

Lessons Learned From Germany's 2001-2006 Labor Market Reforms

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To Marcel
and my parents

Contents

List of Figures	ix
List of Tables	xi
Abbreviations	xii
1 Introduction	1
2 The Laws for Modern Services on the Labor Market in Germany	3
2.1 Introduction	3
2.2 Hartz I	5
2.2.1 New benchmarks of a <i>suitable job</i>	5
2.2.2 Early notification	6
2.2.3 Reversal of the burden of proof	7
2.2.4 More differentiated sanction rules	7
2.2.5 New rules for continuing education	8
2.2.6 Support of older workers	9
2.2.7 ‘Personal-Service-Agenturen’	9
2.2.8 Competition between integration measure providers	10
2.2.9 Controlling and bonus for good performance	10
2.3 Hartz II	11
2.3.1 ‘Minijob’	11
2.3.2 ‘Midijob’	11
2.3.3 Starting a business by an ‘Ich-AG’	12
2.4 Hartz III	13
2.4.1 Self-administration of the Employment Offices	13
2.4.2 Simplification of labor market policy instruments	13
2.4.3 Changes in monetary benefits	14
2.5 Hartz IV	15

2.5.1	Flat unemployment assistance payments	15
2.5.2	'Job-Centers' as contact points	16
2.5.3	Suitable jobs	17
2.5.4	One-Euro jobs	17
2.5.5	Integration agreement	17
2.5.6	Means tests for long-term unemployment benefits	18
2.5.7	Sanctions	18
2.6	Further changes before or after the Hartz reforms	19
2.6.1	Job-AQTIV Laws	19
2.6.2	Changes in the benefit entitlement duration	20
2.7	Further developments and concluding remarks	21
3	Unemployment benefits, distribution, and efficiency	23
3.1	Introduction	23
3.2	Different strands of literature	27
3.2.1	Basics of modeling	28
3.2.2	Basics of estimation	33
3.2.3	Literature on (optimal) UI systems	36
3.3	The model	38
3.3.1	Production, employment, and labor income	39
3.3.2	Optimal behavior	40
3.3.3	Welfare	43
3.4	Equilibrium properties	44
3.4.1	Individual (un)employment probabilities	44
3.4.2	Aggregate unemployment	46
3.4.3	Functional forms and steady state	47
3.5	Structural estimation	48
3.5.1	Exit rates out of unemployment	48
3.5.2	Econometric model	50
3.5.3	Estimation results	54
3.6	Numerical solution I: the pre-reform steady state	56
3.6.1	Equilibrium values in the pre-reform steady state	56
3.6.2	Dynamics on the microeconomic level in steady state	57
3.7	Numerical solution II: the effects of the reforms	60
3.7.1	Decreasing the unemployment assistance benefits b_{UA}	60
3.7.2	Decreasing the entitlement period \bar{s}	63

3.7.3	Decreasing the unemployment assistance benefits b_{UA} and the entitlement period \bar{s} simultaneously	65
3.7.4	Decreasing the unemployment assistance benefits b_{UA} and the entitlement period \bar{s} simultaneously in a grown economy	67
3.8	Numerical solution III: insurance and incentive effects of unemployment assistance benefits b_{UA}	69
3.8.1	An analytical benchmark for the pure insurance effects	69
3.8.2	Quantitative benchmark results of the insurance effects	70
3.8.3	Quantitative benchmark results of the insurance and the incentive effects	73
3.9	Conclusion	75
3.10	Individual contributions to the sections of chapter 3	77
4	Semi-Markov processes in labor market theory	79
4.1	Introduction and underlying setup	79
4.2	Semi-Markov processes - the basics	82
4.2.1	Continuous-time Markov chains	83
4.2.2	Semi-Markov processes	85
4.2.3	Transition probabilities of Semi-Markov processes	87
4.3	Semi-Markov processes with two states	90
4.3.1	Computing transition probabilities for constant arrival rates	91
4.3.2	Computing transition probabilities for general arrival rates	92
4.4	Numerical solution of the transition probabilities	93
4.4.1	Rectangle approximation	94
4.4.2	Trapeze approximation	98
4.5	Numerical results	101
4.5.1	Constant arrival rates - convergence to the analytical solution	102
4.5.2	Duration-dependent arrival rates	110
4.6	Conclusion	113
5	Summary	115
A	Appendix to chapter 3	117
A.1	Wage bargaining	117
A.2	Steady state solution	119
A.3	Data	121
A.4	Initial equilibrium: predicting productivity and vacancy costs	123

A.5	Determining the productivity of specific years	124
A.6	Insurance effects	125
A.7	The insurance and incentive effects in steady state	126
A.7.1	Steady state solution for the insurance effects	126
A.7.2	Steady state solution for the insurance and incentive effects	127
A.8	The Matlab code for comparative statics	130
A.8.1	Preparation	130
A.8.2	Running the programs	130
A.8.3	The solution structure of the programs	136
A.9	Description of the functions using numerical integration	138
A.9.1	muebar.m	138
A.9.2	smcProb.m	139
A.9.3	smcPuu.m	139
A.9.4	unemployment.m	140
A.9.5	SocialWelf.m	142
B	Appendix to chapter 4	143
B.1	The limiting distribution of a Semi-Markov process	143
B.2	The transition probabilities for the trapeze method	144
B.3	The Matlab code for SMP transition probabilities	145
B.3.1	Preparation	145
B.3.2	Running the program	145
	Bibliography	147

List of Figures

2.1	Maximum entitlement duration for UI benefits	21
3.1	Non-parametric hazard functions	49
3.2	Microeconomic variables over the unemployment spell	58
3.3	Cost of search	59
3.4	(Un)employment effects of decreasing long-term benefits	60
3.5	Welfare effects of decreasing long-term benefits	61
3.6	(Un)employment effects of decreasing entitlement duration	63
3.7	Welfare effects of decreasing entitlement duration	64
3.8	(Un)employment effects of decreasing long-term benefits and decreasing entitlement duration	65
3.9	Welfare effects of decreasing long-term benefits and decreasing entitlement duration	66
3.10	(Un)employment effects of decreasing long-term benefits and decreasing entitlement duration while the economy has grown	67
3.11	Welfare effects of decreasing long-term benefits and decreasing entitlement duration while the economy has grown	68
3.12	The insurance effects of long-term benefits regarding (un)employment	71
3.13	The insurance effects of long-term benefits regarding welfare	71
3.14	The insurance effects of long-term benefits	72
3.15	The insurance and incentive effects of long-term benefits regarding (un)employment	73
3.16	The insurance and incentive effects of long-term benefits regarding welfare	74
3.17	The insurance and incentive effects of long-term benefits	75
4.1	Rate diagram for a CTMC with two states	84
4.2	Possible transition paths of a continuous-time SMP	89
4.3	Numerical integration via rectangle method	95
4.4	Numerical integration via trapeze method	98

4.5	Transition probabilities of a CTMC (rectangle method)	103
4.6	Transition probabilities of a CTMC (trapeze method)	104
4.7	Transition probabilities of a CTMC (rectangle method, details)	105
4.8	Transition probabilities of a CTMC (trapeze method, details)	106
4.9	Transition probabilities of a SMP (rectangle and trapeze method)	112

List of Tables

2.1	Developments on the labor market in the reform years	4
3.1	Structural estimation results	54
3.2	Parameters and selected equilibrium values	57
4.1	Transition probabilities from unemployment to unemployment	107
4.2	Transition probabilities from employment to unemployment	109
4.3	Limiting probabilities for the state of unemployment	110
4.4	Transition probabilities for duration-dependent transition rates	111
A.1	Descriptive statistics	122
A.2	Growth rates in Germany between 1999 and 2005	124
A.3	Files and folders in the folder <i>reform</i>	130
A.4	Possible program runs	131
A.5	Naming of the figures from the <i>VGR</i> runs	135
B.1	Files and folders in the folder <i>smp</i>	145

Abbreviations

BGBI.	Bundesgesetzblatt
CMTC	Continuous-Time Markov Chain
DIW	Deutsches Institut für Wirtschaftsforschung
DTMC	Discrete-Time Markov Chain
GDP	Gross Domestic Product
GSOEP	German Socio-Economic Panel
ML	Maximum Likelihood
OECD	Organisation for Economic Co-operation and Development
PSA	Personal-Service-Agentur
SGB	Sozialgesetzbuch
SMP	Semi-Markov Process
UA	Unemployment Assistance
UI	Unemployment Insurance

Chapter 1

Introduction

In recent years, the unemployment rate in Germany has fallen from 10.5% in 2004 to 9.0% in 2007. Having followed legislature developments, a connection between this decrease and the labor market reforms between 2001 and 2006 appears plausible. Among others, the *Laws for Modern Services on the Labor Market*, or Hartz Laws, introduced sweeping changes in labor market institutions, which follow the guideline *support and demand*. The demanding elements are related to the unemployed, who should be helped to avoid or exit unemployment. To this end, some old laws were tightened and several new rules came into force. Support refers to the assistance provided by the unemployment agency to place unemployed workers.

In society, the Hartz Laws caused considerable debate. While liberal groups agreed that, even after the reforms, still insufficient incentives existed for the unemployed to escape unemployment, opponents criticized that the demanding elements were overemphasized. From time to time, this debate arises again. The last big discussion emerged in January 2009 when the Federal Social Court declared the monetary support for children of long-term unemployed granted according to the Hartz Laws as unconstitutional. Furthermore, in addition to social aspects of the reforms, the intentions and effects with respect to microeconomic behavior and macroeconomic performance should be considered and analyzed. The aim of this work is therefore twofold. First, an overview of the most important reform measures and the intended effects is given. Second, two specific and very fundamental amendments, namely the merging of unemployment assistance and social benefits, as well as changes in the duration of unemployment insurance benefits, are analyzed in detail to evaluate their effects on individuals and the entire economy.

Many of the new laws are based on the suggestions of a reform commission, which was appointed by the German government in 2002. The recommendations were compiled in a 350-page report and handed over to the government, which, in turn, used the report

to develop bills for the labor market reforms. Due to the complexity and multitude of the resulting laws, the implementation was partitioned into four sub-packages, which became effective consecutively. In order to give an overview of the reform program, chapter 2 analyzes the *Four Laws for Modern Services on the Labor Market* with respect to the underlying aims as presented by the commission report, as well as other connected reform measures. Furthermore, since controlling with respect to the actual effects is important, we sketch several economic and econometric evaluation studies for different labor market instruments and measures.

There was one amendment in the fourth reform package which was and still is among the most controversially discussed: the merge of unemployment assistance and social benefits. This step actually resulted in lower unemployment assistance benefits for a majority of the benefit claimants. Therefore, the effects of a benefit cut for the long-term unemployed are studied in chapter 3 along with a decrease of entitlement duration for unemployment insurance benefits. To this end, we construct a search and matching model of the labor market and extend it in several ways by essential features of individual behavior on the labor market. For instance, the incentive effect of a two-tier benefit system is considered by allowing for effort adjustments in job search over the unemployment spell. Based on this labor market model, parameters are estimated structurally using data from the German Socio-Economic Panel database. The parameter estimates are then used to evaluate the effects of these specific reform measures and of alternative reform scenarios numerically. Thus, it is possible to examine both the impacts on the microeconomic level (i.e. changes in individual behavior) and the consequences for the economy as a whole (e.g. (un)employment, unemployment insurance contributions, or welfare).

In our labor market model presented in chapter 3, optimal behavior and equilibrium values cannot be solved analytically. Particularly due to the non-stationary exit rate out of unemployment, a solution like in a standard matching framework is not feasible. In order to establish a numerical solution of models like ours from chapter 3, Semi-Markov techniques are needed. The main problem is to compute transition probabilities between the different labor market states, and therefore chapter 4 sheds light on Semi-Markov processes in labor market theory. First, a general introduction to Semi-Markov processes is given and it is shown how transition probabilities and limiting distributions can be determined. Second, numerical results are computed using the Semi-Markov process of labor market states from our model in order to evaluate and assess the chosen numerical computation methods.

Finally, chapter 5 summarizes the main findings of this thesis.

Chapter 2

The Laws for Modern Services on the Labor Market in Germany

2.1 Introduction

In the year 2002, the course for very far-reaching reform measures of the unemployment insurance system in Germany was set. The first step in this direction was the appointment of the so-called *Hartz Commission* by the German government in February 2002. Commission members included representatives from politics, private industries, trade unions, and one professor (of politics), who represented the academic position. The proper name of this group was ‘Kommission für moderne Dienstleistungen am Arbeitsmarkt’, which can be translated as *commission for modern services on the labor market*.¹ The commission’s task was to develop a reform concept in order to make labor market institutions more effective. Many of their proposals were accepted by the government and realized in the *Four Laws for Modern Services on the Labor Markets*, labeled accordingly as the *Hartz Laws* in public. The enacted laws mainly changed the German Social Code, the ‘Sozialgesetzbuch’ (SGB).² Table 2.1 gives an overview of the reform packages.

The purpose of this chapter is to discuss the Four Laws for Modern Services on the Labor Market, referred to as *Hartz I-IV*, and a few other connected amendments. Empha-

¹In public discussion and the media, the commission was referred to as the *Hartz Commission*, named after its President, Peter Hartz, who had been the Human Resources executive from Volkswagen AG.

²The German Social Code consists of 12 code books, labeled as Sozialgesetzbuch I - XII, or SGB I - XII. SGB II addresses basic support for job seekers. It contains the rules for the long-term unemployed and was created by the Fourth Law for Modern Services on the Labor Market in 2003. In SGB III, the rules for the support to find and keep a job are recorded. Until 2003 and before the Fourth Law for Modern Services, SGB III contained the rules for all unemployed. From 2004 on, the regulations for the long-term unemployed were placed in SGB II.

Chronology	Action	Section
Jan 2002	Job-AQTIV Law	2.6.1
Feb 2002	Appointment of the Hartz Commission	2.1
Aug 2002	Proposal of reform measures by the Hartz Commission, basis for the following laws	2.2-5
Jan/May/Jul 2003	First Law for Modern Services on the Labor Market (Hartz I)	2.2
Jan 2003 / Jan 2006	Second Law for Modern Services on the Labor Market (Hartz II)	2.3
Jan 2004	Third Law for Modern Services on the Labor Market (Hartz III)	2.4
Jan 2005	Fourth Law for Modern Services on the Labor Market (Hartz IV)	2.5
Feb 2006	Benefit entitlement drop	2.6.2

Table 2.1: Chronological overview of the developments on the labor market in the reform years. The dates show when the laws became effective.

sis is placed on those measures which are relevant for economics either due to economic modeling possibilities or for normative reasons. Where applicable, respective econometric or economic studies for evaluation are pointed out.

Before we turn to the recent changes, we further go back in history and trace the evolution of the German unemployment insurance system. In Germany, unemployment benefits were first introduced and explicitly distinguished from poor relief in 1914. The benefits for the unemployed started one week after unemployment and were paid for six weeks, see Stolleis (2003). Later, in 1927, national unemployment insurance was established. Contributions were paid equally by employers and employees, payments lasted half a year, and Employment Offices were created, see Stolleis (2003).

After World War II, reinvention and reorganization of the unemployment insurance system was needed. This led to the foundation of the *Federal Employment Office* in Nuremberg and the subordinate local Employment Offices in 1952. The tasks of these agencies included job search and unemployment insurance services. In 1969, when unemployment had been recognized as a problem of skill mismatch, the agencies' tasks were extended to supporting training measures. Those measures were recorded in the Third Social Code Book of Germany, known as Sozialgesetzbuch III (SGB III), see Stolleis (2003). Over the years, specific aspects or rules were enhanced or refined. Dur-

ing the 1980s, the unemployment insurance (UI) entitlement duration for older workers was extended several times, while the wage replacement rate for the unemployed without children was cut. These changes are analyzed by Hunt (1995), who finds that the longer potential duration of UI benefits explains the prolonged unemployment spells in Germany to a certain degree.

In the following years, discussions about the German unemployment benefit system being too generous surfaced regularly. Basic economic literature identifies high benefits as a reason for the lacking incentive among the unemployed to find a job, see Pissarides (2000). Moreover, the inflexible labor market was criticized. For these reasons, the German government formed the Hartz Commission in 2002, which proposed broad labor market reforms in their commission report. These suggestions and the resulting amendments to the former laws are analyzed in this work.

The outline of this chapter follows the official classification: sections 2.2-2.5 present the changes due to the Hartz I-IV laws, respectively. Section 2.6 describes other related amendments of SGB III enacted shortly before or after the Hartz reforms. The changes are all extracted from the 'Bundesgesetzblatt' (BGBl.), which is the official media of law promulgation in Germany. This chapter focuses on the changes of SGB III and the creation of Sozialgesetzbuch II (SGB II) as these two code books were substantially affected by the modifications.

2.2 Hartz I

In this section, the main changes made by the First Law of Modern Services on the Labor Market, known as the Hartz I Law, are presented. All amendments are promulgated in the BGBl. I p. 4607 (2002); most of them changed the Third Social Code Book, SGB III (2002), and connected rules in other code books. The laws came into effect in the beginning of 2003. The following rules all refer to SGB III (2002).

2.2.1 New benchmarks of a *suitable job*

The new labor market policy should be in line with the *support and demand* principle according to the Hartz Commission (2002), p. 45. New benchmarks for *suitable jobs* are one demand to be met by the unemployed because by the First Law of Modern Services on the Labor Market, the rules with respect to acceptable job offers were changed. For example, if a job in Germany is offered, this job should not be rejected due to distance reasons alone, according to the Hartz Commission (2002), p. 93. Rejection of such

an offer is only possible for good reason. An acceptable reason for rejection would be family care or if the offer refers to a part-time job only, for example. However, simply a long distance between the home of the unemployed and the job offered is not accepted, so relocating is considered reasonable. Article 1 No. 15 BGBI. I p. 4607 (2002) implements these suggestions into the Social Code as § 121 SGB III (2003).

The evaluation of this amendment refers to its activation potential regarding the unemployed. The official evaluation of this and other measures are conducted by Mosley et al. (2006). In case studies and polls among Employment Office agents, they find that the application of these stricter rules is rated as increasingly important and inter-regional mobility is very often tested by the Employment Office. Nevertheless, Mosley et al. (2006) find little positive effects. The mobility requirements apply to the unemployed without family and hence only for a subgroup of the unemployed. Besides, mobility of the unemployed has already been assessed as good by agents independent of the sanction threat, and employers have demonstrated a limited interest in hiring workers from a distance. Thus, there was actually no need to tighten this rule from their point of view. Altogether, the effect of this law is regarded as negligible.

2.2.2 Early notification

This new rule also belongs to the group of demands for the unemployed as well as those employed workers threatened by unemployment. In order to speed up the job search, the Hartz Commission (2002), p. 82, suggests that workers should report to the Employment Offices as early as threatened by unemployment. This is realized by article 1 No. 6 BGBI. I p. 4607 (2002) in § 37b SGB III (2003). If early notification is missed, a monetary sanction as proposed by the Hartz Commission (2002), p. 84, is applied, established by article 1 No. 19 BGBI. I p. 4607 (2002) in the Social Code as § 140 SGB III (2003).

This activating measure is also officially evaluated within the report of Mosley et al. (2006). According to the authors, the goal of higher job-to-job placement (a new job before unemployment actually occurs) is not met. Their analysis, based on case studies and surveys among executives from several Employment Offices as well as data from the Federal Employment Office, identifies that only about 7% of early notifiers make a job-to-job transition, and it is not clear whether this results from the workers' efforts or from the earlier placement effort of the Employment Offices.

2.2.3 Reversal of the burden of proof

Under the new laws, the unemployed individual has to prove that he followed the rules of the German Social Code. For example, if the unemployed declines a job or misses a notification, to qualify for support he has to prove that the declined job was not a suitable one or that he missed a notification for good reason. This is suggested by the Hartz Commission (2002), p. 93, and finds its expression in article 1 No. 20 a) BGBI. I p. 4607 (2002) and in § 144 (1) SGB III (2003).

2.2.4 More differentiated sanction rules

The Hartz Commission (2002), p. 99, reports on the rarely imposed sanctions when unemployed do not comply with the rules of the German Social Code. They see one reason for this in the inflexible and long blocking period of 12 weeks, i.e. the retention of benefits for three months. Such a sanction is considered too harsh for relatively minor violations and Employment Office agents hesitated to impose this severe sanction. Consequently, sanctions were no longer credible to the unemployed. In order to restore the credibility of the law and the Employment Offices, the imposition of more flexible sanctions was enabled. It was thus made possible for the agent to adjust the sanction according to the delict. The promulgation of the corresponding law is in article 1 No. 20 b) and c) BGBI. I p. 4607 (2002). In the Social Code, this is included by § 144 (3) and (4) SGB III (2003). For example, if the unemployed refuses a job or an integration measure for the first time, benefits can be blocked for three weeks, see § 144 (4) No. 1 c) SGB III (2003).

Within the framework of the official evaluation of the Hartz Laws, these new rules are analyzed by Mosley et al. (2006). The findings from case studies and evaluations of aggregate data are heterogeneous. In particular, the sanctions due to insufficient search efforts by the unemployed are exercised very differently among Employment Offices. After an increase of total sanction impositions in the years after this reform, the level stabilizes at about the pre-reform level for sanctions due to violation against *suitable job rules*. Hence, the aim to impose more sanctions and establish the credibility has not been achieved so far. In a microeconomic analysis, Mosley et al. (2006) use a random growth difference-in-difference estimator to evaluate the effectiveness of the new sanction rules. They analyze data from the years 2001-2004 and find quite varied impacts on the integration into the general labor market and on the exit into lasting employment. For the states of the former East Germany, the effect of sanctions tends to be stronger than for the states of the former West Germany. Unambiguously positive effects are reported

only for women in the states of the former East Germany. For the other groups, results vary between significant and insignificant over the different periods under consideration. Altogether, the effect of the sanction changes can be assessed as positive, but not consistently positive. Standard models of benefit sanctions study reductions in unemployment benefits rather than the retention of benefits for some time, compare Abbring et al. (2005) or Lalive et al. (2005). Such sanctions will be addressed later.

2.2.5 New rules for continuing education

After having presented the demands towards unemployed workers, we turn to the supporting measures. One aspect of the support provided concerns continuing education. The accumulation of human capital and education has always been highly appreciated, especially in countries like Germany which are poor in natural resources. The Hartz Commission (2002) emphasizes the importance of continuing education on p. 158. Although it is primarily seen as the task of the employer to educate his workers on-the-job, additional external education should further be provided by the Employment Offices in order to avoid unemployment according to the Hartz Commission (2002). The respective rules were completely revised and simplified. The new paragraphs §§ 77-86 SGB III (2003) are promulgated by article 1 No. 14 BGBI. I p. 4607 (2002). One new concept, for example, is that of education vouchers. Vouchers are issued by the Employment Offices and the workers can exchange them for a certified training measure, see § 77 (3) SGB III (2003). The purpose of the vouchers is to increase the self-accountability of the unemployed and through certification, high quality standards should be ensured.

Schneider et al. (2006) evaluate the effects of this reorganization on behalf of the government. They point out that the Employment Offices criticize the harder selection criteria for the assignment of the education vouchers for continuing education on the one hand and that the unemployed are now burdened with the choice of an appropriate program on the other hand. Regarding the effects of continuing education on employment chances, two partial effects can be identified. First, during participation in the program the search effort of the unemployed is lower, leading to worsened employment prospects. Second, the actual impact of the program appears after program completion and is expected to improve employment prospects then. As a result, the duration of continuing education programs was decreased by the reform, hence the real program effect now sets in earlier. Furthermore, Schneider et al. (2006) show that employment probabilities increase for the treatment group (the program participants) compared to the non-treated group (the non-participants) after completion. Altogether, the new rules concerning con-

tinuing education are clearly more effective than the old ones.

The treatment effects of continuing education on the duration of unemployment are analyzed by Lalive et al. (2008) for data from the Swiss labor market. Comparing two different estimators, they are not able to find evidence that continuing education reduces the unemployment spell.

2.2.6 Support of older workers

With regard to demographic changes, the Hartz Commission (2002) considers it necessary to support older workers in particular, p. 117. In the reforms, this is taken into account through several rules. First, if an unemployed aged 55 or older gets a job offer with a lower wage than his wage before unemployment, the Employment Office pays a part of this difference for some time. This is embedded by article 1 No. 43 BGBl. I p. 4607 (2002) in the Social Code as § 421 j SGB III (2003). Second, there should also be an incentive for employers to hire older unemployed. Therefore, the part of the social security contributions normally paid by the employer is covered by the Employment Office in such cases according to § 421 k SGB III (2003). In the Bundesgesetzblatt, this change is also declared in article 1 No. 43 BGBl. I p. 4607 (2002). However, this subsidy was abolished in January 2008.

The official study of the support measures for older workers is done by Zwick et al. (2006). They don't find significant effects on the employment chances of the treated unemployed (with support) compared to the non-treated unemployed (no support), which is, at least to some extent, ascribed to the poor knowledge about and the rare use of these instruments.

2.2.7 'Personal-Service-Agenturen'

In their report, the Hartz Commission (2002), p. 148, proposes the establishment of so-called 'Personal-Service-Agenturen' (PSA). PSAs work on behalf of, though independently of the employment agencies. They employ unemployed and integrate them into the labor market. Thus, PSAs are comparable to temporary employment companies: employers offer temporary jobs there, which are available because employees are on leave, pregnant, or due to seasonal adjustments, for instance. In this way, employers can search for permanent workers also through the PSAs with the advantage of being able to 'test' a possible worker to a certain extent. The primary goal of PSAs is the integration of unemployed in the first labor market. In the Social Code, the rules with respect to PSAs are recorded in § 37c SGB III (2003) through article 1 No. 6 BGBl. I p. 4607 (2002).

Furthermore, according to § 37c (1) SGB III (2003), the PSAs have to offer continuing education to currently jobless workers.

Mosley et al. (2006) compare the outcomes of treated groups and non-treated groups using a Kaplan-Meier estimator. The findings are quite surprising: not only are PSAs a rather expensive labor market instrument, but they are even ineffective. Created in order to integrate unemployed into the labor market, they actually decrease the probability of leaving unemployment in the time period under consideration.

As a consequence, the rules for PSAs were relaxed again in 2006. Employment Offices are no longer forced to establish a PSA. Thus, it is now possible to use this instrument whenever it is considered helpful, which promises improved efficiency.

2.2.8 Competition between integration measure providers

In article 1 No. 43 BGBl. I p. 4607 (2002), a new § 421 i SGB III (2003) is promulgated. According to this new paragraph, a competition between potential providers of labor market integration activities is possible. Also, the compensation of a provider can be performance-based. This rule aims to increase the competition among those providers, make their services more efficient, and lead to new ideas concerning integration measures.

This labor market instrument is evaluated in the official framework by Mosley et al. (2006). In a microeconomic analysis of the average treatment effect (participation in the new integration measure program) they identify few differences between the treatment group and the control group, which did not attend to such a program. Although some positive effects of the program on the subsequent job duration can be identified, this rule is abolished by article 1 No. 64 BGBl. I p. 2917 (2008).

2.2.9 Controlling and bonus for good performance

In order to create an incentive for Employment Office agents to place unemployed into jobs, a performance-based bonus is proposed by the Hartz Commission (2002) on p. 179 ff. This suggestion is realized by article 1 No. 36 BGBl. I p. 4607 (2002) in § 400a SGB III (2003). Furthermore, a controlling of local agencies and the newly established PSAs should be made continuously, compare Hartz Commission (2002), p. 187 ff. In the Social Code, these rules are recorded in § 9 (2) SGB III (2003) and § 11 (3) SGB III (2003) by the promulgation of article 1 No. 4 and No. 4a BGBl. I p. 4607 (2002).

2.3 Hartz II

The Hartz II reform was actually the Second Law for Modern Services on the Labor Market. Like the First Law for Modern Services on the Labor Market, this package is promulgated in BGBl. I p. 4621 (2002), and it essentially became effective in 2003. The most important changes referred to jobs without social security contributions or with reduced social security contributions.

2.3.1 ‘Minijob’

In order to reduce illegal employment, the so-called ‘Minijob’ is suggested by the Hartz Commission (2002), p. 169. For these Minijobs, no social security contributions or taxes have to be paid by the worker. The employer also profits since he pays a flat percentage of social security contributions and taxes. Such marginal jobs already existed before the Hartz II reform, but with a lower wage of 325 Euros. The Hartz Commission (2002) proposes a Minijob wage of 500 Euros per month, but 400 Euros was the wage ultimately accepted by the government. The relief of unemployment insurance contributions with respect to a marginally employed worker is stated in § 27 (2) SGB III (2003). Article 1 No. 3e BGBl. I p. 4621 (2002) embeds the wage of 400 Euros in § 347 No. 5 c) SGB III (2003). The term *marginal employment* and the corresponding wage are defined in Social Code IV³, § 8 (1) SGB IV (2003), which is adjusted by article 2 No. 3 BGBl. I p. 4621 (2002) to the amount of 400 Euros.

In the officially commissioned analysis, Fertig et al. (2006) find in a panel data model with fixed effects that the number of marginal jobs increased substantially due to this reform. Opponents of Minijobs criticize that these would crowd out regular (part-time) employment. However, Fertig et al. (2006) do not find evidence for these objections as with the data available to them, there was no econometric analysis possible with respect to this question. Nevertheless, the intended aim to build a bridge for the unemployed into regular employment via a Minijob cannot be verified, either, although movements between Minijobs and regular employment can be observed.

2.3.2 ‘Midijob’

For an intermediate wage, social security contributions of the worker should increase gradually from a lower level according to the Hartz Commission (2002), p. 170. The logic behind this idea is that a worker earning just slightly more than 400 Euros should not

³SGB IV (2003) contains the common rules and specifications for all parts of the Social Code.

be burdened with the full contributions. Thus, through a ‘Midijob’ a (gradual) transition into a regular job should be possible and more attractive by gradually increasing social security contributions. This intermediate wage is set between 400.01 Euros and 800 Euros by article 2 No. 5 b) BGBI. I p. 4621 (2002) in § 20 SGB IV (2003). The amount of the wage that is subject to social security contributions from the worker’s perspective results from § 344 (4) SGB III (2003) and it is promulgated in article 1 No. 3d a) BGBI. I p. 4621 (2002). In contrast to the worker, the employer must cover the complete contributions without any reliefs in this intermediate wage range.

Fertig et al. (2006) analyze the effects of the Midijob rule in a panel data model with fixed effects. They find that there would have been significantly less jobs in this wage area without the new Midijob. Concerning net movements, there were more workers unemployed before their Midijob than employed. Hence, there is some evidence to support the intended aim, namely the transition into employment. In polls among firms, most employers stated that the marginal employment reforms would not change their employment behavior, compare Fertig et al. (2006). At the same time, however, regular employment decreased and marginal employment and temporary employment increased between 2002 and 2004, suggesting a substitution effect to some extent.

2.3.3 Starting a business by an ‘Ich-AG’

The Hartz Commission (2002), p. 165, also proposes the concept of the ‘Ich-AG’ in order to reduce illegal employment. Accordingly, unemployed workers may receive monetary support for three years when starting their own business. The subsidy for the own business is recorded in § 421 I SGB III (2003) by article 1 No. 5 BGBI. I p. 4621 (2002). It amounted to 600 Euros, 360 Euros, and 240 Euros per month in the first, second, and third year, respectively - and only if the revenues of the Ich-AG remained below 25,000 Euros per year. However, this rule was only temporary and ended in July 2006. Today, only the short-term unemployed can apply for support for their own business, see § 57 SGB III (2008).

Wießner et al. (2006) evaluate the employment effects of this subsidy. The likelihood of not being registered as unemployed six months after support begin is about 60 percent higher for the treated group (the group with the own-business subsidies) than for the control group. Hence, this measure was indeed effective, although the effectiveness of reducing unemployment is decreasing over time.

2.4 Hartz III

The third reform package, the Hartz III Law, became effective in January 2004. This law is announced in BGBl. I p. 2848 (2003) and mainly changed the SGB III (2003).

2.4.1 Self-administration of the Employment Offices

According to the suggestions of the Hartz Commission (2002), the Federal Employment Office and all subordinate offices should change from bureaucratic agencies to market-oriented service providers. In order to support the change, the renaming of the Employment Offices is proposed, see Hartz Commission (2002), p. 223. The names chosen are 'Bundesagentur' instead of 'Bundesanstalt' on the central level and 'Agentur für Arbeit' instead of 'Arbeitsamt' on the local level, compare article 1 No. 4 BGBl. I p. 2848 (2003), for example, or article 1 No. 11 b) BGBl. I p. 2848 (2003). This renaming affected the complete German Social Code.

2.4.2 Simplification of labor market policy instruments

Some reform measures were introduced in order to facilitate the use of labor market policy instruments and to reduce administrative work. For example, job creation programs were simplified by the consolidation of several programs into one measure through article 1 No. 153 BGBl. I p. 2848 (2003). This merge is proposed by the Hartz Commission (2002) on p. 52. Furthermore, subsidies of wages for job creation measures are no longer percentage values, but flat rates depending on the education of the unemployed, compare article 1 No. 144 BGBl. I p. 2848 (2003) and § 264 SGB III (2003).

Schiemann et al. (2006) analyze the survival function in unemployment for program participants and comparable non-participants between 2000 and 2004. In this context, having exited unemployment into employment means having a job for at least six months. They find that the duration of unemployment tends to be higher for the treated unemployed than for the non-treated control group. However, for selected groups and selected regions in some years, there are also positive employment effects due to the measure.

Another labor market instrument are benefits for reduced working hours and other transfer measures due to operational reorganization of the firm or to avoid the dismissal of employees or to make job-to-job transitions possible. Article 1 No. 120 BGBl. I p. 2848 (2003) creates the new §§ 216a and 216b SGB III (2004), which contain the rules. The treatment effects of these measures are evaluated by Schneider et al. (2006) based on micro data. Because such programs already existed before the reform, they do not only

analyze the effects of the program, but also compare the outcomes with the impacts of the rules before the reform. Schneider et al. (2006) can hardly find any significantly positive employment effects of the measures, emphasizing that a long-term analysis would be necessary for a final assessment.

Finally, the monetary integration measures supporting the unemployed with placement difficulties were simplified and restricted. For the disabled unemployed, support is unchanged and remains at 70% of the wage, compare § 219 SGB III (2004), while subsidies for all other unemployed are at maximum of 50% of the wage, compare § 218 SGB III (2004). The paragraphs 217-219 SGB III (2004) are changed by article 1 No. 121 BGBl. I p. 2848 (2003) and the Hartz Commission (2002) suggests the simplification of active labor market instruments on p. 51 and the following pages.

Zwick et al. (2006) analyze the effect of these subsidies on the treatment group compared to the pre-reform era and compared to the non-treated. They find significantly positive long-term effects among those who received subsidies: after three years, the fraction of regularly employed is about 20-40% higher in the treatment group than in the control group. For the new rules in particular, they do not find evidence of better short-term employment effects than for those in the treated group in the pre-reform period. There was not enough data available for the evaluation of possible long-term effects.

A broad survey of econometric methods for the evaluation of active labor market programs can be found in Heckman et al. (1999). These methods can be used for assessing the effectiveness of training, job subsidy, and job search programs. The methods addressed are the estimation of treatment effects by using the outcomes of a treatment group and a control group, by using social experiments, or by using econometric models based on the assumption that behavior is stable.

Saint-Paul (1998) provides a general equilibrium model in order to evaluate the macroeconomic effects of active labor market programs on wages, unemployment, and welfare. By assuming that those programs are effective on the microeconomic level and increase search effort of long-term unemployed, he analyzes the support of those programs by workers. He shows that workers favor active labor market programs if the probability of getting unemployed is high and if the elasticity of labor demand is high.

2.4.3 Changes in monetary benefits

The Hartz Commission (2002), p. 134, proposes a pooling of benefits in order to simplify the rules and to reduce administration efforts. Consequently, extra payments for professional training participants were abolished. They are now simply eligible for un-

employment benefits, compare article 1 No. 62 BGBI. I p. 2848 (2003) and the new §§ 116-119 of SGB III (2005). Furthermore, article 1 No. 86 BGBI. I p. 2848 (2003) eliminates the old rules in §§ 153-159 of SGB III (2004).

Other simplifications were the standardization of the qualifying period for unemployment insurance benefits for all workers and a reduction of the time frame for qualifying. Articles 1 No. 65 and No. 66 BGBI. I p. 2848 (2003) state just one qualifying period of at least 12 months in the last two years before unemployment as opposed to the old §§ 123 and 124 SGB III (2003).

Simplifications were also introduced regarding social security contributions of unemployment benefit receivers. Now, a flat fraction of 21% has to be paid, see article 1 No. 71 BGBI. I p. 2848 (2003), which changes § 133 SGB III (2003).

The monthly exemption for extra income, that can be earned in addition to benefits, is set to 165 Euros in article 1 No. 73 BGBI. I p. 2848 (2003), whereas before, 20% or at least 165 Euros could be earned additionally, see § 141 SGB III (2003). In the area of benefits and monetary sanctions, many more rules were changed by the Hartz III reform. However, as these mainly concern details, we focus now on the next reform package.

2.5 Hartz IV

The last reform package from the Laws for Modern Services on the Labor Market was the one which caused the most public discussion. The main issue raised by the public was the merge of unemployment benefits with social benefits for long-term unemployed, and consequently, the reduction of benefits for the average long-term unemployed individual.⁴ The term *Hartz IV* became a synonym for this change and the perceived injustice. Nevertheless, the Fourth Law for Modern Services included more than just this one measure. The rules for the long-term unemployed are removed from the Third Social Code Book and the Second Social Code Book is created by BGBI. I p. 2848 (2003). Promulgated in December 2003, the law came into force in 2005. The most important rules are presented in the following subsections.

2.5.1 Flat unemployment assistance payments

The amount of unemployment insurance benefits did not change in the course of the Hartz IV reform. Until today, these payments have remained at 60% of the former net wage

⁴Blos and Rudolph (2005) show that the benefits for long-term unemployed actually decreased for a majority of the recipients.

for an unemployed person without children (67% for an unemployed person with at least one child). However, the unemployment assistance payments did change. Before 2005, these benefits were based on the former net wage. A long-term unemployed obtained 53% of the former net wage (57% if he had at least one child), see § 195 SGB III (2003). Beginning in 2005, flat unemployment assistance benefits were introduced in a newly created code book, SGB II (2005). The basic amount, stated in § 20 (2) SGB II (2005), is equal to the amount of social benefits: 345 Euros in the states of the former West Germany and 331 Euros in the states of the former East Germany. Additionally, extra payments are granted for children, partners, housing, and heating costs. Health insurance contributions are also paid by the Federal Employment Office. For details and the exact text of the law, see chapter 3 of SGB II (2005), especially §§ 20, 21, and 23 SGB II (2005).

This change is proposed by the Hartz Commission (2002) on p. 125 in order to simplify the administration of long-term unemployed and social benefit recipients. The subsequent laws are promulgated in article 1 BGBI. I p. 2954 (2003). This amendment is the focus of chapter 3, where a comprehensive evaluation of the benefit decrease is made by means of a search and matching model of the labor market.

2.5.2 ‘Job-Centers’ as contact points

According to article 3 No. 3 BGBI. I p. 2954 (2003), newly created ‘Job-Centers’ should be the contact point for all job seekers,⁵ compare § 9 (1a) SGB III (2005). Furthermore, § 44b SGB II (2005) establishes the foundation of teams within these Job-Centers in order to fulfill their duties according to the Second Social Code Book, see article 1 BGBI. I p. 2954 (2003). As a reaction to these new rules, many local authorities sued against the high financial burden and for their right to self-administration, which they saw restricted by this merging of communal and federal tasks in Job-Centers. In 2007, the Federal Constitutional Court of Germany declared the responsibility of Job-Centers for both long-term unemployed and social benefit recipients as unconstitutional, compare Bundesverfassungsgericht (2007), reference number 2 BvR 2433/04. According to the judgment, the organization has to be restructured by the end of 2010.

⁵Job-Centers should serve as a contact point for job seekers as well as for social benefit recipients in order to merge administration.

2.5.3 Suitable jobs

The basic principle of *support and demand* stated by the Hartz Commission (2002), p. 93, with respect to suitable jobs was already realized by the First Law for Modern Services on the Labor Market in the Third Social Code Book, see subsection 2.2.1. The respective changes for the long-term unemployed followed through the Fourth Law for Modern Services on the Labor Market. Article 1 BGBI. I p. 2954 (2003) implements the rules as § 10 SGB II (2005). For example, unlike the short-term unemployed, the long-term unemployed must now accept, if necessary, significantly ‘worse’ jobs than they had before, see § 10 (2) No. 2 SGB II (2005).

2.5.4 One-Euro jobs

Another much disputed rule concerns the job opportunities stated by article 1 BGBI. I p. 2954 (2003) as § 16 (3) SGB II (2005). According to this rule, job opportunities should be created for unemployed who could not be placed into a regular job. The unemployed then should get an *appropriate allowance* in addition to their unemployment benefits. Since such an appropriate allowance often qualifies as one Euro per working hour, these jobs are also called *One-Euro Jobs*. However, such a low wage is considered inappropriate by a broad spectrum of the population. Another debatable point is that workers with such job opportunities no longer count as unemployed in the unemployment statistics, despite the fact that accepting such a job cannot actually be seen as successful integration into the labor market. The number of workers employed in these job opportunities has stabilized at about 300,000 in recent years, see Bundesagentur für Arbeit - Statistik (2008).

2.5.5 Integration agreement

The Hartz Commission (2002), p. 75, proposes the assignment of a case manager to each long-term unemployed person. One task of the case manager is to create a contract with the unemployed, like the *integration agreement* as established by the Job-AQTIV Laws⁶ in the Third Social Code Book. The contents of this contract include integration measures and benefits provided by the Employment Office, as well as the efforts to be made by the unemployed, and it is refreshed after six months. The aim is to create a binding arrangement in order to activate the unemployed. This contract is established by article 1 BGBI. I p. 2954 (2003) in § 15 SGB II (2005). If an unemployed refuses to

⁶The Job-AQTIV Laws were promulgated in 2001, see subsection 2.6.1.

make this agreement, sanctions will be imposed according to § 31 (1) No. 1 a) and b) SGB II (2005).

2.5.6 Means tests for long-term unemployment benefits

Means tests for long-term unemployed had already existed before the Hartz reforms, see §§ 193 and 194 SGB III (2003), but with the Hartz IV reform, the tests became more severe. For example, more of the accumulated wealth is now taken into account and a smaller amount is left unconsidered, see § 12 SGB II (2005). The complete rules are given by §§ 9 and 11-12 SGB III (2005) and are promulgated in article 1 BGBl. I p. 2954 (2003).

2.5.7 Sanctions

The new laws of SGB II (2005) also contain sanctions to reduce unemployment benefit payments. The Hartz Commission (2002) describes them on p. 93 and on p. 101 and, as a result, § 31 SGB II (2005) is implemented by article 1 BGBl. I p. 2954 (2003). For example, an unemployed individual loses 30% of benefits if he does not complete an integration measure, see § 31 (1) No. 2 SGB II (2005), or 10% if he misses reporting to the Employment Office, compare § 31 (2) SGB II (2005).

Abbring et al. (2005) analyze the effect of sanctions on the exit rates out of unemployed in the Netherlands. For this purpose, they extend a job search model with reservation wages and optimal behavior by a system of sanctions. In addition to the reemployment rate, they estimate a sanction rate depending on observable characteristics (selective imposition of sanctions) and unobservable characteristics (endogeneity of sanctions). They find that benefit sanctions increase search effort not only instantly due to benefit reductions, but also permanently due to higher (perceived) monitoring.

Lalive et al. (2005) present a study about the effectiveness of benefit sanctions in Switzerland. Their setup is also a job search model accounting for the risk of punishment when not exerting enough search effort. In the econometric analysis, Lalive et al. (2005) find positive ex-ante (threat of sanctions) and positive ex-post (sanction actually imposed) effects of the sanction system on the exit rate out of unemployment, which confirms the results of Abbring et al. (2005).

2.6 Further changes before or after the Hartz reforms

In this section, two other labor market reforms, that were not part of the Hartz reforms, are presented. Some of the new rules are in line with the Hartz Laws, while others are contradictory. The first laws considered are the Job-AQTIV Laws, which were enacted before the Hartz reforms. The second law was established afterwards.

2.6.1 Job-AQTIV Laws

In the year before the appointment of the Hartz Commission, the so-called *Job-AQTIV Laws* were promulgated. These laws mainly changed the Third Social Code Book in the areas *activation* and *placement* of the unemployed, as well as training and wage subsidies. Two groups of changes are described here.

The first group concerns placement services of the Employment Offices and the relationship to the unemployed. Under the new laws, all unemployed persons have to sign a contract with the Employment Offices, known as the *integration agreement*⁷, see article 1 No. 4 and No. 13 b) BGBl. I p. 3443 (2001). The contents of this contract are stated by §§ 6 and 35 SGB III (2002).

Another new concept was the cooperation with private job service providers, so more efficient services and placements could be possible. Article 1 No. 15 BGBl. I p. 3443 (2001) introduces § 37a SGB III (2002) in a corresponding manner. If the agency decides to outsource job services to private providers, the unemployed can only decline for an acceptable reason. An unemployed individual, on the other hand, has the right to get private services when still unemployed after six months. Mosley et al. (2006) analyze the effects of private placement services according to this rule using micro data. They find few differences between the treated (those with private placement services) and the non-treated group⁸ with regard to the exit rate out of unemployment. Moreover, the estimated Kaplan-Meier survival function in unemployment of the non-treated control group even lies slightly below the survival function of the treated group. Mosley et al. (2006) also find that the job duration of a placed worker is longer, on average, for the control group than for the treated group. Ultimately, the rule with respect to private services is abolished by article 1 No. 16 BGBl. I p. 2917 (2008).

The second group of changes refers to wage subsidies, of which several were extended or newly established. For example:

⁷For more details, see subsection 2.5.5.

⁸The non-treated group is a control group which gets placement services exclusively from the Employment Offices.

- The wages of unskilled workers are subsidized if the employment contract continues during continuing education, compare article 1 No. 72 BGGl. I p. 3443 (2001).
- New subsidized job creation measures are introduced, see article 1 No. 98 BGGl. I p. 3443 (2001).
- Wage subsidies for handicapped juveniles are enacted by article 1 No. 60 BGGl. I p. 3443 (2001).
- Continuing education rotation: during continuing education of a worker, the employer can hire an unemployed person and obtain wage subsidies from the Employment Office, see article 1 No. 67 BGGl. I p. 3443 (2001).
- Support of older workers: the Employment Office pays continuing education measures if the employment contract continues while an older worker is getting trained, compare article 1 No. 114. BGGl. I p. 3443 (2001).

2.6.2 Changes in the benefit entitlement duration

Along with the Fourth Law of Modern Services on the Labor Market, another reform of the Third Social Code Book is promulgated in article 3 No. 2 BGGl. I p. 3002 (2003). It concerned the entitlement duration of unemployment insurance payments.

Under the previous rule, unemployment insurance benefits were paid for a maximum of 12 months, see § 127 (2) SGB III (2003). From 45 years of age and older, and depending on the number of months unemployment insurance contributions had been paid, the payments could be longer, up to 32 months for a 57-year-old worker. In February 2006, the rule changed especially for the older unemployed, compare article 3 No. 1 BGGl. I p. 3002 (2003). The maximum entitlement period was 18 months for a worker aged 55 years or older. However, because this was judged as too harsh by the public and policy makers, the maximum entitlement length was changed once more in January 2008. Under the current rule, an unemployed person that is at least 58 years old is eligible for a maximum of 24 months of unemployment insurance payments, see § 127 (2) SGB III (2008) for details. Figure 2.1 illustrates the changes in maximum UI benefit entitlement depending on the age of the unemployed over recent years.

The duration of unemployment assistance payments did not change with the recent modifications. As long as he searches for a job and passes the means test, an unemployed person receives unemployment assistance benefits.

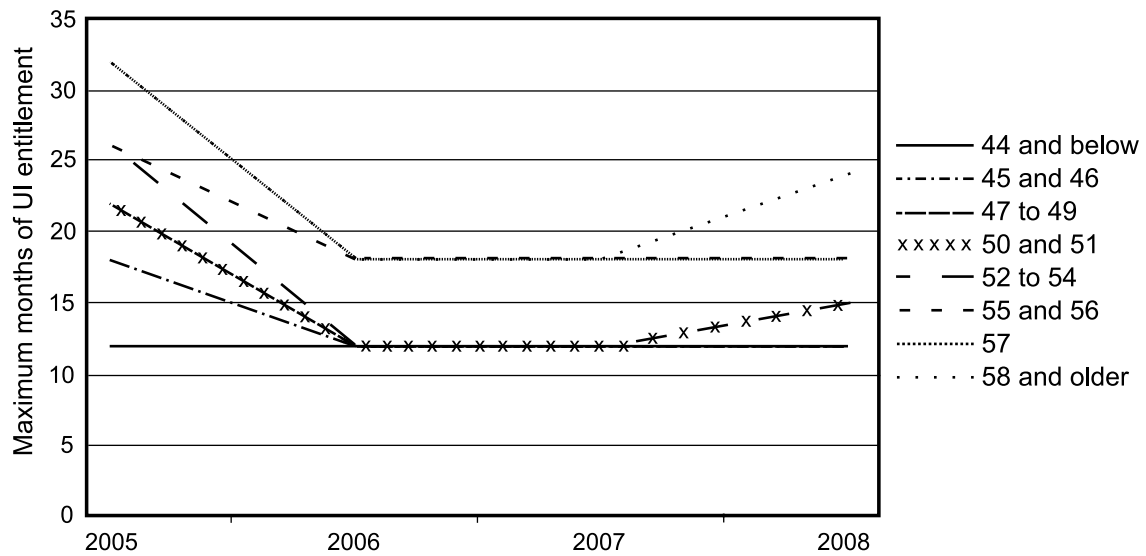


Figure 2.1: The developments in the maximum entitlement duration of UI benefits for different age groups. Sources: § 127 (2) SGB III (2002), article 3 No. 1 BGBI. I p. 3002 (2003), and article 1 No. 7 BGBI. I p. 3245 (2007)

2.7 Further developments and concluding remarks

The previous sections outline the labor market reforms in Germany between 2001 and 2006. Many of the amendments are based on the proposals of the Hartz Commission in order to decrease unemployment and restructure both the Federal Employment Office and the local Employment Offices. This very broad and extensive reform project was enacted within about two years, which is quite a small time frame for such far-reaching changes.

This impression strengthens as formal shortcomings were revealed over time by a steadily increasing number of legal actions, especially against the Hartz IV benefit rules. The successful lawsuit against the benefit rate for children of long-term unemployed, as well as the unconstitutional form of administration in Job-Centers are examples that attracted considerable public attention. Therefore, the initial resentments in huge parts of the German population could not be curtailed and still evoke ongoing discussion.

However, some rules established by the reform were already revised again in the meantime. For instance, the entitlement duration for the unemployment insurance benefit was extended again for older workers beginning in 2008 and unemployment agencies no longer have to create PSAs. Hence, legislature has also identified the necessity for corrections and improvement.

While there is still a wide range of opinions in society reflecting the mixed and diverging feelings about the amendments to the labor market laws, one may wonder whether

the reforms at least had the desired economic effects. The respective evaluations officially commissioned by the German government provide heterogeneous results. Many measures show a partial positive effect on employment and employment prospects, like the changes in continuing education, the new rules for Ich-AGs or the more flexible sanctions. However, some new rules had no or even negative impacts, such as job creation measures or PSAs.

In the next chapter, the implications of two specific and very important reform measures, namely of decreasing UA benefits and decreasing UI benefit entitlement duration, are analyzed within a search and matching framework. In this way, we can assess the microeconomic effects of these changes as well as the impacts for the economy as a whole.

Chapter 3

Unemployment benefits, distribution, and efficiency¹

3.1 Introduction

Continental European unemployment is notorious for its persistence. France, Italy, and Germany have had rising unemployment rates from the 1960s up to 2000 and even onward. There seems to be a consensus now that a combination of shocks and institutional arrangements lies at the origin of these high unemployment rates (Ljungqvist and Sargent, 1998, 2007a, b; Mortensen and Pissarides, 1999; Blanchard and Wolfers, 2000). Neither institutions nor shocks alone can explain the rise in unemployment: institutions have always been there, but unemployment has not - at least not at this level - and shocks have hit many countries, but not all countries have high unemployment rates. The step from this shock-institutions insight towards finding a solution to the European unemployment problem seems to be short: as shocks will always be there, institutions have to be addressed. A common suggestion is therefore to reduce long and generous unemployment benefits.

Is this advisable? Should the length and level of unemployment benefits be reduced in order to reduce unemployment? Finding an answer makes one face a classic efficiency-equity trade-off. While reducing unemployment per se is beneficial, the income of the unemployed and the insurance mechanism implicit in unemployment benefits should not be neglected.

The aim of this chapter is to examine qualitatively and quantitatively the employment

¹The model and the estimation described in this chapter are based on a paper written together with Andrey Launov and Klaus Wälde, compare Launov et al. (2009). Section 3.10 shows my individual contributions in detail.

and welfare effects of a policy reform which reduces the length and level of unemployment benefits. Germany is used as an example of a continental European country since the labor market reforms implemented in 2005/2006 comprised both the reduction of benefits and the cut of the entitlement duration², and because the German unemployment benefit system has a typical two-tier structure as many other OECD countries. Unemployment insurance (UI) payments before the reform were paid for a period of 6 to 32 months followed by unemployment assistance (UA) payments, the latter potentially lasting up to infinity. The experience of Germany with rising unemployment rates over decades is shared by many other countries and, similarly to Germany in 2005/2006, many countries did reduce length and level of unemployment benefits in order to address the issue of high and persistent unemployment, compare OECD (2004).

The Hartz IV reforms have introduced two main modifications. First, the UA payments, formerly proportional to net earnings before the job loss, were replaced by a uniform benefit level. The effect of this new rule on the income of long-term unemployed workers was ambiguous. There were unemployed whose benefit payments were lower before 2005 than after the reform, mainly unemployed workers from the low wage sector. Those were the ‘winners’ of the reform (47 percent of long-term unemployed). On the other hand, there were also long-term unemployed with relatively high wages before entering unemployment. These were affected negatively by the new law and their income has dropped (53 percent of long-term unemployed). Despite the shares of ‘winners’ and ‘losers’ are roughly equal, the gain of the winners has turned out to be lower than the loss of the losers, leading to a loss of the average long-term unemployed of about 7% due to Hartz IV (Blos and Rudolph, 2005; OECD, 2007). Second, for workers who entered unemployment from February 2006 onward, the maximum duration of entitlement to unemployment insurance payments was reduced to 12 months on average (formerly, 14 months was the average).

At first sight, the reforms seem to have worked. The reported unemployment rate dropped between 2004 and 2007 from 10.5% to 9.0%. On the other hand, also growth rates in Germany were comparatively high during this period. While the German economy shrank in 2003, it had recovered in the following years and probably also created new jobs. Given this background, we are left with at least three questions: did the reforms reduce unemployment and increase output? Did they increase the welfare of the unemployed and/or employed workers? Did they increase social welfare or expected utility? In short, the findings of this chapter suggest that post-reform unemployment ben-

²In the following, we refer to these reforms as the Hartz IV reforms since this term became a synonym in public especially for these two amendments.

efits did not reduce the insurance mechanism of UA payments too much and improved the incentive effects.³

These conclusions are reached by using a model which combines various strands of the literature and adds some new and essential features. A general equilibrium matching framework is extended by duration-dependent unemployment benefits, endogenous job search effort, risk-averse households, an exogenous ‘spell effect’, and Semi-Markov tools. Each of these extensions is crucial. The duration dependence of unemployment benefits in our model is important as this is a feature of basically all OECD unemployment benefit systems. Letting agents optimally choose their effort to find a job, the incentive effects of (reforms of) the unemployment benefit system on the search intensity can be analyzed. In a framework with risk-averse households, we can also evaluate the insurance effects of benefits for long-term unemployed. The spell effect allows to obtain - depending on how fast it sets in - rising, falling, or hump-shaped exit rates. Finally, tools from the Semi-Markov literature are required in order to deduce the unemployment rate from individual search. We can thereby compute macroeconomic efficiency effects resulting from microeconomic incentives.

Optimizing the values of the labor market agents provides us with their Bellman equations, which are differential equations that can be solved numerically. Optimal behavior implies an exit rate into employment which is a function of the time spent in unemployment. Thereby, a flexible endogenous distribution of unemployment duration is obtained, which we employ to determine the structural model parameters via Maximum Likelihood estimation.

The main theoretical contribution of our analysis is the explicit treatment of the Semi-Markov nature of optimal individual behavior due to the presence of spell-dependent unemployment benefits: optimal exit rates do not only depend on whether the individual is unemployed (the current state of the worker), but also on how long an individual has been unemployed. While this Semi-Markov aspect has been known for a while, it has not been fully exploited so far in the search literature. Using results from applied mathematics, analytic expressions for individual employment probabilities contingent on the current employment status and the duration of unemployment can be derived. They allow us to compute aggregate unemployment rates using a law of large numbers in this pure idiosyncratic risk economy. Given this link from optimal individual behavior to aggregate outcomes, the distribution and efficiency effects of changes in level and length of unemployment benefits are analyzed.

The main empirical contribution is the careful modeling of exit rates into employ-

³However, it will turn out that the reforms did not lead to a Pareto improvement.

ment. Individual incentives due to falling unemployment benefits imply more search effort and therefore higher exit rates over the unemployment spell. Empirical evidence shows, however, that exit rates tend to fall - at least after some initial increase over the first 3-4 months of unemployment. We therefore combine individual incentive effects with an exogenous time-decreasing spell effect and with unobserved heterogeneity. As is well known, the latter implies *inter alia* falling aggregate exit rates even though individual exit rates are rising or constant. Our structural estimation results then establish the importance of the duration effect and the unobserved heterogeneity effect. We find that the model can replicate empirical stylized facts of first rising and then falling exit rates.

The main policy contribution is the emphasis and structural estimation of the trade-off between insurance and incentive effects of labor market policies. The degree of risk aversion - crucial for understanding the insurance effects - is jointly estimated with exit rates, the spell effect, and other model parameters. A comparative static analysis using the estimated version of the theoretical model then allows to derive precise predictions about the employment and distribution effects of changes in the length and level of unemployment benefits.

The model of this chapter is related to various strands of the literature. From a theoretical perspective, it is built on the search and matching framework of Diamond (1982), Mortensen (1982), and Pissarides (1985), recently surveyed by Rogerson et al. (2005). Duration-dependent unemployment benefits and endogenous search intensity have originally been analyzed by Mortensen (1977) in a one-sided job search model. Equilibrium search and matching models include Cahuc and Lehman (2000) and Fredriksson and Holmlund (2001).⁴ These models, however, are less powerful than our model in explaining the anticipation effect of the reduction in benefits, as exit rates within each benefit regime are constant. There also exists a substantial literature that studies optimal insurance allowing for an arbitrary time path of unemployment benefit payments, for example Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), or Shimer and Werning (2007). The focus of this chapter is more of a positive nature trying to understand the welfare effects of existing systems which have a simpler benefit structure than the ones resulting from an optimization approach.⁵

⁴Albrecht and Vroman (2005), as well as Coles and Masters (2007) also have duration-dependent unemployment benefits, but they do not analyze the implications for individual effort. Albrecht and Vroman (2005) focus on the equilibrium wage dispersion and inefficient job rejection. Coles and Masters (2007) model aggregate uncertainty, which leads to implicit transfers between firms and a stabilizing effect on the unemployment rate over the business cycle.

⁵Acemoglu and Shimer (1999) do consider a general equilibrium model, but their setting is restricted to time-invariant benefits only.

From an empirical perspective, we estimate a parametric duration model, in which duration dependence of the hazard function due to non-stationary benefits is fully described by the equilibrium solution of our theoretical model.⁶ Econometric models with duration-dependent benefits have originally been estimated by van den Berg (1990) and Ferrall (1997).⁷ Van den Berg et al. (2004) and Abbring et al. (2005) extend the setting by introducing non-stationary benefits due to monitoring and sanctions. In contrast to our model, this literature deals with one-sided job search, which makes application of its estimates in a general equilibrium analysis rather difficult. In addition to that, a focus on incentive effects is only partial with the insurance effects remaining largely unaddressed. There also exists a larger empirical equilibrium search literature that deals with unemployment benefit heterogeneity (Bontemps et al., 1999), heterogeneity in workers' abilities (Postel-Vinay and Robin, 2002), and heterogeneity in workers' values of non-participation (Flinn, 2006). Unlike in our model, however, none of these contributions views heterogeneity as being a result of duration dependence.

Finally, Semi-Markov methods are taken from the applied mathematical literature, see e.g. Kulkarni (1995) or Corradi et al. (2004), in order to determine transition probabilities of labor market states and the steady state distribution.

The structure of this chapter is as follows. Section 3.2 presents different strands of literature in more detail. In section 3.3, the theoretical model, institutional setting, behavior of supply and demand sides, as well as the combination of both in economic welfare are pointed out. Section 3.4 describes the equilibrium properties of the model. Section 3.5 illustrates the structural estimation and the underlying data. The simulation results are discussed in three subsequent sections: section 3.6 shows the results from the pre-reform steady state, section 3.7 evaluates the effects of the German labor market reforms in 2005 and 2006, as well as alternative policy scenarios, and section 3.8 analyzes partial insurance and incentive effects of UA benefits. Finally, section 3.9 concludes.

3.2 Different strands of literature

This section gives an overview of literature related to the model we will develop in this chapter. First, we begin with an introduction to labor market modeling. Second, basic articles with respect to related econometric issues are presented. Finally, some more general analyses having either some interesting model features or trying to find answers to questions similar to ours are discussed.

⁶See Lancaster (1990) for a comprehensive introduction.

⁷See also Eckstein and van den Berg (2007) for a literature review on non-stationary empirical models.

3.2.1 Basics of modeling

In his seminal paper, Mortensen (1977) analyzes the job search decision of an unemployed worker who chooses search intensity optimally. In this model, an unemployed worker searches until he gets a job offering a wage higher than his reservation wage. This reservation wage is affected by the alternative income he gets while unemployed (unemployment benefits). The crucial point of this work is that benefits exhaust after a certain unemployment spell. So, if the benefit level increases, there are two effects: i) insured unemployed become choosier with respect to the wage they accept, i.e. the reservation wage increases, but ii) non-insured unemployed search harder as they want to become eligible for the (now) higher benefits again. The total effect is therefore ambiguous and it is not generally clear which of the partial effects is the dominating one. Consequently, one can not say whether unemployment will decrease or increase due to higher benefits. The work of Burdett (1979) is similar. He constructs a job search model with unemployed choosing their search effort optimally and with reservation wages, extended by on-the-job search. In an unemployment insurance system with exhausting benefits, Burdett (1979) shows that the reservation wage decreases and search effort increases over the unemployment spell until the benefit drop is reached where the benefits stay constant equal to zero. Hence, although the incentive mechanism of the unemployment insurance system with exhausting benefits is very similar to the microeconomic part of our model, the analyses by Mortensen (1977) and Burdett (1979) are partial as they ignore job separation into unemployment, wage determination, and macroeconomic effects. Also, the analysis is of a theoretical nature and no empirical test is provided.

In a job search model, Burdett and Mortensen (1998) explain the existence of wage dispersion in an economy where both employed and unemployed workers search for new jobs. Firms search for new workers by posting wages and workers wait for the job offer to arrive. In this framework, the reservation wage of the unemployed can be derived, i.e. the minimum wage which makes a job as attractive as unemployment. If the wage offered exceeds the reservation wage, an unemployed accepts the job. Furthermore, the reservation wage can be used to determine the steady state unemployment rate. Employed workers, on the other hand, search on-the-job for new offers and switch the job whenever a higher wage is available. In this way, an equilibrium wage dispersion can be explained in a framework with ex-ante homogeneous workers. In contrast to our model, there is no explicit search intensity decision of the unemployed in the model of Burdett and Mortensen (1998). In addition, by the one-sided wage posting of the firm, the influence of a worker on wage determination is completely neglected.

The Pissarides (1985) article is an important contribution to matching literature, in

which he explains labor market dynamics following exogenous output shocks. In the basic matching model, unemployed workers and vacancies do not find each other immediately, but face a probability of meeting each other, so there are search frictions in the labor market. When an unemployed worker and a firm with a vacancy meet, they bargain over the wage by splitting the cumulative surplus of the match. Pissarides (1985) shows that labor market tightness, wage, and reservation productivity jump to their new equilibrium and steady state values immediately after information or parameter changes, while unemployment adjusts gradually. However, although the model fits the general predictions and stylized facts of the time series data, it overestimates the quantitative effects. There is no empirical test in this article, but the model has been the basis for many following empirical and theoretical analyses and to some extent also for our model in this chapter.

Moffitt (1983) explains the observation that many eligible individuals do not participate in benefit programs by a stigma effect of benefit reception. The acceptance of benefits might lead to less self-respect and therefore to disutility resulting from participation in welfare programs. Moffitt (1983) models this assumption and performs a test with data from the 'Aid to Families with Dependent Children' program. He finds that this stigma is caused by the mere reception of benefits and not by the amount once the individual is in the program. However, it is more likely that a person participates, the higher the potential benefit is. Also Blumkin et al. (2008) analyze a social stigma effect. The stigma effect in their model is attached to benefit recipients by society, which suspects that a recipient might be undeserving and could actually care for himself. As society is not able to distinguish deserving from undeserving recipients directly, there is a statistical branding as undeserving. This public opinion, combined with a public exposure connected with the reception of benefits (the recipient has to go to the welfare agency and can be seen by neighbors, for example), generates stigma costs that reduce utility derived from consumption. Like in Moffitt (1983), the stigma effect in the model of Blumkin et al. (2008) directly enters the utility function. The spell effect in the setup of our model might, among other things, also reflect a social stigma. As the name 'spell effect' already suggests, this effect captures remaining duration-dependent influences other than the search effort of an unemployed. For example, a social stigma may lead to an increasing spell effect over the unemployment duration since unemployed become more and more ashamed of their situation and try harder to find a job. Such a behavior causes the exit rate to rise over the unemployment spell.

Furthermore, one may think of an employer stigma: employers may perceive a long unemployment duration as a signal and therefore prefer unemployed having smaller un-

employment spells. Such a stigma is analyzed by Vishvanath (1989) in a setup where job offers arrive randomly at unemployed. With a certain probability, the match is then actually realized. A job offer is connected with a wage drawn from a wage offer distribution. A firm knows the unemployment history of the applicant and offers on average a lower wage to an unemployed the more job matching failures he has had before. The unemployed, in turn, knows this relationship and determines his reservation wage so that it decreases over the unemployment spell. Vishvanath (1989) shows further that, given the usual specifications of wage offer distributions, the exit rate out of unemployment depends negatively on the length of the unemployment spell. In this way, falling hazard rates out of unemployment can be explained by true duration dependence due to statistical discrimination of unemployed with longer durations by firms. The drawback of this setup with respect to our model is that the individual does only decide about the reservation wage and not about search effort, since search costs are exogenously given in Vishvanath (1989). Furthermore, no bargaining aspects with respect to the wage are considered and the exogenous wage offer distribution only depends on how often an applicant failed to form a match in his unemployment history. However, the spell effect in our setup, which captures all duration-dependent effects influencing the job arrival rate other than the search effort of an unemployed, might also represent a kind of employer stigma. As it is not clear whether this spell effect decreases right from the start, is hump-shaped, or even stays constant over the duration, it is specified in a flexible way in order to fit the form implied by the data.

Albrecht and Vroman (2005) analyze the effects of time-varying unemployment benefits in a wage-posting search model. Individuals with different amounts of benefit will have different reservation wages and therefore accept different wage offers. A two-tier unemployment benefit system, for example, leads to a two-point wage distribution. There will be high wage offers, which will be accepted by all unemployed, and there will be low wage offers, only accepted by those with low benefits. Simulations show that the increase of the high benefits leads to a lower reservation wage both for the high and the low benefit group due to re-entitlement incentives. However, wage dispersion in this model leads to inefficient job rejection, so a declining benefit scheme is not optimal in this framework. Our model is a matching model with wage bargaining and not a search model with wage posting. However, we also consider a declining benefit system and additionally, we explicitly model search effort and its effects on the individual job arrival rates. Finally, we estimate our model parameters structurally and do not have to choose parameters ‘arbitrarily’.

A search model in a non-stationary environment is described in van den Berg (1990b).

In a structural model, it is shown that the reservation wage changes over the unemployment spell. These changes are induced by altering exogenous variables, which affect the value of being unemployed, like non-stationary benefits, job arrival rates, or the wage offer distribution. However, van den Berg (1990b) does neither consider the possibility of job loss in his model nor wage-setting mechanisms, an endogenous job arrival rate, or macroeconomic effects. All these aspects are included in our structural model.

The nature of wage determination in our setup is wage bargaining between the firm and the worker after they have met and later, in subsection 3.3.2, we will provide an explanation why. However, at this point we will present the bargaining approach and alternative methods of wage determination from the literature. For more detailed descriptions see Mortensen (2007), Mortensen and Nagypal (2007), and Hall and Milgrom (2008).

- Nash bargaining

In general, two possibilities arise when a firm and a worker meet: they break off negotiations or they find an agreement. The threat point of the worker is his outside option U_i , i.e. the value of staying unemployed. Upon agreement, the worker receives the current wage plus the continuation value of the job, $w_i + V_i$. The value of the firm is P_i when producing and the outside option is zero, i.e. zero profit from opening a vacancy. In total, the prospective surplus of a match is $P_i + V_i - U_i$. It will be divided according to a sharing rule or bargaining power β . Hence, the worker's value is his threat point value U_i plus the fraction β of the surplus,

$$w_i + V_i = U_i + \beta [P_i + V_i - U_i] \Leftrightarrow w_i = \beta P_i + [1 - \beta] [U_i - V_i].$$

If workers have no bargaining power, then $\beta = 0$ and the firm posts the wage exactly equal to the reservation wage, $w_i + V_i = U_i$.

- Strategic bargaining

In general, there are three possibilities when a firm and a worker meet: they break up the negotiations, delay bargaining, or find an agreement. In contrast to Nash bargaining, bargaining here can last some periods with alternating bids and counterbids. Delaying the negotiations in a period results in disagreement payoffs of unemployment benefits z for the worker and vacancy expenses of γ for the firm. Abortion of the negotiation results in outcome 0 for the firm and U_i for the worker. With probability δ , bargaining is broken up. Productivity changes with transition probability $\pi_{i,i'}$. Then, there are two indifference conditions,

1. $w_i + V_i = \delta U_i + [1 - \delta] \left[z + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} (w'_i + V_{i'}) \right]$
if the firm proposes w_i and the worker accepts,
2. $P_i - w'_i = [1 - \delta] \left[-\gamma + \frac{1}{1+r} \sum_{i'} \pi_{i,i'} (P_{i'} - w_i) \right]$
if the worker proposes w'_i and the firm accepts.

If the firm can bid first, it will offer w_i from the first indifference condition, which is the wage in the equilibrium and immediately accepted. Nonetheless, the wage is higher than in a Nash equilibrium because the power to delay the negotiation provides the worker with a higher surplus.

However, if there is no possibility of delaying negotiations, $\delta = 1$, then the threat points in both bargaining models are the same.

- Infrequent wage bargaining

Let wages depend on the productivity of the match. Nash bargaining may not occur in the instant of productivity changes, but infrequently with Poisson rate α . At the moment of a productivity shock with new productivity p , the Bellman equation of the firm is

$$rJ_p(w) = p - w - sJ_p(w) + \alpha [J_p - J_p(w)].$$

$J_p(w)$ is the value of the firm with wage w from the last negotiation, J_p is the value after the renegotiation of the wage, which occurs with Poisson rate α , with new productivity p . Hence, the value of the firm depends on the changing productivity. Infrequent bargaining leaves the Nash wage equation unchanged.

- Island matching

Suppose there are different secluded labor markets (“islands”) within the framework of island matching. One vacancy and one unemployed can only be in one labor market at any point in time. These sub-labor markets are competitive. Either there are more workers than jobs, which leads to a wage equal to the reservation wage and unemployment in that market. Or there are more jobs than workers, which implies that the wage is equal to productivity and open jobs exist in that specific market.

Models like the ones just considered above form the basis of the Hartz IV evaluation in this chapter. A broad review of search and matching models in general can be found in Rogerson et al. (2005).

3.2.2 Basics of estimation

In estimation literature, there are two major strands: the non-structural and the structural econometric analyses. The non-structural approaches apply empirical matching techniques, the evaluation of average treatment effects, and difference-in-difference estimations, for example. The advantage of such a proceeding is that the complete effects of a reform measure can be evaluated very specifically. Still, there are also several disadvantages of these approaches. First, when choosing the individuals of the control group, it has to be clear that the individual from the treatment group and the one from the control group are statistical twins, which means that there must be no other important difference between them except for treatment and non-treatment. Second, with such a non-structural approach only an already realized measure can be analyzed and predictions with respect to policy changes are not possible. Third, on the basis of these econometric estimations only conclusions on microeconomic level are possible. Hence, we prefer a structural approach for our analyses, as also alternative policies should be simulated and macroeconomic effects are considered.

In the following, some basic econometric work is presented and related to our model estimated in this chapter.

An empirical analysis on the basis of a reduced-form search model is provided by Lancaster (1979). In this work, several estimation methods are presented by establishing possible specifications of the hazard rate and likelihood functions using the duration densities and distributions. To some extent, this analysis is the starting point for hazard rate and unemployment duration studies. Also the design of the likelihood functions in the estimation of our model is based on the ones from Lancaster (1979). A broader and more detailed introduction to the analysis of transition and duration data can be found in Lancaster (1990).

With respect to the analysis of hazard rates of leaving unemployment, there is ongoing discussion whether they are truly duration-dependent or whether the observed variation stems from unobserved heterogeneity among the unemployed. True duration dependence would arise from incentive effects of non-stationary benefits, for example, and be an issue of theoretical economic modeling, therefore. On the other hand, it is plausible that some unemployed find a job more easily than others for a reason which is not observable by econometricians. Those unemployed would have a higher exit rate out of unemployment than other unemployed. The exit rate for the individuals might then in reality be constant over the individual unemployment duration, but different among unemployed. Not considering this heterogeneity then leads to a misinterpretation as duration dependence. Due to the unobservability, such a heterogeneity is more a matter of econometrics rather than

economic modeling.

In our model, however, we consider both sources of duration dependence. True state dependence is allowed for by duration-dependent modeling of job search effort of an unemployed individual in a world of non-stationary benefit payments like in Mortensen (1977) and by the spell effect described above. Unobserved heterogeneity is included in the likelihood contribution of the unemployed. Those unemployed who left unemployment before the benefit cut may have had an incentive to do so and therefore a higher exit rate level. Such an incentive can be found in the means test of benefits for long-term unemployed. Before becoming eligible to those benefits, comprehensive information about the income situation of an unemployed is collected. We assume that unemployed themselves know right from the start whether they would pass the means test or not, while we do not. It is then plausible that those who know that they would fail, have a higher exit rate out of unemployment. Altogether, in this way we account for true duration dependence and for unobserved heterogeneity. In the following, some econometric studies are described, addressing either true duration dependence, unobserved heterogeneity, or both.

Heckman and Borjas (1980) deal with the question whether the number of previous unemployment spells and their duration positively affect future unemployment probabilities. Based on stochastic process theory, they consider the following different types of state dependence: i) the transition rate depends on the number of unemployment spells experienced before, what Heckman and Borjas (1980) call ‘occurrence dependence’, ii) ‘duration dependence’, where the transition rate to a job depends on the current unemployment spell duration, and iii) ‘lagged duration dependence’, where transition rates depend on previous unemployment spell durations, e.g. if productivity loss due to missing work experience is expected by employers. In this framework, they test the two possible explanations *true state dependence* versus *uncontrolled heterogeneity*. To this end, an introduction to Markov processes is given and extended by the different types of state dependence first. With the derived distribution functions, three different tests of true state dependence can be performed: a conditional exit times test for occurrence dependence, a regression test for all kinds of state dependence, and Maximum Likelihood (ML) tests. For their dataset, Heckman and Borjas (1980) do not find evidence for any of the three state dependences, once considering sample selection bias and heterogeneity bias. In our econometric analysis, duration dependence is taken into account and specified in a way that it can also be constant, i.e. no duration dependence at all. Hence, the specification of the duration dependence intrinsically contains a test of duration-dependent exit rates. The results based on data from the German Socio-Economic Panel database (GSOEP)

provide evidence for the existence of true duration dependence in the way that the exit rate increases over the first months of unemployment and decreases afterwards.

True duration dependence and unobserved heterogeneity are considered by van den Berg and van Ours (1994) in a non-parametric and non-structural way by specifying the exit rate out of unemployment by a mixed proportional hazard function. This function contains a component for each of the features *duration dependence* and *unobserved heterogeneity*. The estimations are run with quarterly data from France, Great Britain, and the Netherlands and the results are inhomogeneous for the different countries and different sexes with respect to duration dependence and with respect to the existence of unobserved heterogeneity. However, the findings show that both effects should not be neglected. In our model, we also account for true duration dependence and unobserved heterogeneity. The advantage of our approach is that the estimation is based on a structural model, which allows to evaluate different policies on microeconomic as well as on macroeconomic level.

The search model of Abbring et al. (2005) has already been mentioned in section 2.5. They study the effects of benefit sanctions and monitoring of job search behavior in the framework of a job search model. Both the job-finding rate and the benefit-sanction rate depend on the search intensity of the unemployed. By optimizing the values of the unemployed, they derive optimal search behavior, the reservation wage, and hazard rates of leaving unemployment. The estimation of the model with data from the Netherlands provides evidence for two effects of sanctions and monitoring: i) the ex-ante effect, meaning that the level of search intensity is higher even if no sanctions were imposed yet due to the mere existence of sanctions in the benefit system, and ii) the ex-post effect, which means that search intensity increases at the moment a sanction is imposed. Van den Berg et al. (2004) also consider the effect of benefit sanctions on the unemployment-to-job transition rates of welfare benefit recipients, who are not entitled to UI benefits. Those sanctions are imposed if the search and registration requirements of the welfare agencies are not met. The model they use is basically taken from an earlier version of the Abbring et al. (2005) paper and estimated with different data from the Netherlands. For this data, van den Berg et al. (2004) also verify a permanently positive effect of an imposed sanction on the exit rate from unemployment to employment. Unlike in our model, the wage offer distribution in Abbring et al. (2005) and van den Berg et al. (2004) is exogenous and there is no job separation, so a job is held forever. Hence, the analyses are restricted to a purely microeconomic level and general equilibrium or macroeconomic effects remain unaddressed.

Eckstein and Wolpin (1995) provide further evidence that hazard rates into employ-

ment decrease with increasing unemployment duration. They analyze the transition to the first job in a search model, where both unemployed and employed workers exert search effort to get matched and an endogenous wage distribution is possible in a model with Nash bargaining. Using quarterly data from U.S. youths between 1979 and 1986, Eckstein and Wolpin (1995) find evidence for a decreasing hazard rate into employment with increasing unemployment duration. Furthermore, they replicate the observation that the mean of the accepted wages decreases with the duration of unemployment in a structural model. As this paper is a purely econometric work, macroeconomic and welfare effects are missing.

3.2.3 Literature on (optimal) UI systems

In a discrete-time one-sided job search model, Shavell and Weiss (1979) analyze whether benefits should decrease or increase during unemployment. To this end, the value of the unemployed is maximized under a fixed governmental budget for benefit expenses. Shavell and Weiss (1979) consider pure insurance effects, where unemployed are not able to influence their job-finding probability by their search intensity, and combined insurance and incentive effects, where probabilities to find a job depend on the effort devoted to search. They study both effects in a world without saving and borrowing first and relax this assumption later.

The findings crucially depend on the assumptions above and can be summarized as follows: i) for exogenous probability/no saving or borrowing, constant benefits over the unemployment spell are optimal, ii) for endogenous probability/no saving or borrowing, a declining benefit scheme is optimal if search effort is unobservable, iii) for exogenous probability/saving and borrowing, an increasing benefit sequence is optimal, and finally, iv) for endogenous probability/saving and borrowing, benefits should increase first and decrease later if search effort is unobservable. The intuitive explanation of i) is that benefits can't influence search effort and consequently consumption smoothing is optimal, while in case ii) incentive effects of benefits have to be considered, which influence search effort and the reservation wage. Case iii) takes into account that initial wealth should be consumed by an unemployed first and then benefits should be provided, even if search effort is exogenous. Finally, case iv) is a mixture of all other cases, so initial wealth leads to increasing benefits first and the incentive effects to decreasing payments finally.

Hopenhayn and Nicolini (1997) extend the work of Shavell and Weiss (1979) by taxation of workers after unemployment. Interpreting unemployment insurance as a contract be-

tween a worker (agent) and the government (principal), Hopenhayn and Nicolini (1997) find that optimal unemployment benefits decrease over the unemployment spell and wage taxes in the subsequent job depend on the prior unemployment spell. They calibrate the model and in this way, more specific conclusions are possible: the hazard rate from unemployment to employment increases before benefits expire, the wage replacement ratio starts at a fairly high value of 99% and decreases with longer unemployment duration, and the tax rate on the wage of the job after unemployment increases with the previous unemployment spell (note that there is a negative tax rate if unemployment lasted less than six weeks).

Also in our model, insurance and incentive effects are studied, but additionally, general equilibrium effects are considered. This has the advantage that, with the benefit budget and the wage distribution being not exogenously given, the reactions of workers and firms, as well as macroeconomic variables can be analyzed, too. The model parameters are estimated structurally, which furthermore enables a quantitative assessment and is preferred over a calibration.

Coles and Masters (2007) compare UI systems with finite benefit payments to UI systems with infinite benefit payments in an economy with stochastically changing job destruction shocks. Unemployed workers do not choose their leisure optimally, but lower unemployment benefits reduce their reservation wages. Firms have all the bargaining power and equilibrium is characterized by the optimal vacancy decision of the firms. With reservation wages decreased, they will post lower equilibrium wages for unemployed. This leads to higher profits for the firms and consequently changes the job arrival rate for all unemployed. So there is no distribution of arrival rates at a point in time. Using this model, they find that finite benefit payments cause lower unemployment and make unemployment less volatile over the business cycle. In contrast to them, we have individual and duration-dependent job arrival rates and economic growth in the business cycle is reflected by productivity changes. The advantage of our model is that it not only accounts for business cycle movements, but also for individual incentive effects. Furthermore, we estimate our parameters structurally and do not rely on calibrated parameters.

Fredriksson and Holmlund (2001) examine the optimal UI benefit system in a search and matching model. There are insured unemployed and non-insured unemployed, both with wage-dependent benefits, and transitions from the group of insured to the group of non-insured occur at a given rate. As a result, benefit duration is not fixed and the optimal search effort, as well as the arrival rate of jobs are constant over time. Wage is determined by Nash bargaining. The optimal UI program according to their setup has decreasing benefits over the unemployment spell because they work as an incentive

for insured unemployed to search harder. For non-insured unemployed, the decreasing benefits cause a re-entitlement effect, so search effort is raised in order to get a job and qualify for the higher benefits again. In contrast to Fredriksson and Holmlund (2001), insured unemployed in our model know when their benefits expire because there are official rules. So this is a more realistic setup from our point of view, which leads to variable optimal search effort over the unemployment spell. Furthermore, we consider the spell effect for unemployed as an alternative source of true duration dependence. A further advantage of our model is the structural estimation of the parameter values, while Fredriksson and Holmlund (2001) use calibrated values.

There are not many job search models with savings due to difficulties to establish optimality. Lentz (2009) determines optimal unemployment benefits in a labor market model allowing for optimal savings, optimal job search effort, and a wage distribution. It can be shown that search effort is lower, the more wealth the unemployed possesses. Furthermore, workers reduce wealth while unemployed and a positive shift of the wage offer distribution results in higher search effort, especially at the lower wage levels. Lentz (2009) analyzes the hazard rate of the model using weekly drawn micro data from Denmark and finds that search effort increases over the unemployment spell as wealth decreases, revealing another kind of true state dependence of the hazard rate. The optimal replacement rate of unemployment benefits is determined to be between 43% and 82%, depending on interest rates and search cost parameters. Although considering savings is an interesting issue for the analysis of unemployment insurance systems as they provide a sort of self-insurance against unemployment, it should be noted that this model neglects some other important aspects. Firm behavior, for example, is left unconsidered and the wage distribution is exogenously given. In addition, duration dependence stemming from incentive effects of non-stationary benefits are ignored. Both features, endogenous wage determination as well as search incentives, are included in the model as described in the following section.

3.3 The model

Starting point is a Diamond-Mortensen-Pissarides-type matching model extended for duration-dependent unemployment benefits, endogenous search effort, risk-averse households, and an exogenous spell effect. To solve it, we use Semi-Markov tools. The separation rate for jobs is constant and there is no on-the-job search. We focus on steady states in our analysis. Households are ex-ante identical, but endogenously heterogeneous in their unemployment duration.

3.3.1 Production, employment, and labor income

The economy has a work force of exogenous constant size N . Employment is endogenous and given by L , so the number of unemployed amounts to $N - L$. Firms produce under perfect competition on the goods market and each worker-firm match produces output A , which is constant. The production process of the worker and the firm can be interrupted by exogenous causes, which occur according to a time-homogeneous Poisson process with a constant arrival rate λ .

Unemployed workers receive UI benefits b_{UI} or UA benefits b_{UA} . Benefits are modeled to reflect institutional arrangements in many European countries. One of the most important features is the dependence of UI benefits on the unemployment duration. Empirical work has repeatedly shown that the length of entitlement to unemployment insurance payments plays a crucial role in determining the unemployment rate, compare Moffitt and Nicholson (1982) or Blanchard and Wolfers (2000). An unemployed with a spell s shorter than \bar{s} receives UI benefits b_{UI} . Afterwards, he gets b_{UA} ,

$$b(s) = \begin{cases} b_{UI} & 0 \leq s \leq \bar{s} \\ b_{UA} & \bar{s} < s \end{cases} . \quad (3.1)$$

We assume a decreasing benefit scheme with $b_{UI} > b_{UA} \geq 0$. Benefits can be paid either at a fixed level or proportional to previous income.

An unemployed worker finds a job according to a time-inhomogeneous Poisson process with arrival rate $\mu(\cdot)$. This rate will also be called job-finding rate, hazard rate, or exit rate to employment. It is allowed to depend on effort $\phi(s(t))$ an individual exerts to find a job. Effort today (in t) depends on the length $s(t)$ this individual has been spending in unemployment since his last job. The spell increases linearly in time and starts in t_0 where the individual has lost the job, $s(t) = t - t_0$. An individual whose duration of unemployment spell $s(t)$ exceeds the length of entitlement to UI benefits \bar{s} , $s(t) \geq t_0 + \bar{s}$, will be called a long-term unemployed.

In addition to effort, the exit rate of an individual will also depend on aggregate labor market conditions and on something which, for simplicity, is called a spell effect. Labor market conditions are captured by labor market tightness θ that differs across steady states, $\theta \equiv V/U$. We assume that effort and tightness are multiplicative: no effort implies permanent unemployment and no vacancies imply that any effort is in vain. The spell effect captures all factors exogenous to the individual which affects his exit rate to employment. This can include stigma, ranking (Blanchard and Diamond, 1994), and gains or losses in individual search productivity. We denote this effect by $\eta(s)$. Assuming that a stigma becomes worse the longer s is, $\eta(s)$ is expected to fall in s . Summarizing, the

exit rate will be of the form $\mu(\phi(s(t))\theta, \eta(s))$.

There is a long discussion in the literature whether aggregate falling exit rates are due to a duration effect (as modeled here by $\eta(s)$) or due to unobserved heterogeneity, compare Kiefer and Neumann (1981), Flinn and Heckman (1982), or, non-parametrically, Heckman and Singer (1984) as well as van den Berg and van Ours (1996).⁸ We take unobserved heterogeneity into account in our empirical part and discuss its effects there.

The outcome of our duration-dependent exit rate will be an endogenous distribution of unemployment duration. Its density is given by (e.g. Ross (1996), ch. 2)

$$f(s) = \mu(\phi(s)\theta, \eta(s)) e^{-\int_0^s \mu(\phi(u)\theta, \eta(u)) du}. \quad (3.2)$$

This density will be crucial later for various purposes, including the estimation of model parameters. It is endogenous to the model as the exit rate $\mu(\phi(s(t))\theta, \eta(s))$ follows from the optimizing behavior of workers and firms.⁹

Unemployment benefit payments to short- and long-term unemployed are financed by a tax rate κ on the gross wage w^g such that the net wage is $w = (1 - \kappa)w^g$. The budget constraint of the government therefore reads

$$\left(b_{UI} \int_0^{\bar{s}} f(s) ds + b_{UA} \int_{\bar{s}}^{\infty} f(s) ds \right) (N - L) = \kappa \frac{w}{1 - \kappa} L, \quad (3.3)$$

where $\int_0^{\bar{s}} f(s) ds (N - L)$ is the number of short-term and $\int_{\bar{s}}^{\infty} f(s) ds (N - L)$ is the number of long-term unemployed. The government adjusts the wage tax κ such that this constraint holds at each point in time.

The wage is determined by bargaining, to which we return below.

3.3.2 Optimal behavior

- Households

Households are infinitely lived and do not save. The present value of having a job is given by $V(w)$ and depends on the current endogenous wage w only. Employed workers enjoy instantaneous utility $u(w, \psi)$, where ψ captures disutility from working.¹⁰ The value $V(w)$ is constant in a steady state as the wage is constant, but differs across steady states. Whenever a worker loses his job, he enters the unemployment benefit system and

⁸See also section 3.2 for details on true duration dependence vs. unobserved heterogeneity in the literature.

⁹Also note that due to drop of benefits at \bar{s} , $f(s)$ will have a more general hurdle structure.

¹⁰This parameter only serves to contrast search effort of unemployed workers and plays no major role.

obtains insurance payments b_{UI} for the full length of \bar{s} . Workers are immediately granted full benefit entitlements, so unemployment benefits are not experience-rated. See the bargaining setup for further discussion. Hence, the value of being unemployed when just having lost the job is given by $V(b_{UI}, 0)$, where 0 stands for a spell of length zero. This leads to a Bellman equation for the employed worker of

$$\rho V(w) = u(w, \psi) + \lambda [V(b_{UI}, 0) - V(w)]. \quad (3.4)$$

The Bellman equation for the unemployed worker reads

$$\begin{aligned} \rho V(b(s), s) = \max_{\phi(s)} & \left\{ u(b(s), \phi(s)) + \frac{dV(b(s), s)}{ds} \right. \\ & \left. + \mu(\phi(s)\theta, \eta(s)) [V(w) - V(b(s), s)] \right\}. \end{aligned} \quad (3.5)$$

The instantaneous utility flow of being unemployed, $\rho V(b(s), s)$, is given by three components. The first component shows the instantaneous utility resulting from consumption of $b(s)$ and effort $\phi(s)$. The second component is a deterministic change of $V(b(s), s)$ as the value of being unemployed changes over time. The third component is a stochastic change that occurs at job-finding rate $\mu(\phi(s)\theta, \eta(s))$. When a job is found, an unemployed gains the difference between the value of being employed $V(w)$ and $V(b(s), s)$.

An optimal choice of effort $\phi(s)$ for (3.5) requires

$$u_{\phi(s)}(b(s), \phi(s)) + \mu_{\phi(s)}(\phi(s)\theta, \eta(s)) [V(w) - V(b(s), s)] = 0, \quad (3.6)$$

where subscripts denote partial derivatives. Equation (3.6) states that the expected utility loss resulting from increasing search effort must be equal to the expected utility gain due to higher effort.

Finally, the value of unemployment an instant before becoming a long-term unemployed has to be identical to the value of being long-term unemployed at \bar{s} ,

$$V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s}). \quad (3.7)$$

- Firms

The value of a job J to a firm is given by its instantaneous profit $A - w/(1 - \kappa)$, which is the difference between revenue A and the gross wage $w/(1 - \kappa)$, reduced by the risk of being driven out of business:

$$\rho J = A - w/(1 - \kappa) - \lambda J, \quad (3.8)$$

where ρ stands for the interest rate, which is identical to the discount rate of households, and where we anticipate that the value of a vacancy is zero.

Given that individual arrival rates are a function of the individual unemployment spell, the expected exit rate out of unemployment is the mean over individual arrival rates, given the endogenous distribution of the unemployment spell $f(s)$ from equation (3.2):

$$\bar{\mu} = \int_0^{\infty} \mu(\phi(s)\theta, \eta(s)) f(s) ds. \quad (3.9)$$

As a consequence, the vacancy filling rate is $\theta^{-1}\bar{\mu}$. The value of a vacant job is $\rho J_0 = -\gamma + \theta^{-1}\bar{\mu}[J - J_0]$, saying that the flow value of a vacancy is given by vacancy costs γ plus the gains from filling this vacancy, which occurs by the vacancy filling rate. With free entry, the value of holding a vacancy is $J_0 = 0$, leading to

$$J = \gamma\theta/\bar{\mu}. \quad (3.10)$$

- Wages

Let wages be determined by Nash bargaining. We assume that the outcome of the bargaining process is such that workers receive a share β of the total surplus of a successful match,

$$V(w) - V(b_{UI}, 0) = \beta \left[J \left(\frac{w}{1 - \kappa} \right) - J_0 + V(w) - V(b_{UI}, 0) \right].$$

The total surplus is the gain of the firm plus the gain of the worker from the match, where the latter depends crucially on the outside option of the worker. The fact that we use $V(b_{UI}, 0)$ as the outside option of the worker means that all workers (even if only working for an instant or, in the limit, if only bargaining) are entitled to full unemployment benefits and hence, b_{UI} over the full length \bar{s} and b_{UA} for $s > \bar{s}$.¹¹ An alternative would consist in specifying $V(b(s), s)$ as the outside option: if the bargain fails, the unemployed worker remains unemployed and continues to receive the benefits he received before the unsuccessful bargaining. This would be theoretically interesting as an endogenous wage distribution would arise as in Albrecht and Vroman (2005), where the distinguishing determinant across workers is the previous unemployment spell. Using an identical outside option for all individuals, however, has the advantage that all workers are homogeneous. Once an unemployed finds a job, all history is deleted, all workers are the same and, independently of their employment history, earn the same wage.¹²

¹¹In the empirical part, the ‘full length’ \bar{s} will be provided by the data. In this sense, entitlement is taken into account.

¹²Our assumption that all workers, even if they have worked only for a second, are entitled to b_{UI} for the full period of length \bar{s} is identical to saying that benefit payments are not experience-rated. While the absence of experience rating is generally distorting the firm’s decision to lay off workers (see e.g. Mongrain and Roberts, 2005), this does not play a role in our setup as the separation rate is exogenous. It would be interesting to study the impact of endogenous separation decisions, but we leave this for future research.

Following the steps as in Pissarides (1985), we end up with a generalized wage equation that reads

$$(1 - \beta) u(w, \psi) + \beta \frac{w}{1 - \kappa} = (1 - \beta) u(b_{UI}, \phi(0)) + \beta [A + \theta\gamma], \quad (3.11)$$

see appendix A.1. The left-hand side corresponds to what in models with risk-neutrality and without taxation is simply the wage rate. If we had $\kappa = 0$ and $u(w, \psi) = w - \psi$, we would obtain just w on the left and additionally $(1 - \beta)\psi$ on the right. Consequently, the worker is not only compensated for the outside option in the case of unemployment, $u(b_{UI}, \phi(0))$, but also for the disutility resulting from work, ψ . The tax rate, that appears in the term $w/(1 - \kappa)$, results from the instantaneous profit of a firm (3.8), which needs to pay a gross wage of $w/(1 - \kappa)$. The right-hand side is a simple generalization of the standard wage equation of Pissarides (1985). Instead of benefits for the unemployed (which we would find on the right for risk-neutral households and no duration dependence of effort), we have instantaneous utility from being unemployed. The impact of the production side is unchanged when compared to the standard wage equation.

Instead of specifying the outside option differently, one could also allow for strategic bargaining. Many recent papers have used strategic bargaining given that either payoffs change over time and Nash bargaining would correspond to myopic behavior (Coles and Wright, 1998; Coles and Muthoo, 2003), that a careful analysis of on-the-job search makes strategic bargaining more appropriate (Cahuc et al., 2006), or that unemployment does not have such a strong effect on bargaining as generally thought (Hall and Milgrom, 2008).¹³ Bruegemann and Moscarini (2007) find, while analyzing a different question, that the quantitative differences between distinct wage-setting rules are small. Given that we want to focus here on the direct incentive effects of non-stationary unemployment benefits on search effort, we feel justified to ‘switch off’ the strategic channel and leave this for future work.

3.3.3 Welfare

When evaluating unemployment policies, we take all agents in our economy into account. There are employed workers with value $V(w)$, unemployed workers with value $V(b(s), s)$ depending on their spell s , and firms with value J . When we compare one policy to another, we look at total output (i.e. employment), distributional effects, and overall welfare. We obtain a social welfare function Ω by aggregating - in the spirit of

¹³Coles and Masters (2004) analyze wage setting by strategic bargaining in a matching setup with non-stationary unemployment benefits. They do not consider endogenous search intensity, however.

Hosios (1990) or Flinn (2006) - over all these welfare levels in a standard Bentham-type utilitarian way,

$$\Omega = L[V(w) + J] + (N - L) \left(\int_0^{\bar{s}} V(b_{UI}, s) f(s) ds + \int_{\bar{s}}^{\infty} V(b_{UA}, s) f(s) ds \right). \quad (3.12)$$

Social welfare is given by the number L of employed workers and occupied jobs times their welfare plus the number of unemployed workers $N - L$ times the average welfare of an unemployed. This average is obtained by integrating over all spells s , where $f(s)$ is the endogenous density (3.2), with exit rates $\mu(\phi(s)\theta, \eta(s))$ that follow from the steady state solution of the model, and the $V(b_i, s)$ are the values of being unemployed with a spell s and benefit payments b_i from equation (3.1).

3.4 Equilibrium properties

3.4.1 Individual (un)employment probabilities

In models with constant job-finding and separation rates, the unemployment rate can easily be derived by assuming that a law of large numbers holds. Aggregate employment dynamics can then be described by $\dot{L} = \mu[N - L] - \lambda L$, which allows to compute unemployment rates. With spell-dependent effort, individual arrival rates $\mu(\cdot)$ are heterogeneous and employment dynamics need to be derived using techniques from the literature on Semi-Markov or renewal processes, see Kulkarni (1995) or Corradi et al. (2004), for example.

The generalization of Semi-Markov processes compared to continuous-time Markov chains consists in allowing the transition rate from one state to another to depend on the time an individual has spent in the current state. We apply this here and let the transition rate from unemployment to employment depend on the time s the individual has been unemployed. Hence, switching from a constant job-finding rate μ to a duration-dependent rate $\mu(s)$ ¹⁴ implies switching from Markov to Semi-Markov processes. These processes are called *semi* as the spell dependence of the job-finding rate $\mu(s)$ is not Markovian. However, these processes are still called *Markov* as, once an individual has found a job, history no longer counts. This is also why these processes are called renewal processes: whenever a transition to a new state occurs, the system starts from the scratch, it is ‘renewed’ and history vanishes.

We start by looking at individual employment probabilities. Let $p_{ij}(\tau, s(t))$ describe

¹⁴For simplification, we use $\mu(s)$ instead of $\mu(\phi(s)\theta, \eta(s))$.

the probability with which an individual who is in state i (either e for employed or u for unemployed) today in t , will be in state $j \in \{e, u\}$ at some future point in time τ , given that his current spell is now $s(t)$. These expressions read, starting with $s(t) = 0$ and taking into account that the separation rate λ remains constant (see section 4.3),

$$p_{uu}(\tau, 0) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) p_{eu}(\tau - v) dv, \quad (3.13a)$$

$$p_{eu}(\tau) = \int_t^\tau e^{-\lambda[v-t]} \lambda p_{uu}(\tau - v, 0) dv. \quad (3.13b)$$

Expressions for complementary transitions are given by $p_{ue}(\tau) = 1 - p_{uu}(\tau)$ and $p_{ee}(\tau) = 1 - p_{eu}(\tau)$, respectively.

These equations have a straightforward intuitive meaning. Consider first the case of τ being not very far in the future with $\tau = t$. Then all integrals are zero and the probability of being unemployed at τ is, if unemployed at t , one from equation (3.13a) and, if employed at t , zero from equation (3.13b). For a $\tau > t$, the part $e^{-\int_t^\tau \mu(s(y))dy}$ in equation (3.13a) gives the probability of remaining in unemployment for the entire period from t to τ . An individual unemployed today can also be unemployed in the future if he remains unemployed from t to v (the probability of which is $e^{-\int_t^v \mu(s(y))dy}$), finds a job in v (which requires multiplication with the exit rate $\mu(s(v))$) and then moves from employment to unemployment again over the remaining interval $\tau - v$ (for which the probability is $p_{eu}(\tau - v)$). As this path is possible for any v between t and τ , the densities for these paths are integrated. The sum of the probability of remaining unemployed all of the time and of finding a job at some v , but being unemployed again at τ then gives the overall probability $p_{uu}(\tau, 0)$ of having no job in τ when having no job in t . Note that there can be an arbitrary number of transitions in and out of employment between v and τ . The interpretation of equation (3.13b) is similar. The probability of remaining employed from t to v is simpler, $e^{-\lambda[v-t]}$, because the separation rate λ is constant.

As we can see, equations (3.13a) and (3.13b) are interdependent: the equation for $p_{uu}(\tau)$ depends on $p_{eu}(\tau - v)$ and the equation for $p_{eu}(\tau)$ depends on $p_{uu}(\tau - v)$. Formally speaking, these equations are integral equations, sometimes called Volterra equations of the first kind (3.13b) and of the second kind (3.13a). Integral equations can sometimes be transformed into differential equations, which then simplifies their solution in practice. In our case, however, no transformation into differential equations is feasible.

After having determined the probability of still being unemployed in τ when becoming unemployed in t and hence having a spell of length $s(t) = 0$, we need an expression for $p_{uu}(\tau, s(t))$. This means, we need the transition probabilities for individuals with

an arbitrary spell $s(t)$ of unemployment. Given the results from equations (3.13a) and (3.13b), this probability is straightforwardly given by

$$p_{uu}(\tau, s(t)) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) p_{eu}(\tau - v) dv. \quad (3.14)$$

An unemployed with spell $s(t)$ in t has different exit rates $\mu(s(y))$, which are known from our analysis of optimal behavior at the individual level. Hence, only the integrals in equation (3.14) are different compared to equation (3.13a), while the probabilities $p_{eu}(\tau - v)$ can be taken from the solution of equations (3.13a) and (3.13b).

3.4.2 Aggregate unemployment

Using our findings in equations (3.13) and (3.14) on $p_{eu}(\tau)$ and $p_{uu}(\tau, s(t))$, we can now derive the expected number of unemployed for any distribution of spell $F(s)$,

$$E_t[N - L_\tau] = [N - L_t] \int_0^\infty p_{uu}(\tau, s(t)) dF(s(t)) + p_{eu}(\tau) L_t. \quad (3.15)$$

Starting at the end of this equation, given there are L_t employed workers in t , the expected number of unemployed workers at some future point τ out of the group of those currently employed in t is given by $p_{eu}(\tau) L_t$. Again, one should keep in mind that the probability $p_{eu}(\tau)$ allows for an arbitrary number of switches between employment and unemployment between t and τ , so it takes the permanent turnover into account.

For the unemployed, we compute the mean over all probabilities of being unemployed in the future, if unemployed today, by integrating over $p_{uu}(\tau, s(t))$ given the current distribution $F(s(t))$. Multiplying this by the number of unemployed today, $N - L_t$, yields the expected number of unemployed at τ out of the pool of unemployed in t . The sum of these two expected quantities is the expected number of unemployed at some future point τ .

The expected unemployment rate at τ then simply is expression (3.15) divided by N . When we focus on a steady state, we let τ approach infinity. In order to obtain a simple expression for the aggregate unemployment rate and to show the link to the textbook equation, we assume a pure idiosyncratic risk model where microeconomic uncertainty vanishes on the aggregate level. Hence, we assume a law of large numbers holds and the population share of unemployed workers equals the average individual probability of being unemployed. This ‘removes’ the expectation operators so that in a steady state, equation (3.15) becomes

$$N - L = [N - L] \int_0^\infty p_{uu}(s(t)) dF(s(t)) + p_{eu}L.$$

We have replaced $L_\tau = L_t$ by the steady state employment level L and the individual probabilities by the steady state expressions $p_{uu}(s(t))$ and p_{eu} . The probability p_{eu} is no longer a function of τ as this probability will not change in steady state, while there will always be a distribution of $p_{uu}(s(t))$, even in steady state.

Solving for the unemployment rates gives

$$\frac{U}{N} = \frac{p_{eu}}{p_{eu} + [1 - \int_0^\infty p_{uu}(s(t)) dF(s(t))]} = \frac{p_{eu}}{p_{eu} + \int_0^\infty p_{ue}(s(t)) dF(s(t))}. \quad (3.16)$$

If we assumed a constant job arrival rate here, we would get $p_{eu} = p_{uu} = \lambda/(\lambda + \mu)$ and $p_{ue} = \mu/(\lambda + \mu)$. Inserting this into our steady state results would yield the standard expression for the unemployment rate, $U/N = \lambda/(\lambda + \mu)$. In our generalized setup, the long-run unemployment rate is given by the ratio of individual probability p_{eu} to be unemployed when employed today divided by this same probability plus $1 - \int_0^\infty p_{uu}(s(t)) dF(s(t))$.

3.4.3 Functional forms and steady state

For estimation purposes and for the numerical solution, we need to specify functional forms for the instantaneous utility function and for the arrival rate. Let the instantaneous utility function of an unemployed worker, used e.g. in equation (3.5), be given by

$$u(b(s), \phi(s)) = \frac{b(s)^{1-\sigma}}{1-\sigma} - \phi(s). \quad (3.17)$$

Search effort is measured in utility terms. The utility function of an employed worker has the same structure, only that consumption is given by w and work effort is the constant ψ .

The arrival rate of jobs $\mu(\phi(s)\theta, \eta(s))$ is assumed to obey

$$\mu(\phi(s)\theta, \eta(s)) = \eta(s) [\phi(s)\theta]^\alpha. \quad (3.18)$$

If one interprets $\eta(s)$ as a productivity of search, one can look at the expression for $\mu(\cdot)$ like at a production function. Input factors are effort $\phi(s)$ and vacancies per unemployed worker θ . With $0 < \alpha < 1$, inputs have decreasing returns. Effort $\phi(s)$ follows from behavior of households and labor market tightness θ is the result of free entry and exit into the creation of vacancies. The spell effect $\eta(s)$ is an exogenous function of the unemployment spell s and its particular parametric form is explained in the next section.

In a steady state, all aggregate variables are constant and there is a stationary distribution of unemployment spells. The solution of the steady state can most easily be found in two steps. Choosing starting values for the wage w and labor market tightness θ , one

can solve for the search effort of an unemployed, the value of being unemployed, and the value of a job, $\phi(b(s), s)$, $V(b(s), s)$, and $V(w)$, respectively. Once these quantities are known, the remaining equations of the model can be used in the second step to solve for the wage rate and tightness, w and θ , and check the initial guess in this way. In doing so, all other endogenous variables are determined as well, like the exit rate $\mu(\phi(s)\theta, \eta(s))$ and the implied density $f(s)$, instantaneous utilities $u(\cdot)$, the tax rate κ , individual employment probabilities p_{ij} , the implied number of short- and long-term unemployed as well as the unemployment rate U/N , the number of vacancies, the value function J for the firm, and social welfare Ω .¹⁵

3.5 Structural estimation

3.5.1 Exit rates out of unemployment

Before we estimate the model using data from the GSOEP¹⁶, the functional form of the spell effect $\eta(s)$ from equation (3.18) has to be specified. In order to do so, we consider the distributional aspects of our data on observed unemployment duration. The specification of the spell effect $\eta(s)$ needs to be sufficiently flexible to be able to capture these aspects.

The left panel of figure 3.1 shows the non-parametric estimate of the hazard function from the entire sample of unemployment durations. The right panel of this figure shows the hazard function for the subsamples of individuals with entitlement length equal to 12 months as the solid line. The dashed line shows the hazard rate of those non-entitled to unemployment insurance b_{UI} .¹⁷ Both panels plot exit rates for the first 2.5 years of unemployment.

From these figures, we can see a clear downward duration dependence of the exit risk. On the one hand, this may be due to the true downward state dependence of an individual hazard rate, see e.g. van den Berg and van Ours (1994) or Eckstein and Wolpin (1995), providing the evidence on this. On the other hand, this may be due to unmeasured heterogeneity, compare Heckman and Singer (1984) or van den Berg and van Ours (1994). Indeed, as far as Germany is concerned there is at least one source of such un-

¹⁵Appendix A.2 provides an explicit presentation of all equations which are given implicitly in the model description above and describes the solution procedure.

¹⁶For more background on the GSOEP and for descriptive statistics, see appendix A.3.

¹⁷See Tanner and Wong (1983) for the definition of the estimator and consistency proof. We use Gaussian kernel. Optimal bandwidth is estimated by cross-validation discussed in Tanner and Wong (1984).

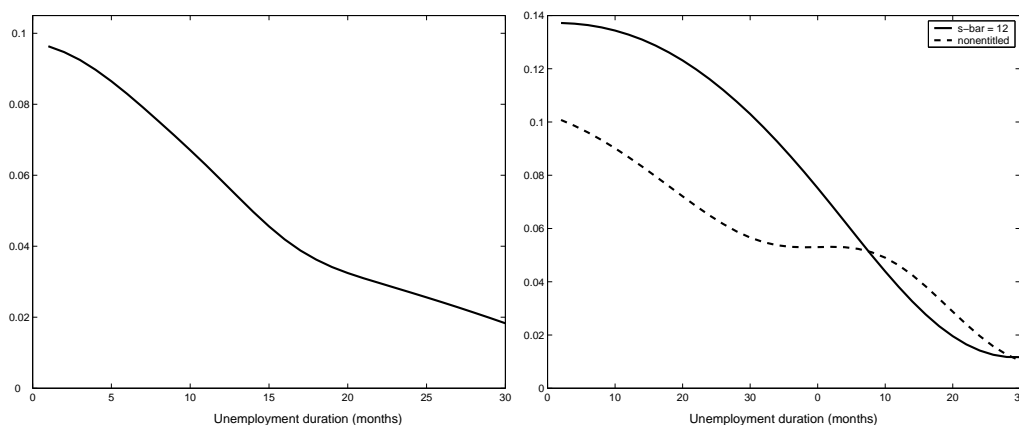


Figure 3.1: Non-parametric hazard functions (entire sample and $\bar{s} = 12$).

observed heterogeneity. Namely, individuals receiving UI benefits may or may not be eligible to UA benefits, once the entitlement period expires. Eligibility to UA benefits is determined by a *means test*, where an individual has to provide lengthy information about income sources of the household, number and age of dependents etc. If the means are sufficient, the person becomes ineligible to UA benefits, but might still claim social assistance, which eventually may or may not be provided. Unobservability in our context means that, if exit out of unemployment occurs before the expiration of entitlement, an econometrician cannot know about the outcome of the test. The individuals themselves, however, are very likely to know what the result of the test will be. Thus, in case they do not expect to pass the test, they would search harder and therefore exit faster into employment. This behavior, if uncontrolled for, results in a decreasing non-parametric estimate of the hazard rate. Clearly, the true individual exit rate in this particular case may as well be constant or increasing up to the expiration of entitlement and constant thereafter, as in Mortensen (1977) and van den Berg (1990b). Finally, both true individual state dependence and unobserved heterogeneity may manifest themselves simultaneously, compare van den Berg and van Ours (1996) and van den Berg and van Ours (1999) for evidence of these effects in U.S. and French data, respectively.

Thus, the individual exit rate derived from the theoretical model should be able to capture two characteristics, namely: i) steady increase before the expiration of entitlement, as in Mortensen (1977), and ii) steady decrease thereafter, as in Heckman and Borjas (1980). Our theoretical exit rates are broadly consistent with both. When we assume that there is no effect other than that of benefits and tightness, so $\eta(s)$ is constant in equation (3.18) due to $\delta_j = 0$ (no stigma), our model predicts exit rates that increase before \bar{s} . If δ_j is very high, exit rates fall over the spell. For intermediate values of δ_j , we get a non-monotonic behavior and exit rates increase in the first months and fall subse-

quently. We are therefore confident that our theoretical exit rates are sufficiently flexible for a successful estimation of the model.

Our aim is to provide a fully structural econometric model in order to estimate the deep parameters of the theoretical model of section 3.3 in presence of possible exogenous individual state dependence and unobserved heterogeneity.

3.5.2 Econometric model

- Specification

We estimate the model parameters applying the ML method. The likelihood function is constructed using the exit rates as implied by the model. For the estimation, we sample data of entrants into unemployment and employment from the GSOEP.¹⁸

The exit rate from unemployment is given by equation (3.18) from the theoretical model. The effort level $\phi(s)$ needs to be replaced by the optimal value as implied by the first-order condition (3.6). So $\phi(s)$ is a function of the spell s , benefits paid at s , the spell effect $\eta(s)$, total entitlement duration \bar{s} , the wage w , and labor market tightness θ . To simplify notation, we group the individual variables given by the dataset into a vector $\mathbf{z} \equiv \{b_{UI}, b_{UA}, \bar{s}, w, \theta\}$. There are additional variables given in the sample, which possibly affect effort and the spell effect. We group these additional variables in a vector \mathbf{x} that contains the rest of personal characteristics. In order to express the econometric model as general as possible, we take those variables into account for the spell effect and the separation rate, with corresponding parameters χ and ζ . Hence, the separation rate reads $\lambda(\mathbf{x})$ and the spell effect is denoted by $\eta(s; \mathbf{x}) = \eta_0(\mathbf{x}) g(s)$. Summarizing, conditional on the vector of observed characteristics, the exit rate from equation (3.18) can be written as

$$\mu_j(s) = \mu(\phi(s; \mathbf{z})\theta, \eta(s; \mathbf{x})) = \eta_0(\mathbf{x}) g(s) [\phi(s; \mathbf{z})\theta]^\alpha, \quad j = 1, 2. \quad (3.19)$$

Effort $\phi(s; \mathbf{z})$ implies an endogenous individual duration dependence due to the anticipation of the benefit reduction and $\eta(s; \mathbf{x})$ is the exogenous individual duration dependence, the spell effect. Finally, j indicates the benefit regime before ($j = 1$) and after ($j = 2$) expiration of unemployment insurance payments.

We have four types of labor market histories in the dataset. The first group are individuals who enter unemployment with the right to claim UI benefits and exit unemployment before the expiration of the entitlement period such that $s \leq \bar{s}$. As argued above, for

¹⁸For more background on the GSOEP and for descriptive statistics, see appendix A.3.

these individuals we do not observe the outcome of the means test for b_{UA} . We do assume, however, that individuals know about the outcome even before applying for b_{UA} . Therefore, let $\phi(s; \mathbf{z}|0)$ indicate the search effort given that $b_{UA} = 0$, which corresponds to the hypothetical test failure. Similarly, let $\phi(s; \mathbf{z}|b_{UA})$ stand for the hypothetical case in which the test is passed and so $b_{UA} > 0$. Finally, let $\xi \equiv \{\alpha, \sigma, \delta_j, \chi, \zeta, \pi\}_{j=1,2}$ denote the vector of parameters to be estimated and let π denote the fraction of the individuals that pass the test. Then, for a single unemployment spell, the individual log-likelihood contribution of an unemployed belonging to this group is

$$\begin{aligned} \ln \ell(\xi) = & \ln \left(\pi [\mu_1(s; \xi, \mathbf{x}, \mathbf{z}|b_{UA})]^{d_u} e^{-\int_0^s \mu_1(u; \xi, \mathbf{x}, \mathbf{z}|b_{UA}) du} \right. \\ & \left. + (1 - \pi) [\mu_1(s; \xi, \mathbf{x}, \mathbf{z}|0)]^{d_u} e^{-\int_0^s \mu_1(u; \xi, \mathbf{x}, \mathbf{z}|0) du} \right). \end{aligned} \quad (3.20a)$$

In this equation and the following, d_u is a dummy variable such that $d_u = 1$ if the unemployment spell is uncensored, d_t is a dummy variable such that $d_t = 1$ if an individual passes the means test, d_j is a dummy variable such that $d_j = 1$ if the employment spell is uncensored, and l is the employment duration.

Second, consider individuals who enter unemployment with the right to claim UI benefits, fail to find a job before entitlement expires, transit to either UA or zero benefit level and thereby reveal the outcome of the means test, and potentially exit unemployment only after the expiration of entitlement, so $s > \bar{s}$. The log-likelihood contribution of these events is given by

$$\begin{aligned} \ln \ell(\xi) = & d_t \ln \pi + (1 - d_t) \ln(1 - \pi) + d_u \ln \mu_2(s; \xi, \mathbf{x}, \mathbf{z}) \\ & - \int_0^{\bar{s}} \mu_1(u; \xi, \mathbf{x}, \mathbf{z}) du - \int_{\bar{s}}^s \mu_2(u; \xi, \mathbf{x}, \mathbf{z}) du. \end{aligned} \quad (3.20b)$$

For individuals who do not have the right to claim UI benefits and enter unemployment receiving lower UA benefits from the very beginning ($d_t = 1$) or not at all ($d_t = 0$), we have

$$\ln \ell(\xi) = d_t \ln \pi + (1 - d_t) \ln(1 - \pi) + d_u \ln \mu_2(s; \xi, \mathbf{x}, \mathbf{z}) - \int_0^s \mu_2(u; \xi, \mathbf{x}, \mathbf{z}) du. \quad (3.20c)$$

For the final group, entrants to employment, the log-likelihood contribution is

$$\ln \ell(\xi) = d_j \ln \lambda(\mathbf{x}) - \lambda(\mathbf{x}) l. \quad (3.20d)$$

Our parametric assumptions about the shape of $g(s)$ is

$$g(s) = e^{-\delta_1 s^{\delta_2}} + 1. \quad (3.21)$$

We choose this parametric form for the spell effect as it should be able to cover all the cases of state dependence illustrated in figure 3.1 even in the absence of unobserved heterogeneity. Indeed, the term in equation (3.21) spawns a variety of shapes for the hazard function: it can be time-invariant, increasing or decreasing, being concave or convex. Thus, even if the influence of unobserved heterogeneity may not be significant, the model is still flexible enough to replicate the non-parametric estimates. For positive δ_i , the individual spell effect $\eta_0(\mathbf{x})g(s)$ is $2\eta_0(\mathbf{x})$ at $s = 0$ and tends to $\eta_0(\mathbf{x})$ for s approaching infinity.¹⁹

The parameterization for $\eta_0(\mathbf{x})$ is the usual $\eta_0(\mathbf{x}) = e^{\mathbf{x}'\chi}$. Similarly, the conditional exit rate out of unemployment is parameterized as $\lambda(\mathbf{x}) = e^{\mathbf{x}'\zeta}$.

- Estimation procedure

The estimation of model parameters uses a part of the numerical solution method for the steady state. We take exogenous values for ρ (0.003, corresponding to an annual interest rate of 3.7%) and bargaining power β (0.5). As described in appendix A.2, for a given wage w and vacancy to unemployment ratio θ , the individual exit rate at any moment of the unemployment spell can be computed. Using individual survey data implies that the wage w for each individual is known and the corresponding θ can be taken from macroeconomic data. Individual exit rates can therefore be computed for each individual job market history in our dataset, given an initial guess for the model parameters ξ . The sum of all log-contributions is then maximized by varying parameters in ξ in order to find the ML estimator.

Note that ξ is estimated without explicitly specifying the wage-setting mechanism. If we used linked employer-employee data, the model could be estimated by using the observable productivity data. This would also allow us to estimate the bargaining power parameter β , as well as provide more information on the discrepancy between the observed wage and an endogenous wage solution implied by the model. For the rest of the parameters unrelated to the wage-setting mechanism, however, both approaches must be equivalent, assuming that wage setting is correctly specified in the second one. Further, computing the steady state solution suggests that estimation with given wage and tightness is faster by a factor of about 4.

¹⁹We experimented with a generalization replacing 2 by some parameter to be estimated. This did not turn out to be viable, however.

- Identification

Altogether, the econometric model described in equations (3.20a)-(3.20d) covers three conceptual features of the observed unemployment duration data: i) endogenous duration dependence of the hazard rate, induced by the anticipation of the future reduction in benefit payments as described by Mortensen (1977) or van den Berg (1990), ii) exogenous duration dependence of the hazard rate induced e.g. by stigma (Vishvanath, 1989; Blanchard and Diamond, 1994), and iii) influence of unobserved heterogeneity as in Heckman and Singer (1984) or van den Berg and van Ours (1994), that is obtained through unobservability of the results of the means test.²⁰ As one can see from the contributions as given by equations (3.20a)-(3.20d), all these effects are readily identifiable. The separation rate parameter vector ζ is always identified by the data on the job duration l and observed characteristics, as becomes obvious from equation (3.20d). Given $\lambda(\mathbf{x})$, the scale parameters χ and the exogenous duration dependence parameters $\{\delta_j\}_{j=1,2}$ are identified from the subsample of non-entitled individuals as described in equation (3.20c) and post-entitlement incremental durations in equation (3.20b), since endogenous duration dependence for these is time-invariant. Given that exogenous duration dependence is pinned down, the parameters α and σ , that shape the endogenous duration dependence induced by anticipation of benefit reduction, are identified by the variables b_{UI} , b_{UA} , and \bar{s} in \mathbf{z} in equation (3.20a). Finally, the fraction π of those who pass the means test is identified by equations (3.20a)-(3.20c).

²⁰Of course, one can also think of some additional sources of unobserved heterogeneity. In this case, the model is extended in a standard way, with heterogeneity entering additively (or multiplicatively) into either η or λ , after which a marginal contribution to the likelihood, with unobserved component integrated out, is considered. However, unlike with unobserved outcome of the means test, this would already be the heterogeneity induced by some unknown source, which makes its modeling less interesting. Moreover, and most importantly, the computational burden of fitting the model with an additional unobserved heterogeneity increases immensely.

	Coeff.	SE	z-Stat.	p-Value
ζ_0	-4.6274	0.0834	-55.4948	0.0000
χ_0	-4.2730	0.2961	-14.4290	0.0000
α	0.5105	0.0533	9.5715	0.0000
σ	0.6076	0.0800	7.5992	0.0000
δ_1	0.0033	0.0003	12.2187	0.0000
δ_2	2.3522	0.0967	24.3270	0.0000
π	0.2029	0.0312	6.4977	0.0000

Table 3.1: Results of the structural estimation of the unconditional model.

3.5.3 Estimation results

- Preliminary discussion

Table 3.1 reports the estimation results²¹ for the specifications excluding observed individual characteristics.²²

As for the estimation results, our main finding is the significance of the exit rate parameter α . This means that changes of the optimal effort path in response to any unemployment benefit reform, be it the reform of $b_{UI,UA}$ or of \bar{s} , will have a significant impact on the exit rate out of unemployment. This finding in particular can contribute to the empirical dispute about the dependence between unemployment benefits and exit decision. Evidence in the literature is conflicting with Hujer and Schneider (1989) and Arulampalam and Stewart (1995) finding minor or negligible dependence, and later Carling et al. (2001) and Røed and Zhang (2003) stating the opposite. With our significant α , any change in the design of unemployment benefit mechanism will induce a significant response on the macroeconomic level.

The next important finding is the role of unobserved heterogeneity. From table 3.1, we can see that π is significant at 5% level, implying that the prospect of not passing the means test significantly increases search effort. Along with that, the results show that the estimates of δ_i are significantly different from zero. This means that, once unobserved

²¹One evaluation of the total likelihood takes over 6.5 minutes with Matlab 6.1 on a laptop with 1.6 GHz CPU and 0.99 GB RAM. The optimization which leads to the estimates in table 3.1 requires 15 hours to converge.

²²We restrict the results shown here to the unconditional results since this is the focus of the theoretical model and the policy simulations, where we do not have observed heterogeneity among individuals.

components are accounted for, there is still true individual state dependence, which is captured by the spell effect. Thus, we find that the unobserved heterogeneity, as well as the exogenous individual downward state dependence are responsible for the declining non-parametric hazard rates in figure 3.1.

- Predicting labor productivity A and vacancy costs γ

After having estimated all the parameters in ξ , labor productivity A and vacancy costs γ ²³ have to be determined. The ML estimation is built on the household side of the model only, but the parameters A and γ are outcomes of the general equilibrium framework of the economy in steady state²⁴, assuming wage and labor market tightness are exogenous. In the econometric analysis, there is variation in the data with respect to exogenously given wages, UI and UA benefits, as well as UI entitlement durations. In order to account for variation of b_{UIi} , b_{UAi} , and \bar{s}_i in the general equilibrium model, the data for a representative agent is used. The representative individual earns the mean wage of the dataset, $w = 1161.21$ Euros²⁵. His UI payments are given by the replacement ratio ρ_{UI} times the mean wage, corrected for the share $\omega = 56.52\%$ ²⁶ of individuals in the dataset, who are entitled to UI payments, $b_{UI} = \omega\rho_{UI}w$. The estimated share of unemployed passing the means test for UA benefits is $\pi = 20.29\%$, compare table 3.1. Hence, also UA payments for the representative agent are the product of the replacement rate ρ_{UA} , the previous wage w , and the share π , $b_{UA} = \pi\rho_{UA}w$. Average sample entitlement to UI payments is 14 months, so \bar{s} for our representative agent is 14.

With these exogenous variables, the parameters A and γ can be determined.²⁷ Productivity A and vacancy costs γ are computed such that the average wage and average tightness in the sample result as endogenous general equilibrium variables in the model. The resulting labor productivity $A = 1227.03$ is just above the wage rate of $w = 1161.21$, leaving some room for firm profits. The costs of a vacancy are given by $\gamma = 112.18$.

Having determined A and γ ²⁸, the pre-reform steady state, the comparative statics,

²³Compare section A.4 in the appendix for the solution structure of the model with respect to A and γ .

²⁴See subsection 3.4.3 for the description of the economy in steady state.

²⁵Monetary values are all expressed in Euros. However, we omit the Euro notation in the following for the sake of clarity.

²⁶See section A.3 in the appendix for descriptive statistics.

²⁷See enclosed CD for the Matlab program.

²⁸As about 90% of the spells in our data are completed between 1997 and 1999, the productivity can be seen as an average value over the years 1997-1999. Using the growth rates of 1997 (1.8%) and 1998 (2.0%), the productivity of 1999 can be determined as $A = 1252.07$. Compare section A.5 in the appendix for details.

and insurance and incentive effects are evaluated in the following sections treating wage and labor market tightness as endogenous.

3.6 Numerical solution I: the pre-reform steady state

With the estimated parameters and equilibrium values given, the next step is a discussion of the pre-reform steady state. First, the steady state values of the situation before the Hartz IV reforms are analyzed and then we turn to the microeconomic dynamics in steady state.

3.6.1 Equilibrium values in the pre-reform steady state

In this section, the structurally estimated, the predicted, and the exogenous parameters, as well as the equilibrium parameters are used in order to characterize the steady state equilibrium of the pre-reform era as implied by the theoretical model. All parameters plus some selected endogenous model variables are provided in table 3.2. As in the estimation procedure, the rate of time preference ρ is chosen to fit the annual interest rate of 3.7%. The bargaining power β is set equal to 0.5. For the numerical solution of the model, we use the parameter estimates from the unconditional estimation model for two reasons. First, as there is no observed heterogeneity in the theoretical model, the respective parameter estimations are more in the spirit of the model, and second, earlier research in the analysis of Launov et al. (2009) revealed that the unconditional and the conditional results do not differ significantly from each other. This suggests that observed heterogeneity does not actually play a big role for the separation rate and the spell effect and hence, it can be neglected when solving the model numerically. The predicted separation rate from the unconditional estimation model is given by $\lambda = 0.01$ and the spell effect parameter by $\eta_0 = 0.014$.²⁹ The other estimated and predicted parameters are directly taken from the previous section.

In the pre-reform steady state, meaning before the Hartz IV reforms, benefit payments of both short-term and long-term unemployed workers depended on the previous wage. The replacement rates are given by $\rho_{UI} = 0.6$ for UI and $\rho_{UA} = 0.53$ for UA payments for those entitled. The benefits for the representative agent, b_{UI} and b_{UA} , are determined in the same way as in subsection 3.5.3 as $b_{UI} = \omega\rho_{UI}w$ and $b_{UA} = \pi\rho_{UA}w$. Average sample entitlement to UI payments is about $\bar{s} = 14$ months and this value is taken for the

²⁹The separation rate is determined in the unconditional model by $\lambda = e^{\zeta_0} = 0.01$ with $\zeta_0 = -4.6274$ and the spell effect parameter by $\eta_0 = e^{\chi_0} = 0.014$ with $\chi_0 = -4.2730$.

exogenous parameters			estimated and predicted parameters							
ρ	β		λ	η_0	σ	δ_1	δ_2	α	A	γ
0.003	0.5		0.010	0.014	0.608	0.003	2.352	0.510	1227.03	112.19
policy parameters			equilibrium values							
ρ_{UI}	ρ_{UA}	\bar{s}	w	θ	u	κ	$\bar{\mu}$			
0.6	0.53	14	1162.21	0.3	16.6%	4.8%	0.06			

Table 3.2: Parameters and selected equilibrium values.

numerical solution of the pre-reform steady state. The outcome of the model for the tax rate, $\kappa = 4.8\%$, and the unemployment rate, $u = 16.6\%$, meets the range of the actual unemployment insurance contribution rate (this is the only purpose of taxes in our model) of 6% and the actual unemployment rate in 1997-1998 of about 12%³⁰. The average job arrival rate in the pre-reform steady state computations of the model is $\bar{\mu} = 0.06$.

In sections 3.7 and 3.8, the comparative statics of different reform scenarios are run. For this purpose, the exogenous, the estimated, and the predicted parameters are taken as given and the change of policy parameters is evaluated in order to understand the effects on the equilibrium values in steady state.

Before that, however, the microeconomic dynamics in steady state will be analyzed.

3.6.2 Dynamics on the microeconomic level in steady state

Although the economy is in steady state, there are still dynamics on the microeconomic level. At any point in time, individuals find and lose jobs. And at any point in the unemployment spell, unemployed adjust search effort optimally. Figure 3.2 illustrates the developments on the microeconomic level in the pre-reform steady state over the unemployment spell. The upper left panel shows the evolution of the exogenous spell effect. The estimation results for the parameters of $\eta(s)$ imply that the spell effect decreases right from the beginning of unemployment, approaching a lower limit of $\eta_0 = 0.014$.

As a consequence, also the value of being unemployed falls over the unemployment spell. If there was no spell effect, so $\eta(s)$ stayed constant, a long-term unemployed would live in a stationary world and the value of being long-term unemployed would be stationary as well. However, with a negative spell effect the job-finding rate - taking optimally chosen effort into account - goes down and the value of being unemployed approaches a lower limit determined by the lower limit of $\eta(s)$. This is shown by the

³⁰Data taken from Bundesagentur für Arbeit - Statistik (2008).

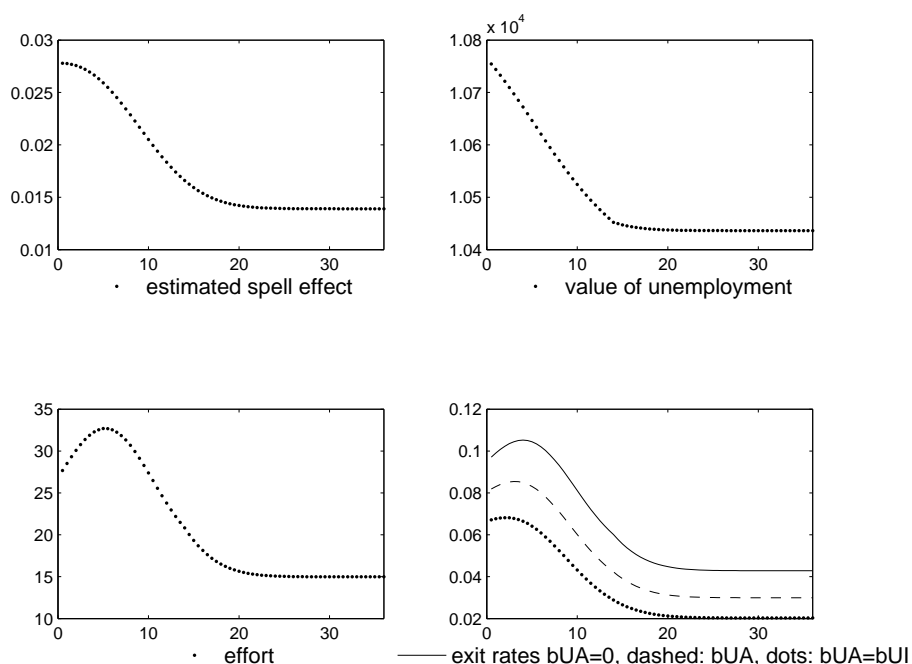


Figure 3.2: Productivity, search effort, exit rate, and value of being unemployed as functions of the spell s (in months).

upper right panel and needs to hold generally as equation (A.8) in the appendix shows.

The optimal reaction of the unemployed worker is shown in the lower left panel by the search effort exerted. Effort unambiguously increases during the first five months of unemployment. While the estimated spell effect is decreasing, this increase reflects the rising incentives to search harder, the closer \bar{s} and the benefit cut come. Optimal effort is the outcome of the interplay between the spell effect (lower $\eta(s)$ reduces optimal effort) and the potential gain from finding a job. This can be seen from the first-order condition in equation (3.6) or, more directly, from equation (A.7) in the appendix. As gains increase due to a falling value of being unemployed, this second effect tends to increase effort. After some time, however, the increase in the gain of finding a job is no longer strong enough to compensate the ‘discouraging’ impacts from the spell effect. Search effort eventually falls and approaches a constant. The fact that unemployed workers finally ‘give up’ is ultimately the effect of the exogenous negative spell effect.

The figure on the lower right shows the exit rates out of unemployment for $b_{UA} = 0$, average b_{UA} , and $b_{UA} = b_{UI}$. Unsurprisingly, the level of the exit rate is smallest for an unemployed eligible to long-term benefits amounting to full UI benefits, $b_{UA} = b_{UI}$ (dotted line). In this case, the incentive effects of the non-stationary unemployment benefit system are completely missing and the exit rate is only affected by the spell effect. Consequently, the job arrival rate evolves qualitatively like the spell effect and decreases

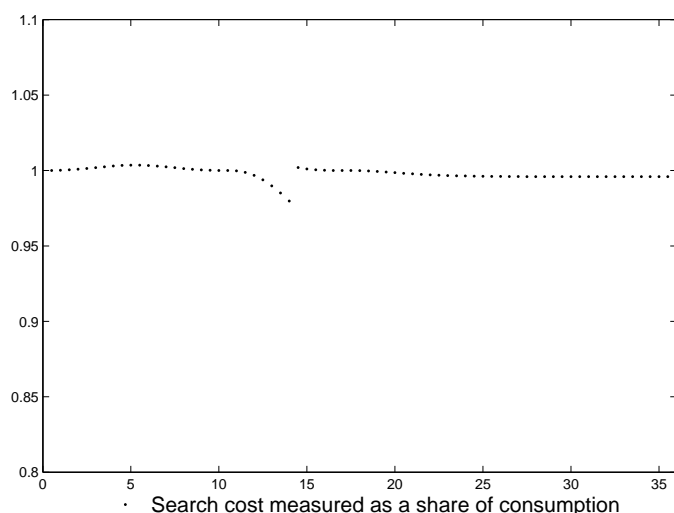


Figure 3.3: Fraction $x(s)$ of consumption loss equivalent to search effort loss.

right from the start. The dashed line shows the job arrival rate for an individual receiving the average b_{UA} , i.e. the benefits of the representative agent that were used in the pre-reform steady state. The rising rate during the first months of unemployment stems from the incentive effects and the resulting search effort increases, which is finally overcome by the spell effect. Third, the subfigure on the lower right of figure 3.2 shows the job arrival rate for $b_{UA} = 0$, i.e. no long-term benefits at all. The exit rates are on the highest level and the increase of the rate is sharpest compared to the other two cases of b_{UA} . The incentive effects for leaving unemployment before \bar{s} are, of course, much higher due to the complete abolishment of long-term benefits. However, also for this case, the spell effect finally causes the exit rate to decrease after about five months of unemployment.

Given the utility function

$$u(b(s), \phi(s)) = \frac{b(s)^{1-\sigma}}{1-\sigma} - \phi(s),$$

one may ask how costly search is in terms of consumption. Or more specific, which percentage consumption reduction $x(s)$ in a world without search effort generates the same utility as in the world where optimal effort costs $\phi(s)$ exist. For this purpose, the share of consumption loss $x(s)$ due to search is determined by

$$\frac{([1 - x(s)] b(s))^{1-\sigma}}{(1-\sigma)} = \frac{b(s)^{1-\sigma}}{(1-\sigma)} - \phi(s).$$

Computing the consumption loss for the average unemployed in the pre-reform steady state shows that it is substantial, ranging between 97 and 102 percent. Figure 3.3 shows the evolution of the loss fraction $x(s)$ over the unemployment spell.

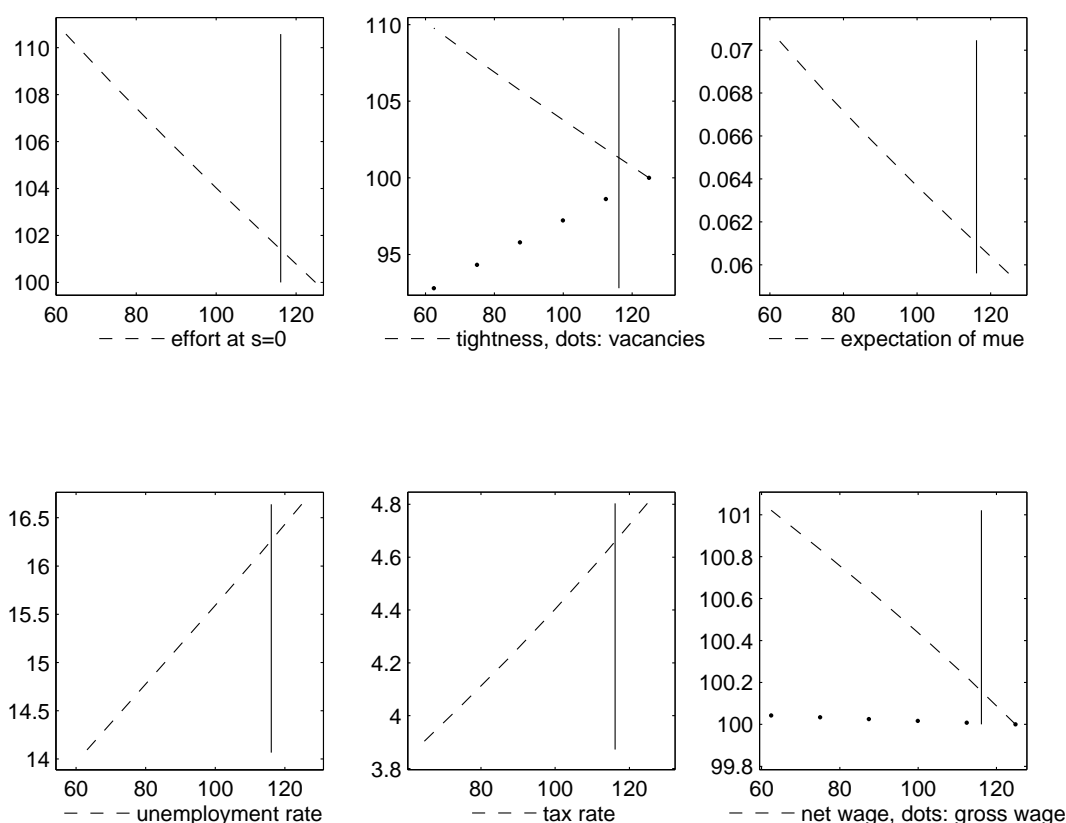


Figure 3.4: (Un)employment effects of decreasing long-term benefits b_{UA} .

3.7 Numerical solution II: the effects of the reforms

The 2005-2006 unemployment benefit reforms in Germany were characterized by a reduction of UA benefits, b_{UA} , and entitlement length, \bar{s} , as described in subsections 2.5.1 and 2.6.2, respectively. Benefits decreased by about 7% on average and mean entitlement length \bar{s} dropped by about two and a half months from 14 to 11.5 months. These changes are now considered in the model. The sectioning is as follows: first, each change is analyzed individually (the change of b_{UA} and then the change of \bar{s}) before both policy parameters are reduced simultaneously. Finally, we account for economic growth of the economy between the pre-reform steady state and the comparative static steady states, which also affects the economic outcomes and welfare positions.

3.7.1 Decreasing the unemployment assistance benefits b_{UA}

Figure 3.4 shows the effects of decreasing UA benefits b_{UA} on the labor market when reading the horizontal axes from right to left. The new levels implied by the reforms are marked by a vertical line. The figure on the upper left shows the search effort of an

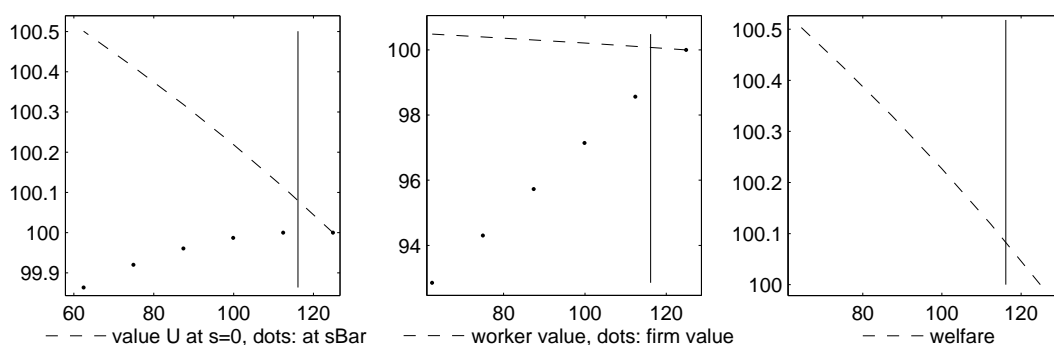


Figure 3.5: Welfare effects of decreasing long-term benefits b_{UA} .

unemployed at the beginning of his unemployment spell. This effort increases as benefits b_{UA} decrease, which illustrates the incentive effects of lower long-term benefits for short-term unemployed. Higher search effort implies a higher job-finding rate and hence, it becomes more likely that a job is found faster, leading to a higher mean exit rate. As a consequence, the unemployment rate in the economy goes down for decreasing UA benefits as shown in figure 3.4 on the lower left. Not surprisingly, less unemployment leads to a higher vacancy-unemployment ratio θ , which means that the labor market becomes tighter for firms, although firms open less vacancies due to the increased gross wage.

With unemployment decreasing, there are more employees financing less unemployed, who, in addition, get lower long-term benefits. Hence, a balanced governmental budget makes the tax rate go down. Net wage, finally, is displayed in the lower right figure and it increases as long-term benefits are cut. From the wage equation (3.11), three parameters influencing the wage can be identified: initial search effort and the tax rate have a negative effect, while the labor market tightness has a positive effect on the bargained net wage. Obviously, the positive effects of a tighter labor market and decreasing taxes dominate the negative effort effect. So without considering any welfare questions, the cut of b_{UA} by the Hartz IV reforms seems to be a good move against the too generous institution.

Now we go on and add welfare measures. Figure 3.5 shows the values of different types of labor market agents and the welfare of the economy. While the newly short-term unemployed win due to the reform, the newly long-term unemployed are worse off in terms of their expected lifetime values. The long-term unemployed are directly hurt by the cut in b_{UA} , so it is no surprise that their value decreases. The short-term unemployed are now subject to higher search pressure because in the near future their benefits may drop from b_{UI} to an even lower level of b_{UA} . However, the higher search effort also increases their job arrival rate. The value of unemployment depends negatively on the

search effort and positively on the exit rate out of unemployment, which can be seen from equations (3.5) and (3.17). So, as figure 3.5 suggests, the expected gains from employment outweigh higher effort costs for the short-term unemployed with $s = 0$. The value of the employed worker is influenced by two forces: positively by both the net wage and unemployment benefits. Due to the reform, net wage increases and UA benefits are cut. For the employed worker, the gains of a higher wage obviously compensate for the prospective loss of becoming unemployed some time in the future, compare equation (3.4). More surprising is the fact that firms lose with dropping b_{UA} , but remembering that gross wage increases slightly explains lower equilibrium profits. In the right subfigure, the overall welfare effect of the reform can be seen. Welfare slightly increases with decreasing UA benefits, not only as a result of welfare gains of the employees and the short-term unemployed, but also as there are the formerly unemployed, who gain since they are now employed.

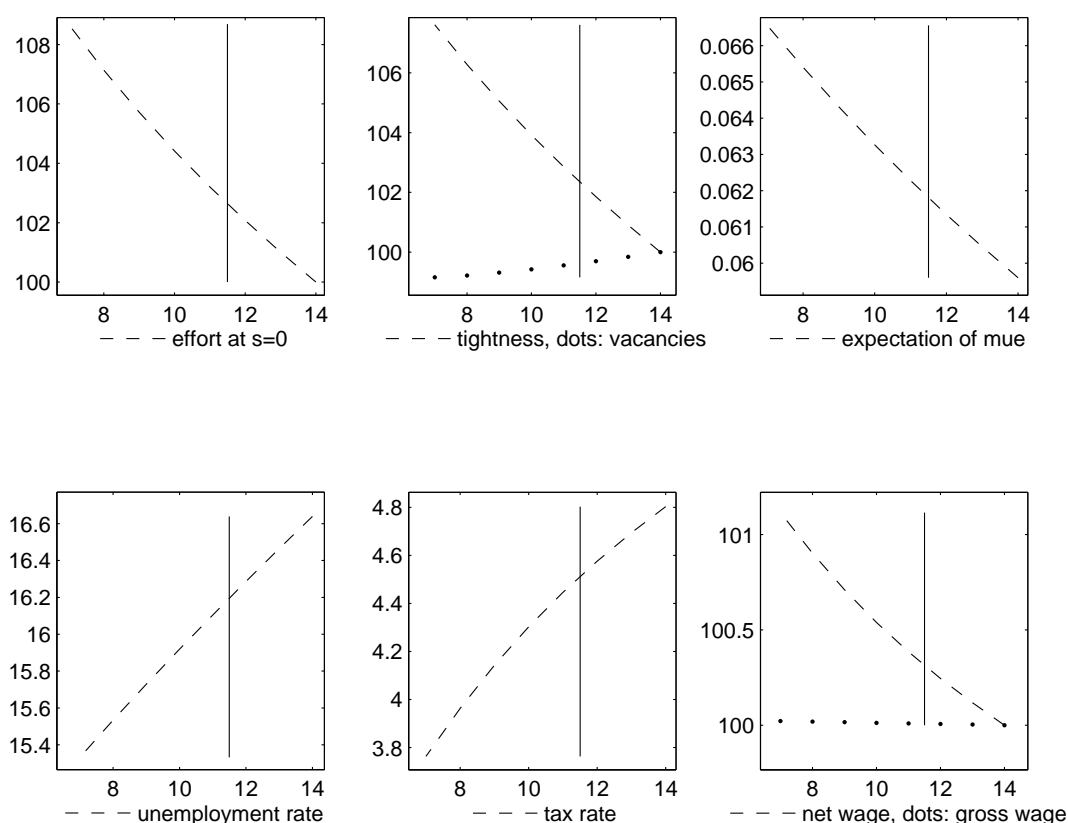


Figure 3.6: (Un)employment effects of decreasing entitlement duration \bar{s} .

3.7.2 Decreasing the entitlement period \bar{s}

In this subsection, the effects of a decreasing entitlement period \bar{s} are discussed. Figure 3.6 shows the effects on the labor market when reading the horizontal axis from right to left. Like for the reduction of b_{UA} , the search effort of a newly unemployed increases, reflecting the incentive effects of an earlier drop to the UA benefits in a two-tier system. Also for \bar{s} it holds that higher effort leads to higher job arrival rates, reflected by the increasing mean job arrival rate $\bar{\mu}$. A higher probability of finding a job faster results in lower unemployment rates as illustrated by the subfigure on the lower left. Consequently, the labor market tightness increases unambiguously, although vacancies drop slightly. For the budget of the government, the effects are similar to those of the long-term benefit reduction. There are now less unemployed to be financed and more workers financing the remaining unemployed. Additionally, the high short-term benefits b_{UI} expire earlier now, which further relieves the governmental budget and leads to tax reductions. Lower tax rates and a higher labor market tightness do also for \bar{s} decreases outweigh negative effort effects in the wage equation, yielding net wage raises. Hardly visible, the gross wage paid by the firms increases slightly. Altogether, the effects of entitlement duration

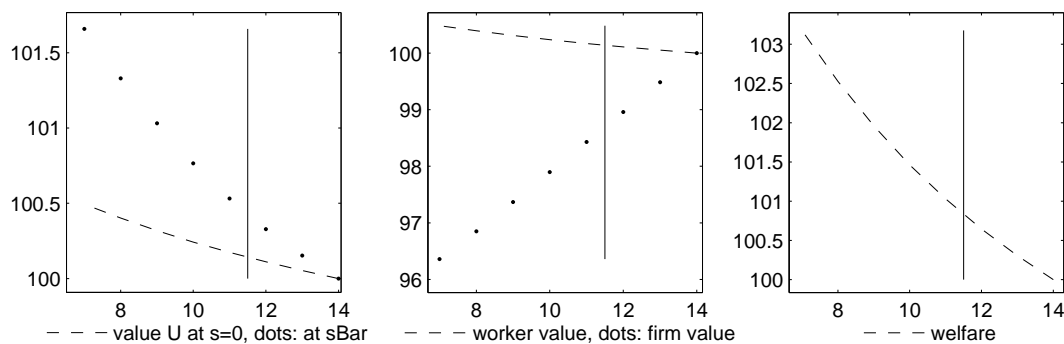


Figure 3.7: Welfare effects of decreasing entitlement duration \bar{s} .

cuts are very similar to the effects of long-term benefit drops.

Figure 3.7 illustrates the implications on individual values and total welfare. The value of the newly short-term unemployed increases, although this group is affected most by the cut. So again, the expected gains of a more likely employment outweigh higher search costs. However, in contrast to dropping b_{UA} , the value of the long-term unemployed at \bar{s} goes up. In this context, a long-term unemployed is defined as an unemployed whose short-term benefits terminated, so the definition adjusts to the current \bar{s} . Hence, the result of an increasing value for the long-term unemployed is driven by the definition of short-term unemployment.

The value of the employed worker increases a little for \bar{s} reductions, too. Hence, the net wage increases still outweigh losses due to possible unemployment and give rise to gains on the employee's side. Firms are again the losers of a reform which cuts benefit entitlement duration, with their value declining right from the start. As the values of all households increase, social welfare increases as well. Again, there are the direct positive effects of the households who keep their labor market status. Additionally, the gains of the unemployed who found work due to the reform now have a higher value since they are employed now. Summarized in the welfare function, the subfigure on the right shows that those gains are high enough to compensate the profit losses of firms.

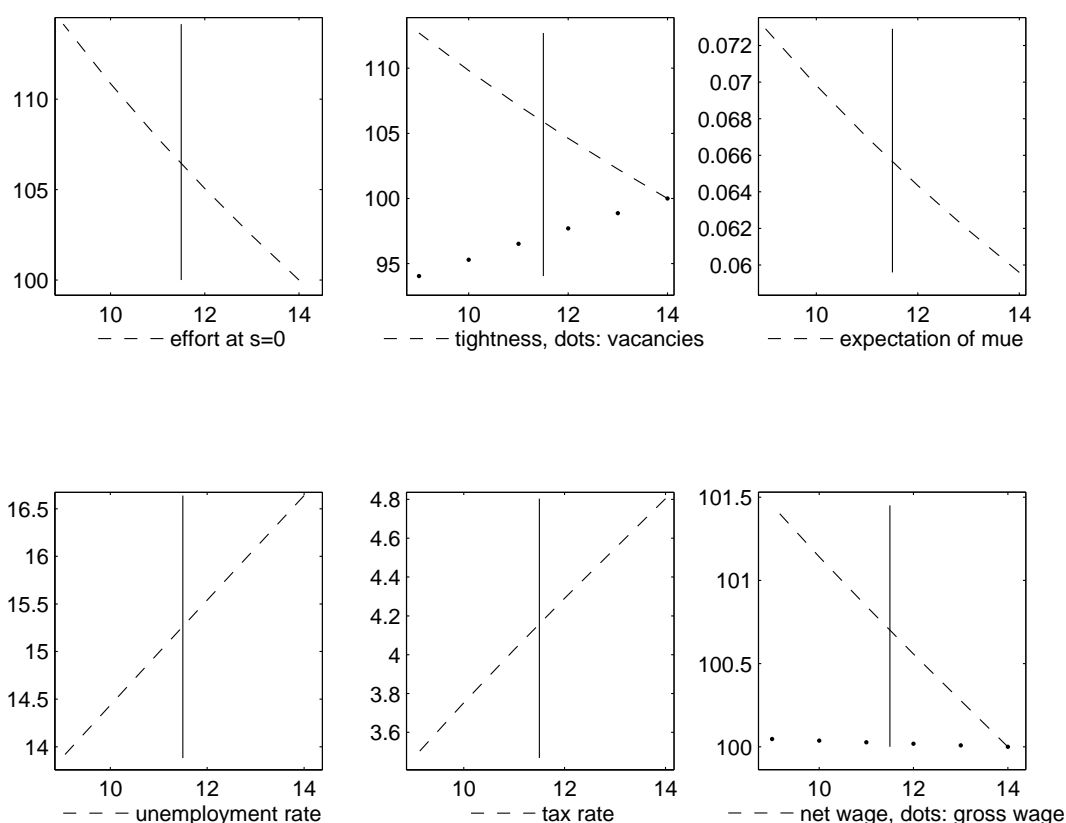


Figure 3.8: (Un)employment effects of decreasing long-term benefits b_{UA} and decreasing entitlement duration \bar{s} simultaneously.

3.7.3 Decreasing the unemployment assistance benefits b_{UA} and the entitlement period \bar{s} simultaneously

Recollecting the results from a reduction of long-term benefits or entitlement duration alone, the expectations with respect to a simultaneous decrease of b_{UA} and \bar{s} are quite clear. The unemployment rate is still expected to go down as now the incentive effects from both sources are acting and still, welfare is supposed to go up. The simultaneous reductions within the numerical analyses are done in what we figuratively call ‘Hartz units’. As already stated earlier, the labor market reforms cut long-term benefits by 7% and entitlement duration by about 2.5 months. In the following, these are the steps that are used for the successive continuation of the comparative statics.

Figure 3.8 shows the (un)employment effects of simultaneously decreasing b_{UA} and \bar{s} when reading the horizontal axis from right to left. The search effort of the newly unemployed increases by more than for the single reductions of b_{UA} or \bar{s} due to the combined incentive to escape unemployment before the lower long-term benefits set in after a shorter entitlement duration. Consequently, also the effects on the mean job arrival

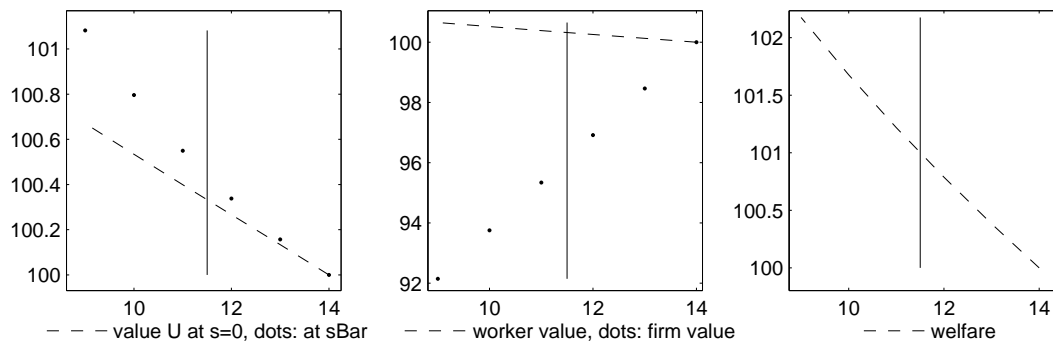


Figure 3.9: Welfare effects of decreasing long-term benefits b_{UA} and decreasing entitlement duration \bar{s} simultaneously.

rate and total unemployment are stronger than for the partial decreases: the former is higher and the latter is lower now. A sharper drop in unemployment leaves even more room for tax reliefs than each single reform measure and, as expected, the net wage increases. For a simultaneous change in ‘Hartz units’, also the gross wage paid by firms increases. As a consequence from equations (3.8) and (3.10), firms open less vacancies, but declining unemployment still leads to a tighter labor market.

The welfare effects of the simultaneous changes are shown in figure 3.9. The value of short-term unemployed at \bar{s} increases as it was the case for partial b_{UA} or \bar{s} reductions. An interesting question is what happens to the value of the long-term unemployed at \bar{s} . It decreased for b_{UA} reductions and increased for \bar{s} reductions. The dominating effect for a simultaneous reform of both policy parameters is the increasing one. So also for this reform scenario, long-term unemployed gain. The value of a job to a firm behaves like before; it goes down as higher gross wages have to be paid, while employed workers are still better off. Not surprisingly, welfare also increases in total. Hence, in the framework of our model a reform like Hartz IV is not only helpful to fight unemployment, but also to increase social welfare.

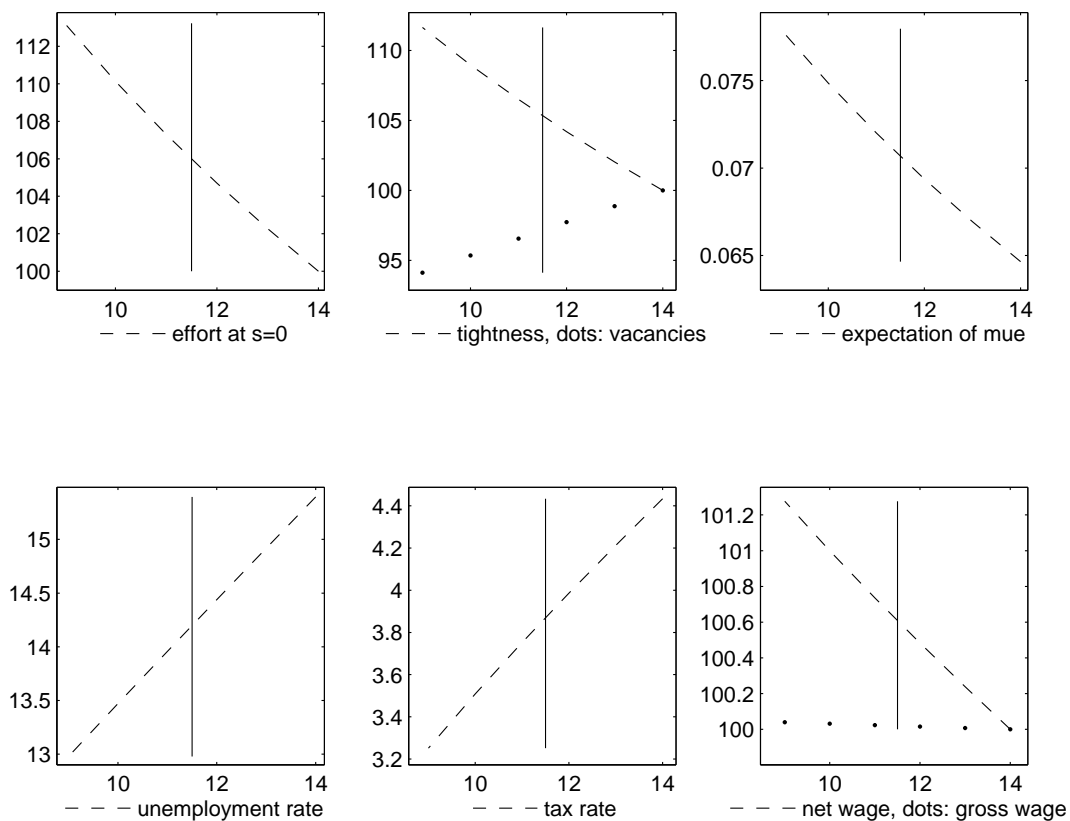


Figure 3.10: (Un)employment effects of decreasing long-term benefits b_{UA} and decreasing entitlement duration \bar{s} simultaneously while the economy has grown.

3.7.4 Decreasing the unemployment assistance benefits b_{UA} and the entitlement period \bar{s} simultaneously in a grown economy

Finally, the joint effects of b_{UA} and \bar{s} decreases are considered in the model economy which experienced economic growth. For Germany, the official growth rates of real GDP can be found in Statistisches Bundesamt (2009). Economic growth is assumed to increase the total factor productivity A . In subsection 3.5.3, we see that average productivity between 1997 and 1999 was 1227.03 and from this value, we can determine productivity in 1999 as 1252.07. For the evaluation of the reforms, which became effective in 2005 and 2006, economic growth between 1999 and 2005 is taken into account now.³¹ The productivity in 2006 is then given by 1333.39.

The implications for the labor market are presented in figure 3.10 in order to show the steady state of the economy in 2006. The incentive effects of simultaneously reduced b_{UA} and \bar{s} are basically the same as in the economy without growth. The search effort at $s = 0$ increases and so does the mean job arrival rate. Not surprisingly, the unemployment rate

³¹Compare section A.5 in the appendix for details.

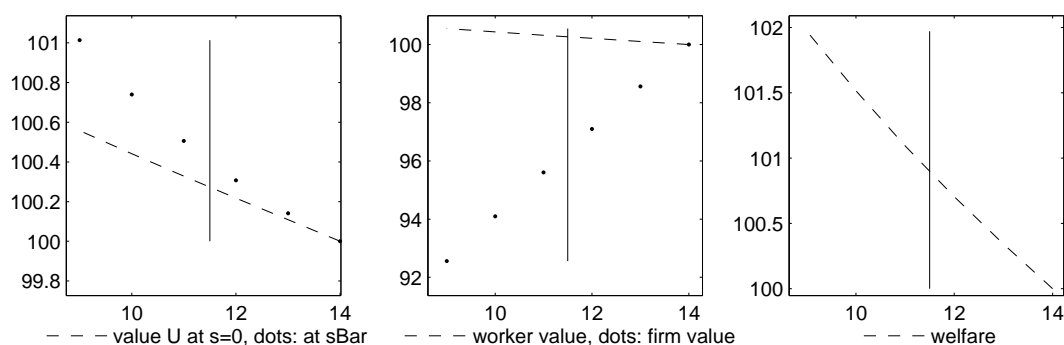


Figure 3.11: Welfare effects of decreasing long-term benefits b_{UA} and decreasing entitlement duration \bar{s} simultaneously while the economy has grown.

keeps decreasing and the decline is even stronger than without economic growth. Firms still have to pay higher gross wages and hence, gains from opening a vacancy will be smaller, which leads to less vacancies. However, in total the decline in unemployment leads to a tighter labor market compared to the situation without growth. Having in mind the exit rate specification as given by equation (3.18), this increase of θ , all other things being equal, results in a higher exit rate and hence in lower unemployment. As a consequence of decreases in unemployment, also the tax rate declines. Remembering the wage equation (3.11), a growth of A increases the right-hand side. However, at the same time tightness is smaller than in the case without growth, which outweighs the increase in A . Consequently, the workers are not able to profit from an increased productivity A and the bargained net wage is comparatively lower.

Again, the welfare effects are analyzed next by figure 3.11. The results are substantially the same as for the case without growth. All household types win and the firms lose. Also here, the gains of the households outweigh the losses of the firms leading to an increase of welfare. However, the results of this subsection suggest that the reforms brought no Pareto improvement and that the positive welfare and unemployment effects were rather small. In the next section, we decompose the over-all effects of UA benefit reforms in order to analyze isolated insurance and incentive effects of unemployment benefits.

3.8 Numerical solution III: insurance and incentive effects of unemployment assistance benefits b_{UA}

Quantitatively, the effects on welfare seem to be small: welfare increased by about 1%. This is not surprising, however, when intertemporal utility is considered. With 16.6% being unemployed in our framework and 1/3 becoming long term unemployed, only 5.53% of the population are affected by the reform. In an intertemporal sense, income is reduced only during 5.53% of one's lifetime. When current income of these 5.53% is cut by 7%, lifetime income is reduced by 0.39%. Hence, the apparently low quantitative effects make sense.

In order to analyze and understand the total effect of changing long-term benefits b_{UA} in detail, the impacts are evaluated partially. First, the pure insurance effects of b_{UA} are studied on a theoretical level before they are assessed in the framework of our labor market model. Furthermore, the combined insurance and incentive effects of benefits for long-term unemployed are evaluated. For all of these variations, we focus on the impacts of a changing b_{UA} .

3.8.1 An analytical benchmark for the pure insurance effects

The above analysis of decreasing b_{UA} in subsection 3.7.1 can be summarized by saying that the labor market reform changes the unemployment rate and total welfare by about one percentage point. To gain intuition about welfare reactions, let us recall the classic result from optimal fair insurance.

An individual can be in one of the two states *employment* or *unemployment* with probability p and $1 - p$, respectively. Earning w in the state of employment and receiving b when unemployed, expected utility is given by $pu(w) + (1 - p)u(b)$. The government finances benefits by a labor tax and equates income with expenditure, $\kappa w / (1 - \kappa)p = b(1 - p)$, where p is both the probability of being employed and the share of the population which is employed. Maximizing expected utility given this constraint requires the marginal utilities in both states to be equal, $u'(w) = u'(b)$, and a tax that implies $w = b$. Identity of benefits to net wage provides perfect insurance and consumption smoothing in the absence of incentive effects.

This result is now replicated for our labor market model. Assume that the path of effort is not influenced by the reform, so it is exogenously given. Assume further that the gross wage and the number of vacancies are not affected as optimal firm behavior is neglected. Then, keeping b_{UI} constant for simplification, a change in b_{UA} is simply

a transfer of income from the state of being employed to the state of being long-term unemployed through the fiscal system by the tax on labor income. Maximizing the social welfare function (3.12) by choosing b_{UA} and letting the tax rate adjust accordingly, yields (see appendix A.6)

$$V'(w) \frac{U_{long}}{N} = \frac{N-L}{N} \int_{\bar{s}}^{\infty} \frac{d}{db_{UA}} V(b_{UA}, s) f(s) ds. \quad (3.22)$$

In the light of standard results of optimal insurance, this expression is easy to understand. Also here, the marginal utilities (intertemporal utilities, the value functions) are compared, but furthermore the distribution of the unemployment spell is taken into account. This happens in two ways: first, through the share of long-term unemployed in the total number of unemployed and second, through the density $f(s)$ - how often and how long this state of being long-term unemployed occurs. The first effect is due to the fact that this maximization problem takes the third state - short-term unemployment - as given. If there were no short-term unemployed and hence, $\frac{U_{long}}{N} = \frac{N-L}{N}$, the optimality condition would read $V'(w) = \int_{\bar{s}}^{\infty} \frac{d}{db_{UA}} V(b_{UA}, s) f(s) ds$. This exactly corresponds to the optimality condition from a static insurance model extended by the distribution as captured by $f(s)$.

In the next subsection, the pure insurance effects are evaluated numerically in our labor market model.

3.8.2 Quantitative benchmark results of the insurance effects

When solving the model described in the previous subsection 3.8.1 numerically, some adjustments with respect to the original setup have to be made. Basically, the labor market model is solved for an exogenous *gross* wage, exogenous number of vacancies, an endogenous tax rate κ , but *exogenous search effort*. The solution structure of the insurance effects model can be found in subsection A.7.1 of the appendix.

As before, the unemployment effects are considered first in figure 3.12, but this time with increasing b_{UA} . With the search intensity path given from the pre-reform steady state, there are no unemployment effects at all. Therefore, dynamics on the microeconomic level are given by the path as visible in figure 3.2. The only endogenous variable in this setup is the tax rate and it is computed, once having solved for all values of the households, by equation (A.19). The tax burden for the employees rises with increasing long-term benefits b_{UA} to keep the government budget balanced. Consequently, the net wage decreases accordingly for the given gross wage.

Figure 3.13 shows the distributional effects of increasing the long-term benefits in an

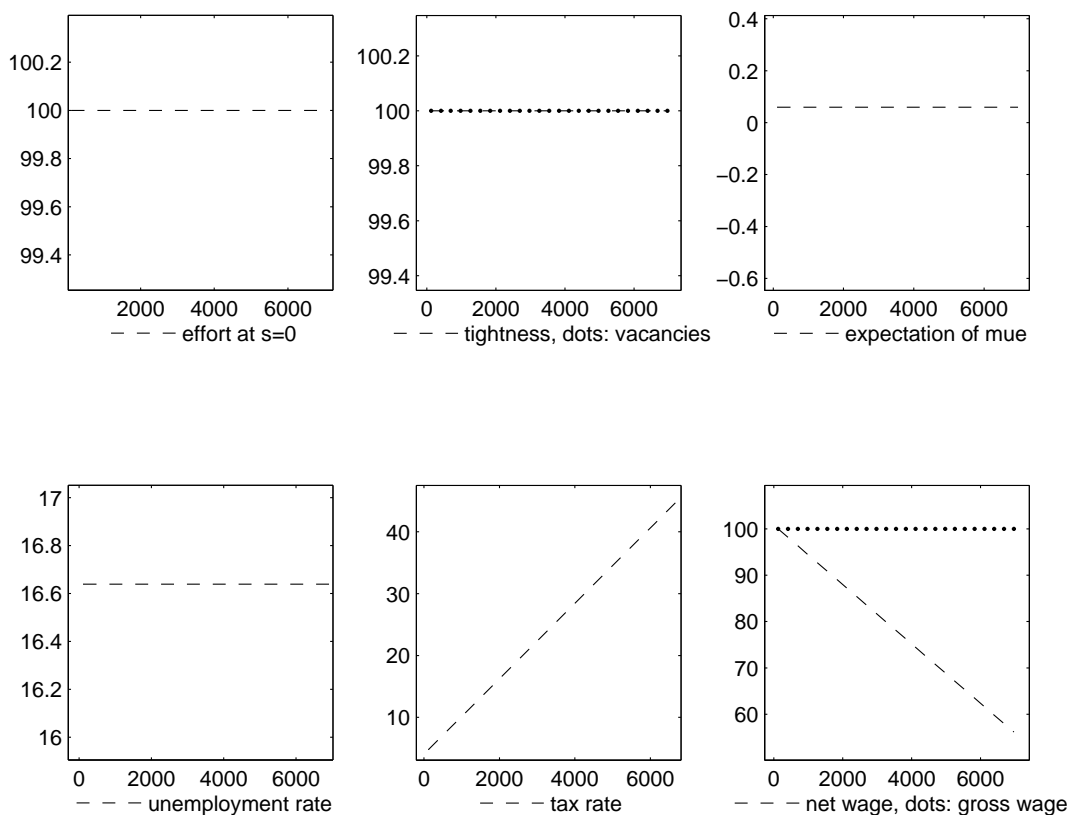


Figure 3.12: The insurance effects of unemployment benefits b_{UA} regarding (un)employment.

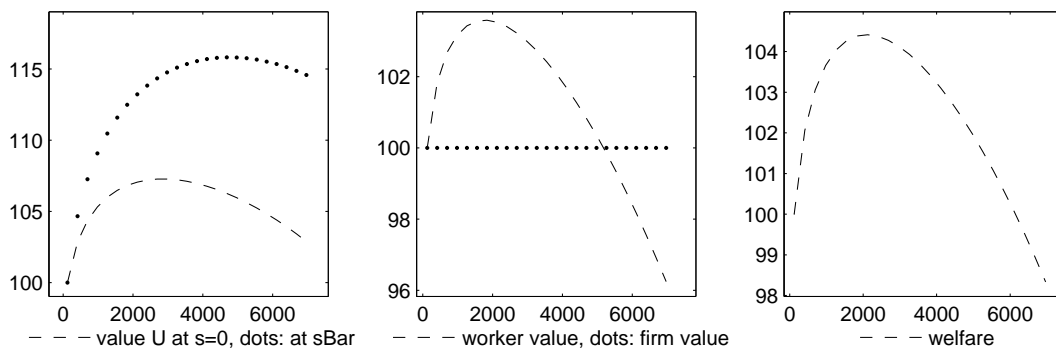


Figure 3.13: The insurance effects of unemployment benefits b_{UA} regarding welfare.

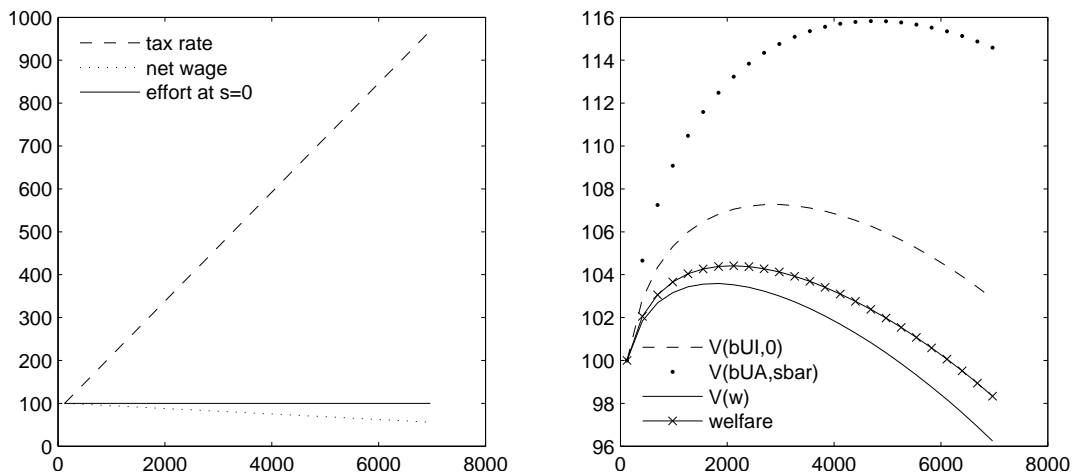


Figure 3.14: The insurance effects of unemployment benefits b_{UA} .

economy when incentive effects are not considered. The firm value stays constant for all b_{UA} as optimal behavior of firms is neglected when analyzing the insurance effects and so vacancies are assumed to be constant. For all individuals, the values are increasing in b_{UA} starting from the representative b_{UA} , they reach a maximum, and finally decrease. Also social welfare follows this pattern. This suggests that there is not enough insurance against unemployment in the pre-reform economy if incentive effects are left unconsidered.

Figure 3.14 illustrates the effect of changes in b_{UA} on the tax rate and the *net* wage in addition to the value of having a job, the value of being short-term and long-term unemployed, and overall welfare. In this way, the optimal b_{UA} resulting from equation (3.22) can be visualized.

The figure shows that UA benefits b_{UA} are required to be *higher* than the net wage w when the pure insurance effects of b_{UA} are considered. The reason is that the existing UI benefits do not provide full insurance with a replacement rate of 60%. Consequently, b_{UA} must overcompensate this loss of the short-term unemployed for full insurance to hold. Furthermore, the figure shows that the optimal b_{UA} is lowest for the employed worker, followed by the optimal b_{UA} of the short-term unemployed, and highest for the long-term unemployed, who is affected directly. The closer an individual is to the UI benefit exhaustion at \bar{s} , the higher is the optimal b_{UA} of this individual. The welfare maximizing b_{UA} lies at about $b_{UA} = 2000$.

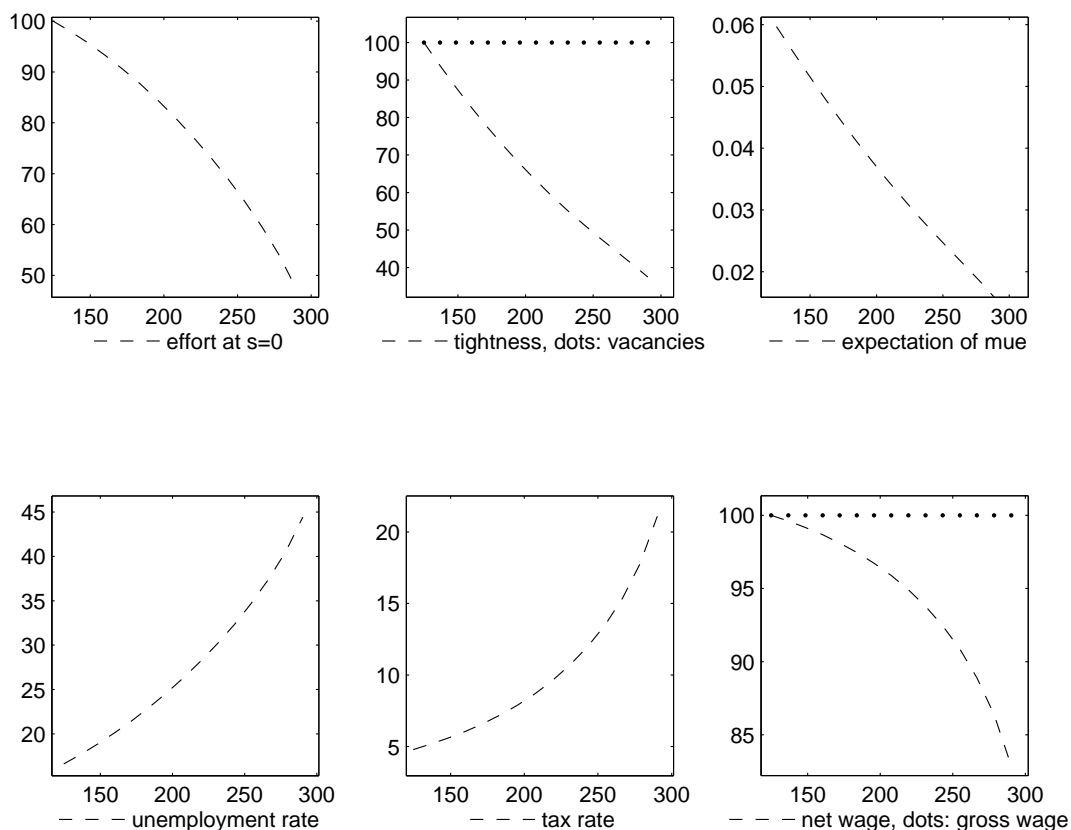


Figure 3.15: The insurance and incentive effects of unemployment benefits b_{UA} on (un)employment.

3.8.3 Quantitative benchmark results of the insurance and the incentive effects

In addition to the insurance effects of b_{UA} , the incentive effects of non-stationary benefits are taken into account now and therefore, search intensity is no longer taken as exogenously given from the pre-reform steady state. The *gross wage* and the number of vacancies are treated as exogenous since optimal firm behavior is still neglected. The formation of optimal household behavior is exactly like in the full equilibrium model. Consequently, unemployment is now endogenous on the macroeconomic level in addition to the tax rate κ . Subsection A.7.2 in the appendix shows the solution structure of this setup.

Figure 3.15 illustrates the unemployment effects of increasing benefits b_{UA} . As long-term benefits increase, search effort at $s = 0$ declines substantially showing the negative incentive effects of higher benefits b_{UA} . Short-term unemployed do no longer fear long-term unemployment so much because the benefit cut at \bar{s} vanishes. This is also reflected in the falling mean exit rate out of unemployment $\bar{\mu}$. The optimizing behavior of firms

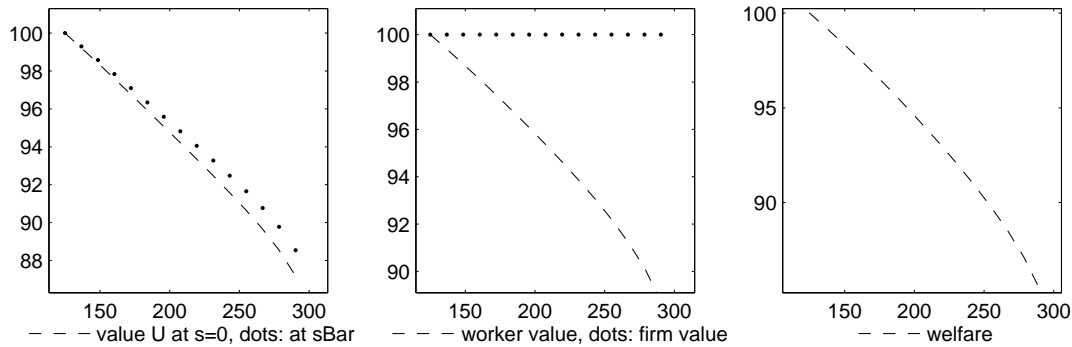


Figure 3.16: The insurance and incentive effects of unemployment benefits b_{UA} on welfare.

is still left unconsidered and therefore, the number of vacancies does not change. With the lower level of search intensity, the unemployment rate goes up and consequently, the labor market tightness reduces according to the binding tightness definition (A.24). The second endogenous macroeconomic variable is the tax rate resulting from the budget constraint of the government. From equation (A.25), it becomes clear that the tax burden increases as there are more unemployed, who get higher benefits. Therefore, net wage goes down, keeping gross wage fixed.

The distribution effects are shown in figure 3.16. Qualitatively, the effects of an increasing b_{UA} , while allowing for incentive effects, are completely reversed compared to subsection 3.8.2: all values decrease right from the start. This reflects the fact that now search effort reacts to the higher benefits and unemployment adjusts accordingly. Unemployed lose due to higher UA benefits as already suggested in section 3.7. Losses resulting from lower expected gains of employment outweigh the gains from higher consumption in the state of long-term unemployment. Consequently, all labor market agents now wish to have a lower b_{UA} .

In order to compare the different welfare levels implied by b_{UA} , the values of all agents are shown in figure 3.17 together with the tax rate and the net wage. Welfare decreases are smallest for employed workers since they are the last to be hurt by the low exit rates out of unemployment. Welfare is lowest for the short-term unemployed since they do not yet profit from the high b_{UA} , but still they are burdened with a low exit rate. Unlike for the pure insurance case of subsection 3.8.2, the UA benefits do no longer have to overcompensate benefits for short-term unemployed, b_{UI} . The UA benefits are lower than the net wage for all types of labor market agents once insurance effects are taken into account. Therefore, full insurance is no longer desired.

The last step to the complete general equilibrium effects as shown in figures 3.4 and

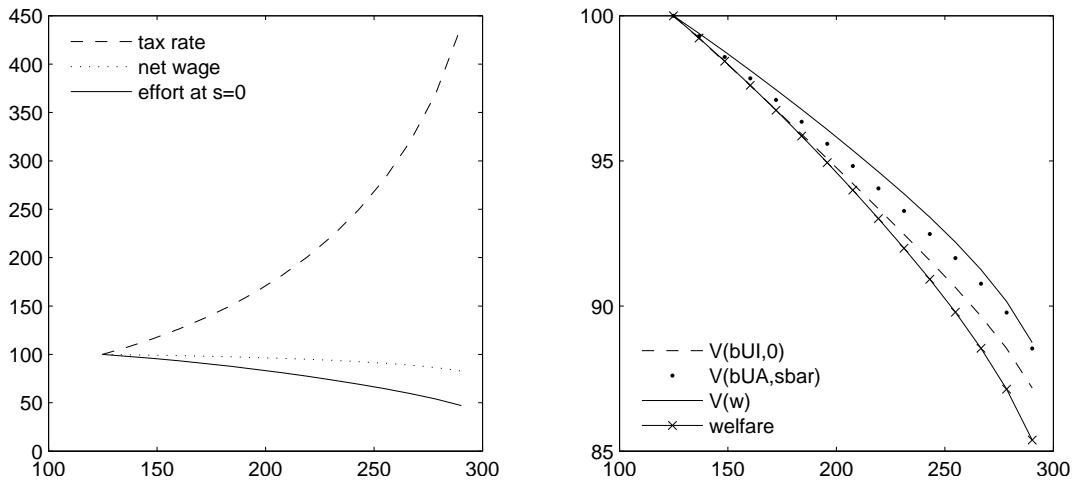


Figure 3.17: The insurance and incentive effects of unemployment benefits b_{UA} .

3.5 is the permission of optimal firm behavior and thus, endogenizing vacancies and gross wage.

3.9 Conclusion

The starting points of this chapter are the motivation of our labor market model and a survey of related literature. We develop an estimable search and matching model with endogenous search effort in a non-stationary unemployment benefit system. The main extension compared to the existing search and matching literature is the endogenous distribution of unemployment duration that arises due to individual choice of search intensity in a non-stationary environment. Based on optimal microeconomic behavior, macroeconomic quantities like the unemployment rate are determined employing tools from Semi-Markov methodology.

Outcome of the optimal behavior of unemployed in the theoretical model is a structural, duration-dependent transition rate from unemployment to employment of an individual being a function of various model parameters. This transition rate is used to determine the unemployment duration density, which is the basis for the structural parameter estimation via Maximum Likelihood. Finally, steady state policy changes are simulated using the parameter estimates.

We find several remarkable results. Concerning the estimation results, it is first discovered that the parameter α from the exit rate function is significant. This means that the duration-dependent search effort affects the exit rate out of unemployment and hence, changing benefits does play a role for the search intensity and for macroeconomic per-

formance. Furthermore, evidence is found for an unobserved heterogeneity among individuals: an estimated fraction of only about 20% of the short-term unemployed pass the means test for long-term benefits, which is likely to be known by the unemployed, but unobserved by the econometrician if exit occurs before \bar{s} . However, this unobserved heterogeneity obviously leads to differences in the individual exit rates out of unemployment. Hence, we verify true individual state dependence of the exit rate as well as state dependence implied by unobserved heterogeneity.

The simulations of different reform scenarios, on the other hand, allow for the assessment of individual and aggregate labor market and welfare effects when the length and level of unemployment benefit payments are changed. As an example of such reforms, the effects of the recent German labor market reforms are evaluated. First, we analyze each reform measure partially before we finally study the combined effects. Total unemployment decreases due to the reform and so does the tax rate. The impacts on the different labor market groups are differentiated: basically, the households win and firms lose. Therefore, the reforms fail to establish a Pareto improvement according to our model, while increasing total welfare. Moreover, partial insurance and incentive effects of long-term benefits are analyzed. Regarding the pure insurance effects, the main finding is that a complete insurance against unemployment due to UI benefits below the net wage requires UA benefits which exceed the net wage. Permitting incentive effects of the benefit system restores the known relations: optimal benefits for long-term unemployed do not establish full insurance against unemployment.

3.10 Individual contributions to the sections of chapter 3

As mentioned earlier, the paper of Launov et al. (2009) builds the basis of this chapter. My individual contributions are as follows:

- 3.1 Introduction: 10%
- 3.2 Different strands of literature: 100%
- 3.3 The model: 33%
- 3.4 Equilibrium properties: 33%
- 3.5 Structural estimation: 5%
- 3.6 Numerical Solution I: 50%
- 3.7 Numerical Solution II: 90%
- 3.8 Numerical Solution III: 60%
- 3.9 Conclusion: 60%
- Matlab source code for the steady state computations (see enclosed CD): 90%

With respect to the paper in its version from April 2009, this chapter was extended by sections 3.2 and 3.6-3.8.

Chapter 4

Semi-Markov processes in labor market theory

4.1 Introduction and underlying setup

Semi-Markov processes are, like all stochastic processes, models of systems or behavior. As extensions of Markov processes and renewal processes, Semi-Markov processes are widely applied and hence, an important methodology for modeling. Semi-Markov processes are used in computer science and engineering, e.g. in queuing theory and server models, see Cohen (1982). In finance, for example, credit rating and reliability models are based upon Semi-Markov theory like in D'Amico et al. (2006). Other applications in business administration are operations research like in Sobel and Heyman (2003), as well as manpower models as described in Mehlman (1979). Moreover, Semi-Markov models are employed in sociology or socioeconomics, see Mills (2004) for a model of the marriage market. In biology and medicine, Semi-Markov processes are used for prognosis and the evolution of diseases, see Beck and Pauker (1983) or Foucher et al. (2005). For demographic questions, models of disability or fertility, Semi-Markov processes are employed, too, see Hoem (1972).

Consequently, Semi-Markov processes are interdisciplinary important and, of course, also economics has discovered the usefulness for modeling issues. Already Markov processes, which can be seen as a special case of Semi-Markov processes, are widely used to describe the different states of an economy or an individual. Depending on the currently occupied state only, there are different transition rates to other states. Possible applications of Markov chains in economics are standard matching models of the labor market as described in Pissarides (2000) or money demand models like in Kiyotaki and Wright (1993). In this chapter, we will focus on the former ones as the methods presented build

the background for the numerical solution of our labor market model in chapter 3.

Typically, the possible states of an individual in the labor market are *unemployment* or *employment* and the transitions between these states are described by Markov processes. For simplification, most of the models in literature take the transition rates between the labor market states to be constant, see the standard matching setup in Pissarides (2000), Pissarides (1985), Mortensen and Pissarides (1994), for example, or Rogerson et al. (2005) for an overview. This simplification may be appropriate for many questions if incentive effects of labor market institutions can be neglected. For other applications, however, this assumption needs generalization. When the behavior of individuals and the incentive effects of unemployment insurance systems are to be analyzed, for example, stationary job arrival rates over the unemployment spell are no longer realistic, see Mortensen (1977) amongst others. In fact, it is plausible that the arrival rate of jobs exhibits true duration dependence. Reasons for this can be found in search effort reactions due to non-stationary benefits or stigmas attached to or perceived by long-term unemployed. Empirical evidence with respect to non-stationary hazard rates can be found in Heckman and Borjas (1980), Meyer (1990), or van den Berg and van Ours (1994), for instance.¹ However, models considering duration-dependent hazard rates are typically restricted to analyze microeconomic behavior only and thus, the Semi-Markov structure is negligible as the first order condition for optimal behavior is unaffected. Therefore in chapter 3, a full equilibrium labor market model is built up with non-stationary exit rates out of unemployment and the parameters of the model are estimated structurally.

Allowing for duration-dependent transition rates has methodological consequences regarding the state distribution of individuals. Analytical solutions for transition probabilities and distributions are no longer feasible for such models having non-analytic and non-stationary transition rates and numerical solution methods are required. Thus, the purpose of this chapter is twofold. First, an accurate, but intuitive definition and classification of Semi-Markov processes among the family of stochastic processes will be given, emphasizing the application to labor market models. Second, it provides a recipe of how to solve for the transition probabilities of Semi-Markov processes, as well as the description of the limiting behavior.

In a first step, this chapter presents the Semi-Markov theory. The properties and transition probabilities, as well as the limiting behavior are discussed on the basis of Pyke (1961a) and Pyke (1961b), Kulkarni (1995), and Ross (1996). While the transition probabilities of continuous-time Markov chains are computed using the Chapman-Kolmogorov equations, which can be solved analytically, for Semi-Markov processes, the correspond-

¹For a discussion of non-stationary hazard rates and possible sources, see subsection 3.2.2.

ing probabilities are based on the renewal argument and convolution theory. An analytical solution is very difficult in this case and impossible for the setup of chapter 3, so the determination of the transition probabilities and of the limiting probabilities is about numerical solution methods and it makes sense to deal with a specific example. Considering the economic model of chapter 3, there exist two groups in the labor market like in the standard model: the unemployed and the employed workers. This makes things as simple as possible, but clearly shows the solution approach at the same time. The job of an employed worker is destroyed at an exogenous separation rate λ and so the waiting time until job destruction is exponentially distributed with parameter λ . An unemployed job seeker with unemployment spell s gets new offers at rate $\mu(\phi(s)\theta, \eta(s))$, where $\phi(s)$ is the job search effort of the unemployed with spell s , θ is the labor market tightness, and $\eta(s)$ is an exogenous spell effect.² Having an unemployment insurance system with non-stationary benefits, it makes sense to assume that an unemployed individual adjusts his search effort over the spell. With increasing unemployment duration, for example, the lower benefits of long-term unemployed get closer. Thus, it is plausible that effort increases before long-term unemployment is realized. Assuming that the job arrival rate $\mu(\phi(s)\theta, \eta(s))$ increases with search effort, this partial effect would lead to an increasing job arrival rate. The duration-dependent spell effect $\eta(s)$ catches remaining duration-dependent factors, which may affect the job arrival rate. This partial effect is discussed in chapter 3 in detail, where it leads to a decreasing job arrival rate for long-term unemployed.

In this chapter, however, we focus on the pure duration dependence and not on its sources. Therefore, we neglect all other arguments but s and reduce the notation to $\mu(s)$ for simplification.

In chapter 3, the steady state behavior of the model economy is analyzed. Using the optimal search effort of an unemployed over the unemployment spell, we derive the densities for the duration in both states. With these densities, the parameters of the structural arrival rate are estimated with micro data from the GSOEP. Based on the parameter estimates, the job arrival rates can be computed as well as transition probabilities and hence, the state distribution for an economy of representative agents can be determined applying the methods derived in this chapter. The knowledge of the state distribution makes it possible to evaluate the Hartz IV reforms in terms of unemployment and welfare effects by models like the one in chapter 3.

The starting point for the calculation of the transition probabilities are interdependent Volterra integral equations of the first and the second kind, which can be derived applying

²Compare subsection 3.3.1 for details on the modeling of the job arrival rate.

the Semi-Markov theory. The key issue is to solve the integrals, which contain unknowns and cannot be solved analytically. To this end, the problem is transformed into a discrete one and numerical solution methods are discussed. The different methods have different advantages and drawbacks. As a rule, the more precise a method is, the longer the computation takes, leading to a time-preciseness trade-off. The different numerical results for the transition probabilities are therefore collected and discussed. First, the special case of constant arrival rates is considered. The Semi-Markov process is a continuous-time Markov chain then, for which the transition probabilities are known. Hence, the numerical solutions can directly be compared to the analytical solution. Permitting non-stationary arrival rates, with the setup taken from chapter 3, a comparison to an analytical solution is no longer possible. Hence, the numerical methods can only be studied independently and with respect to the limiting behavior. As expected, the more complicated method provides the more exact results for the transition probabilities. Since this already applies to smaller step numbers, the computational effort of the more complex method can be outweighed by using less steps. This also applies to the limiting distribution.

The outline of this chapter is as follows. Section 4.2 describes the basics of Semi-Markov processes. From section 4.3 on, we apply the Semi-Markov theory to our labor market model presented in chapter 3, in order to illustrate solution procedures for transition probabilities of Semi-Markov processes. In section 4.4, numerical solution procedures are described. Section 4.5 presents and compares the outcomes of the different numerical methods and section 4.6, finally, concludes with the findings of this chapter.

4.2 Semi-Markov processes - the basics

This section deals with the basics of (Semi-)Markov processes. First of all, like Markov processes, a Semi-Markov process is a stochastic process. A stochastic process collects realizations of one or more random variables over time and the theory of stochastic processes tries to find models which describe such probabilistic systems. One can distinguish between discrete-time processes and continuous-time processes. While the system is observed at discrete points in time only in the first case, there is continuous observation given for the latter. Throughout this chapter, we focus on the continuous-time versions. The starting point of this section is a brief introduction to Markov processes since many well-known concepts also hold for Semi-Markov processes. After that, the definition of Semi-Markov processes will be given and their properties will be outlined. The section concludes with a derivation of conditional transition probabilities of Semi-Markov processes.

4.2.1 Continuous-time Markov chains

Markov chains are stochastic processes and have the property of being memoryless. This means that a continuous-time Markov chain (CTMC) is a sequence of realized states and the transition probability to another state depends on the current state only and not on the history of states. Therefore, for the continuous-time Markov chain the following Markov property holds:

$$P \{X(t+s) = j | X(t) = i, X(u) : 0 \leq u < t\} = P \{X(t+s) = j | X(t) = i\},$$

where $X(t)$ denotes the state of the system at time t and $X(u) : 0 \leq u < t$ denotes all states $X(u)$ in the history from 0 up to t , compare Kulkarni (1995). In other words, this property means that the probability of being in state j at $t+s$, given that the system was in state i at t and the complete history of states, is equal to the probability without the information on the complete history.

The duration period of a CTMC in state i is exponentially distributed with parameter λ_i , so the probability of leaving a state i towards another *arbitrary* state in a spell of s or less is given by

$$F(s) = P \{S \leq s\} = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - e^{-\lambda_i x} & \text{if } x > 0. \end{cases}$$

The state duration period of leaving state i towards a *specific* state j is exponentially distributed with parameter $\lambda_{ij} \geq 0$. By definition, it holds that $\sum_{j \neq i} \lambda_{ij} = \lambda_i$. The parameters λ_i and λ_{ij} are also called transition or hazard rates, which becomes clear when considering the definition of the hazard. The hazard rate is the probability of instantaneously leaving state i at t , given that state i has been occupied till t , see Lancaster (1990). Therefore, the hazard rate for leaving state i to any state is the probability density function of the duration $f(t)$ divided by the survival function in this state i , $1 - F(t)$:

$$\begin{aligned} h(t) &= \frac{f(t)}{1 - F(t)} \\ &= \frac{\lambda_i e^{-\lambda_i t}}{e^{-\lambda_i t}} = \lambda_i. \end{aligned}$$

Equivalently, the hazard rate λ_{ij} for leaving state i and going to state j can be determined. The states of a Markov process and the corresponding transition rates can be visualized in rate diagrams. Figure 4.1 shows the rate diagram for a two-state Markov process. Let the states be state '0' and state '1' and the transition rates λ_{01} and λ_{10} be given by μ and λ^3 , respectively. Clearly, in a process with two states, the rate of leaving i and going to

³The variable λ_i with a subscript denotes general arrival rates, while the variable λ without any subscripts is often used for separation rates in job search models. This notation is kept throughout this chapter.

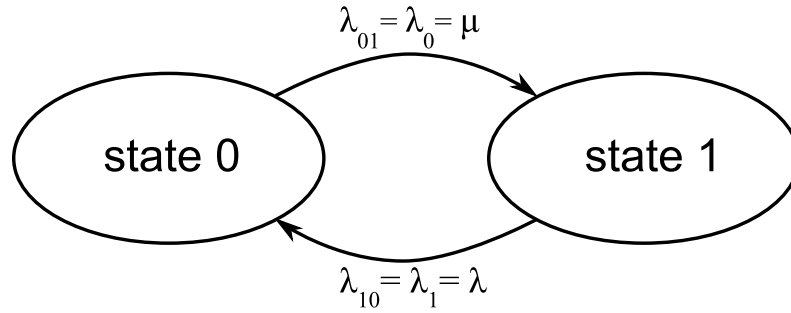


Figure 4.1: Rate diagram for a CTMC with two states (0 and 1). The states are represented by the ovals. The transition rates are given at the arrows that symbolize the transition.

j , λ_{ij} , is identical to the rate of leaving i , $\lambda_i = \sum_{j \neq i} \lambda_{ij}$. Therefore, the rates are simply given by $\lambda_{01} = \lambda_0 = \mu$ and $\lambda_{10} = \lambda_1 = \lambda$.

The transition probability matrix $P = [p_{ij}(t)]$ contains the probabilities that the system which is initially in state i will be in state j at t , $P\{X(t) = j | X(0) = i\}$. In order to compute these transition probabilities, the Chapman-Kolmogorov equations can be used, for details see Ross (1996), for instance. In contrast to discrete-time Markov chains (DTMCs), where the limiting behavior depends on specific properties of the DTMC, the limit of a CTMC transition probability matrix always exists. The limits are given by

$$\lim_{t \rightarrow \infty} p_{jj}(t) = \frac{1}{\lambda_j \eta_{jj}}$$

and

$$\lim_{t \rightarrow \infty} p_{ij}(t) = \frac{f_{ij}}{\lambda_j \eta_{jj}},$$

where f_{ij} is the probability that the spell of state i is less than infinity and a transition occurs to j , $f_{ij} = P\{T_j < \infty | X(0) = i\}$. T_j is the first time the CTMC enters state j and η_{jj} is the expected reoccurrence time of state j , given that the initial state is j , $\eta_{jj} = E[T_j | X(0) = j]$. A proof is provided in Kulkarni (1995).

The interpretation of the limit of $p_{jj}(t)$ is as follows: $1/\lambda_j$ is the expected duration in state j and once the process leaves state j , η_{jj} is the expected time until re-entering state j .

For the limiting probability of ending in j when starting in i , one needs to know how likely a transition from i to j in a period less than infinity is, which is given by $f_{ij} = P\{T_j < \infty | X(0) = i\}$. Once the system enters state j , only the limiting probability for ending in state j upon beginning in state j is needed, which we just determined as $p_{jj}(t) = 1/(\lambda_j \eta_{jj})$. The joint probability is then the product of both probabilities, therefore f_{ij} is multiplied by $1/(\lambda_j \eta_{jj})$.

The limiting probabilities are illustrated by returning to the example from figure 4.1. The rate λ_1 is given by $\lambda_1 = \lambda$ and the rate λ_0 by $\lambda_0 = \mu$. The expected reoccurrence time η_{jj} is given by the sum of the expected duration in both states, $\eta_{jj} = \frac{1}{\lambda_j} + \frac{1}{\lambda_i}$. So, the expected duration in state j , $1/\lambda_j$, and the expected duration in state i , $1/\lambda_i$, after having left state j are added up. Having all this in mind, $\lim_{t \rightarrow \infty} p_{jj}(t) = \frac{1}{\lambda_j \eta_{jj}}$ becomes⁴

$$\lim_{t \rightarrow \infty} p_{00}(t) = \frac{\lambda}{\mu + \lambda} \quad \text{and} \quad \lim_{t \rightarrow \infty} p_{11}(t) = \frac{\mu}{\mu + \lambda}. \quad (4.1)$$

In standard labor market models with the two states *employment* and *unemployment*, this limiting distribution is equal to the equilibrium unemployment rate and employment rate, respectively, which can be shown by using a law of large numbers.

CTMCs whose expected returning time for a state is less than infinity are called *ergodic* and they have an interesting property. Namely, the limiting distribution of the states does not depend on the initial distribution of states, $p_j = \lim_{t \rightarrow \infty} P\{X(t) = j | X(0) = i\}$. In this case, the limiting distribution can be computed by using the so-called balance equations,

$$\sum_{j \in S} p_i \lambda_{ij} = \sum_{j \in S} p_j \lambda_{ji},$$

combined with the condition that all probabilities must sum up to 1, $\sum_{j \in S} p_j = 1$. The idea behind the balance equation is quite simple: in the limit, flows out of state i must equal flows into state i . This property also leads to the well-known expression for the equilibrium unemployment rate in standard matching models with constant arrival rates.

4.2.2 Semi-Markov processes

Also for Semi-Markov processes (SMPs) it holds that only the current state is relevant for the transition rates - and in this sense, there is still memorylessness. However, the transition rates to other states may change over the duration of a state and therefore, the inter-arrival times between subsequent states are no longer exponentially distributed. Thus, the extensions compared to CTMCs are an arbitrary duration distribution and non-stationary transition rates.

A natural way to approach SMPs is through renewal theory, where inter-arrival times between events do not need to be exponentially distributed. For this purpose, it is helpful to define a Markov renewal sequence as a sequence of a bivariate random variable first.

⁴In a system with two states, the remaining limiting probabilities are computed by $\lim_{t \rightarrow \infty} p_{ij}(t) = 1 - \lim_{t \rightarrow \infty} p_{ii}(t)$. Hence, the limiting transition probability from state 1 to state 0 is $\lim_{t \rightarrow \infty} p_{10}(t) = \frac{\lambda}{\mu + \lambda}$ and the limiting transition probability from state 0 to state 1 is $\lim_{t \rightarrow \infty} p_{01}(t) = \frac{\mu}{\mu + \lambda}$.

The two elements of this bivariate random variable are the observation time S_n of the n th transition and the corresponding n th observation Y_n , $n \geq 0, Y_n \in I = \{0, 1, 2, \dots\}$. The joint probability of observing $Y_{n+1} = j$ in an inter-arrival time of $S_{n+1} - S_n \leq x$, conditioned on the observation history, satisfies the Markov property,

$$P \{Y_{n+1} = j, S_{n+1} - S_n \leq x | Y_n = i, S_n, Y_{n-1}, S_{n-1}, \dots, Y_0, 0\} = P \{Y_{n+1}, S_{n+1} - S_n \leq x | Y_n = i\} \equiv G_{ij}(x). \quad (4.2)$$

Finally, a SMP is a stochastic process that records the state of the Markov renewal process at each point in time, see Pyke (1961a).

More formal, let $\{(Y_n, S_n), n \geq 0\}$ be a Markov renewal sequence. Let $N(t)$ be the state with the last completed state spell before t , $N(t) = \sup \{n \geq 0: S_n \leq t\}$, and let $X(t) = Y_{N(t)}$. Then, the stochastic process $\{X(t), t \geq 0\}$ is denoted as a Semi-Markov process. The matrix $G(x) = [G_{ij}(x)]$ as defined in equation (4.2) is called the *kernel* of the SMP, compare Kulkarni (1995).

Next, we discuss some properties of SMPs, which help to classify them. A SMP is *time-homogeneous* if just the interval until the next transition matters for the probability - not when this interval started, or more specific

$$P \{Y_{n+1} = j, S_{n+1} - S_n \leq x | Y_n = i\} = P \{Y_1 = j, S_1 \leq x | Y_0 = i\}.$$

A SMP is called *regular* if there is only a finite number of transitions possible in a finite time period. The SMP is *irreducible* if each state can be reached from any other state; the states are said to communicate with each other in this case. A state j is called *recurrent* if the process returns to this state j in a spell less than infinity and it is called *transient* otherwise (if it never returns). A state is denoted as *positive recurrent* if it is recurrent and the expected returning time to state i , given the process started in i , is less than infinity. For a SMP, a recurrent state i is called *aperiodic* if it is possible to visit this state anytime. *Periodicity* with period d is given if a state i can only be visited at positive multiple integers of d , $d > 1$, see Ross (1996). Therefore, aperiodicity actually means $d = 1$. The initial distribution vector of states $a = [a_i]$ reports the probability that the state of the system is i at the beginning, $a_i = P \{X(0) = i\}$. Finally, a regular SMP is fully specified by the initial distribution of states a and the kernel $G(x) = [G_{ij}(x)]$.

Example. In standard labor market models with two states, all states in the SMP communicate. Furthermore, the SMP is regular, positive recurrent, irreducible, and finally, aperiodic. It is intuitive why: the state *unemployment* is accessible from the state *employment* and vice versa. Hence, the states communicate and the SMP is irreducible. The SMP is regular because the probability of very short durations is less than one. This

means that finding a job or loosing it normally needs some time. It is positive recurrent because the expected ‘revisiting’ duration for an unemployed or an employed is less than infinity. The SMP is aperiodic because obviously $d = 1$ in this two-state process.

Deriving the conditional distribution of the states in a SMP $\{X(t), t \geq 0\}$ at a fixed $t \geq 0$ requires something like the Chapman-Kolmogorov equations, but for SMPs. In doing so, the renewal argument is used to develop integral equations, which is postponed to the next subsection. The numeric methods described in the remainder of this chapter then deal with the computation of these integral equations.

For positive recurrent, irreducible, and aperiodic SMPs, the limiting probability of being in state j when starting in state i is independent of i ,

$$p_j = \lim_{t \rightarrow \infty} P\{X(t) = j | X(0) = i\} = \frac{\pi_j \eta_j}{\sum_{k=0}^{\infty} \pi_k \mu_k}, \quad (4.3)$$

where π is a solution to $\pi = \pi G(\infty)$ and η_k is the expected duration in state k , $k = 0, 1, 2, \dots$, see Kulkarni (1995); also a proof is provided there.

For a labor market model with the two states 1 (employment) and 0 (unemployment), the kernel is given by $G_{10}(\infty) = 1$ and $G_{01}(\infty) = 1$, hence $\pi = (1, 1)$ satisfies the equation $\pi = \pi G(\infty)$. Therefore, equation (4.3) becomes $p_0 = \frac{\eta_0}{\eta_0 + \eta_1}$. The limiting probability of being unemployed is given by the expected duration of the state unemployment divided by the sum of the expected duration in the two states unemployment and employment. According to Cox (1962), this holds for any distribution.

Consequently, the limiting distribution in a two-state labor market model, with duration-dependent transition rates $\mu(\cdot)$ and $\lambda(\cdot)$, becomes⁵

$$p_0 = \frac{\int_0^{\infty} \exp\left\{-\int_0^x \mu(v) dv\right\} dx}{\int_0^{\infty} \exp\left\{-\int_0^x \lambda(v) dv\right\} dx + \int_0^{\infty} \exp\left\{-\int_0^x \mu(v) dv\right\} dx}, \quad (4.4)$$

$$p_1 = 1 - p_0.$$

Equipped with this intuitive, but also formal classification of Semi-Markov processes, the next subsection describes the derivation of the transition probabilities with the integral equations mentioned above.

4.2.3 Transition probabilities of Semi-Markov processes

Now we turn to the transition probabilities of SMPs. This subsection states the general notation and the mathematical basics used throughout this chapter when computing the conditional transition probabilities of a SMP. Pyke (1961a) and Pyke (1961b) are the

⁵See appendix chapter B.1 for a derivation.

seminal articles mentioned in nearly every work about Semi-Markov processes. A very accessible presentation embedded in a general introduction to stochastic processes can be found in Kulkarni (1995).

However, before deriving the equation for the distribution of states, some more definitions and clarifications are needed. Let Y_n denote the state of a system after the n th transition and let this state be i . Let the point in time of the n th transition be denoted by S_n .

The conditional probability of going from state i to state j in a time interval of x or shorter is given by

$$Q_{ij}(x) \equiv P\{Y_{n+1} = j, S_{n+1} - S_n \leq x | Y_n = i\}.$$

Besides the fact that it might not be 1 for $x \rightarrow \infty$, $Q_{ij}(x)$ features all properties of a distribution function, compare Kulkarni (1995). Specifically, $Q_{ij}(x)$ is non-decreasing in x , $\frac{dQ_{ij}(x)}{dx} \geq 0$.

Example. A worker jumps between the two labor market states with the arrival rates being either constant or duration-dependent. As already mentioned earlier, the process is a CTMC in the first case and a SMP in the latter. Such a process is also called alternating renewal process because it alternates between these two states. The probabilities that a jump from i to j occurs in a time period shorter or equal to x is given for these alternative cases by

$$Q_{10}(x) = \begin{cases} 1 - e^{-\lambda x} \\ 1 - e^{-\int_0^x \lambda(y) dy} \end{cases} \text{ for } \begin{cases} \text{constant } \lambda \\ \text{duration-dependent } \lambda(y) \end{cases}, \quad (4.5)$$

$$Q_{01}(x) = \begin{cases} 1 - e^{-\mu x} \\ 1 - e^{-\int_0^x \mu(y) dy} \end{cases} \text{ for } \begin{cases} \text{constant } \mu \\ \text{duration-dependent } \mu(y) \end{cases},$$

assuming that the starting point of the time interval is 0 and the endpoint is x . Due to the homogeneity of the SMP, the probabilities and distributions only depend on the interval length x and not on where the interval is situated on the time axis.⁶ The probabilities of remaining in a given state, the duration distribution, for a certain amount of time x are given in the duration-dependent case by

$$Q_{11}(x) = e^{-\int_0^x \lambda(y) dy}, \quad Q_{00}(x) = e^{-\int_0^x \mu(y) dy}. \quad (4.6)$$

The probability that *any* transition takes place in the spell x is given by summing up the leaving probabilities for each possible state j , $Q_i(x) = \sum_{j \neq i} Q_{ij}(x)$, not taking into

⁶So, it holds that $Q_{ik}(x) = Q_{ik}(\tau|t)$ where $\tau = t + x$.

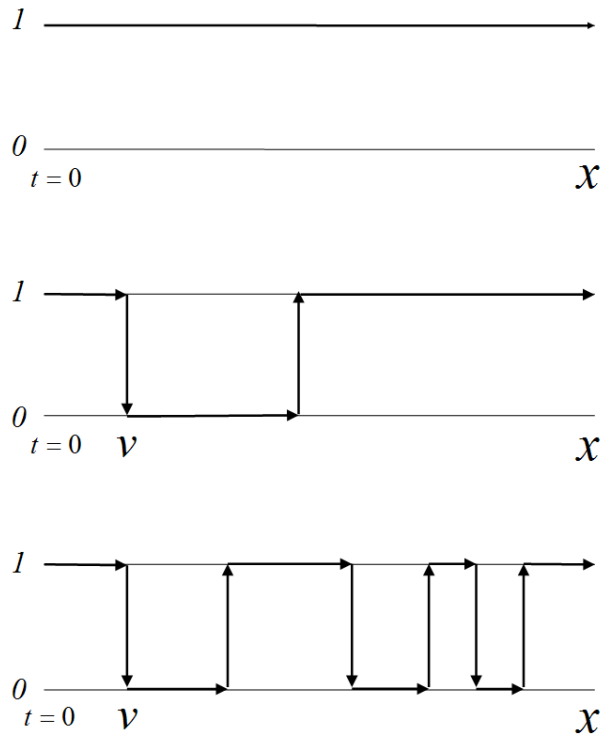


Figure 4.2: Three possible ways of starting in state 1 at $t = 0$ and ending up in state 1 a time period x later.

account transitions from i to i . In a process with two states only, this becomes

$$Q_1(x) = Q_{10}(x), \quad Q_0(x) = Q_{01}(x). \tag{4.7}$$

Having done this preparation, we can now compute the probability of being in state j at x , conditioned on starting from state i today. There is a ‘black box’ on the way from i to j : we know that the system is in state i today and in state j a period x later, but neither do we know when this transition occurs nor whether it occurs directly or via other states. Consequently, all alternative ways of starting in i at $t = 0$ and ending up in j at x have to be taken into account. Figure 4.2 illustrates some possibilities for a continuous-time SMP with two states to start in state i and to end up in state i a time period x later.

Translating all potential transitions that could occur in that ‘black box’ for a multi-state process into mathematics gives the following expression:

$$\begin{aligned} p_{ij}(x) &= \delta_{ij} [1 - Q_i(x)] + \sum_{k \neq i} \int_0^x Q_{ik}(x - v) dp_{kj}(v) \\ &= \delta_{ij} [1 - Q_i(x)] + \sum_{k \neq i} \int_0^x dQ_{ik}(v) p_{kj}(x - v). \end{aligned} \tag{4.8}$$

Integral equations like equation (4.8) are Volterra equations of the first and second kind, see Polyanin and Manzhirov (1998), for example. Equation system (4.8) gives the prob-

ability that the process starting in i will be in j by x , see e.g. Kulkarni (1995) for a proof. The integral $\int_0^x Q_{ik}(x-v) dp_{kj}(v)$ is called the convolution of $Q_{ik}(\cdot)$ and $p_{kj}(\cdot)$, which is denoted by $Q_{ik} * p_{kj}(x)$. In the transition to the second line of equation (4.8), the commutativity of the convolution is used, $Q_{ik} * p_{kj}(x) = p_{kj} * Q_{ik}(x)$.

The interpretation of equation (4.8) is as follows: the first part of the right-hand side is the probability that the system, being in state i , never leaves state i until the end of the period x . In this case, $i = j$ and $\delta_{ij} = 1$, so $1 - Q_i(x)$ is the survival probability in state i . This case corresponds to the upper subfigure of figure 4.2. If $j \neq i$, then $\delta_{ij} = 0$.

The second part of the right-hand side of equation (4.8) collects all cases in which the transition from i to j occurs via another state $k \neq i$, applying the renewal argument. First, the probability that the process stays in state i for a period of length v and then passes to state k is considered, captured by $Q_{ik}(v)$. Passing to this new state k can be interpreted as a renewal of the process because the expected behavior of the process from then on is the same as whenever the process enters k . Hence, the probability that the process which is in state k at v will be in state j at x has to be taken into account, captured by $p_{kj}(x-v)$. As the transition from i to k could occur anytime between 0 and x , all possible transition times have to be covered by the integration over v . The cases, in which the transition occurred via other states is illustrated for $i = j$ in the two lower subfigures of figure 4.2.

Equation (4.8) can be rewritten, provided that $Q_{ik}(v)$ is once differentiable, as

$$p_{ij}(x) = \delta_{ij} [1 - Q_i(x)] + \sum_{k \neq i} \int_0^x p_{kj}(x-v) \frac{dQ_{ik}(v)}{dv} dv. \quad (4.9)$$

This equation is the origin for the following analysis based on labor market applications. As the Q_{ik} are expected to be known and differentiable in economic applications, the starting point here will be equation (4.9) rather than equation (4.8) without loss of generality.

4.3 Semi-Markov processes with two states

As stated earlier, this chapter picks the example of our labor market model from chapter 3. There are the two labor market states *unemployment* (0) and *employment* (1) and thus, four transition probabilities for the future: an unemployed/employed person can either be unemployed or employed at some future point after a spell x . Let these probabilities

be denoted by $p_{ij}(x)$. Writing them out in terms of the general equation (4.9) gives

$$p_{00}(x) = 1 - Q_0(x) + \int_0^x p_{10}(x-v) \frac{dQ_{01}(v)}{dv} dv, \quad (4.10a)$$

$$p_{10}(x) = \int_0^x p_{00}(x-v) \frac{dQ_{10}(v)}{dv} dv, \quad (4.10b)$$

$$p_{11}(x) = 1 - Q_1(x) + \int_0^x p_{01}(x-v) \frac{dQ_{10}(v)}{dv} dv, \quad (4.10c)$$

$$p_{01}(x) = \int_0^x p_{11}(x-v) \frac{dQ_{01}(v)}{dv} dv. \quad (4.10d)$$

In the remainder of this section, we first discuss a special case of a SMP, namely one with constant arrival rates for both states. Since the SMP is also a CTMC in this case, the results for the probabilities from the SMP theory can be compared to the known results from CTMCs. This model is then extended in the way of chapter 3, where there are constant arrival rates in the state of employment and duration-dependent arrival rates in the state of unemployment.

4.3.1 Computing transition probabilities for constant arrival rates

Assuming a continuous-time setup, where the transition rates from one state to the other are constant, the well-known expressions for the transition probabilities of being either unemployed or employed depending on the current state can be derived. Let $p_{ij}(x)$ be the probability that a system being in state i will be in state j at a spell x later. Starting from the Chapman-Kolmogorov backward equations, a system of differential equations can be derived. The solution to this system gives the transition probabilities:

$$\begin{aligned} p_{00}(x) &= \frac{\lambda}{\mu + \lambda} + \frac{\mu}{\mu + \lambda} e^{-[\mu + \lambda]x}, \\ p_{10}(x) &= \frac{\lambda}{\mu + \lambda} - \frac{\lambda}{\mu + \lambda} e^{-[\mu + \lambda]x}, \\ p_{11}(x) &= \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-[\mu + \lambda]x}, \\ p_{01}(x) &= \frac{\mu}{\mu + \lambda} - \frac{\mu}{\mu + \lambda} e^{-[\mu + \lambda]x}, \end{aligned} \quad (4.11)$$

see Ross (1996) or Kulkarni (1995), for example. In the limit as $x \rightarrow \infty$, the second terms of the probability equations approach zero. Hence, the limiting distribution does not depend on the initial distribution of states, so $p_1 = p_{01} = p_{11} = \frac{\mu}{\mu + \lambda}$ and $p_0 = p_{10} = p_{00} = \frac{\lambda}{\mu + \lambda}$. Since CTMCs are special cases of SMPs, we will now show that the transition probabilities (4.11) are special cases of the more general equations (4.10) for transition probabilities of SMPs.

First, the derivative of $Q_{01}(v)$ is prepared,

$$\frac{dQ_{01}(v)}{dv} = \mu e^{-\mu v}. \quad (4.12)$$

Inserting this into the transition probability equation (4.10) for SMPs yields

$$p_{01}(x) = \mu \int_0^x p_{11}(x-v) e^{-\mu v} dv.$$

From subsection 4.2.3, it is known that the convolution of p_{11} and Q_{01} is commutative, that means the convoluted functions and the arguments can be interchanged. Applying this gives

$$p_{01}(x) = \mu \int_0^x p_{11}(v) e^{-\mu[x-v]} dv. \quad (4.13)$$

Next, the time derivative of equation (4.13) with respect to x is computed using the Leibniz rule for integral functions, compare Wälde (2008),

$$\dot{p}_{01}(x) = \mu \left[p_{11}(x) - \mu \int_0^x p_{11}(v) e^{-\mu[x-v]} dv \right].$$

Finally, replacing the convolution by $p_{01}(x)$ from equation (4.13) yields

$$\dot{p}_{01}(x) = \mu [p_{11}(x) - p_{01}(x)] = \mu p_{11}(x) - \mu p_{01}(x). \quad (4.14)$$

This is the expected differential equation which can be derived as well from the Chapman-Kolmogorov backward equations. For the remaining three states, the corresponding differential equations can be determined in the same manner. Solving these differential equations gives the probabilities (4.11). Hence, interpreting the CTMC as a SMP with constant arrival rates leads to the same transition probabilities.

4.3.2 Computing transition probabilities for general arrival rates

From this subsection on, we use duration-dependent job arrival rates as given in our labor market model.⁷

Having non-stationary job arrival rates, the derivatives according to equation (4.5) are given by

$$\begin{aligned} \frac{dQ_{01}(v)}{dv} &= e^{-\int_0^v \mu(y) dy} \frac{d}{dv} \int_0^v \mu(y) dy = e^{-\int_0^v \mu(y) dy} \mu(v), \\ \frac{dQ_{10}(v)}{dv} &= e^{-\int_0^v \lambda dy} \frac{d}{dv} \int_0^v \lambda dy = e^{-\int_0^v \lambda dy} \lambda. \end{aligned}$$

⁷Extending the model additionally by a non-stationary job-to-unemployment transition rate is also possible and would not change the general proceeding.

Together with equation (4.7) and the derivatives, the transition probabilities from equation (4.10) become

$$p_{00}(x) = e^{-\int_0^x \mu(y)dy} + \int_0^x p_{10}(x-v) e^{-\int_0^v \mu(y)dy} \mu(v) dv, \quad (4.15a)$$

$$p_{10}(x) = \int_0^x p_{00}(x-v) e^{-\int_0^v \lambda dy} \lambda dv, \quad (4.15b)$$

$$p_{11}(x) = e^{-\int_0^x \lambda dy} + \int_0^x p_{01}(x-v) e^{-\int_0^v \lambda dy} \lambda dv, \quad (4.15c)$$

$$p_{01}(x) = \int_0^x p_{11}(x-v) e^{-\int_0^v \mu(y)dy} \mu(v) dv. \quad (4.15d)$$

These four equations are central for deriving the transition probabilities of SMPs. Obviously, equations (4.15a) and (4.15b) as well as equations (4.15c) and (4.15d) are interdependent. The equation for $p_{01}(x)$ depends on $p_{11}(x-v)$ and the equation for $p_{11}(x)$, in turn, depends on $p_{01}(x-v)$. The transition probabilities $p_{11}(x)$ and $p_{01}(x)$ can be determined first and then the transition probabilities for the complementary events, $p_{10}(x)$ and $p_{00}(x)$, can be obtained immediately.⁸

One way to solve the probabilities analytically is the Laplace-Stieltjes transform, compare Kulkarni (1995). The striking fact with respect to equations (4.15a)-(4.15d) is that an analytical solution is not feasible in cases like our model because the job arrival rate has no analytical solution. Therefore, the remainder of this chapter deals with the numerical solution of the interdependent integral equations (4.15a)-(4.15d).

4.4 Numerical solution of the transition probabilities

In order to solve the transition probabilities at some point in time x numerically, at least two of the integrals in equations (4.15a)-(4.15d) have to be transformed into discrete integration problems. To this end, the interval of length x is divided into z discretization steps first. The distance between subsequent steps, the step width, is $h = x/z$ and the end point of the interval x is represented by zh . Thus, equations (4.15a) and (4.15b) become

$$p_{00}(zh) = \underbrace{e^{-\int_0^{zh} \mu(ih)d(ih)}}_{\equiv Q_{00}(zh)} + \int_0^{zh} \underbrace{e^{-\int_0^{ih} \mu(kh)d(kh)} \mu(ih) p_{10}(zh-ih) d(ih)}_{\equiv g(ih)} \quad (4.16)$$

⁸After having solved for two probabilities, the remaining two are the probabilities of the complementary events and can be solved by subtracting the respective probability from 1. Thus, an unemployed today can be unemployed at x , for which the probability $p_{00}(x)$ can be computed. The complementary event for the unemployed today would be occupying a job at x . As there are only the two possible states *unemployment* and *employment*, the probability for the latter is given by $p_{01}(x) = 1 - p_{00}(x)$.

and

$$p_{10}(zh) = \int_0^{zh} \underbrace{e^{-\int_0^{ih} \lambda d(kh)} \lambda p_{00}(zh - ih)}_{\equiv f(ih)} d(ih). \quad (4.17)$$

In general and independently from the numerical integration method, the approximation of the integral gets more precise the more steps are used. The drawback of having a better precision with more steps is the prolonged computing time for the integrals.

Furthermore, a numerical integration method has to be chosen in order to approximate the area beneath the function. In this section, two numerical integration methods are presented and compared in the context of the Semi-Markov transition probability problem. In subsection 4.4.1, the very basic rectangle integration method is introduced, while subsection 4.4.2 deals with the trapeze integration. These rules can be subsumed under the Newton-Cotes quadrature formulas. A general presentation can be found in Judd (1998) as well as in Schatzman and Taylor (2002).

4.4.1 Rectangle approximation

This subsection describes the numerical solution of equations (4.15a) and (4.15b) by using the rectangle approximation of integrals. As there exist several variations of the rectangle approximation, the first step is to present the general idea of computing an integral via rectangles as the basis of all variations. Then, one of the variations, the algorithm using left rectangle integration, is discussed in detail.

The general setup

As the name *rectangle approximation* already suggests, it consists of adding up the areas of rectangles beneath a function, say $\gamma(\cdot)$. The width of every rectangle is the step-width h and the height is the function value $\gamma(ih)$ at the current position of the index i . Hence, the rectangle area is computed by $h \cdot \gamma(ih)$.

Possible variations of the rectangle method refer to the function value $\gamma(\cdot)$, which determines the area of the first rectangle. In literature, three methods are distinguished, see Schatzman and Taylor (2002). Figure 4.3 illustrates the different methods.

As for the right rectangle method, the first rectangle is the one with height $\gamma(0)$, hence the area to the right of 0 is computed. Consequently, the rectangles from $i = 0, \dots, z - 1$ are added. The left rectangle method begins with the rectangle of height $\gamma(1h)$ which means that the area to the left of $1h$ is considered. In this case, the rectangles from $i = 1, \dots, z$ are added. For the midpoint rule, the first rectangle taken is the one with height $\gamma(0.5h)$, so the function value in the middle of each interval is used. From figure 4.3

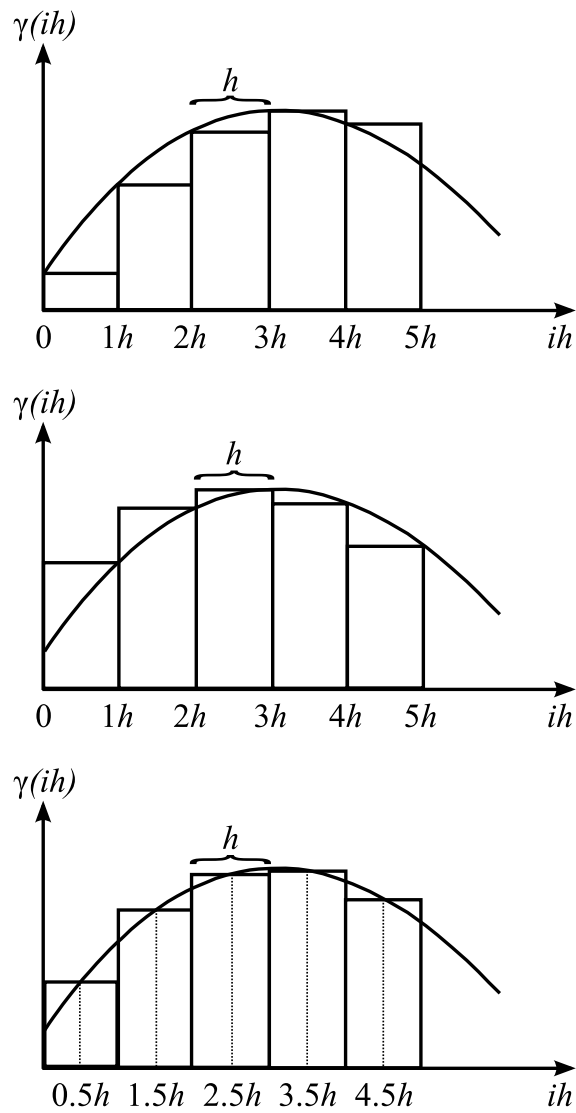


Figure 4.3: The three subfigures show the approximation of the area beneath the function via rectangles and the function values used for the rectangles. The upper figure presents the right rectangle method, the middle figure the left rectangle method, and the figure below the midpoint rule.

becomes clear why the rectangle method is a so-called *open rule*: none of the variations uses both interval endpoints, compare Judd (1998).

In the following, the left rectangle rule is discussed in detail within the Semi-Markov framework. The other two rules can be derived similarly.

Algorithm *Left Rectangles*

As mentioned above, the first function value needed for the left rectangle algorithm is the one at $i = 1$. Hence, by using the left rectangle approximation and z discretization steps the integral becomes

$$\begin{aligned} \int_0^x \gamma(v) dv &= h\gamma(1) + h\gamma(h) + h\gamma(2h) + \dots + h\gamma(zh) \\ &= h \sum_{i=1}^z \gamma(ih), \end{aligned} \quad (4.18)$$

where $zh = x$ is the interval endpoint. Using the numerical integration equation (4.18), the transition probabilities for Semi-Markov processes (4.15a) and (4.15b) become

$$\begin{aligned} p_{00}(zh) &= \underbrace{e^{-h \sum_{i=1}^z \mu(ih)}}_{\equiv Q_{00}(zh)} + h \sum_{i=1}^z \underbrace{e^{-h \sum_{k=1}^i \mu(kh)} \mu(ih)}_{\equiv g(ih)} p_{10}([z-i]h) \\ &= Q_{00}(zh) + h \sum_{i=1}^z g(ih) \end{aligned} \quad (4.19)$$

and

$$\begin{aligned} p_{10}(zh) &= h \sum_{i=1}^z \underbrace{e^{-h \sum_{k=1}^i \lambda} \lambda p_{00}([z-i]h)}_{\equiv f(ih)} \\ &= h \sum_{i=1}^z f(ih). \end{aligned} \quad (4.20)$$

Starting from the given initial values $p_{10}(0) = 0$ and $p_{00}(0) = 1$, the probabilities for any z can be computed successively, which is shown in the following algorithm.

- Initialization for $z = 0$

The initial values $p_{00}(0)$ and $p_{10}(0)$ can be deduced intuitively. If a worker is unemployed today and no time goes by, there is no chance for him to become employed. Consequently, the probability of staying unemployed is equal to one, $p_{00}(0) = 1$. Equivalently, for an employed worker there is no risk of unemployment if no time goes by, which means $p_{10}(0) = 0$. Therefore, the initialization for

the transition probabilities is given by

$$p_{00}(0) = 1,$$

$$p_{10}(0) = 0.$$

- $z = 1$

Starting points are, like at the beginning of every step, the transition probability equations (4.19) and (4.20). Setting $z = 1$ yields

$$\begin{aligned} p_{00}(h) &= Q_{00}(h) + hg(h) \\ &= e^{-\mu(h)h} + he^{-\mu(h)h}\mu(h)p_{10}(0) \end{aligned}$$

and

$$\begin{aligned} p_{10}(h) &= hf(h) \\ &= h\lambda e^{-\lambda h}p_{00}(0). \end{aligned}$$

The computation of the unknowns $p_{10}(h)$ and $p_{00}(h)$, given $p_{10}(0)$ and $p_{00}(0)$, is now straightforward.

- $z = 2$ and subsequent steps

Evaluating equations (4.19) and (4.20) for $z = 2$ and using the definitions of $Q_{11}(ih)$, $Q_{00}(ih)$, $g(ih)$, and $f(ih)$ gives

$$\begin{aligned} p_{00}(2h) &= Q_{00}(2h) + h \sum_{i=1}^2 g(ih) \\ &= e^{-h \sum_{i=1}^2 \mu(ih)} + h \sum_{i=1}^2 e^{-h \sum_{k=1}^i \mu(kh)} \mu(ih) p_{10}([2-i]h) \end{aligned}$$

and

$$\begin{aligned} p_{10}(2h) &= h \sum_{i=1}^2 f(ih) \\ &= h\lambda \sum_{i=1}^2 e^{-h \sum_{k=1}^i \lambda} p_{00}([2-i]h). \end{aligned}$$

The further procedure for $z > 2$ is similar. In this way, the transition probabilities within an interval can be computed step by step until the probabilities for the desired point in time are reached.

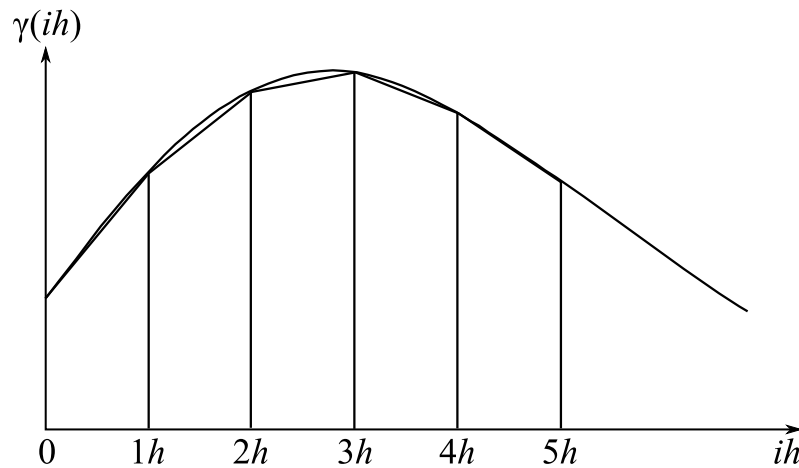


Figure 4.4: When using the trapeze rule, the area beneath the function is determined by adding up the area of the trapezes with step width h as well as side lengths $\gamma(ih)$ and $\gamma([i-1]h)$.

4.4.2 Trapeze approximation

The second approximation rule discussed in this chapter is the trapeze rule. The integral is determined via the sum of trapeze areas beneath the function. Intuitively, the trapeze rule can be derived from the rectangle approximation by adding or subtracting triangles resulting from chords through the end points of the intervals.

The general setup

When using the trapeze approach, there is no longer a differentiation between a *right* or *left* method. As the rule uses both endpoints of the interval, it is called a *closed rule* according to Judd (1998). Figure 4.4 illustrates the trapeze approximation rule.

The trapezes taken for the approximation of the area are constructed by using the width h and the lengths $\gamma([i-1]h)$ and $\gamma(ih)$. As for the rectangle rule, all trapeze areas in the interval are added up. Hence, an integral of a function $\gamma(\cdot)$ becomes

$$\int_0^x \gamma(v) dv = \frac{1}{2}h[\gamma(0) + \gamma(h)] + \frac{1}{2}h[\gamma(h) + \gamma(2h)] + \dots + \frac{1}{2}[\gamma([z-1]h) + \gamma(zh)].$$

Recollection results in

$$\begin{aligned} \int_0^x \gamma(v) dv &= h \left[\frac{1}{2}\gamma(0) + \gamma(h) + \gamma(2h) + \dots + \gamma([z-1]h) + \frac{1}{2}\gamma(zh) \right] \\ &= \frac{1}{2}h\gamma(0) + h \sum_{i=1}^{z-1} \gamma(ih) + \frac{1}{2}h\gamma(zh). \end{aligned} \quad (4.21)$$

Also for this method, the endpoint of the interval $x = zh$ is reached after z discretization steps and $v = ih$ is the time point of the current index position i .

In the following, the application of equation (4.21) for the computation of the transition probabilities (4.16) and (4.17) is described.

Algorithm

The general numerical integration equation (4.21) for the trapeze approximation can be used to substitute the integrals in equations (4.16) and (4.17). The former becomes

$$p_{00}(zh) = Q_{00}(zh) + \frac{1}{2}hg(0) + h \sum_{i=1}^{z-1} g(ih) + \frac{1}{2}hg(zh).$$

In addition to $p_{00}(zh)$, this equation contains a second unknown in $g(0) = \mu(0)p_{10}(zh)$, namely $p_{10}(zh)$. Isolating the two unknowns gives

$$p_{00}(zh) - \frac{1}{2}h \underbrace{\mu(0)p_{10}(zh)}_{g(0)} = Q_{00}(zh) + h \sum_{i=1}^{z-1} g(ih) + \frac{1}{2}hg(zh). \quad (4.22)$$

The full equation without the short-cut functions is written out in the appendix chapter B.2. The second equation (4.17) needs a discrete counterpart for the trapeze case, too. The procedure is equivalent, so after replacing the integrals according to equation (4.21), the probability for the transition from employment to unemployment reads

$$p_{10}(zh) = \frac{1}{2}hf(0) + h \sum_{i=1}^{z-1} f(ih) + \frac{1}{2}hf(zh).$$

This equation also has two unknowns, $p_{10}(zh)$ and $p_{00}(zh)$, because the left expression on the right-hand side, $f(0) = \lambda p_{00}(zh)$, contains the unknown $p_{00}(zh)$. Again, the final step is the isolation of both unknowns,

$$p_{10}(zh) - \frac{1}{2}h \underbrace{\lambda p_{00}(zh)}_{f(0)} = h \sum_{i=1}^{z-1} f(ih) + \frac{1}{2}hf(zh). \quad (4.23)$$

For the full version of this equation, see B.2 of the appendix. Finally, the two unknowns $p_{10}(zh)$ and $p_{00}(zh)$ from equations (4.22) and (4.23) can be determined since the $p_{10}(zh - ih)$ and $p_{00}(zh - ih)$, $i = 1, \dots, z$, are given from previous calculations. In other words, by starting from $p_{10}(0) = 0$ and $p_{00}(0) = 1$, all $p(zh)$ can be solved successively. Equations (4.22) and (4.23) are the starting points of all algorithm steps, but the initialization. The algorithm steps for $z = 0$, $z = 1$, and $z = 2$ are presented in the following.

- Initialization for $z = 0$

The initial transition probabilities from unemployment to unemployment and from employment to unemployment are given by

$$p_{00}(0) = 1$$

and

$$p_{10}(0) = 0,$$

respectively, for the same reason as in subsection 4.4.1 for the rectangle integration method.

- $z = 1$

After the initialization, this is the first computation step. The basis of all computation steps are equations (4.22) and (4.23). Setting $z = 1$ in the former and using the definitions of $Q_{00}(\cdot)$ and $g(\cdot)$ from (4.16) yields the transition probability from unemployment to unemployment at h ,

$$p_{00}(h) - \frac{1}{2}h \underbrace{\mu(0) p_{10}(h)}_{g(0)} = Q_{00}(h) + \frac{1}{2}h \underbrace{Q_{00}(h) \mu(h) p_{10}(0)}_{g(h)}. \quad (4.24)$$

The transition probability from employment to unemployment at h is determined in the same manner, using $f(\cdot)$ from equation (4.17). Setting $z = 1$ in equation (4.23) results in

$$p_{10}(h) - \frac{1}{2}h \underbrace{\lambda p_{00}(h)}_{f(0)} = \frac{1}{2}h \underbrace{e^{-\lambda h} \lambda p_{00}(0)}_{f(h)}. \quad (4.25)$$

Equations (4.24) and (4.25) are the first two equations with the first two unknowns $p_{00}(h)$ and $p_{10}(h)$. The solution is now straightforward.

- $z = 2$ and subsequent steps

The next step is to go on with $z = 2$ and to compute $p_{00}(2h)$ as well as $p_{10}(2h)$ given the results from all previous steps. Equations (4.22) and (4.23) become

$$\begin{aligned} p_{00}(2h) - \frac{1}{2}h \underbrace{\mu(0) p_{10}(2h)}_{g(0)} &= Q_{00}(2h) + \underbrace{h Q_{00}(h) \mu(h) p_{10}(h)}_{g(h)} \\ &+ \frac{1}{2}h \underbrace{Q_{00}(2h) \mu(2h) p_{10}(0)}_{g(2h)} \end{aligned}$$

and

$$p_{10}(2h) - \frac{1}{2}h\underbrace{\lambda p_{00}(2h)}_{f(0)} = \underbrace{he^{-\lambda h}\lambda p_{00}(h)}_{f(h)} + \frac{1}{2}h\underbrace{e^{-\lambda 2h}\lambda p_{00}(0)}_{f(2h)},$$

respectively.

The only two unknowns in step 2 are $p_{10}(2h)$ and $p_{00}(2h)$ on the left-hand side because $p_{10}(0)$ and $p_{00}(0)$ are known from the initialization and $p_{10}(h)$ and $p_{00}(h)$ from the first step. So also this equation system can be solved for the probabilities at $x = 2h$.

The proceeding for the subsequent steps with $z = 3, \dots$ equivalently starts from equations (4.22) and (4.23). The mechanism is always the same: the $p_{00}(zh)$ and $p_{10}(zh)$ are calculated using the $p_{00}(zh - ih)$ and $p_{10}(zh - ih)$, $i = 1, \dots, z$, from the previous steps.

After the theoretical description of possible numerical solution methods, the next section shows the computational results for specific numerical examples.

4.5 Numerical results

Having learned two alternatives of determining transition probabilities in the previous section, this section focuses on how both solutions perform when applying them to specific labor market models.⁹

First, the methods of numerical integration discussed in chapter 4.4, the rectangle and the trapeze method, are compared to the analytically computable transition probabilities in the case of constant arrival rates as given by equations (4.11). In general, it is clear that the trapeze method will perform better than the rectangle method when using the same step width and step number. However, an important question is how much better the trapeze method is when employing it for the solution of our labor market model, considering that the trapeze method is more complex and will need more computation time, consequently. Furthermore, the limiting distribution as derived by equation (4.4) will be tested. Thus, the analytical solution serves as a benchmark for the numerical methods in the case of constant transition rates.

Second, the probabilities for duration-dependent arrival rates are computed with both numerical methods. As there is no longer an analytical solution available in cases like our economic model of chapter 3, the two solutions can only be analyzed independently.

⁹The algorithm of the solution procedure is set up in Matlab. The code is available on the enclosed CD.

However, the limiting distribution can be computed for Semi-Markov processes and, in this way, at least the convergence of both numerical solutions can be evaluated.

4.5.1 Constant arrival rates - convergence to the analytical solution

In order to test the convergence of the transition probabilities computed via the numerical algorithms, constant arrival rates are used. In this special case, the SMP is a CTMC, for which the analytical solution of the transition probabilities is known, see equations (4.11) in subsection 4.3.1. The parameters used for this analysis are taken from Shimer (2005). The monthly values are $\mu = 0.45$ for the job arrival rate and $\lambda = 0.034$ for the job separation rate. The interval endpoint is $x = 500$ months. The limiting distribution is then given by $p_1^A = \frac{\mu}{\mu+\lambda} = 0.93$ and $p_0^A = \frac{\lambda}{\mu+\lambda} = 0.07$ according to subsection 4.3.1.

- Comparison of graphs

Figure 4.5 shows the evolution of the transition probabilities for the analytical solution compared to the numerical solution of the *rectangle* method. Each subfigure presents the probabilities for different step numbers. The probability for the transition from initial unemployment to unemployment is 1 for $t = 0$, the probability for the transition from initial employment to unemployment is 0 as $t = 0$.¹⁰ The analytical solution reaches the limiting distribution at about $t = 20$ months and the two analytical curves can no longer be distinguished from then on. The rectangle probabilities do not seem to converge at all for the displayed step numbers. For 250 steps, the numerical solution using the rectangle method clearly underestimates the probabilities for $t \geq 25$, see the upper subfigure. At the endpoint of the figure at $t = 150$, the numerically approximated probabilities are nearly zero. For 2,000 steps, there is still underestimation of the analytical probabilities, but the magnitude decreases and the difference between the two computation methods at $t = 150$ is much smaller than before.

¹⁰See initialization step for $z = 0$ in the previous section for the explanation.

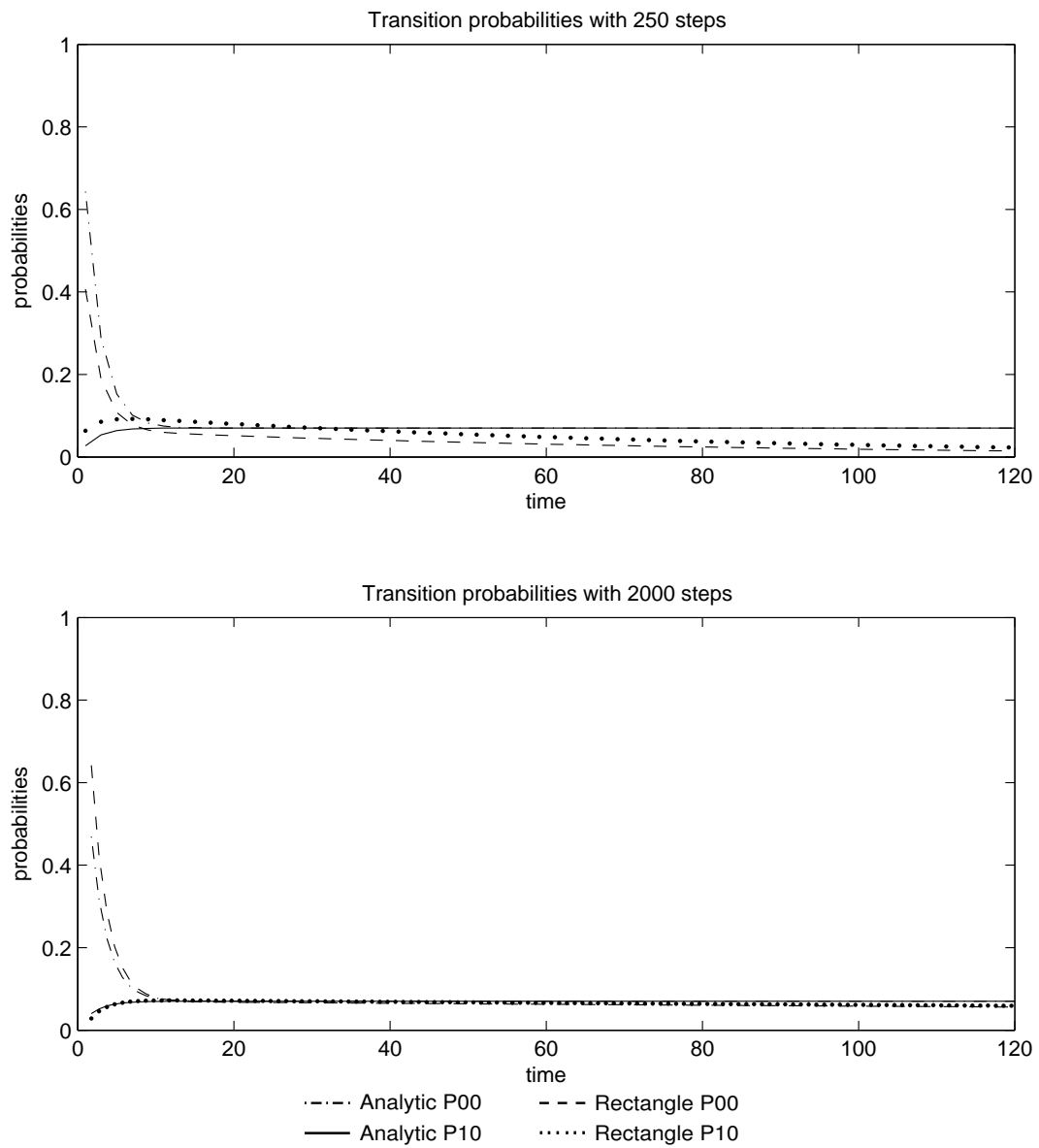


Figure 4.5: Transition probabilities over time for the analytical solution and the rectangle method. The upper figure shows the solution for 250 steps and the figure at the bottom for 2,000 steps.

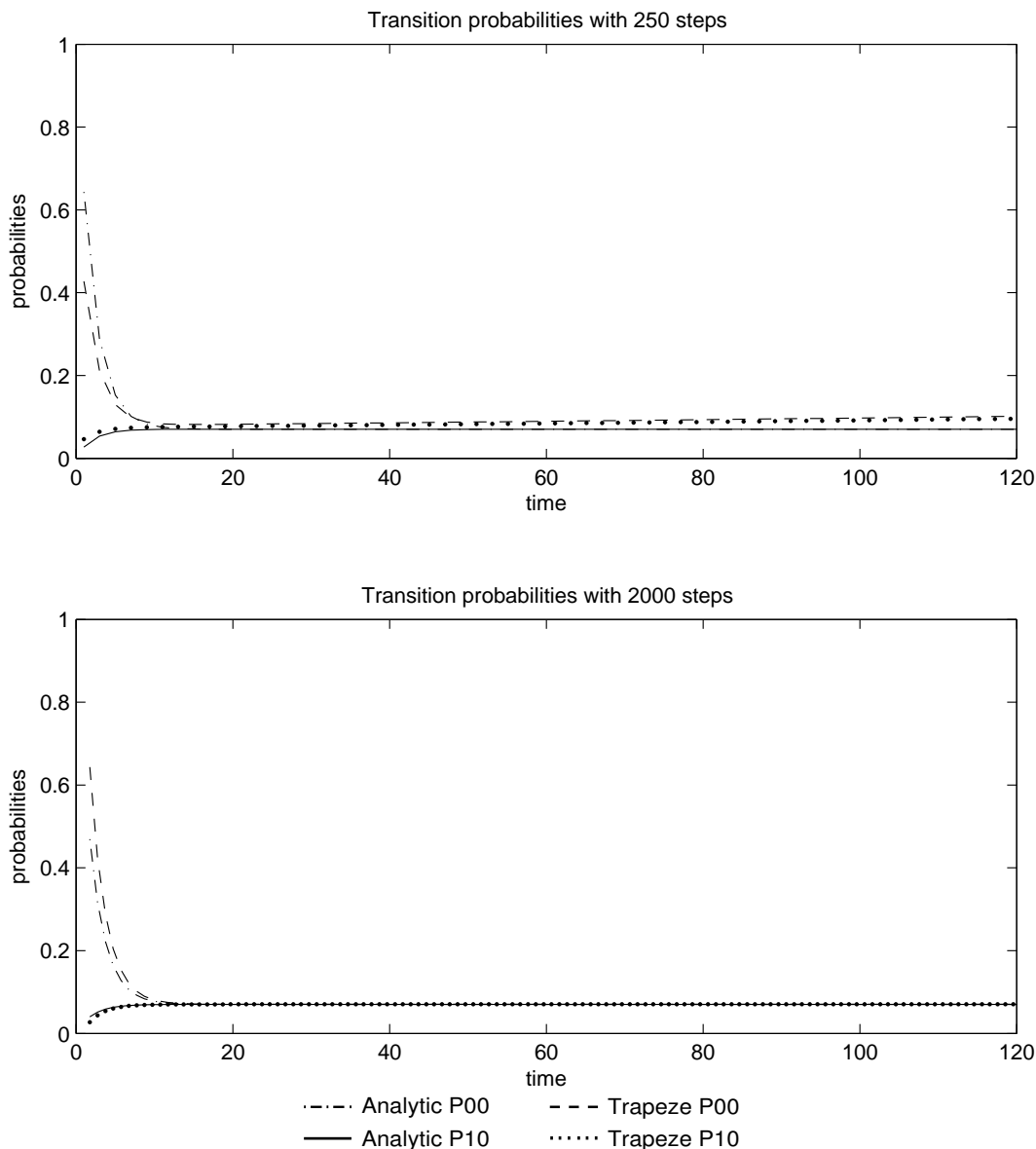


Figure 4.6: Transition probabilities over time for the analytical solution and the trapeze method. The upper figure shows the solution for 250 steps and the figure at the bottom for 2,000 steps.

Figure 4.6 shows the transition probabilities for the analytical solution compared to the numerical solution of the *trapeze* approximation, again for different step numbers. Convergence is much better than for the rectangle solution. Already for 2,000 steps, the trapeze probabilities approach the same limiting value as the analytical solution. As before, the probability for the transition from initial unemployment to unemployment at $t = 0$ is 1, whereas the probability for the transition from initial employment to unemployment at $t = 0$ is 0. The upper subfigure in figure 4.6 shows the curves for 250 steps. After the first 20 months, there is a monotonically increasing overestimation. The trapeze

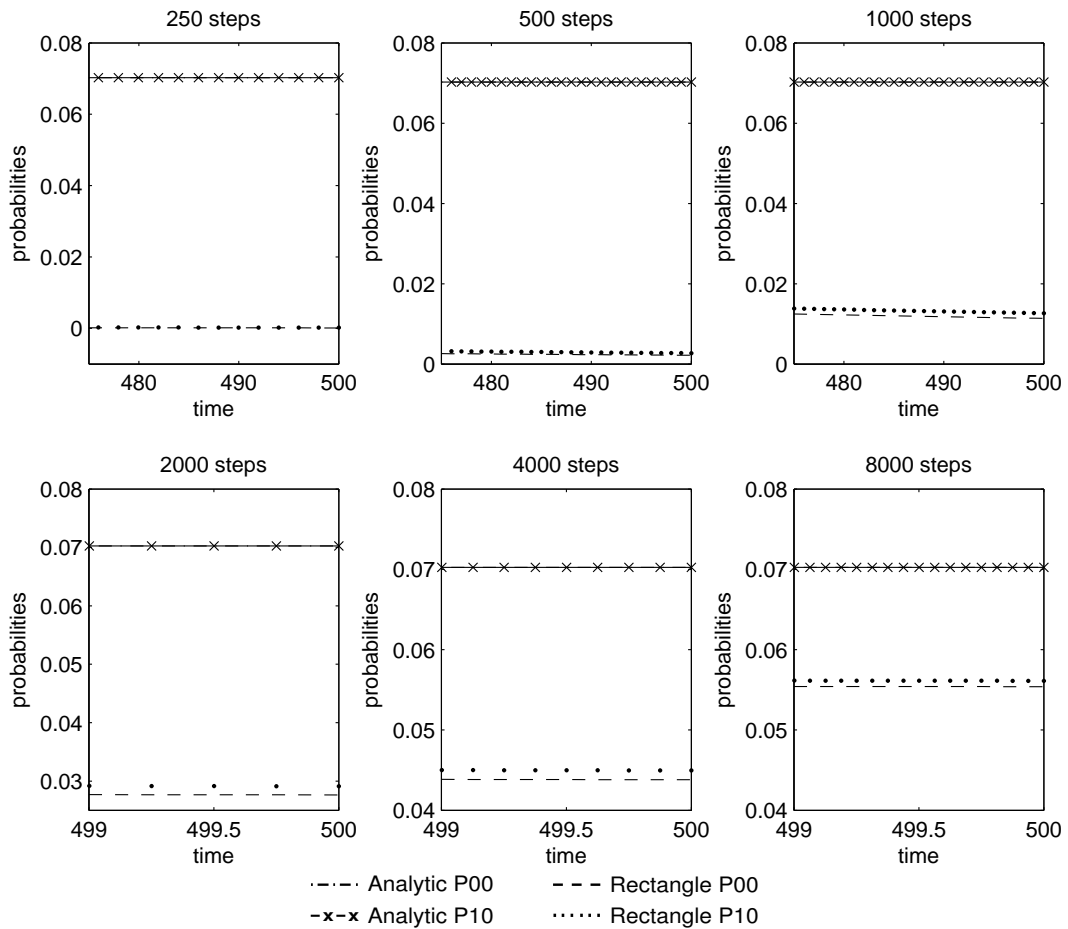


Figure 4.7: Transition probabilities for the analytical solution and the rectangle solution as $t \rightarrow 500$ for different step numbers. The upper figures show the interval $[475, 500]$, the bottom figures show the interval $[499, 500]$.

solution is obviously still much better than the rectangle method described above. The lower subfigure shows the probability evolution for 2,000 steps. The improvement from 250 steps to 2,000 steps is large, especially from $t = 20$ onwards. For this step number, there is nearly no difference between the curves of the analytical solutions and the curves of the numerical trapeze solutions visible. After this overview of the probability evolution, some more detailed figures on the behavior as $t \rightarrow 500$ will be shown.

Figure 4.7 shows the probabilities for the transitions from unemployment to unemployment and from employment to unemployment both for the analytical solution and the *rectangle* approximation zoomed in near the endpoint of the interval. Now, the range of the underestimation of the analytical solution by the rectangle approximation becomes better visible. Clearly, the numerical solution approaches the analytical solution as the step number increases with the errors getting smaller for increasing step numbers.

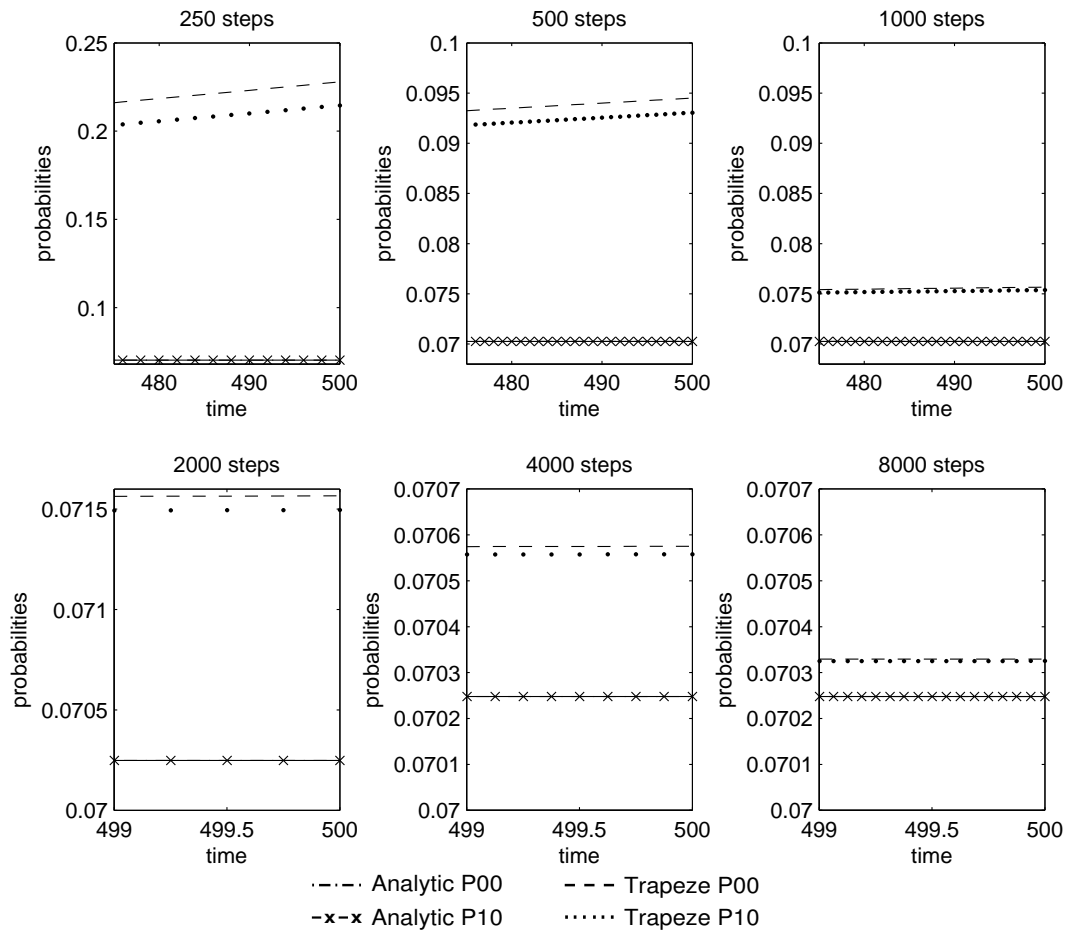


Figure 4.8: Transition probabilities for the analytical solution and the trapeze solution as $t \rightarrow 500$, again for different step numbers. The upper figures show the interval $[475, 500]$, the bottom figures show the interval $[499, 500]$.

Figure 4.8 shows the corresponding probabilities for the *trapeze* approximation compared to the analytical solution. Also these figures verify that, for a bigger step number, the numerical transition probabilities perform better as approximations of the analytical solution. Furthermore, it becomes obvious that the trapeze approximation method overestimates the analytical solution, but, unlike for the rectangle probabilities, already the solutions for 2,000 steps perform quite good. Having an equivalently good approximation in the rectangle case would require 8,000 or more computation steps.

- Comparison by computational results

Table 4.1 and table 4.2 present the computational results for different step numbers and the three methods (analytical, rectangle, trapeze). The solutions and errors of both numerical integration methods are compared to the analytical solution at different points of the interval. While in the former table the results for the transition probabilities from

		250 steps		500 steps		2,000 steps	
		Value	Error	Value	Error	Value	Error
	p_{00}^A	0.070	-	0.070	-	0.070	-
1/5	p_{00}^R	0.019	-0.051	0.035	-0.035	0.058	-0.018
	p_{00}^T	0.097	+0.027	0.076	+0.006	0.071	+0.001
	p_{00}^A	0.070	-	0.070	-	0.070	-
1/2	p_{00}^R	0.003	-0.067	0.012	-0.058	0.044	-0.026
	p_{00}^T	0.134	+0.057	0.083	+0.013	0.071	+0.001
	p_{00}^A	0.070	-	0.070	-	0.070	-
End	p_{00}^R	0.000	-0.07	0.002	-0.068	0.028	-0.042
	p_{00}^T	0.228	+0.158	0.095	+0.025	0.072	+0.002
		4,000 steps		8,000 steps		16,000 steps	
		Value	Error	Value	Error	Value	Error
	p_{00}^A	0.070	-	0.070	-	0.070	-
1/5	p_{00}^R	0.064	-0.006	0.067	-0.003	0.069	-0.001
	p_{00}^T	0.070	-	0.070	-	0.070	-
	p_{00}^A	0.070	-	0.070	-	0.070	-
1/2	p_{00}^R	0.055	-0.015	0.062	-0.008	0.066	-0.004
	p_{00}^T	0.070	-	0.070	-	0.070	-
	p_{00}^A	0.070	-	0.070	-	0.070	-
End	p_{00}^R	0.044	-0.026	0.055	-0.015	0.062	-0.008
	p_{00}^T	0.071	+0.001	0.070	-	0.070	-

Table 4.1: Probabilities for the transition from unemployment to unemployment $p_{00}(\cdot)$ by t_i , where $t_1 = 1/5 \cdot x = 100$, $t_2 = 1/2 \cdot x = 250$, and $t_3 = x = 500$.

unemployment to unemployment, $p_{00}(t)$, are recorded, the latter shows the transition probabilities from employment to unemployment, $p_{10}(t)$.¹¹

The columns present the probabilities for different step numbers, the rows show the probabilities for the three computation methods analytical, rectangle, and trapeze for different points in the interval $[0, 500]$. First, the probabilities at $1/5$ of the interval, $t_1 = 100$, then the probabilities after half of the interval at $t_2 = 250$, and finally, the probabilities at the endpoint $x = 500$ are compared for the three methods.

Table 4.1 shows the probabilities for the transition from initial unemployment to unemployment, $p_{00}(\cdot)$. For 250 (500) steps and after $1/5$ of the time, the rectangle solution underestimates the analytical solution in a range of 73% (50%), whereas the trapeze solution overestimates the analytical solution in a range of 39% (8.6%). So at $t_1 = 100$, the

¹¹Note that the probabilities for the complementary events can easily be determined via $p_{11}(t) = 1 - p_{10}(t)$ and $p_{01}(t) = 1 - p_{00}(t)$, respectively.

trapeze solution performs much better than the rectangle solution. With increasing step numbers, both approximated probabilities continuously get better at $t_1 = 100$ with the trapeze solution being much better than the rectangle solution. Already at 4,000 steps, the deviation of the trapeze probability from the analytical one is 0% within the chosen accuracy of three decimal places. At the interval endpoint $x = 500$ with 250 (500) steps, both probabilities are very bad estimates for the analytical probability with an error of 100% (33%) or higher. As expected, the error decreases with increasing step numbers, so at the interval endpoint with 8,000 steps, there is no longer a significant error for the trapeze solution. The best result for the rectangle solution at the endpoint $x = 500$ with 16,000 steps still delivers an error of 10%, which is disproportionately high given the required amount of computation effort. So in order to get results for the rectangle method, which are equally good like for the trapeze method with 2,000 steps requires 16,000 steps or more.

In the analytic case, convergence is reached at about 20 months. Using adequate step numbers, it also takes both approximation methods around 20 months until convergence to the limiting distribution.

Table 4.2 shows the probabilities for the transition from initial employment to unemployment, $p_{10}(\cdot)$. For 250 (500) steps and after $1/5$ of the time, the underestimation by the rectangle solution is not as big as for the corresponding $p_{00}(1/5)$ probabilities with the error being about 57% (39%). The trapeze solution overestimates the analytical solution in a range of 31% (7%). So at $t_1 = 100$, the trapeze solution again performs much better than the rectangle solution. With increasing step numbers, both approximated probabilities continuously get better at $t_1 = 100$ as it has already been the case for the $p_{00}(\cdot)$ probabilities. This holds for all considered points of time in the interval: starting from the unacceptable 250 and 500 step cases, the results at all observed interval points get better, the more steps are used for the calculation. Again, the results for the trapeze method and 2,000 steps are better than the results for the rectangle method with 16,000 steps.

- Convergence with respect to the limiting distribution

The limiting distribution of the SMP can be determined using equation (4.4). However, the integrals cannot be evaluated analytically as soon as there is no analytic solution for $\mu(\cdot)$. Hence, also for the limiting distribution, the accuracy of the different numerical integration methods is evaluated. The analytical limiting distribution values are $p_1^A = \frac{\mu}{\mu+\lambda} = 0.93$ and $p_0^A = \frac{\lambda}{\mu+\lambda} = 0.07$ according to equation (4.1). For both integration methods, the computed values of the limiting distribution can be taken from

		500 steps		1,000 steps		2,000 steps	
		Value	Error	Value	Error	Value	Error
	p_{10}^A	0.070	-	0.070	-	0.070	-
1/5	p_{10}^R	0.030	-0.04	0.043	-0.027	0.061	-0.009
	p_{10}^T	0.092	+0.022	0.075	+0.005	0.071	+0.001
	p_{10}^A	0.070	-	0.070	-	0.070	-
1/2	p_{10}^R	0.004	-0.066	0.015	-0.055	0.046	-0.024
	p_{10}^T	0.126	+0.056	0.081	+0.011	0.071	+0.001
	p_{10}^A	0.070	-	0.070	-	0.070	-
End	p_{10}^R	0.000	-0.07	0.003	-0.067	0.029	-0.041
	p_{10}^T	0.215	+0.145	0.093	+0.023	0.071	+0.001
		4,000 steps		8,000 steps		16,000 steps	
		Value	Error	Value	Error	Value	Error
	p_{10}^A	0.070	-	0.070	-	0.070	-
1/5	p_{10}^R	0.066	-0.004	0.068	-0.002	0.069	-0.001
	p_{10}^T	0.070	-	0.070	-	0.070	-
	p_{10}^A	0.070	-	0.070	-	0.070	-
1/2	p_{10}^R	0.057	-0.013	0.063	-0.007	0.067	-0.003
	p_{10}^T	0.070	-	0.070	-	0.070	-
	p_{10}^A	0.070	-	0.070	-	0.070	-
End	p_{10}^R	0.045	-0.025	0.056	-0.014	0.063	-0.007
	p_{10}^T	0.071	+0.001	0.070	-	0.070	-

Table 4.2: Probabilities for the transition from employment to unemployment $p_{10}(\cdot)$ by t_i , where $t_1 = 1/5 \cdot x = 100$, $t_2 = 1/2 \cdot x = 250$, and $t_3 = x = 500$.

	p_0^R	p_0^T
250 steps	0.046	0.075
500 steps	0.057	0.071
1,000 steps	0.064	0.071
2,000 steps	0.067	0.070
4,000 steps	0.069	0.070
8,000 steps	0.069	0.070
16,000 steps	0.070	0.070
$p_0^A = 0.070$		

Table 4.3: Limiting probabilities p_0 , computed via the two numerical integration methods at different step numbers. The last line shows the analytical value. The remaining probability of the distribution can be calculated by $p_1 = 1 - p_0$.

table 4.3. Besides the numerical integration method, there is now a second source of inexactness, namely the approximation of infinity by 500. However, as the trapeze method delivers very good estimates of the limiting distribution already for smaller time values, approximating infinity by 500 appears to be reasonable when computing the expectation. All in all, the trapeze method is also for the limiting distribution precise enough given our purpose: using 2,000 steps already results in an error of 0% for three decimals preciseness, while the rectangle method still needs 16,000 steps.

In summary, the trapeze solution performs much better as an approximation for the analytically computed CTMC transition probabilities and the limiting distribution than the rectangle method for the given labor market framework. This better exactness comes along with an extended computation effort since the trapeze method is more complex. However, the increased computation effort due to the higher complexity can be reduced again: the trapeze method requires less steps in order to reach a given accuracy. While for our purposes, 2,000 steps prove to be exact enough when using the trapeze method, we would need 16,000 steps or more to reach acceptable results for the rectangle method. Altogether, the choice of the integration method should be made depending on the complexity and the scope of the underlying project.

4.5.2 Duration-dependent arrival rates

In this subsection, the transition probabilities in a setup with duration-dependent arrival rates for jobs $\mu(\cdot)$ and constant separation rates λ are computed. The $\mu(\cdot)$ are taken from

	$p_{00}^R(\cdot)$	$p_{00}^T(\cdot)$	$p_{10}^R(\cdot)$	$p_{10}^T(\cdot)$
100	0.178	0.179	0.163	0.163
250	0.165	0.167	0.166	0.167
500	0.162	0.167	0.163	0.167

Table 4.4: Transition probabilities for duration-dependent transition rates at different points in time for 2,000 steps.

our labor market model of chapter 3. The parameters used are $\lambda = 0.0098$, 2,000 steps, and again, 500 as the interval endpoint. It is no longer possible to compare the numerical solutions to analytical solutions because an analytical solution is no longer available. However, the evolution of both methods can still be considered and discussed, as well as the convergence to the limiting probabilities.

- Evolution of $p_{ij}(t)$ for increasing t using rectangle and trapeze approximation

Figure 4.9 shows the evolution of the transition probabilities over time using 2,000 steps. The trapeze approximation approaches a limiting value of about 0.167, while the rectangle probabilities slightly keep decreasing. Table 4.4 shows some selected values.

As there is no longer an analytical benchmark for the probabilities, the next step is to compute the limiting distributions by the two numerical integration methods.

- Convergence with respect to the limiting distribution

Using equation (4.4) with the two numerical integration methods and approximating infinity by 500 gives estimates of the limiting distribution for each method. For the rectangle method, it is given by

$$p_0^R = 0.1683, \quad p_1^R = 0.8317, \quad (4.26)$$

while the trapeze method yields

$$p_0^T = 0.1684, \quad p_1^T = 0.8316. \quad (4.27)$$

These values are quite similar and they can be compared to the limiting values from above. For 2,000 steps, the trapeze solution performs again better, as can be seen from table 4.4. The trapeze solution at $t = 500$ of about 0.167 is nearer to both the trapeze limit of 0.1684 and the rectangle limit of 0.1683 than the rectangle solution at $t = 500$. This result is in accordance with the findings from the previous subsection.

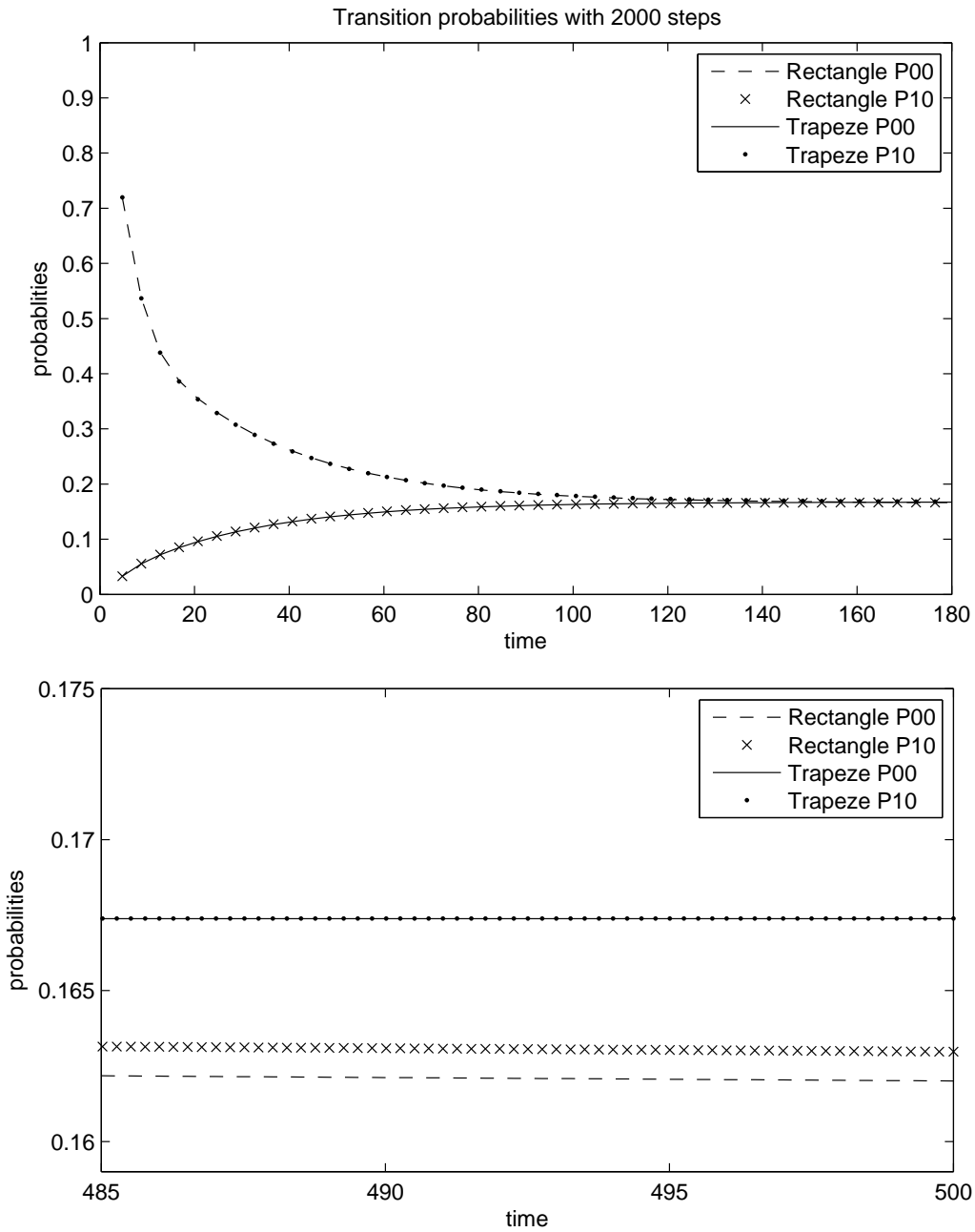


Figure 4.9: Transition probabilities of the labor market model with duration-dependent job arrival rates in the interval $[0, 180]$ (upper subfigure) and in the interval $[485, 500]$ (lower subfigure) for 2,000 steps.

4.6 Conclusion

The use of Semi-Markov processes allows a more realistic description of behavior or states in economic modeling. In labor market theory, duration-dependent transition rates account for microeconomic reactions of individuals over the unemployment spell due to incentive effects of non-stationary benefit schemes, for example. This chapter is devoted to the application of Semi-Markov processes in this area, especially with respect to the derivation of the conditional and unconditional distribution of labor market states. To this end, a basic introduction to Semi-Markov theory is given first. Then, we show how to determine the transition probabilities between labor market states and the limiting distribution of states by means of the labor market model from chapter 3, where a Semi-Markov structure appears in the setup. Since the calculation requires the application of numerical integration methods, two selected methods, the rectangle and the trapeze approximation, are introduced and compared with respect to the accuracy of their numerical results for different step numbers.

Based on a specific labor market example and with constant arrival rates, a step width of about $1/4$ appears to be accurate enough for precise results when using the trapeze rule. For the rectangle method, results are equally acceptable at a step width not more than $1/32$. Regarding the limiting distribution, the trapeze method delivers a very good approximation already at step width $1/4$ with the error being 0% within the chosen preciseness. Also here, the rectangle method requires a much finer step width.

For duration-dependent arrival rates, the transition rates are taken from our labor market model of chapter 3. Also in this case, the transition probabilities of both numerical integration methods approach a limiting value. Again, the trapeze method for the transition probabilities at $x = 500$ converge better to the numerical limiting probabilities computed by both the trapeze method and the rectangle method.

Altogether, the trapeze method is a much more precise method at much smaller step numbers and, therefore, provides higher computation efficiency. Hence, for the transition probabilities of our labor market model it is reasonable to prefer this slightly complexer method over the rectangle method while using less steps.

Chapter 5

Summary

The aim of this thesis is to show the lessons learned from Germany's 2001-2006 labor market reforms. This is done first by a discussion of the new laws and second by a detailed analysis of two particular reform measures.

Chapter 2 presents a comprehensive overview of the reform. We discuss the main law amendments, the four Hartz Laws, with regard to the goals that were intended by the Hartz Commission and the actual outcome in the law. The reforms comprised well-known and widely discussed changes like harder benefit sanctions, marginal employment rules, flat unemployment assistance payments, and One-Euro Jobs. In order to assess their economic effects, we present evaluation studies for the new laws. The results of the official studies commissioned by the government show that many promising changes did not have the desired effects. The 'Personal-Service-Agenturen', for example, were created as state-run temporary employment agencies in order to combine the advantages of placement services of both the Employment Offices and private providers. Nevertheless, it turned out that these 'Personal-Service-Agenturen' were very expensive and inefficient. Positive employment effects are reported for the 'Ich-AG' rules since the likelihood of being employed was considerably increased for the treatment group. The results of these studies, the dissatisfaction among the population as well as shortcomings in the laws have caused the government to change or even abolish several rules again. However, still many controversial issues are left and still many lawsuits against the Hartz Laws are pending.

The detailed analysis of two particular reform measures refers to changes in the benefit system for unemployed, namely the reduction in both unemployment assistance benefits and the entitlement duration for unemployment insurance benefits (Hartz IV). To this end, a broad literature overview is given first in chapter 3, presenting different strands and ideas that build the basis of the evaluation in this chapter. Next, we present our structural search and matching model of the labor market, which is extended by impor-

tant features like endogenous search effort reactions, an exogenous spell effect as well as an endogenous unemployment duration distribution. Moreover, macroeconomic effects are considered in the framework of non-stationary hazard rates by employing techniques from Semi-Markov theory. In this way, we can determine the unemployment rate of the model economy and thus other quantities such as the tax rate and welfare.

When estimating the model parameters structurally by Maximum Likelihood, we account for true duration dependence of the hazard rate as well as duration dependence implied by unobserved heterogeneity and find that both features are crucial for the hazard rate: the exit rate and spell effect parameters as well as the parameter describing the unobserved heterogeneity are significant. Hence, we can infer that search effort reactions and a decreasing spell effect actually play an important role for describing optimal individual behavior, while unobserved heterogeneity leads to differences in individual exit rates out of unemployment.

Given the parameter estimates, different reform scenarios are simulated. When incorporating the Hartz IV reforms into our setup, we can successfully explain the decline in unemployment and unemployment insurance contributions in recent years. Considering welfare positions shows that households gain and firms lose due to the reforms and, while overall welfare increases, no Pareto improvement is realized. Furthermore, we decompose the over-all effects of long-term benefits and analyze the partial insurance and incentive mechanisms in the model.

In order to determine the unemployment rate in chapter 3, specific techniques from the area of Semi-Markov processes are needed. To this end, chapter 4 deals with the application of Semi-Markov processes in labor market theory. First, we give an introduction to continuous-time Markov chains, as they are known in labor market theory and therefore constitute a good starting point, before we turn to Semi-Markov processes. By means of the labor market model presented in chapter 3, it is shown how the transition probabilities and limiting distributions of the different labor market states can be obtained. Since an analytical solution is no longer feasible in this case, numerical solution methods for the transition probabilities are developed. Finally, we test these methods to find out which setup fits best to the requirements of our labor market model.

Summarized in two sentences, starting from a detailed characterization of the labor market reforms in chapter 2 and using the mathematical methods derived in chapter 4, we provide an in-depth analysis of probably the most important reform measures: the reduction of unemployment benefit level and length is evaluated within the framework of a powerful search and matching model in chapter 3, which allows for important features of individual behavior and macroeconomic performance.

Appendix A

Appendix to chapter 3

A.1 Wage bargaining

In this section, we derive the wage equation (3.11). Starting point is the Nash bargaining equation, which determines the division of the job match surpluses,

$$(1 - \beta) [V(w) - V(b_{UI}, 0)] = \beta [J(w^g) - J_0]. \quad (\text{A.1})$$

The bargaining power of the firm is denoted by β . The expression in the square brackets on the left-hand side of the equation is the surplus of a worker and the counterpart, the surplus of a firm, can be found in the square brackets on the right-hand side. The value of a worker earning net wage w is given by $V(w)$ and his fallback position is the value of a newly unemployed, $V(b_{UI}, 0)$, who gets UI benefits b_{UI} and has unemployment duration $s = 0$. The value of a job paying gross wage w^g to employed workers is given by $J(w^g)$, while the value of a vacancy to a firm is denoted by J_0 .

The next step is to determine the time derivative of the bargaining equation,

$$(1 - \beta) \left[\dot{V}(w(t)) - \dot{V}(b_{UI}, 0) \right] = \beta \left[\dot{J} \left(\frac{w}{1 - \kappa} \right) - \dot{J}_0 \right]. \quad (\text{A.2})$$

We go on by finding alternative expressions for the differences in the square brackets of this derivative.

To begin with the household, the Bellman equations of an employed and of an unemployed worker both depend on the net wage and they are given by

$$\rho V(w) = u(w, \psi) + \dot{V}(w) + \lambda [V(b_{UI}, 0) - V(w)] \quad (\text{A.3})$$

and

$$\rho V(b(s), s) = u(b_{UI}, \phi(s)) + \dot{V}(b(s), s) + \mu(\phi(s), \theta, \eta(s)) [V(w) - V(b(s), s)], \quad (\text{A.4})$$

respectively. For the moment, we assume the general case of $V(w)$ possibly changing with t .

Subtracting equation (A.4) from equation (A.3) and using the value of the newly short-term unemployed gives

$$\begin{aligned} \rho [V(w) - V(b_{UI}, 0)] &= u(w, \psi) - u(b_{UI}, \phi(0)) + \dot{V}(w) - \dot{V}(b_{UI}, 0) \\ &\quad - \lambda [V(w) - V(b_{UI}, 0)] \\ &\quad - \mu(\phi(s)\theta, \eta(s)) [V(w) - V(b_{UI}, 0)]. \end{aligned}$$

Rearranging leads to the expression given in the square brackets on the left-hand side of equation (A.2),

$$\begin{aligned} \dot{V}(w) - \dot{V}(b_{UI}, 0) &= [\rho + \lambda + \mu(\phi(s)\theta, \eta(s))] [V(w) - V(b_{UI}, 0)] \\ &\quad - [u(w, \psi) - u(b_{UI}, \phi(0))]. \end{aligned}$$

We proceed in a similar way with the firm values in order to find the difference in the square brackets on the right-hand side of equation (A.2). The values of an occupied and a vacant job to a firm depend on the gross wage $w^g = \frac{w}{1-\kappa}$ and are given by

$$\rho J(w^g) = A - \frac{w}{1-\kappa} + \dot{J}(w^g) + \lambda [J_0 - J(w^g)] \quad (\text{A.5})$$

and

$$\rho J_0 = -\gamma + \dot{J}_0 + q(t) [J(w^g) - J_0], \quad (\text{A.6})$$

respectively, where $q(t)$ is the vacancy filling rate of a firm. Again, we assume for the moment that the value of the firm might change over time.

Subtracting equation (A.6) from equation (A.5) and rearranging gives the desired expression,

$$\begin{aligned} &\rho [J(w^g) - J_0] - [\dot{J}(w^g) - \dot{J}_0] \\ &= A - \frac{w}{1-\kappa} + \gamma - \lambda [J(w^g) - J_0] - q(t) [J(w^g) - J_0] \\ &\Leftrightarrow [\rho + \lambda + q(t)] [J(w^g) - J_0] - [\dot{J}(w^g) - \dot{J}_0] = A - \frac{w}{1-\kappa} + \gamma \\ &\Leftrightarrow \dot{J}(w^g) - \dot{J}_0 = [\rho + \lambda + q(t)] [J(w^g) - J_0] - \left[A - \frac{w}{1-\kappa} + \gamma \right]. \end{aligned}$$

In order to get the wage equation, we can now substitute the terms in the square brackets of (A.2) and get the following modified Nash bargaining equation

$$\begin{aligned} [1 - \beta] \{ &[\rho + \lambda + \mu(\phi(s)\theta, \eta(s))] [V(w) - V(b_{UI}, 0)] - [u(w, \psi) - u(b_{UI}, \phi(0))] \} \\ &= \beta \{ [\rho + \lambda + q(t)] [J(w^g) - J_0] - \left[A - \frac{w}{1-\kappa} + \gamma \right] \}. \end{aligned}$$

Using $V(w) - V(b_{UI}, 0) = \frac{\beta}{1-\beta} [J(w^g) - J_0]$ from the Nash bargaining equation and recollecting results in

$$\begin{aligned} & [\rho + \lambda + \mu(\phi(s)\theta, \eta(s))] \beta [J(w^g) - J_0] - [1 - \beta] [u(w) - u(b_{UI}, \phi(0))] \\ = & \beta \left\{ [\rho + \lambda + q(t)] [J(w^g) - J_0] - \left[A - \frac{w}{1-\kappa} + \gamma \right] \right\} \\ \Leftrightarrow & [1 - \beta] [u(b_{UI}, \phi(0)) - u(w, \psi)] + \beta [\mu(\phi(s)\theta, \eta(s)) - q(t)] [J(w^g) - J_0] \\ & = -\beta \left[A - \frac{w}{1-\kappa} + \gamma \right]. \end{aligned}$$

By using $\mu(\phi(s)\theta, \eta(s)) = \theta q(t)$, $J_0 = 0$, and $q(t) = \frac{\gamma}{J(w^g)}$ we get

$$\begin{aligned} [1 - \beta] [u(b_{UI}, \phi(0)) - u(w, \psi)] + \beta \gamma [\theta - 1] & = -\beta \left[A - \frac{w}{1-\kappa} + \gamma \right] \\ \Leftrightarrow [1 - \beta] [u(b_{UI}, \phi(0)) - u(w, \psi)] - \beta \frac{w}{1-\kappa} & = -\beta [A + \gamma \theta]. \end{aligned}$$

So we finally end up with the wage equation (3.11),

$$[1 - \beta] u(w, \psi) + \beta \frac{w}{1-\kappa} = [1 - \beta] u(b_{UI}, \phi(0)) + \beta [A + \gamma \theta].$$

A.2 Steady state solution

We solve for the steady state of the model by separating the labor market model of chapter 3 into two blocks. Block 1 determines the values of unemployed and employed workers, while block 2 uses the results from block 1 in order to compute macroeconomic variables.

- Block 1: household behavior

Given the functional forms for utility and the spell effect in (3.17) and (3.18), the first-order condition determining optimal search effort (3.6) reads

$$\phi(s) = \{ \alpha \eta(s) \theta^\alpha [V(w) - V(b(s), s)] \}^{\frac{1}{1-\alpha}}. \quad (\text{A.7})$$

It holds for both short- and long-term unemployed. Using this in the Bellman equation for the unemployed (3.5) and rearranging in order to get a differential equation in s gives

$$\dot{V}(b(s), s) = \rho V(b(s), s) - \frac{b(s)^{1-\sigma}}{1-\sigma} + \frac{\alpha-1}{\alpha} [\alpha \eta(s) \theta^\alpha]^{\frac{1}{1-\alpha}} [V(w) - V(b(s), s)]^{\frac{1}{1-\alpha}}, \quad (\text{A.8})$$

which is again valid for both short- and long-term unemployed. As the value of being unemployed an instant before and an instant after becoming a long-term unemployed

is identical, we impose $V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s})$ when solving this differential equation. Finally, since for an infinite unemployment spell the spell effect in (3.18) becomes a constant, $\lim_{s \rightarrow \infty} \eta(s) = \eta_2$, and all other quantities are stationary as well, we get the terminal condition for (A.8) by using $\lim_{s \rightarrow \infty} \dot{V}(b_{UA}, s) = 0$:

$$\rho V(b_{UA}) = \frac{b_{UA}^{1-\sigma}}{1-\sigma} - \frac{\alpha-1}{\alpha} [\alpha \eta_2 \theta^\alpha]^{1-\alpha} [V(w) - V(b_{UA})]^{1-\alpha}. \quad (\text{A.9})$$

With the explicit utility function, the Bellman equation for the employed worker (3.4) can be written as

$$V(w) = \frac{1}{\rho + \lambda} \left(\frac{w^{1-\sigma}}{1-\sigma} - \psi + \lambda V(b_{UI}, 0) \right). \quad (\text{A.10})$$

Now we can insert $V(w)$ as given by equation (A.10) into the Bellman equations of the unemployed, (A.8) and (A.9). Assume further that we know all parameters and, for the time being, some starting values for w and θ . Then, the differential equation (A.8) can be solved starting from an initial value $V(b_{UI}, 0)$. If its solution for $s \rightarrow \infty$ is identical to $V(b_{UA})$ from equation (A.9), the initial guess was right. If it is not, the initial guess $V(b_{UI}, 0)$ has to be adjusted until it is. Hence, with arbitrary exogenous w and θ , we obtain a time path of effort over the unemployment spell, $\phi(b(s), s)$, the spell path of the value of being unemployed, $V(b(s), s)$, and the value of a job, $V(w)$.

- Block 2: wage, tightness and tax rate

Given the equilibrium values $\{\phi(b(s), s), V(b(s), s), V(w)\}$ as functions of w and θ , we now endogenize w and θ .

The Bellman equation for the firm and the free entry result, (3.8) and (3.10), give us

$$\frac{A - \frac{w}{1-\kappa}}{\rho + \lambda} = \gamma \frac{\theta}{\bar{\mu}}. \quad (\text{A.11})$$

Using the utility function (3.17), the bargaining equation (3.11) reads

$$\frac{w^{1-\sigma}}{1-\sigma} - \psi + \frac{\beta}{1-\beta} \frac{w}{1-\kappa} = \left[\frac{b_{UI}^{1-\sigma}}{1-\sigma} - \phi(0) \right] + \frac{\beta}{1-\beta} [A + \theta\gamma], \quad (\text{A.12})$$

where $\phi(0)$ is the optimal search effort at the instant of entry into unemployment, which is given from (A.7). The last two equations require knowledge of the average exit rate $\bar{\mu}$ and the tax rate κ .

The average rate $\bar{\mu}$ is given by equation (3.9), which can easily be computed given that, after having solved block 1, the exit rates $\mu(\cdot)$ are known from equation (3.18) and

the density $f(s)$ can therefore be computed from equation (3.2).¹ The tax rate κ makes the governmental budget constraint (3.3) hold and is given by

$$\kappa = \frac{\frac{b_{UI}U_{short} + b_{UA}U_{long}}{wL}}{1 + \frac{b_{UI}U_{short} + b_{UA}U_{long}}{wL}}. \quad (\text{A.13})$$

Given the density $f(s)$, one can compute the number of short-term and long-term unemployed on the right-hand side of this expression from $U_{short} = U \int_0^{\bar{s}} f(s) ds$ and $U_{long} = U - U_{short}$, where U is the total unknown number of unemployed. However, this unknown number of unemployed can be determined by equation (3.16), using equations (3.13a), (3.13b), and (3.14), which we can solve now given that exit rates are known from block 1.

Hence, we are basically left with equations (A.11) and (A.12) to determine the missing endogenous variables w and θ . After having solved block 1 with a guess of w and θ , we verify whether this guess fulfills equations (A.11) and (A.12). If not, we (Matlab) adjust the guess until a solution is found.

The Matlab code for the steady state solution of the model and the comparative statics can be found in the folder *reform\2 GE* on the enclosed CD (files *goEndo1,2,3.m*).

A.3 Data

The data for the structural estimation comes from the German Socio-Economic Panel (GSOEP). The GSOEP is a panel surveying households on an annual basis. The survey is coordinated by the Deutsches Institut für Wirtschaftsforschung (DIW, Berlin, see www.gsoep.de).

We draw a flow sample of entrants to *employment* and *unemployment* from any of the states *employment*, *unemployment*, and *other state* at each month of years 1997-98. Employment includes job-to-job transitions as well as both full-time and part-time employment. The choice of the year of sampling is determined by the fact that no changes to either benefit level or entitlement length were made between the 1st of January 1997

¹Given the regime change at \bar{s} , the density in equation (3.2) will have a hurdle structure. Denoting the exit rate $\mu(\cdot)$ by $\mu_1(s)$ for short-term unemployed and $\mu_2(s)$ for long-term unemployed, we get

$$f(s) = \begin{cases} \mu_1(s) e^{-\int_0^s \mu_1(u) du} & \text{for } s \leq \bar{s} \\ \frac{\exp\{-\int_0^{\bar{s}} \mu_1(u) du\}}{\exp\{-\int_0^{\bar{s}} \mu_2(u) du\}} \mu_2(s) e^{-\int_0^s \mu_2(u) du} & \text{for } s > \bar{s}. \end{cases}$$

The expression for $s > \bar{s}$ is the probability of surviving \bar{s} with a high level of benefit payments times the density of unemployment duration conditional on the expiration of entitlement, so on $s > \bar{s}$, and transition to a lower level of benefit payments.

Unemployment:	Mean	Std. Dev.		Mean	Std. Dev.
Duration (s)	10.94	14.07	Share of entitled	0.5652	0.4964
UI benefits (b_1)	745.04	289.01	Share of $\bar{s} = 12$ among entitled	0.4462	0.4984
Entitlement (\bar{s})	13.86	6.41	Observed share passing the test	0.2320	0.4232
Wage (w)	1161.21	547.49			
# obs., total	345		# obs., censored	102	

Employment:	Mean	Std. Dev.			
Duration (l), cens.	47.04	29.05			
Duration (l), all	36.48	29.07			
# obs., total	622		# obs., censored	399	

Table A.1: Descriptive statistics of the GSOEP data used for estimation.

and the 1st of January 2005, when the Hartz IV reform came into power. With December 2003 being the last month of our observation period, we end up with a sample that describes a stationary entitlement-benefit environment and provides reliable information on long-term unemployment: about 29.57% of unemployment durations in our sample are right-censored. Among these 29.57%, there are only 6.09% with an (to us) unobservable subsequent state. The remaining 23.48% exit into the *other state*, which exists in the data, but not in the model, and is right-censored in the estimations therefore. For each entrant, we retrieve the duration of stay in the current state since the moment of entry.

Units of measurement are months for the duration data and Euros for the wage and benefit data. Wage is the average monthly wage² for the months employed within a year prior to job loss; prices are those of 2005. Descriptive statistics can be found in table A.1.

It is important to note that GSOEP data do not contain information on the length of entitlement to UI benefits. There exist, however, strict and relatively simple rules that allow to determine the length of entitlement once we know the length of previous job durations and the age of an individual. For this reason, for every person that enters unemployment we also have to retrieve his/her previous job history. In addition to that, previous job history provides us with the record of the last wage earned.

The mean of the vacancy-unemployment ratio between 1997 and 2004 in Germany is $\theta = 0.3$. This value is taken as exogenously given for the ML estimation.

² $w = 1161.21$ Euros is the average monthly net wage before the worker became unemployed, with job being lost during 1997.

A.4 Initial equilibrium: predicting productivity and vacancy costs

- Block 1: household behavior

For the initial equilibrium, wage w and tightness θ are taken as exogenously given. Wage is the sample mean of our GSOEP data and θ is the average vacancy-unemployment ratio for Germany between 1997 and 2004. Using these values, block 1 can be solved as usual, compare section A.2. Hence, with our exogenous w and θ , we obtain the time path of effort over the unemployment spell, $\phi(b(s), s)$, the spell path of the value of being unemployed, $V(b(s), s)$, and the value of a job, $V(w)$.

- Block 2: tax rate, productivity, and vacancy costs

Given the equilibrium values $\{\phi(b(s), s), V(b(s), s), V(w)\}$ and the values for w and θ , we can now determine the tax rate κ , productivity A , and vacancy costs γ .

The unemployment rate is computed using the optimal search strategy of unemployed as given by block 1 and exogenous tightness θ . Given the duration density $f(s)$, one can calculate the number of short-term and long-term unemployed by $U_{short} = U \int_0^{\bar{s}} f(s) ds$ and $U_{long} = U - U_{short}$, where U is the total number of unemployed. The number of unemployed, in turn, follows from equation (3.16) using equations (3.13a), (3.13b), and (3.14), which can be solved now with the exit rates from block 1. Then, the tax rate κ makes the government budget constraint (3.3) hold and is given by

$$\kappa = \frac{\frac{b_{UI}U_{short} + b_{UA}U_{long}}{wL}}{1 + \frac{b_{UI}U_{short} + b_{UA}U_{long}}{wL}}. \quad (\text{A.14})$$

After having determined the tax rate, the bargaining equation (3.11) can be used in order to calculate the auxiliary variable $A_{aux} \equiv A + \theta\gamma$,

$$A_{aux} = \frac{1}{\beta} \left\{ [1 - \beta] u(w) + \beta \frac{w}{1 - \kappa} - [1 - \beta] u(b_{UI}, 0) \right\}.$$

The Bellman equation for the firm and the free entry result, equations (3.8) and (3.10), yield

$$\frac{A - \frac{w}{1 - \kappa}}{\rho + \lambda} = \gamma \frac{\theta}{\bar{\mu}}, \quad (\text{A.15})$$

which becomes

$$\frac{A_{aux} - \theta\gamma - \frac{w}{1 - \kappa}}{\rho + \lambda} = \gamma \frac{\theta}{\bar{\mu}}$$

using the definition of A_{aux} . With this equation, the vacancy costs γ of a firm can be calculated. Having γ , the solution for productivity A can be obtained from the definition

$$A_{aux} \equiv A + \theta\gamma.$$

The Matlab code for the computation of γ and A can be found in the folder *reform\1 compute A and gamma* on the enclosed CD (file *los.m*).

A.5 Determining the productivity of specific years

The predicted value of A is given by 1227.03 and it can be seen as an average A of the years 1997-1999 since 90% of the spells stem from this period. If we now want to know the productivity A of the year 1999, we have to use the growth rates of 1997 and 1998. Starting from 1999, we know that the productivities of 1998 and 1997 can be derived by

$$A_{1998} = \frac{A_{1999}}{1 + g_{1998}}$$

and

$$A_{1997} = \frac{A_{1999}}{(1 + g_{1997})(1 + g_{1998})},$$

respectively, where g_{1997} and g_{1998} denote the growth rates of these years. They are given by $g_{1997} = 0.018$ and $g_{1998} = 0.02$, compare Statistisches Bundesamt (2009). These values can be used to determine the average A between 1997-1999

$$A = \frac{\frac{A_{1999}}{(1+g_{1997})(1+g_{1998})} + \frac{A_{1999}}{1+g_{1998}} + A_{1999}}{3}$$

$$\Leftrightarrow 3A = A_{1999} \left[\frac{1}{(1 + g_{1997})(1 + g_{1998})} + \frac{1}{1 + g_{1998}} + 1 \right].$$

Then, with the values of $A = 1227.03$, g_{1997} , and g_{1998} as given above A_{1999} becomes

$$A_{1999} = 1252.07.$$

If we now want to know the productivity of the year 2006, after the reforms came into power, we have to multiply A_{1999} with all the growth rates between 1999 and 2005. The growth rates are given in table A.2.

year	1999	2000	2001	2002	2003	2004	2005
g_{year}	0.02	0.032	0.012	0.0	-0.02	0.012	0.008

Table A.2: Growth rates in Germany between 1999 and 2005.

With these growth rates, A_{2006} can be determined as

$$A_{2006} = 1333.39.$$

For the Matlab computations, we need to know by how much the productivity A grew from its average value of 1997-1999 to the value of 2005. This growth rate can be computed as

$$\begin{aligned} g &= 1 - \frac{A_{2006}}{A} \\ &= 1 - \frac{1333.39}{1227.03} = 0.0867. \end{aligned}$$

This growth rate is used when we analyze a grown economy, so $WithGrowth = 1$ in *CreateParas.m*, compare Matlab code on the enclosed CD.

A.6 Insurance effects

In this section, we show how to maximize the social welfare function with respect to long-term benefits b_{UA} when neglecting incentive effects. First, we are aware of the relation between the net wage w of a household and the gross wage w^g , which is given by $w = (1 - \kappa) w^g$. Expressing the government budget constraint (3.3) as

$$\left(b_{UI} \int_0^{\bar{s}} f(s) ds + b_{UA} \int_{\bar{s}}^{\infty} f(s) ds \right) (N - L) = \kappa w^g L,$$

we can link the net wage to benefits b_{UA} ,

$$w = (1 - \kappa) w^g = w^g - \left(b_{UI} \int_0^{\bar{s}} f(s) ds + b_{UA} \int_{\bar{s}}^{\infty} f(s) ds \right) \frac{N - L}{L}.$$

The derivative of this net wage equation with respect to b_{UA} for the case of exogenous effort and thereby exogenous unemployment is

$$\frac{dw}{db_{UA}} = - \int_{\bar{s}}^{\infty} f(s) ds \frac{N - L}{L}.$$

This expression can be used in order to maximize the social welfare function (3.12) by choosing b_{UA} ,

$$\frac{d\Omega}{db_{UA}} = LV'(w) \frac{dw}{db_{UA}} + (N - L) \int_{\bar{s}}^{\infty} \frac{d}{db_{UA}} V(b_{UA}, s) f(s) ds.$$

Rearranging this equation results in the condition as given in equation (3.22),

$$\begin{aligned} \frac{d\Omega}{db_{UA}} &= -LV'(w) \int_{\bar{s}}^{\infty} f(s) ds \frac{N - L}{L} + (N - L) \int_{\bar{s}}^{\infty} \frac{d}{db_{UA}} V(b_{UA}, s) f(s) ds \\ &= -V'(w) U_{long} + (N - L) \int_{\bar{s}}^{\infty} \frac{d}{db_{UA}} V(b_{UA}, s) f(s) ds \\ &= N \left[-V'(w) \frac{U_{long}}{N} + \frac{N - L}{N} \int_{\bar{s}}^{\infty} \frac{d}{db_{UA}} V(b_{UA}, s) f(s) ds \right] = 0 \end{aligned}$$

$$\Leftrightarrow V'(w) \frac{U_{long}}{N} = \frac{N-L}{N} \int_{\bar{s}}^{\infty} \frac{d}{db_{UA}} V(b_{UA}, s) f(s) ds. \quad (\text{A.16})$$

Equation (A.16) states the optimality condition for welfare maximizing UA benefits in an economy modeled without incentive effects.

A.7 The insurance and incentive effects in steady state

A.7.1 Steady state solution for the insurance effects

In this subsection, we describe the solution of a model which neglects the incentive effects in order to see where the welfare changes stem from. In order to do so, we keep the path of search effort from the pre-reform steady state as given. Furthermore, vacancies, gross wage, and unemployment are exogenously given. Only the tax rate, linked to the benefits b_{UA} , is endogenous. In order to keep the structure from the previous solutions, we solve for the steady state of the model by separating the model into two blocks.

- Block 1: household behavior

In order to isolate insurance effects, we neglect incentive effects by taking search effort as exogenously given. Hence, we have a path $\phi^{exo}(s)$. For the household, there no longer is a Bellman equation to maximize.

With an exogenous search effort $\phi^{exo}(s)$ and an exogenous gross wage w^g , we can choose a starting value for the tax rate. The net wage can then be computed by $w = [1 - \kappa] w^g$ and the Bellman equations can be solved,

$$\begin{aligned} \rho V(b(s), s) &= u(b(s), s) + \frac{dV(b(s), s)}{ds} + \mu(\phi^{exo}(s) \theta, \eta(s)) [V(w) - V(b(s), s)] \\ \Leftrightarrow \frac{dV(b(s), s)}{ds} &= [\rho + \mu(\phi^{exo}(s) \theta, \eta(s))] V(b(s), s) \\ &\quad - u(b(s), s) - \mu(\phi^{exo}(s) \theta, \eta(s)) V(w). \end{aligned} \quad (\text{A.17})$$

Since $\phi^{exo}(s)$, $\eta(s)$, and θ are given, the Bellman equation is a linear differential equation of the first order, again with the condition that $V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s})$ and the terminal condition $\lim_{s \rightarrow \infty} \dot{V}(b_{UA}, s) = 0$.

The form of the Bellman equation for the employed worker does not change. The net wage is replaced by the gross wage minus taxes and with the explicit utility function, it can be written as

$$V(w) = \frac{1}{\rho + \lambda} \left(\frac{[[1 - \kappa] w^g]^{1-\sigma}}{1 - \sigma} - \psi + \lambda V(b_{UI}, 0) \right). \quad (\text{A.18})$$

$V(w)$ can be inserted into (A.17) and we can use the long-run Bellman equation for the long-term unemployed (which finally gets constant for $s \rightarrow \infty$). The solution of the differential equation can then be obtained using the condition $V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s})$.

- Block 2: tax rate

As there is no optimal behavior of the firm, vacancies are given and there is no wage bargaining. After having solved block 1 with the initial value of κ , we now have to verify whether this κ is consistent with the budget constraint. The tax rate κ can be computed by rearranging the governmental budget constraint (3.3),

$$\left[b_{UI} \int_0^{\bar{s}} f(s) ds + b_{UA} \int_{\bar{s}}^{\infty} f(s) ds \right] [N - L] = \kappa \underbrace{\frac{w}{1 - \kappa}}_{w^g} L$$

$$\Leftrightarrow \kappa = \frac{b_{UI} \int_0^{\bar{s}} f(s) ds + b_{UA} \int_{\bar{s}}^{\infty} f(s) ds}{w^g} \frac{N - L}{L}. \quad (\text{A.19})$$

Since unemployment and the shares of long-term and short-term unemployed stay constant, as well as gross wage and employment, the tax rate directly reacts to the changes in b_{UA} . The solution of equation (A.19) is implemented into the Matlab code by a `fsolve` command.

The Matlab code corresponding to the insurance effects can be found in the folder *reform\4 insurance* on the enclosed CD (files *raiseIns.m* and *runIns.m*).

A.7.2 Steady state solution for the insurance and incentive effects

When allowing for incentive effects in addition to insurance effects, we endogenize search effort. Optimal firm behavior is still neglected and so vacancies and gross wage are given. We solve for the steady state of the model by separating the model into the two known blocks. We give a starting value for the tax rate κ , which implies a net wage $w = [1 - \kappa] w^g$ for a given gross wage, and a starting value for θ . With these starting values, we can compute block 1 and the unemployment rate. Using the definition $\theta = \frac{\bar{V}}{\bar{U}}$ with the given number of vacancies from the pre-reform steady state \bar{V} and the budget constraint of the government, we can then compute θ_1 and κ_1 implied by the model and check our starting values. Matlab will compute this solution by a `fsolve` function.

- Block 1: household behavior

Given the functional forms for utility and search productivity in (3.17) and (3.18), the first-order condition for search effort (3.6) reads

$$\phi(s) = \{\alpha\eta(s)\theta^\alpha [V(w) - V(b(s), s)]\}^{\frac{1}{1-\alpha}}. \quad (\text{A.20})$$

It holds for both short- and long-term unemployed. Inserting this into the Bellman equation for the unemployed (3.5) and expressing it as a differential equation in s gives

$$\dot{V}(b(s), s) = \rho V(b(s), s) - \frac{b(s)^{1-\sigma}}{1-\sigma} + \frac{\alpha-1}{\alpha} [\alpha\eta(s)\theta^\alpha]^{\frac{1}{1-\alpha}} [V(w) - V(b(s), s)]^{\frac{1}{1-\alpha}}, \quad (\text{A.21})$$

which is again valid for both short- and long-term unemployed. As the value of being unemployed an instant before and an instant after becoming a long-term unemployed is identical, we impose $V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s})$ when solving this differential equation. Finally, since for an infinite unemployment spell search productivity in (3.18) becomes a constant, $\lim_{s \rightarrow \infty} \eta(s) = \eta_2$, and all other quantities are stationary as well, we get the known terminal condition for (A.21) by using $\lim_{s \rightarrow \infty} \dot{V}(b_{UA}, s) = 0$:

$$\rho V(b_{UA}) = \frac{b_{UA}^{1-\sigma}}{1-\sigma} - \frac{\alpha-1}{\alpha} [\alpha\eta_2\theta^\alpha]^{\frac{1}{1-\alpha}} [V(w) - V(b_{UA})]^{\frac{1}{1-\alpha}}. \quad (\text{A.22})$$

With the explicit utility function, the Bellman equation for the employed worker (3.4) can be written as

$$V(w) = \frac{1}{\rho + \lambda} \left(\frac{[[1 - \kappa]w^g]^{1-\sigma}}{1-\sigma} - \psi + \lambda V(b_{UI}, 0) \right). \quad (\text{A.23})$$

We can now insert $V(w)$ from (A.23) into (A.21) and (A.22). Imagine further that we know all parameters and assume, for the time being, arbitrary starting values for κ and θ . Then we can solve the differential equation (A.21) starting from some initial value $V(b_{UI}, 0)$ and see whether the solution for $s \rightarrow \infty$ is identical to $V(b_{UA})$ from (A.22). If it is not, we need to adjust our initial guess $V(b_{UI}, 0)$ until it is. Hence, for exogenous κ and θ we obtain the time path of effort over the unemployment spell, $\phi(b(s), s)$, the spell path of the value of being unemployed, $V(b(s), s)$, and the value of a job, $V(w)$. Hence, the solution procedure for optimal household behavior is equivalent to the full equilibrium case of section A.2.

- Block 2: unemployment and tax rate

Given the equilibrium values $\{\phi(b(s), s), V(b(s), s), V(w)\}$ as functions of κ and θ , we now endogenize κ and θ . Using the results from block 1, we can compute the tax

rate κ and the labor market tightness θ implied by the budget constraint of the government (3.3) and the definition of θ ,

$$\theta = \frac{\bar{V}}{U}, \quad (\text{A.24})$$

respectively. The number of vacancies \bar{V} is assumed not to react, and is therefore taken from the pre-reform steady state since optimal behavior from firms is not taken into account when evaluating insurance and incentive effects. The tax rate κ makes the government budget constraint (3.3) hold and can be expressed as

$$\kappa = \frac{\left[b_{UI} \int_0^{\bar{s}} f(s) ds + b_{UA} \int_{\bar{s}}^{\infty} f(s) ds \right] [N - L]}{w^g L}. \quad (\text{A.25})$$

Given the density $f(s)$, one can compute the number of short-term and long-term unemployed on the right-hand side of this expression from $U_{short} = [N - L] \int_0^{\bar{s}} f(s) ds$ and $U_{long} = [N - L] - U_{short}$, where $N - L$ is the total number of unemployed. This number of unemployed follows from (3.16) using (3.13a), (3.13b), and (3.14), which we can now solve given that the exit rates are known from block 1.

Having unemployment, the two equations $\theta = \frac{\bar{V}}{U}$ and (A.25) can be solved in Matlab by a `fsolve` command.

The Matlab code corresponding to the combined insurance and incentive effects can be found in the folder *reform\4 insurance* on the enclosed CD (files *raiseInsInc.m* and *runInsInc.m*).

folders	files
1 compute A and gamma	CreateParas.m
2 GE	DoItAll.m
3 analyze	ReadMe.pdf
4 insurance	
common	
paras	

Table A.3: Files and folders in the folder *reform*.

A.8 The Matlab code for comparative statics

A.8.1 Preparation

The programs for computing the steady state equilibrium and comparative statics of the model can be found in the folder *reform* on the enclosed CD. The folder contains the folders and files as given in table A.3.

A.8.2 Running the programs

The programs are written and tested in Matlab 7.4.0.287 (R2007a). The preset parameters are the parameters we use for our simulations. The structure of the parameters and how you can change them is described in (1). You can either start the whole procedure, see (2), or parts of the program, see (3). After having run comparative statics, we recommend to produce some figures in order to illustrate changes in the model economy, see (4). Part (5) gives some hints on how to proceed with an error prompt.

Table A.4 provides an overview of the different possible program runs and the order in which they have to be run. A more detailed description of each program group can be found in the respective subsection.

If you run *DoItAll.m*, just delete all .txt and .mat files from the folder *reform\paras*, which may be there from previous program runs.

If you run some other comparative static programs as given in the third column, the proceeding depends on the data given in the folder *reform\paras*:

- i) If the folder *reform\paras* is empty except for the folder *graphics*, you will have to run *CreateParas.m* (1.) and *los.m* (2.) before you can continue with the comparative static group (3.) and produce graphics (4.).
- ii) However, if there are data files containing just the data from *CreateParas.m* and *los.m*,

1. parameters	2. pre-reform	3. comp. static	4. graphics
DoItAll.m			DoVGRall.m
CreateParas.m	los.m	goExo.m	VGR0.m
CreateParas.m	los.m	goEndo1.m	VGR1.m
CreateParas.m	los.m	goEndo2.m	VGR2.m
CreateParas.m	los.m	goEndo3.m	VGR3.m
CreateParas.m	los.m	runIns.m	VGRIns.m
CreateParas.m	los.m	runInsInc.m	VGRInsInc.m
CreateParas.m	los.m	raiseIns.m	VGRraiseIns.m
CreateParas.m	los.m	raiseInsInc.m	VGRraiseInsInc.m

Table A.4: Possible program runs.

you can start right away with a program from the comparative static group (3.) and produce graphics afterwards (4.).

iii) Finally, if there are data files containing data from previous comparative static runs, please delete all .txt and .mat files from the folder *reform\paras* or save the data anywhere else and do what is described in i) afterwards.

1. Setting parameters

First of all, in the folder *reform* you find file *CreateParas.m* which helps to produce all the parameters you need in order to start. These are also the values we used for our computations.

However, you can also change the parameters if you like. To this end, open the file *CreateParas.m*. The default values are the parameters that we use for our computations: we reduce the long-term benefits b_{UA} ($redB = 1$), reduce them over 6 steps ($numberB = 6$), and take the parameters estimates from chapter 3 or values that are known from Germany. We recommend to leave the estimated parameters unchanged because otherwise the numerics might no longer work (see 5. Possible error prompts). Especially you can change the following parameters:

- *redB*: determines the sort of comparative statics. $redB=0$ reduces the maximum UI benefit entitlement \bar{s} , $redB=1$ reduces benefits for long-term unemployed b_{UA} , $redB=2$ reduces both in so-called ‘Hartz units’ (for the *raise.m* programs, *redB* determines what is increased - not reduced).
- *numberB*, *numberS*, *numberBS*: depending on the value of *redB*, the value of *numberB*, *numberS*, and *numberBS* determines the number of comparative

static steps with respect to a change of b_{UA} , \bar{s} , and both, respectively. Using more numbers will increase the computation time of the programs, of course, since more steady states have to be determined.

- *WithGrowth*: determines whether the growth of total factor productivity between the pre-reform steady state and the after-reform steady state is taken into account. *WithGrowth=0* means without growth, *WithGrowth=1* incorporates growth.
- *growthrate*: determines the factor by which the pre-reform total factor productivity is multiplied for the after-reform steady state computations, given *WithGrowth=1*.
- *b2*: determines the benefits for long-term unemployed b_{UA} in our two-tier unemployment benefit system.
- *rhoUI*: the wage replacement rate ρ_{UI} for insured unemployed in our two-tier unemployment benefit system.

2. Starting the whole procedure

In order to run all the programs successively, you just have to open *DoItAll.m*, type *DoItAll* into the command window of Matlab and press enter. Please make sure that you deleted all .txt and .mat files in the folder *reform\paras*. The programs will now be run in the right order and depending on your computer capacity, this will take some time.³

First, the *paras.mat* file containing the parameters used will be created by *CreateParas.m* in the folder *reform\paras*. Then, the pre-reform steady state will be computed (*reform\1 compute A and gamma\los.m*) and the results will also be saved in the folder *reform\paras*. After that, the comparative statics will be done. Four different programs will be run: *goExo.m*, *goEndo1.m*, *goEndo2.m*, and *goEndo3.m* from the folder 2 *GE*. All programs do the comparative statics, so they compute steady states for different b_{UA} , \bar{s} , or both. The program *goExo.m* leaves wage, tax rate and labor market tightness exogenous, *goEndo1.m* endogenizes wage, *goEndo2.m* endogenizes wage and tax rate, and in *goEndo3.m*, finally, wage, tax rate, and labor market tightness are endogenous.

³A *DoItAll* run takes approx. 100 minutes on a laptop with an Intel(R) Core(TM)2 Duo CPU T7300 @ 2.00 GHz processor and a memory (RAM) of 2 GB.

3. Starting parts of the programs

If you are interested in special issues, you can also start selected programs. However, if you choose to change parameters, you will have to i) create the parameters (open and run *CreateParas.m*) and ii) run the pre-reform steady state program (open the folder *I compute A and gamma* and run *los.m*) first. Having done this, you have created in the subfolder *paras* all the data you need for the comparative static programs. Please mind that - especially in the *EquilData.txt* file - you should delete data from all former runs except from the first line (which contains data from the *los.m* run) if you want to produce the graphics afterwards. If this is missed, the evaluation program *VGR.m* may not work. The programs from the following list can be run. You may also try to run other Matlab files, but this might lead to error messages.

- *goExo.m*: the macroeconomic steady state variables, wage, labor market tightness, and tax rate, are exogenous. The program does comparative static computations with respect to the parameter you have chosen in *CreateParas.m*.
- *goEndo1.m*: does comparative statics while the wage is treated endogenous. Labor market tightness and tax rate are still exogenous.
- *goEndo2.m*: does comparative statics with the wage and tax rate being endogenous and labor market tightness exogenous.
- *goEndo3.m*: does comparative statics with all three macroeconomic steady state variables, wage, labor market tightness, and tax rate, being endogenous.
- *runIns.m*: evaluates insurance effects in the model while doing comparative statics. To this end, search effort of the unemployed is treated as exogenous and therefore taken from the pre-reform steady state. Furthermore, unemployment, vacancies, and gross wage are exogenous. The only endogenous variable is the tax rate.
- *runInsInc.m*: evaluates insurance and the incentive effects while ignoring effects on the firm's side. Search effort is now endogenous. Additionally to the tax rate, also unemployment is now an endogenous macroeconomic variable. Vacancies and gross wage are still treated as exogenously given.
- *raiseIns.m*: evaluates insurance effects in the model while doing comparative statics with raising b_{UA} , \bar{s} , or both. Short-term benefits b_{UI} are kept constant in the meanwhile. Again, search effort of the unemployed is treated as exogenous and therefore taken from the pre-reform steady state. Furthermore,

unemployment, vacancies, and gross wage are exogenous. The only endogenous variable is the tax rate.

- *raiseInsInc.m*: evaluates insurance and the incentive effects, while ignoring effects on the firm's side and rising the comparative static variable (b_{UA} , \bar{s} , or both). Again, short-term benefits b_{UI} are kept constant in the meanwhile. Search effort is now endogenous. In addition to the tax rate, also the labor market tightness is now an endogenous macroeconomic variable. Vacancies and gross wage are still treated as exogenously given.

4. The resulting data

The data resulting from the comparative static computations can now be evaluated graphically. All the data files needed are in the subfolder *paras*. The file *paras.mat* contains all parameters used. *EquilData.txt* contains selected parameters and steady state values. *MicroData.mat* contains the dynamic evolution of selected microeconomic variables over the unemployment spell from the pre-reform steady state. (*prssDGL.mat* contains the solution of the differential equations from the pre-reform steady state and it is only important for the run of *runIns.m* and *raiseIns.m*.)

Now open the folder *3 analyse*. Depending on whether you did the *DoItAll* run or another single run, evaluate *DoVGRall* (after *DoItAll*) or *VGR1* (for *goEndo1*), *VGR2* (for *goEndo2*), *VGR3* (for *goEndo3*), *VGRIns* (for *runIns*), *VGRInsInc* (for *runInsInc*), *VGRraiseIns* (for *raiseIns*), *VGRraiseInsInc* (for *raiseInsInc*), respectively. If you start any of the *VGR#* programs, please make sure that *EquilData.txt* just contains the *los.m* data in the first row and below only the data from one (!) program run. Otherwise you might get an error prompt and Matlab might abort the program.

The program will now load the steady state data and produce figures: first, for the microeconomic evaluation of effort, stigma, job arrival rate and the value of unemployment over the unemployment spell from the pre-reform steady state; second, for search costs measured in utility units; third, for (un)employment related effects when doing comparative statics; fourth, for distributional effects when doing comparative statics. Actually, two figures will be produced containing distributional effects. The first one contains subfigures already presented at the unemployment effects figure and therefore appear twice since they are interesting for the interpretation of distributional effects. The second figure about distributional effects omits all redundant figures. Furthermore, when having run a program from the

name	comparative static variable
xxx_b2.eps	b_{UA}
xxx_sbar.eps	\bar{s}
xxx_b2sbar.eps	b_{UA} and \bar{s}
xxx_b2A.eps	b_{UA} with increased A
xxx_sbarA.eps	\bar{s} with increased A
xxx_b2sbarA.eps	b_{UA} and \bar{s} with increased A

Table A.5: Naming of figures from the VGR runs.

raise group, the values of the different labor market agents are plotted into one figure. This allows to evaluate the insurance and incentive effects for the different labor market groups at once. All figures will be saved as .eps files in the folder *paras\graphics*.

The figures will be named automatically depending on what was reduced - b_{UA} , \bar{s} , or both - and whether productivity A was increased or not. Table A.5 shows the naming system.

5. Possible error prompts

Please do not change file names, delete files, or move files to other folders as Matlab won't find them anymore and give a corresponding error prompt.

Please do also not change source code unless you know what you do, except for the parameters as described above.

Even if you only change parameters, a program abortion might occur. According to our experience this has often one of the two following sources. First, the defined error tolerance of *fsolve* or *fzero* commands might be too small. In this case, you can try to fix the abortion by increasing error tolerances in *los.m*, *go.m*, *run.m*, *raise.m*, *Vb1BackXXX.m*, *thetaW.m*, or *TaxTheta.m*. Second, the function values of starting values for *fsolve* or *fzero* might not have a different algebraic sign or be complex. In this case, it is helpful to display the starting values and the corresponding function values in order to adjust the starting values. The second error can also follow from the first one, so with an increased error tolerance, the given starting values might work.

A.8.3 The solution structure of the programs

1. Block 1

Block 1 solves for optimal household behavior by solving the Bellman equations of the employed and the unemployed workers as described in section A.2. The following Matlab functions compute block 1:

- *effort.m*: computes the optimal effort of the unemployed.
- *BEbs.m*, *Vb1back.m*: the former states the Bellman equation of the unemployed as a differential equation and the latter solves this differential equation.
- *BELongRun.m*: computes the terminal condition for the Bellman equation of the long-term unemployed, as for $s \rightarrow \infty$ the Bellman equation approaches a constant value.
- *Bew.m*: function for the Bellman equation of the worker.

2. Block 2

Block 2 solves for labor market tightness, wage, and the tax rate and computes the resulting unemployment and welfare. This is done by the following functions:

- *thetaW.m*: in this function, the equations for labor market tightness θ , for the wage w , and the tax rate κ are stated, using the solution of block 1 and the resulting values of the households.
- *smcprob.m*, *smcPuu.m*, *unemployment.m*: these functions use the Semi-Markov techniques in order to compute unemployment endogenously using the optimal job arrival rates resulting from block 1.

3. Additional programs and little helpers

The following programs are additional programs that help to structure the overall program, compute additional variables of interest, or test related numerical issues.

- *CreateParas.m*: this function sets the parameters for the simulations.
- *DoItAll.m*: this program will run the whole procedure. It starts with *CreateParas.m*, goes on to *los.m*, and finally runs *goExo.m*, *goEndo1.m*, *goEndo2.m*, and *goEndo3.m*.
- *go.m*: coordinates together with the remaining *goXXX.m* files (*goExo.m*, *goEndo1.m*, *goEndo2.m*, and *goEndo3.m*) the solution of blocks 1 and 2.

- *muebar.m*: function that computes $\mu(s)$ and the expected value of $\mu(s)$, $\bar{\mu}$.
- *productivity.m*: function in order to compute the exogenously given spell effect.
- *SocialWelf.m*: function that computes the social welfare of the economy given the steady state solutions from block 1 and block 2.
- *los.m*, *gammaA.m*: the former is the central coordination file for the pre-reform steady state, the latter states the functions for the variables of interest in the pre-reform steady state, vacancy costs γ and productivity A .
- *Vb1BackEXO.m*, *BELongRunEXO.m*, *BEbsEXO.m*, *BEwEXO.m*: functions that compute block 1 for the pre-reform steady state; they are used by *los.m* and *gammaA.m*. The structure of these programs corresponds to the structure of those from block 1 with the difference that the wage is exogenous in the pre-reform steady state. This is why they are labeled with an EXO in their file name.
- *run.m*, *raise.m*, *TaxTheta.m*: the former two are central coordination files computing the isolated insurance effects (*runIns.m*, *raiseIns.m*) or the combined insurance and incentive effects (*runInsInc.m*, *raiseInsInc.m*). In the *run* group, the comparative static variable (b_{UA} , \bar{s} , or both) is reduced starting from the pre-reform steady state, while in the *raise* group the comparative static variable is increased. *TaxTheta.m* states the functions for the endogenous variables of interest, tax rate κ and labor market tightness θ .
- *CheckExitflag.m*, *boundtestXXX.m*: the former checks the reason why the *fsolve* procedure was terminated,⁴ the latter is a group of programs (*boundtestBEwEXO.m*, *boundtestBELongRunEXO.m*, *boundtestBEw.m*, *boundtestBELongRun.m*) which tests and chooses suitable starting values for *fzero* and *fsolve* routines.

4. Graphical analysis

The following programs are needed whenever the computations of comparative statics are done and you want to illustrate the changes. They will help to produce graphics.

- *VGR.m*: is the central coordination program for the whole *VGR.m* group (*VGR0.m*, *VGR1.m*, *VGR2.m*, *VGR3.m*, *VGRraiseIns.m*, *VGRraiseInsInc.m*,

⁴The *fsolve* command solves equation systems. Please use the Matlab Help in order to find out more about *fsolve* and *exitflags*.

VGRIns.m, *VGRInsInc.m*, *DoVGRall.m*) and the corresponding comparative statics. The data from the comparative static computations in the *go.m* or the *run.m* family is loaded and *VGR.m* produces and saves the associated figures.

- *UnemplEffects.m*, *DistriEffectsT.m*, *DistriEffectsP.m*, *MicroFigure.m*, *ValueAgents.m*: programs that plot unemployment effects and distribution effects of the comparative static analysis, microeconomic behavior over the unemployment spell, and the values of the agents in one figure, respectively.

A.9 Description of the functions using numerical integration

For solving the steady state solution of the model in Matlab, numerical integration is needed for several calculations. Due to the Semi-Markov structure, the numerical integration routines of Matlab cannot be employed and we have to integrate numerically ‘by hand’. The integration method used is the trapeze approximation and the following subsections describe all programs where this method appears.

A.9.1 *muebar.m*

In *muebar.m*, the average job arrival rate is computed. As there is no analytical solution for $\mu(s)$, the integral of the expectation has to be evaluated numerically. The expectation is given in subsection 3.3.2 by

$$\begin{aligned}\bar{\mu} &= \int_0^{\infty} \mu(s) f(s) ds \\ &= \int_0^{\infty} \mu(s) \mu(s) e^{-\int_0^s \mu(v) dv} ds.\end{aligned}$$

For s out of $[0, 500]$, we know $\mu(s)$ and consequently the numerically approximated value of $f(s)$. So we split the integral in order to compute the expectation.

However, we first integrate the $\mu(\cdot)$ numerically using the trapeze method. The following naming is used in the programs: $\text{mue_int} \equiv \mu(0)$, $\text{mue_int} =$ numerically integrated $\mu(\cdot)$, $\text{mue_int}(0) \equiv 0$, $\tau = zh$. With this notation, the integral of $\mu(\cdot)$ becomes

$$\begin{aligned}\int_0^{\tau} \mu(s(y)) dy &\approx \text{mue_int}(z) \\ &\approx 0.5h \sum_{i=1}^z [\mu(i-1) + \mu(i)] \\ &\approx h [0.5\mu(0) + \mu(1) + \mu(2) + \dots + \mu(z-1) + 0.5\mu(z)].\end{aligned}$$

Using this and the trapeze method for the integral of the expectation from 0 to 500, we get

$$\begin{aligned} \int_0^{500} \mu(s) \mu(s) e^{-\int_0^s \mu(v) dv} ds &\approx 0.5 [\mu(0) \mu(0) e^{-\text{mue.int}(0)} + \mu(1) \mu(1) e^{-\text{mue.int}(1)}] \\ &+ 0.5 [\mu(1) \mu(1) e^{-\text{mue.int}(1)} + \mu(2) \mu(2) e^{-\text{mue.int}(2)}] \\ &+ \dots \\ &+ 0.5 [\mu(\text{steps} - 1) \mu(\text{steps} - 1) e^{-\text{mue.int}(\text{steps}-1)} \\ &+ \mu(\text{steps}) \mu(\text{steps}) e^{-\text{mue.int}(\text{steps})}]. \end{aligned} \quad (\text{A.26})$$

For the second part of the integral, from 500 on, we use the assumption that μ is constant after 500. So the integral becomes

$$\begin{aligned} \int_{500}^{\infty} \mu \mu e^{-\int_0^s \mu(v) dv} ds &= \underbrace{\mu \int_{500}^{\infty} \mu e^{-\int_0^s \mu(v) dv} ds}_{1-F(500)} \\ &= \mu \left[1 - \int_0^{500} f(s) ds \right]. \end{aligned} \quad (\text{A.27})$$

Also the $\int_0^{500} f(s) ds$ is integrated numerically in *muebar.m* using the trapeze method. So $\bar{\mu}$ can be approximated by adding equations (A.26) and (A.27).

A.9.2 smcProb.m

The transition probabilities for a Semi-Markov Process are computed in *smcProb.m* for employed and unemployed starting at this state in t . See section 4.4 for the detailed description of the numerical procedure.

A.9.3 smcPuu.m

In *smcPuu.m*, the probabilities for the transitions from unemployment to unemployment are computed for unemployed with different unemployment durations $s(t)$. Starting point is the equation for the probabilities from unemployment to unemployment for different unemployment durations $s(t)$ as given by equation (3.14) in subsection 3.4.1,

$$p_{uu}(\tau, s(t)) = e^{-\int_t^\tau \mu(s(y)) dy} + \int_t^\tau \underbrace{e^{-\int_t^v \mu(s(y)) dy} \mu(s(v)) p_{eu}(\tau - v)}_{g(v|\tau)} dv. \quad (\text{A.28})$$

In order to compute equation (A.28), the integrals have to be evaluated numerically. The first steps are presented here exemplary for the trapeze method. The initial probability for p_{eu} is given by $p_{eu}^{init} = 0$.

- initialization: $z = 0$

$$p_{uu}^{init} = 1,$$

- step 1: $z = 1$

$$\begin{aligned} p_{uu}(1h) &= p_{uu}(1) = e^{-\text{mue.int}(1)} + 0.5h [g(0|1) + g(1|1)] \\ &= e^{-\text{mue.int}(1)} \\ &\quad + 0.5h [e^{-\text{mue.int}(0)}\mu(0)p_{eu}(1) + e^{-\text{mue.int}(1)}\mu(1)p_{eu}^{init}], \end{aligned}$$

- step 2: $z = 2$

$$\begin{aligned} p_{uu}(2h) &= p_{uu}(2) = e^{-\text{mue.int}(2)} + 0.5h [g(0|2) + g(1|2)] \\ &\quad + 0.5h [g(1|2) + g(2|2)] \\ &= e^{-\text{mue.int}(2)} + 0.5hg(0|2) + hg(1|2) + 0.5hg(2|2) \\ &= e^{-\text{mue.int}(2)} \\ &\quad + 0.5he^{-\text{mue.int}(0)}\mu(0)p_{eu}(2) \\ &\quad + he^{-\text{mue.int}(1)}\mu(1)p_{eu}(1) \\ &\quad + 0.5he^{-\text{mue.int}(2)}\mu(2)p_{eu}^{init}. \end{aligned}$$

A.9.4 unemployment.m

Starting point for the computation of (un)employment in the model is equation (3.16) from subsection 3.4.2,

$$N - L = [N - L] \int_0^\infty p_{uu}(s(t)) dF(s(t)) + p_{eu}L,$$

where p_{eu} is the limit of $p_{eu}(\tau)$ and the $p_{uu}(s(t))$ are the limits of $p_{uu}(\tau, s(t))$ for different $s(t)$. Furthermore, we take the number of potential workers N as given. The endogenous number of employed L can now be determined by isolating L ,

$$\begin{aligned} N - N \int_0^\infty p_{uu}(s(t)) dF(s(t)) &= L - L \int_0^\infty p_{uu}(s(t)) dF(s(t)) + p_{eu}L \\ \Leftrightarrow N \left[1 - \int_0^\infty p_{uu}(s(t)) dF(s(t)) \right] &= L \left[1 - \int_0^\infty p_{uu}(s(t)) dF(s(t)) + p_{eu} \right] \\ \Leftrightarrow L &= \frac{N \left[1 - \int_0^\infty p_{uu}(s(t)) dF(s(t)) \right]}{\left[1 - \int_0^\infty p_{uu}(s(t)) dF(s(t)) + p_{eu} \right]}. \end{aligned}$$

From this equation, we know N and p_{eu} . The integral $\int_0^\infty p_{uu}(s(t)) dF(s(t))$ has to be computed numerically. In order to do so, the interval $[0, \infty[$ is split into two sub-intervals. The first one is $[0, 500]$, the second goes from 500 to infinity. The first interval part can be computed by numerical integration. For each type of unemployed, k , we know the probability $p_{uu}(s(t))$ for his spell s given at the beginning. This probability is approximated by $p_{uu}(500, s(t)) \approx p_{uu}(s(t))$. The density is given by $f(s) = \mu(s) e^{-\int_0^s \mu(v)dv}$. Hence, using the trapeze method we get for the integral

$$\begin{aligned} \int_0^{500} p_{uu}(s(t)) dF(s(t)) &\approx h [0.5p_{uu}(s=0) f(0) + p_{uu}(s=h) f(h) + \dots \\ &\quad + p_{uu}(s=[steps-1]h) f([steps-1]h) \\ &\quad + 0.5p_{uu}(s=steps \cdot h) f(steps \cdot h)]. \end{aligned} \quad (\text{A.29})$$

Now the second part of the integral, the part from 500 on, is determined. We assume that unemployed having a spell of 500 or longer have a constant probability of staying unemployed as the job arrival rate $\mu(\cdot)$ does not change anymore. So we assume that for them $p_{uu}(s(t) > 500) = p_{uu}(s(t) = 500)$. Hence, the integral for the second part becomes

$$\begin{aligned} p_{uu}(s(t) = 500) \int_{500}^\infty dF(s(t)) &= p_{uu}(s(t) = 500) \int_{500}^\infty \mu(s(t)) e^{-\int_0^s \mu(v(t))dv} ds \\ &= \underbrace{p_{uu}(s(t) = 500)}_{p_{uu}} \underbrace{\mu(500)}_{\mu} \int_{500}^\infty e^{-\int_0^s \mu(v(t))dv} ds \\ &= p_{uu} \mu \int_{500}^\infty e^{-[\int_0^{500} \mu(v(t))dv + \int_{500}^s \mu(v(t))dv]} ds. \end{aligned} \quad (\text{A.30})$$

Again, the first part integral in the exponential function can be computed numerically. For the second part, $\int_{500}^s \mu(v(t)) dv$, we know from our assumption that $\mu(v(t)) = const = \mu$. Hence, this integral becomes

$$\int_{500}^s \mu dv = [\mu v]_{500}^s = \mu [s - 500].$$

Using this in equation (A.30), we get

$$\begin{aligned} p_{uu} \mu e^{-\int_0^{500} \mu(v(t))dv} \int_{500}^\infty e^{-\mu[s-500]} ds &= p_{uu} \mu e^{-\int_0^{500} \mu(v(t))dv} \left[-\frac{1}{\mu} e^{-\mu[s-500]} \right]_{500}^\infty \\ &= p_{uu} \mu e^{-\int_0^{500} \mu(v(t))dv} \frac{1}{\mu} \\ &= p_{uu} e^{-\int_0^{500} \mu(v(t))dv}. \end{aligned} \quad (\text{A.31})$$

Equations (A.29) and (A.31) can now be used together with the known p_{uu} , p_{eu} , and mue_int in order to determine employment and unemployment.

A.9.5 SocialWelf.m

Social welfare is given by equation (3.12) in subsection 3.3.3,

$$\Omega = L [V(w) + J] + [N - L] \left[\int_0^{\bar{s}} V(b_1, s) f(s) ds + \int_{\bar{s}}^{\infty} V(b_2, s) f(s) ds \right]. \quad (\text{A.32})$$

Also here, the integrals have to be evaluated numerically. The first integral in the square brackets becomes

$$\begin{aligned} \int_0^{\bar{s}} V(b_1, s) f(s) ds &\approx 0.5h [V(0) f(0) + V(1) f(1)] \\ &+ 0.5h [V(1) f(1) + V(2) f(2)] \\ &+ \dots \\ &+ 0.5h [V(b_1, \text{steps_sbar} - 1) f(\text{steps_sbar} - 1) \\ &+ V(b_1, \text{steps_sbar}) f(\text{steps_sbar})]. \end{aligned}$$

The second integral has to split up again. First, we integrate from \bar{s} to 500 since this is the interval where the value of unemployment still may change. The procedure is equivalent to the one of the first integral,

$$\begin{aligned} \int_{\bar{s}}^{500} V(b_2, s) f(s) ds &\approx 0.5h [V(\text{steps_sbar}) f(\text{steps_sbar}) \\ &+ V(\text{steps_sbar} + 1) f(\text{steps_sbar} + 1)] \\ &+ 0.5h [V(\text{steps_sbar} + 1) f(\text{steps_sbar} + 1) \\ &+ V(\text{steps_sbar} + 2) f(\text{steps_sbar} + 2)] \\ &+ \dots \\ &+ 0.5h [V(\text{steps_sbar} - 1) f(\text{steps_sbar} - 1) \\ &+ V(\text{steps}) f(\text{steps})]. \end{aligned}$$

For the second part of the integral, we can assume that $V(b_2, s)$ stays constant for $s > 500$. Then we can rearrange the integral,

$$\begin{aligned} \int_{500}^{\infty} V(b_2, s) f(s) ds &= V(b_2, 500) \int_{500}^{\infty} f(s) ds \\ &= V(b_2, 500) \left[1 - \int_0^{500} f(s) ds \right]. \end{aligned}$$

The integral of the density can be determined numerically using the density values computed in `muebar.m`.

The numerical approximation of welfare according to equation (A.32) is then straightforward.

Appendix B

Appendix to chapter 4

B.1 The limiting distribution of a Semi-Markov process

From equation (4.3), one can derive the limiting state distribution of a SMP as $p_0 = \frac{\eta_0}{\eta_0 + \eta_1}$ and $p_1 = 1 - p_0$, where η_k is the expected duration in state k , $k = 0, 1$. The duration distributions for a Semi-Markov process in our labor market model of chapter 3 are $f_0(x) = \mu(x) \exp\left\{-\int_0^x \mu(v) dv\right\}$ and $f_1(x) = \lambda(x) \exp\left\{-\int_0^\infty \lambda(v) dv\right\}$ for unemployment and employment, respectively. Then the expected values are

$$\begin{aligned} E[x_0] &= \int_0^\infty x \mu(x) \exp\left\{-\int_0^x \mu(v) dv\right\} dx \\ &= \left[-x \exp\left\{-\int_0^x \mu(v) dv\right\}\right]_0^\infty - \int_0^\infty -\exp\left\{-\int_0^x \mu(v) dv\right\} dx \\ &= \int_0^\infty \exp\left\{-\int_0^x \mu(v) dv\right\} dx \end{aligned}$$

and

$$\begin{aligned} E[x_1] &= \int_0^\infty x \lambda(x) \exp\left\{-\int_0^x \lambda(v) dv\right\} dx \\ &= \left[-x \exp\left\{-\int_0^x \lambda(v) dv\right\}\right]_0^\infty - \int_0^\infty -\exp\left\{-\int_0^x \lambda(v) dv\right\} dx \\ &= \int_0^\infty \exp\left\{-\int_0^x \lambda(v) dv\right\} dx. \end{aligned}$$

Substituting both expectations into $p_0 = \frac{\eta_0}{\eta_0 + \eta_1}$ results in

$$p_0 = \frac{\int_0^\infty \exp\left\{-\int_0^x \mu(v) dv\right\} dx}{\int_0^\infty \exp\left\{-\int_0^x \lambda(v) dv\right\} dx + \int_0^\infty \exp\left\{-\int_0^x \mu(v) dv\right\} dx},$$

which is just the limiting distribution as given by equation (4.4).

B.2 The transition probabilities for the trapeze method

For the sake of clarity, the equations for the transition probabilities of the trapeze method, (4.22) and (4.23), were not fully written out in subsection 4.4.2. As it is helpful for the implementation of the algorithm, the equations are provided here.

Starting point are equations (4.22) and (4.23),

$$p_{00}(zh) - \frac{1}{2}h\underbrace{\mu(0)}_{g(0)}p_{10}(zh) = Q_{00}(zh) + h \sum_{i=1}^{z-1} g(ih) + \frac{1}{2}hg(zh)$$

and

$$p_{10}(zh) - \frac{1}{2}h\underbrace{\lambda p_{00}(zh)}_{f(0)} = h \sum_{i=1}^{z-1} f(ih) + \frac{1}{2}hf(zh).$$

Inserting the definitions $Q_{00}(zh) = \exp\{-h \sum_{i=1}^z \mu(ih)\}$, $Q_{11}(zh) = \exp\{-h \sum_{j=i}^z \lambda\}$, $g(ih) = Q_{00}(ih)\mu(ih)p_{10}([z-i]h)$, and $f(ih) = Q_{11}(ih)\lambda p_{00}([z-i]h)$ yields

$$\begin{aligned} p_{00}(zh) - \frac{1}{2}h\mu(0)p_{10}(zh) &= \exp\left\{-\frac{1}{2}h \sum_{i=1}^z [\mu((i-1)h) + \mu(ih)]\right\} \\ &\quad + h \sum_{i=1}^{z-1} \exp\left\{-\frac{1}{2}h \sum_{j=1}^j [\mu((j-1)h) + \mu(jh)]\right\} \\ &\quad \cdot \mu(ih) \cdot p_{10}([z-i]h) \\ &\quad + \frac{1}{2}h \cdot \exp\left\{-\frac{1}{2}h \sum_{i=1}^z [\mu((i-1)h) + \mu(ih)]\right\} \\ &\quad \cdot \mu(zh) \cdot p_{10}(0) \end{aligned}$$

and

$$\begin{aligned} p_{10}(zh) - \frac{1}{2}\lambda p_{00}(zh) &= h \sum_{i=1}^{z-1} \exp\{-ih\lambda\} \cdot \lambda \cdot p_{00}([z-i]h) \\ &\quad + \frac{1}{2}h \cdot \exp\{-zh\lambda\} \cdot \lambda \cdot p_{00}(0), \end{aligned}$$

respectively.

files	folders
analytic.m	graphics
plotsAR.m	param
plotsAT.m	
rectang.m	
start.m	
trapeze.m	
plots.m	

Table B.1: Files and folders in the folder *smp*.

B.3 The Matlab code for SMP transition probabilities

B.3.1 Preparation

The programs for computing the Semi-Markov probabilities can be found in the folder *smp* on the enclosed CD. The folder *smp* contains the files and folders as given by table B.1.

B.3.2 Running the program

The programs are written and tested in Matlab 7.4.0.287 (R2007a).

Everything you need in order to run the programs is provided in the folder *smp* on the enclosed CD. If you want to change preset parameters, see (1). If you just want to run the program, see (2). A description of resulting data and figures can be found in (3).

1. Setting parameters

First of all, in the folder *smp\param* you find the parameter files you need in order to start right away. The vector of job arrival rates μ and the vector of separation rates λ are saved in the *param* group of .mat files, as well as the number of steps that are used for the computations and the endpoint of the computations x .

The .mat files containing *Const* in their name, *paramConst250*, *paramConst500*, *paramConst1000*, *paramConst2000*, *paramConst4000*, *paramConst8000*, and *paramConst16000*, contain the parameters for the case of constant arrival rates, and hence the parameters on which the computations in subsection 4.5.1 are based. The file *paramVar2000* contains data with duration-dependent job arrival rates μ and leads to the results presented in subsection 4.5.2. Furthermore, all *param* files contain the separation rate vector λ , the step number *steps* and the interval endpoint

x .

If you want, you can replace *mue* or *lam* by your own data for these arrival rates and change the number of steps *steps* and the endpoint of computations x accordingly. In order to do so, open the corresponding *param* file, overwrite the respective values and save your changes. Additionally, you have to open *start* and change the value of *counter*. For constant arrival rates, *counter* begins with the smallest step number (in the preset setup 250) and is doubled after every loop. This proceeding corresponds to the existing step numbers that are used, 250, 500, 1000, 2000... Please make sure to adjust this mechanism if you change parameters. For duration-dependent arrival rates, just change *counter* to the value of the step number you use.

2. Starting the programs

In order to run the programs and to compute the Semi-Markov transition probabilities as well as limiting distributions, just open and run the program *start.m*. The central coordination file *start.m* will first ask whether you want to do the analysis for constant or for duration-dependent job arrival rates $\mu(\cdot)$. After having made that decision, a .txt file is created, which collects some results of interest. Then, the data from the *paras* family is loaded. Using this data, *start.m* runs *rectang.m* and *trapeze.m*, which compute the transition probabilities by the rectangle method and trapeze method, respectively. In the case of constant job arrival rates, the special case of a continuous-time Markov chain, it will additionally run *analytic.m* and determine the analytical transition probabilities for comparison. The evolution of the probabilities over time is plotted into several figures by running *plots.m* (duration-dependent $\mu(\cdot)$) or *plotsAT.m* and *plotsAR.m* (constant μ). Finally, the data is saved into the .txt file. You can find the saved figures and data in the folder *graphics*.

3. The resulting data and figures

The resulting data and figures can be found in the folder *graphics*. The .txt data file presents transition probabilities at selected points in time as well as the limiting probabilities. For a constant job arrival rate μ , the figures are named after the approximation method (*AR* for rectangle and *AT* for trapeze) and the step number. In the case of a duration-dependent $\mu(\cdot)$, the figures are just named *plot* and *plotzoom*.

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