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ORIGINAL PAPER

Coherence and path indistinguishability for the interference of multiple single-mode fields

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Abstract: A well-known result for the interference of two single-mode fields is that the degree of coherence and the degree of indistinguishability are the same when we consider the detection of a single photon. In this article, we present the relation between the degree of coherence, path indistinguishability and the fringe visibility considering interference of multiple numbers of single-mode fields while being interested in the detection of a single photon only. We will also mention how Born's rule of interference for multiple sources is reflected in these results.

Keywords: Path indistinguishability; Coherence; Quantum optics

1. Introduction

Double-slit interference experiment is one of the most remarkable experiments, ever proposed in Physics. From its initial purpose of understanding the nature of light to later explaining the non-trivial nature of quantum mechanics, it has always been of prime interest. Richard P. Feynman's discussions of the double-slit experiment and how the visibility of quantum interference of states is dependent on the path information is very useful for understanding why this is one of the defining features of quantum mechanics [1]. By the first half of the nineteenth century, it was clear that two-point amplitude correlation function was an important tool to understand the nature of the optical phenomena observed till that point. A systematic development of this study was done by E. Wolf. In the classical case where the electric field can be written as sum of positive ($E^{(+)}$) and negative frequency ($E^{(-)}$) part, the second-order correlation function can be defined as the statistical average of the product of these two,

$$\mathcal{C}^{(2)} = \langle E^{(-)}(x_1)E^{(+)}(x_2)\rangle \qquad (1)$$

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The need for higher-order correlation functions became clear after the HBT intensity interferometry in 1956. Mandel did some pioneering works with these newly developed tools by doing semi-classical treatment of light. Here, we note that there is some ambiguity in the nomenclature of the correlation functions. The correlation function of 1 is called second order by some authors [4] and first order by another group of authors [12]. We here follow the convention followed by Mandel and call it second order. In quantum theory of optical coherence, photon correlation function was first defined by Glauber [2, 3]. Using the methods of light detection due to absorption of photons from the field, Glauber was able to derive the most useful measure of partial coherence of the quantized electromagnetic field at the two-point level and generalized it to correlation functions of arbitrary order. In this article, we are interested in the study of the equivalence of coherence and indistinguishability which is an very important result in the perspective of wave-particle duality. Coherence is one of the main criteria for interference of light beams. On the other hand in single-photon interference experiment, the lack of photon's path information plays a crucial role. In his famous article, Mandel [4] has shown that the modulus of degree of second-order coherence is identical to the degree of indistinguishability for the case of two single-mode fields emitted from two sources and by considering the detection a single photon only. The experimental verification of the double-slit thought experiment for single electron [5] and single photon [6] has been successfully performed, and in recent times, the study of coherence [7–9] and experiments of interference for more than two slits [10] has been of prime interest for proving the validity of Born's rule. In this paper, we describe the generalisation of L. Mandel's result for 3 single-mode fields followed by N number of single-mode fields while considering the detection of a single photon only. We also discuss how the equivalent form of Born's rule is obtained in the context of coherence and fringe visibility.

2. Interference of three single-mode fields

Before we discuss the generalised version of the interference, we consider the case of interference of three single-mode fields and the detection of a single photon only. The schematic diagram of the experiment under consideration is shown at Fig. 1. Here, we have three sources and so a photon detected at any point on the detector may take three possible paths. We write the state of the photon before the detection process takes place as

$$|\psi\rangle = \alpha|1\rangle_1|0\rangle_2|0\rangle_3 + \beta|0\rangle_1|1\rangle_2|0\rangle_3 + \gamma|0\rangle_1|0\rangle_2|1\rangle_3 \tag{2}$$

where $|\alpha|^2$, $|\beta|^2$ and $|\gamma|^2$ are the probability of the photon being produced by the first, second and third source, respectively, and $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. Considering this state the density matrix of this pure state can be written as

$$\begin{aligned}\hat{\rho}_{\mathrm{ID}} &= |\psi\rangle\langle\psi| \\ &= \begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix}\end{aligned} \tag{3}$$

On the other hand, we could have an incoherent mixture of states and the corresponding density matrix of this photon represented by a diagonal density matrix of the form

$$\hat{\rho}_{\mathrm{D}} = \begin{pmatrix} |\alpha|^2 & 0 & 0 \\ 0 & |\beta|^2 & 0 \\ 0 & 0 & |\gamma|^2 \end{pmatrix} \tag{4}$$

In the second case, we do not have the off-diagonal terms implying that the intrinsic indistinguishability of the photon path is lost. So, now for the density matrix $\hat{\rho}_D$ in principle we will be able to detect the source of this photon experimentally and the interference pattern will be lost. Following the notation of L. Mandel, here also the subscript of $\hat{\rho}_{ID}$ and $\hat{\rho}_D$ signifies the path indistinguishability and this potential path distinguishability for those two density matrices, respectively. Now in this Hilbert space, we take a general density matrix of the form

$$\hat{\rho} = \sum_{n,m=1}^{3} \rho_{nm}|n\rangle\langle m| \tag{5}$$

and we can decompose it in the terms of $\hat{\rho}_{ID}$ and $\hat{\rho}_D$ to determine the degree of indistinguishability for the system. We write the general density matrix as

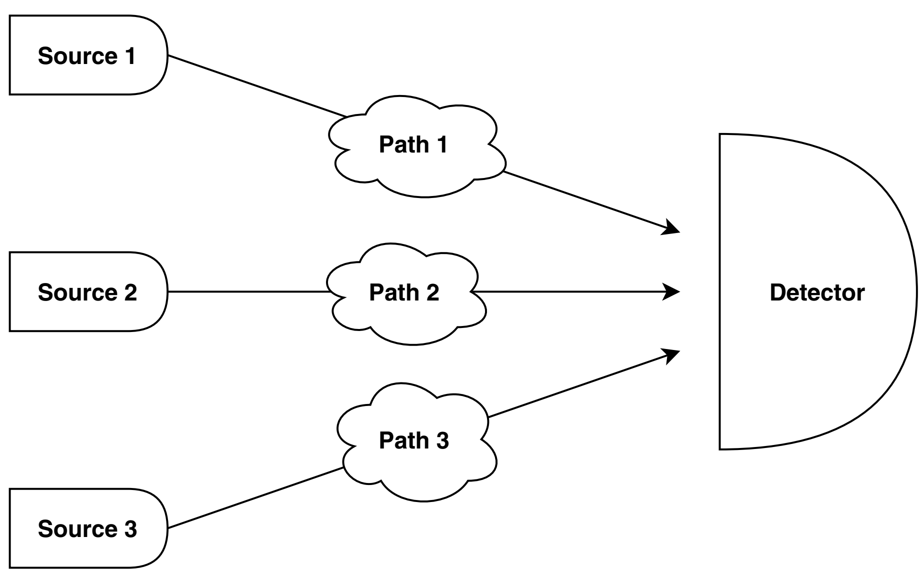

Fig. 1 Schematic diagram of the interference experiment

$$\hat{\rho} = P_{ID}\hat{\rho}_{ID} + P_D\hat{\rho}_D \quad \text{where} \quad P_{ID} + P_D = 1 \tag{6}$$

From Eqs. 6, 5 and 4, we get

$$\rho_{11} = |\alpha|^2, \quad \rho_{22} = |\beta|^2, \quad \rho_{33} = |\gamma|^2 \tag{7}$$

$$\rho_{12} = P_{ID}\alpha\beta^*, \quad \rho_{13} = P_{ID}\alpha\gamma^*, \quad \rho_{23} = P_{ID}\beta\gamma^*. \tag{8}$$

Due to the hermiticity of the density matrix, we can evaluate the lower triangular elements as the complex conjugate of the upper triangular ones. Now using very simple calculation, we see that

$$\begin{aligned} P_{ID} &= \frac{\rho_{12}}{\sqrt{\rho_{11}\rho_{22}}}e^{-i\cdot\arg(\rho_{12})} = \frac{\rho_{13}}{\sqrt{\rho_{11}\rho_{33}}}e^{-i\cdot\arg(\rho_{13})} \\ &= \frac{\rho_{23}}{\sqrt{\rho_{22}\rho_{33}}}e^{-i\cdot\arg(\rho_{23})} \\ &= \frac{|\rho_{21}|}{\sqrt{\rho_{11}\rho_{22}}} = \frac{|\rho_{31}|}{\sqrt{\rho_{11}\rho_{33}}} = \frac{|\rho_{32}|}{\sqrt{\rho_{22}\rho_{33}}} \end{aligned} \tag{9}$$

which indicates that P_{ID} can be written in three equivalent forms where we only have pairwise terms out of the three slits.

Now we calculate the degree of coherence for the system and for that We denote the positive frequency component of the single-mode electric field of the source as

$$\hat{E}^{(+)}(r_j) = K\hat{a}_j \qquad \text{where} j = 1, 2, 3 \tag{10}$$

where K is a constant. The normalised second-order coherence function of the form [11, 12]

$$g^{(1)}(x_i, x_j) = \frac{G^{(1)}(x_i, x_j)}{\sqrt{G^{(1)}(x_i, x_i)G^{(1)}(x_j, x_j)}} \tag{11}$$

where

$$G^{(1)}(x_i, x_j) = \mathrm{Tr}\{\hat{\rho}\hat{E}^{(-)}(x_i)\hat{E}^{(+)}(x_j)\} \tag{12}$$

and $G^{(1)}(x_i, x_j)$ is the general second-order coherence function. For the first pair of sources (x_1, x_2), we get $G^{(1)}(x_1, x_2)$ of the form

$$\begin{aligned} G^{(1)}(x_1, x_2) &= Tr\{\hat{\rho}\hat{E}^{(-)}(x_1)\hat{E}^{(+)}(x_2)\} \\ &= |K|^2\rho_{21} \end{aligned} \tag{13}$$

similarly we get,

$$G^{(1)}(x_1, x_1) = |K|^2\rho_{11} \tag{14}$$

$$G^{(1)}(x_2, x_2) = |K|^2\rho_{22} \tag{15}$$

Now from Eqs. 11, 13, 14 and 15, we get,

$$g^{(1)}(x_1, x_2) = \frac{\rho_{21}}{(\rho_{11}\rho_{22})^{\frac{1}{2}}} \tag{16}$$

We can do similar calculation for other pair of sources and we will get

$$g^{(1)}(x_1, x_3) = \frac{\rho_{31}}{(\rho_{11}\rho_{33})^{\frac{1}{2}}} \tag{17}$$

$$g^{(1)}(x_2, x_3) = \frac{\rho_{32}}{(\rho_{22}\rho_{33})^{\frac{1}{2}}} \tag{18}$$

Now from Eqs. 9, 16, 17 and 18, we see that

$$\begin{aligned} P_{ID} =& |g^{(1)}(x_1, x_2)|, \quad P_{ID} = |g^{(1)}(x_1, x_3)| \quad \text{and} \\ & P_{ID} = |g^{(1)}(x_2, x_3)| \end{aligned} \tag{19}$$

So, we see that the degree of indistinguishability is equal to the degree of coherence even for the case of interference with three sources but in a pair wise manner for all possible combinations of two sources and we also note that the degree of coherence for all pairs of sources are equal to each other.

Now it is very straight forward to show that only the second-order normalised coherence of this system is non-zero. Any higher-order coherence of the following form will be zero for this system;

$$g^{(2)}(x_i, x_j; x_j, x_i) = \frac{G^{(2)}(x_i, x_j; x_j, x_i)}{\sqrt{G^{(1)}(x_i, x_i)G^{(1)}(x_j, x_j)}} = 0$$

where $G^{(2)}(x_i, x_j) = \mathrm{Tr}\{\hat{\rho}\hat{E}^{(-)}(x_i)\hat{E}^{(-)}(x_j)\hat{E}^{(+)}(x_j)\hat{E}^{(+)}(x_i)\}$ is the general fourth-order coherence function [12] or the three-point fourth-order coherence function

$$\begin{aligned} & g^{(3)}(x_i, x_j, x_k; x_k, x_j, x_i) \\ & \quad = \frac{G^{(3)}(x_i, x_j, x_k; x_k, x_j, x_i)}{\sqrt{G^{(1)}(x_i, x_i)G^{(1)}(x_j, x_j)G^{(1)}(x_k, x_k)}} = 0. \end{aligned}$$

So, we see that when we are interested in the detection of a single photon the modulus of degree of indistinguishability and the degree of second-order two-point coherence will be equal for all possible pairs of the three sources.

Now we look at the relation of fringe visibility with the degree of path indistinguishability. Up to an overall scaling and write the total positive component of electric field at the point of detection as,

$$\hat{E}^{(+)} = \hat{a}_1 e^{i\phi_1} + \hat{a}_2 e^{i\phi_2} + \hat{a}_3 e^{i\phi_3} \tag{20}$$

Here, the phases ϕ_1, ϕ_2 and ϕ_3 are acquired during the propagation of the field from source to the point of detection. The overall scaling factor has been chosen to properly normalise correlation function. So, the probability of the photon being detected at this point is

$$\begin{aligned}\mathrm{Tr}(\hat{E}^{(-)}\hat{E}^{(+)}\hat{\rho}) &= \mathrm{Tr}\Big[\Big(\hat{a}_1^{\dagger}e^{-i\phi_1} + \hat{a}_2^{\dagger}e^{-i\phi_2} + \hat{a}_3^{\dagger}e^{-i\phi_3}\Big) \\ &\quad \big(\hat{a}_1 e^{i\phi_1} + \hat{a}_2 e^{i\phi_2} + \hat{a}_3 e^{i\phi_3}\big)\hat{\rho}\Big] \\ &= (\rho_{11} + \rho_{22} + \rho_{33} + 2(|\rho_{21}|\cos(\phi_{21}) \\ &\quad + |\rho_{31}|\cos(\phi_{31}) + |\rho_{32}|\cos(\phi_{32})))\end{aligned} \tag{21}$$

where $\phi_{ij} = \phi_i - \phi_j$. The visibility of interference fringe is defined as [11]

$$\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{22}$$

In Eq. 21, the angle difference ϕ_{ij} can be varied to get $I_{\max}$ and $I_{\min}$. Then from Eqs. 16, 17, 18, 19 and 22, we get

$$\begin{aligned}\mathcal{V} &= \frac{2(|\rho_{21}| + |\rho_{31}| + |\rho_{32}|)}{\rho_{11} + \rho_{22} + \rho_{33}} \\ &= 2\Big(|g^{(1)}(x_1, x_2)|\,(\rho_{11}\rho_{22})^{\frac{1}{2}} + |g^{(1)}(x_1, x_3)|\,(\rho_{11}\rho_{33})^{\frac{1}{2}} \\ &\quad + |g^{(1)}(x_2, x_3)|\,(\rho_{22}\rho_{33})^{\frac{1}{2}}\Big) \\ &= 2P_{ID}\Big((\rho_{11}\rho_{22})^{\frac{1}{2}} + (\rho_{11}\rho_{33})^{\frac{1}{2}} + (\rho_{22}\rho_{33})^{\frac{1}{2}}\Big)\end{aligned} \tag{23}$$

Here, we see that the fringe visibility is related to all three possible combinations of the modulus of two-point coherence functions of slits which is the famous Born's rule of multi-source interference. Very simple calculation shows how according to Born's rule in quantum mechanics, the multi-slit interference experiment's fringe intensity is nothing but the sum of contribution's of all possible pairs of slits [10]. In that paper [10], they experimentally proved the validity of Born's rule for three slit experiments that rules out the possibility of any multi-path, or higher-order interference. We note that for a single-mode field, only the second-order coherence function is nonzero and so only the pairwise two-slit correlations are possible. Following the unit trace of the density matrix and the pairwise two-slit correlation condition for all the possible combination of two slits i and j, it implies that, $\rho_{ii} + \rho_{jj} = 1$ and $\rho_{kk} = 0$ where k designate the last remaining slit. Using the inequality $2\sqrt{I_1 I_2} \le I_1 + I_2$, we can write $2\sqrt{\rho_{11}\rho_{22}} \le \rho_{11} + \rho_{22}$ and similarly for all the combinations we get,

$$\sqrt{\rho_{11}\rho_{22}} \le \frac{1}{2}, \ \sqrt{\rho_{11}\rho_{33}} \le \frac{1}{2} \ \text{ and } \ \sqrt{\rho_{22}\rho_{33}} \le \frac{1}{2} \tag{24}$$

As only two of the ρ_{ii} for $i = 1, 2, 3$, the diagonal elements can be nonzero simultaneously; from Eqs. 23 and 24 it implies that,

$$\mathcal{V} \le P_{ID} \tag{25}$$

This generalises the famous result of Mandel for the case of triple slit interference.

3. Coherence and indistinguishability for interference with N sources

Now we generalise our result for N sources where we are interested in detection of one photon. For the generalisation of the problem, we use a new notation for simplification of the calculation. We denote the state $|1\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3 \otimes |0\rangle_4 .. |0\rangle_N$ simply as $||1\rangle$. So when the photon will be generated by the m th source the state will be denoted as $||m\rangle$ which in our old notation would be $|0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3 \otimes ...|1\rangle_m \otimes ...|0\rangle_N$ and so on. So the state of the photon for this case can be written as

$$|\psi\rangle = \sum_{i=1}^{N} \alpha_i ||i\rangle \tag{26}$$

where $|\alpha_i|^2$ is the probability of the state ψ being in the i th state. Now for this the density matrix can be written as

$$\hat{\rho}_{ID} = |\psi\rangle\langle\psi| = \sum_{i,j=1}^{N} \alpha_i \alpha_j^* ||i\rangle\langle j|| \tag{27}$$

and likewise we denote the diagonal form of the density as

$$\hat{\rho}_D = \sum_{i=1}^{N} \alpha_i \alpha_i^* ||i\rangle\langle i|| = \sum_{i=1}^{N} |\alpha_i|^2 ||i\rangle\langle i|| \tag{28}$$

Now any general density matrix can be written as

$$\hat{\rho} = \sum_{i,j=1}^{N} \rho_{ij} ||i\rangle\langle j|| \tag{29}$$

Now by decomposing this density matrix in terms of the ρ_D and ρ_{ID}, we can write

$$\begin{aligned}\hat{\rho} &= P_{ID}\hat{\rho}_{ID} + P_D\hat{\rho}_D \\ &= P_{ID}\sum_{i,j=1}^{N} \alpha_i\alpha_j^* ||i\rangle\langle j|| + P_D \sum_{i=1}^{N} |\alpha_i|^2 ||i\rangle\langle i||\end{aligned} \tag{30}$$

$$= \sum_{i=1}^{N} |\alpha_i|^2 ||i\rangle\langle i|| + P_{ID} \sum_{i \neq j=1;}^{N} \alpha_i \alpha_j^* ||i\rangle\langle j|| \quad (31)$$

Now by comparing Eqs. 31 and 29, we can write

$$\rho_{ii} = |\alpha_i|^2 \quad \text{and} \quad \rho_{ij} = P_{ID} \alpha_i \alpha_j^* \quad (32)$$

By hermiticity of density matrix, we can write

$$\rho_{ji} = \rho_{ij}^* = P_{ID} \alpha_i^* \alpha_j \quad (33)$$

So, from Eqs. 32 and 33, we can write,

$$\rho_{ij}\rho_{ji} = P_{ID}^2 \rho_{ii}\rho_{jj} \quad (34)$$

From Eqs. 32 and 34, we write

$$\begin{aligned} (\alpha_i \alpha_j^*)^2 &= \frac{\rho_{ij}^2}{P_{ID}^2} \\ &= \frac{\rho_{ij}^2 \times \rho_{ii}\rho_{jj}}{\rho_{ij}\rho_{ji}} \end{aligned} \quad (35)$$

Due to hermiticity of density matrix

$$\rho_{ij} = \rho_{ji} e^{2i\cdot\arg(\rho_{ij})} \quad (36)$$

Using Eqs. 35, 36 and 32, we can get,

$$P_{ID} = \frac{\rho_{ij}}{\sqrt{\rho_{ii}\rho_{jj}}} e^{-i\cdot\arg(\rho_{ij})} = \frac{|\rho_{ij}|}{\sqrt{\rho_{ii}\rho_{jj}}} = \frac{|\rho_{ji}|}{\sqrt{\rho_{ii}\rho_{jj}}} \quad (37)$$

P_{ID} can be expressed in ${}^N C_2$ equivalent ways considering all possible pairs of N sources. This result boils down to Eq. 19 for $N = 3$ case and Mandel's result [4] for $N = 2$ case. Equation 37 suggests that the degree of path indistinguishability is equal for all possible ${}^N C_2$ pairs of sources.

Now to calculate the degree of coherence, we denote the positive frequency part of the single-mode electric fields of these sources as

$$\hat{E}^{(+)}(r_j) = K\hat{a}_j \quad \text{where} \quad j = 1, 2, ...N \quad (38)$$

Now as we have N sources, we will have ${}^N C_2$ pairs of sources for which we can calculate the second-order coherence function. We calculate the general second-order coherence function for a pair of points (x_i, x_j) from Eqs. 11 and 29 as

$$\begin{aligned} G^{(1)}(x_i, x_j) &= |K|^2 \mathrm{Tr}(\hat{a}_i^\dagger \hat{a}_j \hat{\rho}) \\ &= |K|^2 \sum_{n=1}^{N} \left\langle n||\hat{a}_i^\dagger \hat{a}_j \hat{\rho}||n \right\rangle \\ &= |K|^2 \sum_{n=1}^{N} \left\langle n||\hat{a}_i^\dagger \hat{a}_j \left(\sum_{l,k=1}^{N} \rho_{lk} ||l\rangle\langle k|| \right) ||n \right\rangle \\ &= |K|^2 \sum_{k,l=1}^{N} \rho_{lk} \delta_{ki} \delta_{jl} = |K|^2 \rho_{ji} \end{aligned} \quad (39)$$

From Eqs. 39 and 11, we see that

$$g^{(1)}(x_i, x_j) = \frac{\rho_{ji}}{\sqrt{\rho_{ii}\rho_{jj}}}. \quad (40)$$

We also note any higher-order coherence function than this will be zero for this generalised case also. From Eqs. 37 and 40, one sees that these pairwise second-order coherence functions are equal to the modulus of degree of indistinguishability,

$$|g^{(1)}(x_i, x_j)| = P_{ID} \quad (41)$$

Therefore, for N single-mode fields when we consider the detection of a single photon only we see that the degree of coherence is exactly equal to the modulus of degree of indistinguishability when we consider all possible pairs of the sources. Now to relate degree of indistinguishability with the degree of coherence, we denote the total positive component at the point of detection without the scaling factor as

$$\hat{E}^{(+)} = \sum_{m=1}^{N} \hat{a}_m e^{i\phi_m} \quad (42)$$

where ϕ_n is the phase acquired by the field while propagating to the point of detection from the n th source. We denote the intensity as well as the detection probability at this point of detection as expectation of the operator $\hat{E}^{(-)}\hat{E}^{(+)}$ as

$$\mathrm{Tr}\left(\hat{E}^{(-)}\hat{E}^{(+)}\rho\right) = \sum_{i=1}^{N} \rho_{ii} + 2 \sum_{i,j=1; i>j}^{N} |\rho_{ij}| \cos(\phi_{ij}) \quad (43)$$

From Eqs. 22, 2 and 43, we write the visibility as

$$\mathcal{V} = 2 \frac{\sum_{i,j=1; i>j}^{N} |\rho_{ij}|}{\sum_{i=1}^{N} |\rho_{ii}|} = 2 \sum_{i,j=1; i>j}^{N} |g^{(1)}(x_i, x_j)| (\rho_{ii}\rho_{jj})^{\frac{1}{2}}$$
$$\text{where } i > j \quad (44)$$

$$\leq P_{ID} \sum_{i,j=1;i>j}^{N} (\rho_{ii} + \rho_{jj}) \tag{45}$$

$$\leq P_{ID} \tag{46}$$

Equation 44 is the general form for interference fringe visibility of N single-mode fields. Equations 45 and 46 are the generalised relations between the fringe visibility, degree of coherence and degree of indistinguishability. The multi-slit interference of single-mode field being dependent on all the possible combinations of 2-slits was mentioned by Sorkin [13]. Here also as discussed for the case of three slits, for a state of the form Eq. 26, only the second-order correlation and two of the diagonal elements of the density matrix can be nonzero. Using this property, we get relation between visibility, degree of coherence and the probability of path indistinguishability.

4. Conclusions

In this article, we present a generalised relation between degree of indistinguishability and the degree of coherence. We also see how these quantities are related to the visibility of interference fringe for N single-mode fields for detection of one photon only. We note all the multi-path interference can be thought as contributions from all possible two-slit correlations as expected from Born's rule also. These results present a good picture or relations between wave and particle nature of photon.

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