmissible solutions until 1,000 proper solutions were reached for the models with three common factors and three composites. An inadmissible solution is defined as estimation for which the PLS-PM algorithm does not converge, at least one standardized loading or one construct reliability of greater than 1 is produced, or for which the construct correlation matrix or the model-implied observed variable's correlation matrix are not positive definite.

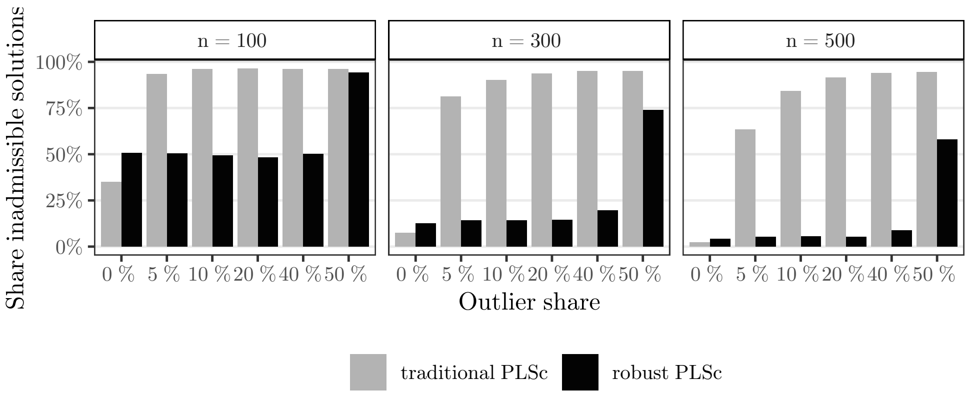

Figure 3.9: Occurence of inadmissible solutions for the model containing three common factors

Figure 3.9 shows the shares of inadmissible solutions for the model containing three common factors. The largest number of inadmissible solutions is produced by PLSc based on the Pearson correlation.[9] In this case, neither the sample size nor the share of outliers significantly influences the share of inadmissible solutions. In contrast, robust PLSc produces fewer inadmissible solutions than does traditional PLSc in every condition except for samples without outliers. Although robust PLSc produces numerous inadmissible solutions in case of 100 observations, its results improve for samples of size 300 and 500. Robust PLSc only produces a large number of inadmissible solutions for 50% of outliers in the sample.

[9] Additionally, to examine whether the large number of inadmissible solutions is a PLSc-specific problem, we estimated the model with three common factors, 100 observations and 20% outlier by maximum likelihood using the `sem()` function of the *lavaan* package (Rosseel, 2012). As a result, we observed a similar share of indadmissible solutions.

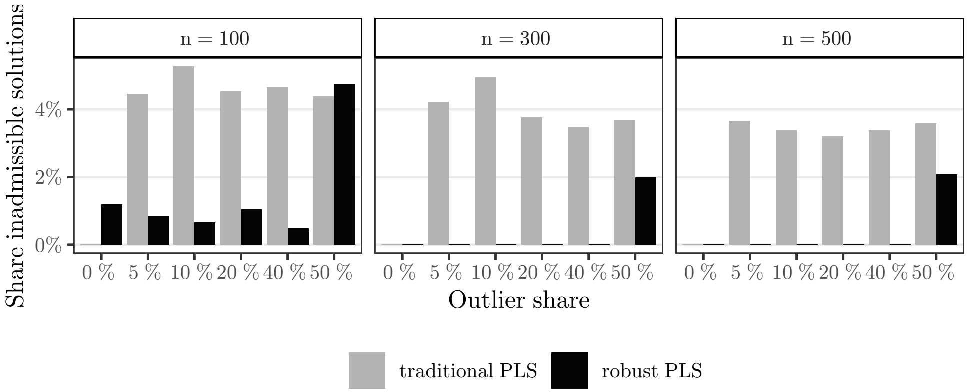

Figure 3.10: Occurence of inadmissible solutions for the model containing three composites

Figure 3.10 shows the share of inadmissible solutions for the model consisting of three composites. In general, the share of inadmissible solutions is lower than that for the model with three common factors. While almost no inadmissible solutions are produced by robust PLS, except in case of the outlier share of 50%, the share of inadmissible solutions of PLS-PM is substantial when outliers are present and is almost unaffected by the sample size and the outlier share. However, in case of no outliers, almost no inadmissible solutions are produced by PLS-PM.

3.5 Empirical examples

In this section, we illustrate the relevance of robust PLS/PLSc for empirical research. In doing so, we adopt the Corporate Reputation Model adapted from Hair et al. (2017b) and evaluate the influence of an incorrectly prepared dataset on the estimation results. Additionally, using the open- and closed-book dataset from Mardia et al. (1979), we compare the results of robust PLSc to those obtained by the robust covariance-based estimator suggested by Yuan and Bentler (1998a).

3.5.1 Example: Corporate reputation

The Corporate Reputation Model explains customer satisfaction (CUSA) and customer loyalty (CUSL) by corporate reputation. Corporate reputation is measured using the following two dimensions: (i) company's competence (COMP) which represents the cognitive evaluation of the company, and (ii) company's likeability (LIKE) which captures the affective judgments. Furthermore, the following four theoretical concepts explain the two dimensions of corporate reputation: (i) social responsibility (CSOR), (ii) quality of company's products and customer orientation (QUAL), (iii) economic and managerial performance (PERF) and (iv) company's attractiveness (ATTR).

The concepts CSOR, PERF, QUAL and ATTR are modeled as composites while the concepts LIKE, COMP, CUSA and CUSL are modeled as common factors. In total, 31 observed variables are used for the concept's operationalization. Each observed variable is measured on a 7-point scale ranging from 1 to 7. The dataset is publicly available and comprises 344 observations per observed variable including 8 observations with missing values in at least one observed variable coded as -99 (SmartPLS, 2019).

The conceptual model is illustrated in Figure 3.11. To preserve clarity, we omit the measurement and structural errors as well as the correlations among the observed variables. For detailed information about the underlying theory and the used questionnaire, it is referred to Hair et al. (2017b).

We estimate the model by robust and traditional PLS-PM/PLSc based on the dataset with and without missing values. Ignoring missing values, i.e., analyzing a dataset containing missing values, represents a situation where a researcher does not inspect the dataset for missing values a priori to the analysis. Consequently, the missing values which are coded as -99 are treated as actual observations and can therefore be regarded as outliers since they are obviously different from the rest of the observations.

In case of no missing values, the missing values are assumed to be completely missing at random and are removed prior to the estimation. As a consequence, they do not pose a threat for the analysis.

To obtain consistent estimates, the model is estimated by PLSc, i.e., mode B is applied for composites and mode A with a correction for attenuation is employed for common factors . Additionally, the factor weighting scheme is used for inner weighting and statistical inferences are based on bootstrap percentile confidence intervals employing 999 bootstrap runs. Table 3.2 presents the path coefficient estimates and

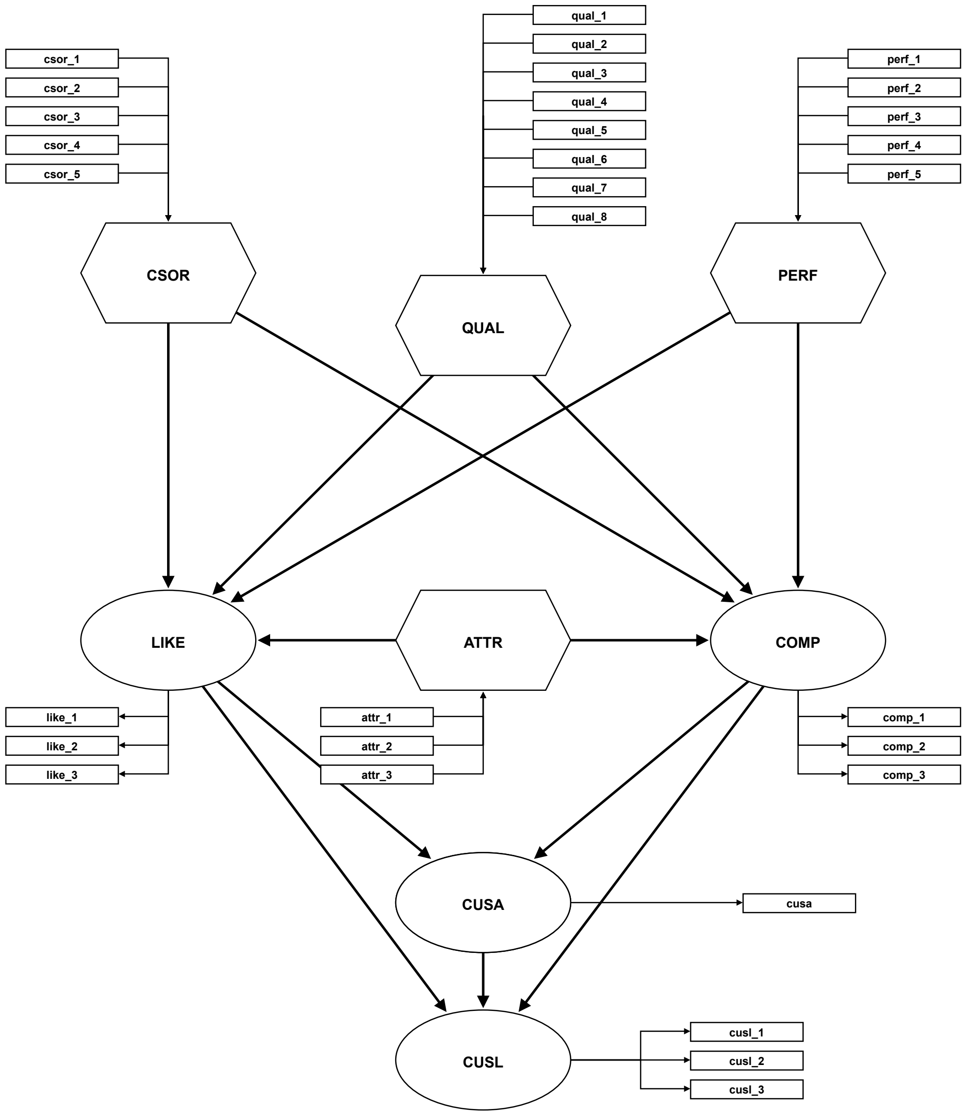

Figure 3.11: Corporate Reputation Model

their significances.[10]

Although PLSc and robust PLSc produce quite similar path coefficient estimates in case of the dataset containing outliers, there are some noteworthy differences leading to contrary interpretations. While PLSc produces a non-significant effect with a negative sign of LIKE on CUSA ($\hat{\beta} = -0.151$), employing robust PLSc results in

[10]The estimated weights and factor loadings can be found in Table 8.4 in the Appendix.

Table 3.2: Path coefficient estimates for the Corporate Reputation Model

			With outliers		Without outliers	
			traditional PLSc	robust PLSc	traditional PLSc	robust PLSc
QUAL	$\rightarrow$	COMP	0.482**	0.412**	0.486**	0.402**
PERF	$\rightarrow$	COMP	0.345**	0.398**	0.339**	0.388**
CSOR	$\rightarrow$	COMP	0.058	-0.008	0.060	0.008
ATTR	$\rightarrow$	COMP	0.098°	0.204*	0.097°	0.210*
QUAL	$\rightarrow$	LIKE	0.414**	0.463**	0.413**	0.482**
PERF	$\rightarrow$	LIKE	0.128°	0.152	0.127	0.123
CSOR	$\rightarrow$	LIKE	0.197**	0.227*	0.209**	0.204*
ATTR	$\rightarrow$	LIKE	0.182**	0.133°	0.173*	0.181°
COMP	$\rightarrow$	CUSA	0.252	0.203	0.033	0.221
LIKE	$\rightarrow$	CUSA	-0.151	0.454*	0.555**	0.449*
COMP	$\rightarrow$	CUSL	0.049	-0.054	-0.116	-0.147
LIKE	$\rightarrow$	CUSL	0.031	0.507**	0.533**	0.601**
CUSA	$\rightarrow$	CUSL	0.698**	0.504**	0.499**	0.497**

**: significant on a 1% level; *: significant on a 5% level;
°: significant on a 10% level

a clear positive effect ($\hat{\beta} = 0.454, f^2 = 0.085$). Moreover, the effect of LIKE on CUSL is non-significant under PLSc ($\hat{\beta} = 0,031$) indicating no effect, robust PLSc produces a moderate positive effect ($\hat{\beta} = 0.507, f^2 = 0.237$). In case of no outliers, both estimators lead to similar results with no contradictions in the interpretation.

3.5.2 Example: Open- and closed-book

This section compares the results of robust PLSc to the outlier-robust covariance-based (robust CB) estimator proposed by Yuan and Bentler (1998a). The latter employs M- and S- estimators to obtain robust estimates for the observed variables' variance-covariance matrix as input for the maximum likelihood (ML) estimator. For the comparison, we replicate the empirical example in Yuan and Bentler (1998a) using the open- and closed-book dataset from Mardia et al. (1979).

The dataset contains test scores of 88 students on five examinations. The first two observable variables (score on Mechanics and Vectors) are linked to the first factor (closed-book exam) and the last three observable variables (score on Algebra, Analysis and Statistics) depend on the second factor (open-book exam). For more details, see Tanaka et al. (1991).

Table 3.3 presents the estimated factor correlation ($\hat{\rho}$) for the different estimators.

Table 3.3: Open and closed book example: estimated factor correlation

Estimator	$\hat{\rho}$
PLSc	0.791
ML	0.818
robust PLSc	0.853
robust CB estimator	$[0.856; 0.896]^*$

*: Depending on the weighting factor, the estimate ranges from 0.856 to 0.896.

The ML and robust CB estimates are taken from Yuan and Bentler (1998a). Since the M- and S- estimator depend on a weighting factor, the parameter estimates depend on that weighting factor as well. As a consequence, the estimated factor correlation ranges from 0.856 to 0.896 for the robust CB estimator.

In general, the PLSc and the ML estimate and the robust PLSc and the robust ML estimates, respectively, are very similar indicating that robust PLSc performs similarly as the robust CB estimator. Moreover, the difference between robust PLSc and its traditional counterpart is 0.062, while the difference between the ML estimator and its robust version ranges from 0.038 to 0.078. This is in line with Yuan and Bentler's conclusion that no extreme influential observations are present in the dataset leading to similar results for robust and non-robust estimators.

3.6 Discussion

Outliers are a major threat to the validity of results of empirical analyses, with VB estimators being no exception. Identifying and removing outliers, if practiced at all, often entails a set of practical problems. Using methods that are robust against outliers is thus a preferable alternative.

Given the frequent occurrence of outliers in empirical research practice, it appears surprising that the behavior of traditional PLS-PM and PLSc has not yet been studied under this circumstance. The first important insight from our simulation study is that neither traditional PLS-PM nor PLSc is suitable for datasets containing outliers; both methods produce distorted estimates when outliers are present. Strikingly, even a small number of outliers can greatly distort the results of traditional PLS-PM/PLSc.

This observation underscores the need for a methodological advancement and highlights the relevance of addressing outliers in empirical research using PLS-PM/PLSc.

As a solution, we introduced the robust PLS/PLSc estimators to deal with outliers without the need to manually remove them. The robust PLS/PLSc estimators use the MCD estimator as input to the PLS-PM algorithm. This modular construction of the new method permits the PLS-PM algorithm and the correction for attenuation applied in PLSc to remain untouched and thus allow for an straightforward implementation.

The computational experiment in the form of a Monte Carlo simulation showed that both robust PLS and robust PLSc can deal with large shares of unsystematic outlier and that their results are hardly affected by the model complexity and the number of observed variables contaminated by outliers. The proposed method's estimates are almost undistorted for the outlier share of up to 40%. The share of outliers would need to reach or exceed 50% of observations for the robust PLS/PLSc to break down. This finding is unsurprising, as this level matches the asymptotic BP of the employed MCD correlation estimator. Our findings are relatively stable with regard to outlier extent and model complexity. Even for systematic outliers, our Monte Carlo simulation provides first evidence that robust PLS/PLSc yield undistorted estimates. However, the BP is slightly lower compared to the situation with unsystematic outliers. This is not surprisingly since the asymptotic Breakdown Point of an estimator is defined on basis of randomly generated contamination.

Although robust PLSc produces a large number of inadmissible solutions in case of small sample sizes, it still produces a smaller number of such solutions than does its non-robust counterpart. Furthermore, robust PLS produces only a notable number of inadmissible solutions for samples with the outlier share of 50%, while its traditional counterpart also produces higher numbers of inadmissible results for smaller outlier shares. Generally, as the sample size increases, the number of inadmissible results decreases and as expected, the estimates become more precise.

It is worth noting that if the data do not contain outliers, PLS-PM and PLSc outperform their robust counterparts with regard to efficiency, i.e., by producing undistorted estimates with smaller standard errors. This finding is unsurprising because the Pearson correlation equals the maximum likelihood correlation estimate under normality, which is known to be asymptotically efficient (Anderson and Olkin, 1985). Moreover, the MCD estimator is based only on a fraction of the original dataset, while the Pearson correlation takes the whole dataset into account.

The practical relevance of robust PLS/PLSc in empirical research is demonstrated by two empirical examples which additionally emphasize the problem of ignoring outliers. By means of the Corporate Reputation example, it is shown that not addressing outliers can affect the sign and magnitude of the estimates, and thus, also their statistical significance. This is particular problematic as researchers can draw wrong conclusions when generalizing their results. Additionally, the open- and closed-book example shows that robust PLSc produces similar results as the robust covariance-based estimator suggested by Yuan and Bentler (1998a) providing initial evidence that both estimators perform similarly well. While the latter is likely to be more efficient in case of pure common factor models as it is based on a maximum likelihood estimator, robust PLSc is likely to be advantageous in situations in which researchers face models containing both common factors and composites.

Although robust PLS and PLSc produce almost undistorted estimates when the outliers arise randomly and initial evidence is obtained that they are robust against systematic outliers, future research should investigate the behavior of these estimators in case of outliers that arise from a second population, e.g., from an underlying population that the researcher is unaware of or uninterested in. Moreover, since robust PLS and PLSc are outperformed by their traditional counterparts when no outliers are present, future research should develop statistical criteria and tests to decide whether the influence of outliers is such that the use of a robust method is recommendable. As robust covariance-based estimators have already been introduced, future research should compare their performance to robust PLS/PLSc. Furthermore, the large number of inadmissible solutions produced by PLSc if outliers are present, should be investigated. Even though an initial simulation has shown that the large number of inadmissible results is not a PLSc-specific problem, future research should examine whether a use of other correction factors (Dijkstra, 2013b) or an empirical Bayes approach (Dijkstra, 2018) could improve its performance in presence of outliers. It may be fruitful to depart from robust PLS/PLSc in exploring all these new research directions.

Chapter 4

Performing out-of-sample predictions based on models estimated by ordinal consistent partial least squares

4.1 Introduction [1]

Partial least squares path modeling (PLS-PM) was developed as a computationally efficient estimator for structural equation models containing common factors (Wold, 1974, 1982). It is based on the PLS-PM algorithm that first creates weights to form linear combinations of observed observed variables that serve as proxies for latent variables. Subsequently, it uses these proxies to estimate the model parameters. Since PLS-PM estimates are known to be only consistent at large (e.g., Hui and Wold, 1982), PLS-PM was enhanced. This enhancement, known as consistent partial

[1] This chapter is based on joint work with Gabriele Cantaluppi and Florian Schuberth. At the time this dissertation was submitted, the paper was under review for possible publication in: H. Latan, J.F. Hair & R. Noonan (Eds.), Partial least squares path modeling: Basic concepts, methodological issues, and applications (2nd ed.). Cham, Switzerland: Springer.

least squares (PLSc), applies a correction for attenuation in order to obtain consistent estimates for structural models containing common factors (Dijkstra, 2013b; Dijkstra and Henseler, 2015b). In the course of this, PLS-PM was extended to deal with non-linear and non-recursive structural models, as well as with correlated measurement errors within blocks of observed variables (Dijkstra and Schermelleh-Engel, 2014; Dijkstra and Henseler, 2015a; Rademaker et al., 2019). PLS-PM has now become a widely used estimator in various fields including marketing research (Hair et al., 2011), information systems research (Benitez et al., 2020), and tourism research (Müller et al., 2018). Moreover, during the last decade the evaluation of the predictive power of models estimated by PLS-PM has gained more and more interest (Carrión et al., 2016). For instance, PLSpredict was proposed as an approach to perform out-of-sample predictions based on a model estimated by PLS-PM (Shmueli et al., 2016). Guidelines on how to use PLSpredict have been introduced (Shmueli et al., 2019). Nowadays, PLS-PM is even regarded as the preferred approach to structural equation modeling "when the statistical objective is prediction" (Hair et al., 2017a, p. 454).

As recognized early in the PLS-PM literature the "the standard procedures cannot be used for the categorical and ordinal-scaled variables that are often encountered in the behavioral sciences" (Lohmöller, 1989, p. 155). Nevertheless, most of the developments of PLS-PM such as PLSc assume that the observed variables are measured on a metric scale. To relax this metric scale assumption, ordinal partial least squares (OrdPLS, Cantaluppi, 2012) and ordinal consistent partial least squares (OrdPLSc, Schuberth et al., 2018b) were designed to deal with ordinal categorical observed variables in a psychometric way. OrdPLS and OrdPLSc are similar to PLS-PM and PLSc, respectively; however, the Pearson correlations are replaced by polychoric/polyserial correlations. Consequently, researchers who want to apply PLSc and PLS-PM who also striving for consistent estimates in the case of ordinal categorical observed variables are advised to apply OrdPLSc and OrdPLS, respectively.

Although the development of OrdPLS and OrdPLSc is promising in terms of parameter recovery (Schuberth et al., 2018b), very little research has been conducted to assess their suitability for performing model-based out-of-sample predictions. Performing predictions based on models estimated by PLS-PM, known as PLSpredict (Shmueli et al., 2016), is well established (Shmueli et al., 2019); however, an approach to perform predictions in the context of OrdPLS and OrdPLSc when all observed variables are ordinal categorical, was only recently proposed (Cantaluppi and Schuberth, 2019). In this chapter, we propose OrdPLScpredict and OrdPLSpredict which are extensions of the approach Cantaluppi and Schuberth (2019) suggested to deal with both continuous and ordinal categorical observed variables. Additionally, we provide guidelines on how to perform OrdPLScpredict and OrdPLSpredict using the open source R package *cSEM* (Rademaker and Schuberth, 2020).

The remainder of the chapter is organized as follows: Section 4.2 presents OrdPLS and OrdPLSc. Section 4.3 gives an overview of performing out-of-sample predictions using PLScpredict and PLSpredict. In Section 4.4, we propose OrdPLScpredict and OrdPLSpredict, two approaches to perform out-of-sample predictions using models estimated by OrdPLS and OrdPLSc, respectively. In Section 4.5, we conduct a Monte Carlo simulation to evaluate the performance of our two proposed approaches. Section 4.6 provides guidelines on how to perform predictions using the open source R package *cSEM*. Our chapter closes with a discussion given in Section 4.7.

4.2 Ordinal (consistent) partial least squares path modeling

Wold (1966) originally developed PLS-PM as an approach for principal component analysis and (generalized) canonical correlation analysis, which at the time was still known as nonlinear iterative least squares and nonlinear iterative partial least squares, respectively (Tenenhaus et al., 2005). In fact, PLS-PM can emulate several of Kettenring's (1971) approaches for generalized canonical correlation analysis (Tenenhaus et al., 2005). A few years later, Wold proposed PLS-PM as a computational efficient estimator for structural models containing common factors (Wold, 1974, 1982). In this case, weights are determined by the PLS-PM algorithm to form proxies and subsequently these proxies are used to estimate the relationships between the common factors. As various researchers emphasized, PLS-PM estimates for this type of model are only consistent at large; i.e., only if both the number of observations and the number of observed variables converge to infinity, will PLS-PM estimates converge in probability to the respective population parameters (e.g., Hui and Wold, 1982; Dijkstra, 1985). However, recently various studies have shown that PLS-PM produces consistent estimates for models containing interrelated composites (Dijkstra, 2017;

Cho and Choi, 2020; Henseler, 2021).[2]

In its most modern appearance known as consistent partial least squares (Dijkstra and Henseler, 2015a,b, PLSc), it produces consistent parameter estimates for structural models containing common factors and composites. Similar to PLS-PM, PLSc relies on the PLS-PM algorithm to determine the weights to build proxies for the constructs. In cases that constructs are modeled as common factors, it applies a correction for attenuation to correlations affected by common factors. In this way, it is ensured that the construct correlation matrix is consistently estimated, and thus, consistent path coefficient estimates can be obtained. Moreover, in contrast to PLS-PM which always relies on ordinary least squares (OLS) to estimate the model parameters, PLSc applies two-stage least squares (2SLS) in the case of non-recursive structural models. Finally, a recent development allows PLSc to deal with correlated random measurement errors within a block of observed variables measuring a common factor (Rademaker et al., 2019).

PLS-PM including PLSc assumes that the observed variables are measured on a metric scale. To overcome this limitation, various modifications of PLS-PM have been developed to cope with non-metric variables such as partial maximum likelihood partial least squares (Jakobowicz and Derquenne, 2007) and non-metric partial least squares (Russolillo, 2012). A further approach that was developed to deal with ordinal categorical observed variables in a classic psychometric way, is OrdPLS (Cantaluppi, 2012). OrdPLS is similar to PLS-PM, but applies polychoric and polyserial correlations as input for the PLS-PM algorithm to take the nature of ordinal categorical observed variables into account. Consequently, the original PLS-PM algorithm remains untouched. In the same way as PLS-PM was extended by PLSc, OrdPLS was extended by OrdPLSc to consistently estimate structural models containing common factors and ordinal categorical observed variables (Schuberth and Cantaluppi, 2017; Schuberth et al., 2018b). Figure 4.1 illustrates the four steps of OrdPLSc: (i) calculating the polychoric/polyserial correlations, (ii) performing the PLS-PM algorithm, (iii) correcting for attenuation if some constructs are modeled as common factors, and (iv) estimating the path coefficients by OLS and 2SLS, respectively. Below we

[2]These composites are not only a weighted linear combination of variables, but also convey all the information between its observed variables and other variables in the model.

elaborate each of the four steps[3].

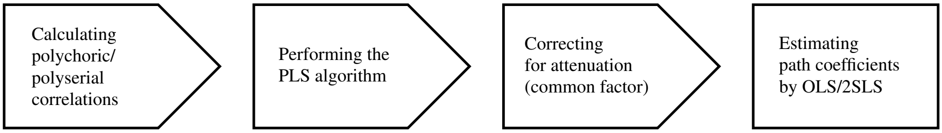

Figure 4.1: Ordinal Consistent Partial Least Squares (adapted from Schuberth et al. (2018b))

4.2.1 Calculating polychoric/polyserial correlations

Following Pearson's idea of a polytomous variable, we assume an ordinal categorical observed variable x to be the result of a categorized unobservable standard normally distributed random variable x^* (Pearson, 1900, 1913)

$$x = x_m \quad \text{if} \quad \tau_{m-1} \leq x^* < \tau_m \quad m = 1, \ldots, M \tag{4.1}$$

where the threshold parameters $\tau_0, \ldots, \tau_M$ determine the observed categories. The first and last threshold are fixed: $\tau_0 = -\infty$ and $\tau_M = \infty$. Moreover, we assume the thresholds to be strictly increasing: $\tau_0 < \tau_1 < \ldots < \tau_M$.[4]

Figure 4.2 depicts the idea of an underlying continuous variable, i.e., for observed variable x, category x_m is observed if the realisation of the underlying continuous variable x^* falls between the thresholds τ_{m-1} and τ_m.

Since we assume an ordinal categorical variable to be determined by an underlying continuous variable, it is more appropriate to consider the correlation between these underlying continuous variables for evaluating the linear relationship of interest. This is achieved by using the polychoric correlation (Drasgow, 1986). In cases where the correlation between an ordinal categorical variable and a metric variable is calculated the polyserial correlation can be used (Lee and Poon, 1986).

[3]Note that the following subsections contain large parts adapted from Schuberth et al. (2018b) which is published under the Creative Commons Attribution 4.0 International License (https://creativecommons.org/licenses/by/4.0/).

[4]In empirical work two consecutive threshold parameters can be equal, $\tau_{m-1} = \tau_m$, if the corresponding category x_m is not observed.

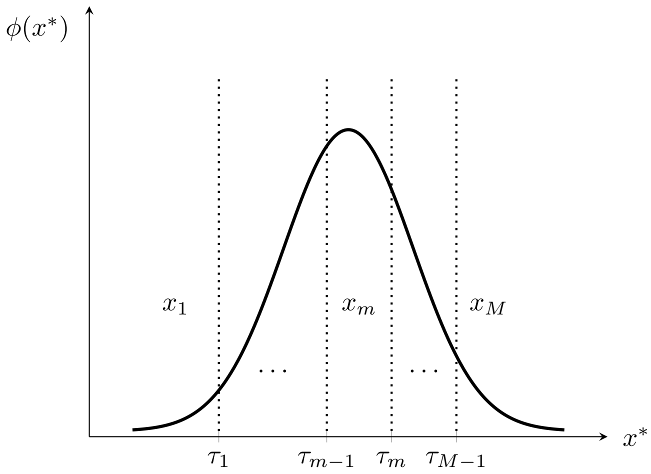

Figure 4.2: Pearson's idea of an ordinal categorical variable (taken from Schuberth et al. (2018b))

4.2.2 Performing the PLS-PM algorithm

The second step of OrdPLS and OrdPLSc involves applying the PLS-PM algorithm to the sample correlation matrix $\boldsymbol{S}$ of dimension $K \times K$ calculated in the previous step. For simplicity, the K_j observed variables belonging to one construct η_j, i.e., a common factor or a composite, are grouped to form the block j with $j = 1, \dots, J$ and where $\sum_{j=1}^{J} K_j = K$, i.e., each observed variable belongs exactly to one block.

The PLS-PM algorithm is an iterative algorithm which starts with initial arbitrary weights $\hat{\boldsymbol{w}}_j^{(0)}$ $(K_j \times 1)$. The initial weights are chosen in such a way that they satisfy the following condition: $\hat{\boldsymbol{w}}_j^{(0)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(0)} = 1$ for each block j where the $(K_j \times K_j)$ matrix $\boldsymbol{S}_{jj}$ contains the sample correlations of the observed variables of block j. This condition holds for all weights in each iteration i and can be achieved by using a scaling factor $(\hat{\boldsymbol{w}}_j^{(i)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(i)})^{-\frac{1}{2}}$ for the weights $\hat{\boldsymbol{w}}_j^{(i)}$ in each iteration i.

The PLS-PM algorithm aims to determine weights to build proxies for the J constructs. This can be done in three ways, identified as *Mode A*, *Mode B*, and *Mode C*. In the case of Mode A, the weights, also known as correlation weights, are determined as follows:

$$\hat{\boldsymbol{w}}_j^{(i+1)} \propto \sum_{l=1}^{J} \boldsymbol{S}_{jl} \hat{\boldsymbol{w}}_l^{(i)} e_{jl}^{(i)} \quad \text{with} \quad \hat{\boldsymbol{w}}_j^{(i+1)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(i+1)} = 1. \tag{4.2}$$

In the case of *Mode B*, the weights, also known as regression weights, are calculated as follows:

$$\hat{\boldsymbol{w}}_j^{(i+1)} \propto \boldsymbol{S}_{jj}^{-1} \sum_{l=1}^{J} \boldsymbol{S}_{jl} \hat{\boldsymbol{w}}_l^{(i)} e_{jl}^{(i)} \quad \text{with} \quad \hat{\boldsymbol{w}}_j^{(i+1)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(i+1)} = 1. \tag{4.3}$$

Mode C, also known as *MIMIC mode*, is a mixture of mode A and B, which we do not consider here. The inner weights e_{jl} can be obtained in three different ways, following the *centroid* (Wold, 1982), *factorial* (Lohmöller, 1989), and *path* weighting scheme. All inner weighting schemes produce essentially the same results (Noonan and Wold, 1982), hence, we consider the path weighting scheme here.[5] For the path weighting scheme, the inner weight e_{ij} is chosen as follows:

$$e_{jl}^{(i)} = \begin{cases} \hat{\boldsymbol{w}}_j^{(i)\prime} \boldsymbol{S}_{jl} \hat{\boldsymbol{w}}_l^{(i)} & \text{if } \eta_l \text{ is a consequence of } \eta_j \\ \hat{\beta}_l & \text{if } \eta_l \text{ is an antecedent of } \eta_j \\ 0 & \text{otherwise} \end{cases} \tag{4.4}$$

As the Equation (4.4) shows, the inner weight e_{jl} equals the covariance between the proxies of the constructs η_j and η_l if construct η_l is a consequence of the construct η_j. In contrast, if the construct η_l is an antecedent of the construct η_j, the inner weight e_{jl} is equal to the regression coefficient $\hat{\beta}_l$ of a multiple regression of the construct η_j on its antecedents. Otherwise, if the two constructs are not connected via the structural model, the inner weight is set to 0.

Since the PLS-PM algorithm has no single optimization criterion to be minimized, the new weights $\hat{\boldsymbol{w}}_j^{(i+1)}$ are checked for significant changes compared with the weights $\hat{\boldsymbol{w}}_j^{(i)}$ in the previous iteration step. If there is a significant change in the weights, the algorithm starts again. Otherwise, the final weights $\hat{\boldsymbol{w}}_j$ equal the stable weights determined in the last iteration.

Finally, the standardized composite loadings, i.e., the correlations between a proxy and its observed variables, are calculated as:

$$\hat{\boldsymbol{\lambda}}_j = \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j \tag{4.5}$$

If we apply OrdPLS, the standardized factor loading estimates, i.e., the estimated correlations between a common factor and its observed variables, are calculated in

[5] Note that the choice of inner weighting scheme can substantially affect the estimates in the case of models containing second-order constructs (Becker et al., 2012; Schuberth et al., 2020). For more details on the other inner weighting schemes, see Tenenhaus et al. (2005).

the same way. Thus, the polychoric/polyserial correlation matrix is taken into account and therefore the calculation considers correlations with the underlying continuous variable that correspond to ordinal observed variables of a common factor. As for PLS-PM, the factor loading estimates of OrdPLS are not consistent.

4.2.3 Correcting for attenuation if constructs are modeled as common factors

OrdPLS creates composites as proxies for constructs. Consequently, its estimates are biased if the constructs are modeled as common factors. To overcome this issue in the context of PLS-PM, Dijkstra and Henseler (2015a,b) proposed PLSc which applies a correction to obtain consistent parameter estimates. OrdPLSc applies the same correction to obtain consistent estimates for models containing common factors. The correction exploits the linearity between population factor loadings and the population weights, $\boldsymbol{\lambda}_j = c_j \boldsymbol{w}_j$ and requires that each common factor be measured by at least two observed variables. The estimated correction factor for block j satisfies the following condition

$$\text{plim } \hat{c}_j = \sqrt{\boldsymbol{\lambda}_j' \boldsymbol{\Sigma}_{jj} \boldsymbol{\lambda}_j}, \tag{4.6}$$

where $\boldsymbol{\lambda}_j$ is a column vector of length K_j containing the population loadings of common factor η_j and $\boldsymbol{\Sigma}_{jj}$ is the $(K_j \times K_j)$ population correlation matrix of the observed variables of block j.[6] The correction factor $\hat{c}_j$ can be obtained by

$$\hat{c}_j^2 = \frac{\hat{\boldsymbol{w}}_j' (\boldsymbol{S}_{jj} - \text{diag}(\boldsymbol{S}_{jj})) \hat{\boldsymbol{w}}_j}{\hat{\boldsymbol{w}}_j' (\hat{\boldsymbol{w}}_j \hat{\boldsymbol{w}}_j' - \text{diag}(\hat{\boldsymbol{w}}_j \hat{\boldsymbol{w}}_j')) \hat{\boldsymbol{w}}_j}. \tag{4.7}$$

It is chosen in such a way that the Euclidean distance between

$$\boldsymbol{S}_{jj} - \text{diag}(\boldsymbol{S}_{jj}) \quad \text{and} \quad (c_j \hat{\boldsymbol{w}}_j)(c_j \hat{\boldsymbol{w}}_j)' - \text{diag}((c_j \hat{\boldsymbol{w}}_j)(c_j \hat{\boldsymbol{w}}_j')) \tag{4.8}$$

is minimized (Dijkstra and Henseler, 2015a). For other ways to obtain correction factors, the interested reader is referred to Dijkstra (2013b). Finally. the standardized factor loadings of block j can be consistently estimated as

$$\hat{\boldsymbol{\lambda}}_j = \hat{c}_j \hat{\boldsymbol{w}}_j. \tag{4.9}$$

[6] Here we do not consider the use of *mode B* for common factors. For a consistent version of PLS-PM using mode B, the interested reader is referred to Dijkstra (2011).

4.2.4 Estimating path coefficients by OLS/2SLS

In the last step, we estimate the path coefficients based on the proxies' correlation matrix, i.e., $\hat{\boldsymbol{W}}'\boldsymbol{S}\hat{\boldsymbol{W}}$, where the matrix $\boldsymbol{W}$ of dimension $K \times J$ contains all the weight estimates. In OrdPLS, this matrix is directly applied to estimate the parameters of the structural model by OLS. In contrast, if constructs are modeled as common factors, OrdPLSc applies a correction for attenuation to the proxies' correlation matrix before calculating the path coefficients. The correlation between the two common factors η_j and η_l where $j \neq l$ can be consistently estimated by:

$$\widehat{\text{cor}}(\eta_j, \eta_l) = \frac{\hat{\boldsymbol{w}}_j' \boldsymbol{S}_{jl} \hat{\boldsymbol{w}}_l}{\sqrt{\hat{c}_j^2 \hat{\boldsymbol{w}}_j' \hat{\boldsymbol{w}}_j \hat{c}_l^2 \hat{\boldsymbol{w}}_l' \hat{\boldsymbol{w}}_l}} \tag{4.10}$$

Similarly, if construct η_j is modeled as a common factor and construct η_l as a composite, the consistently estimated correlation is obtained by

$$\widehat{\text{cor}}(\eta_j, \eta_l) = \frac{\hat{\boldsymbol{w}}_j' \boldsymbol{S}_{jl} \hat{\boldsymbol{w}}_l}{\sqrt{\hat{c}_j^2 \hat{\boldsymbol{w}}_j' \hat{\boldsymbol{w}}_j}}. \tag{4.11}$$

In the case of both constructs being modeled as composites, no correction of the correlation is required because we assume that the correlation between two composites is not affected by attenuation. Finally, in OrdPLSc the path coefficients are estimated by OLS or 2SLS depending on the structure of the underlying structural model.

4.3 Model-based predictions using PLS-PM and PLSc

In the context of PLS-PM, out-of-sample predictions have increasingly gained attention (Evermann and Tate, 2014; Carrión et al., 2016; Sarstedt and Danks, 2021; Shmueli et al., 2016, 2019). To perform such out-of-sample prediction, a procedure called PLSpredict was introduced (Shmueli et al., 2016). In PLSpredict, endogenous values are predicted by exogenous values based on a model estimated by PLS-PM. In cases where the model parameters are estimated by PLSc, we label the procedure PLScpredict. In the following exposition we present the steps of PLSpredict and PLScpredict.

We start out with two datasets, namely the train dataset $\boldsymbol{X}_{\text{train}}$ and the test dataset $\boldsymbol{X}_{\text{test}}$. The train dataset contains observations for all observed variables and is used to estimate the model parameters by PLS-PM or PLSc, i.e., the weights

$\hat{\boldsymbol{w}}_j$, $j = 1, \ldots, J$, the loadings $\hat{\boldsymbol{\lambda}}_j$, and the path coefficients of the exogenous and endogenous constructs, which are captured in the matrices $\hat{\boldsymbol{\Gamma}}$ and $\hat{\boldsymbol{B}}$, respectively. Subsequently, out-of-sample predictions can be performed based on the estimated model and the observations given in the test dataset. The test dataset comprises N observations for at least all observed variables connected to exogenous constructs. Importantly, the observations of the test dataset are not used during the estimation of the model.

In the context of PLSpredict, we can distinguish different types of predictions (Lohmöller, 1989; Shmueli et al., 2016): (i) *valid predictions* in which predictions for scores of exogenous constructs are obtained by observations of their associated observed variables, (ii) *structural predictions* in which predictions for scores of endogenous constructs are obtained by exogenous construct scores, (iii) *communal predictions* in which predictions for values of observed variables associated with endogenous constructs are obtained by scores of their associated constructs, (iv) *redundant predictions* in which predictions for values of the observed variables associated with endogenous constructs are obtained by exogenous construct scores and the estimated structural model, (v) *latent predictions* in which predictions for scores of endogenous constructs are obtained by observations of the observed variables associated with exogenous constructs and the estimated structural model, and (vi) *operative predictions* in which predictions for values of observed variables associated with endogenous constructs are obtained by observations for the observed variables associated with exogenous constructs and the estimated structural model.

Obviously, operative predictions are the most general case in that they involve all the steps of the other types of predictions. Additionally, predictions can only be evaluated if they are performed on item level, i.e., if values of the observed variables are predicted. Against this background, we will now focus on operative predictions. Note that other types of predictions can be obtained by starting or stopping the approach we describe below at a later or earlier stage.

To obtain operative predictions, valid predictions have to be performed first. To do this, we standardize the N observations of the test dataset $\boldsymbol{X}_{\text{test}}$ for the observed variables associated with the exogenous constructs using the corresponding moments estimated on the basis of the train datasets (Shmueli et al., 2016). Subsequently, for all J_{exo} exogenous constructs, we predict scores as the weighted sum of their associated observed variables using the observations from the test dataset. Consequently, the predicted scores of the exogenous constructs are obtained as follows:

$$\hat{\boldsymbol{\eta}}_{j,\text{exo}} = \boldsymbol{X}_{j,\text{test}}\hat{\boldsymbol{w}}_j \qquad j = 1, \ldots, J_{\text{exo}} \tag{4.12}$$

In a next step, we use the predicted scores of the exogenous constructs to predict the scores of the J_{end} endogenous constructs in accordance with the structural model, i.e. we perform structural predictions:

$$\hat{\boldsymbol{\eta}}_{\text{end}} = \hat{\boldsymbol{\eta}}_{\text{exo}}\hat{\boldsymbol{\Gamma}}'(\boldsymbol{I} - \hat{\boldsymbol{B}}')^{-1}, \tag{4.13}$$

where $\hat{\boldsymbol{\eta}}_{\text{end}}$ is a matrix of dimension $N \times J_{\text{end}}$ that contains the predictions for the scores of the endogenous constructs in its columns.

Finally, in the last step, we use the scores of the endogenous constructs to predict values of the observed variables connected to endogenous constructs, i.e. we perform communal predictions:

$$\hat{\boldsymbol{X}}_{\text{end}} = \hat{\boldsymbol{\eta}}_{\text{end}}\hat{\boldsymbol{\Lambda}}'_{\text{end}} \tag{4.14}$$

where the matrix $\hat{\boldsymbol{\Lambda}}_{\text{end}}$ contains the estimated loadings of the observed variables connected to endogenous constructs in its columns. To obtain the final predictions for continuous observed variables, the values in $\hat{\boldsymbol{X}}_{\text{end}}$ are brought back to their original scale using the mean and standard deviation of the train dataset (see Shmueli et al. (2016)). In cases where ordinal categorical observed variables are associated with endogenous constructs, Cantaluppi and Schuberth (2019) proposed rounding the predicted values to an integer. Thereby, we obtain predictions that are in line with the domain of the ordinal categorical observed variables.

To evaluate the model's predictive power, the test dataset must contain observations for all observed variables. In such a case, the observed values of the endogenous constructs can be compared to their predicted counterparts (Shmueli, 2010). As predictive performance measures, the mean absolute error (MAE), the root mean squared error (RMSE) (Evermann and Tate, 2014), and the concordance can be used to evaluate the predictive power of the model. Note that the latter, that summarizes the number of 'exact' predictions, is only useful to assess predictions' accuracy in the case of categorical observed variables. Also, contributions to MAE and RMSE related to ordered categorical observed variables can be appropriately interpreted as penalties of 0 in the presence of exact concordances between actual and predicted categories, as penalties of 1 if a category $h-1$ or $h+1$ is predicted for the observed category h, and as penalties of 2(MAE) or 4(RMSE) if a category $h-2$ or $h+2$ is predicted for category h, and so on.

4.4 Model-based predictions using OrdPLS and OrdPLSc (OrdPLSpredict and OrdPLScpredict)

In this section, we present an approach to performing predictions based on a model estimated by OrdPLS and OrdPLSc, which we label OrdPLSpredict and OrdPLScpredict, respectively. This approach allows for both ordinal categorical and continuous observed variables. In fact, our approach generalizes the idea Cantaluppi and Schuberth (2019) presented to perform predictions based on a model estimated by OrdPLS or OrdPLSc in cases where all observed variables are ordinal categorical.

We rely on the idea presented in section 4.2.1 that an ordinal categorical observed variable x is the outcome of a polytomized standard normally distributed unobservable random variable x^*, see Equation (4.1). In cases with more than one categorical observed variable, we assume that the categorical observed variables $\boldsymbol{x}$ are the outcome of categorized underlying multivariate standard normally distributed latent random variables $\boldsymbol{x}^*$. Consequently, the observations of the ordinal categorical observed variables $\boldsymbol{x}_j$ belonging to construct j are the outcome of columnwise transformations (as expressed by Equation (4.1)) of the observations of the underlying multivariate normally distributed random variables that are stacked in the matrix $\boldsymbol{X}_j^*$, expressed as:

$$\boldsymbol{X}_j^* \quad \rightarrow \quad \boldsymbol{X}_j \tag{4.15}$$

As shown in Section 4.2, OrdPLS and OrdPLSc can deal with both ordinal categorical and continuous observed variables. Note that the transformation is only performed for the observations of the ordinal categorical observed variables and not for those of continuous observed variables.

As in PLSpredict, the first step is to estimate the model parameters. In the context of OrdPLSpredict and OrdPLScpredict this is done by OrdPLS and OrdPLSc, respectively, based on the train dataset which contains observations for at least one ordinal categorical observed variable. Otherwise, if the train dataset contains no ordinal categorical observed variable, there is no need to apply OrdPLS or OrdPLSc.

Next, the estimated model and the observations of the test dataset are used to perform out-of-sample predictions. For this purpose, the test dataset must at least contain observations for the observed variables associated with the exogenous constructs which are stored in the matrix $\boldsymbol{X}_{\text{test, exo}}$.[7] We assume the observations of ordinal categorical observed variables of the test dataset to be the columnwise transformations of a multivariate *truncated* normally distributed dataset, as stated by Equation (4.1):

$$\boldsymbol{X}_{\text{test, exo}}^{\text{Trunc},*} \quad \rightarrow \quad \boldsymbol{X}_{\text{test, exo}} \tag{4.16}$$

The observations from the multivariate truncated normal distribution $\boldsymbol{X}_{\text{test, exo}}^{\text{Trunc},*}$ are standardized and have the same correlation matrix as the polychoric correlation between the observed variables connected to the train data's exogenous constructs. For each subject in the test dataset, given the expressed categories, the domain of $\boldsymbol{X}_{\text{test, exo}}^{\text{Trunc},*}$ is defined by the corresponding pairs of thresholds in the set of thresholds $(\boldsymbol{\tau}_{j-1}, \boldsymbol{\tau}_j)$. These are obtained from the polychoric correlation matrix used for model parameter estimation, i.e., the one based on the train dataset $\boldsymbol{X}_{\text{train}}$. If the test dataset contains additional observations for the observed variables associated with endogenous constructs, the model's predictive performance can be evaluated by comparing the observed variables' observed values to their predicted counterparts.

In the following explication, we present the steps OrdPLSpredict and OrdPLScpredict take to perform out-of-sample predictions. Similar to PLSpredict and PLScpredict, the only difference between OrdPLSpredict and OrdPLScpredict is that the former uses OrdPLS estimates, while the latter employs OrdPLSc estimates.

1. Standardize the test dataset $\boldsymbol{X}_{\text{test, exo}}$ using the means and standard deviations of the train dataset. Note that only the continuous observed variables are standardized, i.e., the ordinal categorical observed variables comprised in the test dataset remain untouched.

2. Predict the scores of the exogenous constructs. In PLSpredict, scores of construct j are obtained as linear combinations of the observed variables $\boldsymbol{x}_j$ and

[7] In cases where only values of a subset of the observed variables associated with endogenous constructs are predicted, a subset of the observed variables associated with the exogenous constructs might be sufficient.

the corresponding weight estimates, regardless of the observed variables' measurement scale. In contrast, in OrdPLSpredict and OrdPLScpredict the nature of ordinal categorical observed variables is explicitly taken into account. Since the number of ordinal categorical observed variables associated with exogenous constructs can differ, three cases have to be distinguished, namely ones in which (i) all observed variables are continuous, (ii) all observed variables are ordinal categorical, and (iii) there is a mixture of continuous and ordinal categorical observed variables.

Considering cases in which all observed variables associated with exogenous constructs are continuous, construct scores are obtained as in PLSpredict:

$$\hat{\boldsymbol{\eta}}_{j,\text{exo}} = \boldsymbol{X}_{j,\text{test},\text{exo}} \hat{\boldsymbol{w}}_j, \qquad j = 1, \ldots, J_{\text{exo}} \tag{4.17}$$

where $\hat{\boldsymbol{w}}_j$ are the weight estimates obtained by OrdPLS/OrdPLSc based on the train data.

Considering cases in which all observed variables associated with exogenous constructs are ordinal categorical, the unknown values of the unobservable variables underlying these observed variables (see Equation (4.1)) need to be aggregated. Specifically, the exogenous constructs' scores can be calculated as linear combinations of multivariate truncated normally distributed random variables $x^{\text{Trunc},*}_{j,\text{test},\text{exo}}$ which are continuous. They also have the domain $(\boldsymbol{\tau}_{j-1}, \boldsymbol{\tau}_j)$ defined by the threshold parameters of the polychoric correlations based on the train dataset $\boldsymbol{X}_{\text{train}}$, conditional on categories that characterize the manifest test data set regarding ordinal categorical observed variables. Consequently, we obtain the construct scores as follows:

$$\hat{\boldsymbol{\eta}}_{j,\text{exo}} = \boldsymbol{X}^{\text{Trunc},*}_{j,\text{test},\text{exo}} \hat{\boldsymbol{w}}_j \qquad j = 1, \ldots, J_{\text{exo}} \tag{4.18}$$

As Equation (4.18) shows, the distribution of the construct scores is a linear combination of multivariate truncated normally distributed random variables with OrdPLS/OrdPLSc weight estimates based on the train data. The distribution of the construct scores has no simple form but can be approximated by simulation. To simulate this distribution for each subject, we generate $n_{\text{pred}} = 100$ drawings from a multivariate truncated normal distribution with a variance-covariance matrix that equals the polychoric correlation matrix of the train dataset and truncation limits that equal the threshold parameter estimates of this polychoric correlation matrix. As a consequence, we obtain n_{pred} draws in total for the unobservable variables underlying the ordinal categorical observed variables associated with exogenous constructs $\boldsymbol{X}^{\text{Trunc},*,p}_{j,\text{test},\text{exo}}$ for $p = 1, \ldots, n_{\text{pred}}$, and thus, n_{pred} sets of predicted scores for each exogenous construct:

$$\hat{\boldsymbol{\eta}}^{p}_{j,\text{exo}} = \boldsymbol{X}^{\text{Trunc},*,p}_{j,\text{test},\text{exo}} \hat{\boldsymbol{w}}_j \qquad j = 1, \ldots, J_{\text{exo}}, \quad p = 1, \ldots, n_{\text{pred}} \tag{4.19}$$

Considering a case in which there is a mixture of continuous and ordinal categorical observed variables associated with exogenous constructs, we generate n_{pred} drawings from a multivariate truncated normal distribution for both the categorical and the continuous observed variables to obtain construct scores. The variance-covariance matrix of the multivariate truncated normal distribution equals the estimated correlation matrix of the observed variables based on the train dataset, which can contain polychoric, polyserial and Pearson correlations. We take the continuous observed variables into account during the simulation to preserve the correlation structure. However, their generated values in $\boldsymbol{X}^{\text{Trunc},*,p}_{j,\text{test},\text{exo}}$ are replaced by the corresponding observations from the test data. For the ordinal categorical observed variables, the truncation limits are appropriately chosen, conditional on categories that characterize the test data set by using the threshold estimates obtained by the polychoric/polyserial correlations based on the train data. In contrast, for the continuous observed variables we use arbitrary lower and upper truncation limits, e.g. -10 and 10. Consequently, we obtain n_{pred} datasets for the observed variables connected to exogenous constructs where the observations of the continuous observed variables equal the observations from the test data, while for the categorical observed variables we use the generated dataset of the multivariate truncated normal distribution. Based on the resulting samples, we calculate the n_{pred} scores for each exogenous construct as follows:

$$\hat{\boldsymbol{\eta}}^{p}_{j,\text{exo}} = \boldsymbol{X}^{\text{Trunc},*,p}_{j,\text{test},\text{exo}} \hat{\boldsymbol{w}}_j \qquad j = 1, \ldots, J_{\text{exo}}, \quad p = 1, \ldots, n_{\text{pred}} \tag{4.20}$$

3. Predict the endogenous constructs' scores using the exogenous constructs' scores in accordance with the structural model. Using the n_{pred} predicted scores of the exogenous constructs, n_{pred} scores for the endogenous constructs can be

predicted via the structural model:

$$\hat{\boldsymbol{\eta}}^p_{\text{end}} = \hat{\boldsymbol{\eta}}^p_{\text{exo}} \hat{\boldsymbol{\Gamma}}' (\boldsymbol{I} - \hat{\boldsymbol{B}}')^{-1} \qquad p = 1, \ldots, n_{\text{pred}} \tag{4.21}$$

For the case in which only continuous observed variables are connected to exogenous constructs, the matrix with the predicted construct scores $\hat{\boldsymbol{\eta}}^p_{\text{exo}}$ is replaced by $\hat{\boldsymbol{\eta}}_{\text{exo}}$ from Equation (4.17). Consequently, we do not obtain n_{pred} matrices containing predicted scores for the endogenous constructs, but only one matrix $\hat{\boldsymbol{\eta}}_{\text{end}}$.

4. Predict the values of the observed variables belonging to the endogenous constructs. Here, two cases need to be distinguished, namely ones in which (i) an observed variable belonging to an endogenous construct is continuous, and (ii) an observed variable belonging to an endogenous construct is ordinal categorical.

   If the observed variable $\boldsymbol{x}_k$ belonging to the j-th endogenous construct is continuous, first n_{pred} predictions are obtained by multiplying the construct scores with the estimated loading from the train dataset:

$$\hat{\boldsymbol{x}}^p_{k,\text{end}} = \hat{\boldsymbol{\eta}}^p_{j,\text{end}} \hat{\lambda}_{j,k,\text{end}} \qquad j = 1, \ldots, J_{\text{end}} \quad k = 1, \ldots, K_j \quad p = 1, \ldots, n_{\text{pred}} \tag{4.22}$$

   In contrast, if the observed variable associated with an endogenous construct is ordinal categorical, predictions for the continuous unobservable variables underlying the ordinal categorical observed variable have to be obtained first. As we have predicted n_{pred} scores for an endogenous construct, we also obtain n_{pred} predictions for the unobservable variable underlying the ordinal categorical observed variable. This we do by multiplying the endogenous construct's scores with the estimated loading corresponding to the k-th observed variable $\boldsymbol{x}_k$ of the j-th endogenous construct η_j:

$$\hat{\boldsymbol{x}}^{*,p}_{k,\text{end}} = \hat{\boldsymbol{\eta}}^p_{j,\text{end}} \hat{\lambda}_{j,k,\text{end}} \qquad j = 1, \ldots, J_{\text{end}} \quad k = 1, \ldots, K_j \quad p = 1, \ldots, n_{\text{pred}} \tag{4.23}$$

   Obviously, the only difference between the procedure for continuous and ordinal categorical variables is that for the latter the values of the unobservable variable underlying the ordinal categorical variable are predicted.

Finally, to obtain one prediction for each observation of the test dataset, the n_{pred} predictions for the observed variables of the endogenous constructs need to be summarized. For this purpose, Cantaluppi and Boari (2016) proposed the *mean*, the *median*, or the *mode* approach. In the case of the mean approach, the i-th value of a continuous observed variable is predicted as the mean of the n_{pred} draws, expressed as:

$$\hat{x}^*_{k,i,\text{end}} = \frac{1}{n_{\text{pred}}} \sum_{p=1}^{n_{\text{pred}}} \hat{x}^{*,p}_{k,i,\text{end}} \qquad i = 1, \ldots, N \tag{4.24}$$

The median approach works similar to the mean approach; however, instead of using the mean to summarize the n_{pred} predictions, the median is used. While for continuous observed variables the summarized values equal the final predictions, for ordinal categorical variables, the summarized values are transformed into categorical values according to Equation (4.1) using the estimated thresholds based on the train data.

As a third approach to summarizing the n_{pred} predictions, we can use the mode approach. It uses the maximum of the predicted unobservable variable's empirical density on the intervals defined by the thresholds. Consequently, this approach cannot be used for continuous observed variables.

Finally, the continuous observed variables' predicted values are brought back to their original scale using the mean and standard deviation of the train data.

4.5 Monte Carlo simulation

To assess the performance of OrdPLSpredict and OrdPLScpredict, we conducted a Monte Carlo simulation. Specifically, we compared the accuracy of predictions for continuous and ordinal categorical observed variables obtained by OrdPLScpredict, OrdPLSpredict, PLScpredict, and PLSpredict. For OrdPLSpredict and OrdPLScpredict, we used the *mean* and the *median* approach to obtain the final predictions of the observed variables. Since PLSpredict and PLScpredict produce real values as predictions, even for ordinal categorical observed variables, we rounded the predicted values for the categorical observed variables to an integer.

4.5.1 Simulation design

To compare the various approaches' performance, we considered a population model consisting of one exogenous common factor η_1 and two endogenous common factors η_2 and η_3. We assumed all common factors to be standardized and related via a structural model, as follows:

$$\eta_2 = 0.6 \cdot \eta_1 + \zeta_1 \tag{4.25}$$

$$\eta_3 = 0.0 \cdot \eta_1 + 0.6 \cdot \eta_2 + \zeta_2 \tag{4.26}$$

Additionally, we measured each of the three common factors by three observed variables; therefore, x_{11}, x_{12}, and x_{13} loaded on η_1 with the factor loadings of 0.8, 0.7, and 0.6, respectively; x_{21}, x_{22}, and x_{23} loaded on η_2 each with a factor loading of 0.7; and x_{31}, x_{32}, and x_{33} loaded on η_3 with factor loadings of 0.5, 0.7, and 0.9, respectively. Similar to the common factors, the observed variables were assumed to be standardized. Moreover, all structural disturbance terms ζ_j and random measurement errors ε_i were assumed to be uncorrelated. Similarly, the common factors η_j were assumed to be uncorrelated with the random measurement errors. Finally, we assumed the exogenous common factor η_1 to be uncorrelated with the structural disturbance terms ζ_1 and ζ_2. Consequently, we could give the population correlation matrix of the observed variables as follows:

$$\boldsymbol{\Sigma} = \begin{array}{c} \begin{array}{ccccccccc} \underline{x_{11}} & \underline{x_{12}} & \underline{x_{13}} & \underline{x_{21}} & \underline{x_{22}} & \underline{x_{23}} & \underline{x_{31}} & \underline{x_{32}} & \underline{x_{33}} \end{array} \\ \begin{pmatrix} 1.000 & & & & & & & & \\ 0.560 & 1.000 & & & & & & & \\ 0.480 & 0.420 & 1.000 & & & & & & \\ 0.336 & 0.294 & 0.252 & 1.000 & & & & & \\ 0.336 & 0.294 & 0.252 & 0.490 & 1.000 & & & & \\ 0.336 & 0.294 & 0.252 & 0.490 & 0.490 & 1.000 & & & \\ 0.144 & 0.126 & 0.108 & 0.210 & 0.210 & 0.210 & 1.000 & & \\ 0.202 & 0.176 & 0.151 & 0.294 & 0.294 & 0.294 & 0.350 & 1.000 & \\ 0.259 & 0.227 & 0.194 & 0.378 & 0.378 & 0.378 & 0.450 & 0.630 & 1.000 \end{pmatrix} \end{array} \tag{4.27}$$

To examine the approaches' performance in predicting ordinal categorical observed variables' values, the values of observed variables x_{11}, x_{13}, x_{21}, x_{23}, and x_{33} were transformed as described in section 4.2. In doing so, we varied the number of categories between four and five, and we considered two sets of threshold parameters that Rhemtulla et al. (2012) proposed. In the case of symmetrically distributed threshold parameters, all five observed variables were categorized using the following threshold parameters: -∞, -1.25, 0, 1.25, ∞ and -∞, -1.5, -0.5, 0.5, 1.5, ∞, respectively. Similarly, for the extreme asymmetric threshold parameter distribution, we set the thresholds to -∞, 0.28, 0.71, 1.23, ∞ in the case of four categories, and to -∞, 0.05, 0.44, 0.84, 1.34, ∞ in the case of five categories.

To assess the approaches' predictive performance, we considered test datasets containing $N = 100$ observations. Additionally, we focused on the following three predictive performance measures: (i) the mean absolute error (MAE), (ii) the root mean squared error (RMSE), and (iii) the concordance. The MAE is the average absolute deviation of the predicted value of an observed variable from its observed counterpart, $\frac{1}{N}\sum_{i=1}^{N}|\hat{x}_i - x_i|$, where N is the sample size of the test dataset. The RMSE is the square root of the average squared deviation of the predicted value from its observed counterpart, $\sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{x}_i - x_i)^2}$ and the concordance gives the share of exact predictions for an ordinal categorical observed variable. Note that the concordance measure can only be reasonably calculated for categorical observed variables. Further, while small values for the MAE and the RMSE indicate an accurate prediction, for the concordance measure large values are desirable.

The complete Monte Carlo simulation is carried out in the statistical programming environment R (R Core Team, 2020). To assess the influence of the train dataset's sample size on the approaches' predictive performance, we varied the sample sizes of the train dataset from 200, 500, and 1000 observations. Hence, in total, we had 24 conditions: three different sample sizes of the test dataset (200, 500, and 1000 observations) $\times$ two different numbers of categories for the ordinal categorical observed variables (four and five categories) $\times$ two threshold parameter distributions (symmetric and extreme asymmetric distributions) $\times$ two ways to obtain the final predictions for OrdPLScpredict and OrdPLSpredict (mean and median approach). For each condition, we conducted 500 simulation runs. In each run, we drew a dataset from the multivariate standard normal distribution with a mean vector of $\mathbf{0}$ and the correlation matrix shown in Equation (4.27) using the `mvrnorm()` function of the *MASS* package (Venables and Ripley, 2002). The number of draws equaled the train dataset's sample size from the corresponding condition plus the 100 observations of the test dataset. Subsequently, we categorized the observations for the variables $x_{11}, x_{13}, x_{21}, x_{23}$, and x_{33} to obtain categorical variables using threshold parameters from the corresponding condition. To estimate the model by PLS-PM, PLSc, OrdPLS, and OrdPLSc, we used the `csem()` function of the R package *cSEM* (version 0.4.0.9000, Rademaker and Schuberth, 2020). In doing so, the path weighting scheme was used for inner weighting and Mode A was used to calculate the weights to form the proxies for the common factors. Additionally, we replaced inadmissible estimations, i.e., each condition was based on 500 valid estimations. An inadmissible estimation suffers from at least one of the following problems: (i) the PLS-PM algorithm has not converged, (ii) at least one reliability estimate is larger than 1, (iii) at least one absolute factor loading estimate is larger than 1, (iv) the model-implied construct correlation matrix is not positive semi-definite, and (v) the model-implied observed variables' correlation matrix is not positive semi-definite. Next, by using the `predict()` function of the R package *cSEM*, we obtained the predictions for the observed variables associated with endogenous constructs using OrdPLScpredict, OrdPLSpredict, PLScpredict, and PLSpredict.

4.5.2 Simulation results

In this section, we present the results of our Monte Carlo simulation. Since the results for the ordinal categorical observed variables and the continuous observed variables, respectively, are very similar, we only present the results for the ordinal categorical observed variable x_{23} and for the continuous observed variable x_{31}. Further, the results for four and five categories are very similar. Therefore, we only report the results for four categories. Furthermore, the results are only slightly affected by the train dataset's sample size. Hence, we only report the results for 500 observations. Finally, the results for the mean and median approaches used to obtain the predictions with OrdPLScpredict and OrdPLSpredict hardly differ. Therefore, we report only the results for the mean approach. The complete results are given in the Appendix.

Figure 4.3 shows the average values over the 500 simulation runs for the three predictive performance measures, namely, the concordance, the MAE, and the RMSE. Since the information value of the concordance measure is limited for continuous observed variables, we report it only for the ordinal categorical observed variable x_{23}.

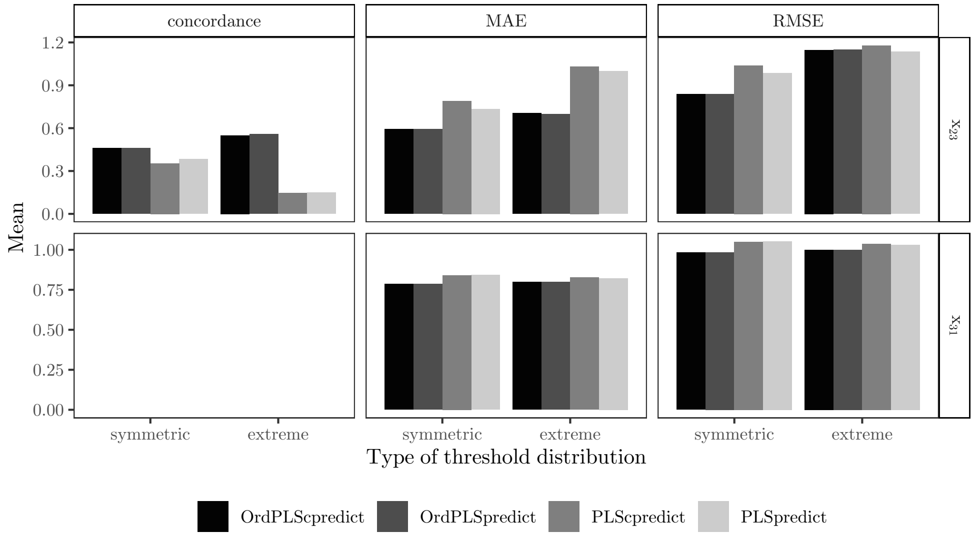

Figure 4.3: Average performance measures

Considering the ordinal categorical observed variable x_{23}, OrdPLScpredict and OrdPLSpredict produce very similar results. The same is observed for PLScpredict and PLSpredict. Also, the four approaches produce very similar results in terms of average RMSEs for an extreme asymmetric threshold distribution. Considering the average MAE and the average RMSE for a symmetric threshold distribution, OrdPLScpredict and OrdPLSpredict outperform PLSpredict and PLScpredict. Similarly, OrdPLScpredict and OrdPLSpredict outperform the other approaches in terms of the average concordance. This is particularly obvious in the case of the extreme asymmetric threshold parameter distribution. Considering the continuous observed variable x_{31}, all approaches perform very similar in terms of average MAE and average RMSE.

4.5.3 Simulation insights

The results of our Monte Carlo simulation show that for continuous observed variables, OrdPLScpredict and OrdPLSpredict perform very similar to PLScpredict and PLSpredict. Considering the ordinal categorical observed variables, no clear picture emerges, and the advantage of an approach depends on the employed predictive performance measure. While for the average RMSE and an extreme asymmetric threshold distribution all four approaches produce similar results, OrdPLScpredict and Ord-

PLSpredict outperform the other two approaches regarding the average concordance and the average MAE. In line with the findings of Cantaluppi and Schuberth (2019), this is particularly the case for extremely skewed categorical observed variables. Similarly, correcting for attenuation bias as is done in OrdPLSc and PLSc, does not lead to more accurate predictions.

4.6 Guidelines on performing predictions using the R package *cSEM*

To illustrate how researchers can apply OrdPLScpredict, OrdPLSpredict, PLScpredict, and PLSpredict, we provide guidelines for the open source R package *cSEM*. In doing so, we focus on a model that Hwang and Takane (2004) studied. We display their model in Figure 4.4. To preserve clarity, we have omitted the measurement error terms and the structural error terms. For a motivation of the model, the interested reader is referred to the article of Hwang and Takane (2004).

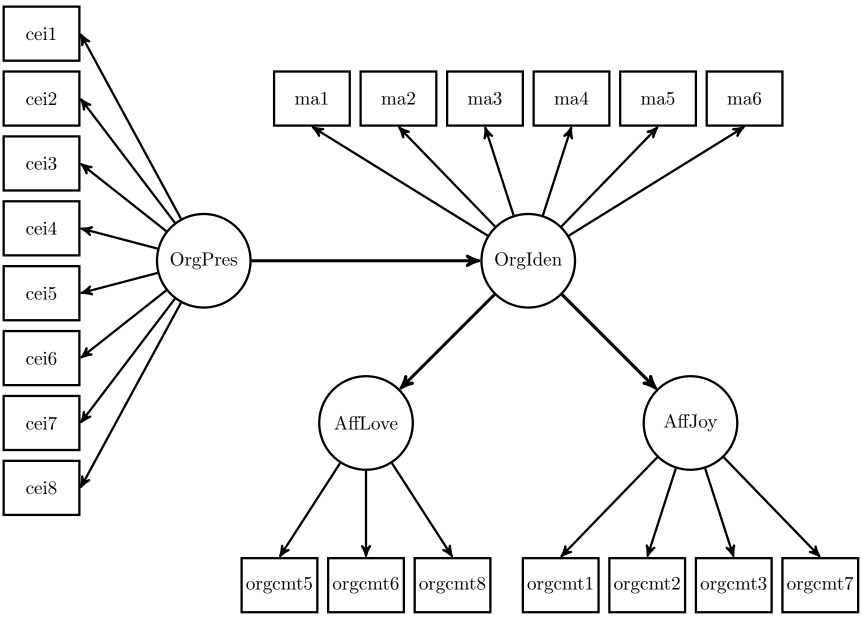

Figure 4.4: Model from Hwang and Takane (2004)

As Figure 4.4 shows, the model consists of the following four concepts modeled as common factors: organizational prestige (OrgPres), organizational identification (OrgIden), affective commitment (Love) (Afflove), and affective commitment (Joy) (AffJoy). Bergami and Bagozzi (2000) give an elaboration of the concepts. The considered dataset is part of the survey data used in Bergami and Bagozzi's (2000) study. It consists of 305 observations for the 21 observed variables. Each observed variable is measured on a 5 point scale ranging from 1 (=strongly disagree) to 5 (=strongly agree), i.e., all observed variables are categorical. A detailed description of the observed variables can be found in Henseler (2021, Table 6.1).

As a first step, we need to estimate the model parameters. For this purpose, we can use the `csem()` function of the *cSEM* R package. In general, the `csem()` function requires a dataset and a model as input.

To specify models in *cSEM*, lavaan syntax (Rosseel, 2012) is used. Specifically, the '=∼' operator is used to specify the relationship between observed variables and common factors, the '<∼' operator is used to specify observed variables forming a composite, and the '∼' operator is used to specify the structural model. The specification for the model illustrated in Figure 4.4 is given as follows:

```
.model="
#Measurement models
OrgPres =~ cei1 + cei2 + cei3 + cei4 + cei5 + cei6 + cei7 + cei8
OrgIden =~ ma1 + ma2 + ma3 + ma4 + ma5 + ma6
AffJoy  =~ orgcmt1 + orgcmt2 + orgcmt3 + orgcmt7
AffLove =~ orgcmt5 + orgcmt 6 + orgcmt8

# Structural model
OrgIden ~ OrgPres
AffLove ~ OrgIden
AffJoy  ~ OrgIden
"
```

The dataset we use here is publicly available and also provided in the *cSEM* R package. However, as it is provided in the *cSEM* package all observed variables are labeled as *numeric*. In this case, the `csem()` function uses the Pearson correlations to estimate the model parameters, i.e., PLS-PM or PLSc is employed. To use the polychoric/polyserial correlations, and thus to apply OrdPLS or OrdPLSc, the ordinal categorical observed variables need to be labeled as *ordered factors*, as shown in the following:

```r
# Load the data from the cSEM package
data(BergamiBagozzi2000)

# Transform the numerical indicators into factors
data_new <- data.frame(cei1    = as.ordered(BergamiBagozzi2000$cei1),
                       cei2    = as.ordered(BergamiBagozzi2000$cei2),
                       cei3    = as.ordered(BergamiBagozzi2000$cei3),
                       cei4    = as.ordered(BergamiBagozzi2000$cei4),
                       cei5    = as.ordered(BergamiBagozzi2000$cei5),
                       cei6    = as.ordered(BergamiBagozzi2000$cei6),
                       cei7    = as.ordered(BergamiBagozzi2000$cei7),
                       cei8    = as.ordered(BergamiBagozzi2000$cei8),
                       ma1     = as.ordered(BergamiBagozzi2000$ma1),
                       ma2     = as.ordered(BergamiBagozzi2000$ma2),
                       ma3     = as.ordered(BergamiBagozzi2000$ma3),
                       ma4     = as.ordered(BergamiBagozzi2000$ma4),
                       ma5     = as.ordered(BergamiBagozzi2000$ma5),
                       ma6     = as.ordered(BergamiBagozzi2000$ma6),
                       orgcmt1 = as.ordered(BergamiBagozzi2000$orgcmt1),
                       orgcmt2 = as.ordered(BergamiBagozzi2000$orgcmt2),
                       orgcmt3 = as.ordered(BergamiBagozzi2000$orgcmt3),
                       orgcmt5 = as.ordered(BergamiBagozzi2000$orgcmt5),
                       orgcmt6 = as.ordered(BergamiBagozzi2000$orgcmt6),
                       orgcmt7 = as.ordered(BergamiBagozzi2000$orgcmt7),
                       orgcmt8 = as.ordered(BergamiBagozzi2000$orgcmt8))
```

Finally, to estimate the model parameters, the dataset and the specified model are provided as input to the `csem()` function as the following shows:

```r
res <- csem(.model = .model, .data = data_new[1:250,], .resample_method = "bootstrap")
```

Since we wanted to evaluate our model's predictive performance, we use only the first 250 observations of the dataset for the estimation. Note that *cSEM* applies a correction for attenuation by default if common factors are included in the model, i.e., PLSc or OrdPLSc is used. Further, by default, *cSEM* uses the path weighting scheme to calculate the inner weights. In our case, the specified model was estimated by OrdPLSc since at least one observed variable is labeled as factor and the model comprises at least one common factor. If the user aims for statistical inference about the parameter estimates, the argument '.resample_method' has to be set to either 'bootstrap' or 'jackknife', otherwise no standard errors will be estimated. In our case, we used bootstrap for statistical inference. By default, 499 bootstrap runs are conducted. A summary of the estimated model can be obtained via the `summarize()` function.

To assess the estimated model's predictive performance, the `predict()` function is used. Evaluating the predictive performance of a model requires benchmark predictions. For that purpose, the '.benchmark' argument of the `predict()` function can be used to determine how benchmark predictions are obtained. In cases where the original model was estimated by OrdPLSc or OrdPLS, the benchmark predictions are rounded for the categorical observed variables if PLS-PM or PLSc were used to estimate the benchmark model. If predictions based on OrdPLSpredict were to be used as benchmark, the '.benchmark' argument must be set to 'PLS-PM', the argument '.treat_as_continuous' must be set to 'FALSE', and the argument '.disattenuate' has to be set to 'FALSE' to prevent a correction for attenuation. In the case of OrdPLScpredict and OrdPLSpredict, by default $n_{\text{pred}} = 100$ draws are performed for the multivariate truncated normally distributed unobservable variables underlying the categorical observed variables associated with exogenous constructs. To summarize the n_{pred} predictions for the observed variables of the endogenous constructs, the 'mean', 'median', or 'mode' approach can be used. The approach can be chosen separately for the target predictions and the benchmark predictions using the arguments '.approach_score_target' and '.approach_score_benchmark', respectively.

The `predict()` function allows the user to provide a test dataset via the '.test_data' argument. If no test dataset is provided, k fold cross-validation is applied, i.e., the dataset from the original estimation is randomly split into k (approximately) equal parts. Subsequently, the values of each part are predicted based on a model estimated on the basis of the remaining parts. To adjust the number of cross-validation folds the '.cv_folds' argument is used. By default this argument is set to 10. To minimize the effect of random splitting in k fold cross-validation, the k fold cross-validation is repeated several times (Shmueli et al., 2019). In the `predict()` function, the number of repetitions is adjusted via the argument '.r'. If a test dataset is provided, no k fold cross-validation is conducted and predictions are performed based on the observations of the test dataset.

For the considered empirical example, we used PLScpredict as benchmark, and provided the last 55 observations of the original dataset as test dataset, i.e., only the observations of the test dataset were predicted. Additionally, we used the 'median' approach to obtain predictions in OrdPLScpredict. The results of the `predict()` function are as follows.

```
pred = predict(.object = res, .benchmark = "PLS-PM",.test_data = data_new[(251):305,],
               .treat_as_continuous = TRUE, .approach_score_target = "median")
```

The output contains some general information in the top. In addition, it provides the MAE and the RMSE of the predictions for all observed variables associated with endogenous constructs:

```
## ------------------------------------------------------------------------------
## ----------------------------------- Overview -----------------------------------
##
##  Number of obs. training          = 250
##  Number of obs. test              = 55
##  Number of cv folds               = NA
##  Number of repetitions            = 1
##  Handle inadmissibles             = stop
##  Estimator target                 = 'OrdPLS'
##  Estimator benchmark              = 'PLS-PM'
##  Disattenuation target            = 'TRUE'
##  Disattenuation benchmark         = 'TRUE'
##
## ------------------------------ Prediction metrics ------------------------------
##
##
##   Name        MAE target  MAE benchmark  RMSE target RMSE benchmark   Q2_predict
##   ma1             0.5091         1.3455       0.7628         1.5255       0.0101
##   ma2             0.4545         1.3818       0.7006         1.5255       0.0125
##   ma3             0.5636         1.0000       0.8202         1.2863       0.0000
##   ma4             0.6000         1.6909       0.8842         1.8635      -0.1832
##   ma5             0.6909         1.6545       0.9909         1.8537      -0.2624
##   ma6             0.5636         1.2182       0.8202         1.4460       0.0148
##   orgcmt5         0.4000         0.9818       0.6876         1.1442       0.0000
##   orgcmt6         0.4727         0.6545       0.7135         0.9145       0.0000
##   orgcmt8         0.7455         0.9091       1.0180         1.1755      -0.2475
##   orgcmt1         0.6727         1.1818       0.9045         1.3817       0.0000
##   orgcmt2         0.5818         1.1273       0.8090         1.3212       0.0000
##   orgcmt3         0.5455         1.1273       0.7862         1.3484       0.0495
##   orgcmt7         0.5636         0.8364       0.8202         1.0445      -0.0916
## ------------------------------------------------------------------------------
```

Considering our example's MAE and RMSE, the results show that OrdPLScpredict outperforms PLScpredict for both measures and all observed variables. In contrast to the MAE and the RMSE, the concordance is not reported by default. However, it can be accessed as follows:

```r
pred$Prediction_metrics
```

```
##       Name MAE_target MAE_benchmark RMSE_target RMSE_benchmark  Q2_predict
## 1      ma1  0.5090909     1.3454545   0.7627701      1.5255401  0.01012658
## 2      ma2  0.4545455     1.3818182   0.7006490      1.5255401  0.01253918
## 3      ma3  0.5636364     1.0000000   0.8201995      1.2862914  0.00000000
## 4      ma4  0.6000000     1.6909091   0.8842048      1.8635255 -0.18318966
## 5      ma5  0.6909091     1.6545455   0.9908674      1.8537431 -0.26240458
## 6      ma6  0.5636364     1.2181818   0.8201995      1.4459976  0.01478353
## 7  orgcmt5  0.4000000     0.9818182   0.6875517      1.1441551  0.00000000
## 8  orgcmt6  0.4727273     0.6545455   0.7135061      0.9145292  0.00000000
## 9  orgcmt8  0.7454545     0.9090909   1.0180195      1.1755076 -0.24751656
## 10 orgcmt1  0.6727273     1.1818182   0.9045340      1.3816986  0.00000000
## 11 orgcmt2  0.5818182     1.1272727   0.8090398      1.3211565  0.00000000
## 12 orgcmt3  0.5454545     1.1272727   0.7862454      1.3483997  0.04947368
## 13 orgcmt7  0.5636364     0.8363636   0.8201995      1.0444659 -0.09159483
##    concordance_target concordance_benchmark
## 1           0.5272727            0.10909091
## 2           0.5636364            0.05454545
## 3           0.4909091            0.27272727
## 4           0.4727273            0.05454545
## 5           0.4363636            0.07272727
## 6           0.4909091            0.16363636
## 7           0.6363636            0.18181818
## 8           0.5454545            0.43636364
## 9           0.3818182            0.30909091
## 10          0.4000000            0.18181818
## 11          0.4545455            0.18181818
## 12          0.4909091            0.20000000
## 13          0.4909091            0.29090909
```

This shows that the concordance is larger for all observed variables in the case of OrdPLScpredict, which indicates more accurate predictions than those obtained by PLScpredict. These results are also in line with the findings of our Monte Carlo simulation.

In general, the *cSEM* R package provides users with a lot of flexibility. For more details about the package, we refer the interested reader to the manual. Also, additional tutorials using the *cSEM* package can be found in Henseler (2021).

4.7 Discussion

The past decade has seen increased scholarly attention to evaluating the predictive power of models estimated by PLS-PM (e.g., Carrión et al., 2016; Shmueli et al., 2016, 2019). This is mainly due to the *causal-predictive* nature of PLS-PM (Chin et al., 2020). However, as Schuberth et al. (forthcoming) has emphasized, if PLS-PM is applied in the context of explanatory modeling, i.e., in theory testing, researchers should not rely solely on predictive metrics for model evaluation, but should consider all possible means known from explanatory modeling for model assessment, including overall model fit assessment.

In this chapter, we have focused on the predictive power of models estimated by OrdPLS and OrdPLSc. Specifically, we presented OrdPLScpredict and OrdPLSpredict. The two approaches are similar to those known from PLS-PM and PLSc to perform predictions, namely PLSpredict and PLScpredict. In contrast to PLSpredict and PLScpredict, our two proposed approaches take the nature of ordinal categorical observed variables into account. Additionally, our approaches resemble those Cantaluppi and Schuberth (2019) recently proposed. However, our two proposed approaches are not limited to models containing only ordinal categorical observed variables.

The results of our Monte Carlo simulation to evaluate OrdPLScpredict's performance provides several interesting insights. First, all approaches, i.e., OrdPLScpredict, OrdPLSpredict, PLScpredict, and PLSpredict perform very similar in cases where values of continuous observed variables are predicted. Second, considering the concordance evaluation metric and the MAE, OrdPLScpredict outperforms the other approaches in cases where values of ordinal categorical observed variables are predicted. This is especially the case with highly skewed ordinal categorical observed variables. In contrast, there is no big difference between the approaches' performance in terms of RMSE in case of an extreme asymmetric threshold distribution. Third, comparing the performance of OrdPLScpredict and OrdPLSpredict to the performance of PLScpredict and PLSpredict, the results show that not correcting for attenuation, even if the parameter estimates are not consistent, does not lead to a worse predictive performance.

A crucial point in predictive research is the principle that estimation should be based solely on the train dataset, while prediction should be based solely on the test dataset (James et al., 2017). In OrdPLSpredict and OrdPLScpredict, we simulate values for the observed variables connected to exogenous constructs from a multivariate truncated normal distribution if ordinal categorical variables are present. Specifically, we use a variance-covariance matrix that equals the estimated correlation matrix and truncation limits that equal the estimated thresholds of the train dataset. Although using estimated threshold parameters based on the train dataset is the only feasible solution in various situations, e.g., in cases of small test datasets, future research should evaluate ways of rendering the current prediction method more robust to situations in which the test data set's correlation structure slightly differs from the one observed on the training data set. For instance, research could consider the effect on a model's predictive performance if the test dataset's correlation matrix instead the train dataset's is used for simulating the scores of exogenous constructs. Further, simulation studies are limited regarding their design. Consequently, future research should evaluate the effect of our chosen simulation parameters. Specifically, the effect of the test data sample size and model complexity should be evaluated. Finally, the results of OrdPLScpredict should be compared to predictions based on other estimates such as GSCA or canonical correlation analysis.

Chapter 5

Using confirmatory composite analysis to assess composites in human development research

5.1 Introduction[1]

Human development research often relies on aggregated variables or composites to operationalize theoretical concepts of interest (e.g., Blau, 1998; Davis et al., 2004). Already in 1983, Rushton et al. (1983) recognized the aggregation principle's relevance in the context of human development research. For instance, composite indices such as the United Nations Development Program's Human Development Index (Hopkins, 1991; UNDP, 1990) or the Centre for Global Development and Foreign Policy's Commitment to Development Index (Lee et al., 2020) are frequently applied in human development research (e.g., Chowdhury and Squire, 2006; Harttgen and Klasen, 2011; Noorbakhsh, 1998). In all such instances, the theoretical concept of interest is

[1] This chapter is based on joint work with Florian Schuberth and Jörg Henseler. By the time this dissertation was submitted, the paper was under review for possible publication in the International Journal of Behavioral Development.

represented by a composite, i.e., a linear combination of more elementary variables.

Although human development research often deals with composites, these constructs are mostly assessed by confirmatory factor analysis (CFA, Jöreskog, 1979). For instance, in the existing human development literature, CFA was used to assess work and job withdrawal (Blau, 1998), which were both modeled as composites. CFA is not only a standard tool in human development research; it is also frequently applied in other research fields such as psychology (DiStefano and Hess, 2005; MacCallum and Austin, 2000), business management (Mak and Sockel, 2001), and criminology (Williams et al., 2007). In CFA, which is a covariance structure analysis (CSA) technique, theoretical concepts are modeled as common factors and not as composites (Jöreskog, 1969, 1970a). Common factors are unobserved variables that are assumed to explain the variance-covariance structure of observed variables. As all CSA techniques, CFA entails the following four steps: (i) model specification, (ii) model identification, (iii) model estimation, and (iv) model assessment (e.g., Bollen, 1989).

Considering the situation outlined above, it would be illogical for researchers to employ CFA as a statistical tool for construct validation if they want to model theoretical concepts that function according to a composite. To avoid the misuse of CFA in cases where a theoretical concept is modeled as a composite, researchers are faced with the question of how to assess composites with the same degree of rigor as they are accustomed to when studying common factors with CFA.

Against this background, this chapter presents a novel kind of CSA devoted to the analysis of interrelated composites: confirmatory composite analysis (CCA, Schuberth et al., 2018a). Since CCA is a CSA technique, it follows the same four steps, namely model specification, model identification, model estimation and model assessment (Henseler and Schuberth, 2020; Hubona et al., 2021; Schuberth, 2021). Originally, it was suggested to use MAXVAR, an approach to generalized canonical correlation analysis (Kettenring, 1971), however, a recently proposed specification allows us to employ the maximum likelihood estimator known from CFA (Henseler and Schuberth, 2021b; Schuberth, forthcoming). Overall, CCA shows the same benefits for assessing theoretical concepts modeled as composites as CFA shows for theoretical concepts modeled as common factors. Hence, CCA is a suitable approach for assessing composites as it overcomes the drawback CFA has in assessing composites.

The remainder of this chapter is structured as follows: The following section emphasizes the need for a proper method to assess composites in the context of human development research by highlighting the important role of composites in this discipline. Subsequently, we present CCA and describe its steps. Following this description, we provide an illustrative example in the context of human development research. Finally, the chapter closes with concluding remarks.

5.2 The need for a proper method to assess composites in human development

Various fields in human development research rely on the aggregation of more elementary variables, i.e., they frequently use composites. For instance, fear, anger and joy were modeled as composites to study their effects on children's emotional development (Kochanska, 2001). Similarly, core-self evaluation was proposed to be modeled as composite which comprises self-esteem, generalized self-efficacy, locus of control, avoidance motivation and approach motivation (Johnson et al., 2008). Another example is socio-economic status which is "composed of items relating to parental educational attainment, occupational prestige, and family income" (p. 86S, Wright et al., 2017). Similarly, work withdrawal and job withdrawal are modeled as composites composed of unfavorable job behaviors, lateness and absence, and turnover intent, desire to retire and intended retirement age, respectively (Blau, 1998). Moreover, composites often appear as indices, so-called composite indices. The arguably most prominent composite index in the context of human development research is the Human Development Index (HDI), which measures the development status of a country and is composed of the life expectancy index, the gross domestic product index and the education index (UNDP, 1990). Due to several criticism of the HDI, the Modified Human Development Index was introduced (Noorbakhsh, 1998). Additionally, alternative indices, such as the Composite Global Well-Being Index (Chaaban et al., 2015), have been proposed. Besides the HDI, the Gender Development Index, the Human Poverty Index (UNDP, 1990), the Inequality in Human Development Index, the Gender Inequality Index, and the Multi-dimensional Poverty Index (UNDP, 2010) are popular composite indices in human development research. Alongside composite indices used to evaluate the development status of countries, composite indices are also used in other contexts, such as in assessing the quality of universities (Murias et al., 2008). For a literature overview on composites in higher education research we refer to Table I in Asif and Searcy (2014). Further, composite indices are applied on the individual level to evaluate children's development status. Such indices include the Mental Development Index (Bayley and Reuner, 1969) which focuses on the status of cognitive and language development (Lowe et al., 2011) or the Early Development Index (Janus and Offord, 2000) which evaluates a child's development status in deciding on school readiness.

In general, composites are the outcome of a dimension reduction (Dijkstra and Henseler, 2008). Thus, combining variables into a composite does carry information loss. However, currently researchers do not assess whether the benefits of dimension reduction, such as studying a single variable instead of multiple variables combined, sufficiently compensate for the disadvantage of losing information. Similarly, researchers lack statistical methods to assess whether a block of observed variables acts as a whole. Both these issues can be addressed by means of CCA. Against this background, we present CCA in the following section.

5.3 Confirmatory composite analysis and its step-by-step application

A common method used to assess theoretical concepts is CSA – a family of generalized methods – in which hypotheses concerning variance-covariance matrices' underlying structure can be assessed. Although CSA literature has already mentioned the possibility of assessing composites (e.g., Schönemann and Steiger, 1976) and whether a composite acts as a single variable (Borsboom et al., 2003), to our knowledge the application of composites in CSA in human development research is still rather limited.

To address this gap, we present a recently developed CSA method, namely CCA (Schuberth et al., 2018a). CCA was first sketched by Jörg Henseler and Theo K. Dijkstra (Henseler et al., 2014) when they used the iterative partial least squares path algorithm (Wold, 1975) for model estimation. Subsequently, it was fully elaborated by Schuberth et al. (2018a) who suggested the employment of MAXVAR, one of Kettenring's (1971) approaches to generalized canonical correlation analysis to estimate the model parameters. However, a more recently introduced specification allows for conducting a CCA using ML (Henseler and Schuberth, 2021b; Hubona et al., 2021; Schuberth, forthcoming). Although CCA has been introduced in various fields, such as business research (Henseler, 2021), managerial science (Schuberth, 2021), and information systems research (Hubona et al., 2021), it has not yet been presented to the field of human development research.

CCA is similar to CFA (Jöreskog, 1979) in which the theoretical concepts are modeled as common factors that explain the variance-covariance structure of a set of observed variables. In contrast to CFA, CCA models theoretical concepts as composites which emerge from a set of observed variables. In its application, CCA follows the same four steps as CFA, which we elaborate in the next sections.

5.3.1 Model specification in CCA

In a first step of CCA, a composite model has to be specified (Cho et al., in press; Dijkstra, 2013a, 2017). Considering K observed variables, the observed variables that belong to one composite η_j are stored in block $\boldsymbol{x}_j$ with K_j observed variables which are allowed to covary freely. Following the composite model, we assume that each observed variable belongs to one block and that the composites convey all of the information between their blocks. The composition of a composite η_j can be understood as a prescription of dimension reduction (Dijkstra and Henseler, 2008), which is typically expressed as follows:

$$\eta_j = \boldsymbol{w}_j'\boldsymbol{x}_j \qquad j = 1, \ldots, J \tag{5.1}$$

where $\boldsymbol{w}_j$ is a vector of weights and J is the number of composites. Specifying a composite in terms of weights is done very intuitively because it directly reflects how the ingredients compose the composite. However, such specification prevents researchers from estimating a composite model with common CFA software such as lavaan (Rosseel, 2012), AMOS (Arbuckle, 2014) and Mplus (Muthén and Muthén, 1998-2017).

To overcome this issue, we rely on a specification that was introduced recently, which expresses the relations between a composite and its observed variables in terms of composite loadings (Schuberth, forthcoming). In doing so, not only one composite, but as many composites as observed variables are extracted per block. These composites are uncorrelated among each other and thus, together they span the same space as the observed variables do. Consequently, Equation (5.1) is rewritten as:

$$\begin{pmatrix} \eta_j \\ \boldsymbol{\nu}_j \end{pmatrix} = \boldsymbol{W}_j' \boldsymbol{x}_j \tag{5.2}$$

We follow Henseler (2021) in denoting the composite of interest η_j as an emergent variable to emphasize that it emerges from its observed variables and conveys all the information between its observed variables and the other variables in the model. In contrast, the remaining composites $\boldsymbol{\nu}_j$, which are labeled as excrescent variables, have no surplus meaning and just serve the purpose of spanning the remaining space of the observed variables. Hence, the excrescent variables capture the remaining variances and covariances among the observed variables of one block that are not accounted for by the emergent variable. Moreover, the excrescent variables are assumed to be uncorrelated with one another and uncorrelated with the emergent variable.

Equation (5.2) makes it apparent that the relationship between composites and their observed variables can be expressed in terms of composite loadings $\boldsymbol{\Lambda}_j$ instead of weights $\boldsymbol{W}_j$:

$$\boldsymbol{x}_j = (\boldsymbol{W}_j')^{-1} \begin{pmatrix} \eta_j \\ \boldsymbol{\nu}_j \end{pmatrix} = \boldsymbol{\Lambda}_j \begin{pmatrix} \eta_j \\ \boldsymbol{\nu}_j \end{pmatrix} \tag{5.3}$$

Since the transposed weight matrix $\boldsymbol{W}_j'$ is quadratic and of full rank, it can be inverted. As a consequence, the intra-block variance-covariance matrix, i.e., the variance-covariance matrix of a block of observed variables, can be displayed as follows:

$$\boldsymbol{\Sigma}_{jj} = \boldsymbol{\Lambda}_j \boldsymbol{\Phi}_{jj} \boldsymbol{\Lambda}_j' \tag{5.4}$$

The matrix $\boldsymbol{\Phi}_{jj}$ equals the variance-covariance matrix of block j's emergent and excrescent variables. Since the excrescent variables $\boldsymbol{\nu}_j$ are uncorrelated with one another and uncorrelated with the emergent variable η_j, $\boldsymbol{\Phi}_{jj}$ is a diagonal matrix.

In addition to extracting composites from the blocks of observed variables, i.e., emergent and excrescent variables, their covariances need to be specified. While the emergent variables are typically allowed to covary freely, the excrescent variables do not covary with any other variables in the model apart from their corresponding observed variables. Consequently, the covariances between observed variables of two different blocks $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ are stored in the inter-block covariance matrix $\boldsymbol{\Sigma}_{ij}$:

$$\boldsymbol{\Sigma}_{ij} = \boldsymbol{\Lambda}_i \boldsymbol{\Phi}_{ij} \boldsymbol{\Lambda}_j' \tag{5.5}$$

where the matrix $\boldsymbol{\Phi}_{ij}$ contains the covariances between the emergent and excrescent variables of the i-th and j-th block. The complete observed variables' variance-covariance matrix $\boldsymbol{\Sigma}(\boldsymbol{\theta})$ implied by the model is a block matrix with the intra-block variance-covariance matrices on the main diagonal and the inter-block covariance matrices on the off-diagonal. The vector $\boldsymbol{\theta}$ contains all model parameters, specifically the composite loadings and the covariances between the emergent variables.

To illustrate this way of specifying composites, we consider a situation in which a researcher wants to study two correlated composites η_1 and η_2, where the two composites are made up of three and four observed variables, respectively. Following the specification described above, each composite is replaced by a set of emergent and excrescent variables as displayed in Figure 5.1. Specifically, the first composite is replaced by one emergent variable η_1 and two excrescent variables ν_{11} and ν_{12}, while the second composite is replaced by one emergent variable η_2 and three excrescent variables ν_{21}, ν_{22}, and ν_{23}.

In Figure 5.1, observed variables are depicted as rectangles. The emergent and excrescent variables are displayed as hexagons to distinguish them from common factors, which are typically expressed as ovals. However, most CFA software with a graphical user interface such as AMOS (Arbuckle, 2014) model these variables as common factors and thus display them as ovals. Consequently, we need to constrain variances of measurement errors associated with observed variables to zero. Further, the relations between the variables are depicted by different types of arrows. While single-headed arrows display linear regression coefficients, double-headed arrows illustrate covariances.

5.3.2 Model identification in CCA

Once the model has been specified, we need to ensure that the model is identified, i.e., that there is a unique solution for the model parameters. In general, models can be either under-identified, just-identified or over-identified (Brown, 2015). If a model is under-identified – also referred to as a not-identified model – more than

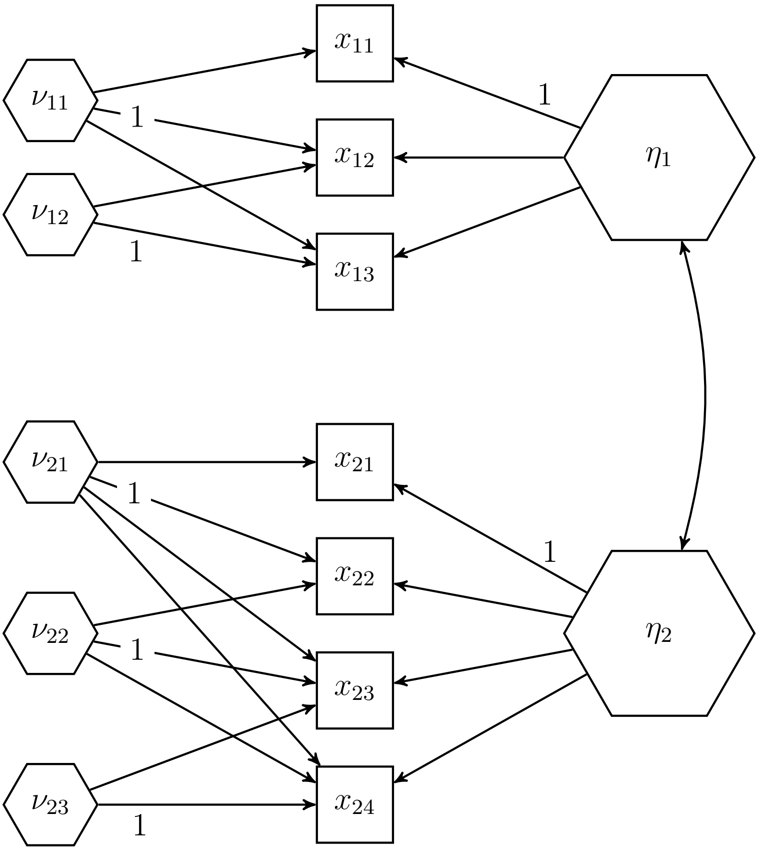

Figure 5.1: Specification of a composite model

one set of parameters is consistent with the model constraints. Consequently, there is no unique solution for the model parameters so that only limited conclusions can be drawn from the model. In contrast, a just-identified model provides a unique solution for the model parameters and shows the same number of free parameters as non-redundant elements of the observed variables' variance-covariance matrix, i.e., it shows zero degrees of freedom. In empirical analysis, such models cannot be used to evaluate the overall model fit since they perfectly fit the data. An over-identified model also provides a unique solution for the model parameters; however, it shows more non-redundant elements in the observed variables' variance-covariance matrix than model parameters and thus puts contraints on the variance-covariance matrix, i.e., it shows a positive number of degrees of freedom. These constraints can be exploited in empirical studies to assess the overall model fit.

To achieve model identification in CCA, several additional constraints need to be imposed. In the following exposition, we provide concise guidelines; for a more technical explanation of the identification of composite models, see Schuberth (forthcoming). First, the variances of the emergent and excrescent variables need to be determined. Hence, we recommend that one composite loading for each emergent and excrescent variable be constrained to one. In doing so, one needs to ensure that an observed variable serves not multiple times as scaling variable. For our example specification in Figure 5.1, x_{11}, x_{12}, and x_{13} serve as scaling variables for η_1, ν_{11}, and ν_{12}, respectively. A similar action was taken for the second block of observed variables. Second, further composite loadings of the excrescent variables need to be fixed to avoid over-parameterization. For this reason, we recommend that excrescent variables' composite loadings be fixed at zero in the following way: For the first excrescent variable, no additional constraints are imposed; for the second excrescent variable, we fix one of the composite loadings at zero; for the third excrescent variable, we fix two composite loadings at zero; for the fourth excrescent variable, we fix three composite loadings at zero; and so forth. Consequently, for the last excrescent variable of each block, one composite loading will remain unconstrained. As Figure 5.1 shows, in our example specification the first composite loading of the excrescent variable ν_{12} was fixed at zero. A similar action was taken for the other excrescent variables.

In the situation described above, the degrees of freedom are obtained as follows:

$$
\begin{aligned}
\mathrm{df} = & \underbrace{0.5 \cdot K \cdot (K+1)}_{\substack{\text{elements of the lower triangle} \\ \text{including the main diagonal} \\ \text{of the empirical} \\ \text{variance-covariance matrix}}} - \underbrace{\sum_{j=1}^{J}\left(K_j - 1 + \frac{K_j(K_j-1)}{2}\right)}_{\text{number of free composite loadings}} \\
& - \underbrace{0.5 \cdot J \cdot (J+1)}_{\substack{\text{number of free variances and covariances} \\ \text{between the emergent variables}}} - \underbrace{(K-J)}_{\substack{\text{number of free variances} \\ \text{of the excrescent variables}}} \\
= & 0.5 \cdot \left(K(K-2) + J(3-J) - \sum_{j=1}^{J} K_j^2 \right)
\end{aligned}
\tag{5.6}
$$

With Equation (5.6) we can show that our example specification depicted in Figure 5.1 has 6 degrees of freedom:

$$
\mathrm{df} = 0.5 \cdot (7 \cdot 5 + 2 \cdot 1 - 3^2 - 4^2) = 6 \tag{5.7}
$$

5.3.3 Model estimation in CCA

After ensuring identification of the model, the next step of estimating the model parameters can be taken. For this purpose, a variety of estimators implemented in common CFA software can be used, such as the ML estimator (Jöreskog, 1969; Schuberth, forthcoming) and the generalized least squares estimator (Browne, 1974). Besides these estimators, as originally proposed in CCA, estimators that emerged outside the realm of CFA can be applied, e.g., MAXVAR (Schuberth et al., 2018a) and partial least squares path modeling (Henseler and Schuberth, 2020). However, note that these estimators require a different model specification in terms of composite weights without excrescent variables.

5.3.4 Model assessment in CCA

In the last step of CCA, the model is assessed. This involves assessing the overall model fit and the parameter estimates. Overall model fit refers to the comparison of the observed variables' sample variance-covariance matrix $\boldsymbol{S}$ and their estimated model-implied variance-covariance matrix $\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}})$. In CFA literature, the dominant ways of assessing the overall model fit are statistical tests for exact model fit and fit indices to assess the model's approximate fit (e.g., Schermelleh-Engel et al., 2003). In CCA, overall model fit assessment helps to evaluate whether composites fully convey the information between blocks, and thus whether their ingredients act as a whole instead of a mere loose collection of parts. Hence, this assessment examines the trade-off between the benefits of dimension reduction and losing information by forming a composite. If the estimated model's fit is regarded as unacceptable, forming composites is most likely not justified since the information loss is not tolerable and "more information can be extracted from the data" (Jöreskog, 1969, p. 201). Therefore, researchers are advised to consider the observed variables individually.

Besides overall model fit assessment, the parameter estimates need to be assessed. CFA software provides estimates and their standard errors for the parameters from the specification described above, i.e., the covariances between the emergent variables and the composite loadings. Hence, inference about these parameters can be drawn in the common way. In contrast, weight estimates are not directly provided. Researchers who are interested in the weights, e.g., who want to evaluate the contribution each observed variable makes to a composite or to calculate composite scores, can exploit the relationship between the weights and composite loadings as described in Equation (5.3). Consequently, the weight estimates can be obtained as the inverse of the estimated composite loading matrix:

$$\hat{\boldsymbol{W}}_j = (\hat{\boldsymbol{\Lambda}}_j')^{-1} \tag{5.8}$$

However, to obtain standard errors of the weight estimates, requires more effort. For instance, researchers could obtain them via bootstrap or using the delta method as proposed by Schuberth (forthcoming).

5.4 Illustrative example

To illustrate the use of CCA in human development research, we study the relations between political instability, agricultural inequality, and industrial development using the Russet data (Russett, 1964)[2]. The Russet data contains observations from 47 countries for 10 variables. While *Agricultural Inequality* is a composite made up of the percentage of farmers that own half of the land (*farm*), the inequality of land distribution (*gini*), and the percentage of farmers that rent all their land (*rent*), the composite *Political Instability* is composed of variables indicating the stability of a democracy (*demo*), the instability of the executive (*inst*), whether a country's form of government is a dictatorship (*dict*), the number of violent internal war incidents (*ecks*), and people killed as a result of civic group violence (*deat*). Similarly, the composite *Industrial Development* is formed of the gross national product per capita (*gnpr*) and the percentage of labor force employed in agriculture (*labo*). Note that some of the variables have been transformed before the analysis as suggested by Tenenhaus and Tenenhaus (2011).[3] Equation 5.9 gives the observed variables'

[2]The data can be accessed using the R package cSEM (Rademaker and Schuberth, 2020).

[3]The original Russett dataset is publicly available, see e.g., Gifi (1990). Moreover the transformed dataset is available in the R package cSEM (Rademaker and Schuberth, 2020).

empirical correlations:

$$\begin{array}{c} \\ \boldsymbol{\Sigma} = \end{array} \begin{array}{c} \begin{array}{cccccccccc} \underline{\text{farm}} & \underline{\text{gini}} & \underline{\text{rent}} & \underline{\text{demo}} & \underline{\text{inst}} & \underline{\text{ecks}} & \underline{\text{dict}} & \underline{\text{deat}} & \underline{\text{gnpr}} & \underline{\text{labo}} \end{array} \\ \begin{pmatrix} 1.00 & & & & & & & & & \\ 0.94 & 1.00 & & & & & & & & \\ 0.46 & 0.39 & 1.00 & & & & & & & \\ -0.43 & -0.36 & 0.17 & 1.00 & & & & & & \\ 0.12 & 0.14 & 0.08 & -0.34 & 1.00 & & & & & \\ 0.35 & 0.29 & 0.09 & -0.60 & 0.33 & 1.00 & & & & \\ 0.28 & 0.21 & 0.01 & -0.59 & 0.02 & 0.39 & 1.00 & & & \\ 0.41 & 0.43 & 0.29 & -0.49 & 0.08 & 0.63 & 0.53 & 1.00 & & \\ -0.37 & -0.30 & -0.06 & 0.63 & -0.14 & -0.55 & -0.62 & -0.51 & 1.00 & \\ 0.30 & 0.26 & -0.21 & -0.71 & 0.25 & 0.49 & 0.66 & 0.52 & -0.82 & 1.00 \end{pmatrix} \end{array} \tag{5.9}$$

This model and the Russet data have already been showcased in other studies focusing on composites (e.g., Tenenhaus and Tenenhaus, 2011). Hence, for a detailed explanation of the observed variables, we refer to Tenenhaus and Tenenhaus (2011) and Russett (1964).

As we explained in the section “Model Specification in CCA” above, each composite is replaced by a set of emergent and excrescent variables and the corresponding composite loadings. The complete model specification is displayed in Figure 5.2.

As Figure 5.2 illustrates, *Agricultural Inequality*, *Political Instability*, and *Industrial Development* are allowed to covary freely.

To ensure identification, one composite loading of each emergent and excrescent variable was set to 1[4]. Further, we restricted additional composite loadings of the excrescent variables to zero, as explained in the section “Model Identification in CCA” and illustrated in Figure 5.2. Consequently, the model displayed in Figure 5.2 has 21 degrees of freedom.

[4]For the emergent variable “Political Instability” the composite loading for the observed variable “demo” was set to -1 to ensure the correct orientation of “Political Instability”.

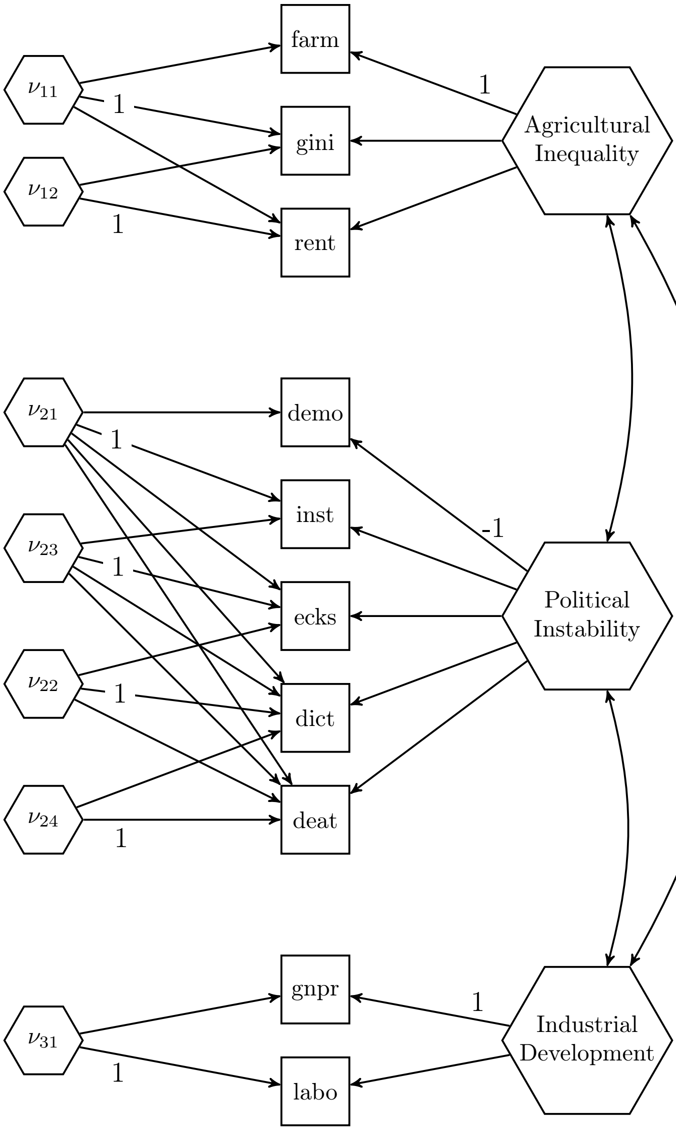

Figure 5.2: A confirmatory composite analysis of political instability, agricultural inequality, and industrial development

To obtain the model results, we used the ML estimator as implemented in Mplus (Muthén and Muthén, 1998-2017, Version 8), using the following syntax:

```
TITLE:
  Russett;
DATA:
  FILE IS Russettcor.dat;
  TYPE = CORRELATION;
  NOBSERVATIONS = 47;
VARIABLE:
  NAMES ARE gini farm rent gnpr labo inst
            ecks deat demo dict;

MODEL:
  !Specify the emergent variable
  !Agricultural Inequality (AI)
  AI BY farm@1 gini*0 rent*0;

  !Specify the two corresponding
  !excrescent variables (n1 and n2)
  !including constraints on composite loadings
  !to avoid overparametrization
  n1 BY farm*0 gini@1 rent*0;
  n2 BY farm@0 gini*0 rent@1;

  !Specify the emergent variable
  !Political Instability (PI)
  PI BY demo@-1 inst*0 ecks*0 deat*0 dict*0;
```

```
!Specify the four corresponding
!excrescent variables (n3, n4, n5 and n6)
!including constraints on composite loadings
!to avoid overparametrization
n3 BY demo*0 inst@1 ecks*0 deat*0 dict*0;
n4 BY demo@0 inst*0 ecks@1 deat*0 dict*0;
n5 BY demo@0 inst@0 ecks*0 deat*0 dict@1;
n6 BY demo@0 inst@0 ecks@0 deat@1 dict*0;

!Specify the emergent variable
!Industrial Development (ID)
ID BY gnpr@1 labo*0;

!Specify the corresponding
!excrescent variable n7
!including constraints on composite loadings
!to avoid overparametrization
n7 BY gnpr*0 labo@1;

!Specify covariances among emergent
!variables
AI WITH PI ID;
PI WITH ID;
```

```
  !Constrain covariances of excrescent
  !variables with other variables
  n7 WITH AI@0 PI@0 ID@0 n1-n6@0;
  n6 WITH AI@0 PI@0 ID@0 n1-n5@0;
  n5 WITH AI@0 PI@0 ID@0 n1-n4@0;
  n4 WITH AI@0 PI@0 ID@0 n1-n3@0;
  n3 WITH AI@0 PI@0 ID@0 n1-n2@0;
  n2 WITH AI@0 PI@0 ID@0 n1@0;
  n1 WITH AI@0 PI@0 ID@0;

  !Specify the variances of
  !the observed variables
  gini@0 farm@0 rent@0 gnpr@0 labo@0
  inst@0 ecks@0 deat@0 demo@0 dict@0;
OUTPUT:
  STANDARDIZED;
```

As Schuberth (forthcoming) observed, the default Mplus starting values often lead to convergence issues. Hence, we set all starting values of the composite loadings at zero as shown in the syntax above.

The estimation in Mplus converged normally and estimates for the composite loadings and the covariances among the emergent variables are provided. Additionally, the results of the chi square test used for overall model fit assessment indicate that our model is misspecified ($\chi^2 = 61.03$, df=21, p<0.01). As a consequence, the information loss through the dimension reduction is most likely not tolerable and the observed variables should be studied individually instead of combining them into composites. To locate the misspecification, researchers can follow CSA guidelines and, for instance, inspect residuals, i.e., the differences between the sample and estimated model-implied variance-covariance matrix (e.g., Kline, 2015). Even though the misfit of the model was not acceptable, we continue here to demonstrate how the weights can be obtained from the composite loadings.

Considering *Agricultural Inequality*, the estimated composite loading matrix is given as follows:

$$\begin{array}{cc} & \begin{array}{ccc} AI & \nu_1 & \nu_2 \end{array} \\ \boldsymbol{\Lambda} = & \begin{pmatrix} 1.00 & 0.93 & 0.00 \\ 0.82 & 1.00 & -0.69 \\ -0.26 & 1.18 & 1.00 \end{pmatrix} \begin{array}{l} \textit{farm} \\ \textit{gini} \\ \textit{rent} \end{array} \end{array} \tag{5.10}$$

The first column contains the composite loadings of *Agricultural Inequality*, while the second and third columns contain the composite loadings of the respective excrescent variables.

The weights that form *Agricultural Inequality* can be obtained by the inverse of the transposed composite loading matrix as shown in Equation (5.8):

$$\begin{array}{cc} & \begin{array}{ccc} AI & \nu_1 & \nu_2 \end{array} \\ \boldsymbol{W} = \left(\boldsymbol{\Lambda}^{-1}\right)' = & \begin{pmatrix} 1.49 & -0.53 & 1.01 \\ -0.76 & 0.82 & -1.17 \\ -0.53 & 0.57 & 0.19 \end{pmatrix} \begin{array}{l} \textit{farm} \\ \textit{gini} \\ \textit{rent} \end{array} \end{array} \tag{5.11}$$

This means that *Agricultural Inequality* is built by its observed variables in the following way:

$$AI = 1.49 \cdot \textit{farm} - 0.76 \cdot \textit{gini} - 0.53 \cdot \textit{rent} \tag{5.12}$$

The weights that form *Political Instability* and *Industrial Development* can be determined analogically. Standardized weights can be obtained in a similar way; however, instead of using the original composite loadings, the standardized composite loadings need to be transformed. For *Agricultural Inequality* the standardized weights are as follows: 2.03 (*farm*), −1.04 (*gini*), and −0.72 (*rent*).

5.5 Concluding remarks

Researchers often inappropriately assess composites in human development research using CFA. To address this issue, we present a recently developed approach to CSA – namely CCA – which allows for assessing composites with the same rigor as researchers who assess common factors in CFA. In doing so, we explain how to specify a composite model by means of emergent and excrescent variables. Additionally, we show how such models can be identified and how parameter estimates can be obtained using common CFA software. Finally, we elaborate on the assessment of composite models, which helps researchers to evaluate whether the observed variables of a block form a whole or act as a mere pile of parts, and thus should be studied individually.

Besides explaining the steps of CCA, we demonstrate its use by means of an illustrative example using Mplus. We deliberately chose Mplus to specify and estimate the model, for the following reasons: First, Mplus is a widely used CFA software. Second, Mplus shows a relatively good convergence behavior, whereas other SEM software such as AMOS (Arbuckle, 2014) or the R package lavaan (Rosseel, 2012) face bigger difficulties (Schuberth, forthcoming). Although the use of common CFA software facilitates the application of CCA, it has the drawback that it provides no weight estimates and corresponding standard errors directly. While weight estimates can be obtained via a straightforward transformation of the estimated composite loadings, future research needs to show ways of obtaining the corresponding standard errors in a simple fashion.

Finally, although researchers can use CCA to assess composite models, in empirical applications, they are likely to face both common factors and composites in their models. To deal with such situations, the model specification we have presented, can be adapted. In this case, one could speak of confirmatory composite and factor analysis (CCFA). Future research should provide concise guidelines for CCFA.

Chapter 6

A maximum likelihood estimator for composite models

6.1 Introduction [1]

Structural equation modeling (SEM) is a widely acknowledged method in social and behavioural sciences, which includes educational research (Khine, 2013), criminology (Higgins, 2002), counseling psychology (Fassinger, 1987), marketing research (Steenkamp and Baumgartner, 2000), psychology (MacCallum and Austin, 2000), international business research (Hult et al., 2006) and information systems research (Urbach et al., 2010). This method is frequently applied in these research areas due to its ability to model and assess theories comprising abstract concepts (Bagozzi and Phillips, 1982). Researchers are able not only to specify how observed variables are related to their constructs (Bollen and Bauldry, 2011), but also to model complex relationships, such as nonlinear relationships, between the constructs (Klein and Moosbrugger, 2000).

Originally, in SEM abstract concepts are modeled as common factors, i.e., as

[1] This chapter is based on joint work with Florian Schuberth and Jörg Henseler. At the time this dissertation was submitted, the paper was under review at a methodological journal.

latent variables that explain the covariance structures among their associated observed variables (Jöreskog, 1970a). This type of modeling is well established in the literature (Bollen, 1989; Schumacker and Lomax, 2009) and various approaches have been developed to estimate its model parameters, such as generalized least squares (GLS, Jöreskog and Goldberger, 1972), weighted least squares (WLS, Browne, 1984), model-implied instrumental variable estimation (MIIV, Bollen, 1996), factor score regression in combination with a correction for attenuation (Devlieger and Rosseel, 2017), consistent partial least squares (PLSc, Dijkstra and Henseler, 2015a,b; Dijkstra and Schermelleh-Engel, 2014), and generalized structured component analysis with unique terms for accommodating measurement error (GSCAm, Hwang et al., 2017). Arguably, the most often applied estimator is the full information maximum likelihood estimator (ML, Jöreskog, 1970b), including its robust versions (Yuan and Bentler, 1998a, 2007). This could be explained by its favourable statistical properties, such as consistency, asymptotical efficiency, and asymptotical normality, given that its underlying assumptions are met (Davidson and Mackinnon, 1993).

Over the last few decades, the composite model has gained popularity as a second type of modeling in the context of SEM (e.g., Conway and Kovacs, 2015; Dijkstra, 2017; Edwards and Bagozzi, 2000; Fornell and Bookstein, 1982; Grace and Bollen, 2008; Sarstedt et al., 2016). In the composite model, theoretical concepts are modeled as composites, i.e, as linear combinations of observed variables that are related via a structural model. Yet, the number of approaches with which composite models can be estimated is rather limited. The possibly most well-known estimator for composite models is partial least squares path modeling (PLS-PM, Wold, 1975). Although PLS-PM produces consistent estimates, it shows crucial limitations. For instance, PLS-PM is limited regarding model specification, e.g., parameters cannot be constrained. Additionally, there is no formula for standard errors and statistical inference relies on non-parametric approaches such as bootstrapping. To date, no estimator with the same capacities as the ML estimator known from SEM with common factors is available.

Against this background, we contribute a full information ML estimator for the composite model. As is common for ML estimators, our proposed estimator is consistent, asymptotically efficient, and asymptotically normal. Moreover, it overcomes the current limitations of existing estimators such as PLS-PM. Generally, compared to

PLS-PM, it allows for higher flexibility in terms of model specification. Further, the overall fit of composite models estimated by ML can be assessed by a likelihood ratio test, which makes it a prominent candidate for explanatory statistical modeling and theory testing. Finally, one can obtain the standard errors of ML estimators using the information matrix, thus no bootstrap is required.

The remainder of the chapter is structured as follows. Section 6.2 presents the composite model and gives an overview of extant approaches that extract composites, also showing their limitations for the composite model. In Section 6.3, we contribute a full information ML estimator for composite models. To demonstrate its finite sample behaviour and to compare its performance to PLS-PM, we conduct a Monte Carlo simulation in Section 6.4. Finally, the chapter closes with a discussion and conclusion in Sections 6.5 and 6.6.

6.2 The composite model and approaches to extract composites

For a long time, the composite model has not been recognized as a statistical model with meaningful and testable implications. Traditionally, composites have merely been regarded as mathematical operations or tools for dimension reduction and not as statistical entities representing concepts. For instance, researchers do not ascribe any conceptual unity to the components of a composite (Bollen and Diamantopoulos, 2017), and methods dedicated to composites such as canonical correlation analysis (CCA, Hotelling, 1936) do not have a history of assessing the overall model fit (Fan, 1997). This situation changed when theoretical frameworks developed that provide justification for employing composites to model theoretical concepts (e.g., Henseler, 2017; Henseler and Schuberth, 2021a; Rigdon, 2012; Sarstedt et al., 2016) and when the composite model was formally introduced (Dijkstra, 2013a, 2015, 2017). Since the composite model forms the basis of our ML estimator we elaborately describe it in the following section.

6.2.1 The composite model

A composite η_j is a linear combination of observed variables:

$$\eta_j = \boldsymbol{w}_j' \boldsymbol{x}_j, \tag{6.1}$$

where the vector $\boldsymbol{x}_j' = (x_{j1}, x_{j2}, \ldots, x_{jK_j})$ contains the K_j observed variables of block j making up the composite η_j and the vector $\boldsymbol{w}_j' = (w_{j1}, w_{j2}, \ldots, w_{jK_j})$ contains the corresponding weights. We assume that the J blocks of observed variables are disjoint, i.e., each observed variable is connected to only one composite. Therefore, the total number of observed variables equals the sum of the observed variables per block $K = \sum_{j=1}^{J} K_j$. As is common in the composite model, the observed variables are centered and the variances and covariances among the observed variables of one block are unconstrained (Dijkstra, 2017). Figure 6.1 shows one block of observed variables with its corresponding composite.

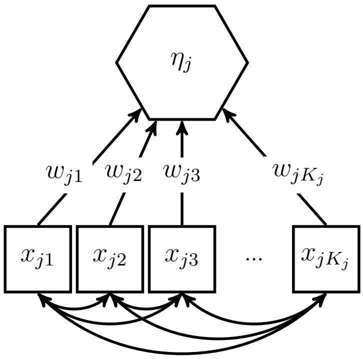

Figure 6.1: A block of observed variables and its associated composite

In the composite model, the composites convey all the information between the blocks of observed variables. In its simplest form, all composites are allowed to be freely correlated, i.e., the variance-covariance matrix of the composites is unconstrained. This is the same as the model that is typically studied in confirmatory composite analysis (Henseler and Schuberth, 2020; Schuberth et al., 2018a). Further, the composites can also be related via a structural model. For that purpose, we distinguish between exogenous composites $\boldsymbol{\eta}_{\text{ex}}$ and endogenous composites $\boldsymbol{\eta}_{\text{en}}$. In contrast to endogenous composites, exogenous composites are not explained by other composites in the structural model. The structural model specifies how the composites are related and can be expressed as follows:

$$\boldsymbol{\eta}_{\text{en}} = \boldsymbol{B}\boldsymbol{\eta}_{\text{en}} + \boldsymbol{\Gamma}\boldsymbol{\eta}_{\text{ex}} + \boldsymbol{\zeta} \tag{6.2}$$

The matrix $\boldsymbol{\Gamma}$ contains the coefficients of the exogenous composites where γ_{ij} shows the influence of $\eta_{\text{ex},j}$ on $\eta_{\text{en},i}$. Similarly, the coefficients of the endogenous composites are captured in the matrix $\boldsymbol{B}$, i.e., b_{ij} shows the effect of $\eta_{\text{en},j}$ on $\eta_{\text{en},i}$. Here we assume that no composite explains itself, i.e., the diagonal elements of $\boldsymbol{B}$ are all equal to 0. Additionally, we assume the structural error terms $\boldsymbol{\zeta}$ have zero mean and account for the remaining variance in a dependent composite that cannot be explained by its independent composites. Figure 6.2 shows an exemplary composite model embedded in a structural model with three blocks of observed variables, one exogenous and two endogenous composites.

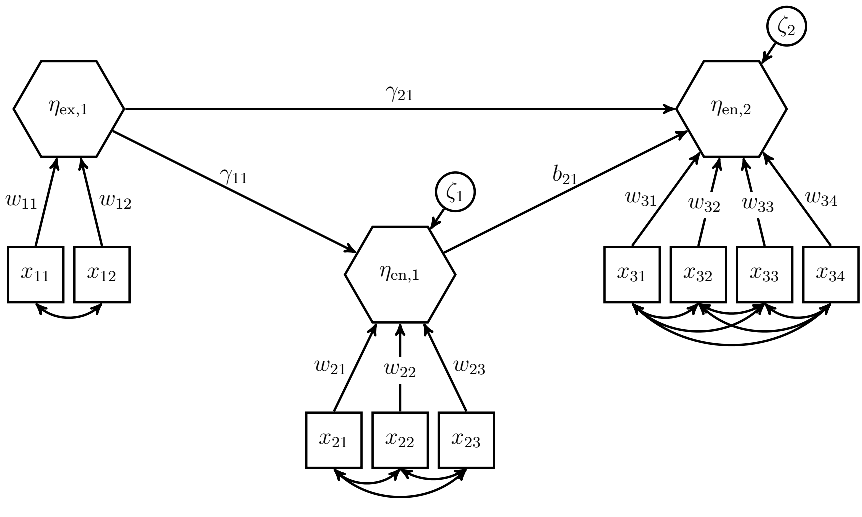

Figure 6.2: Three composites embedded in a structural model

6.2.2 Existing approaches to extract composites

Several approaches that have been developed to extract composites might be suitable to estimate composite models. In this section, we discuss the arguably most well-known approaches, which include principal component analysis (PCA, Pearson, 1901), (generalized) canonical correlation analysis ((G)CCA, Hotelling, 1936; Horst, 1961a,b; Kettenring, 1971), PLS-PM (Wold, 1975), generalized structured component analysis (GSCA, Hwang and Takane, 2004), and ML representing the composites in terms of loadings (Henseler, 2021; Schuberth, forthcoming).

PCA is one famous approach to extract composites from a dataset (Pearson, 1901). Originally, in PCA, several orthogonal composites, or principal components as they are labeled in PCA, could be extracted. To use PCA in estimating the composite model, Tenenhaus (2008) suggested the first principal component for each block of observed variables should be extracted and subsequently the structural model should be estimated using these components. A disadvantage of this approach is that the structural model is not taken into account when weights are estimated. Consequently, we expect that this approach will not produce consistent estimates for the composite model.

Besides PCA, GCCA was proposed as an approach to extract composites, or as they are called in GCCA, canonical variates, from several blocks of observed variables. Originally, in GCCA several composites are extracted stagewise from the blocks of observed variables. The composites of different stages are formed in such a way that they are mutually orthogonal. In contrast, in the case of two blocks of observed variables as studied in canonical correlation analysis (CCA), the composites of one stage are maximally correlated (Hotelling, 1936). Similarly, in the case of more than two blocks of observed variables, the composites of one stage are maximally related. The exact objective function depends on the chosen approach (see e.g., Kettenring, 1971). In addition to Kettenring's (1971) approaches to GCCA, others proposed an ML approach (Gu et al., 2019; Ogasawara, 2007). In principle, all these approaches can be directly applied to the composite model in its simplest form, i.e., where the composites are freely correlated and not embedded in a structural model. For the composite model comprising a structural model, a viable approach is to extract only the first-stage composites from the blocks of observed variables and subsequently to use their variance-covariance matrix to estimate the structural model (Dijkstra, 2017). A drawback all GCCA approaches have for the composite model is that the structural model is not considered during the weights' estimation because the composites are allowed to correlate freely. Consequently, the structural model's constraints cannot be taken into account.

The arguably most widely used estimator for composite models is PLS-PM which relies on a two step procedure (Lohmöller, 1989; Wold, 1975). In the first step, the weights are estimated by the iterative partial least squares algorithm to calculate the composites and their variance-covariance matrix. Subsequently, in the second step, the path coefficients of the composite model are estimated based on the composites' variance-covariance matrix. Although PLS-PM produces consistent estimates for the composite model (Dijkstra, 2017), it is very restrictive in terms of model specification. For instance, covariances among the exogenous composites cannot be constrained. Similarly, weights and path coefficients cannot be constrained. Finally, several statistical properties of PLS-PM estimates, such as asymptotic efficiency, have not been derived yet.

GSCA was proposed as an alternative to PLS-PM (Cho and Choi, 2020; Hwang and Takane, 2004). It also produces consistent estimates for the composite model. In contrast to PLS-PM, it optimizes a global criterion to obtain the parameter estimates. Specifically, it minimizes all sum squared residuals by an alternating least squares algorithm to obtain the parameter estimates. While GSCA is quite flexible in terms of model specification, currently no tests are available to assess the estimated model's overall fit.

Only recently, a specification was proposed that allows us to estimate composite models by means of ML known from SEM with common factors (Henseler, 2021). This specification was inspired by the composite factor model (Henseler et al., 2014) and the ML estimator proposed for GCCA (Ogasawara, 2007). In this specification, the relationships between a composite and its associated observed variables are expressed by loadings, i.e., covariances between the composites and their observed variables, and not by weights. As a result, no weights are estimated and statistical inference about the weights cannot be drawn directly.

6.3 A full information maximum likelihood-based approach for estimating composite models

In this section, we contribute a full information ML estimator for the composite model to the literature. This ML estimator is specifically tailored for the composite model and thus overcomes the limitations of other approaches that can be used to estimate composite models. Additionally, since it is an ML estimator, it has known desirable statistical properties, such as asymptotic efficiency. It has been designed analogous to the full information ML estimator for common factor models (e.g., Jöreskog, 1970b).

6.3.1 Full information maximum likelihood estimator for composite models

The parameters of the composite model as shown in Subsection 6.2.1 cannot be identified without further assumptions. Therefore, we impose the following constraints to ensure that our ML is able to produce unique parameter estimates for the composite model. First, we scale each weight vector $\boldsymbol{w}_j$ to ensure a unit variance for the composites, i.e., $\boldsymbol{w}_j'\boldsymbol{\Sigma}_{jj}\boldsymbol{w}_j = 1$.[2] Second, we assume that no composite η_j is isolated in the structural model, i.e., each composite needs to be connected to at least one other composite. Otherwise, an infinite number of weight sets for this composite exists that satisfies the scaling condition. Additionally, it needs to be ensured that the structural model is identified. For this purpose, we assume that the structural error terms $\boldsymbol{\zeta}$ are uncorrelated with the exogenous composites $\boldsymbol{\eta}_{\text{ex}}$. For simplicity, we further assume that the error terms are mutually uncorrelated, i.e., we consider only recursive structural models. Consequently, the corresponding path diagram equals a directed acyclic graph. Notably, this assumption can be relaxed (Dijkstra, 2017). Furthermore, we assume that $\boldsymbol{I} - \boldsymbol{B}$ is nonsingular so that $(\boldsymbol{I} - \boldsymbol{B})^{-1}$ exists. Consequently, the structural model from Equation 6.2 that relates the composites can be written as follows:

$$\boldsymbol{\eta}_{\text{en}} = \boldsymbol{\Pi}\boldsymbol{\eta}_{\text{ex}} + (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{\zeta} \tag{6.3}$$

with $\boldsymbol{\Pi} = (\boldsymbol{I} - \boldsymbol{B})^{-1}\boldsymbol{\Gamma}$. Finally, we assume some regularity conditions to hold, such as that weight vectors $\boldsymbol{w}_j$ are not allowed to consist of zeros only and the variance-covariance matrix of a block of observed variables $\boldsymbol{x}_j$ cannot be singular.

Against these assumptions, we can derive the variance-covariance matrix implied by the composite model. Considering the K_j observed variables of block j, their variance-covariance matrix $\boldsymbol{\Sigma}_{jj}$, also known as *intra-block* variance-covariance matrix,

[2] Alternative ways to fix the scale of the composites would be to fix one weight per composite to a specific value or to fix the length of the weight vector.

is usually unconstrained and has the following general form:

$$\mathbf{\Sigma}_{jj} = \begin{pmatrix} \sigma_{11}^{(j)} & & & \\ \sigma_{12}^{(j)} & \sigma_{22}^{(j)} & & \\ \vdots & \vdots & \ddots & \\ \sigma_{1K_j}^{(j)} & \sigma_{2K_j}^{(j)} & \dots & \sigma_{K_jK_j}^{(j)} \end{pmatrix} \tag{6.4}$$

where $\sigma_{ik}^{(j)}$ equals the covariance between the observed variables x_{ji} and x_{jk}. Moreover, the covariances between the observed variables of two different blocks i and j, which are captured in the *inter-block* covariance matrix $\mathbf{\Sigma}_{ij}$, can be written as:

$$\mathbf{\Sigma}_{ij} = \mathbf{\Sigma}_{ii}\boldsymbol{w}_i\boldsymbol{w}_j'\mathbf{\Sigma}_{jj} \cdot r_{ij}. \tag{6.5}$$

Due to the scaling condition, r_{ij} equals the correlation between the composites η_i and η_j. Notably, the weights $\boldsymbol{w}_j$ producing these matrices are the same across all inter-block covariance matrices $\mathbf{\Sigma}_{ij}$ with $i = 1, \dots, J$ and $i \neq j$.

The exact structure of the composites' correlation matrix $\boldsymbol{R}$ depends on the coefficients in $\boldsymbol{B}$ and $\mathbf{\Gamma}$, and the correlation matrix of the exogenous composites $\mathbf{\Phi}$. The variance-covariance-matrix of the structural error terms $\mathbf{\Psi}$ is a diagonal matrix of which the elements are determined by the model parameters because all composites are standardized. Since the structural error terms are uncorrelated with the exogenous variables, the composites' correlation matrix can be written as follows:

$$\boldsymbol{R} = \begin{pmatrix} \mathbf{\Phi} & \mathbf{\Phi}\mathbf{\Pi}' \\ \mathbf{\Pi}\mathbf{\Phi} & \mathbf{\Pi}\mathbf{\Phi}'\mathbf{\Pi}' + (\boldsymbol{I} - \boldsymbol{B})^{-1}\mathbf{\Psi}(\boldsymbol{I} - \boldsymbol{B})'^{-1} \end{pmatrix} \tag{6.6}$$

The complete variance-covariance matrix of the observed variables $\mathbf{\Sigma}(\boldsymbol{\theta})$ can be expressed as a partitioned matrix:[3]

$$\mathbf{\Sigma}(\boldsymbol{\theta}) = \begin{pmatrix} \mathbf{\Sigma}_{11} & & & \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} & & \\ \vdots & \vdots & \ddots & \\ \mathbf{\Sigma}_{J1} & \mathbf{\Sigma}_{J2} & \dots & \mathbf{\Sigma}_{JJ} \end{pmatrix} \tag{6.7}$$

and can be rewritten as:

$$\mathbf{\Sigma}(\boldsymbol{\theta}) = \boldsymbol{A} + \boldsymbol{A}\boldsymbol{w}\boldsymbol{w}'\boldsymbol{A} \circ \boldsymbol{D}, \tag{6.8}$$

[3] We can show that this variance-covariance matrix is positive-definite if, and only if, the following two conditions hold: (i) all intra-block variance-covariance matrices are positive-definite, and (ii) the variance-covariance matrix of the composite is positive-definite (Dijkstra, 2015, 2017).

where the vector $\boldsymbol{w}' = (\boldsymbol{w}_1', \boldsymbol{w}_2', \ldots, \boldsymbol{w}_J')$ contains all weights used to build the composites $\boldsymbol{\eta}$ and the operator "$\circ$" denotes the Hadamard product. Additionally, the block-diagonal matrix $\boldsymbol{A}$ contains the intra-block variance-covariance matrices on its diagonal and the matrix $\boldsymbol{D}$ is a block matrix with $\boldsymbol{D}_{ii} = \mathbf{0} \in \mathbb{R}^{K_i \times K_i}$ and $\boldsymbol{D}_{ij} = r_{ij} \cdot \boldsymbol{\iota}_i \boldsymbol{\iota}_j' \in \mathbb{R}^{K_i \times K_j}$ with $i, j = 1, \ldots, J$ and $i \neq j$. It is obvious, that the observed variables' variance-covariance matrix depends on the model parameters $\boldsymbol{\theta}$, i.e., the weights, the intra-block variances and covariances, and the covariances among the composites. The latter depends on the covariances among the exogenous composites and the path coefficients in $\boldsymbol{B}$ and $\boldsymbol{\Gamma}$ (and on the variances of the structural error terms, if a different scaling condition is used for the composites) if the composites are embedded in a structural model.

To develop our full information ML estimator, additional distributional assumptions about the observed variables $\boldsymbol{x}$ are required. Analogous to the ML estimators known from SEM with common factors, we assume that the observed variables are multivariate normally distributed $\boldsymbol{x} \sim N(\mathbf{0}, \boldsymbol{\Sigma}(\boldsymbol{\theta}))$. Consequently, the variance-covariance matrix $\boldsymbol{S}$ of a sample with $n+1$ observations drawn from this distribution follows a Wishart distribution. Then, given that the model is correctly specified, the parameters in $\boldsymbol{\theta}$ are estimated consistently by maximizing the corresponding loglikelihood function (Lawley and Maxwell, 1963):

$$\log \mathrm{L} = -\frac{1}{2} n \left[\log|\boldsymbol{\Sigma}(\boldsymbol{\theta})| + \mathrm{tr}(\boldsymbol{S}\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1})\right] \tag{6.9}$$

with $|\boldsymbol{A}| = \det(\boldsymbol{A})$. Maximizing the loglikelihood function is equivalent to minimizing the following fit function:

$$\mathrm{F} = \log|\boldsymbol{\Sigma}(\boldsymbol{\theta})| + \mathrm{tr}(\boldsymbol{S}\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}) - \log|\boldsymbol{S}| - K, \tag{6.10}$$

where the constants $\log|\boldsymbol{S}|$ and K are included to obtain a value of 0 for F if $\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}})$ equals $\boldsymbol{S}$. Since the variance of each composite is fixed to one by scaling its weights, the fit function needs to be minimized under this scaling condition, i.e., $\boldsymbol{w}_j' \boldsymbol{\Sigma}_{jj} \boldsymbol{w}_j = 1$, $\forall j = 1, \ldots J$. Consequently, the ML estimates are obtained by solving the first order conditions (FOCs) which, with respect to the fit function F, are given as:

$$\frac{\partial \mathrm{F}}{\partial \theta_k} = \frac{\partial \log|\boldsymbol{\Sigma}(\boldsymbol{\theta})|}{\partial \theta_k} + \frac{\partial \mathrm{tr}(\boldsymbol{S}\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1})}{\partial \theta_k} \tag{6.11}$$

$$= \sum_{i=1}^{K} \sum_{j=i+1}^{K} \{\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}\}_{i,j} \cdot \frac{\partial \sigma_{ij}}{\partial \theta_k} - \sum_{i=1}^{k} \sum_{j=i+1}^{k} \{\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}\boldsymbol{S}\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}\}_{ij} \cdot \frac{\partial \sigma_{ij}}{\partial \theta_i} \overset{!}{=} 0 \tag{6.12}$$

To solve the FOCs, iterative algorithms such as the SOLNP algorithm (Ye, 1987) can be used. We suggest to use PLS-PM estimates as starting values for the iterative algorithm. Similarly, is done in PLSe1 (Bentler and Huang, 2014; Huang, 2013), where PLSc estimates are used as starting values for the Newton Raphson procedure to obtain consistent and asymptotically efficient factor model estimates.

An advantage of ML estimators is that the estimates' asymptotic variance-covariance matrix can be obtained by the inverse of the Fisher information matrix. The Fisher information matrix equals the Cramér-Rao boundary and is equal to the negative expectation of the second derivatives of the loglikelihood function (Davidson and Mackinnon, 1993). To the best of our knowledge, for our ML estimator no closed form of the second derivatives exist and therefore there is no closed form of the Fisher information matrix. Consequently, the asymptotic variance-covariance matrix has to be calculated individually for each model.

6.3.2 Testing the overall model fit

In empirical research, it can be crucial to assess whether an estimated composite model is consistent with the collected data. Our proposed ML estimator provides the opportunity to assess the composite model's overall fit by means of a likelihood ratio test. Specifically, it tests the following null hypothesis:

$$H_0 : \boldsymbol{\Sigma}(\boldsymbol{\theta}) = \boldsymbol{\Sigma} \tag{6.13}$$

where $\boldsymbol{\Sigma}(\boldsymbol{\theta})$ denotes the model-implied variance-covariance matrix of the observed variables based on the population parameters, and $\boldsymbol{\Sigma}$ is the population variance-covariance matrix of the observed variables. The likelihood ratio test statistic can be computed as follows (Jöreskog, 1969; Wilks, 1938):

$$T = n \cdot F(\hat{\boldsymbol{\theta}}) \tag{6.14}$$

where $n + 1$ is the sample size and F the fit function from Equation 6.10.

Under the null hypothesis, the likelihood ratio test statistic T is asymptotically χ^2 distributed with k degrees of freedom, where k equals the number of non-redundant elements in the sample variance-covariance matrix of the observed variables minus the number of free model parameters (Wilks, 1938). In our case, the number of free model parameters equals the number of weights plus the number of free elements in $\boldsymbol{B}$ and $\boldsymbol{\Gamma}$ plus the number of non-redundant elements in $\boldsymbol{\Phi}$ and in the intra-block variance-covariance matrices minus the number of blocks of observed variables. The number of blocks is subtracted from the free model parameters because for each block of observed variables one weight is determined by the others due to the scaling condition.

6.4 Monte Carlo simulation

The asymptotic properties of ML estimators are well-known. Particularly, ML estimators are consistent and asymptotically efficient, i.e., as the sample size increases their estimates converge in probability to the true counterpart and the variance-covariance matrix of the estimates converges towards the Cramér-Rao bound (Davidson and Mackinnon, 1993). To investigate the finite sample behaviour of our proposed estimator, we conducted a Monte Carlo simulation. To assess the performance of our proposed ML estimator, we compared it to the performance of PLS-PM, the dominant estimator for composite models.

6.4.1 Simulation design

In designing our Monte Carlo simulation, we considered a population model with three composites, namely, one exogenous composite $\eta_{\text{ex},1}$, and two endogenous composites $\eta_{\text{en},1}$ and $\eta_{\text{en},2}$. The three composites are formed by two, three and four observed variables, respectively. The population intra-block variance-covariance matrices for the three blocks of observed variables are given in Equation 6.15. Notably, the following values are rounded to the second decimal.

$$\mathbf{\Sigma}_{11} = \begin{pmatrix} 3.00 & 2.52 \\ 2.52 & 5.00 \end{pmatrix}, \mathbf{\Sigma}_{22} = \begin{pmatrix} 4.00 & 0.87 & 2.40 \\ 0.87 & 3.00 & 0.83 \\ 2.40 & 0.83 & 9.00 \end{pmatrix}, \mathbf{\Sigma}_{33} = \begin{pmatrix} 2.00 & 0.63 & 1.20 & 1.70 \\ 0.63 & 5.00 & 2.53 & 3.19 \\ 1.20 & 2.53 & 8.00 & 2.12 \\ 1.70 & 3.19 & 2.12 & 9.00 \end{pmatrix} \tag{6.15}$$

Additionally, the population weights to form the composites are set as follows: $w_{11} = 0.35$ and $w_{12} = 0.22$ for $\eta_{\text{ex},1}$, $w_{21} = 0.20$, $w_{22} = 0.29$ and $w_{23} = 0.17$ for $\eta_{\text{en},1}$ and $w_{31} = 0.28$, $w_{32} = 0.09$, $w_{33} = 0.18$ and $w_{34} = 0.10$ for $\eta_{\text{en},2}$. Note that the population weights are chosen in such a way that the corresponding composite shows a unit variance. As structural model, we opt for a full mediation model in which $\eta_{en,1}$ fully mediates the effect of $\eta_{\text{ex},1}$ on $\eta_{\text{en},2}$. The population path coefficients are set as follows: $\gamma_{11} = 0.3$ and $b_{21} = 0.4$. Since the composites have a unit variance, the variances of the structural error terms are given as: $\psi_{11} = 0.91$ and $\psi_{22} = 0.62$. The complete population variance-covariance of the observed variables is given in Equation 6.16.

$$\mathbf{\Sigma} = \begin{array}{c} \begin{array}{ccccccccc} \underline{x_{11}} & \underline{x_{12}} & \underline{x_{21}} & \underline{x_{22}} & \underline{x_{23}} & \underline{x_{31}} & \underline{x_{32}} & \underline{x_{33}} & \underline{x_{34}} \end{array} \\ \begin{pmatrix} 3.00 & & & & & & & & \\ 2.52 & 5.00 & & & & & & & \\ 0.70 & 0.87 & 4.00 & & & & & & \\ 0.57 & 0.70 & 0.87 & 3.00 & & & & & \\ 1.07 & 1.33 & 2.40 & 0.83 & 9.00 & & & & \\ 0.19 & 0.24 & 0.58 & 0.47 & 0.89 & 2.00 & & & \\ 0.27 & 0.33 & 0.81 & 0.66 & 1.24 & 0.63 & 5.00 & & \\ 0.42 & 0.52 & 1.27 & 1.03 & 1.95 & 1.20 & 2.53 & 8.00 & \\ 0.39 & 0.49 & 1.18 & 0.96 & 1.81 & 1.70 & 3.19 & 2.12 & 9.00 \end{pmatrix} \end{array} \tag{6.16}$$

Figure 6.3 displays the population model. For more clarity, we omit the covariances among the observed variables belonging to one block, i.e., the intra-block covariances.

A model that equals the population model given in Figure 6.3 is estimated by each estimator 1,000 times using sample sample sizes of 100, 200, 500 and 1000 to investigate our proposed ML estimator's finite sample behaviour. To assess and compare

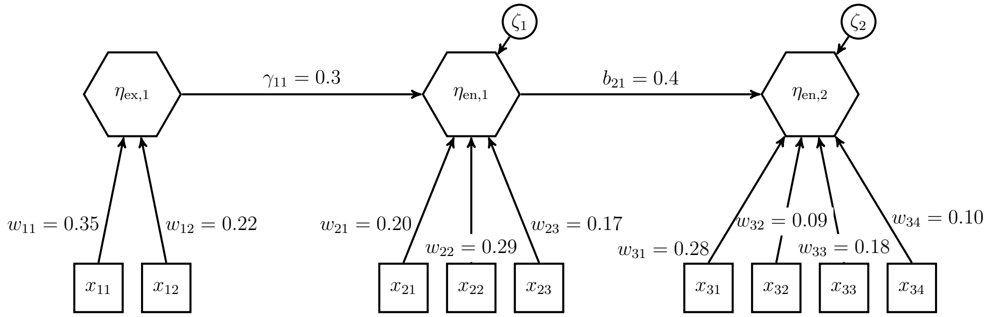

Figure 6.3: Population model of the Monte Carlo simulation

the performance of our ML estimator and PLS-PM, we consider the estimated bias, and the estimated root mean square error (RMSE) of the standardized parameters. The estimated bias and RMSE are obtained as follows:

$$\widehat{\text{Bias}} = \frac{1}{N}\sum_{i=1}^{N}\hat{\theta}_i - \theta \tag{6.17}$$

$$\widehat{\text{RMSE}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{\theta}_i - \theta)^2} \tag{6.18}$$

where θ represents a generic population parameter, $\hat{\theta}_i$ is its corresponding estimate from the i-th Monte Carlo simulation run, and N denotes the total number of Monte Carlo simulation runs. While the estimated bias indicates how much, on average, an estimate differs from its population counterpart, the RMSE combines the bias and the uncertainty involved in an estimate, namely its standard error. Since both estimators are consistent, we expect that the bias of the estimates will diminish for increasing sample size. Since ML estimates are asymptotically efficient, we expect that compared to PLS-PM, our approach will produce estimates with smaller standard errors for large sample sizes. Therefore, the ML estimates are expected to show a smaller RMSE than estimates produced by PLS-PM for large sample sizes. Further, we compare the two estimators regarding Fisher consistency, i.e., whether they are able to retrieve the population parameters when the population variance-covariance matrix is used as input. Since ML estimators are Fisher consistent (Dijkstra, 1983), we expect both estimators to be Fisher consistent.

We carried out the complete Monte Carlo simulation in the statistical programming environment R (R Core Team, 2020). The datasets were drawn from a multivariate normal distribution with zero mean, and the variance-covariance matrix from Equation 6.16 using the `mvrnorm()` function of the *MASS* package (Venables and Ripley, 2002). To employ our proposed estimator, we used our own implementation.To minimize the loglikelihood function under the constraint that each composite has unit variance, we used the `solnp()` function of the *Rsolnp* package (Ghalanos and Theussl, 2015), which is based on the SOLNP algorithm to solve non-linear systems of equations comprising constraints (Ye, 1987). As starting values for the SOLNP algorithm, we used the PLS-PM estimates obtained by the *matrixpls* package's `matrixpls()` function (Rönkkö, 2021) based on the sample variance-covariance matrix of the observed variables. To obtain the unstandardized model parameter estimates of PLS-PM, we again used the *matrixpls* package's `matrixpls()` function (Rönkkö, 2021)[4]. To fix the orientation of each composite and to avoid sign ambiguity of the weight vector, we choose the sign of each weight vector in such a way that the dominant observed variable of a block is positively correlated with its composite. Specifically, we choose x_{12} for $\eta_{\text{ex},1}$, x_{23} for $\eta_{\text{en},1}$ and x_{33} for $\eta_{\text{en},2}$ as dominant observed variables, which are the observed variables that show the highest positive population correlation with the composite.

6.4.2 Simulation results

Considering Fisher consistency, Table 6.1 shows that our proposed ML estimator, as well as PLS-PM, are able to retrieve the population parameters when the population variance-covariance matrix is provided as input. The covariances among the observed variables of one block are omitted to preserve clarity. As Table 6.1 shows, both estimators are Fisher consistent for the considered model.

Besides the results for Fisher consistency, Figure 6.4 presents bar plots for the estimated bias and the RMSE. Since the results are very similar across the various parameters, we report only the results for the covariance between x_{22} and x_{23} ($\sigma_{23}^{(2)}$), the weight of x_{11} (w_{11}) and the path coefficient of $\eta_{\text{ex},1}$ on $\eta_{\text{en},1}$ (γ_{11}). Overall, for each approach, the bias and the RMSE are calculated based on 1,000 valid estimations per considered sample size, i.e., no inadmissible solutions or convergence issues

[4]Note that for PLS-PM the correlation matrix of the observed variables is often used as input for the estimation. Consequently, the parameter estimates are standardized. However, we used the variance-covariance matrix of the observed variables as input.

Table 6.1: Fisher consistency

Parameter	Population parameter	ML	PLS-PM
w_{11}	0.35	0.35	0.35
w_{12}	0.22	0.22	0.22
w_{21}	0.20	0.20	0.20
w_{22}	0.29	0.29	0.29
w_{23}	0.17	0.17	0.17
w_{31}	0.28	0.28	0.28
w_{32}	0.09	0.09	0.09
w_{33}	0.18	0.18	0.18
w_{34}	0.10	0.10	0.10
γ_{11}	0.30	0.30	0.30
b_{21}	0.40	0.40	0.40

were encountered during the estimations. The complete results are provided in the Appendix.

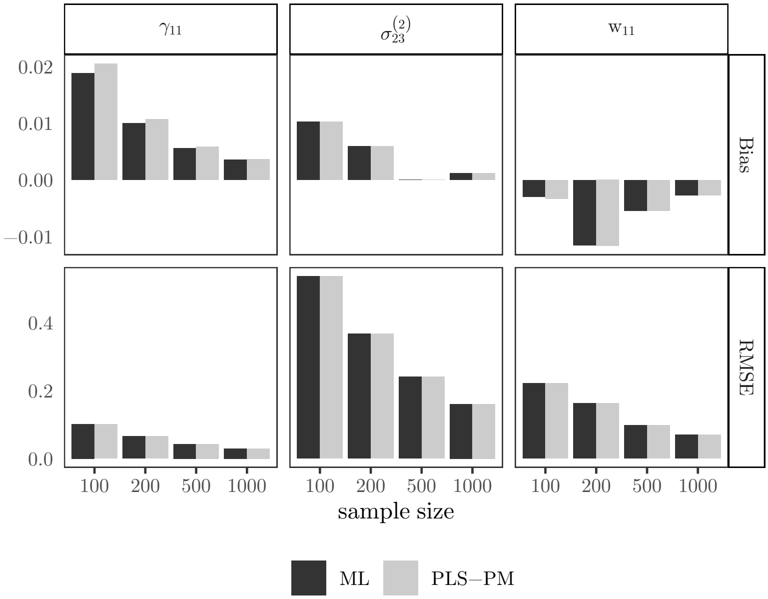

Figure 6.4: Simulation results

As can be seen from Figure 6.4, our proposed ML estimator produces estimates that, on average, are close to the population value. Even for a smaller sample size, i.e., 100 observations per sample, the estimates are hardly biased. This also holds for the PLS-PM estimates. While the bias of the ML estimates is slightly smaller than the bias of PLS-PM estimates for the path coefficient and the weight, the bias of the intra-block covariance for the two estimators does not differ. For increasing sample sizes, the bias and the difference between the two estimators diminish. Considering the RMSE of the ML estimates, it decreases for all parameters for an increasing sample size. Moreover, the RMSE is very similar for the estimates obtained by ML and PLS-PM. In fact, there is almost no difference between the two estimators.

6.4.3 Simulation insights

Our Monte Carlo simulation results largely confirm our expectations, i.e. the bias of the estimates diminishes and the RMSE of the estimates decreases for an increasing sample size. This is in line with the ML estimator's properties, i.e., with its consistency and asymptotic efficiency. Additionally, a comparison of the ML estimator and PLS-PM shows that their estimates hardly differ.

These results lead to two conclusions. Firstly, our proposed ML estimator does not perform worse than existing estimators for composite models in finite samples, particularly in small samples. Secondly, PLS-PM performs similarly to ML in terms of RMSE, although asymptotic efficiency has not been proven for PLS-PM. Consequently, for our considered model, the two estimators can be used equivalently.

6.5 Discussion

In this chapter, we designed an ML estimator that overcomes the limitations of existing estimators for composite models such as the lack of a test for overall model fit and limitations in terms of model specification. Specifically, we present the variance-covariance matrix of the observed variables implied by the composite model, which forms the basis of our ML estimator. Further, we present a fit function that is minimized by our ML estimator to obtain consistent estimates for the composite model. To assess the overall fit of composite models, we present a likelihood ratio test that can be used for this purpose. Additionally, the likelihood ratio test allows us to test nested models, which can be beneficial in assessing measurement invariance of composites (Henseler et al., 2016b). Finally, besides constraining the variances of the composites, the ML principle allows for imposing further parameter constraints in the estimation. Thus, it overcomes a limitation of PLS-PM for composite models.

Our Monte Carlo simulation results show that our ML estimator behaves as expected in finite samples. Specifically, the bias diminishes and the RMSE decreases for an increasing sample size. Moreover, our proposed ML estimator shows a bias in finite samples similar to that of PLS-PM. Since the statistical properties of PLS-PM have not been completely explored (Hanafi, 2007; Henseler, 2010), the results provide empirical evidence for the efficiency of PLS-PM.

Naturally, simulation studies are limited by their design. Therefore, we advise future research to examine and compare the performance of our ML estimator in more complex settings, e.g., in non-recursive structural models. Similarly, future research is advised to compare our proposed estimator to PLS-PM in cases of model misspecification. Due to its two step nature PLS-PM is a limited information estimator in comparison to our proposed ML approach. Hence, we expect PLS-PM to outperform our proposed approach in cases of model misspecification. Additionally, researchers most likely deal with a combination of composites and common factors in empirical research (Hwang et al., 2021). Hence, a promising avenue for future research would be to adjust our ML estimator to models that consist of both composites and common factors. In such a case, the performance should be compared to that of consistent partial least squares (PLSc, Dijkstra and Henseler, 2015a), an approach that can deal with models containing both common factors and composites. Further, the ML estimates are currently obtained by an iterative algorithm. Considering this, an issue that requires further attention is the choice of starting values. We propose to use PLS-PM estimates as starting values, which is similar to PLSe1 proposed for common factor models. A set of improper starting values can result in optimizer non-convergence or even prevent the optimization. While our simulation used the PLS-PM estimates as starting values, future research should propose ways to determine starting values that do not require an iterative algorithm. Moreover, we deliberately focused on the unidimensional composite model. Future research should try to extend our approach to the case where more than one composite per block of observed variables should to be extracted, i.e., to the multidimensional case. Finally, future research should investigate how current enhancements to the ML estimator for common factor models can be adopted to our ML estimators such as dealing with missing values.

6.6 Conclusion

In this chapter, we contribute a full information ML estimator for composite models, which overcomes the limitations of existing approaches to estimate composite models. In analogy to PLSe1, it can be regarded as a consistent and asymptotically efficient PLS-PM estimator if PLS-PM estimates are used as starting values to solve the FOCs of our ML estimator. As shown in our Monte Carlo simulation study, our ML estimator performs similar to existing estimators for composite models such as PLS-PM in finite samples. However, in contrast to PLS-PM, it allows for more flexibility in terms of model specification and for assessing the overall fit of composite models by means of a likelihood ratio test.

Chapter 7

Epilogue

This thesis has presented several methodological advances in the field of composite-based structural equation modeling (SEM). Traditionally, theoretical concepts are modeled as common factors and thus this type of SEM is often referred to as factor-based SEM (Bollen, 1989; Jöreskog, 1970b). Factor-based SEM is well established and widely used in various disciplines such as psychology (Raykov et al., 1991), education (Khine, 2013), criminology (Higgins, 2002), and business (Shook et al., 2004). In contrast, composite-based SEM has only recently been recognized as a second type of SEM. Nevertheless, composite-based SEM has gained increasing attention over the last two decades and has been applied in various disciplines such as marketing (Hair et al., 2011), psychology (Karimi and Meyer, 2014), or information systems (Al-Emran et al., 2018). This thesis further contributes to composite-based SEM literature, specifically by having introduced new methods, improved existing methods, critically discussed existing methods, and having provided guidelines for the presented methods.

Originally, composites were regarded as the outcome of dimension reduction procedures such as principal component analysis (Pearson, 1901). The introduction of partial least squares path modeling (PLS-PM, Wold, 1975) marks an important cornerstone in the maturation of composite-based SEM. To start with, PLS-PM was introduced in the light of principal component analysis (Wold, 1966, 1975). Subsequently, it was introduced as an approach that could estimate path models with latent variables (Wold, 1982). As a result, it gained increasing attention (Lohmöller, 1989; Tenenhaus et al., 2005) and various improvements were developed (Becker et al., 2018; Dijkstra and Henseler, 2015a,b; Henseler, 2007; Henseler and Sarstedt, 2013; Henseler et al., 2014; Huang, 2013; Rademaker et al., 2019; Sarstedt et al., 2011; Tenenhaus et al., 2004). Although the early literature already disclosed the limitations of PLS-PM (Dijkstra, 1981), they were not taken into account for a long time, but only considered later (Rönkkö and Evermann, 2013; Rönkkö et al., 2016). As a consequence, other estimators that built on composites to estimate structural equation models, such as generalized structured component analysis (GSCA, Hwang and Takane, 2004), generalized structured component analysis with uniqueness terms for accommodating measurement error (GSCAm, Hwang et al., 2017), or integrated generalized structured component analysis (IGSCA, Hwang et al., 2021) have become more attractive and composite-based SEM was established as a second type of SEM.

In this light, two different strands have contributed to the spread of composite-based SEM. The first strand involves the original application of composites in SEM, namely composite-based methods to estimate models in which theoretical concepts are modeled as common factors as is the case in PLSc, GSCAm, IGSCA and other variance-based estimators. At first, using composites was questionable because taking the composites as proxies for common factors lead to inconsistent estimates. Yet, the introduction of consistent estimators for common factor models that built on composites increased their acceptance in the field of SEM and expanded the world of composite-based estimators for common factor models.

The second strand of composite-based SEM to be established, differs from the first that uses composites to estimate the model parameters of common factor models. With the introduction of the composite model (Dijkstra, 2013b, 2017) and confirmatory composite analysis (Henseler and Schuberth, 2021b; Schuberth et al., 2018a) as a method of assessing these models and thus assessing whether the components act as a whole or rather merely as loose parts, composites can be embedded in models which enable the modeling of theoretical concepts (Henseler and Schuberth, 2021a). This was strengthened when new theoretical frameworks such as the concept-proxy framework (Rigdon, 2012) and the synthesis theory (Henseler and Schuberth, 2021a) were introduced. All these instances assume that the composites are not only linear combinations of observed variables, but also convey all information between their observed variables and all other variables in the model. This thesis contributes to both strands of composite-based SEM in a methodological sense, and thus contributes to the wider distribution of composite-based SEM.

Chapter 2 provides guidelines for conducting Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation models using the R package *cSEM* (Rademaker and Schuberth, 2020). The R package *cSEM* was chosen as the tool to perform these Monte Carlo simulations because it was introduced by researchers in the field of composite-based SEM and is open to other researchers' contributions, making the latest developments related to composite-based SEM publicly available. Moreover, *cSEM* is now widely applied in research and is gaining increasing attention in academia (Chuah et al., 2021; Fabbricatore et al., 2021; Klesel et al., forthcoming). This chapter illustrates the guidelines by means of an illustrative Monte Carlo simulation that investigates PLS-PM's and PLSc's finite sample behavior, particularly regarding the consequences of sample correlations among measurement errors on statistical inference. This example was included in particular because PLS-PM is arguably the most dominant approach to obtaining parameter estimates for composite models, and thus the most prominent approach in composite-based SEM. Consequently, using PLS-PM for illustration provides researchers with a perfect outline for their own Monte Carlo simulations using *cSEM*.

Chapter 3 presents versions of PLS-PM and its consistent version PLSc, which are robust in responding to outlier distortion. Thus, the chapter contributes to the composite-based SEM literature by offering an improvement to an existing estimator. Empirical data is rarely clean in the sense that researchers lack knowledge of how to clean – often referred to as manipulate – data in a sense that model parameters are appropriately estimable. By demonstrating that PLS-PM and PLSc lead to biased estimates with high variances when unsystematic outliers are present in a dataset, we contribute to the critical discussion of composite-based SEM methods' performance. As a remedy, we introduce robust PLS and robust PLSc (Schamberger et al., 2020) which do not show such a bias and high variances for samples with unsystematic outliers. Both robust PLS and robust PLSc use the minimum covariance determinant (MCD) estimator to obtain the observed variables' correlation matrix. Their traditional counterparts use the Pearson correlation for this purpose. We give an overview of other possible estimators for the observed variables' correlations. Using a Monte Carlo simulation, we show that robust PLS and robust PLSc can handle samples containing unsystematic outliers of almost 50%. In addition, we illustrate the use of robust PLS and robust PLSc by means of an empirical example. Robust PLS and robust PLSc open the possibility for applied researchers to solve their analytical problems, which has already been demonstrated in the literature (Ramírez-Correa et al., 2020). In addition, the introduction of robust PLS and robust PLSc has served as inspiration for other researchers to develop robust versions for PLS-PM and PLSc. For example, Abdullaha et al. (2020) replaced the MCD correlation with the Spearman correlation to obtain a robust version of PLS-PM. Given the higher breakdown point of the MCD correlation compared to the Spearman correlation, it can be surmised that the robust versions of PLS-PM and PLSc presented in Chapter 3 perform better than the versions presented by Abdullaha et al. (2020) in terms of robustness to outliers. Nevertheless, this should be further verified, e.g., by means of a Monte Carlo simulation. In addition, robust PLS and robust PLSc allow researchers to compare the results of the traditional estimators with those of the robust estimators to see if outliers are even a problem in their data set. Future research should develop valid test procedures for this purpose.

Chapter 4 presents a way of performing out-of-sample predictions based on models' estimates by ordinal partial least squares (OrdPLS) and ordinal consistent partial least squares (OrdPLSc). In recent years, performing out-of sample predictions has gained increasing attention in PLS-PM literature (Shmueli et al., 2016, 2019). In contrast, out-of-sample predictions cannot be performed using traditional factor-based techniques. Consequently, discussing the predictive performance of composite-based SEM methods also contributes to their spread. Nevertheless, current guidelines for performing out-of sample predictions based on models estimated with PLS-PM or PLSc – called PLSpredict and PLScpredict respectively – cannot explicitly account for the ordinal categorical nature of the observed variables. As a remedy, Chapter 4 presents approaches for performing out-of sample predictions based on models estimated by OrdPLS and OrdPLSc, namely OrdPLSpredict and OrdPLScpredict, which allow us to account for the ordinal categorical nature of the observed variables. The performance of OrdPLSpredict and OrdPLScpredict are compared to PLSpredict and

PLScpredict, which ignore the categorical nature of the variables by means of a Monte Carlo simulation using the R package *cSEM*. The results show that OrdPLSpredict and OrdPLScpredict outperform their traditional counterparts when the MAE or the concordance is used to evaluate the predictive performance. Besides presenting OrdPLSpredict and OrdPLScpredict, this chapter provides guidelines on OrdPLScpredict using the R package *cSEM*, also giving an empirical example. OrdPLSpredict and OrdPLScpredict open the possibility for researchers to take into account the scaling of observable variables when making predictions. Furthermore, OrdPLSpredict and OrdPLScpredict could serve as inspiration for other researchers to develop further prediction methods for other composite-based estimators.

Chapters 5 and 6 intentionally contribute to the second strand of composite-based SEM in using the composite model and assuming the composites to comprise theoretical concepts. Specifically, Chapter 5 presents a method to assess composites in human development research, namely confirmatory composite analysis (CCA). The chapter explains the different steps of CCA: model specification, model identification, model estimation, and model assessment. In addition, we use the Henseler-Ogasawara specification (Henseler, 2021; Schuberth, forthcoming) of the composite model which allows us to estimate the model with a maximum likelihood estimator known from factor-based SEM (Jöreskog, 1969). Moreover, this chapter provides guidelines for assessing these kinds of models and illustrates CCA by means of an empirical example. Still, CCA using the Henseler-Ogasawara specification has some limitations. First, the specification of the composite model in terms of loadings might be less intuitive than the specification in terms of weights that directly allows for the interpretation of composites as linear combinations of their observed variables. Second, although standard tools of factor-based SEM can be applied to estimate composite models using the Henseler-Ogasawara specification, some tools like the R package *lavaan* run into convergence issues (Schuberth, forthcoming). Further, the choice for appropriate starting values is not straightforward. Future research should explore new tactics for setting starting values for composite models. In addition, future research should develop simple ways to obtain the standard errors of the composite weights to allow users to perform inference on the parameter estimates.

Chapter 6 presents an alternative specification of the composite model that also allows the model parameters to be estimated using a maximum likelihood estimation.

As in Chapter 5, we assume the composites to comprise theoretical concepts in Chapter 6. Unlike the Henseler-Ogasawara specification, this specification allows direct estimation of weights, thus it is more intuitive than the other with respect to the interpretation of composites. We derive an ML estimator along the lines of the ML estimator for common factor models (Jöreskog, 1967). Besides the ML estimator, we derive a closed form for the model-implied variance covariance matrix for composite models and present a way to test the overall model fit. Moreover, the results based on this ML estimator are compared to those of a traditional estimator for composite models, namely PLS-PM, using a Monte Carlo simulation, which shows that the estimators yield similar results in terms of bias and efficiency.

Chapters 5 and 6 provide researchers with the possibility to assess composites with the same degree of rigor as known from factor-based SEM. Moreover, both the presented ML estimator and the Henseler-Ogasawara specification offer researchers the possibility to apply many developments known from the ML estimator for common factor models to composite models. Future research should therefore investigate which and to what extent these methods are transferable from factor-based SEM to composite-based SEM.

Overall, this thesis concludes that by introducing the composite model and other theoretical frameworks such as the concept-proxy framework (Rigdon, 2012) and the synthesis theory (Henseler and Schuberth, 2021a), the use of composite-based SEM has increased. Even so, traditional composite-based techniques are also still gaining attention and contributing to the spread of composite-based SEM. Moreover, other strands like out-of-sample predictions have gained increasing attention in various research fields. Consequently, we can expect that other strands of composite-based SEM will develop and methodological advances will continue to contribute to the spread and use of composite-based SEM.

Chapter 8

Appendix

8.1 Conducting Monte Carlo simulations with PLS-PM and other variance-based estimators for structural equation model

8.1.1 Observed variables' population correlations for the population model with 4 constructs

Table 8.1: Observed variables' population correlations for the population model with 4 constructs

x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	x_{41}	x_{42}	x_{43}
1.000											
0.720	1.000										
0.630	0.560	1.000									
0.000	0.000	0.000	1.000								
0.000	0.000	0.000	0.500	1.000							
0.000	0.000	0.000	0.500	0.500	1.000						
0.000	0.000	0.000	0.108	0.096	0.084	1.000					
0.000	0.000	0.000	0.216	0.192	0.168	0.200	1.000				
0.000	0.000	0.000	0.216	0.192	0.168	0.000	0.400	1.000			
0.196	0.174	0.152	0.000	0.000	0.000	0.000	0.000	0.000	1.000		
0.184	0.163	0.143	0.000	0.000	0.000	0.000	0.000	0.000	0.250	1.000	
0.200	0.178	0.155	0.000	0.000	0.000	0.000	0.000	0.000	0.400	0.160	1.000

8.1.2 Simulation code for the population model with 4 constructs

```
library(cSEM)
library(cSEM.DGP)
# Model definition in lavaan syntax for the csem function
model <- '
# Relations between the constructs and the observed variables
eta1 =~ x11 + x12 + x13
eta2 <~ x21 + x22 + x23
eta3 <~ x31 + x32 + x33
eta4 <~ x41 + x42 + x43

# Relations between the constructs
eta2 ~ eta1 +  eta3
eta1 ~ eta4'

# Population model with the population values in lavaan syntax
model_dgp <- '
# Relations between the constructs and the observed variables
eta1 =~ 0.9*x11 + 0.8*x12 + 0.7*x13
eta2 <~ 0.6*x21 + 0.4*x22 + 0.2*x23
eta3 <~ 0.3*x31 + 0.5*x32 + 0.6*x33
eta4 <~ 0.4*x41 + 0.5*x42 + 0.5*x43

# Intra block correlations of the observed variables
x21 ~~ 0.5*x22 + 0.5*x23
x22 ~~ 0.5*x23

x31 ~~ 0.2*x32 + 0.0*x33
x32 ~~ 0.4*x33

x41 ~~ 0.25*x42 + 0.4*x43
x42 ~~ 0.16*x43

# Relations between the constructs
eta2 ~ 0.0*eta1 + 0.3*eta3
eta1 ~ 0.3*eta4'
```

```
# Lists for the simulation results
res_PLS <- list()
res_PLSc <- list()
i <- 1
j <- 0
set.seed(123)
while(i < 501){
  data <- generateData(.model = model_dgp, .N = 200)
  res_PLSc_temp <- csem(.model = model, .data = data)
  res_PLS_temp <- csem(.model = model, .data = data, .disattenuate = FALSE)

  if(sum(verify(res_PLSc_temp)) == 0 && sum(verify(res_PLS_temp)) == 0){
    res_PLSc[[i]] <- csem(.model = model, .data = data, .resample_method = "bootstrap",
                          .handle_inadmissibles = "replace", .seed = 123)
    res_PLS[[i]] <- csem(.model = model, .data = data, .resample_method = "bootstrap",
                         .disattenuate = FALSE, .handle_inadmissibles = "replace",
                         .seed = 123)
    i <- i+1
  }else{
    j <- j +1
  }
}
```

8.1.3 Simulation results

Table 8.2: Share of significant path coefficients for the 95% confidence interval

Number of constructs	n	Estimator	Share of significant path coefficients
2	200	PLS-PM	0.080
		PLSc	0.102
	500	PLS-PM	0.062
		PLSc	0.088
4	200	PLS-PM	0.030
		PLSc	0.024
	500	PLS-PM	0.038
		PLSc	0.044

Table 8.3: Share of significant path coefficients for the t-tests

Number of constructs	n	Estimator	Share of sign. path coeff.		
			$\alpha = 1\%$	$\alpha = 5\%$	$\alpha = 10\%$
2	200	PLS-PM	0.040	0.094	0.134
		PLSc	0.056	0.108	0.152
	500	PLS-PM	0.042	0.084	0.104
		PLSc	0.044	0.088	0.118
4	200	PLS-PM	0.008	0.026	0.060
		PLSc	0.006	0.022	0.056
	500	PLS-PM	0.006	0.042	0.090
		PLSc	0.006	0.042	0.086

8.2 Robust partial least squares path modeling

8.2.1 Empirical example

Table 8.4: Estimated weights and factor loadings for the Corporate Reputation Model with and without missing values

Parameter	with missing values		without missing values	
	traditional PLSc	robust PLSc	traditional PLSc	robust PLSc
w_{11}	0.205**	0.168°	0.203**	0.119
w_{12}	0.038	-0.020	0.054	-0.018
w_{13}	0.102°	0.083	0.095	0.084
w_{14}	-0.007	0.086	-0.011	0.097
w_{15}	0.159**	0.140°	0.156**	0.166°
w_{16}	0.399**	0.410**	0.398**	0.415**
w_{17}	0.230**	0.119°	0.228**	0.119°
w_{18}	0.194**	0.224**	0.205**	0.219**
w_{21}	0.463**	0.479**	0.463**	0.507**
w_{22}	0.179**	0.174*	0.171*	0.128*
w_{23}	0.197**	0.144*	0.188**	0.169*
w_{24}	0.342**	0.218*	0.351**	0.216*
w_{25}	0.199**	0.235*	0.201**	0.221*
w_{31}	0.309**	0.270**	0.275**	0.250*
w_{32}	0.038	0.031	0.035	0.037
w_{33}	0.406**	0.349*	0.418**	0.403*
w_{34}	0.081	0.099	0.095	0.128
w_{35}	0.413**	0.427**	0.420**	0.366**
w_{41}	0.419**	0.446**	0.420**	0.449**
w_{42}	0.199**	0.156*	0.203**	0.175*
w_{43}	0.655**	0.637**	0.655**	0.622**
λ_{51}	0.792**	0.911**	0.824**	0.915**
λ_{52}	0.679**	0.738**	0.668**	0.736**
λ_{53}	0.715**	0.772**	0.687**	0.776**
λ_{61}	0.859**	0.917**	0.857**	0.920**
λ_{62}	0.755**	0.811**	0.758**	0.823**
λ_{63}	0.749**	0.829**	0.745**	0.823**
λ_{71}	1.000**	1.000**	1.000**	1.000**
λ_{81}	0.009	0.878**	0.788**	0.882**
λ_{82}	0.708**	0.881**	0.849**	0.893**
λ_{83}	0.834**	0.752**	0.739**	0.755**

**: significant on a 1% level; *: significant on a 5% level; °: significant on a 10% level

8.2.2 Additional population models

Population model containing five common factors

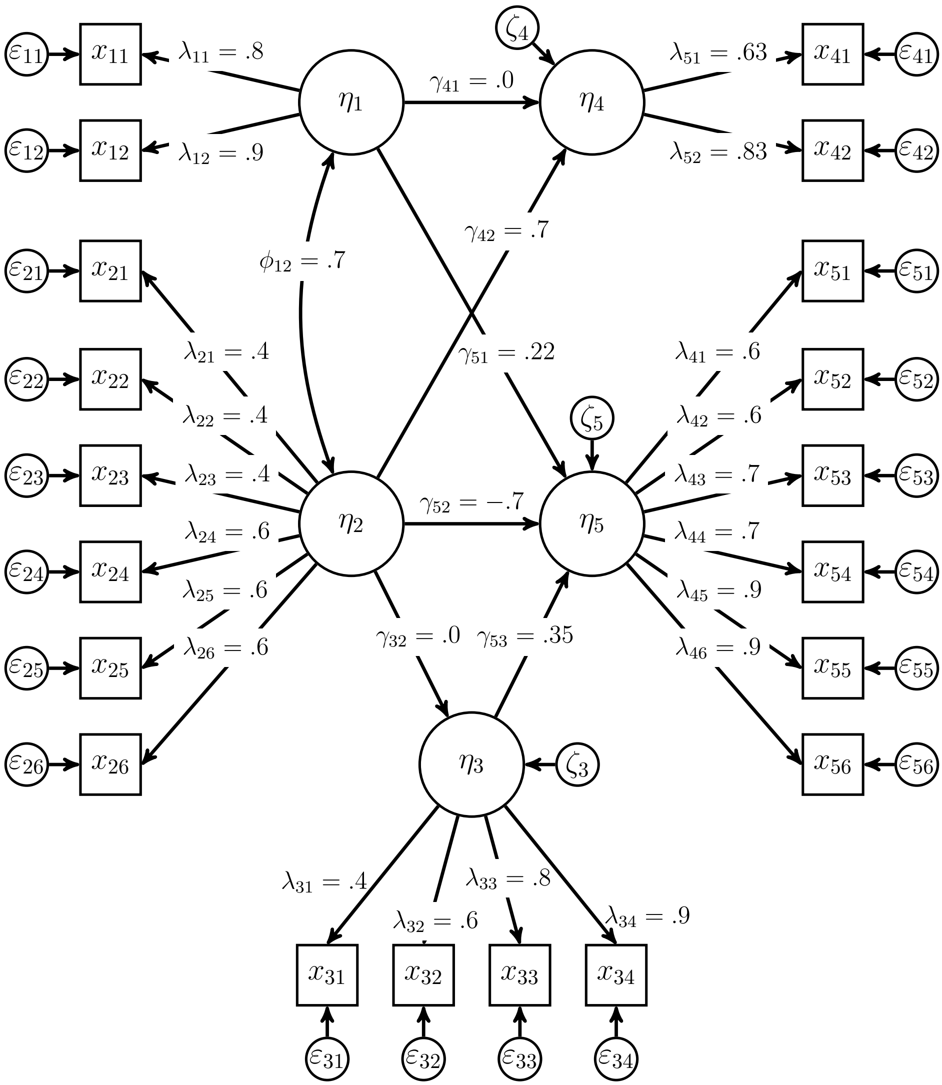

Figure 8.1: Population model containing five common factors

Observed variables' population correlation matrix for the population model containing five common factors:[1]

$$
\Sigma = \begin{array}{cccccccccccccccccccc}
x_{11} & x_{12} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{31} & x_{32} & x_{33} & x_{34} & x_{41} & x_{42} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56}
\end{array}
$$

$$
\Sigma = \left(\begin{array}{cccccccccccccccccccc}
1.000 & & & & & & & & & & & & & & & & & & & \\
0.720 & 1.000 & & & & & & & & & & & & & & & & & & \\
0.224 & 0.252 & 1.000 & & & & & & & & & & & & & & & & & \\
0.224 & 0.252 & 0.160 & 1.000 & & & & & & & & & & & & & & & & \\
0.224 & 0.252 & 0.160 & 0.160 & 1.000 & & & & & & & & & & & & & & & \\
0.336 & 0.378 & 0.240 & 0.240 & 0.240 & 1.000 & & & & & & & & & & & & & & \\
0.336 & 0.378 & 0.240 & 0.240 & 0.240 & 0.360 & 1.000 & & & & & & & & & & & & & \\
0.336 & 0.378 & 0.240 & 0.240 & 0.240 & 0.360 & 0.360 & 1.000 & & & & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & & & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.240 & 1.000 & & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.320 & 0.480 & 1.000 & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.360 & 0.540 & 0.720 & 1.000 & & & & & & & & \\
0.235 & 0.265 & 0.168 & 0.168 & 0.168 & 0.252 & 0.252 & 0.252 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & & & & & & & \\
0.314 & 0.353 & 0.224 & 0.224 & 0.224 & 0.336 & 0.336 & 0.336 & 0.000 & 0.000 & 0.000 & 0.000 & 0.480 & 1.000 & & & & & & \\
-0.139 & -0.157 & -0.134 & -0.134 & -0.134 & -0.202 & -0.202 & -0.202 & 0.072 & 0.108 & 0.144 & 0.162 & -0.141 & -0.188 & 1.000 & & & & & \\
-0.139 & -0.157 & -0.134 & -0.134 & -0.134 & -0.202 & -0.202 & -0.202 & 0.072 & 0.108 & 0.144 & 0.162 & -0.141 & -0.188 & 0.360 & 1.000 & & & & \\
-0.162 & -0.183 & -0.157 & -0.157 & -0.157 & -0.235 & -0.235 & -0.235 & 0.084 & 0.126 & 0.168 & 0.189 & -0.165 & -0.220 & 0.420 & 0.420 & 1.000 & & & \\
-0.162 & -0.183 & -0.157 & -0.157 & -0.157 & -0.235 & -0.235 & -0.235 & 0.084 & 0.126 & 0.168 & 0.189 & -0.165 & -0.220 & 0.420 & 0.420 & 0.490 & 1.000 & & \\
-0.209 & -0.235 & -0.202 & -0.202 & -0.202 & -0.302 & -0.302 & -0.302 & 0.108 & 0.162 & 0.216 & 0.243 & -0.212 & -0.282 & 0.540 & 0.540 & 0.630 & 0.630 & 1.000 & \\
-0.209 & -0.235 & -0.202 & -0.202 & -0.202 & -0.302 & -0.302 & -0.302 & 0.108 & 0.162 & 0.216 & 0.243 & -0.212 & -0.282 & 0.540 & 0.540 & 0.630 & 0.630 & 0.810 & 1.000
\end{array}\right) \tag{8.1}
$$

[1] All correlations are rounded on three decimal places.

Population model containing five composites

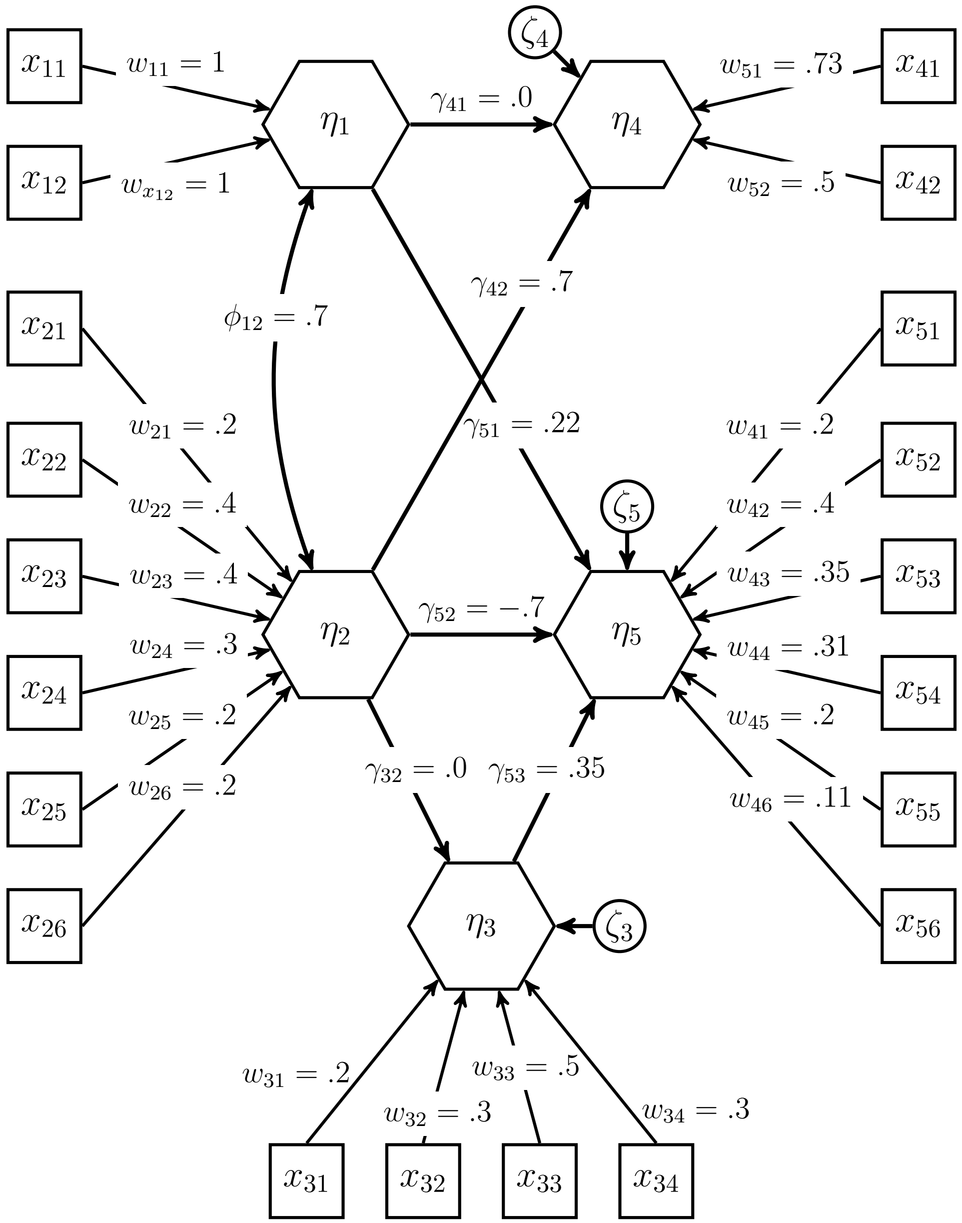

Figure 8.2: Population model containing five composites

Observed variable's population matrix for the population model containing five composites:[2]

$$\boldsymbol{\Sigma} = \begin{array}{c}
\begin{array}{cccccccccccccccccccc}
x_{11} & x_{12} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{31} & x_{32} & x_{33} & x_{34} & x_{41} & x_{42} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56}
\end{array} \\
\left(\begin{array}{cccccccccccccccccccc}
1.000 & & & & & & & & & & & & & & & & & & & \\
-0.500 & 1.000 & & & & & & & & & & & & & & & & & & \\
0.192 & 0.192 & 1.000 & & & & & & & & & & & & & & & & & \\
0.230 & 0.230 & 0.250 & 1.000 & & & & & & & & & & & & & & & & \\
0.230 & 0.230 & 0.250 & 0.160 & 1.000 & & & & & & & & & & & & & & & \\
0.228 & 0.228 & 0.250 & 0.240 & 0.240 & 1.000 & & & & & & & & & & & & & & \\
0.205 & 0.205 & 0.245 & 0.240 & 0.240 & 0.360 & 1.000 & & & & & & & & & & & & & \\
0.182 & 0.182 & 0.240 & 0.240 & 0.240 & 0.360 & 0.360 & 1.000 & & & & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & & & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.100 & 1.000 & & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.200 & 0.400 & 1.000 & & & & & & & & & \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.200 & 0.600 & 0.750 & 1.000 & & & & & & & & \\
0.215 & 0.215 & 0.337 & 0.405 & 0.405 & 0.400 & 0.360 & 0.320 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & & & & & & & \\
0.176 & 0.176 & 0.276 & 0.331 & 0.331 & 0.327 & 0.294 & 0.261 & 0.000 & 0.000 & 0.000 & 0.000 & 0.300 & 1.000 & & & & & & \\
-0.082 & -0.082 & -0.174 & -0.209 & -0.209 & -0.206 & -0.186 & -0.165 & 0.066 & 0.119 & 0.150 & 0.152 & -0.195 & -0.159 & 1.000 & & & & & \\
-0.105 & -0.105 & -0.222 & -0.267 & -0.267 & -0.264 & -0.237 & -0.211 & 0.085 & 0.152 & 0.192 & 0.194 & -0.250 & -0.204 & 0.240 & 1.000 & & & & \\
-0.100 & -0.100 & -0.212 & -0.255 & -0.255 & -0.252 & -0.227 & -0.201 & 0.081 & 0.145 & 0.184 & 0.186 & -0.238 & -0.195 & 0.245 & 0.360 & 1.000 & & & \\
-0.089 & -0.089 & -0.188 & -0.225 & -0.225 & -0.222 & -0.200 & -0.178 & 0.072 & 0.128 & 0.162 & 0.164 & -0.211 & -0.172 & 0.235 & 0.240 & 0.235 & 1.000 & & \\
-0.081 & -0.081 & -0.170 & -0.205 & -0.205 & -0.202 & -0.182 & -0.162 & 0.065 & 0.117 & 0.148 & 0.149 & -0.191 & -0.156 & 0.360 & 0.240 & 0.240 & 0.245 & 1.000 & \\
-0.072 & -0.072 & -0.152 & -0.183 & -0.183 & -0.180 & -0.162 & -0.144 & 0.058 & 0.104 & 0.132 & 0.133 & -0.171 & -0.140 & 0.360 & 0.250 & 0.250 & 0.245 & 0.250 & 1.000
\end{array}\right)
\end{array} \tag{8.2}$$

[2]All correlations are rounded on three decimal places.

8.2.3 Complete results of the Monte Carlo simulation

Tables 8.5 - 8.20 show the complete results of the Monte Carlo simulation[3], where:

- Par.: Model parameter
- Appr.: estimator used during the estimation (robust or traditional[4] PLS/PLSc).

The experimental design was full factorial and we varied the following experimental conditions:

- concept operationalization (all constructs are specified either as composites or common factors),
- model complexity (model containing either three or five constructs),
- sample size ($n = 100$, 300 and 500),
- share of outliers (0%, 5%, 10%, 20%, 40% and 50%),
- kind of outliers (unsystematic and systematic) and
- number of observed variables that are contaminated by outliers (either all or two observed variables).

The unsystematic outliers were drawn from a continuous univariate uniform distribution with a lower bound -10 and an upper bound 10 while the systematic outliers were drawn from the same distribution, but with a lower bound 2 and an upper bound 5.

[3] All results are rounded on three decimal places.

[4] Note that for simplicity, traditional PLS-PM is referred to as traditional PLS in the following tables.

Unsystematic outliers

This subsection shows the complete results for unsystematic outliers.

Table 8.5: Results for traditional and robust PLSc for three common factors and unsystematic outliers in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.018	-0.038	-0.198	-0.317	-0.457	-0.401	0.097	0.534	0.614	0.649	0.628	0.620
		rob.	0.034	0.046	0.030	0.033	0.034	-0.131	0.124	0.132	0.124	0.125	0.126	0.626
	300	trad.	0.003	0.057	-0.067	-0.216	-0.333	-0.350	0.060	0.329	0.429	0.545	0.561	0.567
		rob.	0.006	0.010	0.003	0.003	0.006	-0.016	0.069	0.065	0.068	0.068	0.075	0.349
	500	trad.	-0.000	0.087	0.007	-0.131	-0.287	-0.317	0.045	0.204	0.321	0.464	0.536	0.541
		rob.	-0.001	0.003	0.001	0.003	-0.003	-0.062	0.051	0.051	0.053	0.055	0.058	0.240
γ_{31}	100	trad.	0.039	0.051	-0.121	-0.181	-0.221	-0.251	0.152	1.006	0.827	0.752	0.757	0.862
		rob.	0.064	0.062	0.052	0.049	0.060	-0.082	0.238	0.232	0.229	0.221	0.207	0.829
	300	trad.	0.010	0.132	-0.049	-0.164	-0.227	-0.204	0.081	0.919	0.719	0.729	0.744	0.718
		rob.	0.010	0.005	0.011	0.017	0.020	0.016	0.095	0.093	0.093	0.090	0.107	0.549
	500	trad.	0.004	0.141	0.044	-0.093	-0.193	-0.209	0.062	0.586	0.658	0.759	0.693	0.684
		rob.	0.004	0.004	0.004	0.006	0.006	-0.039	0.069	0.074	0.072	0.073	0.078	0.368
γ_{32}	100	trad.	0.013	0.182	0.075	0.093	0.034	0.097	0.176	1.034	0.814	0.766	0.778	0.860
		rob.	0.013	0.007	0.006	0.010	-0.001	0.048	0.271	0.265	0.274	0.248	0.249	0.813
	300	trad.	-0.001	0.075	0.151	0.107	0.013	0.072	0.092	0.957	0.747	0.763	0.724	0.731
		rob.	0.001	0.014	0.006	0.001	-0.000	0.014	0.109	0.103	0.110	0.107	0.119	0.612
	500	trad.	-0.000	0.017	0.107	0.119	0.069	0.062	0.069	0.646	0.721	0.780	0.694	0.688
		rob.	0.002	0.005	0.005	-0.000	-0.002	0.029	0.078	0.080	0.080	0.081	0.090	0.433
λ_{11}	100	trad.	-0.026	-0.395	-0.524	-0.611	-0.694	-0.698	0.066	0.376	0.391	0.400	0.364	0.365
		rob.	-0.037	-0.033	-0.032	-0.030	-0.034	-0.302	0.082	0.085	0.081	0.084	0.082	0.383
	300	trad.	-0.010	-0.431	-0.539	-0.674	-0.729	-0.737	0.046	0.301	0.323	0.324	0.301	0.289
		rob.	-0.010	-0.007	-0.005	-0.009	-0.010	-0.224	0.052	0.051	0.052	0.053	0.058	0.267
	500	trad.	-0.002	-0.419	-0.549	-0.656	-0.737	-0.758	0.036	0.258	0.289	0.287	0.274	0.258
		rob.	-0.002	-0.004	-0.003	-0.003	-0.004	-0.185	0.040	0.041	0.039	0.042	0.047	0.224
λ_{12}	100	trad.	-0.000	-0.337	-0.431	-0.545	-0.590	-0.602	0.089	0.383	0.398	0.390	0.362	0.342
		rob.	-0.003	-0.009	0.003	-0.005	-0.002	-0.246	0.110	0.108	0.109	0.116	0.111	0.420
	300	trad.	-0.001	-0.358	-0.493	-0.591	-0.622	-0.638	0.054	0.300	0.332	0.334	0.291	0.287
		rob.	-0.001	0.001	0.003	0.000	-0.001	-0.213	0.062	0.063	0.063	0.062	0.071	0.307
	500	trad.	-0.001	-0.353	-0.483	-0.592	-0.645	-0.646	0.044	0.254	0.294	0.291	0.269	0.267
		rob.	-0.001	-0.001	0.000	-0.001	-0.000	-0.168	0.048	0.047	0.047	0.052	0.058	0.252
λ_{13}	100	trad.	0.011	-0.264	-0.337	-0.443	-0.502	-0.504	0.105	0.403	0.421	0.408	0.384	0.359
		rob.	0.013	0.015	0.006	0.014	0.019	-0.183	0.137	0.135	0.139	0.139	0.130	0.424
	300	trad.	0.006	-0.304	-0.397	-0.484	-0.538	-0.549	0.064	0.308	0.321	0.314	0.317	0.290
		rob.	0.006	0.004	-0.001	0.003	0.006	-0.170	0.075	0.070	0.075	0.074	0.082	0.336
	500	trad.	-0.000	-0.304	-0.415	-0.493	-0.543	-0.553	0.050	0.258	0.288	0.294	0.271	0.249
		rob.	-0.000	0.004	0.002	0.002	-0.000	-0.151	0.055	0.056	0.058	0.060	0.067	0.287
λ_{21}	100	trad.	-0.015	-0.308	-0.363	-0.447	-0.502	-0.489	0.131	0.401	0.416	0.410	0.371	0.358
		rob.	-0.021	-0.001	-0.006	-0.017	-0.014	-0.209	0.169	0.165	0.156	0.157	0.171	0.440
	300	trad.	0.002	-0.306	-0.413	-0.495	-0.526	-0.539	0.076	0.305	0.326	0.328	0.291	0.285
		rob.	0.001	0.000	0.002	-0.007	-0.001	-0.219	0.087	0.086	0.089	0.091	0.098	0.317
	500	trad.	0.001	-0.325	-0.414	-0.492	-0.554	-0.565	0.057	0.255	0.282	0.287	0.268	0.257
		rob.	0.001	-0.001	0.002	-0.007	-0.005	-0.199	0.062	0.065	0.064	0.066	0.074	0.287
λ_{22}	100	trad.	0.000	-0.290	-0.372	-0.436	-0.493	-0.496	0.124	0.410	0.431	0.410	0.370	0.354
		rob.	-0.011	-0.022	-0.013	-0.014	-0.015	-0.207	0.161	0.148	0.158	0.154	0.164	0.428
	300	trad.	-0.002	-0.320	-0.398	-0.509	-0.534	-0.549	0.073	0.303	0.328	0.334	0.292	0.283
		rob.	-0.004	0.001	0.003	0.002	-0.007	-0.212	0.084	0.086	0.087	0.090	0.099	0.319
	500	trad.	-0.004	-0.311	-0.419	-0.507	-0.548	-0.574	0.058	0.258	0.285	0.283	0.265	0.261
		rob.	-0.005	-0.002	-0.003	0.001	0.003	-0.204	0.065	0.063	0.065	0.066	0.074	0.284
λ_{23}	100	trad.	-0.015	-0.265	-0.364	-0.462	-0.504	-0.503	0.121	0.421	0.412	0.409	0.386	0.353
		rob.	-0.022	-0.011	-0.022	-0.011	-0.017	-0.213	0.160	0.156	0.161	0.162	0.161	0.439
	300	trad.	-0.006	-0.334	-0.425	-0.488	-0.510	-0.540	0.077	0.308	0.328	0.312	0.307	0.275
		rob.	-0.004	-0.000	-0.008	-0.002	-0.002	-0.203	0.087	0.086	0.087	0.089	0.096	0.303
	500	trad.	0.001	-0.320	-0.414	-0.492	-0.544	-0.553	0.058	0.258	0.283	0.297	0.267	0.251
		rob.	0.000	-0.001	0.001	-0.003	-0.010	-0.181	0.067	0.064	0.066	0.068	0.078	0.271
λ_{31}	100	trad.	-0.042	-0.388	-0.448	-0.546	-0.606	-0.607	0.164	0.429	0.412	0.393	0.378	0.343
		rob.	-0.060	-0.058	-0.058	-0.065	-0.051	-0.269	0.190	0.195	0.193	0.199	0.198	0.417
	300	trad.	-0.017	-0.432	-0.515	-0.592	-0.650	-0.632	0.094	0.316	0.342	0.330	0.296	0.297
		rob.	-0.019	-0.011	-0.009	-0.017	-0.016	-0.271	0.107	0.111	0.113	0.112	0.129	0.331
	500	trad.	-0.002	-0.426	-0.530	-0.597	-0.647	-0.658	0.079	0.299	0.313	0.298	0.281	0.260
		rob.	-0.003	-0.008	-0.004	-0.005	-0.010	-0.277	0.089	0.089	0.087	0.094	0.099	0.316
λ_{32}	100	trad.	-0.036	-0.363	-0.457	-0.547	-0.573	-0.582	0.158	0.410	0.402	0.404	0.357	0.359
		rob.	-0.063	-0.054	-0.071	-0.063	-0.072	-0.263	0.202	0.200	0.210	0.206	0.211	0.411
	300	trad.	-0.005	-0.422	-0.527	-0.585	-0.621	-0.649	0.092	0.343	0.331	0.319	0.302	0.281
		rob.	-0.011	-0.008	-0.018	-0.011	-0.019	-0.281	0.102	0.111	0.118	0.113	0.125	0.346
	500	trad.	-0.006	-0.431	-0.539	-0.594	-0.655	-0.664	0.080	0.295	0.318	0.301	0.273	0.250
		rob.	-0.008	-0.010	-0.005	-0.010	-0.009	-0.253	0.089	0.091	0.089	0.093	0.101	0.324
λ_{33}	100	trad.	0.004	-0.280	-0.343	-0.458	-0.490	-0.483	0.181	0.397	0.407	0.415	0.371	0.356
		rob.	-0.009	-0.034	-0.033	-0.024	-0.034	-0.222	0.218	0.248	0.247	0.241	0.247	0.450
	300	trad.	0.007	-0.353	-0.423	-0.484	-0.553	-0.549	0.114	0.339	0.335	0.322	0.303	0.295
		rob.	0.009	-0.001	0.004	0.008	0.007	-0.218	0.128	0.139	0.135	0.137	0.144	0.359
	500	trad.	-0.001	-0.360	-0.447	-0.517	-0.552	-0.556	0.094	0.305	0.303	0.289	0.279	0.259
		rob.	-0.001	0.008	0.001	0.002	0.004	-0.225	0.104	0.104	0.105	0.108	0.117	0.335

Table 8.6: Results for traditional and robust PLSc for three common factors and unsystematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.027	0.061	0.073	0.065	0.073	0.077	0.096	0.115	0.124	0.121	0.121	0.132
		rob.	0.036	0.033	0.040	0.032	0.044	0.094	0.125	0.128	0.125	0.127	0.126	0.157
	300	trad.	0.001	0.018	0.016	0.014	0.018	0.010	0.059	0.068	0.068	0.068	0.067	0.052
		rob.	0.003	0.006	0.003	0.002	0.011	0.002	0.069	0.067	0.065	0.068	0.077	0.055
	500	trad.	-0.001	0.006	0.009	0.007	0.010	0.013	0.045	0.054	0.056	0.054	0.052	0.052
		rob.	-0.001	0.000	0.001	-0.002	0.004	0.011	0.050	0.051	0.054	0.053	0.055	0.067
γ_{31}	100	trad.	0.047	0.076	0.084	0.073	0.072	0.078	0.150	0.186	0.223	0.197	0.221	0.218
		rob.	0.066	0.071	0.077	0.071	0.071	0.094	0.233	0.213	0.218	0.223	0.221	0.313
	300	trad.	0.008	0.012	0.012	0.016	0.017	0.001	0.078	0.088	0.092	0.088	0.088	0.069
		rob.	0.008	0.008	0.008	0.015	0.016	0.001	0.090	0.093	0.096	0.097	0.107	0.079
	500	trad.	0.004	0.009	0.009	0.005	0.008	0.003	0.061	0.069	0.069	0.067	0.068	0.069
		rob.	0.005	0.004	0.006	0.003	0.007	0.005	0.069	0.070	0.071	0.071	0.081	0.089
γ_{32}	100	trad.	0.007	-0.021	-0.035	-0.012	-0.019	-0.025	0.179	0.219	0.263	0.232	0.264	0.262
		rob.	0.014	-0.002	-0.017	0.011	-0.000	-0.025	0.275	0.265	0.260	0.259	0.269	0.364
	300	trad.	0.003	-0.001	0.000	-0.003	-0.003	0.003	0.092	0.102	0.107	0.105	0.104	0.077
		rob.	0.005	0.007	0.005	-0.002	-0.001	0.004	0.108	0.108	0.108	0.112	0.120	0.085
	500	trad.	0.001	-0.003	-0.005	0.002	-0.001	0.003	0.070	0.075	0.080	0.078	0.079	0.081
		rob.	0.002	0.003	-0.001	0.003	0.001	0.004	0.079	0.078	0.079	0.077	0.091	0.099
λ_{11}	100	trad.	-0.027	-0.057	-0.061	-0.055	-0.054	-0.055	0.069	0.095	0.093	0.086	0.079	0.084
		rob.	-0.034	-0.037	-0.040	-0.035	-0.042	-0.081	0.086	0.081	0.084	0.082	0.088	0.122
	300	trad.	-0.004	-0.020	-0.018	-0.015	-0.019	-0.012	0.046	0.058	0.056	0.049	0.049	0.037
		rob.	-0.005	-0.009	-0.006	-0.009	-0.017	-0.007	0.050	0.050	0.051	0.053	0.061	0.043
	500	trad.	-0.001	-0.011	-0.009	-0.011	-0.008	-0.012	0.036	0.045	0.044	0.040	0.038	0.038
		rob.	-0.002	-0.005	-0.003	-0.005	-0.005	-0.011	0.041	0.040	0.041	0.041	0.047	0.053
λ_{12}	100	trad.	-0.000	-0.030	-0.027	-0.024	-0.022	-0.016	0.088	0.110	0.102	0.094	0.088	0.087
		rob.	-0.003	-0.001	-0.003	0.001	-0.004	-0.032	0.109	0.111	0.109	0.104	0.105	0.128
	300	trad.	0.002	-0.010	-0.012	-0.009	-0.004	-0.004	0.054	0.065	0.058	0.050	0.048	0.040
		rob.	0.003	0.002	0.003	-0.002	-0.003	0.001	0.061	0.064	0.064	0.064	0.068	0.052
	500	trad.	-0.002	-0.006	-0.004	-0.005	-0.008	-0.005	0.043	0.046	0.043	0.038	0.038	0.038
		rob.	-0.003	-0.002	0.001	0.002	-0.003	-0.002	0.048	0.045	0.049	0.049	0.056	0.061
λ_{13}	100	trad.	0.010	-0.250	-0.370	-0.477	-0.585	-0.612	0.101	0.222	0.212	0.208	0.203	0.200
		rob.	0.018	0.017	0.007	0.004	-0.039	-0.237	0.123	0.134	0.133	0.136	0.168	0.263
	300	trad.	-0.001	-0.271	-0.381	-0.496	-0.588	-0.550	0.063	0.126	0.123	0.120	0.121	0.094
		rob.	-0.001	0.001	-0.003	-0.010	-0.048	-0.024	0.073	0.071	0.076	0.078	0.097	0.065
	500	trad.	0.002	-0.286	-0.392	-0.491	-0.585	-0.618	0.051	0.096	0.093	0.092	0.091	0.098
		rob.	0.003	-0.001	-0.006	-0.016	-0.060	-0.171	0.056	0.056	0.055	0.061	0.078	0.104
λ_{21}	100	trad.	-0.009	-0.274	-0.386	-0.490	-0.596	-0.608	0.120	0.226	0.215	0.213	0.199	0.197
		rob.	-0.018	-0.025	-0.029	-0.034	-0.073	-0.268	0.145	0.167	0.163	0.163	0.188	0.261
	300	trad.	-0.001	-0.280	-0.390	-0.492	-0.591	-0.552	0.076	0.123	0.122	0.121	0.123	0.092
		rob.	-0.003	-0.006	-0.011	-0.014	-0.067	-0.032	0.086	0.084	0.089	0.090	0.113	0.076
	500	trad.	-0.003	-0.288	-0.393	-0.491	-0.593	-0.619	0.056	0.094	0.094	0.093	0.099	0.095
		rob.	-0.003	-0.006	-0.006	-0.013	-0.071	-0.194	0.063	0.067	0.066	0.069	0.084	0.113
λ_{22}	100	trad.	-0.012	-0.037	-0.045	-0.047	-0.048	-0.053	0.126	0.143	0.137	0.143	0.132	0.138
		rob.	-0.013	-0.018	-0.020	-0.018	-0.027	-0.066	0.157	0.159	0.164	0.167	0.159	0.172
	300	trad.	-0.003	-0.011	-0.014	-0.014	-0.012	-0.007	0.071	0.083	0.079	0.072	0.075	0.056
		rob.	-0.004	-0.000	-0.003	-0.002	-0.004	-0.001	0.080	0.084	0.081	0.087	0.101	0.071
	500	trad.	0.000	-0.004	-0.006	-0.009	-0.011	-0.009	0.057	0.066	0.061	0.062	0.057	0.056
		rob.	0.000	-0.001	-0.000	-0.002	-0.008	-0.005	0.063	0.069	0.064	0.070	0.072	0.080
λ_{23}	100	trad.	-0.002	-0.042	-0.047	-0.048	-0.045	-0.051	0.121	0.137	0.137	0.139	0.132	0.137
		rob.	-0.004	-0.018	-0.016	-0.022	-0.026	-0.065	0.149	0.164	0.163	0.167	0.161	0.172
	300	trad.	-0.002	-0.015	-0.016	-0.013	-0.016	-0.011	0.075	0.083	0.080	0.079	0.073	0.057
		rob.	-0.000	-0.003	-0.003	-0.002	-0.010	-0.005	0.087	0.084	0.085	0.091	0.098	0.070
	500	trad.	0.002	-0.007	-0.008	-0.009	-0.008	-0.010	0.058	0.063	0.061	0.060	0.055	0.056
		rob.	-0.000	-0.001	-0.003	-0.002	0.000	-0.009	0.066	0.064	0.065	0.068	0.075	0.081
λ_{31}	100	trad.	-0.034	-0.037	-0.034	-0.033	-0.030	-0.038	0.156	0.153	0.160	0.153	0.148	0.153
		rob.	-0.059	-0.062	-0.048	-0.057	-0.054	-0.065	0.197	0.199	0.179	0.191	0.188	0.201
	300	trad.	-0.009	-0.010	-0.015	-0.008	-0.013	-0.008	0.097	0.093	0.098	0.095	0.090	0.083
		rob.	-0.013	-0.012	-0.022	-0.013	-0.019	-0.010	0.109	0.111	0.110	0.113	0.113	0.096
	500	trad.	-0.009	-0.008	-0.005	-0.006	-0.003	-0.010	0.082	0.084	0.080	0.081	0.081	0.079
		rob.	-0.009	-0.009	-0.002	-0.009	-0.008	-0.018	0.092	0.094	0.087	0.094	0.100	0.102
λ_{32}	100	trad.	-0.034	-0.044	-0.035	-0.038	-0.043	-0.032	0.149	0.157	0.154	0.159	0.157	0.156
		rob.	-0.057	-0.063	-0.057	-0.057	-0.066	-0.060	0.189	0.189	0.186	0.187	0.207	0.202
	300	trad.	-0.014	-0.011	-0.008	-0.008	-0.013	-0.005	0.099	0.099	0.103	0.097	0.098	0.081
		rob.	-0.016	-0.014	-0.016	-0.014	-0.014	-0.011	0.107	0.115	0.115	0.116	0.121	0.093
	500	trad.	-0.002	-0.005	-0.007	-0.002	-0.009	-0.006	0.080	0.081	0.080	0.079	0.079	0.081
		rob.	-0.005	-0.005	-0.007	-0.001	-0.012	-0.008	0.086	0.089	0.090	0.091	0.096	0.105
λ_{33}	100	trad.	0.005	-0.001	-0.021	-0.004	-0.005	0.008	0.172	0.196	0.207	0.197	0.190	0.170
		rob.	-0.018	-0.008	-0.035	-0.005	-0.017	-0.007	0.223	0.222	0.247	0.212	0.221	0.222
	300	trad.	0.006	0.001	0.007	0.003	0.011	0.001	0.121	0.121	0.113	0.117	0.111	0.094
		rob.	0.009	-0.001	0.015	0.003	0.002	0.004	0.134	0.134	0.127	0.141	0.145	0.113
	500	trad.	0.002	0.006	0.004	-0.002	0.003	0.008	0.090	0.093	0.097	0.093	0.092	0.091
		rob.	0.004	0.005	-0.002	-0.003	0.004	0.007	0.100	0.108	0.107	0.106	0.119	0.125

Table 8.7: Results for traditional and robust PLS for three composites and unsystematic outliers in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.018	-0.133	-0.235	-0.366	-0.475	-0.486	0.075	0.495	0.479	0.418	0.330	0.303
		rob.	0.031	0.031	0.034	0.038	0.036	-0.050	0.112	0.108	0.105	0.099	0.100	0.565
	300	trad.	0.006	-0.150	-0.256	-0.378	-0.445	-0.466	0.044	0.222	0.246	0.238	0.187	0.174
		rob.	0.010	0.010	0.007	0.010	0.010	-0.063	0.050	0.049	0.052	0.049	0.056	0.257
	500	trad.	0.004	-0.182	-0.284	-0.387	-0.450	-0.456	0.033	0.167	0.182	0.178	0.144	0.131
		rob.	0.004	0.005	0.006	0.006	0.008	-0.078	0.037	0.038	0.037	0.038	0.043	0.143
γ_{31}	100	trad.	0.016	-0.106	-0.162	-0.206	-0.266	-0.278	0.120	0.402	0.425	0.392	0.314	0.293
		rob.	0.028	0.031	0.019	0.024	0.017	-0.120	0.205	0.191	0.188	0.184	0.185	0.468
	300	trad.	0.006	-0.090	-0.166	-0.219	-0.268	-0.276	0.065	0.271	0.262	0.232	0.189	0.170
		rob.	0.009	0.009	0.009	0.008	0.012	-0.077	0.075	0.074	0.076	0.076	0.090	0.296
	500	trad.	0.002	-0.105	-0.163	-0.225	-0.267	-0.275	0.048	0.214	0.208	0.181	0.146	0.133
		rob.	0.004	0.006	0.006	0.005	0.006	-0.072	0.054	0.055	0.055	0.055	0.066	0.219
γ_{32}	100	trad.	0.033	0.104	0.067	0.014	-0.014	0.018	0.137	0.378	0.416	0.401	0.320	0.289
		rob.	0.052	0.058	0.058	0.053	0.051	0.105	0.238	0.218	0.211	0.207	0.203	0.435
	300	trad.	0.012	0.097	0.083	0.043	0.019	0.000	0.068	0.289	0.289	0.243	0.189	0.177
		rob.	0.013	0.015	0.020	0.015	0.016	0.091	0.079	0.077	0.079	0.082	0.094	0.315
	500	trad.	0.004	0.086	0.056	0.027	0.014	0.009	0.052	0.227	0.228	0.188	0.147	0.131
		rob.	0.006	0.006	0.008	0.008	0.015	0.067	0.057	0.059	0.058	0.060	0.070	0.227
w_{11}	100	trad.	-0.017	-0.360	-0.395	-0.365	-0.353	-0.355	0.185	0.610	0.588	0.563	0.540	0.540
		rob.	-0.030	-0.035	-0.043	-0.042	-0.045	-0.414	0.296	0.301	0.261	0.276	0.277	0.617
	300	trad.	-0.008	-0.291	-0.341	-0.346	-0.356	-0.349	0.097	0.543	0.534	0.530	0.535	0.528
		rob.	-0.010	-0.012	-0.001	-0.006	-0.013	-0.210	0.114	0.116	0.112	0.121	0.134	0.534
	500	trad.	-0.001	-0.226	-0.269	-0.328	-0.332	-0.353	0.075	0.513	0.514	0.524	0.515	0.529
		rob.	-0.001	-0.003	0.000	-0.009	-0.008	-0.111	0.085	0.086	0.084	0.091	0.099	0.429
w_{12}	100	trad.	-0.012	-0.240	-0.208	-0.171	-0.196	-0.137	0.194	0.581	0.598	0.561	0.539	0.536
		rob.	-0.045	-0.052	-0.020	-0.028	-0.021	-0.184	0.302	0.301	0.290	0.289	0.290	0.626
	300	trad.	-0.004	-0.138	-0.167	-0.175	-0.138	-0.148	0.109	0.550	0.540	0.535	0.515	0.521
		rob.	-0.005	0.006	-0.008	-0.005	-0.008	-0.151	0.126	0.125	0.127	0.131	0.139	0.549
	500	trad.	-0.007	-0.097	-0.124	-0.153	-0.136	-0.152	0.081	0.508	0.508	0.507	0.522	0.523
		rob.	-0.007	0.001	-0.006	-0.000	-0.003	-0.056	0.090	0.095	0.092	0.097	0.108	0.426
w_{13}	100	trad.	-0.003	-0.061	-0.008	0.013	0.020	0.021	0.208	0.598	0.592	0.561	0.561	0.538
		rob.	-0.012	0.004	-0.010	0.001	-0.016	-0.059	0.320	0.306	0.296	0.284	0.304	0.601
	300	trad.	0.002	-0.027	0.042	0.017	0.031	0.033	0.106	0.559	0.549	0.551	0.530	0.532
		rob.	0.004	-0.007	-0.005	-0.003	0.006	-0.073	0.125	0.130	0.128	0.134	0.146	0.538
	500	trad.	0.004	-0.003	0.030	0.073	0.032	0.069	0.085	0.494	0.515	0.524	0.525	0.509
		rob.	0.002	-0.005	-0.001	0.003	0.003	-0.035	0.094	0.091	0.094	0.098	0.111	0.443
w_{21}	100	trad.	-0.005	-0.102	-0.072	-0.063	-0.064	-0.041	0.189	0.511	0.548	0.531	0.552	0.525
		rob.	-0.026	-0.021	-0.026	-0.029	-0.033	-0.101	0.285	0.286	0.268	0.265	0.281	0.527
	300	trad.	-0.008	-0.087	-0.069	-0.057	-0.032	-0.063	0.105	0.504	0.513	0.520	0.513	0.537
		rob.	-0.010	-0.008	-0.005	-0.006	-0.011	-0.089	0.123	0.126	0.124	0.129	0.136	0.486
	500	trad.	-0.002	-0.095	-0.098	-0.041	-0.053	-0.048	0.078	0.469	0.494	0.512	0.521	0.511
		rob.	-0.003	-0.000	-0.000	-0.002	-0.001	-0.032	0.090	0.094	0.093	0.095	0.108	0.409
w_{22}	100	trad.	-0.021	-0.293	-0.286	-0.235	-0.250	-0.286	0.194	0.571	0.560	0.544	0.538	0.533
		rob.	-0.046	-0.029	-0.035	-0.058	-0.053	-0.305	0.298	0.290	0.272	0.278	0.291	0.573
	300	trad.	-0.006	-0.221	-0.224	-0.241	-0.248	-0.258	0.105	0.539	0.540	0.533	0.520	0.512
		rob.	-0.006	-0.006	-0.004	-0.009	-0.005	-0.188	0.120	0.127	0.131	0.130	0.143	0.499
	500	trad.	-0.000	-0.139	-0.189	-0.242	-0.245	-0.232	0.083	0.485	0.522	0.522	0.508	0.513
		rob.	-0.002	-0.004	-0.004	-0.006	-0.006	-0.116	0.093	0.093	0.094	0.100	0.106	0.408
w_{23}	100	trad.	-0.023	-0.342	-0.366	-0.354	-0.352	-0.346	0.179	0.555	0.569	0.564	0.524	0.550
		rob.	-0.051	-0.067	-0.047	-0.023	-0.040	-0.338	0.277	0.279	0.271	0.251	0.270	0.552
	300	trad.	-0.004	-0.268	-0.312	-0.335	-0.347	-0.367	0.099	0.505	0.525	0.529	0.534	0.541
		rob.	-0.005	-0.008	-0.013	-0.009	-0.015	-0.246	0.112	0.117	0.118	0.120	0.134	0.518
	500	trad.	-0.006	-0.203	-0.278	-0.316	-0.313	-0.345	0.073	0.469	0.512	0.514	0.524	0.529
		rob.	-0.006	-0.006	-0.006	-0.003	-0.007	-0.121	0.084	0.087	0.086	0.094	0.099	0.403
w_{31}	100	trad.	-0.043	-0.257	-0.239	-0.190	-0.138	-0.150	0.357	0.580	0.589	0.575	0.538	0.539
		rob.	-0.094	-0.068	-0.059	-0.085	-0.105	-0.213	0.464	0.440	0.445	0.440	0.439	0.605
	300	trad.	-0.016	-0.166	-0.157	-0.169	-0.156	-0.161	0.196	0.568	0.547	0.541	0.526	0.533
		rob.	-0.021	-0.017	-0.010	-0.031	-0.018	-0.182	0.229	0.237	0.237	0.252	0.259	0.546
	500	trad.	-0.007	-0.169	-0.168	-0.154	-0.140	-0.131	0.155	0.557	0.540	0.531	0.521	0.531
		rob.	-0.009	-0.009	-0.008	-0.019	-0.014	-0.165	0.178	0.174	0.186	0.181	0.207	0.540
w_{32}	100	trad.	-0.067	-0.319	-0.248	-0.256	-0.272	-0.258	0.305	0.563	0.559	0.540	0.538	0.522
		rob.	-0.105	-0.110	-0.118	-0.119	-0.101	-0.323	0.414	0.395	0.402	0.393	0.406	0.557
	300	trad.	-0.026	-0.272	-0.239	-0.283	-0.283	-0.240	0.172	0.522	0.526	0.530	0.542	0.532
		rob.	-0.034	-0.024	-0.028	-0.023	-0.034	-0.272	0.203	0.213	0.202	0.208	0.235	0.549
	500	trad.	-0.011	-0.229	-0.244	-0.256	-0.238	-0.254	0.142	0.501	0.511	0.531	0.508	0.515
		rob.	-0.016	-0.022	-0.011	-0.017	-0.024	-0.190	0.156	0.155	0.158	0.162	0.178	0.491
w_{33}	100	trad.	-0.069	-0.294	-0.310	-0.267	-0.272	-0.257	0.339	0.565	0.569	0.557	0.543	0.539
		rob.	-0.120	-0.125	-0.130	-0.087	-0.105	-0.337	0.433	0.430	0.435	0.414	0.416	0.578
	300	trad.	-0.011	-0.302	-0.279	-0.258	-0.239	-0.255	0.185	0.555	0.539	0.541	0.526	0.513
		rob.	-0.017	-0.033	-0.034	-0.026	-0.039	-0.238	0.219	0.220	0.226	0.225	0.243	0.558
	500	trad.	-0.013	-0.236	-0.208	-0.244	-0.234	-0.239	0.148	0.528	0.516	0.516	0.525	0.513
		rob.	-0.015	-0.008	-0.024	-0.010	-0.018	-0.210	0.167	0.163	0.172	0.176	0.193	0.521

Table 8.8: Results for traditional and robust PLS for three composites and unsystematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.022	0.016	0.011	-0.000	0.000	-0.008	0.070	0.076	0.082	0.075	0.076	0.078
		rob.	0.039	0.029	0.038	0.029	0.034	0.043	0.101	0.103	0.103	0.100	0.104	0.122
	300	trad.	0.010	-0.010	-0.012	-0.015	-0.018	-0.021	0.042	0.044	0.045	0.045	0.045	0.044
		rob.	0.014	0.011	0.011	0.013	0.005	-0.003	0.048	0.048	0.050	0.051	0.056	0.055
	500	trad.	0.005	-0.014	-0.018	-0.021	-0.023	-0.024	0.034	0.034	0.034	0.033	0.034	0.035
		rob.	0.006	0.006	0.005	0.004	0.000	-0.010	0.037	0.037	0.039	0.039	0.042	0.045
γ_{31}	100	trad.	0.012	0.003	0.004	0.010	0.008	0.009	0.122	0.127	0.131	0.117	0.124	0.127
		rob.	0.030	0.024	0.019	0.023	0.013	0.012	0.207	0.204	0.195	0.174	0.183	0.193
	300	trad.	0.006	0.003	0.003	-0.001	0.001	-0.001	0.066	0.061	0.064	0.063	0.063	0.062
		rob.	0.009	0.008	0.009	0.007	0.010	0.004	0.077	0.076	0.076	0.076	0.084	0.086
	500	trad.	0.003	-0.002	0.000	-0.003	-0.001	-0.002	0.048	0.048	0.047	0.049	0.050	0.048
		rob.	0.004	0.002	0.006	0.002	0.006	0.007	0.054	0.056	0.056	0.059	0.062	0.065
γ_{32}	100	trad.	0.037	0.039	0.049	0.044	0.041	0.041	0.136	0.137	0.139	0.131	0.134	0.136
		rob.	0.060	0.055	0.057	0.058	0.059	0.054	0.229	0.218	0.213	0.200	0.197	0.202
	300	trad.	0.007	0.014	0.014	0.015	0.016	0.017	0.071	0.064	0.068	0.067	0.067	0.065
		rob.	0.011	0.015	0.015	0.013	0.016	0.021	0.082	0.079	0.082	0.082	0.091	0.090
	500	trad.	0.006	0.011	0.010	0.013	0.010	0.010	0.051	0.049	0.049	0.051	0.053	0.049
		rob.	0.008	0.010	0.009	0.013	0.010	0.011	0.057	0.055	0.057	0.059	0.066	0.068
w_{11}	100	trad.	-0.022	-0.006	0.008	0.038	0.057	0.055	0.189	0.205	0.200	0.177	0.174	0.183
		rob.	-0.049	-0.050	-0.037	-0.031	-0.014	-0.065	0.294	0.298	0.257	0.266	0.264	0.314
	300	trad.	-0.000	0.044	0.055	0.069	0.067	0.076	0.102	0.100	0.091	0.096	0.092	0.093
		rob.	-0.003	-0.002	-0.006	-0.002	0.009	0.037	0.124	0.113	0.111	0.121	0.127	0.135
	500	trad.	-0.004	0.048	0.055	0.069	0.070	0.072	0.076	0.073	0.072	0.072	0.071	0.072
		rob.	-0.006	-0.006	-0.006	0.001	0.012	0.040	0.086	0.086	0.088	0.091	0.092	0.098
w_{12}	100	trad.	-0.012	0.010	0.040	0.056	0.045	0.048	0.202	0.215	0.203	0.196	0.197	0.206
		rob.	-0.039	-0.035	-0.019	-0.012	-0.022	-0.037	0.320	0.303	0.295	0.276	0.277	0.308
	300	trad.	-0.010	0.047	0.060	0.059	0.068	0.060	0.108	0.110	0.103	0.104	0.102	0.105
		rob.	-0.011	-0.004	-0.002	-0.009	0.016	0.025	0.129	0.126	0.122	0.128	0.136	0.142
	500	trad.	-0.004	0.048	0.064	0.064	0.069	0.069	0.084	0.080	0.080	0.081	0.079	0.081
		rob.	-0.001	-0.003	0.007	-0.000	0.017	0.033	0.093	0.090	0.093	0.100	0.105	0.110
w_{13}	100	trad.	-0.001	-0.078	-0.114	-0.158	-0.176	-0.186	0.208	0.332	0.282	0.222	0.201	0.193
		rob.	0.003	-0.001	-0.019	-0.032	-0.049	-0.066	0.317	0.314	0.303	0.292	0.321	0.454
	300	trad.	-0.001	-0.102	-0.131	-0.154	-0.175	-0.186	0.113	0.136	0.115	0.104	0.100	0.098
		rob.	-0.002	-0.007	-0.004	-0.000	-0.040	-0.078	0.129	0.124	0.122	0.136	0.147	0.160
	500	trad.	0.002	-0.100	-0.130	-0.160	-0.177	-0.182	0.085	0.099	0.090	0.078	0.076	0.073
		rob.	0.000	0.003	-0.007	-0.008	-0.033	-0.081	0.096	0.098	0.098	0.099	0.110	0.113
w_{21}	100	trad.	-0.005	-0.083	-0.131	-0.212	-0.240	-0.253	0.192	0.320	0.288	0.244	0.216	0.223
		rob.	-0.003	-0.029	-0.014	-0.034	-0.038	-0.061	0.282	0.280	0.262	0.271	0.291	0.427
	300	trad.	-0.008	-0.110	-0.159	-0.211	-0.261	-0.266	0.107	0.135	0.129	0.121	0.114	0.118
		rob.	-0.010	-0.006	-0.007	-0.013	-0.049	-0.103	0.123	0.124	0.125	0.133	0.149	0.170
	500	trad.	-0.002	-0.120	-0.162	-0.206	-0.256	-0.264	0.080	0.096	0.098	0.090	0.091	0.087
		rob.	-0.003	-0.004	-0.005	-0.013	-0.046	-0.104	0.089	0.094	0.090	0.097	0.111	0.122
w_{22}	100	trad.	-0.028	-0.002	0.031	0.058	0.077	0.069	0.199	0.218	0.213	0.202	0.195	0.193
		rob.	-0.063	-0.038	-0.031	-0.034	-0.011	-0.060	0.293	0.287	0.281	0.278	0.282	0.304
	300	trad.	0.000	0.051	0.068	0.088	0.087	0.086	0.104	0.111	0.108	0.109	0.109	0.107
		rob.	-0.002	-0.006	-0.007	0.002	0.016	0.035	0.125	0.124	0.124	0.124	0.140	0.146
	500	trad.	-0.007	0.054	0.074	0.083	0.093	0.092	0.083	0.084	0.083	0.082	0.083	0.083
		rob.	-0.008	-0.008	0.001	0.001	0.021	0.051	0.095	0.094	0.094	0.098	0.105	0.111
w_{23}	100	trad.	-0.015	-0.050	-0.056	-0.037	-0.042	-0.031	0.183	0.199	0.215	0.197	0.198	0.197
		rob.	-0.038	-0.048	-0.057	-0.044	-0.078	-0.105	0.267	0.274	0.260	0.255	0.274	0.285
	300	trad.	-0.010	-0.013	-0.013	-0.019	-0.009	-0.009	0.099	0.106	0.105	0.109	0.108	0.107
		rob.	-0.011	-0.010	-0.007	-0.017	-0.013	-0.011	0.119	0.116	0.114	0.120	0.129	0.140
	500	trad.	0.001	-0.003	-0.008	-0.006	-0.009	-0.007	0.075	0.080	0.079	0.081	0.082	0.083
		rob.	0.000	-0.000	-0.008	-0.005	-0.009	-0.012	0.085	0.089	0.086	0.092	0.097	0.107
w_{31}	100	trad.	-0.051	-0.045	-0.081	-0.059	-0.042	-0.066	0.376	0.370	0.365	0.376	0.367	0.364
		rob.	-0.071	-0.100	-0.102	-0.080	-0.071	-0.108	0.462	0.434	0.435	0.446	0.444	0.470
	300	trad.	-0.019	-0.015	-0.008	-0.024	-0.019	-0.016	0.202	0.206	0.202	0.206	0.202	0.207
		rob.	-0.019	-0.024	-0.012	-0.025	-0.033	-0.027	0.233	0.244	0.232	0.240	0.260	0.269
	500	trad.	-0.007	-0.015	-0.015	-0.013	-0.002	-0.011	0.156	0.161	0.160	0.163	0.160	0.162
		rob.	-0.010	-0.015	-0.016	-0.015	-0.008	-0.025	0.172	0.182	0.180	0.184	0.192	0.214
w_{32}	100	trad.	-0.054	-0.061	-0.052	-0.055	-0.074	-0.057	0.309	0.321	0.321	0.316	0.333	0.324
		rob.	-0.123	-0.074	-0.091	-0.093	-0.118	-0.124	0.418	0.395	0.392	0.402	0.393	0.425
	300	trad.	-0.017	-0.022	-0.028	-0.019	-0.019	-0.033	0.174	0.185	0.178	0.183	0.186	0.188
		rob.	-0.030	-0.031	-0.032	-0.025	-0.029	-0.049	0.201	0.218	0.200	0.209	0.238	0.238
	500	trad.	-0.012	-0.010	-0.002	-0.005	-0.017	-0.008	0.138	0.142	0.138	0.144	0.136	0.145
		rob.	-0.015	-0.015	-0.004	-0.008	-0.023	-0.019	0.155	0.159	0.156	0.166	0.170	0.186
w_{33}	100	trad.	-0.085	-0.095	-0.064	-0.081	-0.085	-0.075	0.343	0.358	0.348	0.337	0.355	0.358
		rob.	-0.136	-0.122	-0.092	-0.135	-0.111	-0.127	0.454	0.423	0.416	0.431	0.426	0.451
	300	trad.	-0.017	-0.021	-0.018	-0.014	-0.017	-0.009	0.190	0.196	0.198	0.197	0.187	0.193
		rob.	-0.023	-0.026	-0.026	-0.027	-0.034	-0.021	0.217	0.226	0.226	0.229	0.244	0.247
	500	trad.	-0.012	-0.008	-0.015	-0.016	-0.013	-0.015	0.145	0.153	0.152	0.155	0.149	0.152
		rob.	-0.012	-0.011	-0.021	-0.023	-0.016	-0.015	0.158	0.166	0.171	0.181	0.184	0.196

Table 8.9: Results for traditional and robust PLSc for five common factors and unsystematic outliers in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.005	-0.193	-0.260	-0.313	-0.436	-0.456	0.082	0.562	1.065	1.888	2.446	4.790
		rob.	0.010	0.002	0.001	0.007	0.010	-0.359	0.109	0.107	0.108	0.111	0.103	0.910
	300	trad.	0.002	-0.090	-0.206	-0.273	-0.454	-0.514	0.052	0.240	0.342	0.494	0.910	0.889
		rob.	0.001	0.002	0.004	0.005	0.001	-0.054	0.056	0.059	0.056	0.059	0.064	0.347
	500	trad.	0.001	-0.081	-0.163	-0.265	-0.439	-0.533	0.040	0.196	0.264	0.411	0.508	0.708
		rob.	0.002	0.000	0.003	0.002	0.002	-0.020	0.043	0.043	0.043	0.046	0.049	0.194
γ_{32}	100	trad.	-0.016	-0.072	-0.074	-0.041	-0.078	-0.080	0.164	0.486	0.417	0.524	0.645	0.731
		rob.	-0.054	-0.054	-0.045	-0.038	-0.038	-0.053	0.233	0.243	0.225	0.239	0.220	0.712
	300	trad.	-0.007	-0.037	-0.018	-0.039	-0.057	-0.043	0.079	0.817	0.173	0.193	0.303	0.311
		rob.	-0.004	-0.004	-0.010	-0.011	-0.016	-0.043	0.097	0.096	0.095	0.101	0.109	0.241
	500	trad.	-0.008	-0.007	-0.012	-0.028	-0.039	-0.045	0.061	0.093	0.108	0.133	0.203	0.239
		rob.	-0.002	-0.006	-0.007	-0.009	-0.010	-0.004	0.068	0.068	0.071	0.074	0.078	0.159
γ_{41}	100	trad.	-0.015	0.225	0.234	0.222	-0.084	0.087	0.229	0.633	0.752	0.895	7.039	0.809
		rob.	0.025	0.014	0.013	-0.005	-0.024	0.342	0.319	0.306	0.316	0.308	0.297	2.953
	300	trad.	-0.012	0.024	0.105	0.105	0.151	0.123	0.123	0.453	0.432	0.469	0.486	0.597
		rob.	-0.023	-0.010	-0.017	-0.026	-0.021	-0.032	0.154	0.150	0.150	0.153	0.176	0.606
	500	trad.	-0.005	0.049	0.072	0.112	0.117	0.128	0.098	0.375	0.390	0.392	0.462	0.469
		rob.	-0.005	-0.013	-0.007	-0.016	-0.012	-0.163	0.102	0.111	0.112	0.113	0.129	0.425
γ_{42}	100	trad.	0.019	-0.220	-0.258	-0.330	-0.151	-0.433	0.228	0.632	0.735	0.900	7.034	0.844
		rob.	-0.009	0.001	0.003	0.013	0.030	-0.485	0.304	0.288	0.299	0.295	0.278	2.990
	300	trad.	0.012	-0.063	-0.122	-0.159	-0.234	-0.300	0.126	0.413	0.371	0.419	0.445	0.606
		rob.	0.019	0.014	0.020	0.029	0.024	-0.194	0.153	0.147	0.153	0.153	0.172	0.600
	500	trad.	0.006	-0.069	-0.092	-0.154	-0.180	-0.220	0.098	0.345	0.334	0.334	0.397	0.432
		rob.	0.007	0.013	0.011	0.019	0.014	-0.067	0.106	0.111	0.109	0.116	0.129	0.412
γ_{51}	100	trad.	0.007	-0.172	-0.243	-0.268	-0.358	-0.772	0.223	0.728	0.980	1.161	1.111	14.454
		rob.	-0.004	0.004	-0.009	0.025	0.005	-0.390	0.322	0.302	0.309	0.321	0.290	2.423
	300	trad.	-0.008	-0.017	-0.134	-0.159	-0.268	-0.241	0.116	0.497	0.492	0.502	0.455	0.807
		rob.	-0.003	0.002	-0.000	0.001	0.004	0.027	0.142	0.140	0.143	0.137	0.159	0.638
	500	trad.	-0.015	-0.033	-0.074	-0.178	-0.231	-0.277	0.086	0.419	0.518	0.394	0.459	0.424
		rob.	-0.011	-0.008	-0.006	-0.003	-0.011	0.095	0.098	0.099	0.100	0.102	0.119	0.460
γ_{52}	100	trad.	-0.045	0.159	0.243	0.305	0.463	-0.002	0.214	0.973	1.359	1.256	1.271	14.462
		rob.	-0.045	-0.051	-0.035	-0.075	-0.056	0.419	0.318	0.308	0.310	0.313	0.295	2.532
	300	trad.	-0.015	0.006	0.103	0.161	0.281	0.341	0.113	0.452	0.440	0.468	0.444	0.964
		rob.	-0.026	-0.025	-0.020	-0.026	-0.033	0.104	0.140	0.140	0.143	0.137	0.157	0.640
	500	trad.	-0.005	0.025	0.058	0.171	0.239	0.312	0.086	0.390	0.476	0.342	0.420	0.437
		rob.	-0.011	-0.014	-0.015	-0.020	-0.015	0.038	0.099	0.096	0.098	0.102	0.119	0.452
γ_{53}	100	trad.	-0.015	-0.010	-0.020	-0.002	-0.006	-0.149	0.116	0.698	1.015	0.902	0.610	2.334
		rob.	-0.000	0.008	0.007	-0.005	0.003	0.086	0.195	0.187	0.178	0.195	0.169	1.343
	300	trad.	-0.038	-0.031	-0.041	-0.048	-0.052	-0.033	0.061	0.116	0.127	0.150	0.198	0.534
		rob.	-0.035	-0.031	-0.035	-0.037	-0.032	-0.017	0.070	0.071	0.070	0.074	0.081	0.174
	500	trad.	-0.045	-0.040	-0.045	-0.052	-0.053	-0.053	0.046	0.079	0.089	0.097	0.140	0.187
		rob.	-0.044	-0.043	-0.042	-0.043	-0.040	-0.027	0.051	0.054	0.055	0.055	0.061	0.126
λ_{11}	100	trad.	0.007	-0.170	-0.273	-0.368	-0.457	-0.484	0.059	0.276	0.451	0.417	0.417	0.397
		rob.	0.016	0.014	0.012	0.014	0.011	-0.179	0.076	0.078	0.076	0.078	0.073	0.528
	300	trad.	-0.000	-0.250	-0.357	-0.413	-0.519	-0.536	0.036	0.179	0.269	0.244	0.297	0.326
		rob.	-0.000	-0.000	0.002	0.000	0.002	-0.065	0.042	0.043	0.042	0.044	0.048	0.186
	500	trad.	-0.001	-0.278	-0.379	-0.454	-0.516	-0.550	0.030	0.136	0.148	0.194	0.260	0.281
		rob.	0.000	0.000	0.002	0.001	0.001	-0.046	0.032	0.031	0.033	0.034	0.037	0.125
λ_{12}	100	trad.	-0.010	-0.237	-0.214	-0.457	-0.553	-0.573	0.056	0.361	3.704	0.378	0.447	0.426
		rob.	-0.018	-0.020	-0.019	-0.018	-0.013	-0.262	0.069	0.066	0.071	0.069	0.065	0.416
	300	trad.	-0.000	-0.285	-0.387	-0.488	-0.571	-0.606	0.036	0.178	0.208	0.263	0.303	0.317
		rob.	-0.000	0.000	-0.002	0.002	-0.002	-0.127	0.040	0.042	0.042	0.042	0.045	0.177
	500	trad.	-0.002	-0.307	-0.403	-0.513	-0.596	-0.619	0.029	0.151	0.172	0.208	0.254	0.271
		rob.	0.000	0.000	-0.000	-0.001	-0.003	-0.091	0.032	0.031	0.032	0.032	0.036	0.131
λ_{21}	100	trad.	-0.003	-0.114	-0.205	-0.255	-0.291	-0.297	0.102	0.276	0.287	0.285	0.251	0.249
		rob.	0.000	-0.011	-0.005	-0.009	-0.003	-0.201	0.148	0.152	0.150	0.148	0.137	0.399
	300	trad.	-0.005	-0.141	-0.196	-0.243	-0.303	-0.319	0.062	0.131	0.132	0.137	0.158	0.159
		rob.	-0.000	-0.002	-0.003	0.004	-0.006	-0.144	0.069	0.071	0.072	0.071	0.087	0.244
	500	trad.	0.001	-0.148	-0.198	-0.253	-0.304	-0.313	0.048	0.092	0.092	0.097	0.116	0.128
		rob.	-0.001	-0.004	0.000	-0.004	-0.005	-0.124	0.054	0.053	0.052	0.057	0.062	0.172
λ_{22}	100	trad.	0.001	-0.130	-0.187	-0.241	-0.297	-0.314	0.107	0.276	0.301	0.278	0.247	0.240
		rob.	0.000	-0.000	-0.009	-0.002	-0.006	-0.223	0.144	0.154	0.146	0.139	0.140	0.406
	300	trad.	0.000	-0.142	-0.190	-0.239	-0.302	-0.317	0.061	0.123	0.129	0.134	0.157	0.162
		rob.	-0.003	-0.001	-0.005	-0.003	-0.004	-0.148	0.067	0.071	0.070	0.075	0.083	0.246
	500	trad.	-0.001	-0.151	-0.197	-0.251	-0.304	-0.323	0.047	0.091	0.091	0.098	0.118	0.122
		rob.	-0.003	-0.002	0.000	-0.002	-0.000	-0.138	0.053	0.052	0.056	0.058	0.064	0.173
λ_{23}	100	trad.	-0.001	-0.100	-0.187	-0.244	-0.298	-0.306	0.104	0.264	0.295	0.277	0.255	0.241
		rob.	-0.005	0.005	-0.001	-0.005	-0.003	-0.198	0.151	0.152	0.143	0.152	0.144	0.411
	300	trad.	-0.001	-0.139	-0.187	-0.244	-0.298	-0.326	0.061	0.127	0.130	0.138	0.156	0.156
		rob.	-0.002	-0.004	-0.005	-0.004	-0.003	-0.138	0.072	0.072	0.072	0.072	0.081	0.249
	500	trad.	-0.003	-0.145	-0.202	-0.247	-0.296	-0.321	0.049	0.094	0.092	0.104	0.119	0.124
		rob.	-0.002	-0.001	-0.004	-0.003	-0.002	-0.118	0.052	0.052	0.053	0.056	0.062	0.171
λ_{24}	100	trad.	-0.008	-0.187	-0.292	-0.386	-0.465	-0.492	0.090	0.289	0.304	0.284	0.255	0.246
		rob.	0.000	-0.003	-0.009	-0.009	0.002	-0.347	0.122	0.121	0.120	0.115	0.109	0.410
	300	trad.	-0.003	-0.206	-0.285	-0.372	-0.448	-0.488	0.051	0.129	0.140	0.141	0.153	0.166
		rob.	-0.005	-0.005	-0.005	-0.005	-0.008	-0.184	0.056	0.057	0.058	0.057	0.066	0.237
	500	trad.	-0.003	-0.215	-0.297	-0.373	-0.449	-0.487	0.039	0.096	0.096	0.103	0.121	0.122
		rob.	-0.000	-0.003	-0.004	-0.003	-0.003	-0.166	0.041	0.045	0.044	0.044	0.052	0.164
λ_{25}	100	trad.	-0.007	-0.189	-0.269	-0.387	-0.460	-0.486	0.085	0.282	0.295	0.282	0.267	0.250
		rob.	0.002	-0.003	-0.008	-0.006	0.003	-0.323	0.117	0.118	0.121	0.118	0.113	0.416
	300	trad.	-0.002	-0.214	-0.290	-0.370	-0.454	-0.488	0.050	0.129	0.138	0.144	0.158	0.164
		rob.	-0.005	-0.003	-0.004	-0.006	-0.005	-0.175	0.056	0.058	0.060	0.063	0.065	0.246
	500	trad.	-0.002	-0.217	-0.294	-0.381	-0.455	-0.479	0.038	0.096	0.093	0.103	0.121	0.124
		rob.	-0.001	-0.001	-0.002	-0.004	-0.007	-0.162	0.042	0.043	0.043	0.044	0.050	0.165
λ_{26}	100	trad.	-0.011	-0.188	-0.290	-0.376	-0.460	-0.488	0.086	0.279	0.292	0.273	0.255	0.247
		rob.	-0.002	0.001	0.000	-0.003	-0.006	-0.362	0.118	0.120	0.121	0.120	0.117	0.403
	300	trad.	-0.003	-0.210	-0.283	-0.365	-0.448	-0.490	0.048	0.128	0.135	0.141	0.156	0.156
		rob.	-0.006	-0.005	-0.002	-0.004	-0.008	-0.177	0.058	0.060	0.057	0.060	0.067	0.257
	500	trad.	0.001	-0.220	-0.296	-0.369	-0.453	-0.483	0.038	0.097	0.092	0.105	0.120	0.126
		rob.	-0.000	-0.006	-0.002	-0.003	-0.004	-0.160	0.043	0.043	0.042	0.046	0.052	0.169

λ_{31}	100	trad.	-0.014	-0.103	-0.157	-0.254	-0.320	-0.331	0.282	0.415	0.418	0.396	0.342	0.336
		rob.	-0.012	-0.027	-0.020	-0.018	-0.031	-0.206	0.332	0.344	0.341	0.322	0.340	0.518
	300	trad.	0.001	-0.136	-0.207	-0.289	-0.343	-0.369	0.177	0.233	0.250	0.223	0.212	0.213
		rob.	0.007	-0.002	0.001	0.001	-0.016	-0.153	0.186	0.201	0.201	0.202	0.229	0.366
	500	trad.	-0.008	-0.140	-0.222	-0.287	-0.331	-0.357	0.129	0.174	0.179	0.168	0.159	0.160
		rob.	-0.006	-0.005	-0.004	-0.008	0.001	-0.146	0.148	0.153	0.147	0.152	0.176	0.287
λ_{32}	100	trad.	-0.025	-0.105	-0.107	-0.091	-0.064	-0.038	0.220	0.244	0.239	0.235	0.231	0.210
		rob.	-0.056	-0.082	-0.064	-0.083	-0.071	-0.143	0.281	0.294	0.291	0.289	0.274	0.268
	300	trad.	-0.006	-0.024	-0.033	-0.016	-0.014	-0.020	0.130	0.144	0.147	0.133	0.127	0.135
		rob.	-0.003	-0.010	-0.008	-0.009	-0.010	-0.069	0.156	0.156	0.147	0.162	0.180	0.187
	500	trad.	-0.001	-0.013	-0.015	-0.013	-0.009	-0.006	0.104	0.105	0.107	0.100	0.104	0.099
		rob.	-0.003	-0.012	-0.002	0.003	-0.003	-0.041	0.112	0.118	0.116	0.128	0.140	0.159
λ_{33}	100	trad.	-0.086	-0.169	-0.163	-0.148	-0.116	-0.118	0.180	0.225	0.233	0.229	0.189	0.202
		rob.	-0.145	-0.149	-0.140	-0.143	-0.127	-0.241	0.239	0.246	0.235	0.251	0.231	0.277
	300	trad.	-0.025	-0.047	-0.050	-0.044	-0.034	-0.035	0.104	0.125	0.129	0.108	0.103	0.100
		rob.	-0.029	-0.034	-0.034	-0.034	-0.043	-0.110	0.114	0.127	0.116	0.125	0.141	0.183
	500	trad.	-0.009	-0.024	-0.023	-0.017	-0.019	-0.021	0.081	0.090	0.090	0.081	0.078	0.077
		rob.	-0.014	-0.011	-0.013	-0.019	-0.025	-0.070	0.088	0.087	0.087	0.089	0.104	0.146
λ_{34}	100	trad.	-0.111	-0.215	-0.217	-0.199	-0.154	-0.159	0.151	0.219	0.232	0.229	0.182	0.183
		rob.	-0.191	-0.191	-0.188	-0.196	-0.181	-0.290	0.218	0.233	0.217	0.252	0.221	0.282
	300	trad.	-0.044	-0.070	-0.071	-0.061	-0.056	-0.057	0.085	0.116	0.123	0.098	0.089	0.089
		rob.	-0.054	-0.057	-0.053	-0.065	-0.068	-0.149	0.093	0.097	0.091	0.104	0.108	0.184
	500	trad.	-0.023	-0.039	-0.036	-0.034	-0.033	-0.035	0.061	0.079	0.077	0.069	0.068	0.068
		rob.	-0.029	-0.027	-0.032	-0.035	-0.043	-0.092	0.072	0.072	0.074	0.077	0.085	0.140
λ_{41}	100	trad.	-0.026	0.014	0.013	0.033	0.033	0.002	0.096	0.213	0.180	0.321	0.510	0.447
		rob.	-0.013	-0.002	-0.002	-0.011	-0.018	0.018	0.126	0.128	0.122	0.117	0.118	0.340
	300	trad.	-0.031	-0.026	-0.016	0.000	0.038	0.044	0.053	0.067	0.078	0.100	0.199	0.181
		rob.	-0.032	-0.029	-0.030	-0.032	-0.032	0.004	0.064	0.062	0.064	0.065	0.069	0.108
	500	trad.	-0.028	-0.030	-0.025	-0.014	0.024	0.039	0.043	0.050	0.060	0.082	0.119	0.158
		rob.	-0.030	-0.031	-0.031	-0.032	-0.033	-0.017	0.045	0.047	0.047	0.051	0.053	0.081
λ_{42}	100	trad.	-0.030	-0.062	-0.073	-0.126	-0.117	-0.134	0.090	0.193	0.257	0.644	0.414	0.578
		rob.	-0.041	-0.046	-0.045	-0.040	-0.035	-0.107	0.111	0.113	0.111	0.117	0.109	0.398
	300	trad.	-0.028	-0.028	-0.042	-0.056	-0.090	-0.114	0.057	0.075	0.087	0.108	0.173	0.194
		rob.	-0.029	-0.031	-0.031	-0.034	-0.032	-0.054	0.066	0.064	0.066	0.068	0.073	0.115
	500	trad.	-0.029	-0.028	-0.031	-0.039	-0.073	-0.098	0.044	0.059	0.070	0.092	0.126	0.146
		rob.	-0.030	-0.030	-0.029	-0.028	-0.028	-0.041	0.046	0.049	0.049	0.054	0.059	0.090
λ_{51}	100	trad.	0.002	0.003	-0.008	-0.001	-0.002	0.001	0.130	0.173	0.192	0.206	0.216	0.204
		rob.	-0.002	-0.013	-0.002	-0.001	-0.004	-0.029	0.178	0.182	0.181	0.180	0.168	0.272
	300	trad.	-0.003	-0.002	0.000	0.001	0.010	0.005	0.077	0.095	0.104	0.118	0.126	0.131
		rob.	-0.003	-0.001	-0.000	0.001	-0.002	-0.002	0.088	0.088	0.092	0.095	0.105	0.155
	500	trad.	-0.001	-0.005	0.001	-0.004	-0.001	-0.002	0.061	0.074	0.079	0.091	0.103	0.111
		rob.	-0.002	0.004	-0.003	0.000	-0.000	-0.002	0.064	0.068	0.069	0.070	0.078	0.112
λ_{52}	100	trad.	-0.004	-0.002	0.002	-0.001	-0.011	0.007	0.137	0.169	0.187	0.207	0.218	0.201
		rob.	0.002	-0.011	-0.006	-0.011	-0.009	-0.029	0.183	0.188	0.184	0.176	0.170	0.267
	300	trad.	0.001	-0.004	-0.002	-0.002	-0.003	0.005	0.076	0.093	0.109	0.121	0.125	0.136
		rob.	0.001	-0.003	-0.005	-0.002	-0.002	-0.001	0.089	0.088	0.090	0.091	0.099	0.147
	500	trad.	-0.004	-0.000	-0.001	-0.002	0.003	0.001	0.063	0.072	0.082	0.092	0.107	0.108
		rob.	-0.003	-0.001	0.001	-0.002	-0.002	-0.005	0.065	0.066	0.067	0.072	0.079	0.115
λ_{53}	100	trad.	-0.003	-0.008	-0.019	-0.024	-0.015	-0.027	0.118	0.142	0.168	0.173	0.183	0.186
		rob.	-0.009	-0.013	-0.023	-0.007	-0.016	-0.063	0.154	0.151	0.151	0.151	0.149	0.230
	300	trad.	0.001	0.000	0.001	-0.003	-0.001	-0.002	0.068	0.082	0.088	0.102	0.112	0.112
		rob.	-0.006	-0.004	-0.008	-0.005	-0.008	-0.005	0.079	0.079	0.081	0.080	0.087	0.129
	500	trad.	-0.003	-0.005	-0.004	-0.000	-0.006	-0.003	0.053	0.063	0.069	0.083	0.091	0.091
		rob.	0.001	-0.005	-0.005	-0.005	-0.001	-0.002	0.059	0.057	0.060	0.062	0.066	0.099
λ_{54}	100	trad.	-0.009	-0.012	-0.014	-0.014	-0.027	-0.021	0.116	0.155	0.166	0.173	0.179	0.178
		rob.	-0.010	-0.003	-0.011	-0.019	-0.007	-0.056	0.150	0.149	0.157	0.164	0.150	0.238
	300	trad.	-0.003	0.002	-0.001	-0.003	-0.008	-0.004	0.070	0.084	0.091	0.102	0.113	0.120
		rob.	-0.006	0.000	-0.004	-0.005	-0.006	-0.005	0.078	0.079	0.077	0.082	0.087	0.133
	500	trad.	-0.002	-0.004	0.001	-0.001	-0.002	-0.003	0.054	0.065	0.070	0.078	0.091	0.091
		rob.	-0.004	0.000	-0.004	-0.001	-0.002	0.001	0.058	0.058	0.058	0.064	0.068	0.097
λ_{55}	100	trad.	-0.024	-0.048	-0.062	-0.069	-0.071	-0.075	0.066	0.100	0.125	0.110	0.117	0.120
		rob.	-0.048	-0.047	-0.045	-0.049	-0.037	-0.117	0.095	0.093	0.093	0.098	0.087	0.198
	300	trad.	-0.008	-0.014	-0.019	-0.021	-0.030	-0.037	0.044	0.053	0.057	0.063	0.070	0.075
		rob.	-0.008	-0.010	-0.008	-0.010	-0.013	-0.033	0.048	0.049	0.051	0.049	0.055	0.088
	500	trad.	-0.004	-0.005	-0.010	-0.014	-0.017	-0.023	0.034	0.042	0.046	0.055	0.056	0.060
		rob.	-0.003	-0.005	-0.004	-0.006	-0.008	-0.022	0.038	0.038	0.038	0.039	0.042	0.061
λ_{56}	100	trad.	-0.022	-0.048	-0.061	-0.066	-0.082	-0.073	0.068	0.097	0.123	0.120	0.129	0.124
		rob.	-0.046	-0.048	-0.046	-0.046	-0.043	-0.107	0.091	0.091	0.092	0.094	0.089	0.190
	300	trad.	-0.005	-0.015	-0.017	-0.022	-0.028	-0.034	0.044	0.051	0.056	0.064	0.070	0.074
		rob.	-0.005	-0.012	-0.008	-0.008	-0.014	-0.039	0.049	0.049	0.049	0.047	0.054	0.091
	500	trad.	-0.005	-0.005	-0.010	-0.014	-0.021	-0.017	0.036	0.040	0.044	0.052	0.058	0.058
		rob.	-0.005	-0.006	-0.002	-0.004	-0.008	-0.020	0.037	0.039	0.037	0.040	0.043	0.063

Table 8.10: Results for traditional and robust PLSc for five common factors and unsystematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.007	0.008	0.015	0.026	0.025	0.010	0.082	0.111	0.114	0.122	0.127	0.127
		rob.	0.012	0.007	0.007	0.004	0.005	-0.005	0.110	0.108	0.108	0.110	0.125	0.154
	300	trad.	0.002	0.005	0.007	0.023	0.038	0.044	0.051	0.071	0.087	0.099	0.111	0.119
		rob.	0.005	0.006	0.001	0.006	0.004	0.036	0.056	0.058	0.057	0.061	0.071	0.105
	500	trad.	0.001	0.006	0.008	0.018	0.041	0.045	0.039	0.060	0.070	0.093	0.108	0.113
		rob.	0.003	0.003	0.001	-0.001	0.004	0.028	0.043	0.044	0.045	0.045	0.054	0.088
γ_{32}	100	trad.	-0.025	-0.029	-0.022	-0.025	-0.018	-0.021	0.159	0.161	0.162	0.159	0.163	0.163
		rob.	-0.046	-0.036	-0.029	-0.045	-0.045	-0.041	0.237	0.243	0.234	0.229	0.230	0.248
	300	trad.	-0.014	-0.007	-0.008	-0.007	-0.010	-0.004	0.077	0.080	0.081	0.083	0.082	0.077
		rob.	-0.012	-0.007	-0.009	-0.013	-0.009	-0.012	0.092	0.091	0.094	0.094	0.108	0.103
	500	trad.	-0.009	-0.004	-0.001	-0.006	-0.002	-0.004	0.062	0.063	0.059	0.062	0.063	0.061
		rob.	-0.002	-0.004	-0.006	-0.010	-0.006	-0.008	0.068	0.070	0.069	0.070	0.077	0.076
γ_{41}	100	trad.	-0.020	-0.006	0.011	0.020	0.045	0.030	0.222	0.302	0.344	0.371	0.404	0.373
		rob.	-0.018	-0.000	0.008	0.006	-0.004	0.025	0.341	0.311	0.309	0.300	0.378	0.436
	300	trad.	-0.013	-0.017	-0.023	-0.007	0.014	0.010	0.122	0.164	0.199	0.240	0.296	0.334
		rob.	-0.012	-0.018	-0.015	-0.026	-0.026	-0.044	0.145	0.155	0.156	0.159	0.185	0.314
	500	trad.	-0.010	-0.013	-0.013	-0.016	-0.004	-0.005	0.097	0.122	0.132	0.182	0.253	0.257
		rob.	-0.009	-0.011	-0.007	-0.009	-0.013	-0.048	0.108	0.111	0.110	0.120	0.136	0.208
γ_{42}	100	trad.	0.022	0.020	0.008	-0.002	-0.028	-0.020	0.220	0.279	0.318	0.343	0.375	0.349
		rob.	0.021	0.012	-0.004	0.005	0.023	-0.008	0.320	0.294	0.295	0.280	0.345	0.391
	300	trad.	0.013	0.016	0.025	0.009	-0.004	-0.003	0.125	0.156	0.192	0.228	0.282	0.320
		rob.	0.014	0.019	0.018	0.029	0.028	0.042	0.144	0.157	0.153	0.160	0.177	0.295
	500	trad.	0.011	0.014	0.010	0.016	0.006	0.008	0.099	0.114	0.125	0.173	0.243	0.245
		rob.	0.012	0.010	0.008	0.008	0.010	0.048	0.105	0.108	0.108	0.120	0.133	0.197
γ_{51}	100	trad.	0.017	0.049	0.042	0.062	0.057	0.041	0.219	0.311	0.357	0.434	0.419	0.434
		rob.	0.016	0.020	0.002	0.005	0.018	-0.031	0.344	0.329	0.313	0.328	0.368	0.459
	300	trad.	-0.007	0.019	0.031	0.062	0.099	0.145	0.112	0.177	0.208	0.284	0.384	0.430
		rob.	0.005	-0.000	-0.006	-0.000	0.013	0.088	0.143	0.142	0.136	0.152	0.174	0.331
	500	trad.	-0.010	-0.001	0.009	0.044	0.125	0.142	0.087	0.123	0.157	0.265	0.384	0.391
		rob.	-0.011	-0.004	-0.009	-0.016	0.002	0.075	0.096	0.104	0.100	0.106	0.124	0.255
γ_{52}	100	trad.	-0.051	-0.084	-0.083	-0.102	-0.098	-0.078	0.219	0.298	0.340	0.423	0.398	0.409
		rob.	-0.059	-0.064	-0.051	-0.050	-0.066	-0.037	0.355	0.334	0.316	0.330	0.351	0.430
	300	trad.	-0.016	-0.039	-0.055	-0.086	-0.124	-0.167	0.110	0.164	0.201	0.273	0.373	0.417
		rob.	-0.031	-0.025	-0.018	-0.028	-0.039	-0.116	0.144	0.140	0.133	0.149	0.165	0.315
	500	trad.	-0.012	-0.020	-0.031	-0.068	-0.147	-0.162	0.087	0.117	0.146	0.260	0.380	0.382
		rob.	-0.013	-0.018	-0.013	-0.007	-0.023	-0.094	0.094	0.102	0.095	0.102	0.122	0.242
γ_{53}	100	trad.	-0.023	-0.024	-0.016	-0.020	-0.015	-0.019	0.124	0.123	0.130	0.138	0.132	0.136
		rob.	0.012	0.008	0.008	-0.007	0.001	-0.008	0.193	0.193	0.188	0.188	0.177	0.203
	300	trad.	-0.042	-0.043	-0.041	-0.039	-0.043	-0.040	0.059	0.065	0.066	0.069	0.073	0.079
		rob.	-0.037	-0.036	-0.036	-0.036	-0.032	-0.038	0.070	0.070	0.072	0.071	0.083	0.091
	500	trad.	-0.044	-0.044	-0.044	-0.047	-0.044	-0.047	0.046	0.050	0.048	0.051	0.059	0.061
		rob.	-0.041	-0.045	-0.042	-0.042	-0.040	-0.043	0.051	0.053	0.054	0.053	0.063	0.063
λ_{11}	100	trad.	0.006	-0.011	-0.018	-0.035	-0.036	-0.028	0.061	0.100	0.113	0.123	0.126	0.130
		rob.	0.017	0.016	0.017	0.009	-0.020	-0.039	0.077	0.076	0.078	0.080	0.113	0.143
	300	trad.	-0.001	-0.001	-0.001	-0.021	-0.029	-0.036	0.038	0.070	0.087	0.105	0.116	0.119
		rob.	0.002	0.003	0.001	-0.001	-0.003	-0.025	0.042	0.041	0.043	0.044	0.058	0.102
	500	trad.	0.001	-0.002	-0.001	-0.007	-0.030	-0.038	0.029	0.055	0.072	0.096	0.111	0.117
		rob.	0.001	0.000	0.000	0.000	0.002	-0.015	0.031	0.033	0.033	0.034	0.042	0.088
λ_{12}	100	trad.	-0.007	-0.337	-0.465	-0.591	-0.725	-0.759	0.054	0.126	0.129	0.132	0.156	0.153
		rob.	-0.020	-0.021	-0.025	-0.032	-0.283	-0.638	0.070	0.068	0.072	0.079	0.220	0.245
	300	trad.	0.003	-0.364	-0.498	-0.626	-0.731	-0.768	0.036	0.079	0.072	0.074	0.075	0.083
		rob.	-0.000	-0.006	-0.008	-0.020	-0.112	-0.510	0.041	0.039	0.044	0.046	0.079	0.153
	500	trad.	0.000	-0.366	-0.506	-0.633	-0.742	-0.777	0.030	0.061	0.056	0.057	0.056	0.059
		rob.	0.000	-0.004	-0.006	-0.020	-0.105	-0.471	0.031	0.032	0.033	0.037	0.055	0.125
λ_{21}	100	trad.	-0.003	-0.166	-0.219	-0.279	-0.337	-0.351	0.105	0.140	0.134	0.134	0.129	0.126
		rob.	-0.005	-0.008	-0.011	-0.014	-0.177	-0.312	0.158	0.148	0.148	0.148	0.204	0.196
	300	trad.	-0.003	-0.160	-0.224	-0.286	-0.333	-0.348	0.059	0.084	0.083	0.074	0.072	0.075
		rob.	-0.001	-0.004	-0.006	-0.021	-0.081	-0.270	0.070	0.074	0.074	0.077	0.099	0.134
	500	trad.	-0.000	-0.163	-0.223	-0.287	-0.338	-0.347	0.049	0.065	0.061	0.059	0.059	0.061
		rob.	-0.003	-0.005	-0.009	-0.020	-0.075	-0.250	0.052	0.051	0.055	0.056	0.072	0.107
λ_{22}	100	trad.	0.001	-0.007	-0.003	-0.009	-0.003	-0.005	0.108	0.112	0.112	0.109	0.112	0.109
		rob.	-0.008	0.006	0.005	-0.001	-0.012	-0.000	0.146	0.147	0.146	0.140	0.138	0.150
	300	trad.	-0.005	-0.004	-0.005	-0.002	-0.002	-0.000	0.063	0.062	0.064	0.064	0.064	0.061
		rob.	-0.006	-0.001	-0.004	-0.003	-0.001	-0.006	0.072	0.073	0.075	0.072	0.080	0.075
	500	trad.	-0.001	-0.003	-0.001	-0.001	-0.004	-0.002	0.047	0.050	0.050	0.050	0.049	0.049
		rob.	-0.003	-0.001	-0.003	-0.001	-0.001	-0.005	0.053	0.053	0.053	0.058	0.060	0.061
λ_{23}	100	trad.	-0.004	-0.006	0.002	-0.005	-0.008	-0.006	0.110	0.108	0.111	0.112	0.112	0.111
		rob.	-0.012	-0.010	0.004	0.001	-0.003	-0.004	0.157	0.153	0.147	0.148	0.144	0.156
	300	trad.	0.002	-0.004	-0.003	-0.003	-0.002	-0.001	0.061	0.065	0.062	0.065	0.064	0.066
		rob.	-0.002	-0.002	-0.003	0.001	-0.005	-0.006	0.072	0.070	0.072	0.071	0.082	0.078
	500	trad.	-0.002	-0.004	0.002	-0.003	0.002	-0.001	0.048	0.049	0.047	0.050	0.049	0.050
		rob.	-0.000	-0.002	-0.001	-0.002	-0.004	-0.002	0.052	0.054	0.054	0.055	0.061	0.061
λ_{24}	100	trad.	-0.006	-0.009	-0.008	-0.007	-0.007	-0.006	0.086	0.092	0.093	0.088	0.095	0.092
		rob.	-0.002	-0.001	-0.001	-0.006	-0.011	-0.013	0.122	0.116	0.123	0.117	0.117	0.121
	300	trad.	-0.005	-0.005	-0.005	-0.004	-0.005	-0.004	0.049	0.051	0.054	0.052	0.050	0.053
		rob.	-0.006	-0.006	-0.002	-0.004	-0.006	-0.008	0.056	0.058	0.059	0.057	0.061	0.066
	500	trad.	-0.001	-0.001	-0.004	-0.001	-0.004	-0.003	0.039	0.041	0.041	0.042	0.040	0.040
		rob.	-0.002	-0.002	-0.001	-0.002	-0.004	-0.008	0.043	0.044	0.043	0.046	0.050	0.049
λ_{25}	100	trad.	-0.009	-0.011	-0.008	-0.009	-0.007	-0.006	0.090	0.089	0.094	0.092	0.088	0.091
		rob.	-0.010	-0.003	-0.013	-0.008	-0.004	-0.010	0.123	0.120	0.115	0.114	0.118	0.129
	300	trad.	-0.003	-0.005	-0.006	-0.002	-0.002	-0.003	0.049	0.053	0.053	0.054	0.051	0.053
		rob.	-0.004	-0.003	-0.006	-0.002	-0.007	-0.011	0.058	0.056	0.058	0.058	0.066	0.064
	500	trad.	-0.002	-0.003	-0.004	-0.003	-0.002	-0.002	0.040	0.041	0.040	0.040	0.041	0.042
		rob.	-0.000	-0.001	-0.004	0.001	-0.003	-0.009	0.043	0.043	0.044	0.045	0.052	0.050
λ_{26}	100	trad.	-0.001	-0.003	-0.004	-0.014	-0.012	-0.010	0.088	0.088	0.091	0.095	0.088	0.092
		rob.	-0.013	-0.003	0.003	-0.003	0.000	-0.013	0.125	0.126	0.115	0.114	0.116	0.126
	300	trad.	-0.003	-0.005	-0.005	-0.005	0.000	-0.004	0.050	0.052	0.054	0.054	0.052	0.053
		rob.	-0.004	-0.005	-0.003	-0.005	-0.007	-0.008	0.057	0.056	0.059	0.063	0.066	0.062
	500	trad.	-0.002	-0.002	-0.003	-0.000	-0.004	-0.003	0.039	0.040	0.043	0.040	0.040	0.040
		rob.	-0.003	-0.004	-0.002	-0.003	-0.005	-0.005	0.043	0.044	0.042	0.047	0.049	0.049

λ_{31}	100	trad.	-0.015	-0.034	-0.014	-0.006	-0.018	-0.021	0.287	0.293	0.287	0.268	0.281	0.281
		rob.	-0.030	-0.032	-0.010	-0.040	-0.011	-0.004	0.344	0.334	0.334	0.350	0.317	0.334
	300	trad.	-0.006	-0.002	-0.001	-0.005	-0.000	-0.005	0.170	0.173	0.165	0.182	0.171	0.176
		rob.	-0.004	-0.001	0.008	-0.007	-0.009	-0.003	0.196	0.198	0.195	0.211	0.218	0.205
	500	trad.	0.005	0.003	0.001	0.001	-0.007	0.004	0.132	0.135	0.130	0.131	0.130	0.134
		rob.	-0.001	-0.005	-0.003	0.006	0.008	0.008	0.141	0.150	0.155	0.153	0.165	0.158
λ_{32}	100	trad.	-0.026	-0.029	-0.039	-0.028	-0.036	-0.036	0.231	0.230	0.222	0.227	0.240	0.247
		rob.	-0.094	-0.069	-0.064	-0.057	-0.065	-0.071	0.292	0.293	0.284	0.274	0.278	0.282
	300	trad.	-0.011	-0.007	-0.004	-0.005	-0.002	-0.010	0.136	0.136	0.135	0.139	0.131	0.134
		rob.	-0.008	-0.005	-0.008	-0.006	-0.014	-0.012	0.154	0.154	0.157	0.158	0.177	0.164
	500	trad.	-0.004	-0.001	-0.004	-0.008	-0.003	0.000	0.101	0.100	0.104	0.102	0.105	0.107
		rob.	0.004	-0.009	-0.003	-0.009	-0.004	-0.005	0.115	0.115	0.117	0.121	0.129	0.129
λ_{33}	100	trad.	-0.092	-0.086	-0.083	-0.074	-0.092	-0.079	0.195	0.187	0.190	0.181	0.189	0.182
		rob.	-0.143	-0.144	-0.147	-0.134	-0.140	-0.134	0.242	0.247	0.238	0.226	0.245	0.228
	300	trad.	-0.019	-0.020	-0.020	-0.026	-0.026	-0.017	0.103	0.105	0.102	0.104	0.104	0.102
		rob.	-0.032	-0.032	-0.035	-0.037	-0.037	-0.033	0.111	0.118	0.119	0.129	0.135	0.125
	500	trad.	-0.013	-0.017	-0.014	-0.008	-0.008	-0.017	0.079	0.082	0.082	0.078	0.077	0.083
		rob.	-0.020	-0.008	-0.017	-0.016	-0.023	-0.026	0.088	0.088	0.095	0.093	0.098	0.100
λ_{34}	100	trad.	-0.126	-0.126	-0.123	-0.116	-0.114	-0.124	0.174	0.174	0.164	0.154	0.160	0.168
		rob.	-0.199	-0.198	-0.193	-0.191	-0.181	-0.174	0.235	0.235	0.228	0.229	0.216	0.210
	300	trad.	-0.039	-0.043	-0.046	-0.043	-0.042	-0.043	0.078	0.085	0.084	0.084	0.081	0.083
		rob.	-0.049	-0.055	-0.056	-0.058	-0.067	-0.061	0.094	0.094	0.095	0.098	0.110	0.099
	500	trad.	-0.023	-0.023	-0.023	-0.022	-0.022	-0.025	0.063	0.065	0.065	0.065	0.068	0.064
		rob.	-0.030	-0.029	-0.031	-0.035	-0.038	-0.037	0.069	0.072	0.074	0.074	0.079	0.079
λ_{41}	100	trad.	-0.028	-0.020	-0.021	-0.019	-0.023	-0.022	0.090	0.094	0.092	0.096	0.099	0.095
		rob.	-0.008	0.001	-0.009	-0.014	-0.009	-0.007	0.125	0.119	0.121	0.118	0.117	0.124
	300	trad.	-0.029	-0.029	-0.032	-0.031	-0.031	-0.032	0.056	0.055	0.053	0.058	0.056	0.055
		rob.	-0.034	-0.027	-0.029	-0.029	-0.026	-0.030	0.064	0.059	0.061	0.063	0.068	0.067
	500	trad.	-0.032	-0.030	-0.030	-0.029	-0.032	-0.029	0.041	0.041	0.042	0.043	0.042	0.043
		rob.	-0.029	-0.031	-0.030	-0.031	-0.031	-0.030	0.047	0.046	0.046	0.050	0.052	0.053
λ_{42}	100	trad.	-0.029	-0.037	-0.038	-0.033	-0.031	-0.033	0.091	0.095	0.092	0.094	0.092	0.094
		rob.	-0.042	-0.039	-0.042	-0.027	-0.044	-0.045	0.114	0.114	0.112	0.107	0.113	0.114
	300	trad.	-0.027	-0.030	-0.029	-0.032	-0.028	-0.030	0.056	0.060	0.058	0.060	0.060	0.056
		rob.	-0.026	-0.029	-0.031	-0.031	-0.029	-0.030	0.067	0.065	0.069	0.071	0.074	0.070
	500	trad.	-0.030	-0.028	-0.027	-0.031	-0.028	-0.028	0.046	0.047	0.045	0.044	0.044	0.046
		rob.	-0.029	-0.031	-0.027	-0.031	-0.029	-0.030	0.049	0.051	0.048	0.053	0.058	0.056
λ_{51}	100	trad.	0.001	-0.002	-0.002	0.004	-0.002	-0.008	0.134	0.133	0.131	0.129	0.134	0.132
		rob.	0.008	0.002	-0.009	-0.010	-0.001	-0.001	0.177	0.173	0.181	0.182	0.177	0.175
	300	trad.	-0.005	-0.001	-0.001	0.001	0.001	0.002	0.077	0.078	0.078	0.080	0.079	0.079
		rob.	0.000	-0.005	-0.001	-0.005	-0.005	-0.004	0.088	0.091	0.096	0.091	0.104	0.093
	500	trad.	0.001	-0.005	0.002	0.000	0.000	-0.004	0.059	0.062	0.062	0.061	0.064	0.063
		rob.	-0.001	-0.004	-0.003	-0.006	0.001	-0.002	0.067	0.067	0.067	0.070	0.078	0.075
λ_{52}	100	trad.	0.000	-0.001	-0.008	-0.010	-0.001	-0.002	0.130	0.134	0.133	0.131	0.137	0.142
		rob.	-0.009	-0.007	-0.000	-0.005	-0.009	0.003	0.179	0.185	0.182	0.179	0.176	0.171
	300	trad.	0.000	-0.007	-0.002	-0.001	-0.002	-0.002	0.081	0.081	0.082	0.081	0.080	0.079
		rob.	-0.004	-0.003	-0.002	-0.002	-0.008	-0.006	0.090	0.089	0.091	0.088	0.098	0.094
	500	trad.	0.001	-0.001	-0.000	0.001	-0.002	0.000	0.059	0.062	0.063	0.059	0.063	0.062
		rob.	0.000	-0.005	0.001	-0.000	-0.002	-0.002	0.067	0.068	0.068	0.070	0.079	0.075
λ_{53}	100	trad.	-0.003	-0.011	-0.008	-0.002	-0.007	-0.014	0.117	0.115	0.114	0.109	0.117	0.122
		rob.	-0.014	-0.017	-0.007	-0.013	-0.021	-0.016	0.156	0.162	0.153	0.146	0.152	0.154
	300	trad.	-0.006	-0.003	-0.006	-0.002	-0.003	-0.004	0.067	0.067	0.070	0.065	0.068	0.067
		rob.	-0.005	-0.005	0.000	-0.001	-0.004	-0.000	0.081	0.080	0.078	0.079	0.086	0.083
	500	trad.	-0.001	0.001	-0.003	-0.002	-0.002	-0.002	0.051	0.054	0.053	0.052	0.055	0.053
		rob.	-0.006	0.001	-0.002	-0.001	-0.003	-0.005	0.058	0.060	0.059	0.060	0.066	0.063
λ_{54}	100	trad.	-0.009	-0.003	-0.005	-0.011	-0.008	-0.002	0.117	0.119	0.114	0.111	0.117	0.118
		rob.	-0.016	-0.017	-0.008	-0.012	-0.015	-0.008	0.163	0.156	0.152	0.147	0.154	0.149
	300	trad.	0.001	-0.004	-0.004	-0.006	-0.004	-0.002	0.067	0.067	0.072	0.068	0.070	0.070
		rob.	0.002	-0.000	-0.006	-0.004	-0.006	-0.001	0.075	0.081	0.079	0.083	0.086	0.085
	500	trad.	0.000	-0.003	-0.001	-0.002	-0.002	-0.001	0.051	0.050	0.052	0.052	0.055	0.054
		rob.	-0.002	-0.001	-0.004	0.000	0.000	-0.006	0.057	0.059	0.059	0.058	0.068	0.066
λ_{55}	100	trad.	-0.024	-0.024	-0.023	-0.026	-0.024	-0.028	0.065	0.067	0.064	0.066	0.070	0.071
		rob.	-0.048	-0.045	-0.045	-0.043	-0.044	-0.049	0.096	0.089	0.092	0.089	0.091	0.093
	300	trad.	-0.007	-0.009	-0.006	-0.006	-0.006	-0.008	0.044	0.043	0.042	0.044	0.044	0.043
		rob.	-0.009	-0.011	-0.010	-0.011	-0.012	-0.011	0.048	0.051	0.051	0.049	0.054	0.053
	500	trad.	-0.004	-0.002	-0.005	-0.004	-0.003	-0.006	0.034	0.034	0.033	0.033	0.035	0.034
		rob.	-0.004	-0.005	-0.003	-0.007	-0.007	-0.007	0.037	0.038	0.038	0.040	0.042	0.041
λ_{56}	100	trad.	-0.027	-0.026	-0.023	-0.025	-0.030	-0.029	0.071	0.067	0.068	0.071	0.069	0.070
		rob.	-0.048	-0.047	-0.055	-0.039	-0.046	-0.044	0.093	0.093	0.093	0.085	0.094	0.089
	300	trad.	-0.006	-0.009	-0.006	-0.007	-0.007	-0.009	0.042	0.046	0.042	0.044	0.043	0.043
		rob.	-0.011	-0.008	-0.012	-0.009	-0.011	-0.008	0.047	0.050	0.049	0.050	0.053	0.049
	500	trad.	-0.004	-0.003	-0.005	-0.003	-0.006	-0.005	0.034	0.035	0.034	0.034	0.034	0.035
		rob.	-0.004	-0.006	-0.005	-0.005	-0.006	-0.006	0.038	0.038	0.038	0.038	0.043	0.041

Table 8.11: Results for traditional and robust PLS for five composites and unsystematic outliers in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.005	-0.172	-0.319	-0.507	-0.623	-0.664	0.051	0.332	0.445	0.456	0.374	0.332
		rob.	0.011	0.012	0.009	0.008	0.008	-0.245	0.071	0.071	0.072	0.071	0.066	0.576
	300	trad.	0.001	-0.331	-0.447	-0.551	-0.635	-0.654	0.030	0.172	0.221	0.218	0.195	0.185
		rob.	0.002	0.005	0.002	0.004	0.002	-0.047	0.034	0.033	0.035	0.035	0.039	0.180
	500	trad.	0.001	-0.382	-0.489	-0.585	-0.646	-0.661	0.022	0.139	0.134	0.138	0.132	0.132
		rob.	0.001	-0.000	0.002	0.002	0.003	-0.061	0.025	0.026	0.026	0.026	0.028	0.099
γ_{32}	100	trad.	-0.072	-0.093	-0.109	-0.076	-0.068	-0.027	0.146	0.334	0.356	0.324	0.274	0.270
		rob.	-0.114	-0.121	-0.106	-0.114	-0.096	-0.115	0.230	0.239	0.234	0.216	0.212	0.496
	300	trad.	-0.025	-0.025	-0.027	-0.024	-0.020	-0.026	0.064	0.116	0.127	0.103	0.112	0.108
		rob.	-0.032	-0.034	-0.032	-0.034	-0.044	-0.057	0.077	0.078	0.078	0.082	0.092	0.253
	500	trad.	-0.012	-0.013	-0.011	-0.013	-0.015	-0.014	0.048	0.073	0.066	0.066	0.068	0.072
		rob.	-0.014	-0.016	-0.016	-0.017	-0.025	-0.025	0.053	0.053	0.055	0.058	0.063	0.139
γ_{41}	100	trad.	-0.005	-0.047	0.020	0.062	0.057	0.044	0.101	0.237	0.224	0.187	0.150	0.152
		rob.	-0.007	0.003	0.005	0.005	0.000	-0.030	0.153	0.155	0.148	0.137	0.136	0.324
	300	trad.	-0.004	0.038	0.060	0.065	0.048	0.041	0.060	0.115	0.109	0.092	0.083	0.086
		rob.	-0.006	-0.001	-0.002	0.000	-0.001	-0.102	0.068	0.065	0.068	0.072	0.076	0.189
	500	trad.	-0.002	0.065	0.077	0.073	0.049	0.042	0.045	0.089	0.073	0.067	0.059	0.062
		rob.	-0.001	-0.001	-0.001	-0.002	-0.000	-0.075	0.049	0.050	0.052	0.054	0.059	0.131
γ_{42}	100	trad.	0.010	-0.141	-0.289	-0.418	-0.513	-0.560	0.088	0.229	0.237	0.201	0.169	0.188
		rob.	0.019	0.010	0.011	0.005	0.011	-0.313	0.134	0.139	0.132	0.123	0.119	0.342
	300	trad.	0.006	-0.176	-0.292	-0.410	-0.527	-0.561	0.052	0.082	0.089	0.090	0.088	0.089
		rob.	0.008	0.006	0.004	0.004	0.005	-0.154	0.059	0.058	0.060	0.062	0.068	0.182
	500	trad.	0.002	-0.197	-0.296	-0.407	-0.526	-0.563	0.038	0.058	0.054	0.055	0.060	0.068
		rob.	0.002	0.003	0.004	0.005	0.002	-0.166	0.041	0.042	0.046	0.046	0.051	0.121
γ_{51}	100	trad.	-0.068	-0.147	-0.220	-0.254	-0.249	-0.249	0.116	0.193	0.188	0.165	0.161	0.167
		rob.	-0.118	-0.132	-0.135	-0.119	-0.105	-0.180	0.179	0.173	0.168	0.166	0.153	0.243
	300	trad.	-0.037	-0.203	-0.239	-0.254	-0.254	-0.247	0.062	0.098	0.092	0.082	0.079	0.077
		rob.	-0.045	-0.045	-0.041	-0.043	-0.047	-0.082	0.072	0.075	0.071	0.075	0.086	0.160
	500	trad.	-0.028	-0.221	-0.250	-0.259	-0.251	-0.248	0.048	0.077	0.064	0.059	0.056	0.057
		rob.	-0.030	-0.030	-0.032	-0.033	-0.036	-0.077	0.052	0.053	0.055	0.053	0.062	0.121
γ_{52}	100	trad.	0.029	0.210	0.349	0.467	0.559	0.592	0.103	0.202	0.222	0.206	0.204	0.214
		rob.	0.054	0.060	0.066	0.059	0.039	0.389	0.172	0.175	0.177	0.157	0.149	0.333
	300	trad.	0.007	0.258	0.359	0.456	0.558	0.587	0.055	0.073	0.081	0.081	0.092	0.097
		rob.	0.012	0.013	0.011	0.012	0.014	0.208	0.064	0.067	0.066	0.070	0.073	0.166
	500	trad.	0.004	0.276	0.364	0.459	0.556	0.590	0.044	0.056	0.050	0.050	0.055	0.064
		rob.	0.005	0.004	0.006	0.006	0.009	0.210	0.047	0.049	0.049	0.049	0.055	0.114
γ_{53}	100	trad.	-0.057	-0.115	-0.118	-0.074	-0.048	-0.034	0.109	0.179	0.195	0.182	0.167	0.163
		rob.	-0.069	-0.068	-0.077	-0.064	-0.082	-0.141	0.178	0.185	0.196	0.185	0.169	0.255
	300	trad.	-0.049	-0.054	-0.054	-0.039	-0.035	-0.026	0.047	0.077	0.083	0.070	0.069	0.063
		rob.	-0.050	-0.051	-0.049	-0.048	-0.051	-0.090	0.060	0.054	0.056	0.060	0.067	0.150
	500	trad.	-0.050	-0.052	-0.045	-0.044	-0.038	-0.036	0.035	0.049	0.045	0.046	0.041	0.041
		rob.	-0.050	-0.050	-0.047	-0.048	-0.050	-0.059	0.039	0.038	0.040	0.041	0.045	0.091
w_{11}	100	trad.	-0.011	-0.260	-0.426	-0.547	-0.579	-0.587	0.112	0.390	0.484	0.562	0.563	0.590
		rob.	-0.013	-0.005	-0.016	-0.014	-0.011	-0.324	0.166	0.168	0.164	0.160	0.151	0.502
	300	trad.	-0.001	-0.247	-0.337	-0.449	-0.556	-0.548	0.066	0.230	0.341	0.458	0.549	0.547
		rob.	-0.001	-0.005	0.000	-0.000	-0.005	-0.058	0.075	0.076	0.076	0.079	0.085	0.256
	500	trad.	0.001	-0.240	-0.291	-0.369	-0.489	-0.515	0.051	0.187	0.252	0.377	0.504	0.515
		rob.	-0.000	0.002	0.002	0.003	0.002	-0.007	0.055	0.057	0.055	0.058	0.066	0.191
w_{12}	100	trad.	-0.006	-0.258	-0.391	-0.482	-0.555	-0.554	0.116	0.345	0.465	0.526	0.574	0.551
		rob.	-0.007	-0.019	-0.011	-0.011	-0.006	-0.369	0.172	0.165	0.162	0.157	0.149	0.546
	300	trad.	0.000	-0.249	-0.330	-0.441	-0.529	-0.552	0.065	0.227	0.329	0.460	0.539	0.552
		rob.	0.001	-0.001	-0.001	-0.003	-0.002	-0.054	0.076	0.074	0.076	0.077	0.087	0.272
	500	trad.	0.000	-0.234	-0.315	-0.401	-0.508	-0.538	0.050	0.176	0.273	0.382	0.505	0.541
		rob.	0.000	-0.001	-0.003	-0.001	-0.002	0.007	0.055	0.054	0.058	0.059	0.067	0.177
w_{21}	100	trad.	0.001	0.030	0.011	-0.011	-0.022	-0.059	0.075	0.306	0.369	0.402	0.410	0.403
		rob.	0.001	-0.006	0.000	-0.000	-0.003	0.002	0.118	0.120	0.118	0.112	0.101	0.415
	300	trad.	-0.001	0.080	0.105	0.097	0.023	0.015	0.041	0.191	0.223	0.259	0.329	0.353
		rob.	-0.002	-0.000	0.001	0.000	-0.001	0.045	0.047	0.046	0.048	0.049	0.055	0.234
	500	trad.	-0.001	0.087	0.116	0.117	0.079	0.038	0.031	0.148	0.155	0.183	0.266	0.307
		rob.	-0.001	0.001	-0.002	0.000	-0.000	0.043	0.034	0.033	0.035	0.038	0.041	0.169
w_{22}	100	trad.	0.001	-0.031	-0.100	-0.138	-0.205	-0.245	0.075	0.286	0.385	0.393	0.388	0.411
		rob.	-0.004	-0.020	-0.008	-0.009	-0.007	-0.107	0.122	0.116	0.114	0.109	0.102	0.419
	300	trad.	-0.002	0.004	-0.016	-0.041	-0.128	-0.166	0.041	0.176	0.208	0.254	0.328	0.345
		rob.	-0.004	-0.002	-0.001	-0.002	-0.000	0.049	0.047	0.046	0.046	0.049	0.055	0.233
	500	trad.	-0.000	0.008	-0.000	0.006	-0.062	-0.113	0.031	0.133	0.151	0.168	0.262	0.309
		rob.	0.000	0.002	-0.001	0.000	-0.001	0.069	0.035	0.033	0.036	0.037	0.041	0.152
w_{23}	100	trad.	-0.006	-0.034	-0.085	-0.155	-0.215	-0.213	0.072	0.305	0.369	0.401	0.394	0.401
		rob.	-0.004	-0.001	-0.006	-0.008	-0.013	-0.126	0.112	0.111	0.110	0.110	0.101	0.408
	300	trad.	-0.000	0.008	-0.007	-0.044	-0.107	-0.161	0.041	0.187	0.217	0.260	0.321	0.355
		rob.	-0.001	-0.005	-0.003	-0.002	-0.002	0.054	0.046	0.047	0.047	0.048	0.055	0.219
	500	trad.	-0.000	0.009	0.009	0.002	-0.056	-0.111	0.030	0.137	0.148	0.170	0.264	0.314
		rob.	-0.001	-0.000	-0.002	0.000	0.001	0.080	0.033	0.034	0.036	0.035	0.041	0.150
w_{24}	100	trad.	0.000	0.015	-0.037	-0.062	-0.132	-0.153	0.078	0.316	0.395	0.394	0.394	0.400
		rob.	-0.010	-0.002	-0.001	-0.007	-0.009	-0.068	0.123	0.118	0.118	0.120	0.106	0.451
	300	trad.	-0.002	0.047	0.052	0.045	-0.040	-0.071	0.043	0.194	0.219	0.254	0.327	0.372
		rob.	-0.002	0.003	-0.001	-0.000	0.000	0.050	0.050	0.048	0.050	0.051	0.056	0.252
	500	trad.	-0.002	0.062	0.087	0.088	0.034	-0.007	0.033	0.144	0.147	0.172	0.257	0.320
		rob.	-0.002	-0.001	0.003	-0.001	-0.001	0.068	0.035	0.035	0.036	0.037	0.043	0.167
w_{25}	100	trad.	-0.004	0.044	0.021	0.009	-0.048	-0.050	0.078	0.324	0.378	0.400	0.405	0.404
		rob.	-0.005	-0.006	-0.010	0.001	-0.000	-0.035	0.123	0.121	0.121	0.117	0.107	0.429
	300	trad.	0.002	0.087	0.107	0.092	0.059	0.007	0.044	0.189	0.223	0.269	0.326	0.335
		rob.	0.004	-0.004	-0.003	-0.002	-0.001	0.036	0.049	0.047	0.049	0.049	0.059	0.242
	500	trad.	0.002	0.086	0.127	0.130	0.100	0.050	0.032	0.151	0.150	0.175	0.265	0.321
		rob.	0.002	-0.001	0.000	0.000	-0.001	0.044	0.036	0.035	0.036	0.038	0.043	0.174
w_{26}	100	trad.	-0.100	-0.031	-0.011	-0.020	-0.045	-0.053	0.078	0.314	0.387	0.394	0.398	0.396
		rob.	-0.103	-0.092	-0.106	-0.103	-0.098	-0.097	0.122	0.119	0.123	0.119	0.112	0.433
	300	trad.	-0.102	0.025	0.046	0.064	0.029	-0.019	0.042	0.201	0.227	0.263	0.332	0.357
		rob.	-0.100	-0.098	-0.098	-0.102	-0.101	-0.062	0.050	0.050	0.049	0.052	0.056	0.249
	500	trad.	-0.099	0.024	0.064	0.082	0.058	0.023	0.033	0.155	0.163	0.190	0.277	0.311
		rob.	-0.099	-0.102	-0.101	-0.103	-0.101	-0.056	0.036	0.037	0.037	0.038	0.042	0.170

w_{31}	100	trad.	-0.021	-0.021	-0.048	-0.067	-0.119	-0.156	0.353	0.602	0.648	0.592	0.530	0.493
		rob.	-0.076	-0.054	-0.050	-0.056	-0.055	-0.000	0.439	0.437	0.455	0.427	0.407	0.721
	300	trad.	-0.010	-0.069	-0.103	-0.145	-0.173	-0.171	0.194	0.313	0.332	0.282	0.286	0.265
		rob.	-0.021	-0.006	-0.025	-0.017	-0.014	-0.001	0.236	0.229	0.231	0.242	0.268	0.529
	500	trad.	-0.002	-0.072	-0.110	-0.136	-0.168	-0.183	0.141	0.214	0.206	0.192	0.180	0.173
		rob.	-0.003	-0.006	-0.004	-0.007	-0.018	-0.069	0.157	0.171	0.169	0.164	0.204	0.365
w_{32}	100	trad.	-0.081	-0.115	-0.134	-0.112	-0.113	-0.125	0.413	0.398	0.382	0.415	0.441	0.465
		rob.	-0.136	-0.117	-0.092	-0.085	-0.098	-0.193	0.544	0.561	0.558	0.545	0.543	0.424
	300	trad.	-0.009	-0.042	-0.033	-0.015	-0.040	-0.023	0.244	0.233	0.246	0.243	0.284	0.258
		rob.	-0.018	-0.032	-0.033	-0.026	-0.040	-0.089	0.284	0.270	0.279	0.284	0.322	0.317
	500	trad.	-0.006	-0.011	-0.015	-0.003	-0.019	-0.008	0.175	0.184	0.184	0.191	0.187	0.184
		rob.	-0.005	-0.016	-0.014	-0.004	-0.015	-0.034	0.189	0.196	0.205	0.214	0.226	0.253
w_{33}	100	trad.	-0.126	-0.199	-0.244	-0.216	-0.191	-0.180	0.531	0.488	0.461	0.518	0.570	0.571
		rob.	-0.179	-0.233	-0.197	-0.248	-0.203	-0.355	0.669	0.680	0.657	0.668	0.644	0.492
	300	trad.	-0.044	-0.040	-0.037	-0.022	-0.032	-0.004	0.286	0.301	0.299	0.298	0.322	0.309
		rob.	-0.050	-0.043	-0.041	-0.044	-0.083	-0.153	0.335	0.328	0.329	0.348	0.389	0.403
	500	trad.	-0.024	-0.008	-0.001	-0.007	0.008	0.003	0.217	0.223	0.216	0.221	0.215	0.225
		rob.	-0.027	-0.024	-0.022	-0.031	-0.023	-0.070	0.232	0.240	0.246	0.251	0.267	0.313
w_{34}	100	trad.	-0.050	-0.172	-0.150	-0.155	-0.124	-0.086	0.598	0.549	0.526	0.583	0.609	0.645
		rob.	-0.110	-0.092	-0.164	-0.095	-0.104	-0.183	0.797	0.780	0.776	0.788	0.734	0.575
	300	trad.	-0.012	-0.004	-0.014	-0.011	0.007	-0.014	0.346	0.335	0.341	0.352	0.386	0.354
		rob.	-0.024	-0.020	-0.014	-0.029	-0.010	-0.096	0.403	0.386	0.384	0.404	0.456	0.432
	500	trad.	-0.006	0.002	0.014	0.020	0.027	0.025	0.253	0.258	0.259	0.263	0.253	0.270
		rob.	-0.010	-0.007	-0.014	-0.012	-0.020	-0.014	0.270	0.283	0.301	0.293	0.315	0.356
w_{41}	100	trad.	-0.011	-0.025	-0.078	-0.155	-0.253	-0.328	0.095	0.208	0.328	0.444	0.564	0.618
		rob.	-0.017	-0.007	-0.011	-0.012	-0.006	-0.190	0.143	0.134	0.136	0.134	0.124	0.501
	300	trad.	0.000	-0.001	-0.011	-0.033	-0.113	-0.154	0.056	0.085	0.119	0.202	0.394	0.472
		rob.	0.001	-0.003	-0.003	-0.004	-0.003	-0.016	0.061	0.063	0.064	0.062	0.071	0.153
	500	trad.	-0.001	-0.005	-0.001	-0.011	-0.041	-0.137	0.042	0.060	0.081	0.124	0.273	0.421
		rob.	-0.000	-0.002	-0.001	-0.001	0.002	-0.010	0.045	0.045	0.046	0.048	0.053	0.104
w_{42}	100	trad.	0.006	-0.027	-0.037	-0.052	-0.118	-0.139	0.110	0.246	0.354	0.446	0.576	0.622
		rob.	0.006	-0.008	-0.001	-0.003	-0.008	-0.119	0.164	0.163	0.167	0.161	0.152	0.524
	300	trad.	-0.004	-0.009	-0.004	-0.014	-0.078	-0.115	0.067	0.106	0.141	0.251	0.447	0.513
		rob.	-0.005	-0.002	-0.002	0.002	-0.002	-0.009	0.074	0.074	0.077	0.073	0.085	0.186
	500	trad.	-0.002	0.001	-0.008	-0.005	-0.049	-0.038	0.050	0.073	0.099	0.148	0.322	0.410
		rob.	-0.002	-0.001	-0.003	-0.003	-0.006	-0.000	0.054	0.054	0.055	0.059	0.063	0.124
w_{51}	100	trad.	-0.012	-0.038	-0.040	-0.051	-0.073	-0.071	0.178	0.285	0.327	0.357	0.390	0.391
		rob.	-0.023	-0.029	-0.025	-0.021	-0.026	-0.101	0.276	0.273	0.277	0.255	0.233	0.441
	300	trad.	-0.001	-0.012	-0.005	-0.017	-0.033	-0.016	0.096	0.121	0.152	0.182	0.232	0.222
		rob.	-0.001	-0.002	0.000	-0.009	-0.017	-0.022	0.111	0.109	0.115	0.119	0.126	0.235
	500	trad.	-0.006	0.001	-0.001	-0.010	-0.023	-0.003	0.070	0.092	0.104	0.129	0.163	0.162
		rob.	-0.004	-0.001	0.001	-0.004	-0.004	-0.010	0.077	0.081	0.080	0.083	0.093	0.150
w_{52}	100	trad.	-0.020	-0.078	-0.098	-0.146	-0.139	-0.166	0.163	0.264	0.326	0.352	0.371	0.389
		rob.	-0.046	-0.042	-0.051	-0.056	-0.047	-0.197	0.242	0.244	0.253	0.238	0.227	0.410
	300	trad.	-0.003	-0.008	-0.021	-0.012	-0.043	-0.046	0.086	0.115	0.142	0.169	0.221	0.203
		rob.	-0.005	-0.009	-0.010	-0.003	-0.007	-0.049	0.104	0.101	0.103	0.104	0.119	0.216
	500	trad.	-0.002	-0.006	-0.006	-0.012	-0.015	-0.027	0.067	0.084	0.102	0.128	0.144	0.140
		rob.	-0.002	-0.005	-0.003	-0.005	-0.007	-0.014	0.073	0.071	0.075	0.083	0.090	0.143
w_{53}	100	trad.	-0.029	-0.061	-0.120	-0.130	-0.142	-0.141	0.165	0.267	0.333	0.367	0.366	0.363
		rob.	-0.051	-0.039	-0.051	-0.038	-0.035	-0.164	0.252	0.246	0.254	0.244	0.224	0.396
	300	trad.	-0.009	-0.012	-0.018	-0.032	-0.043	-0.038	0.090	0.116	0.150	0.169	0.215	0.223
		rob.	-0.013	-0.011	-0.008	-0.011	-0.010	-0.036	0.106	0.103	0.108	0.105	0.122	0.210
	500	trad.	-0.003	-0.005	-0.012	-0.013	-0.017	-0.016	0.069	0.086	0.101	0.126	0.151	0.150
		rob.	-0.004	-0.004	-0.006	-0.002	-0.003	-0.022	0.076	0.074	0.081	0.079	0.093	0.148
w_{54}	100	trad.	-0.014	-0.070	-0.096	-0.122	-0.145	-0.116	0.165	0.254	0.315	0.351	0.371	0.364
		rob.	-0.036	-0.046	-0.044	-0.037	-0.034	-0.152	0.244	0.236	0.255	0.242	0.224	0.401
	300	trad.	-0.009	-0.006	-0.005	-0.021	-0.048	-0.034	0.089	0.114	0.147	0.161	0.210	0.204
		rob.	-0.012	-0.003	-0.002	-0.006	-0.009	-0.042	0.103	0.100	0.102	0.104	0.116	0.218
	500	trad.	-0.000	-0.008	-0.010	-0.010	-0.021	-0.018	0.067	0.087	0.103	0.125	0.149	0.147
		rob.	-0.000	-0.004	-0.006	-0.006	-0.010	-0.012	0.073	0.075	0.076	0.082	0.091	0.140
w_{55}	100	trad.	-0.008	-0.029	-0.051	-0.069	-0.076	-0.081	0.165	0.279	0.325	0.346	0.371	0.371
		rob.	-0.035	-0.026	-0.023	-0.014	-0.018	-0.097	0.255	0.260	0.251	0.254	0.229	0.396
	300	trad.	0.000	-0.002	-0.019	-0.020	-0.013	-0.025	0.089	0.120	0.146	0.179	0.214	0.211
		rob.	0.000	-0.006	-0.010	-0.008	-0.003	-0.027	0.105	0.104	0.109	0.109	0.123	0.214
	500	trad.	0.001	-0.005	-0.002	-0.013	-0.002	-0.014	0.071	0.090	0.104	0.131	0.150	0.160
		rob.	-0.000	-0.002	-0.005	-0.002	-0.000	-0.009	0.077	0.080	0.076	0.082	0.090	0.146
w_{56}	100	trad.	-0.004	-0.017	-0.041	-0.048	-0.037	-0.042	0.165	0.284	0.332	0.371	0.366	0.382
		rob.	-0.012	-0.013	-0.019	-0.029	-0.008	-0.046	0.262	0.254	0.253	0.243	0.225	0.415
	300	trad.	-0.005	-0.008	-0.004	-0.005	0.003	-0.007	0.089	0.123	0.151	0.179	0.226	0.227
		rob.	-0.004	-0.005	-0.005	-0.002	0.000	0.005	0.105	0.110	0.111	0.114	0.121	0.223
	500	trad.	-0.006	-0.004	-0.004	0.002	-0.006	-0.001	0.069	0.090	0.104	0.130	0.158	0.161
		rob.	-0.007	-0.003	-0.002	-0.003	-0.003	-0.008	0.075	0.079	0.082	0.085	0.093	0.149

Table 8.12: Results for traditional and robust PLS for five composites and unsystematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.007	-0.031	-0.047	-0.053	-0.063	-0.064	0.050	0.057	0.060	0.059	0.060	0.062
		rob.	0.009	0.011	0.009	0.007	-0.031	-0.050	0.074	0.070	0.072	0.070	0.082	0.085
	300	trad.	0.002	-0.045	-0.055	-0.063	-0.071	-0.069	0.030	0.033	0.033	0.036	0.035	0.035
		rob.	0.003	0.001	0.001	-0.003	-0.021	-0.058	0.033	0.033	0.034	0.036	0.041	0.042
	500	trad.	0.001	-0.045	-0.056	-0.065	-0.069	-0.071	0.024	0.026	0.026	0.028	0.028	0.027
		rob.	0.002	0.001	0.000	-0.003	-0.019	-0.057	0.027	0.025	0.026	0.026	0.032	0.033
γ_{32}	100	trad.	-0.075	-0.066	-0.064	-0.079	-0.067	-0.064	0.137	0.143	0.141	0.143	0.140	0.146
		rob.	-0.120	-0.103	-0.115	-0.109	-0.099	-0.101	0.227	0.242	0.231	0.216	0.227	0.241
	300	trad.	-0.025	-0.022	-0.020	-0.026	-0.018	-0.020	0.065	0.067	0.065	0.063	0.067	0.065
		rob.	-0.033	-0.031	-0.028	-0.036	-0.028	-0.030	0.075	0.078	0.078	0.079	0.091	0.080
	500	trad.	-0.014	-0.010	-0.014	-0.015	-0.014	-0.012	0.048	0.049	0.048	0.047	0.047	0.049
		rob.	-0.017	-0.012	-0.018	-0.019	-0.022	-0.020	0.054	0.056	0.053	0.055	0.060	0.059
γ_{41}	100	trad.	-0.001	0.099	0.119	0.139	0.151	0.153	0.105	0.101	0.106	0.104	0.102	0.102
		rob.	-0.005	0.005	0.006	0.010	0.098	0.147	0.156	0.145	0.150	0.152	0.145	0.147
	300	trad.	-0.003	0.103	0.134	0.146	0.154	0.160	0.058	0.060	0.057	0.055	0.057	0.058
		rob.	-0.002	-0.000	0.005	0.011	0.055	0.140	0.066	0.067	0.069	0.069	0.077	0.071
	500	trad.	-0.000	0.108	0.133	0.148	0.156	0.157	0.044	0.046	0.045	0.044	0.045	0.043
		rob.	-0.000	0.000	0.004	0.012	0.056	0.132	0.048	0.051	0.051	0.052	0.061	0.053
γ_{42}	100	trad.	0.009	-0.097	-0.122	-0.140	-0.155	-0.159	0.090	0.092	0.097	0.093	0.095	0.089
		rob.	0.017	0.009	0.006	0.006	-0.094	-0.140	0.135	0.125	0.134	0.130	0.133	0.136
	300	trad.	0.004	-0.111	-0.143	-0.154	-0.164	-0.170	0.050	0.054	0.051	0.053	0.052	0.054
		rob.	0.004	0.001	-0.003	-0.008	-0.055	-0.147	0.057	0.059	0.059	0.061	0.069	0.067
	500	trad.	0.002	-0.116	-0.143	-0.159	-0.167	-0.169	0.038	0.040	0.041	0.040	0.041	0.039
		rob.	0.003	0.001	-0.004	-0.012	-0.057	-0.141	0.041	0.043	0.044	0.045	0.055	0.049
γ_{51}	100	trad.	-0.078	-0.161	-0.188	-0.204	-0.210	-0.207	0.112	0.115	0.113	0.110	0.113	0.108
		rob.	-0.127	-0.129	-0.123	-0.134	-0.204	-0.237	0.177	0.169	0.172	0.162	0.170	0.171
	300	trad.	-0.036	-0.139	-0.167	-0.181	-0.186	-0.188	0.064	0.064	0.063	0.060	0.061	0.063
		rob.	-0.043	-0.042	-0.048	-0.056	-0.100	-0.176	0.073	0.075	0.073	0.075	0.081	0.074
	500	trad.	-0.030	-0.139	-0.159	-0.175	-0.185	-0.185	0.048	0.048	0.048	0.047	0.049	0.048
		rob.	-0.030	-0.035	-0.033	-0.045	-0.093	-0.163	0.053	0.054	0.055	0.057	0.065	0.061
γ_{52}	100	trad.	0.027	0.118	0.149	0.165	0.174	0.174	0.101	0.107	0.106	0.107	0.104	0.105
		rob.	0.055	0.058	0.060	0.069	0.140	0.178	0.172	0.174	0.173	0.166	0.168	0.169
	300	trad.	0.009	0.120	0.147	0.162	0.167	0.174	0.057	0.058	0.057	0.056	0.057	0.058
		rob.	0.012	0.013	0.016	0.024	0.068	0.157	0.066	0.067	0.064	0.069	0.077	0.070
	500	trad.	0.002	0.121	0.145	0.163	0.172	0.174	0.045	0.045	0.045	0.044	0.045	0.046
		rob.	0.002	0.008	0.009	0.019	0.068	0.147	0.049	0.049	0.049	0.051	0.059	0.057
γ_{53}	100	trad.	-0.060	-0.058	-0.056	-0.054	-0.048	-0.054	0.109	0.110	0.118	0.114	0.111	0.117
		rob.	-0.076	-0.085	-0.075	-0.064	-0.060	-0.065	0.195	0.195	0.188	0.171	0.178	0.202
	300	trad.	-0.049	-0.047	-0.045	-0.048	-0.043	-0.044	0.045	0.048	0.048	0.047	0.047	0.047
		rob.	-0.051	-0.050	-0.047	-0.049	-0.044	-0.042	0.054	0.058	0.057	0.057	0.062	0.058
	500	trad.	-0.051	-0.048	-0.047	-0.047	-0.047	-0.048	0.034	0.036	0.036	0.038	0.035	0.036
		rob.	-0.051	-0.049	-0.049	-0.048	-0.048	-0.048	0.038	0.040	0.040	0.040	0.043	0.046
w_{11}	100	trad.	-0.002	-0.010	-0.007	-0.010	-0.014	-0.002	0.113	0.114	0.116	0.118	0.120	0.119
		rob.	-0.003	-0.013	-0.016	-0.011	-0.016	-0.007	0.157	0.156	0.163	0.161	0.156	0.167
	300	trad.	0.000	0.000	-0.003	-0.001	-0.000	-0.001	0.065	0.067	0.065	0.067	0.067	0.067
		rob.	-0.002	-0.001	-0.001	0.002	-0.003	-0.001	0.074	0.071	0.076	0.078	0.083	0.079
	500	trad.	-0.003	-0.001	-0.001	0.001	-0.002	-0.000	0.051	0.050	0.051	0.052	0.053	0.050
		rob.	-0.004	-0.000	-0.000	-0.000	-0.001	0.001	0.055	0.054	0.059	0.059	0.065	0.061
w_{12}	100	trad.	-0.006	-0.010	-0.009	-0.003	-0.004	-0.006	0.115	0.114	0.117	0.118	0.119	0.112
		rob.	-0.009	-0.011	-0.011	-0.006	-0.012	-0.018	0.163	0.157	0.165	0.157	0.161	0.162
	300	trad.	0.002	0.003	-0.000	-0.003	0.002	-0.002	0.064	0.066	0.066	0.069	0.065	0.067
		rob.	0.001	0.002	-0.001	-0.002	0.003	-0.002	0.073	0.074	0.074	0.080	0.082	0.079
	500	trad.	0.000	-0.002	0.001	0.001	-0.002	-0.001	0.050	0.050	0.053	0.051	0.053	0.052
		rob.	0.001	-0.002	0.001	0.001	-0.002	-0.002	0.055	0.056	0.056	0.059	0.064	0.060
w_{21}	100	trad.	-0.001	-0.047	-0.083	-0.125	-0.159	-0.167	0.077	0.132	0.108	0.099	0.092	0.094
		rob.	0.000	-0.010	-0.001	-0.010	-0.059	-0.140	0.123	0.118	0.115	0.115	0.208	0.172
	300	trad.	-0.002	-0.056	-0.088	-0.128	-0.161	-0.171	0.041	0.067	0.059	0.055	0.051	0.051
		rob.	-0.003	-0.002	0.003	-0.006	-0.026	-0.102	0.046	0.048	0.047	0.051	0.068	0.107
	500	trad.	0.000	-0.059	-0.095	-0.127	-0.159	-0.170	0.031	0.049	0.044	0.040	0.038	0.038
		rob.	0.000	-0.001	-0.002	-0.005	-0.022	-0.097	0.034	0.035	0.037	0.038	0.052	0.089
w_{22}	100	trad.	-0.007	-0.134	-0.205	-0.274	-0.333	-0.343	0.074	0.110	0.107	0.095	0.093	0.091
		rob.	-0.019	-0.015	-0.015	-0.017	-0.143	-0.290	0.117	0.119	0.117	0.112	0.194	0.175
	300	trad.	-0.000	-0.144	-0.209	-0.272	-0.332	-0.349	0.040	0.058	0.053	0.051	0.052	0.051
		rob.	-0.001	-0.001	-0.005	-0.012	-0.062	-0.236	0.047	0.048	0.048	0.050	0.063	0.106
	500	trad.	-0.001	-0.150	-0.214	-0.274	-0.331	-0.349	0.030	0.044	0.041	0.039	0.039	0.039
		rob.	-0.001	-0.002	-0.004	-0.013	-0.061	-0.208	0.033	0.036	0.036	0.037	0.048	0.082
w_{23}	100	trad.	-0.007	0.055	0.070	0.094	0.103	0.101	0.076	0.087	0.093	0.091	0.092	0.093
		rob.	-0.013	-0.001	-0.011	0.002	0.051	0.085	0.116	0.116	0.114	0.114	0.132	0.134
	300	trad.	-0.001	0.063	0.086	0.100	0.106	0.106	0.041	0.049	0.049	0.050	0.051	0.050
		rob.	-0.002	-0.002	0.003	0.005	0.031	0.088	0.048	0.047	0.047	0.049	0.058	0.062
	500	trad.	-0.001	0.069	0.087	0.102	0.110	0.109	0.031	0.037	0.037	0.039	0.041	0.038
		rob.	-0.001	0.001	0.001	0.007	0.033	0.088	0.034	0.034	0.034	0.036	0.045	0.046
w_{24}	100	trad.	-0.003	0.063	0.093	0.102	0.109	0.111	0.079	0.092	0.094	0.100	0.101	0.103
		rob.	-0.003	-0.006	0.001	-0.003	0.061	0.100	0.125	0.125	0.126	0.117	0.141	0.152
	300	trad.	-0.000	0.072	0.092	0.104	0.118	0.120	0.042	0.052	0.054	0.055	0.056	0.056
		rob.	0.000	-0.000	0.000	0.006	0.036	0.102	0.049	0.050	0.050	0.052	0.061	0.069
	500	trad.	-0.001	0.072	0.097	0.107	0.116	0.119	0.031	0.039	0.040	0.041	0.043	0.040
		rob.	-0.001	0.001	0.004	0.006	0.034	0.095	0.035	0.037	0.036	0.037	0.046	0.051
w_{25}	100	trad.	0.002	0.054	0.077	0.093	0.100	0.107	0.078	0.095	0.097	0.099	0.102	0.104
		rob.	0.001	-0.002	-0.005	0.002	0.053	0.094	0.127	0.131	0.125	0.116	0.141	0.159
	300	trad.	0.001	0.066	0.083	0.097	0.104	0.105	0.041	0.052	0.052	0.055	0.055	0.056
		rob.	0.001	0.003	0.002	0.008	0.032	0.088	0.047	0.050	0.049	0.053	0.062	0.066
	500	trad.	0.000	0.065	0.083	0.097	0.104	0.108	0.033	0.039	0.040	0.042	0.042	0.041
		rob.	-0.001	0.000	0.000	0.006	0.030	0.086	0.036	0.035	0.036	0.038	0.047	0.052
w_{26}	100	trad.	-0.101	-0.047	-0.031	-0.024	-0.012	-0.010	0.078	0.090	0.102	0.099	0.106	0.104
		rob.	-0.100	-0.103	-0.098	-0.098	-0.043	-0.019	0.121	0.125	0.125	0.121	0.146	0.159
	300	trad.	-0.101	-0.045	-0.026	-0.014	-0.009	-0.007	0.042	0.053	0.054	0.057	0.057	0.056
		rob.	-0.100	-0.100	-0.099	-0.095	-0.074	-0.021	0.048	0.050	0.049	0.053	0.062	0.066
	500	trad.	-0.099	-0.043	-0.025	-0.014	-0.008	-0.008	0.032	0.039	0.043	0.043	0.044	0.043
		rob.	-0.098	-0.100	-0.097	-0.093	-0.072	-0.028	0.035	0.036	0.037	0.039	0.046	0.051

w_{31}	100	trad.	-0.019	-0.035	-0.034	-0.055	-0.041	-0.038	0.330	0.344	0.348	0.352	0.344	0.351
		rob.	-0.038	-0.068	-0.081	-0.070	-0.054	-0.084	0.417	0.448	0.443	0.425	0.427	0.457
	300	trad.	-0.013	-0.001	-0.008	-0.023	-0.012	-0.023	0.195	0.195	0.190	0.199	0.191	0.192
		rob.	-0.017	-0.005	-0.012	-0.027	-0.014	-0.032	0.229	0.229	0.228	0.232	0.242	0.236
	500	trad.	0.004	-0.005	-0.005	-0.003	-0.014	-0.005	0.144	0.139	0.147	0.144	0.147	0.151
		rob.	0.003	-0.005	-0.007	-0.005	-0.022	-0.009	0.158	0.155	0.163	0.168	0.187	0.180
w_{32}	100	trad.	-0.051	-0.083	-0.048	-0.087	-0.073	-0.065	0.423	0.434	0.436	0.429	0.442	0.430
		rob.	-0.085	-0.095	-0.078	-0.135	-0.113	-0.115	0.540	0.562	0.546	0.540	0.508	0.557
	300	trad.	-0.005	-0.019	-0.002	-0.018	-0.013	-0.009	0.237	0.236	0.228	0.238	0.230	0.235
		rob.	-0.017	-0.021	-0.003	-0.028	-0.023	-0.028	0.277	0.277	0.267	0.290	0.293	0.292
	500	trad.	-0.013	0.003	-0.012	-0.009	-0.014	-0.019	0.184	0.173	0.177	0.175	0.179	0.175
		rob.	-0.013	-0.002	-0.015	-0.013	-0.026	-0.024	0.206	0.195	0.199	0.199	0.236	0.219
w_{33}	100	trad.	-0.096	-0.141	-0.117	-0.104	-0.097	-0.130	0.514	0.521	0.510	0.527	0.520	0.519
		rob.	-0.202	-0.238	-0.200	-0.199	-0.163	-0.260	0.669	0.672	0.677	0.669	0.659	0.658
	300	trad.	-0.019	-0.023	-0.019	-0.022	-0.021	-0.018	0.282	0.280	0.277	0.288	0.274	0.262
		rob.	-0.032	-0.039	-0.029	-0.022	-0.037	-0.028	0.325	0.328	0.315	0.334	0.346	0.321
	500	trad.	-0.024	-0.011	-0.014	-0.014	-0.017	-0.009	0.210	0.222	0.203	0.213	0.212	0.210
		rob.	-0.029	-0.015	-0.018	-0.017	-0.023	-0.023	0.228	0.244	0.233	0.245	0.264	0.260
w_{34}	100	trad.	-0.093	-0.030	-0.080	-0.053	-0.085	-0.060	0.608	0.599	0.584	0.606	0.615	0.607
		rob.	-0.133	-0.105	-0.152	-0.080	-0.122	-0.072	0.770	0.767	0.787	0.765	0.749	0.752
	300	trad.	-0.037	-0.028	-0.038	-0.021	-0.026	-0.025	0.342	0.323	0.330	0.334	0.325	0.318
		rob.	-0.038	-0.037	-0.050	-0.040	-0.043	-0.032	0.396	0.381	0.373	0.389	0.412	0.393
	500	trad.	-0.004	-0.024	-0.009	-0.013	-0.001	-0.010	0.261	0.257	0.250	0.247	0.260	0.241
		rob.	-0.008	-0.026	-0.011	-0.018	-0.006	-0.012	0.286	0.289	0.280	0.284	0.329	0.306
w_{41}	100	trad.	-0.004	-0.004	-0.001	-0.006	-0.005	-0.004	0.094	0.102	0.094	0.102	0.106	0.100
		rob.	-0.010	-0.011	-0.009	-0.006	-0.013	-0.010	0.141	0.140	0.133	0.130	0.148	0.149
	300	trad.	-0.001	-0.001	-0.002	-0.002	-0.003	-0.000	0.056	0.056	0.057	0.056	0.057	0.058
		rob.	-0.001	-0.003	-0.004	-0.001	-0.001	0.000	0.063	0.063	0.063	0.062	0.068	0.068
	500	trad.	-0.003	-0.001	-0.000	-0.001	-0.003	-0.003	0.042	0.043	0.044	0.045	0.043	0.044
		rob.	-0.004	-0.001	-0.001	-0.001	-0.003	-0.002	0.046	0.046	0.047	0.048	0.053	0.051
w_{42}	100	trad.	-0.003	-0.007	-0.006	-0.004	-0.006	-0.004	0.113	0.124	0.115	0.120	0.127	0.118
		rob.	-0.007	-0.006	-0.003	-0.008	-0.004	-0.007	0.169	0.164	0.159	0.155	0.172	0.171
	300	trad.	-0.001	-0.002	-0.002	-0.003	-0.001	-0.004	0.066	0.067	0.070	0.068	0.068	0.069
		rob.	-0.001	0.000	0.000	-0.003	-0.004	-0.007	0.075	0.074	0.075	0.074	0.082	0.082
	500	trad.	0.001	-0.002	-0.003	-0.001	-0.000	0.000	0.049	0.051	0.052	0.054	0.052	0.052
		rob.	0.002	-0.002	-0.002	-0.001	-0.001	-0.001	0.053	0.054	0.057	0.057	0.064	0.063
w_{51}	100	trad.	-0.007	-0.011	-0.005	-0.022	-0.006	-0.008	0.179	0.178	0.190	0.187	0.198	0.195
		rob.	-0.017	-0.025	-0.026	-0.033	-0.014	-0.026	0.265	0.270	0.266	0.252	0.268	0.283
	300	trad.	-0.003	-0.001	-0.002	-0.008	-0.002	-0.001	0.095	0.100	0.103	0.100	0.099	0.109
		rob.	-0.006	-0.003	-0.002	-0.010	-0.004	-0.007	0.111	0.112	0.112	0.119	0.122	0.126
	500	trad.	0.001	-0.004	0.000	0.001	-0.003	-0.002	0.069	0.076	0.078	0.077	0.078	0.077
		rob.	0.001	-0.005	-0.001	0.003	-0.003	-0.004	0.075	0.081	0.082	0.085	0.091	0.092
w_{52}	100	trad.	-0.025	-0.025	-0.026	-0.028	-0.031	-0.031	0.167	0.168	0.170	0.173	0.176	0.175
		rob.	-0.056	-0.058	-0.052	-0.052	-0.059	-0.069	0.255	0.253	0.248	0.250	0.249	0.272
	300	trad.	-0.010	-0.009	-0.014	-0.003	-0.007	-0.008	0.086	0.092	0.098	0.094	0.095	0.097
		rob.	-0.010	-0.008	-0.017	-0.009	-0.011	-0.013	0.102	0.102	0.108	0.108	0.114	0.117
	500	trad.	-0.007	-0.005	-0.006	-0.003	-0.007	-0.007	0.066	0.073	0.075	0.071	0.070	0.074
		rob.	-0.009	-0.005	-0.006	-0.005	-0.009	-0.008	0.073	0.075	0.082	0.080	0.085	0.089
w_{53}	100	trad.	-0.022	-0.023	-0.026	-0.026	-0.024	-0.021	0.166	0.170	0.184	0.184	0.180	0.184
		rob.	-0.042	-0.052	-0.047	-0.041	-0.049	-0.037	0.246	0.265	0.259	0.239	0.256	0.270
	300	trad.	-0.006	-0.004	-0.005	-0.010	-0.011	-0.007	0.087	0.095	0.095	0.097	0.096	0.094
		rob.	-0.009	-0.007	-0.009	-0.009	-0.013	-0.011	0.101	0.103	0.108	0.109	0.119	0.112
	500	trad.	-0.002	-0.003	-0.003	-0.006	-0.002	-0.001	0.068	0.074	0.073	0.073	0.077	0.074
		rob.	-0.002	-0.003	-0.002	-0.007	-0.003	-0.002	0.074	0.077	0.078	0.079	0.091	0.089
w_{54}	100	trad.	-0.011	-0.013	-0.031	-0.021	-0.026	-0.024	0.158	0.169	0.179	0.172	0.178	0.176
		rob.	-0.039	-0.043	-0.055	-0.057	-0.050	-0.055	0.248	0.261	0.257	0.241	0.244	0.256
	300	trad.	-0.001	-0.008	-0.001	-0.003	-0.007	-0.007	0.091	0.091	0.093	0.093	0.096	0.094
		rob.	-0.005	-0.008	0.001	-0.005	-0.013	-0.010	0.105	0.103	0.100	0.104	0.115	0.112
	500	trad.	-0.001	-0.002	-0.003	-0.007	-0.003	-0.003	0.066	0.071	0.074	0.073	0.070	0.072
		rob.	-0.001	-0.002	-0.005	-0.007	-0.004	-0.004	0.073	0.074	0.077	0.079	0.083	0.087
w_{55}	100	trad.	-0.020	-0.014	-0.014	-0.011	-0.012	-0.013	0.166	0.175	0.189	0.180	0.184	0.181
		rob.	-0.038	-0.024	-0.025	-0.024	-0.030	-0.024	0.261	0.260	0.266	0.259	0.254	0.271
	300	trad.	-0.002	-0.000	-0.009	-0.005	-0.001	-0.008	0.091	0.097	0.100	0.101	0.098	0.101
		rob.	-0.004	-0.005	-0.008	-0.003	-0.002	-0.008	0.108	0.107	0.107	0.110	0.120	0.120
	500	trad.	-0.006	-0.003	-0.002	-0.001	-0.004	-0.004	0.070	0.074	0.077	0.075	0.078	0.077
		rob.	-0.006	-0.003	-0.003	-0.003	-0.006	-0.006	0.077	0.080	0.080	0.082	0.093	0.091
w_{56}	100	trad.	-0.007	-0.009	0.001	0.006	-0.012	-0.005	0.165	0.177	0.185	0.180	0.185	0.188
		rob.	-0.010	-0.018	0.003	0.009	-0.016	-0.015	0.258	0.265	0.255	0.258	0.260	0.272
	300	trad.	-0.004	-0.006	0.002	-0.003	-0.001	-0.001	0.091	0.099	0.096	0.101	0.097	0.104
		rob.	-0.003	-0.004	-0.001	-0.002	-0.002	0.004	0.105	0.108	0.108	0.110	0.119	0.125
	500	trad.	-0.002	-0.002	-0.004	-0.003	0.001	-0.001	0.071	0.072	0.074	0.078	0.075	0.076
		rob.	-0.003	-0.001	-0.002	-0.003	-0.002	0.000	0.078	0.076	0.079	0.086	0.091	0.091

Systematic outliers

This subsection shows the complete results for systematic outliers.

Table 8.13: Results for traditional and robust PLSc for three common factors and systematic outliers in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.018	0.269	0.347	0.410	0.450	0.459	0.097	0.052	0.033	0.022	0.016	0.015
		rob.	0.034	0.035	0.037	0.042	0.248	0.455	0.124	0.125	0.123	0.130	0.201	0.019
	300	trad.	0.002	0.263	0.344	0.407	0.449	0.458	0.059	0.029	0.021	0.014	0.010	0.009
		rob.	0.003	0.006	0.005	0.005	0.094	0.454	0.067	0.066	0.070	0.071	0.114	0.010
	500	trad.	0.001	0.260	0.342	0.407	0.449	0.458	0.046	0.023	0.016	0.011	0.007	0.007
		rob.	0.001	-0.001	-0.003	0.002	0.075	0.455	0.051	0.053	0.052	0.056	0.083	0.008
γ_{31}	100	trad.	0.039	0.172	0.212	0.231	0.247	0.254	0.152	0.153	0.174	0.190	0.286	0.330
		rob.	0.064	0.053	0.063	0.056	0.145	0.230	0.238	0.225	0.243	0.227	0.266	0.440
	300	trad.	0.006	0.154	0.196	0.221	0.239	0.234	0.080	0.089	0.091	0.102	0.132	0.151
		rob.	0.008	0.004	0.010	0.008	0.068	0.219	0.090	0.089	0.093	0.097	0.117	0.150
	500	trad.	0.004	0.156	0.195	0.221	0.238	0.240	0.063	0.067	0.072	0.080	0.098	0.115
		rob.	0.003	0.003	0.004	0.006	0.055	0.225	0.069	0.071	0.071	0.073	0.094	0.112
γ_{32}	100	trad.	0.013	0.257	0.314	0.369	0.400	0.401	0.176	0.162	0.180	0.194	0.286	0.330
		rob.	0.013	0.016	0.000	0.023	0.227	0.423	0.271	0.269	0.288	0.263	0.327	0.440
	300	trad.	0.003	0.260	0.325	0.376	0.405	0.421	0.089	0.094	0.094	0.104	0.133	0.151
		rob.	0.004	0.009	0.003	0.007	0.082	0.434	0.105	0.106	0.103	0.111	0.146	0.151
	500	trad.	0.002	0.257	0.326	0.376	0.406	0.415	0.068	0.069	0.074	0.081	0.098	0.115
		rob.	0.002	0.002	0.004	0.005	0.069	0.428	0.075	0.076	0.079	0.082	0.107	0.112
λ_{11}	100	trad.	-0.026	-0.004	0.014	0.031	0.036	0.032	0.066	0.039	0.026	0.017	0.014	0.013
		rob.	-0.037	-0.032	-0.036	-0.038	0.010	0.042	0.082	0.083	0.084	0.088	0.071	0.018
	300	trad.	-0.007	-0.000	0.016	0.031	0.036	0.031	0.045	0.023	0.016	0.011	0.008	0.008
		rob.	-0.008	-0.006	-0.008	-0.008	-0.001	0.038	0.052	0.051	0.052	0.053	0.049	0.010
	500	trad.	-0.003	0.000	0.016	0.031	0.036	0.031	0.036	0.017	0.012	0.008	0.006	0.006
		rob.	-0.003	-0.003	-0.003	-0.003	-0.002	0.037	0.040	0.041	0.042	0.042	0.041	0.007
λ_{12}	100	trad.	-0.000	0.072	0.100	0.123	0.131	0.127	0.089	0.043	0.031	0.019	0.015	0.014
		rob.	-0.003	-0.007	0.003	0.006	0.071	0.137	0.110	0.110	0.105	0.102	0.100	0.019
	300	trad.	0.000	0.071	0.100	0.121	0.130	0.127	0.057	0.026	0.018	0.011	0.008	0.008
		rob.	0.000	0.001	0.001	-0.002	0.021	0.133	0.065	0.062	0.067	0.065	0.066	0.010
	500	trad.	-0.000	0.071	0.098	0.122	0.131	0.127	0.043	0.020	0.013	0.009	0.007	0.006
		rob.	0.001	-0.000	0.000	-0.001	0.020	0.131	0.049	0.047	0.049	0.051	0.053	0.007
λ_{13}	100	trad.	0.011	0.143	0.183	0.213	0.226	0.223	0.105	0.049	0.032	0.022	0.016	0.015
		rob.	0.013	0.017	0.018	0.016	0.129	0.231	0.137	0.133	0.125	0.140	0.137	0.019
	300	trad.	0.003	0.139	0.181	0.212	0.225	0.223	0.062	0.029	0.019	0.013	0.009	0.009
		rob.	0.004	-0.001	0.003	0.006	0.047	0.227	0.069	0.074	0.077	0.077	0.081	0.010
	500	trad.	0.001	0.139	0.181	0.212	0.225	0.223	0.049	0.021	0.015	0.010	0.007	0.007
		rob.	0.000	-0.002	0.001	-0.000	0.039	0.226	0.056	0.056	0.057	0.059	0.064	0.008
λ_{21}	100	trad.	-0.015	0.116	0.164	0.201	0.218	0.217	0.131	0.055	0.038	0.024	0.016	0.017
		rob.	-0.021	-0.025	-0.007	-0.005	0.109	0.224	0.169	0.162	0.153	0.162	0.148	0.019
	300	trad.	0.002	0.116	0.162	0.200	0.218	0.217	0.076	0.032	0.022	0.014	0.010	0.009
		rob.	0.001	-0.004	-0.003	-0.002	0.035	0.221	0.088	0.085	0.090	0.088	0.091	0.010
	500	trad.	0.001	0.118	0.162	0.199	0.218	0.217	0.058	0.025	0.017	0.010	0.008	0.007
		rob.	0.000	-0.002	-0.003	-0.002	0.030	0.219	0.065	0.066	0.060	0.067	0.070	0.008
λ_{22}	100	trad.	0.000	0.117	0.161	0.199	0.219	0.217	0.124	0.054	0.037	0.024	0.016	0.016
		rob.	-0.011	-0.015	-0.017	-0.018	0.110	0.223	0.161	0.165	0.152	0.151	0.161	0.020
	300	trad.	-0.001	0.117	0.163	0.199	0.218	0.217	0.073	0.031	0.021	0.014	0.009	0.010
		rob.	-0.002	-0.004	-0.001	-0.001	0.033	0.220	0.086	0.086	0.086	0.092	0.088	0.010
	500	trad.	0.000	0.117	0.162	0.200	0.218	0.217	0.060	0.026	0.016	0.011	0.007	0.007
		rob.	0.000	-0.001	-0.001	0.000	0.027	0.220	0.068	0.067	0.066	0.068	0.070	0.008
λ_{23}	100	trad.	-0.015	0.114	0.162	0.199	0.218	0.218	0.121	0.054	0.038	0.024	0.017	0.016
		rob.	-0.022	-0.013	-0.012	-0.007	0.106	0.225	0.160	0.160	0.156	0.155	0.159	0.020
	300	trad.	-0.008	0.118	0.162	0.199	0.218	0.217	0.077	0.032	0.021	0.014	0.010	0.009
		rob.	-0.008	-0.002	-0.002	-0.006	0.035	0.220	0.085	0.087	0.088	0.090	0.092	0.010
	500	trad.	-0.002	0.117	0.162	0.198	0.218	0.217	0.059	0.025	0.017	0.011	0.008	0.007
		rob.	-0.003	-0.002	-0.003	-0.005	0.027	0.219	0.066	0.064	0.065	0.066	0.072	0.008
λ_{31}	100	trad.	-0.042	0.055	0.090	0.116	0.127	0.126	0.164	0.056	0.036	0.023	0.016	0.016
		rob.	-0.060	-0.063	-0.063	-0.058	0.031	0.135	0.190	0.200	0.194	0.212	0.192	0.019
	300	trad.	-0.008	0.056	0.090	0.116	0.127	0.124	0.097	0.032	0.021	0.013	0.009	0.009
		rob.	-0.010	-0.016	-0.012	-0.009	0.010	0.129	0.108	0.108	0.115	0.114	0.097	0.010
	500	trad.	-0.008	0.058	0.089	0.116	0.128	0.124	0.077	0.028	0.017	0.010	0.007	0.007
		rob.	-0.006	-0.011	-0.008	-0.001	0.012	0.128	0.086	0.094	0.090	0.091	0.082	0.008
λ_{32}	100	trad.	-0.036	0.055	0.089	0.116	0.128	0.125	0.158	0.055	0.036	0.022	0.017	0.016
		rob.	-0.063	-0.060	-0.064	-0.070	0.035	0.134	0.202	0.204	0.213	0.212	0.177	0.020
	300	trad.	-0.012	0.059	0.090	0.117	0.128	0.124	0.100	0.033	0.020	0.013	0.009	0.009
		rob.	-0.017	-0.009	-0.011	-0.009	0.007	0.129	0.116	0.109	0.113	0.113	0.104	0.010
	500	trad.	-0.003	0.059	0.089	0.116	0.127	0.124	0.078	0.027	0.017	0.010	0.008	0.007
		rob.	-0.004	-0.001	-0.007	-0.010	0.006	0.128	0.086	0.091	0.088	0.092	0.087	0.008
λ_{33}	100	trad.	0.004	0.140	0.182	0.209	0.224	0.223	0.181	0.059	0.039	0.025	0.018	0.016
		rob.	-0.009	-0.032	-0.018	-0.034	0.110	0.230	0.218	0.243	0.250	0.248	0.197	0.021
	300	trad.	0.002	0.138	0.178	0.210	0.224	0.222	0.123	0.038	0.024	0.014	0.009	0.010
		rob.	0.002	-0.001	-0.002	-0.005	0.046	0.226	0.135	0.138	0.136	0.142	0.132	0.010
	500	trad.	0.001	0.139	0.179	0.210	0.224	0.222	0.093	0.028	0.017	0.011	0.008	0.007
		rob.	-0.001	-0.000	0.001	0.000	0.035	0.225	0.104	0.103	0.106	0.106	0.107	0.008

Table 8.14: Results for traditional and robust PLSc for three common factors and systematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.023	0.123	0.200	0.302	0.336	0.321	0.094	0.095	0.098	0.099	0.118	0.131
		rob.	0.038	0.035	0.046	0.064	0.296	0.308	0.120	0.121	0.122	0.116	0.130	0.131
	300	trad.	-0.001	0.105	0.186	0.312	0.408	0.393	0.060	0.058	0.057	0.064	0.065	0.096
		rob.	-0.003	0.010	0.011	0.033	0.285	0.378	0.068	0.067	0.067	0.068	0.154	0.102
	500	trad.	0.002	0.104	0.184	0.310	0.430	0.427	0.047	0.046	0.045	0.048	0.044	0.064
		rob.	0.002	0.004	0.010	0.028	0.269	0.417	0.053	0.050	0.050	0.053	0.164	0.071
γ_{31}	100	trad.	0.059	0.080	0.131	0.344	0.473	0.527	0.155	0.199	0.302	0.657	1.139	1.019
		rob.	0.080	0.055	0.044	0.068	0.355	0.373	0.249	0.221	0.232	0.206	0.804	0.934
	300	trad.	0.008	0.044	0.090	0.286	1.011	1.036	0.082	0.100	0.131	0.409	1.334	1.292
		rob.	0.010	0.010	0.011	0.014	0.490	0.830	0.090	0.093	0.095	0.098	0.924	1.349
	500	trad.	0.005	0.035	0.089	0.253	1.278	1.359	0.066	0.078	0.097	0.232	1.248	1.406
		rob.	0.006	0.004	0.009	0.012	0.523	1.132	0.072	0.070	0.074	0.074	0.903	1.278
γ_{32}	100	trad.	-0.013	-0.044	-0.101	-0.325	-0.424	-0.445	0.184	0.229	0.334	0.696	1.158	1.053
		rob.	-0.018	0.017	0.035	0.015	-0.264	-0.243	0.278	0.261	0.263	0.252	0.839	0.972
	300	trad.	0.003	-0.053	-0.114	-0.333	-1.062	-1.077	0.091	0.113	0.144	0.419	1.341	1.311
		rob.	0.004	0.002	0.002	-0.001	-0.503	-0.862	0.105	0.105	0.109	0.110	0.942	1.366
	500	trad.	0.000	-0.049	-0.114	-0.305	-1.366	-1.447	0.070	0.086	0.107	0.241	1.242	1.401
		rob.	-0.001	0.001	-0.000	-0.003	-0.560	-1.216	0.077	0.078	0.082	0.083	0.929	1.276
λ_{11}	100	trad.	-0.026	-0.128	-0.196	-0.271	-0.271	-0.250	0.068	0.086	0.100	0.126	0.159	0.164
		rob.	-0.035	-0.034	-0.032	-0.040	-0.218	-0.224	0.087	0.083	0.084	0.086	0.164	0.174
	300	trad.	-0.007	-0.124	-0.212	-0.322	-0.407	-0.385	0.044	0.053	0.068	0.088	0.128	0.171
		rob.	-0.007	-0.007	-0.014	-0.022	-0.270	-0.373	0.050	0.051	0.053	0.054	0.187	0.183
	500	trad.	-0.002	-0.126	-0.209	-0.319	-0.451	-0.463	0.037	0.043	0.050	0.069	0.102	0.146
		rob.	-0.003	-0.005	-0.009	-0.016	-0.264	-0.460	0.041	0.039	0.041	0.042	0.196	0.156
λ_{12}	100	trad.	-0.001	-0.104	-0.160	-0.229	-0.230	-0.202	0.087	0.104	0.110	0.129	0.157	0.163
		rob.	-0.006	-0.005	-0.006	-0.009	-0.176	-0.172	0.115	0.111	0.116	0.106	0.163	0.173
	300	trad.	-0.000	-0.106	-0.185	-0.283	-0.357	-0.336	0.054	0.062	0.071	0.088	0.120	0.162
		rob.	-0.001	-0.002	-0.005	-0.013	-0.233	-0.324	0.064	0.063	0.063	0.067	0.172	0.173
	500	trad.	-0.000	-0.110	-0.184	-0.281	-0.399	-0.410	0.044	0.049	0.057	0.071	0.096	0.137
		rob.	-0.000	-0.002	-0.005	-0.012	-0.231	-0.406	0.049	0.050	0.049	0.048	0.176	0.147
λ_{13}	100	trad.	0.015	0.027	0.031	0.024	-0.081	-0.156	0.101	0.121	0.135	0.162	0.263	0.298
		rob.	0.025	0.019	0.019	0.037	-0.055	-0.188	0.127	0.136	0.132	0.129	0.270	0.333
	300	trad.	0.002	0.039	0.060	0.076	0.068	0.004	0.066	0.066	0.078	0.091	0.135	0.213
		rob.	0.003	0.007	0.012	0.029	0.090	-0.009	0.074	0.073	0.073	0.079	0.129	0.246
	500	trad.	0.004	0.041	0.062	0.079	0.107	0.083	0.051	0.052	0.060	0.074	0.093	0.155
		rob.	0.003	0.005	0.012	0.028	0.121	0.084	0.056	0.056	0.056	0.055	0.077	0.175
λ_{21}	100	trad.	-0.013	-0.007	-0.005	-0.019	-0.116	-0.182	0.127	0.130	0.143	0.160	0.253	0.295
		rob.	-0.018	-0.017	-0.007	0.020	-0.087	-0.211	0.169	0.149	0.152	0.143	0.268	0.330
	300	trad.	-0.004	0.017	0.024	0.027	0.021	-0.036	0.075	0.076	0.080	0.093	0.134	0.211
		rob.	-0.006	0.002	0.006	0.028	0.062	-0.044	0.085	0.091	0.086	0.086	0.134	0.243
	500	trad.	0.001	0.016	0.026	0.029	0.054	0.037	0.059	0.057	0.061	0.074	0.092	0.151
		rob.	0.001	-0.000	0.010	0.028	0.097	0.040	0.066	0.065	0.064	0.066	0.087	0.169
λ_{22}	100	trad.	0.003	-0.101	-0.161	-0.222	-0.207	-0.181	0.120	0.127	0.131	0.132	0.155	0.157
		rob.	-0.009	-0.009	-0.017	-0.030	-0.159	-0.156	0.153	0.156	0.155	0.152	0.170	0.175
	300	trad.	-0.001	-0.103	-0.179	-0.260	-0.317	-0.302	0.072	0.075	0.078	0.087	0.110	0.142
		rob.	-0.003	-0.002	-0.011	-0.018	-0.213	-0.291	0.082	0.087	0.084	0.087	0.158	0.152
	500	trad.	-0.001	-0.106	-0.175	-0.264	-0.357	-0.360	0.056	0.055	0.063	0.068	0.087	0.119
		rob.	-0.003	-0.002	-0.007	-0.018	-0.216	-0.356	0.062	0.062	0.066	0.066	0.156	0.128
λ_{23}	100	trad.	-0.013	-0.100	-0.158	-0.221	-0.202	-0.185	0.127	0.121	0.131	0.136	0.147	0.155
		rob.	-0.017	-0.011	-0.020	-0.022	-0.149	-0.159	0.166	0.158	0.154	0.148	0.169	0.172
	300	trad.	-0.006	-0.107	-0.178	-0.261	-0.319	-0.297	0.076	0.074	0.078	0.087	0.108	0.145
		rob.	-0.008	-0.004	-0.009	-0.016	-0.216	-0.286	0.088	0.087	0.087	0.086	0.157	0.156
	500	trad.	0.002	-0.106	-0.175	-0.263	-0.357	-0.361	0.056	0.060	0.060	0.068	0.086	0.120
		rob.	0.004	-0.004	-0.007	-0.016	-0.214	-0.357	0.063	0.066	0.065	0.070	0.156	0.130
λ_{31}	100	trad.	-0.034	-0.039	-0.040	-0.046	-0.039	-0.044	0.142	0.161	0.166	0.169	0.169	0.173
		rob.	-0.051	-0.053	-0.056	-0.051	-0.050	-0.057	0.186	0.185	0.188	0.165	0.194	0.184
	300	trad.	-0.009	-0.013	-0.011	-0.012	-0.021	-0.022	0.096	0.099	0.107	0.109	0.123	0.133
		rob.	-0.015	-0.016	-0.015	-0.013	-0.020	-0.027	0.106	0.107	0.112	0.112	0.128	0.151
	500	trad.	-0.009	-0.003	-0.008	-0.006	-0.014	-0.021	0.083	0.084	0.089	0.093	0.111	0.117
		rob.	-0.009	-0.005	-0.007	-0.010	-0.011	-0.025	0.091	0.089	0.091	0.093	0.107	0.130
λ_{32}	100	trad.	-0.033	-0.040	-0.046	-0.040	-0.042	-0.036	0.145	0.169	0.166	0.164	0.168	0.165
		rob.	-0.054	-0.061	-0.059	-0.039	-0.056	-0.047	0.186	0.181	0.183	0.166	0.197	0.179
	300	trad.	-0.009	-0.011	-0.009	-0.010	-0.026	-0.019	0.097	0.104	0.106	0.115	0.132	0.123
		rob.	-0.012	-0.015	-0.013	-0.010	-0.023	-0.028	0.111	0.114	0.110	0.106	0.125	0.139
	500	trad.	-0.000	-0.004	-0.005	-0.010	-0.010	-0.016	0.078	0.083	0.087	0.093	0.105	0.117
		rob.	-0.003	-0.003	-0.007	-0.009	-0.007	-0.018	0.086	0.089	0.090	0.095	0.111	0.124
λ_{33}	100	trad.	-0.001	-0.012	0.003	-0.012	-0.008	-0.006	0.188	0.205	0.188	0.209	0.205	0.198
		rob.	-0.015	-0.010	-0.013	-0.012	-0.023	-0.011	0.218	0.227	0.218	0.221	0.234	0.224
	300	trad.	0.002	0.003	0.003	-0.001	0.006	0.003	0.112	0.125	0.130	0.139	0.156	0.149
		rob.	0.006	0.005	0.007	0.001	0.004	0.001	0.128	0.137	0.137	0.133	0.158	0.169
	500	trad.	0.001	-0.003	0.003	0.004	-0.000	0.010	0.093	0.098	0.100	0.107	0.130	0.132
		rob.	0.002	-0.004	0.004	0.005	-0.005	0.006	0.106	0.107	0.109	0.106	0.131	0.149

Table 8.15: Results for traditional and robust PLS for three composites and systematic outliers in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.021	0.254	0.331	0.388	0.420	0.425	0.071	0.034	0.024	0.016	0.011	0.011
		rob.	0.034	0.037	0.029	0.033	0.282	0.432	0.097	0.096	0.103	0.098	0.200	0.014
	300	trad.	0.006	0.248	0.326	0.385	0.419	0.422	0.045	0.021	0.014	0.009	0.007	0.007
		rob.	0.008	0.010	0.012	0.010	0.037	0.425	0.052	0.052	0.050	0.052	0.095	0.007
	500	trad.	0.004	0.247	0.325	0.385	0.418	0.422	0.034	0.016	0.012	0.007	0.005	0.005
		rob.	0.005	0.007	0.005	0.008	0.019	0.423	0.037	0.037	0.038	0.038	0.052	0.007
γ_{31}	100	trad.	0.016	0.137	0.158	0.171	0.183	0.181	0.124	0.103	0.109	0.104	0.103	0.103
		rob.	0.020	0.022	0.025	0.017	0.129	0.186	0.211	0.203	0.189	0.170	0.191	0.156
	300	trad.	0.005	0.132	0.156	0.168	0.177	0.182	0.063	0.061	0.061	0.061	0.060	0.058
		rob.	0.007	0.009	0.007	0.007	0.025	0.181	0.073	0.072	0.078	0.078	0.093	0.066
	500	trad.	0.004	0.130	0.155	0.171	0.180	0.181	0.049	0.046	0.047	0.048	0.045	0.045
		rob.	0.006	0.006	0.006	0.006	0.016	0.180	0.054	0.053	0.057	0.058	0.070	0.049
γ_{32}	100	trad.	0.030	0.301	0.375	0.429	0.455	0.463	0.138	0.105	0.110	0.105	0.104	0.103
		rob.	0.057	0.057	0.047	0.066	0.309	0.465	0.232	0.227	0.225	0.194	0.258	0.156
	300	trad.	0.015	0.296	0.373	0.429	0.458	0.459	0.068	0.062	0.061	0.061	0.060	0.058
		rob.	0.017	0.012	0.016	0.019	0.044	0.465	0.081	0.081	0.083	0.083	0.123	0.066
	500	trad.	0.008	0.296	0.373	0.425	0.455	0.461	0.050	0.048	0.048	0.048	0.045	0.045
		rob.	0.008	0.008	0.009	0.006	0.022	0.464	0.059	0.060	0.060	0.061	0.076	0.048
w_{11}	100	trad.	-0.004	-0.148	-0.166	-0.195	-0.220	-0.222	0.185	0.124	0.108	0.096	0.078	0.077
		rob.	-0.030	-0.064	-0.048	-0.025	-0.163	-0.225	0.291	0.300	0.282	0.263	0.212	0.110
	300	trad.	-0.006	-0.137	-0.166	-0.193	-0.218	-0.222	0.100	0.070	0.061	0.051	0.046	0.043
		rob.	-0.006	-0.006	-0.006	-0.002	-0.025	-0.221	0.112	0.116	0.114	0.117	0.136	0.050
	500	trad.	-0.004	-0.134	-0.169	-0.193	-0.215	-0.222	0.074	0.053	0.047	0.041	0.034	0.033
		rob.	-0.007	-0.005	-0.005	-0.001	-0.011	-0.223	0.085	0.085	0.087	0.091	0.100	0.036
w_{12}	100	trad.	-0.024	-0.017	-0.037	-0.044	-0.045	-0.048	0.197	0.125	0.111	0.095	0.081	0.074
		rob.	-0.053	-0.000	-0.021	-0.029	-0.037	-0.050	0.301	0.296	0.304	0.274	0.213	0.110
	300	trad.	-0.003	-0.023	-0.041	-0.044	-0.045	-0.047	0.110	0.072	0.063	0.052	0.045	0.042
		rob.	-0.006	-0.006	-0.006	-0.005	-0.000	-0.048	0.127	0.128	0.123	0.125	0.137	0.049
	500	trad.	0.001	-0.023	-0.034	-0.044	-0.047	-0.049	0.080	0.054	0.049	0.042	0.034	0.033
		rob.	0.002	0.002	-0.002	-0.007	0.000	-0.049	0.091	0.090	0.095	0.100	0.103	0.036
w_{13}	100	trad.	-0.005	0.084	0.093	0.104	0.119	0.125	0.199	0.124	0.114	0.100	0.080	0.077
		rob.	0.004	-0.014	-0.011	-0.017	0.071	0.124	0.306	0.297	0.315	0.282	0.224	0.113
	300	trad.	0.000	0.084	0.099	0.105	0.118	0.125	0.111	0.072	0.067	0.054	0.044	0.041
		rob.	-0.000	-0.002	-0.001	-0.007	-0.000	0.124	0.130	0.130	0.134	0.129	0.142	0.048
	500	trad.	-0.003	0.082	0.097	0.105	0.118	0.128	0.081	0.054	0.048	0.040	0.035	0.033
		rob.	-0.002	-0.006	-0.000	-0.002	-0.003	0.128	0.092	0.093	0.095	0.101	0.110	0.036
w_{21}	100	trad.	-0.011	0.080	0.070	0.062	0.057	0.054	0.202	0.098	0.084	0.074	0.067	0.069
		rob.	-0.033	-0.025	-0.020	-0.023	0.025	0.055	0.295	0.291	0.290	0.263	0.193	0.098
	300	trad.	-0.002	0.079	0.072	0.065	0.057	0.055	0.105	0.055	0.047	0.042	0.039	0.040
		rob.	-0.004	-0.008	-0.003	-0.002	0.004	0.054	0.124	0.123	0.125	0.124	0.131	0.046
	500	trad.	-0.003	0.079	0.072	0.063	0.054	0.057	0.082	0.045	0.037	0.033	0.032	0.031
		rob.	-0.004	-0.005	-0.001	-0.003	-0.000	0.059	0.091	0.094	0.095	0.095	0.107	0.033
w_{22}	100	trad.	-0.016	-0.142	-0.162	-0.177	-0.173	-0.169	0.194	0.118	0.104	0.089	0.079	0.072
		rob.	-0.038	-0.037	-0.054	-0.049	-0.127	-0.174	0.296	0.298	0.283	0.278	0.215	0.108
	300	trad.	-0.002	-0.137	-0.160	-0.177	-0.172	-0.168	0.111	0.072	0.057	0.049	0.044	0.042
		rob.	-0.005	-0.002	-0.003	-0.011	-0.021	-0.169	0.127	0.132	0.127	0.131	0.142	0.049
	500	trad.	-0.002	-0.136	-0.161	-0.175	-0.171	-0.169	0.081	0.054	0.043	0.040	0.035	0.033
		rob.	-0.001	-0.003	-0.005	-0.003	-0.008	-0.170	0.093	0.091	0.092	0.099	0.108	0.035
w_{23}	100	trad.	-0.027	-0.120	-0.156	-0.183	-0.211	-0.215	0.182	0.105	0.092	0.081	0.074	0.069
		rob.	-0.051	-0.068	-0.048	-0.031	-0.158	-0.218	0.278	0.289	0.278	0.251	0.209	0.103
	300	trad.	-0.010	-0.118	-0.156	-0.184	-0.209	-0.214	0.104	0.065	0.052	0.044	0.040	0.040
		rob.	-0.010	-0.014	-0.014	-0.008	-0.023	-0.216	0.119	0.123	0.114	0.121	0.132	0.046
	500	trad.	-0.004	-0.117	-0.153	-0.184	-0.208	-0.216	0.072	0.047	0.040	0.035	0.032	0.031
		rob.	-0.007	-0.006	-0.004	-0.008	-0.014	-0.218	0.082	0.084	0.086	0.091	0.101	0.034
w_{31}	100	trad.	-0.047	-0.062	-0.068	-0.082	-0.076	-0.072	0.356	0.150	0.126	0.102	0.087	0.081
		rob.	-0.079	-0.104	-0.060	-0.091	-0.076	-0.072	0.465	0.444	0.455	0.428	0.290	0.116
	300	trad.	-0.009	-0.059	-0.077	-0.078	-0.075	-0.068	0.201	0.084	0.068	0.059	0.047	0.047
		rob.	-0.009	-0.027	-0.033	-0.010	-0.036	-0.069	0.224	0.239	0.242	0.242	0.253	0.054
	500	trad.	-0.019	-0.054	-0.072	-0.079	-0.075	-0.069	0.162	0.067	0.053	0.045	0.036	0.036
		rob.	-0.024	-0.004	-0.011	-0.004	-0.007	-0.068	0.177	0.175	0.174	0.185	0.195	0.039
w_{32}	100	trad.	-0.057	-0.056	-0.080	-0.097	-0.113	-0.120	0.303	0.123	0.104	0.090	0.081	0.073
		rob.	-0.128	-0.111	-0.141	-0.111	-0.126	-0.122	0.414	0.402	0.399	0.398	0.272	0.104
	300	trad.	-0.020	-0.049	-0.074	-0.096	-0.113	-0.121	0.177	0.070	0.059	0.050	0.045	0.045
		rob.	-0.025	-0.021	-0.030	-0.029	-0.027	-0.122	0.203	0.208	0.213	0.204	0.215	0.051
	500	trad.	-0.012	-0.053	-0.076	-0.095	-0.114	-0.120	0.141	0.054	0.045	0.040	0.036	0.034
		rob.	-0.013	-0.022	-0.016	-0.019	-0.019	-0.121	0.158	0.159	0.156	0.163	0.175	0.036
w_{33}	100	trad.	-0.074	-0.092	-0.117	-0.128	-0.143	-0.141	0.348	0.140	0.114	0.094	0.083	0.080
		rob.	-0.121	-0.089	-0.119	-0.088	-0.145	-0.146	0.447	0.428	0.442	0.408	0.298	0.114
	300	trad.	-0.025	-0.091	-0.106	-0.131	-0.142	-0.143	0.194	0.081	0.062	0.054	0.050	0.045
		rob.	-0.035	-0.026	-0.013	-0.037	-0.038	-0.144	0.227	0.220	0.217	0.222	0.237	0.050
	500	trad.	-0.001	-0.090	-0.110	-0.131	-0.140	-0.143	0.151	0.062	0.050	0.042	0.037	0.035
		rob.	-0.003	-0.015	-0.013	-0.022	-0.032	-0.143	0.167	0.167	0.165	0.170	0.185	0.038

Table 8.16: Results for traditional and robust PLS for three composites and systematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
γ_{21}	100	trad.	0.020	0.060	0.130	0.230	0.299	0.307	0.072	0.066	0.063	0.047	0.033	0.029
		rob.	0.036	0.033	0.034	0.054	0.308	0.321	0.107	0.111	0.109	0.105	0.060	0.046
	300	trad.	0.008	0.040	0.115	0.228	0.295	0.301	0.042	0.040	0.036	0.025	0.016	0.016
		rob.	0.011	0.012	0.017	0.027	0.290	0.310	0.049	0.047	0.049	0.050	0.057	0.018
	500	trad.	0.004	0.036	0.112	0.227	0.295	0.301	0.033	0.032	0.027	0.018	0.013	0.012
		rob.	0.005	0.007	0.009	0.021	0.293	0.308	0.038	0.038	0.037	0.038	0.043	0.013
γ_{31}	100	trad.	0.019	0.006	-0.043	-0.120	-0.152	-0.151	0.122	0.137	0.166	0.197	0.209	0.211
		rob.	0.025	0.026	0.026	0.020	-0.121	-0.109	0.213	0.198	0.195	0.188	0.341	0.332
	300	trad.	0.008	-0.005	-0.055	-0.153	-0.195	-0.200	0.064	0.069	0.102	0.118	0.120	0.117
		rob.	0.011	0.007	0.013	0.008	-0.171	-0.194	0.075	0.075	0.075	0.079	0.146	0.138
	500	trad.	0.000	-0.007	-0.064	-0.155	-0.207	-0.215	0.048	0.052	0.077	0.091	0.093	0.094
		rob.	0.000	0.006	0.003	0.005	-0.194	-0.212	0.056	0.054	0.057	0.057	0.111	0.104
γ_{32}	100	trad.	0.032	0.026	0.024	0.030	0.036	0.027	0.136	0.144	0.163	0.179	0.201	0.206
		rob.	0.056	0.054	0.057	0.042	0.053	0.040	0.223	0.222	0.207	0.197	0.335	0.326
	300	trad.	0.007	-0.002	-0.015	-0.007	0.007	0.010	0.066	0.073	0.083	0.105	0.114	0.115
		rob.	0.011	0.014	0.009	0.010	0.008	0.011	0.077	0.082	0.079	0.084	0.134	0.132
	500	trad.	0.009	-0.005	-0.022	-0.022	-0.003	-0.000	0.051	0.054	0.061	0.079	0.088	0.089
		rob.	0.010	0.008	0.010	0.006	0.001	0.002	0.057	0.060	0.059	0.060	0.097	0.102
w_{11}	100	trad.	-0.008	-0.148	-0.344	-0.536	-0.584	-0.589	0.184	0.245	0.289	0.225	0.162	0.161
		rob.	-0.056	-0.059	-0.049	-0.085	-0.555	-0.566	0.312	0.295	0.265	0.268	0.237	0.214
	300	trad.	-0.001	-0.115	-0.322	-0.547	-0.602	-0.599	0.096	0.126	0.172	0.124	0.087	0.090
		rob.	-0.001	-0.004	-0.014	-0.039	-0.573	-0.603	0.113	0.113	0.114	0.118	0.147	0.100
	500	trad.	-0.002	-0.113	-0.327	-0.548	-0.601	-0.603	0.076	0.094	0.125	0.094	0.067	0.065
		rob.	0.000	-0.010	-0.013	-0.037	-0.584	-0.605	0.087	0.089	0.089	0.091	0.113	0.069
w_{12}	100	trad.	-0.018	-0.123	-0.264	-0.395	-0.422	-0.416	0.207	0.230	0.234	0.191	0.140	0.136
		rob.	-0.036	-0.030	-0.040	-0.077	-0.397	-0.400	0.317	0.295	0.294	0.285	0.198	0.190
	300	trad.	-0.001	-0.100	-0.255	-0.409	-0.432	-0.424	0.103	0.117	0.142	0.104	0.073	0.072
		rob.	-0.007	-0.011	-0.012	-0.034	-0.412	-0.426	0.120	0.122	0.120	0.129	0.116	0.081
	500	trad.	-0.003	-0.095	-0.260	-0.414	-0.431	-0.427	0.083	0.091	0.107	0.077	0.056	0.054
		rob.	-0.004	-0.004	-0.011	-0.032	-0.421	-0.428	0.093	0.090	0.092	0.093	0.085	0.058
w_{13}	100	trad.	-0.016	0.241	0.484	0.686	0.752	0.746	0.212	0.331	0.394	0.313	0.202	0.230
		rob.	-0.005	0.012	0.013	0.094	0.685	0.683	0.325	0.314	0.317	0.333	0.336	0.363
	300	trad.	-0.009	0.264	0.566	0.766	0.795	0.791	0.113	0.173	0.188	0.119	0.022	0.066
		rob.	-0.007	0.002	0.016	0.070	0.759	0.790	0.129	0.128	0.130	0.150	0.151	0.067
	500	trad.	0.001	0.268	0.592	0.780	0.799	0.798	0.083	0.129	0.128	0.041	0.015	0.012
		rob.	-0.003	0.008	0.020	0.073	0.779	0.798	0.094	0.097	0.097	0.109	0.099	0.013
w_{21}	100	trad.	-0.021	0.174	0.385	0.573	0.647	0.645	0.194	0.320	0.379	0.308	0.201	0.232
		rob.	-0.026	-0.022	0.009	0.070	0.583	0.588	0.293	0.280	0.297	0.301	0.324	0.363
	300	trad.	-0.003	0.211	0.475	0.652	0.687	0.686	0.108	0.162	0.169	0.114	0.018	0.065
		rob.	-0.004	0.013	0.027	0.066	0.658	0.686	0.125	0.125	0.123	0.142	0.125	0.066
	500	trad.	-0.007	0.215	0.502	0.665	0.691	0.693	0.083	0.122	0.109	0.033	0.013	0.011
		rob.	-0.004	0.009	0.030	0.076	0.674	0.693	0.094	0.090	0.093	0.104	0.085	0.012
w_{22}	100	trad.	-0.018	-0.123	-0.282	-0.435	-0.486	-0.490	0.201	0.237	0.250	0.200	0.142	0.136
		rob.	-0.054	-0.054	-0.055	-0.079	-0.457	-0.472	0.306	0.289	0.287	0.287	0.208	0.194
	300	trad.	0.000	-0.091	-0.266	-0.451	-0.500	-0.496	0.109	0.129	0.154	0.108	0.073	0.077
		rob.	0.000	-0.010	-0.014	-0.032	-0.475	-0.499	0.129	0.127	0.125	0.132	0.122	0.086
	500	trad.	0.002	-0.080	-0.274	-0.458	-0.499	-0.502	0.083	0.092	0.111	0.080	0.056	0.056
		rob.	-0.000	0.001	-0.015	-0.037	-0.483	-0.503	0.093	0.092	0.094	0.094	0.096	0.060
w_{23}	100	trad.	-0.015	-0.096	-0.239	-0.406	-0.491	-0.514	0.180	0.206	0.214	0.175	0.135	0.129
		rob.	-0.052	-0.043	-0.059	-0.081	-0.465	-0.503	0.296	0.284	0.266	0.257	0.195	0.180
	300	trad.	-0.012	-0.064	-0.210	-0.409	-0.501	-0.516	0.099	0.101	0.130	0.099	0.075	0.071
		rob.	-0.017	-0.016	-0.022	-0.031	-0.479	-0.522	0.116	0.114	0.117	0.122	0.124	0.080
	500	trad.	-0.007	-0.061	-0.206	-0.409	-0.497	-0.518	0.077	0.079	0.095	0.075	0.054	0.054
		rob.	-0.009	-0.014	-0.013	-0.024	-0.483	-0.521	0.087	0.088	0.089	0.086	0.095	0.059
w_{31}	100	trad.	-0.055	-0.055	-0.120	-0.230	-0.271	-0.291	0.358	0.401	0.486	0.564	0.619	0.619
		rob.	-0.081	-0.068	-0.078	-0.084	-0.243	-0.282	0.451	0.457	0.446	0.441	0.634	0.637
	300	trad.	-0.008	-0.024	-0.049	-0.151	-0.235	-0.243	0.200	0.219	0.328	0.506	0.581	0.580
		rob.	-0.014	-0.025	-0.013	-0.024	-0.217	-0.241	0.228	0.229	0.236	0.247	0.575	0.587
	500	trad.	-0.006	-0.011	-0.032	-0.111	-0.185	-0.245	0.154	0.181	0.256	0.460	0.548	0.568
		rob.	-0.005	-0.010	-0.023	-0.012	-0.194	-0.262	0.175	0.180	0.180	0.191	0.545	0.580
w_{32}	100	trad.	-0.083	-0.072	-0.153	-0.230	-0.325	-0.351	0.321	0.349	0.445	0.521	0.550	0.575
		rob.	-0.131	-0.116	-0.116	-0.086	-0.319	-0.339	0.423	0.406	0.404	0.398	0.553	0.582
	300	trad.	-0.025	-0.026	-0.055	-0.200	-0.296	-0.338	0.177	0.199	0.291	0.477	0.552	0.568
		rob.	-0.032	-0.032	-0.027	-0.027	-0.284	-0.356	0.211	0.213	0.210	0.220	0.540	0.567
	500	trad.	-0.010	-0.015	-0.031	-0.150	-0.245	-0.281	0.139	0.149	0.239	0.419	0.516	0.541
		rob.	-0.013	-0.019	-0.010	-0.016	-0.243	-0.289	0.157	0.154	0.155	0.160	0.513	0.546
w_{33}	100	trad.	-0.049	-0.117	-0.157	-0.263	-0.312	-0.336	0.333	0.375	0.459	0.554	0.597	0.609
		rob.	-0.105	-0.125	-0.106	-0.132	-0.332	-0.314	0.421	0.432	0.427	0.432	0.627	0.620
	300	trad.	-0.020	-0.019	-0.070	-0.172	-0.316	-0.311	0.191	0.213	0.321	0.482	0.587	0.575
		rob.	-0.026	-0.017	-0.036	-0.032	-0.293	-0.325	0.222	0.215	0.226	0.236	0.579	0.591
	500	trad.	-0.015	-0.012	-0.032	-0.133	-0.245	-0.262	0.146	0.158	0.230	0.450	0.542	0.572
		rob.	-0.022	-0.011	-0.005	-0.015	-0.244	-0.269	0.168	0.164	0.160	0.176	0.547	0.582

Table 8.17: Results for traditional and robust PLSc for five common factors and systematic outliers in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.011	0.151	0.202	0.240	0.265	0.271	0.085	0.041	0.027	0.018	0.013	0.012
		rob.	0.010	0.008	0.006	0.011	0.263	0.269	0.109	0.111	0.109	0.113	0.016	0.015
	300	trad.	0.004	0.150	0.199	0.239	0.266	0.273	0.050	0.023	0.016	0.011	0.008	0.008
		rob.	0.001	0.005	0.002	0.004	0.255	0.273	0.057	0.056	0.057	0.060	0.055	0.009
	500	trad.	0.000	0.149	0.199	0.239	0.267	0.273	0.038	0.018	0.013	0.008	0.006	0.006
		rob.	0.002	0.000	0.001	-0.001	0.243	0.273	0.042	0.043	0.043	0.046	0.075	0.006
γ_{32}	100	trad.	-0.030	0.139	0.370	0.634	0.750	0.791	0.161	0.410	0.531	0.512	0.474	0.412
		rob.	-0.054	-0.049	-0.052	-0.032	0.582	0.622	0.233	0.245	0.223	0.253	0.623	0.594
	300	trad.	-0.003	0.253	0.725	0.869	0.920	0.930	0.079	0.355	0.273	0.097	0.041	0.072
		rob.	-0.006	-0.013	-0.008	-0.014	0.874	0.923	0.092	0.092	0.093	0.093	0.223	0.071
	500	trad.	-0.003	0.292	0.784	0.872	0.920	0.928	0.062	0.305	0.108	0.055	0.039	0.036
		rob.	-0.006	-0.006	-0.006	-0.006	0.838	0.935	0.063	0.067	0.071	0.070	0.271	0.144
γ_{41}	100	trad.	-0.028	0.113	0.174	0.312	0.369	0.329	0.231	0.315	0.415	0.563	0.956	1.352
		rob.	0.025	-0.004	0.009	-0.012	0.350	0.293	0.319	0.315	0.319	0.334	1.172	1.552
	300	trad.	-0.008	0.116	0.191	0.271	0.333	0.368	0.132	0.170	0.219	0.289	0.463	0.606
		rob.	-0.015	-0.025	-0.016	-0.020	0.330	0.354	0.150	0.147	0.151	0.157	0.520	0.697
	500	trad.	-0.007	0.128	0.192	0.264	0.322	0.356	0.098	0.128	0.166	0.225	0.366	0.427
		rob.	-0.013	-0.002	-0.007	-0.012	0.259	0.346	0.108	0.107	0.110	0.115	0.365	0.470
γ_{42}	100	trad.	0.030	-0.400	-0.547	-0.761	-0.892	-0.888	0.223	0.318	0.422	0.567	0.952	1.353
		rob.	-0.009	0.016	0.001	0.014	-0.855	-0.844	0.304	0.307	0.307	0.321	1.165	1.539
	300	trad.	0.009	-0.422	-0.584	-0.749	-0.879	-0.933	0.131	0.176	0.217	0.288	0.466	0.606
		rob.	0.020	0.027	0.017	0.025	-0.842	-0.921	0.148	0.151	0.150	0.154	0.547	0.698
	500	trad.	0.007	-0.437	-0.589	-0.745	-0.873	-0.929	0.097	0.130	0.166	0.227	0.365	0.428
		rob.	0.012	0.002	0.008	0.011	-0.755	-0.919	0.105	0.107	0.109	0.113	0.428	0.469
γ_{51}	100	trad.	0.028	-0.035	-0.011	0.031	0.041	0.186	0.233	0.303	0.452	0.626	1.201	3.452
		rob.	-0.004	0.017	0.004	-0.014	-0.101	-0.011	0.322	0.309	0.308	0.314	1.351	2.543
	300	trad.	-0.004	-0.037	0.013	0.061	0.094	0.009	0.112	0.165	0.259	0.352	0.594	0.741
		rob.	-0.002	-0.003	-0.005	-0.000	0.046	0.018	0.143	0.144	0.133	0.150	0.668	0.829
	500	trad.	-0.009	-0.044	0.020	0.069	0.103	0.070	0.088	0.124	0.191	0.289	0.419	0.544
		rob.	-0.007	-0.014	-0.008	-0.008	0.095	0.125	0.100	0.101	0.101	0.106	0.457	0.605
γ_{52}	100	trad.	-0.062	0.148	0.021	-0.152	-0.063	-0.348	0.230	0.396	0.793	1.140	1.839	9.797
		rob.	-0.045	-0.057	-0.043	-0.027	0.131	0.165	0.318	0.314	0.303	0.317	1.746	2.832
	300	trad.	-0.017	0.115	-0.248	-0.555	-0.610	-0.312	0.112	0.295	0.590	0.852	1.422	1.590
		rob.	-0.024	-0.020	-0.019	-0.027	-0.463	-0.178	0.143	0.145	0.130	0.149	1.398	1.565
	500	trad.	-0.014	0.113	-0.294	-0.587	-0.754	-0.589	0.084	0.259	0.512	0.849	1.227	1.365
		rob.	-0.013	-0.008	-0.014	-0.016	-0.663	-0.624	0.098	0.097	0.098	0.105	1.198	1.461
γ_{53}	100	trad.	-0.023	0.139	0.265	0.387	0.270	0.415	0.127	0.212	0.520	0.798	1.152	6.783
		rob.	-0.000	0.003	0.007	0.016	0.318	0.239	0.195	0.191	0.176	0.184	0.997	1.089
	300	trad.	-0.039	0.107	0.295	0.541	0.582	0.376	0.062	0.171	0.443	0.675	1.127	1.219
		rob.	-0.036	-0.036	-0.037	-0.036	0.477	0.238	0.070	0.071	0.072	0.073	1.052	1.240
	500	trad.	-0.046	0.106	0.288	0.567	0.725	0.597	0.048	0.157	0.422	0.691	1.047	1.117
		rob.	-0.042	-0.044	-0.043	-0.040	0.624	0.574	0.050	0.052	0.051	0.055	0.974	1.156
λ_{11}	100	trad.	0.002	0.078	0.105	0.126	0.135	0.130	0.061	0.036	0.026	0.018	0.015	0.015
		rob.	0.016	0.015	0.015	0.018	0.142	0.137	0.076	0.077	0.078	0.076	0.020	0.020
	300	trad.	0.001	0.076	0.104	0.125	0.134	0.130	0.037	0.020	0.016	0.011	0.008	0.008
		rob.	0.002	0.002	0.002	0.002	0.131	0.132	0.043	0.041	0.042	0.043	0.029	0.009
	500	trad.	0.001	0.075	0.104	0.125	0.134	0.129	0.029	0.017	0.012	0.008	0.006	0.006
		rob.	-0.000	-0.001	-0.001	0.001	0.124	0.131	0.030	0.032	0.033	0.035	0.038	0.007
λ_{12}	100	trad.	-0.007	0.020	0.034	0.043	0.042	0.037	0.057	0.034	0.024	0.018	0.015	0.015
		rob.	-0.018	-0.017	-0.018	-0.013	0.051	0.044	0.069	0.068	0.070	0.065	0.019	0.020
	300	trad.	-0.000	0.022	0.034	0.042	0.042	0.037	0.037	0.020	0.015	0.010	0.008	0.008
		rob.	-0.002	-0.001	-0.002	-0.002	0.043	0.038	0.041	0.040	0.041	0.042	0.015	0.010
	500	trad.	-0.001	0.023	0.033	0.043	0.042	0.036	0.028	0.016	0.012	0.008	0.006	0.006
		rob.	-0.001	0.002	0.001	0.000	0.041	0.038	0.030	0.031	0.032	0.034	0.018	0.007
λ_{21}	100	trad.	-0.005	0.260	0.362	0.444	0.490	0.495	0.102	0.069	0.048	0.032	0.023	0.022
		rob.	0.000	-0.009	0.004	0.011	0.495	0.501	0.148	0.149	0.147	0.159	0.033	0.030
	300	trad.	-0.001	0.262	0.369	0.446	0.489	0.495	0.061	0.041	0.026	0.017	0.013	0.012
		rob.	-0.004	-0.005	-0.003	0.002	0.472	0.497	0.072	0.072	0.071	0.073	0.100	0.012
	500	trad.	-0.001	0.261	0.369	0.446	0.490	0.494	0.049	0.031	0.021	0.013	0.009	0.009
		rob.	-0.001	-0.004	-0.003	0.001	0.449	0.496	0.051	0.051	0.055	0.054	0.137	0.009
λ_{22}	100	trad.	0.001	0.261	0.364	0.445	0.490	0.495	0.108	0.068	0.048	0.033	0.022	0.022
		rob.	0.000	0.004	0.002	0.012	0.495	0.502	0.144	0.153	0.146	0.159	0.032	0.029
	300	trad.	-0.002	0.260	0.367	0.445	0.489	0.494	0.060	0.041	0.027	0.018	0.012	0.012
		rob.	-0.003	-0.002	0.001	-0.002	0.471	0.497	0.071	0.074	0.072	0.075	0.104	0.012
	500	trad.	0.001	0.259	0.369	0.445	0.489	0.493	0.047	0.032	0.019	0.013	0.009	0.009
		rob.	-0.000	-0.002	-0.003	-0.003	0.448	0.496	0.052	0.054	0.053	0.056	0.140	0.009
λ_{23}	100	trad.	-0.010	0.261	0.365	0.446	0.490	0.495	0.110	0.069	0.047	0.033	0.023	0.021
		rob.	-0.005	0.001	-0.009	0.012	0.496	0.503	0.151	0.142	0.147	0.160	0.032	0.030
	300	trad.	-0.002	0.263	0.369	0.445	0.490	0.494	0.062	0.040	0.027	0.017	0.012	0.011
		rob.	-0.002	-0.005	-0.001	-0.002	0.471	0.497	0.068	0.069	0.070	0.071	0.101	0.013
	500	trad.	-0.003	0.261	0.369	0.446	0.490	0.493	0.049	0.031	0.021	0.013	0.009	0.009
		rob.	-0.005	-0.001	-0.000	0.001	0.449	0.496	0.051	0.052	0.053	0.056	0.138	0.009
λ_{24}	100	trad.	-0.008	0.181	0.237	0.279	0.307	0.307	0.085	0.054	0.040	0.028	0.020	0.020
		rob.	0.000	0.002	0.000	0.004	0.312	0.315	0.122	0.117	0.113	0.123	0.028	0.028
	300	trad.	-0.003	0.181	0.230	0.277	0.305	0.307	0.050	0.032	0.023	0.015	0.011	0.011
		rob.	-0.002	-0.003	-0.002	0.001	0.294	0.309	0.057	0.057	0.059	0.058	0.067	0.012
	500	trad.	-0.003	0.181	0.230	0.278	0.305	0.306	0.039	0.025	0.017	0.012	0.009	0.008
		rob.	-0.003	-0.001	-0.000	-0.001	0.281	0.308	0.043	0.042	0.044	0.045	0.086	0.009
λ_{25}	100	trad.	-0.007	0.181	0.233	0.280	0.307	0.307	0.088	0.054	0.041	0.027	0.021	0.020
		rob.	0.002	-0.001	-0.003	0.005	0.313	0.313	0.117	0.119	0.114	0.126	0.029	0.027
	300	trad.	-0.002	0.181	0.231	0.279	0.305	0.307	0.051	0.031	0.022	0.015	0.011	0.011
		rob.	-0.004	-0.003	-0.002	-0.005	0.294	0.309	0.058	0.056	0.057	0.060	0.066	0.012
	500	trad.	-0.001	0.181	0.230	0.278	0.305	0.306	0.037	0.025	0.017	0.012	0.008	0.008
		rob.	0.001	-0.002	-0.005	-0.003	0.280	0.308	0.042	0.043	0.044	0.045	0.091	0.008
λ_{26}	100	trad.	-0.011	0.180	0.235	0.281	0.307	0.306	0.085	0.055	0.040	0.028	0.020	0.020
		rob.	-0.002	-0.001	-0.002	0.001	0.313	0.312	0.118	0.119	0.119	0.124	0.029	0.027
	300	trad.	-0.002	0.179	0.231	0.279	0.305	0.306	0.050	0.033	0.022	0.015	0.011	0.011
		rob.	-0.001	-0.004	-0.003	-0.006	0.294	0.309	0.058	0.057	0.056	0.061	0.065	0.011
	500	trad.	-0.002	0.180	0.229	0.278	0.305	0.307	0.039	0.027	0.017	0.011	0.008	0.008
		rob.	-0.002	-0.003	-0.004	-0.002	0.280	0.308	0.043	0.042	0.043	0.045	0.089	0.009

λ_{31}	100	trad.	-0.016	0.046	0.197	0.332	0.391	0.431	0.284	0.528	0.549	0.494	0.465	0.412
		rob.	-0.012	-0.027	-0.024	-0.019	0.248	0.283	0.332	0.331	0.335	0.354	0.642	0.616
	300	trad.	0.009	0.226	0.411	0.490	0.523	0.526	0.166	0.451	0.239	0.088	0.043	0.047
		rob.	0.000	-0.002	0.001	0.002	0.500	0.530	0.197	0.198	0.189	0.203	0.156	0.068
	500	trad.	-0.008	0.319	0.441	0.497	0.527	0.530	0.130	0.380	0.103	0.055	0.040	0.037
		rob.	0.000	-0.002	-0.001	-0.008	0.479	0.531	0.146	0.149	0.141	0.157	0.175	0.044
λ_{32}	100	trad.	-0.037	-0.216	-0.378	-0.516	-0.588	-0.593	0.238	0.240	0.248	0.217	0.193	0.188
		rob.	-0.056	-0.079	-0.064	-0.082	-0.496	-0.504	0.281	0.302	0.289	0.284	0.250	0.251
	300	trad.	-0.015	-0.284	-0.610	-0.623	-0.625	-0.635	0.137	0.235	0.133	0.061	0.072	0.077
		rob.	-0.013	-0.011	-0.004	-0.005	-0.608	-0.641	0.154	0.157	0.158	0.154	0.151	0.096
	500	trad.	-0.005	-0.292	-0.645	-0.628	-0.617	-0.622	0.108	0.219	0.070	0.049	0.042	0.049
		rob.	-0.003	-0.010	-0.002	0.007	-0.568	-0.625	0.113	0.120	0.119	0.120	0.186	0.058
λ_{33}	100	trad.	-0.093	-0.318	-0.524	-0.699	-0.778	-0.789	0.183	0.272	0.279	0.241	0.214	0.204
		rob.	-0.145	-0.140	-0.149	-0.152	-0.672	-0.686	0.239	0.245	0.247	0.240	0.277	0.280
	300	trad.	-0.019	-0.392	-0.809	-0.837	-0.831	-0.834	0.100	0.286	0.154	0.056	0.071	0.085
		rob.	-0.030	-0.032	-0.032	-0.035	-0.803	-0.841	0.115	0.121	0.117	0.121	0.183	0.103
	500	trad.	-0.006	-0.399	-0.852	-0.838	-0.822	-0.820	0.081	0.271	0.073	0.044	0.042	0.049
		rob.	-0.015	-0.013	-0.018	-0.019	-0.753	-0.825	0.089	0.088	0.090	0.095	0.232	0.058
λ_{34}	100	trad.	-0.121	-0.377	-0.605	-0.790	-0.878	-0.886	0.162	0.285	0.301	0.252	0.222	0.219
		rob.	-0.191	-0.194	-0.194	-0.195	-0.761	-0.770	0.218	0.229	0.232	0.230	0.292	0.292
	300	trad.	-0.038	-0.448	-0.908	-0.939	-0.933	-0.936	0.081	0.306	0.164	0.055	0.074	0.086
		rob.	-0.054	-0.053	-0.054	-0.059	-0.902	-0.942	0.095	0.101	0.095	0.095	0.198	0.107
	500	trad.	-0.023	-0.457	-0.953	-0.938	-0.924	-0.922	0.064	0.293	0.077	0.044	0.041	0.049
		rob.	-0.030	-0.031	-0.028	-0.038	-0.846	-0.929	0.073	0.071	0.073	0.076	0.258	0.059
λ_{41}	100	trad.	-0.026	0.006	0.021	0.038	0.058	0.062	0.094	0.120	0.132	0.163	0.171	0.167
		rob.	-0.013	-0.015	-0.014	-0.015	0.023	0.030	0.126	0.120	0.120	0.119	0.230	0.226
	300	trad.	-0.032	-0.017	-0.009	0.011	0.036	0.053	0.054	0.076	0.091	0.108	0.134	0.139
		rob.	-0.030	-0.030	-0.029	-0.026	0.038	0.054	0.060	0.063	0.065	0.065	0.130	0.139
	500	trad.	-0.029	-0.027	-0.022	-0.004	0.027	0.038	0.041	0.062	0.072	0.095	0.117	0.128
		rob.	-0.031	-0.031	-0.031	-0.029	0.024	0.042	0.046	0.048	0.046	0.049	0.117	0.137
λ_{42}	100	trad.	-0.036	-0.065	-0.073	-0.107	-0.127	-0.142	0.091	0.118	0.143	0.152	0.167	0.197
		rob.	-0.041	-0.039	-0.042	-0.039	-0.144	-0.158	0.111	0.114	0.111	0.111	0.231	0.248
	300	trad.	-0.029	-0.039	-0.045	-0.066	-0.087	-0.104	0.058	0.088	0.102	0.120	0.136	0.142
		rob.	-0.032	-0.032	-0.034	-0.034	-0.092	-0.109	0.067	0.068	0.067	0.065	0.142	0.143
	500	trad.	-0.030	-0.028	-0.035	-0.051	-0.083	-0.090	0.046	0.074	0.087	0.107	0.125	0.133
		rob.	-0.031	-0.028	-0.027	-0.029	-0.078	-0.094	0.048	0.050	0.051	0.049	0.123	0.142
λ_{51}	100	trad.	0.001	0.002	-0.004	-0.057	-0.106	-0.174	0.138	0.178	0.211	0.311	0.350	0.390
		rob.	-0.002	-0.002	0.003	-0.011	-0.142	-0.168	0.178	0.179	0.179	0.179	0.363	0.382
	300	trad.	-0.006	0.005	-0.004	0.002	-0.079	-0.120	0.077	0.113	0.177	0.233	0.325	0.356
		rob.	-0.005	-0.000	0.002	-0.004	-0.103	-0.133	0.088	0.090	0.087	0.092	0.337	0.356
	500	trad.	-0.004	-0.002	-0.008	0.002	-0.035	-0.062	0.061	0.086	0.138	0.181	0.273	0.313
		rob.	-0.003	-0.005	-0.003	0.004	-0.033	-0.081	0.068	0.068	0.068	0.070	0.273	0.333
λ_{52}	100	trad.	0.002	-0.006	-0.009	-0.055	-0.136	-0.149	0.136	0.170	0.230	0.301	0.362	0.390
		rob.	0.002	-0.014	-0.005	-0.004	-0.131	-0.158	0.183	0.180	0.177	0.177	0.380	0.388
	300	trad.	-0.006	0.004	-0.003	-0.017	-0.076	-0.119	0.079	0.112	0.177	0.237	0.323	0.363
		rob.	-0.007	-0.004	-0.004	0.003	-0.090	-0.146	0.092	0.088	0.092	0.090	0.335	0.370
	500	trad.	-0.001	0.001	0.004	-0.005	-0.035	-0.053	0.061	0.084	0.140	0.184	0.276	0.321
		rob.	-0.003	0.000	0.001	-0.006	-0.050	-0.109	0.068	0.066	0.064	0.070	0.289	0.346
λ_{53}	100	trad.	-0.008	-0.007	-0.025	-0.102	-0.180	-0.215	0.124	0.150	0.185	0.286	0.344	0.358
		rob.	-0.009	-0.016	-0.014	-0.005	-0.183	-0.194	0.154	0.158	0.152	0.153	0.339	0.354
	300	trad.	-0.002	0.003	-0.012	-0.031	-0.105	-0.173	0.068	0.098	0.148	0.195	0.285	0.337
		rob.	-0.004	-0.002	-0.001	-0.004	-0.129	-0.183	0.081	0.078	0.077	0.079	0.300	0.341
	500	trad.	-0.001	0.003	-0.008	-0.017	-0.053	-0.100	0.052	0.074	0.132	0.157	0.234	0.275
		rob.	-0.005	-0.003	0.000	-0.002	-0.066	-0.117	0.060	0.059	0.057	0.062	0.247	0.289
λ_{54}	100	trad.	-0.012	-0.011	-0.023	-0.092	-0.174	-0.193	0.120	0.153	0.196	0.295	0.342	0.347
		rob.	-0.010	-0.012	-0.020	-0.016	-0.177	-0.185	0.150	0.160	0.144	0.153	0.346	0.353
	300	trad.	0.000	-0.003	-0.011	-0.042	-0.101	-0.162	0.070	0.098	0.144	0.213	0.297	0.325
		rob.	-0.003	-0.003	-0.011	-0.002	-0.133	-0.206	0.077	0.078	0.081	0.081	0.311	0.350
	500	trad.	-0.002	0.004	-0.004	-0.010	-0.078	-0.104	0.053	0.073	0.118	0.162	0.248	0.283
		rob.	-0.003	-0.003	-0.001	-0.005	-0.071	-0.110	0.060	0.060	0.056	0.060	0.239	0.299
λ_{55}	100	trad.	-0.030	-0.052	-0.082	-0.166	-0.257	-0.290	0.070	0.096	0.135	0.237	0.311	0.333
		rob.	-0.048	-0.048	-0.041	-0.044	-0.254	-0.286	0.095	0.096	0.087	0.097	0.296	0.325
	300	trad.	-0.007	-0.023	-0.048	-0.090	-0.170	-0.240	0.043	0.063	0.091	0.134	0.218	0.292
		rob.	-0.008	-0.010	-0.010	-0.013	-0.192	-0.267	0.048	0.048	0.051	0.051	0.258	0.308
	500	trad.	-0.004	-0.012	-0.032	-0.061	-0.109	-0.171	0.034	0.047	0.074	0.103	0.162	0.226
		rob.	-0.002	-0.005	-0.007	-0.006	-0.120	-0.196	0.037	0.038	0.037	0.039	0.174	0.240
λ_{56}	100	trad.	-0.028	-0.049	-0.076	-0.176	-0.254	-0.302	0.068	0.093	0.139	0.236	0.295	0.328
		rob.	-0.046	-0.048	-0.047	-0.048	-0.262	-0.291	0.091	0.092	0.090	0.092	0.310	0.316
	300	trad.	-0.005	-0.020	-0.051	-0.089	-0.173	-0.230	0.044	0.062	0.091	0.129	0.222	0.285
		rob.	-0.008	-0.009	-0.010	-0.009	-0.193	-0.260	0.048	0.050	0.050	0.050	0.249	0.294
	500	trad.	-0.006	-0.013	-0.033	-0.058	-0.113	-0.167	0.035	0.048	0.074	0.093	0.170	0.228
		rob.	-0.003	-0.003	-0.004	-0.006	-0.116	-0.192	0.038	0.038	0.037	0.040	0.170	0.248

Table 8.18: Results for traditional and robust PLSc for five common factors and systematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.004	0.061	0.088	0.096	0.069	0.059	0.087	0.091	0.096	0.107	0.120	0.119
		rob.	0.002	0.011	0.013	0.045	0.041	0.039	0.112	0.109	0.115	0.115	0.150	0.143
	300	trad.	-0.000	0.077	0.121	0.142	0.117	0.109	0.050	0.055	0.061	0.071	0.095	0.101
		rob.	0.003	0.006	0.012	0.043	0.107	0.092	0.059	0.057	0.058	0.062	0.100	0.108
	500	trad.	0.001	0.077	0.130	0.158	0.143	0.126	0.039	0.045	0.049	0.056	0.084	0.094
		rob.	0.000	0.005	0.013	0.038	0.133	0.122	0.042	0.043	0.044	0.047	0.088	0.097
γ_{32}	100	trad.	-0.025	-0.026	-0.032	-0.029	-0.023	-0.019	0.155	0.159	0.159	0.164	0.160	0.169
		rob.	-0.044	-0.042	-0.044	-0.028	-0.057	-0.060	0.241	0.238	0.251	0.238	0.248	0.242
	300	trad.	-0.007	-0.009	-0.007	-0.015	-0.010	-0.012	0.079	0.078	0.084	0.080	0.085	0.086
		rob.	-0.011	-0.009	-0.012	-0.008	-0.017	-0.014	0.095	0.093	0.094	0.094	0.098	0.097
	500	trad.	-0.007	-0.009	-0.003	-0.005	-0.008	-0.005	0.061	0.061	0.061	0.063	0.061	0.062
		rob.	-0.009	-0.005	-0.003	-0.004	-0.014	-0.004	0.067	0.067	0.069	0.074	0.070	0.068
γ_{41}	100	trad.	-0.029	-0.140	-0.179	-0.217	-0.129	-0.063	0.241	0.329	0.386	0.460	0.427	0.415
		rob.	0.024	-0.013	-0.014	-0.044	-0.040	-0.022	0.315	0.329	0.334	0.399	0.703	0.570
	300	trad.	-0.020	-0.165	-0.286	-0.384	-0.260	-0.191	0.130	0.205	0.281	0.361	0.434	0.416
		rob.	-0.018	-0.019	-0.028	-0.084	-0.209	-0.167	0.147	0.154	0.161	0.194	0.398	0.360
	500	trad.	-0.008	-0.150	-0.306	-0.409	-0.305	-0.224	0.096	0.154	0.245	0.315	0.353	0.374
		rob.	-0.011	-0.009	-0.030	-0.057	-0.294	-0.222	0.106	0.104	0.110	0.134	0.376	0.378
γ_{42}	100	trad.	0.038	0.120	0.154	0.192	0.121	0.067	0.233	0.303	0.365	0.430	0.403	0.390
		rob.	-0.015	0.020	0.026	0.033	0.060	0.030	0.306	0.308	0.311	0.379	0.682	0.541
	300	trad.	0.022	0.138	0.245	0.344	0.237	0.173	0.132	0.194	0.268	0.345	0.413	0.398
		rob.	0.017	0.018	0.027	0.070	0.188	0.151	0.146	0.155	0.163	0.188	0.376	0.337
	500	trad.	0.008	0.124	0.268	0.363	0.276	0.206	0.097	0.148	0.234	0.301	0.340	0.357
		rob.	0.011	0.010	0.028	0.051	0.265	0.203	0.106	0.107	0.110	0.132	0.359	0.362
γ_{51}	100	trad.	0.018	0.178	0.261	0.301	0.211	0.200	0.229	0.349	0.414	0.545	0.486	0.492
		rob.	-0.005	0.024	0.034	0.096	0.086	0.049	0.339	0.334	0.317	0.477	0.596	0.709
	300	trad.	-0.007	0.177	0.380	0.522	0.417	0.388	0.113	0.207	0.333	0.420	0.511	0.518
		rob.	-0.010	-0.000	0.007	0.080	0.386	0.341	0.141	0.139	0.149	0.216	0.500	0.516
	500	trad.	-0.014	0.171	0.394	0.598	0.562	0.459	0.086	0.155	0.270	0.416	0.543	0.518
		rob.	-0.013	-0.005	0.007	0.056	0.510	0.469	0.095	0.096	0.107	0.139	0.512	0.571
γ_{52}	100	trad.	-0.051	-0.195	-0.272	-0.318	-0.244	-0.228	0.229	0.340	0.394	0.524	0.464	0.468
		rob.	-0.044	-0.071	-0.072	-0.139	-0.130	-0.108	0.335	0.331	0.317	0.462	0.576	0.692
	300	trad.	-0.015	-0.181	-0.374	-0.519	-0.424	-0.397	0.112	0.198	0.324	0.408	0.496	0.506
		rob.	-0.017	-0.025	-0.028	-0.096	-0.396	-0.350	0.139	0.136	0.146	0.212	0.483	0.501
	500	trad.	-0.006	-0.173	-0.384	-0.587	-0.562	-0.466	0.085	0.149	0.261	0.407	0.533	0.506
		rob.	-0.011	-0.017	-0.027	-0.070	-0.510	-0.474	0.095	0.096	0.105	0.133	0.499	0.563
γ_{53}	100	trad.	-0.018	-0.023	-0.032	-0.029	-0.027	-0.023	0.118	0.132	0.148	0.155	0.143	0.155
		rob.	0.010	-0.003	-0.001	0.001	-0.006	-0.015	0.185	0.195	0.191	0.191	0.220	0.247
	300	trad.	-0.043	-0.040	-0.041	-0.052	-0.044	-0.039	0.062	0.067	0.082	0.093	0.096	0.094
		rob.	-0.037	-0.037	-0.039	-0.038	-0.049	-0.038	0.071	0.071	0.074	0.083	0.110	0.109
	500	trad.	-0.046	-0.048	-0.045	-0.047	-0.050	-0.049	0.047	0.054	0.061	0.076	0.081	0.073
		rob.	-0.049	-0.045	-0.043	-0.043	-0.050	-0.046	0.051	0.052	0.053	0.056	0.083	0.089
λ_{11}	100	trad.	0.008	-0.045	-0.062	-0.063	-0.044	-0.038	0.059	0.087	0.101	0.110	0.121	0.120
		rob.	0.016	0.012	0.002	-0.020	-0.036	-0.036	0.080	0.081	0.082	0.104	0.141	0.140
	300	trad.	0.001	-0.051	-0.078	-0.092	-0.077	-0.074	0.038	0.049	0.060	0.073	0.092	0.100
		rob.	0.001	-0.001	-0.004	-0.013	-0.069	-0.059	0.042	0.042	0.043	0.049	0.104	0.105
	500	trad.	0.001	-0.051	-0.082	-0.104	-0.099	-0.089	0.030	0.038	0.046	0.054	0.078	0.089
		rob.	0.001	-0.002	-0.003	-0.011	-0.090	-0.084	0.032	0.032	0.033	0.036	0.083	0.091
λ_{12}	100	trad.	-0.009	-0.168	-0.273	-0.414	-0.600	-0.660	0.054	0.100	0.118	0.136	0.163	0.167
		rob.	-0.017	-0.019	-0.023	-0.105	-0.561	-0.656	0.070	0.072	0.078	0.178	0.239	0.255
	300	trad.	-0.002	-0.171	-0.273	-0.412	-0.588	-0.643	0.036	0.058	0.069	0.082	0.080	0.088
		rob.	-0.001	-0.002	-0.004	-0.022	-0.580	-0.643	0.039	0.040	0.042	0.065	0.097	0.099
	500	trad.	-0.000	-0.172	-0.274	-0.408	-0.579	-0.642	0.027	0.047	0.053	0.061	0.066	0.066
		rob.	0.001	-0.001	-0.004	-0.011	-0.574	-0.640	0.031	0.031	0.031	0.037	0.072	0.071
λ_{21}	100	trad.	-0.007	0.017	-0.002	-0.050	-0.168	-0.207	0.106	0.107	0.120	0.141	0.164	0.172
		rob.	0.001	0.011	0.018	0.042	-0.149	-0.224	0.150	0.150	0.147	0.159	0.229	0.245
	300	trad.	-0.005	0.019	0.015	-0.032	-0.139	-0.177	0.063	0.060	0.070	0.085	0.090	0.101
		rob.	-0.000	0.007	0.017	0.053	-0.136	-0.184	0.070	0.071	0.071	0.074	0.109	0.109
	500	trad.	-0.002	0.023	0.015	-0.023	-0.124	-0.172	0.047	0.049	0.051	0.063	0.075	0.078
		rob.	-0.001	0.009	0.020	0.057	-0.124	-0.172	0.054	0.054	0.055	0.055	0.083	0.085
λ_{22}	100	trad.	-0.004	-0.017	-0.023	-0.022	-0.014	-0.004	0.110	0.106	0.106	0.107	0.106	0.107
		rob.	0.003	0.003	-0.007	0.003	-0.013	-0.005	0.149	0.146	0.150	0.144	0.153	0.151
	300	trad.	-0.000	-0.019	-0.024	-0.022	-0.015	-0.013	0.063	0.062	0.062	0.062	0.064	0.065
		rob.	0.001	-0.001	-0.002	-0.004	-0.014	-0.014	0.069	0.068	0.073	0.072	0.070	0.071
	500	trad.	-0.001	-0.019	-0.024	-0.024	-0.019	-0.010	0.049	0.048	0.047	0.046	0.048	0.048
		rob.	-0.001	-0.001	0.001	-0.005	-0.017	-0.014	0.051	0.052	0.053	0.053	0.054	0.053
λ_{23}	100	trad.	0.000	-0.009	-0.022	-0.016	-0.010	-0.014	0.107	0.105	0.109	0.106	0.110	0.112
		rob.	0.002	0.000	-0.014	-0.010	-0.001	-0.002	0.147	0.147	0.156	0.151	0.150	0.145
	300	trad.	-0.003	-0.022	-0.026	-0.025	-0.015	-0.010	0.065	0.064	0.061	0.061	0.062	0.063
		rob.	-0.003	-0.004	-0.004	-0.007	-0.016	-0.016	0.071	0.068	0.071	0.073	0.071	0.074
	500	trad.	-0.002	-0.019	-0.024	-0.024	-0.017	-0.010	0.048	0.051	0.049	0.048	0.050	0.050
		rob.	-0.001	-0.002	-0.003	-0.004	-0.017	-0.016	0.051	0.053	0.052	0.055	0.054	0.055
λ_{24}	100	trad.	-0.008	-0.032	-0.031	-0.030	-0.023	-0.018	0.086	0.091	0.091	0.090	0.093	0.089
		rob.	-0.005	-0.008	-0.014	-0.019	-0.015	-0.012	0.119	0.120	0.120	0.114	0.123	0.123
	300	trad.	-0.002	-0.025	-0.037	-0.036	-0.023	-0.020	0.050	0.049	0.050	0.052	0.053	0.054
		rob.	-0.005	-0.000	-0.007	-0.010	-0.028	-0.015	0.059	0.056	0.058	0.061	0.059	0.058
	500	trad.	-0.000	-0.028	-0.038	-0.039	-0.027	-0.021	0.039	0.039	0.038	0.040	0.041	0.042
		rob.	-0.002	-0.001	-0.003	-0.009	-0.025	-0.020	0.042	0.042	0.043	0.045	0.044	0.045
λ_{25}	100	trad.	-0.004	-0.027	-0.032	-0.028	-0.021	-0.012	0.083	0.091	0.090	0.087	0.088	0.089
		rob.	-0.012	-0.003	-0.005	-0.017	-0.019	-0.012	0.125	0.119	0.125	0.120	0.116	0.121
	300	trad.	-0.005	-0.029	-0.037	-0.038	-0.024	-0.017	0.052	0.051	0.052	0.052	0.051	0.055
		rob.	-0.003	-0.005	-0.002	-0.016	-0.025	-0.018	0.056	0.056	0.059	0.060	0.059	0.061
	500	trad.	-0.001	-0.023	-0.035	-0.038	-0.024	-0.017	0.039	0.040	0.040	0.040	0.040	0.043
		rob.	0.001	-0.002	-0.005	-0.012	-0.025	-0.021	0.043	0.042	0.045	0.044	0.045	0.045
λ_{26}	100	trad.	-0.012	-0.028	-0.032	-0.024	-0.022	-0.015	0.084	0.093	0.091	0.089	0.087	0.089
		rob.	-0.007	-0.012	-0.013	-0.017	-0.017	-0.015	0.120	0.116	0.123	0.120	0.124	0.129
	300	trad.	0.001	-0.025	-0.038	-0.036	-0.025	-0.023	0.049	0.051	0.052	0.053	0.052	0.053
		rob.	-0.004	-0.006	-0.006	-0.013	-0.022	-0.018	0.056	0.057	0.058	0.063	0.060	0.060
	500	trad.	-0.002	-0.028	-0.038	-0.038	-0.025	-0.018	0.039	0.039	0.041	0.040	0.040	0.043
		rob.	0.000	-0.001	-0.005	-0.008	-0.025	-0.019	0.043	0.043	0.044	0.043	0.046	0.044

λ_{31}	100	trad.	0.005	-0.011	-0.012	-0.002	0.003	-0.004	0.287	0.278	0.274	0.295	0.281	0.271
		rob.	-0.026	-0.008	-0.029	-0.019	-0.030	-0.022	0.339	0.330	0.332	0.339	0.345	0.335
	300	trad.	-0.006	0.003	0.001	0.002	-0.003	0.003	0.170	0.170	0.170	0.177	0.164	0.168
		rob.	-0.012	-0.008	-0.009	-0.001	-0.003	0.004	0.206	0.208	0.199	0.198	0.199	0.191
	500	trad.	-0.003	-0.011	-0.001	0.001	-0.004	0.005	0.126	0.128	0.134	0.134	0.137	0.128
		rob.	-0.003	0.003	-0.002	-0.005	-0.002	-0.003	0.145	0.149	0.142	0.153	0.144	0.144
λ_{32}	100	trad.	-0.036	-0.040	-0.035	-0.046	-0.029	-0.029	0.240	0.236	0.228	0.245	0.226	0.242
		rob.	-0.078	-0.078	-0.083	-0.074	-0.072	-0.062	0.291	0.295	0.288	0.287	0.295	0.293
	300	trad.	-0.006	-0.006	-0.003	-0.008	0.001	-0.002	0.131	0.139	0.132	0.141	0.135	0.132
		rob.	-0.006	-0.000	-0.011	-0.002	-0.013	-0.006	0.155	0.154	0.158	0.163	0.153	0.153
	500	trad.	-0.001	-0.000	-0.006	-0.009	-0.006	-0.007	0.101	0.103	0.108	0.102	0.110	0.109
		rob.	-0.005	-0.005	0.000	0.002	0.004	-0.008	0.114	0.117	0.118	0.120	0.113	0.116
λ_{33}	100	trad.	-0.091	-0.079	-0.076	-0.099	-0.093	-0.089	0.177	0.182	0.186	0.197	0.187	0.189
		rob.	-0.148	-0.157	-0.157	-0.138	-0.155	-0.146	0.254	0.250	0.255	0.233	0.253	0.248
	300	trad.	-0.018	-0.019	-0.021	-0.023	-0.022	-0.021	0.102	0.102	0.103	0.109	0.103	0.098
		rob.	-0.025	-0.035	-0.032	-0.034	-0.028	-0.031	0.120	0.122	0.122	0.124	0.116	0.110
	500	trad.	-0.010	-0.009	-0.010	-0.009	-0.012	-0.014	0.079	0.079	0.083	0.080	0.081	0.078
		rob.	-0.013	-0.015	-0.017	-0.018	-0.013	-0.013	0.087	0.087	0.089	0.091	0.083	0.087
λ_{34}	100	trad.	-0.127	-0.126	-0.117	-0.127	-0.118	-0.123	0.164	0.169	0.165	0.169	0.160	0.181
		rob.	-0.201	-0.187	-0.196	-0.181	-0.211	-0.196	0.235	0.230	0.239	0.215	0.234	0.231
	300	trad.	-0.040	-0.045	-0.042	-0.044	-0.044	-0.043	0.083	0.083	0.085	0.084	0.081	0.080
		rob.	-0.061	-0.057	-0.056	-0.063	-0.056	-0.051	0.099	0.094	0.097	0.100	0.094	0.094
	500	trad.	-0.024	-0.023	-0.026	-0.023	-0.021	-0.024	0.062	0.064	0.064	0.067	0.067	0.065
		rob.	-0.027	-0.032	-0.029	-0.037	-0.033	-0.026	0.068	0.072	0.073	0.077	0.072	0.072
λ_{41}	100	trad.	-0.023	-0.025	-0.024	-0.019	-0.018	-0.016	0.092	0.095	0.097	0.090	0.097	0.093
		rob.	-0.004	-0.005	0.001	-0.009	0.001	0.006	0.127	0.130	0.126	0.122	0.123	0.121
	300	trad.	-0.027	-0.034	-0.030	-0.028	-0.029	-0.030	0.053	0.057	0.056	0.056	0.055	0.056
		rob.	-0.028	-0.034	-0.032	-0.029	-0.029	-0.028	0.064	0.062	0.062	0.063	0.063	0.065
	500	trad.	-0.031	-0.029	-0.031	-0.029	-0.031	-0.028	0.042	0.043	0.043	0.043	0.045	0.043
		rob.	-0.028	-0.032	-0.030	-0.027	-0.028	-0.031	0.046	0.048	0.047	0.051	0.047	0.047
λ_{42}	100	trad.	-0.037	-0.034	-0.032	-0.035	-0.029	-0.036	0.090	0.094	0.096	0.095	0.098	0.093
		rob.	-0.044	-0.045	-0.048	-0.038	-0.043	-0.036	0.118	0.116	0.113	0.128	0.117	0.115
	300	trad.	-0.029	-0.027	-0.028	-0.027	-0.027	-0.025	0.060	0.060	0.061	0.061	0.059	0.059
		rob.	-0.028	-0.025	-0.029	-0.029	-0.027	-0.027	0.066	0.068	0.066	0.067	0.067	0.068
	500	trad.	-0.030	-0.029	-0.030	-0.028	-0.026	-0.028	0.042	0.048	0.047	0.048	0.047	0.047
		rob.	-0.030	-0.030	-0.029	-0.027	-0.026	-0.027	0.051	0.050	0.050	0.048	0.052	0.052
λ_{51}	100	trad.	0.000	0.001	0.000	-0.004	-0.003	-0.008	0.140	0.139	0.136	0.139	0.142	0.135
		rob.	-0.012	-0.001	-0.006	0.000	-0.003	0.002	0.186	0.182	0.190	0.180	0.182	0.179
	300	trad.	0.003	-0.003	-0.003	-0.001	-0.005	-0.002	0.079	0.079	0.084	0.080	0.079	0.083
		rob.	-0.002	0.001	-0.004	0.001	0.001	-0.001	0.088	0.089	0.091	0.095	0.090	0.094
	500	trad.	-0.003	-0.001	-0.004	-0.001	-0.004	-0.003	0.060	0.062	0.064	0.062	0.057	0.060
		rob.	0.001	-0.002	-0.003	0.000	-0.004	0.002	0.070	0.068	0.068	0.070	0.067	0.067
λ_{52}	100	trad.	0.000	0.007	-0.002	0.007	0.002	0.004	0.134	0.137	0.136	0.140	0.134	0.136
		rob.	-0.010	-0.013	-0.008	-0.006	-0.006	0.001	0.185	0.187	0.183	0.180	0.189	0.189
	300	trad.	-0.008	-0.004	-0.007	0.001	-0.000	-0.004	0.080	0.076	0.082	0.079	0.078	0.080
		rob.	0.005	-0.004	-0.007	-0.005	0.000	-0.003	0.093	0.088	0.092	0.094	0.086	0.092
	500	trad.	0.000	0.001	-0.003	-0.004	-0.000	-0.001	0.061	0.061	0.064	0.064	0.060	0.062
		rob.	-0.002	-0.003	-0.006	-0.003	0.002	0.002	0.064	0.071	0.067	0.072	0.066	0.068
λ_{53}	100	trad.	-0.012	-0.005	-0.003	-0.002	-0.003	-0.006	0.114	0.115	0.116	0.115	0.113	0.115
		rob.	-0.009	-0.016	-0.013	-0.004	-0.008	-0.008	0.154	0.155	0.160	0.153	0.161	0.151
	300	trad.	-0.004	-0.000	-0.003	-0.001	-0.000	-0.004	0.068	0.068	0.069	0.071	0.071	0.067
		rob.	-0.001	-0.007	-0.002	0.001	-0.004	-0.004	0.076	0.078	0.081	0.080	0.081	0.076
	500	trad.	-0.001	-0.002	-0.004	0.001	0.001	-0.001	0.053	0.054	0.055	0.054	0.052	0.054
		rob.	-0.004	-0.003	-0.003	-0.001	-0.002	-0.003	0.057	0.061	0.058	0.062	0.055	0.061
λ_{54}	100	trad.	-0.005	-0.009	-0.000	-0.014	-0.009	0.004	0.120	0.114	0.119	0.124	0.117	0.117
		rob.	-0.011	-0.007	-0.005	-0.018	-0.009	-0.011	0.146	0.152	0.163	0.157	0.153	0.153
	300	trad.	-0.000	-0.002	-0.003	-0.006	-0.004	-0.001	0.068	0.069	0.071	0.072	0.070	0.068
		rob.	-0.004	-0.003	-0.002	-0.002	-0.000	-0.002	0.080	0.079	0.079	0.082	0.081	0.075
	500	trad.	-0.001	0.000	-0.001	-0.004	-0.002	0.001	0.051	0.054	0.054	0.052	0.056	0.054
		rob.	0.002	-0.005	-0.001	-0.004	0.001	-0.002	0.057	0.058	0.059	0.063	0.059	0.057
λ_{55}	100	trad.	-0.025	-0.029	-0.028	-0.030	-0.027	-0.026	0.071	0.070	0.071	0.072	0.072	0.071
		rob.	-0.049	-0.048	-0.048	-0.050	-0.049	-0.048	0.092	0.091	0.095	0.093	0.093	0.093
	300	trad.	-0.005	-0.004	-0.005	-0.006	-0.008	-0.007	0.044	0.044	0.045	0.044	0.043	0.043
		rob.	-0.010	-0.006	-0.010	-0.009	-0.010	-0.011	0.048	0.050	0.050	0.051	0.049	0.049
	500	trad.	-0.003	-0.003	-0.004	-0.003	-0.001	-0.004	0.035	0.037	0.035	0.035	0.036	0.034
		rob.	-0.005	-0.004	-0.005	-0.003	-0.006	-0.005	0.038	0.037	0.038	0.040	0.038	0.039
λ_{56}	100	trad.	-0.030	-0.029	-0.027	-0.029	-0.027	-0.028	0.072	0.070	0.070	0.071	0.070	0.070
		rob.	-0.048	-0.047	-0.049	-0.045	-0.044	-0.040	0.090	0.094	0.094	0.091	0.089	0.089
	300	trad.	-0.007	-0.008	-0.006	-0.009	-0.006	-0.007	0.044	0.045	0.044	0.044	0.043	0.043
		rob.	-0.011	-0.010	-0.008	-0.012	-0.009	-0.010	0.049	0.049	0.048	0.052	0.050	0.049
	500	trad.	-0.003	-0.003	-0.001	-0.003	-0.002	-0.002	0.033	0.035	0.035	0.036	0.034	0.035
		rob.	-0.005	-0.003	-0.005	-0.004	-0.003	-0.006	0.039	0.037	0.040	0.039	0.038	0.037

Table 8.19: Results for the model with five composites and systematic outlier in all observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.007	0.182	0.219	0.242	0.250	0.248	0.050	0.018	0.012	0.009	0.007	0.008
		rob.	0.013	0.008	0.009	0.015	0.253	0.250	0.073	0.074	0.071	0.075	0.011	0.012
	300	trad.	0.003	0.181	0.219	0.243	0.250	0.248	0.029	0.010	0.007	0.005	0.004	0.004
		rob.	0.004	0.002	0.002	0.003	0.253	0.251	0.034	0.033	0.035	0.036	0.021	0.005
	500	trad.	0.002	0.181	0.219	0.242	0.250	0.248	0.023	0.008	0.006	0.004	0.003	0.003
		rob.	0.002	0.001	0.001	0.002	0.247	0.250	0.026	0.027	0.026	0.026	0.040	0.004
γ_{32}	100	trad.	-0.066	0.324	0.592	0.753	0.841	0.857	0.144	0.391	0.297	0.196	0.122	0.058
		rob.	-0.113	-0.116	-0.104	-0.100	0.804	0.845	0.238	0.230	0.233	0.248	0.314	0.221
	300	trad.	-0.023	0.459	0.644	0.768	0.841	0.852	0.066	0.168	0.049	0.019	0.013	0.012
		rob.	-0.031	-0.031	-0.030	-0.029	0.841	0.859	0.080	0.077	0.078	0.082	0.070	0.013
	500	trad.	-0.012	0.479	0.641	0.766	0.840	0.851	0.048	0.091	0.022	0.015	0.010	0.009
		rob.	-0.015	-0.015	-0.021	-0.018	0.820	0.856	0.053	0.053	0.058	0.057	0.145	0.010
γ_{41}	100	trad.	0.001	-0.493	-0.603	-0.615	-0.406	-0.300	0.104	0.205	0.265	0.343	0.378	0.348
		rob.	0.005	0.001	0.006	-0.021	-0.437	-0.358	0.159	0.155	0.147	0.167	0.575	0.526
	300	trad.	-0.001	-0.519	-0.673	-0.700	-0.474	-0.362	0.059	0.112	0.143	0.183	0.213	0.206
		rob.	-0.001	-0.002	-0.004	-0.001	-0.571	-0.401	0.066	0.069	0.067	0.072	0.284	0.250
	500	trad.	0.001	-0.524	-0.679	-0.722	-0.508	-0.374	0.044	0.086	0.111	0.135	0.163	0.162
		rob.	0.001	-0.000	-0.001	-0.003	-0.589	-0.406	0.047	0.048	0.051	0.053	0.211	0.185
γ_{42}	100	trad.	0.007	0.170	0.186	0.121	-0.156	-0.283	0.087	0.199	0.270	0.356	0.401	0.371
		rob.	0.010	0.011	0.005	0.024	-0.117	-0.227	0.137	0.133	0.133	0.137	0.618	0.576
	300	trad.	0.004	0.187	0.246	0.195	-0.094	-0.231	0.051	0.107	0.140	0.184	0.222	0.215
		rob.	0.006	0.006	0.006	0.003	0.013	-0.194	0.058	0.059	0.059	0.062	0.297	0.262
	500	trad.	0.001	0.187	0.250	0.213	-0.068	-0.224	0.038	0.080	0.110	0.135	0.169	0.167
		rob.	0.001	0.001	0.003	0.004	0.033	-0.192	0.040	0.042	0.045	0.046	0.198	0.193
γ_{51}	100	trad.	-0.072	0.161	0.136	0.055	-0.088	-0.098	0.111	0.243	0.309	0.363	0.365	0.331
		rob.	-0.123	-0.120	-0.121	-0.101	-0.045	-0.088	0.175	0.173	0.177	0.175	0.545	0.520
	300	trad.	-0.032	0.287	0.322	0.210	-0.009	-0.073	0.065	0.124	0.185	0.253	0.247	0.220
		rob.	-0.037	-0.045	-0.045	-0.042	0.041	-0.061	0.074	0.073	0.074	0.076	0.319	0.260
	500	trad.	-0.030	0.312	0.385	0.311	0.033	-0.051	0.047	0.092	0.122	0.186	0.211	0.187
		rob.	-0.032	-0.032	-0.030	-0.033	0.081	-0.042	0.052	0.053	0.053	0.057	0.254	0.211
γ_{52}	100	trad.	0.024	-0.069	0.025	0.154	0.413	0.452	0.105	0.286	0.408	0.482	0.481	0.444
		rob.	0.051	0.050	0.062	0.048	0.354	0.444	0.178	0.173	0.180	0.182	0.701	0.685
	300	trad.	0.002	-0.198	-0.214	-0.049	0.282	0.389	0.057	0.121	0.208	0.345	0.361	0.316
		rob.	0.004	0.012	0.014	0.011	0.224	0.387	0.066	0.065	0.068	0.066	0.426	0.365
	500	trad.	0.005	-0.220	-0.281	-0.185	0.211	0.344	0.041	0.086	0.126	0.232	0.327	0.290
		rob.	0.006	0.006	0.006	0.005	0.153	0.350	0.046	0.048	0.049	0.052	0.376	0.319
γ_{53}	100	trad.	-0.061	-0.191	-0.225	-0.209	-0.232	-0.249	0.110	0.194	0.197	0.223	0.241	0.248
		rob.	-0.073	-0.084	-0.077	-0.079	-0.227	-0.239	0.186	0.191	0.195	0.180	0.373	0.397
	300	trad.	-0.052	-0.155	-0.137	-0.151	-0.205	-0.228	0.046	0.123	0.119	0.144	0.157	0.151
		rob.	-0.051	-0.051	-0.049	-0.048	-0.208	-0.231	0.056	0.052	0.059	0.058	0.171	0.170
	500	trad.	-0.049	-0.143	-0.118	-0.108	-0.178	-0.213	0.034	0.089	0.087	0.111	0.132	0.124
		rob.	-0.050	-0.050	-0.048	-0.050	-0.176	-0.222	0.038	0.038	0.040	0.042	0.143	0.132
w_{11}	100	trad.	-0.000	-0.316	-0.386	-0.429	-0.459	-0.464	0.114	0.075	0.067	0.061	0.065	0.064
		rob.	-0.006	-0.002	-0.007	-0.013	-0.459	-0.470	0.155	0.170	0.158	0.162	0.108	0.117
	300	trad.	0.000	-0.318	-0.384	-0.430	-0.457	-0.462	0.066	0.041	0.034	0.032	0.032	0.032
		rob.	0.001	-0.006	0.000	-0.002	-0.454	-0.465	0.077	0.076	0.074	0.074	0.047	0.036
	500	trad.	0.000	-0.315	-0.383	-0.430	-0.455	-0.459	0.049	0.032	0.027	0.024	0.023	0.024
		rob.	0.001	-0.001	-0.001	-0.003	-0.442	-0.461	0.053	0.055	0.055	0.057	0.082	0.026
w_{12}	100	trad.	-0.005	-0.317	-0.385	-0.432	-0.454	-0.458	0.116	0.074	0.066	0.060	0.065	0.064
		rob.	-0.013	-0.010	-0.015	-0.022	-0.460	-0.461	0.165	0.172	0.155	0.158	0.108	0.116
	300	trad.	-0.001	-0.315	-0.385	-0.430	-0.455	-0.458	0.065	0.041	0.034	0.032	0.032	0.031
		rob.	0.000	-0.002	-0.006	-0.000	-0.454	-0.460	0.075	0.075	0.073	0.077	0.053	0.037
	500	trad.	-0.002	-0.317	-0.383	-0.430	-0.457	-0.461	0.050	0.032	0.027	0.025	0.024	0.024
		rob.	-0.003	-0.003	-0.001	-0.004	-0.445	-0.462	0.054	0.057	0.054	0.060	0.079	0.026
w_{21}	100	trad.	-0.002	-0.035	-0.039	-0.033	-0.031	-0.026	0.071	0.077	0.080	0.081	0.088	0.088
		rob.	-0.001	-0.005	-0.006	-0.001	-0.031	-0.032	0.116	0.118	0.118	0.108	0.144	0.146
	300	trad.	0.001	-0.036	-0.038	-0.035	-0.028	-0.025	0.042	0.041	0.040	0.044	0.045	0.044
		rob.	0.001	-0.002	0.001	-0.001	-0.032	-0.026	0.048	0.046	0.047	0.048	0.050	0.050
	500	trad.	-0.000	-0.038	-0.038	-0.037	-0.028	-0.026	0.030	0.031	0.032	0.033	0.034	0.033
		rob.	-0.000	-0.001	0.000	-0.000	-0.029	-0.028	0.032	0.034	0.036	0.036	0.038	0.036
w_{22}	100	trad.	-0.005	-0.094	-0.120	-0.157	-0.177	-0.182	0.076	0.076	0.075	0.077	0.086	0.090
		rob.	-0.010	-0.017	-0.011	-0.009	-0.174	-0.178	0.120	0.118	0.115	0.110	0.143	0.148
	300	trad.	-0.001	-0.086	-0.120	-0.151	-0.179	-0.185	0.040	0.039	0.041	0.041	0.043	0.044
		rob.	-0.002	-0.002	-0.004	-0.003	-0.173	-0.185	0.046	0.047	0.046	0.048	0.053	0.050
	500	trad.	0.000	-0.086	-0.119	-0.149	-0.175	-0.185	0.032	0.031	0.031	0.031	0.033	0.033
		rob.	0.000	-0.001	-0.001	-0.001	-0.168	-0.185	0.034	0.036	0.035	0.037	0.044	0.036
w_{23}	100	trad.	-0.003	-0.088	-0.121	-0.153	-0.177	-0.186	0.076	0.075	0.078	0.080	0.085	0.084
		rob.	-0.006	-0.005	-0.007	-0.011	-0.179	-0.187	0.118	0.118	0.119	0.111	0.142	0.149
	300	trad.	-0.003	-0.086	-0.120	-0.149	-0.174	-0.185	0.040	0.039	0.040	0.042	0.042	0.044
		rob.	-0.003	0.000	-0.002	-0.001	-0.169	-0.184	0.046	0.047	0.048	0.049	0.050	0.051
	500	trad.	-0.001	-0.087	-0.120	-0.147	-0.177	-0.185	0.030	0.030	0.031	0.030	0.032	0.034
		rob.	-0.000	-0.002	-0.001	-0.002	-0.169	-0.184	0.034	0.034	0.034	0.035	0.045	0.037
w_{24}	100	trad.	-0.001	-0.064	-0.094	-0.110	-0.122	-0.123	0.078	0.084	0.085	0.088	0.088	0.087
		rob.	-0.005	0.003	-0.009	-0.006	-0.123	-0.125	0.127	0.123	0.123	0.122	0.151	0.149
	300	trad.	-0.001	-0.064	-0.087	-0.107	-0.121	-0.121	0.042	0.041	0.043	0.044	0.048	0.047
		rob.	-0.001	-0.000	-0.001	-0.001	-0.120	-0.121	0.049	0.048	0.049	0.051	0.055	0.053
	500	trad.	-0.001	-0.064	-0.089	-0.108	-0.122	-0.123	0.032	0.033	0.033	0.034	0.035	0.035
		rob.	-0.001	-0.001	-0.001	-0.001	-0.116	-0.123	0.036	0.037	0.037	0.039	0.044	0.038
w_{25}	100	trad.	-0.004	-0.040	-0.042	-0.047	-0.041	-0.044	0.078	0.086	0.083	0.087	0.086	0.092
		rob.	-0.011	-0.001	-0.003	-0.008	-0.048	-0.045	0.126	0.128	0.129	0.117	0.139	0.153
	300	trad.	-0.002	-0.038	-0.046	-0.048	-0.042	-0.039	0.043	0.042	0.044	0.043	0.046	0.047
		rob.	-0.002	-0.001	-0.001	0.001	-0.042	-0.040	0.050	0.050	0.052	0.050	0.053	0.052
	500	trad.	-0.001	-0.036	-0.044	-0.047	-0.043	-0.038	0.033	0.033	0.033	0.034	0.035	0.036
		rob.	-0.001	0.002	-0.001	-0.001	-0.044	-0.038	0.036	0.036	0.037	0.037	0.038	0.039
w_{26}	100	trad.	-0.099	-0.112	-0.103	-0.082	-0.066	-0.054	0.076	0.079	0.083	0.089	0.094	0.093
		rob.	-0.097	-0.112	-0.097	-0.100	-0.070	-0.061	0.124	0.123	0.122	0.114	0.149	0.152
	300	trad.	-0.100	-0.113	-0.101	-0.086	-0.066	-0.056	0.040	0.045	0.046	0.046	0.048	0.046
		rob.	-0.099	-0.102	-0.097	-0.099	-0.072	-0.059	0.045	0.052	0.049	0.048	0.056	0.053
	500	trad.	-0.098	-0.112	-0.100	-0.086	-0.064	-0.053	0.031	0.031	0.034	0.035	0.037	0.035
		rob.	-0.098	-0.101	-0.099	-0.098	-0.072	-0.056	0.034	0.034	0.037	0.037	0.041	0.038

w_{31}	100	trad.	-0.051	0.421	0.684	0.759	0.782	0.790	0.346	0.681	0.413	0.238	0.136	0.064
		rob.	-0.058	-0.067	-0.067	-0.059	0.719	0.754	0.437	0.449	0.444	0.454	0.344	0.238
	300	trad.	-0.016	0.729	0.796	0.801	0.799	0.798	0.195	0.317	0.063	0.009	0.007	0.006
		rob.	-0.015	-0.017	-0.017	-0.015	0.794	0.798	0.225	0.231	0.228	0.236	0.068	0.007
	500	trad.	-0.007	0.779	0.802	0.802	0.800	0.800	0.148	0.169	0.010	0.007	0.005	0.004
		rob.	-0.009	-0.019	0.001	-0.003	0.778	0.799	0.163	0.167	0.171	0.173	0.133	0.005
w_{32}	100	trad.	-0.075	-0.251	-0.285	-0.299	-0.293	-0.298	0.430	0.242	0.172	0.123	0.094	0.093
		rob.	-0.164	-0.119	-0.117	-0.111	-0.296	-0.296	0.569	0.543	0.547	0.514	0.149	0.141
	300	trad.	-0.014	-0.273	-0.288	-0.294	-0.296	-0.296	0.240	0.142	0.092	0.065	0.051	0.049
		rob.	-0.017	-0.035	-0.024	-0.012	-0.295	-0.296	0.279	0.275	0.273	0.293	0.068	0.055
	500	trad.	-0.010	-0.266	-0.288	-0.292	-0.299	-0.298	0.181	0.107	0.074	0.051	0.039	0.038
		rob.	-0.015	-0.021	-0.011	-0.023	-0.293	-0.299	0.200	0.200	0.205	0.220	0.067	0.041
w_{33}	100	trad.	-0.097	-0.490	-0.565	-0.563	-0.540	-0.536	0.525	0.345	0.228	0.157	0.118	0.113
		rob.	-0.243	-0.205	-0.205	-0.225	-0.529	-0.528	0.695	0.686	0.672	0.677	0.185	0.177
	300	trad.	-0.023	-0.560	-0.570	-0.561	-0.542	-0.534	0.279	0.187	0.116	0.081	0.061	0.059
		rob.	-0.038	-0.038	-0.033	-0.046	-0.539	-0.532	0.330	0.332	0.320	0.340	0.078	0.064
	500	trad.	-0.020	-0.562	-0.567	-0.558	-0.543	-0.536	0.210	0.140	0.089	0.064	0.046	0.045
		rob.	-0.019	-0.019	-0.017	-0.027	-0.531	-0.535	0.235	0.231	0.245	0.263	0.116	0.048
w_{34}	100	trad.	-0.077	-0.331	-0.376	-0.362	-0.343	-0.337	0.597	0.376	0.245	0.174	0.128	0.126
		rob.	-0.062	-0.121	-0.114	-0.092	-0.337	-0.330	0.782	0.782	0.772	0.775	0.199	0.194
	300	trad.	-0.026	-0.368	-0.385	-0.363	-0.344	-0.337	0.336	0.186	0.125	0.089	0.071	0.067
		rob.	-0.032	-0.019	-0.032	-0.039	-0.344	-0.337	0.389	0.384	0.405	0.386	0.090	0.074
	500	trad.	-0.006	-0.375	-0.380	-0.364	-0.340	-0.333	0.249	0.151	0.098	0.071	0.053	0.052
		rob.	-0.011	-0.001	-0.023	-0.008	-0.331	-0.332	0.277	0.282	0.295	0.310	0.104	0.056
w_{41}	100	trad.	-0.004	-0.037	-0.068	-0.121	-0.217	-0.251	0.093	0.213	0.297	0.408	0.523	0.557
		rob.	-0.014	-0.010	-0.011	-0.009	-0.286	-0.258	0.135	0.142	0.135	0.135	0.587	0.575
	300	trad.	-0.004	-0.007	-0.008	-0.036	-0.080	-0.151	0.053	0.108	0.147	0.227	0.353	0.445
		rob.	-0.003	-0.002	-0.001	-0.000	-0.108	-0.160	0.061	0.061	0.060	0.064	0.394	0.457
	500	trad.	-0.001	-0.008	-0.022	-0.024	-0.049	-0.080	0.039	0.088	0.124	0.172	0.286	0.345
		rob.	-0.000	-0.001	-0.003	-0.003	-0.054	-0.104	0.042	0.045	0.048	0.049	0.297	0.377
w_{42}	100	trad.	-0.003	-0.010	-0.024	-0.053	-0.077	-0.071	0.111	0.255	0.344	0.436	0.544	0.552
		rob.	0.001	-0.006	-0.002	-0.006	-0.076	-0.101	0.157	0.169	0.163	0.163	0.596	0.588
	300	trad.	-0.001	-0.008	-0.019	-0.019	-0.068	-0.047	0.065	0.133	0.183	0.266	0.414	0.454
		rob.	-0.003	-0.003	-0.004	-0.004	-0.061	-0.063	0.075	0.074	0.072	0.076	0.434	0.489
	500	trad.	-0.001	-0.000	0.008	-0.008	-0.048	-0.055	0.047	0.108	0.145	0.206	0.343	0.400
		rob.	-0.002	-0.001	0.001	-0.000	-0.046	-0.047	0.051	0.055	0.058	0.059	0.344	0.419
w_{51}	100	trad.	-0.020	-0.046	-0.099	-0.091	-0.098	-0.102	0.172	0.337	0.404	0.435	0.459	0.469
		rob.	-0.026	-0.037	-0.021	-0.016	-0.119	-0.084	0.271	0.272	0.262	0.268	0.465	0.492
	300	trad.	-0.003	-0.014	-0.036	-0.074	-0.102	-0.082	0.098	0.198	0.271	0.372	0.421	0.427
		rob.	-0.000	-0.001	-0.005	-0.002	-0.104	-0.113	0.110	0.108	0.106	0.114	0.428	0.434
	500	trad.	-0.002	-0.005	-0.018	-0.043	-0.098	-0.097	0.074	0.158	0.204	0.302	0.405	0.417
		rob.	-0.001	0.003	0.000	-0.005	-0.084	-0.102	0.083	0.080	0.082	0.084	0.404	0.424
w_{52}	100	trad.	-0.025	-0.091	-0.185	-0.225	-0.279	-0.289	0.167	0.318	0.383	0.408	0.450	0.448
		rob.	-0.060	-0.049	-0.058	-0.068	-0.281	-0.282	0.254	0.251	0.248	0.245	0.472	0.467
	300	trad.	-0.005	-0.028	-0.055	-0.135	-0.224	-0.237	0.089	0.183	0.250	0.339	0.416	0.420
		rob.	-0.007	-0.010	-0.006	-0.009	-0.235	-0.238	0.102	0.102	0.104	0.112	0.419	0.436
	500	trad.	-0.002	-0.018	-0.041	-0.074	-0.181	-0.208	0.067	0.142	0.202	0.282	0.390	0.402
		rob.	-0.003	-0.006	-0.010	-0.005	-0.186	-0.226	0.074	0.073	0.076	0.081	0.387	0.411
w_{53}	100	trad.	-0.012	-0.093	-0.145	-0.211	-0.252	-0.273	0.179	0.332	0.388	0.421	0.459	0.435
		rob.	-0.046	-0.045	-0.048	-0.058	-0.237	-0.269	0.261	0.251	0.257	0.232	0.474	0.467
	300	trad.	-0.006	-0.037	-0.053	-0.130	-0.191	-0.210	0.091	0.193	0.249	0.338	0.423	0.424
		rob.	-0.010	-0.010	-0.007	-0.009	-0.201	-0.215	0.107	0.102	0.107	0.112	0.428	0.422
	500	trad.	-0.004	-0.015	-0.030	-0.079	-0.158	-0.185	0.069	0.146	0.200	0.281	0.382	0.402
		rob.	-0.005	-0.003	0.000	-0.004	-0.167	-0.189	0.075	0.077	0.078	0.080	0.387	0.410
w_{54}	100	trad.	-0.029	-0.098	-0.120	-0.164	-0.196	-0.172	0.161	0.301	0.367	0.421	0.437	0.440
		rob.	-0.053	-0.033	-0.045	-0.032	-0.219	-0.184	0.251	0.243	0.248	0.244	0.464	0.445
	300	trad.	-0.005	-0.022	-0.058	-0.101	-0.173	-0.206	0.090	0.188	0.246	0.332	0.386	0.405
		rob.	-0.007	-0.005	-0.008	-0.010	-0.163	-0.227	0.105	0.102	0.096	0.106	0.388	0.412
	500	trad.	-0.005	-0.018	-0.032	-0.063	-0.124	-0.163	0.067	0.144	0.192	0.271	0.367	0.395
		rob.	-0.003	-0.005	-0.004	0.001	-0.134	-0.178	0.073	0.076	0.073	0.081	0.372	0.411
w_{55}	100	trad.	-0.003	-0.044	-0.071	-0.091	-0.138	-0.132	0.172	0.314	0.398	0.417	0.451	0.450
		rob.	-0.013	-0.021	-0.023	-0.021	-0.100	-0.138	0.269	0.260	0.254	0.252	0.472	0.472
	300	trad.	-0.007	-0.016	-0.037	-0.067	-0.106	-0.123	0.092	0.191	0.259	0.345	0.416	0.440
		rob.	-0.010	-0.004	-0.006	-0.010	-0.102	-0.127	0.108	0.106	0.106	0.113	0.424	0.448
	500	trad.	0.003	-0.014	-0.015	-0.033	-0.097	-0.104	0.071	0.153	0.209	0.289	0.388	0.400
		rob.	0.001	-0.007	-0.008	-0.003	-0.090	-0.099	0.077	0.083	0.078	0.086	0.389	0.401
w_{56}	100	trad.	-0.006	-0.038	-0.047	-0.040	-0.071	-0.050	0.172	0.319	0.390	0.409	0.440	0.446
		rob.	-0.015	-0.021	-0.016	-0.003	-0.046	-0.062	0.256	0.264	0.255	0.253	0.479	0.471
	300	trad.	-0.005	-0.013	-0.013	-0.011	-0.049	-0.063	0.094	0.194	0.268	0.340	0.417	0.434
		rob.	-0.005	-0.004	-0.002	0.001	-0.052	-0.040	0.108	0.108	0.107	0.109	0.427	0.438
	500	trad.	-0.005	-0.008	-0.009	-0.032	-0.058	-0.065	0.072	0.153	0.199	0.283	0.386	0.421
		rob.	-0.006	-0.002	0.001	-0.007	-0.051	-0.067	0.078	0.081	0.077	0.081	0.378	0.424

Table 8.20: Results for the model with five composites and systematic outliers in two observed variables

Par.	n	Appr.	Mean Value						Standard Deviation					
			0%	5%	10%	20%	40%	50%	0%	5%	10%	20%	40%	50%
ϕ_{12}	100	trad.	0.006	-0.030	-0.042	-0.046	-0.059	-0.058	0.050	0.056	0.059	0.058	0.060	0.060
		rob.	0.010	0.012	0.004	-0.017	-0.050	-0.049	0.073	0.072	0.074	0.080	0.087	0.088
	300	trad.	0.002	-0.035	-0.047	-0.058	-0.064	-0.066	0.030	0.032	0.034	0.035	0.034	0.034
		rob.	0.002	0.003	0.001	-0.009	-0.064	-0.066	0.034	0.032	0.035	0.040	0.040	0.040
	500	trad.	0.001	-0.037	-0.049	-0.059	-0.066	-0.068	0.022	0.024	0.027	0.026	0.027	0.026
		rob.	0.001	-0.000	-0.001	-0.006	-0.066	-0.067	0.024	0.024	0.027	0.028	0.029	0.029
γ_{32}	100	trad.	-0.072	-0.069	-0.077	-0.068	-0.074	-0.078	0.138	0.138	0.141	0.142	0.144	0.137
		rob.	-0.108	-0.108	-0.122	-0.105	-0.114	-0.137	0.236	0.231	0.232	0.240	0.244	0.230
	300	trad.	-0.023	-0.020	-0.021	-0.019	-0.017	-0.020	0.064	0.064	0.065	0.065	0.065	0.067
		rob.	-0.029	-0.028	-0.030	-0.030	-0.024	-0.028	0.075	0.078	0.081	0.080	0.078	0.077
	500	trad.	-0.010	-0.014	-0.010	-0.013	-0.012	-0.012	0.047	0.049	0.047	0.048	0.049	0.048
		rob.	-0.012	-0.017	-0.014	-0.019	-0.014	-0.015	0.052	0.055	0.055	0.056	0.054	0.052
γ_{41}	100	trad.	-0.007	0.089	0.113	0.121	0.145	0.143	0.102	0.101	0.096	0.100	0.103	0.101
		rob.	-0.008	0.005	0.009	0.067	0.141	0.133	0.156	0.156	0.147	0.155	0.153	0.152
	300	trad.	-0.000	0.090	0.118	0.135	0.148	0.153	0.059	0.060	0.058	0.058	0.058	0.056
		rob.	-0.000	-0.002	0.011	0.031	0.147	0.150	0.067	0.067	0.066	0.077	0.067	0.064
	500	trad.	0.001	0.092	0.121	0.137	0.149	0.152	0.046	0.047	0.045	0.043	0.045	0.043
		rob.	-0.000	0.004	0.012	0.030	0.147	0.150	0.051	0.052	0.052	0.054	0.049	0.048
γ_{42}	100	trad.	0.013	-0.085	-0.112	-0.123	-0.148	-0.147	0.090	0.093	0.085	0.093	0.094	0.092
		rob.	0.020	0.010	0.004	-0.055	-0.138	-0.133	0.134	0.140	0.132	0.142	0.138	0.138
	300	trad.	0.003	-0.097	-0.122	-0.145	-0.158	-0.163	0.050	0.054	0.052	0.053	0.053	0.053
		rob.	0.004	0.002	-0.006	-0.029	-0.157	-0.160	0.057	0.060	0.058	0.070	0.062	0.060
	500	trad.	0.001	-0.097	-0.125	-0.146	-0.159	-0.163	0.040	0.043	0.041	0.041	0.039	0.040
		rob.	0.003	-0.003	-0.006	-0.025	-0.158	-0.161	0.044	0.045	0.045	0.051	0.044	0.044
γ_{51}	100	trad.	-0.074	-0.150	-0.179	-0.186	-0.201	-0.217	0.112	0.113	0.110	0.116	0.109	0.108
		rob.	-0.124	-0.126	-0.140	-0.173	-0.238	-0.244	0.172	0.176	0.173	0.184	0.174	0.174
	300	trad.	-0.037	-0.125	-0.151	-0.168	-0.181	-0.186	0.063	0.063	0.063	0.063	0.062	0.059
		rob.	-0.042	-0.045	-0.052	-0.075	-0.183	-0.189	0.073	0.075	0.076	0.081	0.074	0.071
	500	trad.	-0.030	-0.120	-0.147	-0.164	-0.178	-0.177	0.049	0.049	0.049	0.047	0.047	0.046
		rob.	-0.033	-0.036	-0.043	-0.063	-0.178	-0.178	0.053	0.054	0.055	0.058	0.052	0.051
γ_{52}	100	trad.	0.027	0.113	0.141	0.152	0.165	0.182	0.103	0.106	0.105	0.105	0.105	0.107
		rob.	0.058	0.056	0.073	0.118	0.174	0.188	0.180	0.172	0.170	0.189	0.179	0.179
	300	trad.	0.010	0.105	0.129	0.151	0.166	0.170	0.057	0.057	0.057	0.056	0.059	0.057
		rob.	0.013	0.017	0.018	0.043	0.165	0.170	0.066	0.066	0.070	0.074	0.068	0.067
	500	trad.	0.006	0.102	0.129	0.148	0.165	0.165	0.044	0.044	0.045	0.043	0.043	0.042
		rob.	0.008	0.009	0.014	0.033	0.165	0.164	0.047	0.048	0.050	0.052	0.047	0.046
γ_{53}	100	trad.	-0.058	-0.049	-0.056	-0.050	-0.054	-0.047	0.106	0.107	0.111	0.114	0.108	0.105
		rob.	-0.061	-0.074	-0.081	-0.068	-0.062	-0.060	0.182	0.198	0.191	0.197	0.198	0.208
	300	trad.	-0.048	-0.048	-0.045	-0.044	-0.044	-0.046	0.047	0.047	0.046	0.048	0.049	0.048
		rob.	-0.048	-0.049	-0.048	-0.047	-0.045	-0.045	0.057	0.055	0.056	0.057	0.058	0.058
	500	trad.	-0.047	-0.046	-0.047	-0.048	-0.048	-0.048	0.035	0.035	0.037	0.037	0.036	0.036
		rob.	-0.046	-0.047	-0.048	-0.049	-0.047	-0.048	0.039	0.039	0.041	0.042	0.040	0.041
w_{11}	100	trad.	-0.008	-0.001	-0.012	-0.005	-0.004	-0.005	0.113	0.115	0.117	0.116	0.120	0.116
		rob.	-0.017	-0.007	-0.018	-0.012	-0.012	-0.009	0.164	0.162	0.159	0.161	0.172	0.169
	300	trad.	0.001	-0.002	-0.002	-0.001	-0.000	-0.005	0.065	0.066	0.067	0.067	0.067	0.065
		rob.	0.000	-0.003	-0.001	-0.003	0.002	-0.002	0.074	0.076	0.075	0.076	0.077	0.075
	500	trad.	-0.001	-0.001	-0.001	0.000	0.002	-0.001	0.051	0.053	0.051	0.050	0.051	0.054
		rob.	-0.001	-0.001	-0.002	-0.003	0.002	-0.001	0.056	0.056	0.059	0.056	0.055	0.059
w_{12}	100	trad.	-0.004	-0.009	-0.009	-0.005	-0.007	-0.011	0.116	0.114	0.113	0.117	0.117	0.118
		rob.	-0.015	-0.013	-0.013	-0.022	-0.024	-0.012	0.167	0.159	0.156	0.167	0.173	0.169
	300	trad.	-0.003	0.003	0.000	0.002	-0.004	0.001	0.062	0.064	0.066	0.065	0.067	0.065
		rob.	-0.003	0.002	-0.001	-0.001	-0.005	-0.001	0.071	0.072	0.072	0.074	0.076	0.074
	500	trad.	-0.003	0.002	0.002	-0.000	-0.002	-0.001	0.052	0.049	0.052	0.052	0.052	0.052
		rob.	-0.002	0.003	-0.000	-0.004	-0.002	-0.002	0.058	0.055	0.057	0.060	0.057	0.057
w_{21}	100	trad.	-0.006	-0.159	-0.210	-0.266	-0.282	-0.257	0.076	0.092	0.113	0.132	0.160	0.158
		rob.	-0.004	-0.016	-0.029	-0.176	-0.289	-0.267	0.118	0.117	0.134	0.210	0.265	0.256
	300	trad.	0.000	-0.152	-0.215	-0.264	-0.271	-0.272	0.040	0.053	0.062	0.073	0.079	0.088
		rob.	0.001	-0.008	-0.022	-0.062	-0.273	-0.273	0.047	0.051	0.054	0.081	0.095	0.101
	500	trad.	-0.002	-0.152	-0.213	-0.259	-0.276	-0.263	0.032	0.042	0.046	0.056	0.064	0.066
		rob.	-0.001	-0.008	-0.019	-0.051	-0.277	-0.265	0.035	0.038	0.039	0.055	0.070	0.072
w_{22}	100	trad.	-0.005	-0.094	-0.128	-0.150	-0.205	-0.247	0.074	0.098	0.116	0.131	0.163	0.163
		rob.	-0.008	-0.007	-0.014	-0.085	-0.199	-0.249	0.119	0.119	0.119	0.189	0.263	0.271
	300	trad.	-0.001	-0.092	-0.123	-0.151	-0.212	-0.233	0.040	0.055	0.060	0.072	0.081	0.087
		rob.	-0.002	-0.002	-0.011	-0.030	-0.207	-0.235	0.046	0.048	0.049	0.065	0.096	0.103
	500	trad.	-0.001	-0.094	-0.125	-0.154	-0.207	-0.241	0.031	0.042	0.048	0.057	0.064	0.065
		rob.	-0.001	-0.006	-0.011	-0.029	-0.204	-0.241	0.034	0.036	0.037	0.042	0.071	0.071
w_{23}	100	trad.	0.000	0.063	0.078	0.088	0.096	0.098	0.076	0.084	0.088	0.091	0.090	0.095
		rob.	-0.009	0.002	-0.001	0.044	0.089	0.090	0.118	0.117	0.115	0.134	0.142	0.144
	300	trad.	0.000	0.062	0.083	0.098	0.104	0.108	0.040	0.045	0.048	0.048	0.051	0.049
		rob.	-0.000	-0.000	0.004	0.019	0.103	0.107	0.047	0.047	0.049	0.057	0.058	0.057
	500	trad.	0.001	0.064	0.086	0.098	0.107	0.107	0.031	0.035	0.037	0.037	0.038	0.039
		rob.	0.000	0.002	0.005	0.014	0.107	0.108	0.034	0.035	0.035	0.039	0.041	0.044
w_{24}	100	trad.	-0.003	0.057	0.078	0.103	0.102	0.110	0.078	0.090	0.092	0.094	0.099	0.100
		rob.	-0.009	-0.007	0.005	0.059	0.087	0.102	0.124	0.123	0.123	0.148	0.156	0.155
	300	trad.	0.000	0.064	0.084	0.099	0.116	0.115	0.042	0.047	0.052	0.053	0.054	0.055
		rob.	0.000	0.001	0.006	0.016	0.115	0.116	0.048	0.047	0.048	0.059	0.062	0.063
	500	trad.	-0.002	0.067	0.083	0.101	0.112	0.117	0.033	0.037	0.038	0.041	0.042	0.042
		rob.	-0.002	0.003	0.004	0.015	0.112	0.116	0.036	0.035	0.036	0.041	0.046	0.045
w_{25}	100	trad.	0.000	0.053	0.070	0.081	0.100	0.098	0.078	0.092	0.093	0.099	0.103	0.102
		rob.	-0.003	-0.007	0.005	0.040	0.094	0.088	0.127	0.119	0.123	0.146	0.157	0.158
	300	trad.	-0.001	0.058	0.075	0.089	0.100	0.100	0.041	0.047	0.048	0.055	0.056	0.054
		rob.	-0.001	0.002	0.004	0.016	0.100	0.098	0.048	0.047	0.050	0.060	0.065	0.063
	500	trad.	0.001	0.054	0.077	0.089	0.101	0.103	0.031	0.039	0.040	0.040	0.043	0.042
		rob.	0.001	0.000	0.006	0.014	0.100	0.103	0.034	0.037	0.038	0.041	0.046	0.047
w_{26}	100	trad.	-0.103	-0.053	-0.034	-0.029	-0.013	-0.011	0.080	0.092	0.093	0.098	0.100	0.100
		rob.	-0.102	-0.103	-0.098	-0.063	-0.019	-0.013	0.124	0.125	0.128	0.147	0.166	0.162
	300	trad.	-0.100	-0.052	-0.034	-0.019	-0.015	-0.011	0.043	0.049	0.052	0.056	0.053	0.056
		rob.	-0.100	-0.099	-0.096	-0.083	-0.016	-0.011	0.050	0.049	0.050	0.056	0.062	0.067
	500	trad.	-0.101	-0.051	-0.035	-0.021	-0.013	-0.012	0.031	0.038	0.041	0.040	0.043	0.043
		rob.	-0.101	-0.099	-0.097	-0.088	-0.014	-0.013	0.035	0.036	0.037	0.040	0.046	0.048

w_{31}	100	trad.	-0.041	-0.047	-0.036	-0.030	-0.036	-0.020	0.332	0.365	0.353	0.349	0.344	0.335
		rob.	-0.038	-0.075	-0.065	-0.062	-0.056	-0.072	0.431	0.455	0.445	0.444	0.455	0.446
	300	trad.	-0.007	-0.018	-0.021	-0.014	-0.022	-0.012	0.189	0.193	0.190	0.190	0.190	0.199
		rob.	-0.013	-0.021	-0.026	-0.014	-0.025	-0.013	0.216	0.228	0.226	0.229	0.227	0.238
	500	trad.	-0.008	-0.007	-0.011	-0.006	-0.006	-0.013	0.143	0.149	0.143	0.144	0.150	0.154
		rob.	-0.009	-0.006	-0.013	-0.008	-0.006	-0.012	0.157	0.168	0.165	0.167	0.164	0.166
w_{32}	100	trad.	-0.075	-0.058	-0.060	-0.055	-0.076	-0.048	0.446	0.426	0.434	0.426	0.442	0.423
		rob.	-0.119	-0.091	-0.089	-0.072	-0.105	-0.137	0.560	0.557	0.546	0.539	0.556	0.549
	300	trad.	-0.008	-0.022	-0.021	-0.031	-0.013	-0.011	0.246	0.245	0.230	0.235	0.230	0.222
		rob.	-0.011	-0.030	-0.034	-0.042	-0.021	-0.013	0.284	0.286	0.265	0.279	0.267	0.256
	500	trad.	-0.016	-0.020	-0.010	0.002	-0.009	-0.014	0.172	0.171	0.183	0.180	0.186	0.175
		rob.	-0.018	-0.022	-0.013	-0.002	-0.010	-0.016	0.188	0.197	0.204	0.206	0.202	0.194
w_{33}	100	trad.	-0.110	-0.132	-0.115	-0.128	-0.130	-0.104	0.528	0.499	0.514	0.540	0.527	0.491
		rob.	-0.204	-0.219	-0.216	-0.225	-0.243	-0.231	0.685	0.645	0.663	0.676	0.675	0.669
	300	trad.	-0.018	-0.031	-0.034	-0.019	-0.028	-0.018	0.280	0.285	0.278	0.274	0.282	0.275
		rob.	-0.029	-0.037	-0.047	-0.032	-0.027	-0.031	0.319	0.344	0.333	0.340	0.319	0.322
	500	trad.	-0.002	-0.030	-0.019	-0.017	-0.014	-0.004	0.207	0.207	0.226	0.212	0.212	0.219
		rob.	-0.006	-0.029	-0.024	-0.019	-0.021	-0.009	0.224	0.228	0.254	0.250	0.235	0.244
w_{34}	100	trad.	-0.066	-0.050	-0.076	-0.081	-0.045	-0.091	0.608	0.570	0.618	0.620	0.612	0.604
		rob.	-0.130	-0.114	-0.129	-0.134	-0.107	-0.085	0.780	0.739	0.767	0.754	0.779	0.769
	300	trad.	-0.038	-0.011	-0.005	-0.013	-0.015	-0.030	0.329	0.343	0.328	0.324	0.337	0.325
		rob.	-0.045	-0.025	-0.007	-0.022	-0.033	-0.040	0.379	0.404	0.391	0.387	0.391	0.379
	500	trad.	-0.014	0.013	-0.006	-0.017	-0.013	-0.016	0.237	0.246	0.273	0.257	0.253	0.255
		rob.	-0.016	0.003	-0.009	-0.026	-0.013	-0.019	0.257	0.275	0.307	0.301	0.283	0.289
w_{41}	100	trad.	-0.010	-0.005	-0.005	-0.006	-0.002	-0.004	0.101	0.099	0.101	0.101	0.102	0.101
		rob.	-0.018	-0.017	-0.015	-0.011	-0.009	-0.011	0.146	0.137	0.134	0.149	0.151	0.146
	300	trad.	-0.004	-0.004	-0.003	-0.005	-0.002	-0.003	0.053	0.056	0.058	0.058	0.056	0.057
		rob.	-0.004	-0.004	-0.003	-0.004	-0.002	-0.003	0.061	0.062	0.063	0.064	0.065	0.064
	500	trad.	-0.002	-0.002	-0.000	-0.000	-0.002	-0.002	0.041	0.042	0.045	0.043	0.045	0.045
		rob.	-0.002	-0.002	-0.001	0.000	-0.002	-0.003	0.045	0.046	0.047	0.047	0.049	0.048
w_{42}	100	trad.	0.001	-0.002	-0.004	-0.003	-0.007	-0.005	0.119	0.117	0.119	0.121	0.124	0.123
		rob.	0.000	0.003	-0.001	-0.010	-0.010	-0.006	0.170	0.157	0.159	0.170	0.179	0.177
	300	trad.	0.001	0.001	-0.001	0.002	-0.001	-0.000	0.063	0.066	0.069	0.068	0.069	0.068
		rob.	0.000	0.001	-0.001	-0.001	-0.002	-0.000	0.072	0.074	0.075	0.075	0.079	0.077
	500	trad.	-0.001	-0.001	-0.002	-0.002	-0.000	-0.000	0.050	0.051	0.053	0.053	0.054	0.054
		rob.	-0.001	-0.001	-0.002	-0.004	-0.001	0.001	0.054	0.055	0.056	0.057	0.059	0.058
w_{51}	100	trad.	-0.019	-0.021	-0.008	-0.023	-0.019	-0.025	0.178	0.176	0.185	0.187	0.190	0.187
		rob.	-0.036	-0.040	-0.021	-0.042	-0.035	-0.048	0.267	0.273	0.271	0.274	0.289	0.291
	300	trad.	-0.001	-0.011	-0.003	-0.006	-0.003	0.001	0.097	0.100	0.103	0.097	0.101	0.098
		rob.	-0.003	-0.014	-0.007	-0.006	-0.003	0.000	0.112	0.113	0.114	0.111	0.118	0.114
	500	trad.	0.004	-0.003	0.000	-0.004	-0.004	-0.003	0.073	0.075	0.076	0.078	0.079	0.077
		rob.	0.002	-0.001	0.001	-0.003	-0.005	-0.004	0.079	0.081	0.081	0.087	0.086	0.083
w_{52}	100	trad.	-0.015	-0.029	-0.022	-0.028	-0.024	-0.021	0.162	0.168	0.174	0.170	0.173	0.172
		rob.	-0.052	-0.063	-0.045	-0.068	-0.070	-0.060	0.253	0.248	0.239	0.257	0.268	0.268
	300	trad.	-0.007	-0.003	-0.007	-0.010	-0.010	-0.006	0.086	0.096	0.094	0.091	0.094	0.099
		rob.	-0.008	-0.006	-0.008	-0.014	-0.011	-0.011	0.099	0.107	0.106	0.106	0.110	0.113
	500	trad.	-0.004	-0.009	-0.004	-0.007	-0.006	-0.001	0.068	0.073	0.073	0.073	0.071	0.074
		rob.	-0.005	-0.011	-0.007	-0.009	-0.006	-0.002	0.074	0.076	0.078	0.081	0.079	0.082
w_{53}	100	trad.	-0.021	-0.016	-0.032	-0.020	-0.016	-0.027	0.159	0.172	0.169	0.178	0.180	0.179
		rob.	-0.046	-0.030	-0.054	-0.061	-0.053	-0.057	0.254	0.260	0.248	0.259	0.271	0.273
	300	trad.	-0.008	-0.008	-0.008	-0.005	-0.007	-0.014	0.093	0.096	0.095	0.094	0.096	0.096
		rob.	-0.011	-0.009	-0.008	-0.006	-0.011	-0.015	0.108	0.105	0.104	0.110	0.109	0.113
	500	trad.	-0.005	0.000	-0.007	-0.001	-0.005	-0.003	0.069	0.074	0.072	0.073	0.075	0.077
		rob.	-0.007	-0.002	-0.008	-0.003	-0.004	-0.004	0.076	0.078	0.078	0.082	0.083	0.084
w_{54}	100	trad.	-0.016	-0.008	-0.012	-0.015	-0.021	-0.011	0.163	0.172	0.173	0.179	0.173	0.171
		rob.	-0.046	-0.038	-0.029	-0.029	-0.054	-0.033	0.250	0.247	0.236	0.268	0.267	0.258
	300	trad.	-0.003	-0.004	-0.010	-0.003	-0.004	-0.005	0.087	0.091	0.092	0.090	0.096	0.098
		rob.	-0.001	-0.001	-0.010	-0.003	-0.006	-0.010	0.099	0.101	0.102	0.101	0.112	0.116
	500	trad.	-0.002	-0.002	-0.003	-0.003	-0.001	-0.011	0.069	0.073	0.073	0.073	0.074	0.072
		rob.	-0.001	-0.002	-0.002	-0.004	-0.001	-0.012	0.075	0.077	0.076	0.080	0.082	0.079
w_{55}	100	trad.	-0.013	-0.014	-0.016	-0.008	-0.018	-0.014	0.163	0.179	0.174	0.181	0.181	0.177
		rob.	-0.022	-0.028	-0.022	-0.010	-0.030	-0.030	0.261	0.268	0.260	0.273	0.280	0.274
	300	trad.	-0.006	-0.001	-0.003	-0.006	-0.002	-0.003	0.096	0.095	0.098	0.100	0.099	0.101
		rob.	-0.007	-0.003	-0.004	-0.011	-0.002	-0.002	0.109	0.108	0.111	0.115	0.116	0.114
	500	trad.	-0.009	-0.003	-0.003	-0.003	-0.002	-0.003	0.070	0.071	0.078	0.078	0.077	0.076
		rob.	-0.009	-0.004	-0.002	-0.004	-0.004	-0.002	0.076	0.077	0.083	0.084	0.083	0.084
w_{56}	100	trad.	-0.007	-0.010	-0.009	-0.010	-0.009	-0.011	0.170	0.172	0.191	0.187	0.183	0.188
		rob.	-0.018	-0.014	-0.018	-0.022	-0.005	-0.013	0.275	0.253	0.254	0.266	0.278	0.282
	300	trad.	-0.001	-0.001	0.003	0.002	-0.004	-0.002	0.096	0.095	0.101	0.100	0.099	0.100
		rob.	-0.002	-0.001	0.001	0.002	-0.005	-0.004	0.108	0.107	0.109	0.112	0.114	0.115
	500	trad.	0.003	-0.000	0.001	0.001	-0.000	0.003	0.072	0.073	0.074	0.072	0.078	0.079
		rob.	0.004	0.001	0.000	-0.001	-0.001	0.002	0.080	0.077	0.078	0.080	0.084	0.088

8.2.4 Inadmissible solutions

Table 8.21 - 8.28 show the share of inadmissible solutions for the different simulation conditions.

Unsystematic Outliers

Table 8.21: Inadmissible solutions with unsystematic outliers in all observed variables for three common factors/ three composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factors/ PLSc	100	trad.	0.349	0.934	0.960	0.964	0.962	0.960
		rob.	0.508	0.504	0.492	0.484	0.501	0.943
common factors/ PLSc	300	trad.	0.074	0.811	0.900	0.938	0.950	0.950
		rob.	0.126	0.141	0.142	0.144	0.196	0.738
common factors/ PLSc	500	trad.	0.022	0.635	0.842	0.915	0.938	0.944
		rob.	0.042	0.053	0.055	0.053	0.089	0.579
composites/ PLS	100	trad.	0.000	0.045	0.053	0.045	0.046	0.044
		rob.	0.012	0.009	0.007	0.010	0.005	0.047
composites/ PLS	300	trad.	0.000	0.042	0.049	0.038	0.035	0.037
		rob.	0.000	0.000	0.000	0.000	0.000	0.020
composites/ PLS	500	trad.	0.000	0.037	0.034	0.032	0.034	0.036
		rob.	0.000	0.000	0.000	0.000	0.000	0.021

Table 8.22: Inadmissible solutions with unsystematic outliers in all observed variables for five common factors/ five composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factors/ PLSc	100	trad.	0.370	0.907	0.934	0.942	0.945	0.945
		rob.	0.649	0.642	0.644	0.622	0.570	0.962
common factors/ PLSc	300	trad.	0.128	0.679	0.779	0.842	0.879	0.885
		rob.	0.174	0.159	0.158	0.190	0.224	0.770
common factors/ PLSc	500	trad.	0.072	0.534	0.648	0.758	0.821	0.836
		rob.	0.076	0.086	0.085	0.081	0.120	0.525
composites/ PLS	100	trad.	0.004	0.043	0.073	0.090	0.121	0.110
		rob.	0.007	0.007	0.010	0.005	0.004	0.121
composites/ PLS	300	trad.	0.000	0.007	0.011	0.021	0.038	0.049
		rob.	0.000	0.000	0.000	0.001	0.000	0.017
composites/ PLS	500	trad.	0.000	0.000	0.004	0.002	0.015	0.029
		rob.	0.000	0.000	0.000	0.000	0.001	0.005

Table 8.23: Inadmissible solutions with unsystematic outliers in two observed variables for three common factors/ three composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factors/ PLSc	100	trad.	0.342	0.364	0.330	0.338	0.153	0.311
		rob.	0.502	0.511	0.494	0.476	0.250	0.462
common factors/ PLSc	300	trad.	0.084	0.089	0.095	0.079	0.039	0.025
		rob.	0.155	0.133	0.139	0.139	0.096	0.078
common factors/ PLSc	500	trad.	0.032	0.038	0.030	0.028	0.011	0.028
		rob.	0.055	0.039	0.048	0.046	0.039	0.107
composites/ PLS	100	trad.	0.000	0.020	0.011	0.003	0.002	0.005
		rob.	0.004	0.004	0.006	0.003	0.010	0.020
composites/ PLS	300	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.000	0.000	0.000	0.000	0.000	0.000
composites/ PLS	500	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.000	0.000	0.000	0.000	0.000	0.000

Table 8.24: Inadmissible solutions with unsystematic outliers in two observed variables for five common factors/ five composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factors/ PLSc	100	trad.	0.366	0.463	0.530	0.645	0.746	0.765
		rob.	0.632	0.627	0.634	0.612	0.708	0.852
common factors/ PLSc	300	trad.	0.144	0.123	0.182	0.276	0.498	0.577
		rob.	0.173	0.178	0.170	0.190	0.227	0.407
common factors/ PLSc	500	trad.	0.071	0.063	0.089	0.151	0.376	0.457
		rob.	0.104	0.077	0.084	0.106	0.113	0.165
composites/ PLS	100	trad.	0.004	0.001	0.003	0.005	0.001	0.003
		rob.	0.008	0.010	0.009	0.006	0.007	0.019
composites/ PLS	300	trad.	0.000	0.000	0.000	0.000	0.001	0.001
		rob.	0.000	0.000	0.000	0.000	0.000	0.001
composites/ PLS	500	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.000	0.000	0.000	0.000	0.000	0.000

Systematic Outliers

Table 8.25: Inadmissible solutions with systematic outliers in all observed variables for three common factors/ three composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factor/PLSc	100	trad.	0.349	0.032	0.008	0.000	0.001	0.000
		rob.	0.508	0.497	0.490	0.483	0.300	0.023
common factor/PLSc	300	trad.	0.087	0.000	0.000	0.000	0.000	0.000
		rob.	0.124	0.145	0.141	0.142	0.127	0.000
common factor/PLSc	500	trad.	0.021	0.000	0.000	0.000	0.000	0.000
		rob.	0.045	0.049	0.057	0.061	0.042	0.000
composites/ PLS	100	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.007	0.005	0.002	0.008	0.002	0.000
composites/ PLS	300	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.000	0.000	0.000	0.000	0.000	0.000
composites/ PLS	500	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.000	0.000	0.000	0.000	0.000	0.000

Table 8.26: Inadmissible solutions with systematic outliers in all observed variables for five common factors/ five composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factors/ PLSc	100	trad.	0.358	0.696	0.890	0.960	0.969	0.968
		rob.	0.649	0.627	0.614	0.642	0.962	0.966
common factors/ PLSc	300	trad.	0.146	0.733	0.933	0.962	0.981	0.982
		rob.	0.184	0.155	0.174	0.155	0.981	0.980
common factors/ PLSc	500	trad.	0.071	0.800	0.945	0.959	0.979	0.983
		rob.	0.071	0.085	0.097	0.091	0.979	0.984
composites/ PLS	100	trad.	0.004	0.007	0.002	0.001	0.000	0.000
		rob.	0.006	0.016	0.007	0.011	0.000	0.001
composites/ PLS	300	trad.	0.000	0.000	0.002	0.002	0.001	0.000
		rob.	0.000	0.000	0.000	0.000	0.000	0.000
composites/ PLS	500	trad.	0.000	0.000	0.000	0.001	0.000	0.003
		rob.	0.000	0.000	0.000	0.000	0.002	0.000

Table 8.27: Inadmissible solutions with systematic outliers in two observed variables for three common factors/ three composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factor/PLSc	100	trad.	0.354	0.308	0.324	0.442	0.523	0.781
		rob.	0.511	0.479	0.482	0.460	0.539	0.824
common factor/PLSc	300	trad.	0.078	0.082	0.100	0.143	0.429	0.794
		rob.	0.129	0.153	0.145	0.133	0.332	0.804
common factor/PLSc	500	trad.	0.027	0.030	0.043	0.055	0.413	0.821
		rob.	0.040	0.049	0.057	0.061	0.236	0.826
composites/ PLS	100	trad.	0.001	0.030	0.065	0.052	0.034	0.029
		rob.	0.003	0.002	0.007	0.012	0.035	0.039
composites/ PLS	300	trad.	0.000	0.002	0.037	0.022	0.003	0.002
		rob.	0.000	0.000	0.000	0.000	0.006	0.005
composites/ PLS	500	trad.	0.000	0.000	0.017	0.009	0.001	0.002
		rob.	0.000	0.000	0.000	0.000	0.002	0.001

Table 8.28: Inadmissible solutions with systematic outliers in two observed variables for five common factors/ five composites

Construct types/ Estimator	n	Appr.	Share of inadmissible solutions					
			0%	5%	10%	20%	40%	50%
common factors/ PLSc	100	trad.	0.383	0.424	0.551	0.713	0.825	0.834
		rob.	0.643	0.633	0.628	0.736	0.911	0.923
common factors/ PLSc	300	trad.	0.125	0.123	0.197	0.464	0.646	0.646
		rob.	0.167	0.163	0.183	0.203	0.680	0.692
common factors/ PLSc	500	trad.	0.059	0.068	0.086	0.339	0.558	0.558
		rob.	0.082	0.093	0.103	0.097	0.599	0.598
composites/ PLS	100	trad.	0.002	0.001	0.007	0.000	0.000	0.001
		rob.	0.011	0.007	0.011	0.005	0.012	0.013
composites/ PLS	300	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.000	0.001	0.001	0.000	0.000	0.000
composites/ PLS	500	trad.	0.000	0.000	0.000	0.000	0.000	0.000
		rob.	0.000	0.000	0.000	0.000	0.000	0.000

8.3 Performing out-of-sample predictions based on models estimated by ordinal consistent partial least squares

Table 8.29: Average performance of PLSpredict, PLScpredict, OrdPLSpredict and OrdPLScpredict in the case of ordinal categorical observed variables

Categ.	n	Distribution	Approach	MAE						RMSE						concordance		
				x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	x_{21}	x_{23}	x_{33}
4	200	extreme	OrdPLScpredict (mean)	0.707	0.757	0.718	0.778	0.785	0.733	1.156	0.941	1.175	0.972	0.975	1.200	0.554	0.552	0.551
			OrdPLScpredict (median)	0.708	0.756	0.719	0.778	0.784	0.733	1.156	0.941	1.174	0.971	0.975	1.200	0.551	0.550	0.549
			OrdPLSpredict (mean)	0.704	0.755	0.714	0.777	0.783	0.729	1.166	0.939	1.177	0.971	0.973	1.212	0.564	0.559	0.565
			OrdPLSpredict (median)	0.704	0.755	0.714	0.777	0.783	0.730	1.165	0.939	1.176	0.971	0.973	1.213	0.563	0.557	0.565
			PLSpredict	0.994	0.839	0.998	0.804	0.825	0.982	1.131	1.041	1.141	1.004	1.027	1.113	0.150	0.156	0.149
			PLScpredict	1.025	0.893	1.024	0.810	0.848	1.011	1.176	1.105	1.174	1.011	1.055	1.153	0.154	0.153	0.150
4	200	symmetric	OrdPLScpredict (mean)	0.599	0.754	0.612	0.794	0.791	0.635	0.842	0.942	0.850	0.990	0.989	0.877	0.457	0.445	0.434
			OrdPLScpredict (median)	0.599	0.754	0.612	0.794	0.791	0.635	0.841	0.942	0.850	0.990	0.989	0.877	0.457	0.444	0.434
			OrdPLSpredict (mean)	0.600	0.752	0.611	0.794	0.790	0.637	0.841	0.940	0.849	0.989	0.988	0.880	0.456	0.445	0.433
			OrdPLSpredict (median)	0.600	0.752	0.611	0.794	0.790	0.637	0.842	0.940	0.849	0.989	0.988	0.879	0.456	0.445	0.433
			PLSpredict	0.728	0.973	0.741	0.844	0.874	0.710	0.978	1.193	0.989	1.054	1.089	0.961	0.388	0.379	0.397
			PLScpredict	0.768	1.061	0.790	0.848	0.899	0.731	1.020	1.285	1.041	1.059	1.117	0.982	0.369	0.358	0.387
4	500	extreme	OrdPLScpredict (mean)	0.699	0.746	0.705	0.799	0.774	0.711	1.140	0.934	1.147	0.999	0.967	1.178	0.549	0.549	0.562
			OrdPLScpredict (median)	0.700	0.746	0.705	0.798	0.774	0.711	1.140	0.934	1.144	0.999	0.967	1.177	0.547	0.547	0.562
			OrdPLSpredict (mean)	0.699	0.746	0.699	0.798	0.774	0.710	1.152	0.933	1.150	0.999	0.966	1.197	0.557	0.559	0.579
			OrdPLSpredict (median)	0.701	0.746	0.699	0.798	0.773	0.711	1.152	0.933	1.148	0.999	0.966	1.198	0.556	0.557	0.579
			PLSpredict	0.989	0.843	0.998	0.823	0.809	0.964	1.129	1.047	1.135	1.031	1.010	1.092	0.157	0.149	0.151
			PLScpredict	1.014	0.898	1.032	0.827	0.825	0.992	1.165	1.110	1.179	1.038	1.028	1.129	0.159	0.147	0.152
4	500	symmetric	OrdPLScpredict (mean)	0.598	0.748	0.594	0.786	0.781	0.626	0.837	0.932	0.837	0.983	0.980	0.870	0.455	0.461	0.441
			OrdPLScpredict (median)	0.598	0.748	0.594	0.786	0.781	0.625	0.837	0.932	0.837	0.983	0.980	0.870	0.456	0.461	0.442
			OrdPLSpredict (mean)	0.599	0.747	0.594	0.787	0.780	0.624	0.838	0.931	0.837	0.984	0.980	0.869	0.455	0.460	0.443
			OrdPLSpredict (median)	0.599	0.747	0.595	0.787	0.781	0.624	0.838	0.931	0.838	0.984	0.980	0.870	0.455	0.460	0.443
			PLSpredict	0.723	0.965	0.735	0.842	0.885	0.712	0.980	1.190	0.984	1.053	1.101	0.957	0.397	0.382	0.391
			PLScpredict	0.774	1.046	0.790	0.841	0.915	0.725	1.031	1.276	1.039	1.050	1.135	0.973	0.369	0.353	0.387
4	1000	extreme	OrdPLScpredict (mean)	0.672	0.763	0.697	0.782	0.791	0.728	1.113	0.956	1.150	0.976	0.985	1.189	0.567	0.559	0.545
			OrdPLScpredict (median)	0.674	0.763	0.697	0.781	0.791	0.730	1.113	0.955	1.148	0.976	0.985	1.190	0.564	0.557	0.544
			OrdPLSpredict (mean)	0.671	0.763	0.693	0.781	0.791	0.727	1.125	0.955	1.155	0.975	0.985	1.208	0.577	0.567	0.562
			OrdPLSpredict (median)	0.671	0.764	0.695	0.781	0.791	0.727	1.125	0.956	1.156	0.975	0.985	1.208	0.576	0.565	0.563
			PLSpredict	0.979	0.869	0.995	0.803	0.823	0.959	1.118	1.078	1.137	1.003	1.028	1.093	0.157	0.154	0.160
			PLScpredict	1.019	0.937	1.026	0.809	0.840	0.977	1.170	1.157	1.175	1.011	1.050	1.122	0.156	0.153	0.165
4	1000	symmetric	OrdPLScpredict (mean)	0.594	0.750	0.583	0.767	0.774	0.616	0.832	0.938	0.822	0.958	0.969	0.855	0.456	0.464	0.444
			OrdPLScpredict (median)	0.594	0.750	0.584	0.767	0.774	0.613	0.832	0.938	0.822	0.958	0.969	0.853	0.457	0.464	0.445
			OrdPLSpredict (mean)	0.592	0.749	0.582	0.767	0.774	0.617	0.832	0.936	0.821	0.958	0.969	0.857	0.458	0.465	0.443
			OrdPLSpredict (median)	0.594	0.749	0.582	0.767	0.773	0.616	0.832	0.936	0.821	0.958	0.969	0.856	0.457	0.465	0.444
			PLSpredict	0.737	0.980	0.724	0.811	0.852	0.686	0.987	1.205	0.976	1.019	1.062	0.936	0.382	0.390	0.412
			PLScpredict	0.786	1.063	0.781	0.813	0.872	0.691	1.038	1.294	1.033	1.020	1.085	0.942	0.358	0.361	0.409
5	200	extreme	OrdPLScpredict (mean)	0.993	0.753	0.995	0.790	0.781	1.015	1.462	0.938	1.468	0.987	0.977	1.495	0.418	0.423	0.415
			OrdPLScpredict (median)	0.992	0.753	0.997	0.790	0.781	1.016	1.455	0.938	1.464	0.987	0.976	1.491	0.414	0.417	0.410
			OrdPLSpredict (mean)	0.986	0.752	0.992	0.790	0.780	1.011	1.456	0.935	1.467	0.986	0.975	1.499	0.419	0.424	0.420
			OrdPLSpredict (median)	0.986	0.752	0.993	0.790	0.780	1.011	1.451	0.935	1.461	0.986	0.975	1.494	0.415	0.419	0.416
			PLSpredict	1.347	0.900	1.366	0.832	0.842	1.263	1.572	1.115	1.576	1.038	1.047	1.463	0.149	0.132	0.142
			PLScpredict	1.439	0.985	1.422	0.841	0.871	1.335	1.687	1.212	1.649	1.048	1.080	1.547	0.145	0.133	0.141
5	200	symmetric	OrdPLScpredict (mean)	0.744	0.768	0.731	0.800	0.786	0.759	1.008	0.961	0.986	0.994	0.988	1.019	0.390	0.388	0.379
			OrdPLScpredict (median)	0.745	0.768	0.731	0.800	0.786	0.758	1.009	0.961	0.986	0.994	0.988	1.017	0.389	0.389	0.379
			OrdPLSpredict (mean)	0.745	0.767	0.732	0.799	0.785	0.759	1.005	0.961	0.985	0.993	0.987	1.015	0.387	0.387	0.377
			OrdPLSpredict (median)	0.747	0.767	0.732	0.799	0.785	0.757	1.006	0.961	0.985	0.993	0.987	1.014	0.386	0.387	0.378
			PLSpredict	1.029	1.055	1.015	0.894	0.921	0.914	1.311	1.287	1.294	1.113	1.147	1.197	0.274	0.277	0.324
			PLScpredict	1.092	1.154	1.111	0.906	0.953	0.999	1.377	1.389	1.389	1.127	1.182	1.289	0.256	0.245	0.297
5	500	extreme	OrdPLScpredict (mean)	0.965	0.748	0.950	0.794	0.762	1.016	1.446	0.934	1.427	0.993	0.950	1.512	0.437	0.443	0.430
			OrdPLScpredict (median)	0.965	0.747	0.947	0.794	0.762	1.013	1.440	0.933	1.419	0.993	0.950	1.504	0.432	0.440	0.427
			OrdPLSpredict (mean)	0.960	0.747	0.944	0.793	0.762	1.020	1.448	0.933	1.424	0.992	0.950	1.531	0.442	0.449	0.437
			OrdPLSpredict (median)	0.962	0.747	0.944	0.793	0.762	1.019	1.442	0.933	1.419	0.992	0.949	1.525	0.437	0.444	0.433
			PLSpredict	1.295	0.894	1.298	0.821	0.812	1.242	1.509	1.105	1.511	1.032	1.009	1.438	0.147	0.144	0.143
			PLScpredict	1.346	0.960	1.365	0.826	0.829	1.303	1.578	1.182	1.594	1.038	1.030	1.511	0.151	0.137	0.140
5	500	symmetric	OrdPLScpredict (mean)	0.749	0.741	0.750	0.786	0.796	0.745	1.003	0.921	1.006	0.990	0.995	0.995	0.378	0.379	0.378
			OrdPLScpredict (median)	0.748	0.741	0.750	0.786	0.795	0.745	1.002	0.921	1.006	0.990	0.995	0.995	0.378	0.379	0.378
			OrdPLSpredict (mean)	0.744	0.740	0.750	0.786	0.795	0.748	0.996	0.919	1.002	0.990	0.994	1.000	0.380	0.377	0.379
			OrdPLSpredict (median)	0.744	0.740	0.749	0.786	0.795	0.748	0.995	0.919	1.001	0.990	0.994	1.000	0.379	0.377	0.380
			PLSpredict	1.067	1.053	1.058	0.870	0.926	0.938	1.354	1.279	1.344	1.090	1.152	1.219	0.267	0.270	0.311
			PLScpredict	1.137	1.159	1.155	0.866	0.946	1.030	1.426	1.390	1.444	1.085	1.175	1.316	0.249	0.243	0.281
5	1000	extreme	OrdPLScpredict (mean)	0.962	0.745	1.008	0.790	0.772	1.028	1.429	0.933	1.490	0.986	0.968	1.503	0.432	0.427	0.403
			OrdPLScpredict (median)	0.960	0.744	1.009	0.789	0.772	1.029	1.424	0.933	1.484	0.985	0.968	1.498	0.430	0.420	0.397
			OrdPLSpredict (mean)	0.959	0.744	1.005	0.790	0.772	1.028	1.432	0.932	1.491	0.986	0.968	1.512	0.435	0.430	0.407
			OrdPLSpredict (median)	0.958	0.744	1.006	0.790	0.772	1.028	1.426	0.932	1.487	0.986	0.968	1.507	0.431	0.424	0.403
			PLSpredict	1.315	0.903	1.309	0.820	0.826	1.239	1.530	1.120	1.533	1.022	1.037	1.439	0.144	0.155	0.150
			PLScpredict	1.407	1.000	1.365	0.826	0.850	1.301	1.643	1.229	1.600	1.029	1.066	1.512	0.137	0.148	0.146
5	1000	symmetric	OrdPLScpredict (mean)	0.754	0.751	0.730	0.785	0.780	0.748	1.005	0.939	0.986	0.980	0.974	1.015	0.373	0.391	0.393
			OrdPLScpredict (median)	0.754	0.751	0.729	0.785	0.780	0.747	1.004	0.939	0.984	0.980	0.974	1.014	0.372	0.391	0.394
			OrdPLSpredict (mean)	0.753	0.749	0.719	0.785	0.780	0.739	1.002	0.937	0.973	0.980	0.973	1.005	0.373	0.396	0.399
			OrdPLSpredict (median)	0.753	0.749	0.720	0.785	0.780	0.738	1.003	0.937	0.975	0.980	0.973	1.005	0.373	0.396	0.399
			PLSpredict	1.043	1.067	1.008	0.865	0.915	0.938	1.346	1.297	1.299	1.081	1.130	1.225	0.289	0.290	0.314
			PLScpredict	1.141	1.173	1.111	0.867	0.941	1.021	1.437	1.407	1.403	1.083	1.160	1.309	0.251	0.254	0.282

8.4 A maximum likelihood estimator for composite models

Table 8.30: Simulation results

Par.	Estimator	Mean				Standard Deviation				RMSE			
		$n = 100$	$n = 200$	$n = 500$	$n = 1000$	$n = 100$	$n = 200$	$n = 500$	$n = 1000$	$n = 100$	$n = 200$	$n = 500$	$n = 1000$
$\sigma_{11}^{(1)}$	ML	-0.018	0.006	0.001	0.001	0.405	0.310	0.200	0.133	0.406	0.310	0.199	0.133
	PLS	-0.018	0.006	0.001	0.001	0.405	0.310	0.200	0.133	0.406	0.310	0.199	0.133
$\sigma_{12}^{(1)}$	ML	-0.020	0.010	0.003	-0.002	0.454	0.319	0.209	0.148	0.454	0.319	0.209	0.148
	PLS	-0.020	0.010	0.003	-0.002	0.454	0.319	0.209	0.148	0.454	0.319	0.209	0.148
$\sigma_{22}^{(1)}$	ML	-0.020	0.020	0.008	-0.003	0.713	0.478	0.310	0.227	0.713	0.478	0.310	0.227
	PLS	-0.020	0.020	0.008	-0.003	0.713	0.478	0.310	0.227	0.713	0.478	0.310	0.227
$\sigma_{11}^{(2)}$	ML	-0.013	0.009	-0.002	0.003	0.577	0.402	0.247	0.179	0.577	0.401	0.247	0.179
	PLS	-0.013	0.009	-0.002	0.003	0.577	0.402	0.247	0.179	0.577	0.401	0.247	0.179
$\sigma_{12}^{(2)}$	ML	-0.007	0.008	-0.001	-0.003	0.347	0.261	0.160	0.117	0.347	0.261	0.160	0.117
	PLS	-0.007	0.008	-0.001	-0.003	0.347	0.261	0.160	0.117	0.347	0.261	0.160	0.117
$\sigma_{13}^{(2)}$	ML	-0.010	0.008	-0.006	-0.004	0.656	0.452	0.284	0.198	0.656	0.452	0.284	0.198
	PLS	-0.010	0.008	-0.006	-0.004	0.656	0.452	0.284	0.198	0.656	0.452	0.284	0.198
$\sigma_{22}^{(2)}$	ML	0.012	0.010	-0.004	0.005	0.422	0.299	0.191	0.134	0.422	0.299	0.191	0.134
	PLS	0.012	0.010	-0.004	0.005	0.422	0.299	0.191	0.134	0.422	0.299	0.191	0.134
$\sigma_{23}^{(2)}$	ML	0.010	0.006	0.000	0.001	0.537	0.367	0.241	0.161	0.537	0.367	0.241	0.161
	PLS	0.010	0.006	0.000	0.001	0.537	0.367	0.241	0.161	0.537	0.367	0.241	0.161
$\sigma_{33}^{(2)}$	ML	0.014	0.025	-0.006	-0.011	1.262	0.891	0.574	0.412	1.262	0.891	0.573	0.412
	PLS	0.014	0.025	-0.006	-0.011	1.263	0.891	0.574	0.412	1.262	0.891	0.573	0.412
$\sigma_{11}^{(3)}$	ML	0.006	0.002	-0.002	-0.001	0.280	0.197	0.124	0.091	0.280	0.197	0.124	0.091
	PLS	0.006	0.002	-0.002	-0.001	0.280	0.197	0.124	0.091	0.280	0.197	0.124	0.091
$\sigma_{12}^{(3)}$	ML	0.003	0.007	-0.004	-0.003	0.327	0.226	0.150	0.106	0.327	0.226	0.150	0.106
	PLS	0.003	0.007	-0.004	-0.003	0.327	0.226	0.150	0.106	0.327	0.226	0.150	0.106
$\sigma_{13}^{(3)}$	ML	0.004	0.011	-0.002	-0.010	0.422	0.304	0.191	0.135	0.422	0.304	0.191	0.136
	PLS	0.004	0.011	-0.002	-0.010	0.422	0.304	0.191	0.135	0.422	0.304	0.191	0.136
$\sigma_{14}^{(3)}$	ML	-0.002	0.007	-0.006	0.001	0.453	0.325	0.209	0.149	0.453	0.324	0.209	0.149
	PLS	-0.002	0.007	-0.006	0.001	0.453	0.325	0.209	0.149	0.453	0.324	0.209	0.149
$\sigma_{22}^{(3)}$	ML	0.009	-0.008	0.011	0.008	0.725	0.497	0.309	0.227	0.725	0.497	0.309	0.227
	PLS	0.009	-0.008	0.011	0.008	0.725	0.497	0.309	0.227	0.725	0.497	0.309	0.227
$\sigma_{23}^{(3)}$	ML	-0.021	-0.003	-0.007	-0.006	0.696	0.461	0.306	0.213	0.696	0.461	0.306	0.213
	PLS	-0.021	-0.003	-0.007	-0.006	0.696	0.461	0.306	0.213	0.696	0.461	0.306	0.213
$\sigma_{24}^{(3)}$	ML	0.014	0.009	0.008	-0.001	0.710	0.515	0.320	0.242	0.710	0.514	0.320	0.242
	PLS	0.014	0.009	0.008	-0.001	0.710	0.515	0.320	0.242	0.710	0.514	0.320	0.242
$\sigma_{33}^{(3)}$	ML	-0.013	-0.010	0.003	0.001	1.117	0.791	0.524	0.353	1.117	0.791	0.524	0.352
	PLS	-0.013	-0.010	0.003	0.001	1.117	0.791	0.524	0.353	1.117	0.791	0.524	0.352
$\sigma_{34}^{(3)}$	ML	-0.001	0.005	-0.019	-0.010	0.873	0.618	0.389	0.275	0.872	0.618	0.390	0.275
	PLS	-0.001	0.005	-0.019	-0.010	0.873	0.618	0.389	0.275	0.873	0.618	0.390	0.275
$\sigma_{44}^{(3)}$	ML	0.018	0.046	0.013	-0.017	1.260	0.914	0.552	0.410	1.259	0.914	0.552	0.410
	PLS	0.018	0.046	0.013	-0.017	1.260	0.914	0.552	0.410	1.259	0.914	0.552	0.410
w_{11}	ML	-0.003	-0.012	-0.005	-0.003	0.222	0.162	0.098	0.070	0.222	0.162	0.098	0.070
	PLS	-0.003	-0.012	-0.005	-0.003	0.221	0.162	0.098	0.070	0.221	0.163	0.098	0.070
w_{12}	ML	-0.025	-0.004	0.000	0.000	0.192	0.126	0.078	0.057	0.193	0.126	0.078	0.057
	PLS	-0.025	-0.004	0.000	0.000	0.192	0.126	0.078	0.057	0.193	0.126	0.078	0.057
w_{21}	ML	-0.004	-0.003	-0.002	-0.002	0.115	0.073	0.045	0.032	0.115	0.073	0.045	0.032
	PLS	-0.004	-0.003	-0.002	-0.002	0.114	0.073	0.045	0.032	0.114	0.073	0.045	0.032
w_{22}	ML	-0.009	-0.003	-0.001	-0.001	0.121	0.075	0.046	0.032	0.122	0.075	0.046	0.032
	PLS	-0.010	-0.003	-0.001	-0.001	0.121	0.074	0.046	0.032	0.121	0.074	0.046	0.032
w_{23}	ML	-0.013	-0.004	-0.001	0.001	0.070	0.046	0.029	0.021	0.071	0.046	0.029	0.021
	PLS	-0.013	-0.004	-0.001	0.000	0.070	0.046	0.029	0.021	0.071	0.046	0.029	0.021
w_{31}	ML	-0.023	-0.002	-0.004	-0.004	0.182	0.121	0.078	0.055	0.183	0.121	0.078	0.055
	PLS	-0.023	-0.002	-0.004	-0.004	0.182	0.121	0.078	0.055	0.183	0.121	0.078	0.055
w_{32}	ML	-0.008	0.000	0.000	0.001	0.127	0.089	0.053	0.038	0.127	0.089	0.053	0.038
	PLS	-0.008	0.000	0.000	0.001	0.127	0.089	0.053	0.038	0.127	0.089	0.053	0.038
w_{33}	ML	-0.018	-0.010	-0.004	-0.001	0.084	0.061	0.036	0.027	0.086	0.062	0.036	0.027
	PLS	-0.018	-0.010	-0.004	-0.001	0.084	0.061	0.036	0.027	0.086	0.062	0.036	0.027
w_{34}	ML	-0.005	-0.007	0.000	-0.001	0.093	0.065	0.041	0.030	0.093	0.065	0.041	0.030
	PLS	-0.005	-0.007	0.000	-0.001	0.093	0.065	0.041	0.030	0.093	0.065	0.041	0.030
γ_{12}	ML	0.019	0.010	0.006	0.004	0.100	0.065	0.042	0.029	0.102	0.066	0.043	0.030
	PLS	0.021	0.011	0.006	0.004	0.099	0.065	0.042	0.029	0.101	0.066	0.043	0.030
b_{23}	ML	0.042	0.020	0.007	0.004	0.089	0.054	0.037	0.026	0.098	0.058	0.038	0.026
	PLS	0.041	0.020	0.007	0.003	0.089	0.054	0.037	0.026	0.098	0.058	0.038	0.026

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