Essays on Strategic Behavior and Dynamic Oligopoly Competition

Inauguraldissertation

zur Erlangung des akademischen Grades
eines Doktors der Wirtschaftswissenschaften an der
wirtschaftswissenschaftlichen Fakultät
der Julius-Maximilians-Universität Würzburg

Vorgelegt von:
Alexander Steinmetz
Würzburg, im Oktober 2009

Betreuer der Arbeit:
Professor Norbert Schulz, Ph.D.

Danksagung

Diese Arbeit habe ich als wissenschaftlicher Mitarbeiter des Lehrstuhls für Industrieökonomik von Professor Norbert Schulz, Ph.D. sowie als Stipendiat des Bavarian Graduate Program in Economics „Incentives“ angefertigt. An dieser Stelle möchte ich mich bei allen Personen bedanken, die mich bei der Erstellung dieser Arbeit unterstützt und begleitet haben.

Norbert Schulz danke ich besonders für das mir entgegengebrachte Vertrauen. Er hat mir viele Freiräume gelassen und mich gleichzeitig ständig unterstützt und ermutigt. Ich habe sehr gern an seinem Lehrstuhl gearbeitet.

Ebenso danke ich Monika Schnitzer dafür meine Zweitbetreuung übernommen zu haben, sowie für konstruktive Kritik und wertvolle Ratschläge.

Víctor Aguirregabiria danke ich herzlich dafür, dass er mich an die University of Toronto einlud und dort während meines Forschungssemesters intensiv betreute. Die Arbeit mit ihm in Toronto war überaus lehrreich und hat mir sehr viel Spaß gemacht.

Teile der Arbeit wurden bereits auf Konferenzen und Workshops vorgestellt. Ich danke den Zuhörern dieser Vorträge für hilfreiche Kritik und Motivation.

Weiterhin danke ich allen Kollegen, die durch Kommentare und Anregungen zur Qualität der Arbeit beigetragen haben. Sie haben insbesondere durch ihre freundschaftliche Unterstützung während unzähliger Kaffeepausen und Mensabesuche dafür gesorgt, dass mir die Arbeit an der Universität immer Spaß gemacht hat. Ausdrücklich erwähnen möchte ich hierbei meinen Freund und Kollegen Robin, der in unserem gemeinsamen Büro meine spießigen Eigenarten ertragen musste. Aus zahlreichen Gründen gäbe es diese Arbeit ohne ihn so nicht.

Mein ganz besonderer Dank gilt meinen Eltern für ihre ständige und uneingeschränkte Unterstützung in allen Bereichen des Lebens. Meine Familie, meine Eltern und meine Schwester gaben mir jederzeit Bestärkung, Zuversicht und Halt.

Weitere Personen außerhalb des universitären Umfelds haben dazu beigetragen, dass ich diese Arbeit verwirklichen konnte. Mein Dank gilt dabei allen Freunden aus Karlsruhe und Würzburg, die mich immer wieder von meiner Arbeit abgelenkt und so für den nötigen Ausgleich gesorgt haben.

Von ganzem Herzen danke ich Hanna für alles, was sie für mich ist und tut.

Würzburg, im April 2010.

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There is only one way to be perfect, but many ways to be imperfect.

– Paul R. Krugman –

Chapter 1

Introduction

Imperfect competition is inherently capable of giving rise to a great range of strategic behavior. Especially, oligopolies provide scope for many things to happen. The field of oligopoly competition encompasses a rich variety of business behavior to apply economic principles, but the very richness of strategies defies simple and general theories.

The study of oligopolistic industries lies at the heart of the field of industrial organization. Theories about the behavior of large firms in concentrated markets shed light on a broad range of strategies, performance, and policies.

As a central topic in microeconomics oligopoly theory has a long history.[1] Sir Thomas More seems to have invented the term oligopoly in his *Utopia* (More, 1518, p. 103) and noted that prices need not fall to competitive levels simply due to the presence of more than one seller when there are only a few. More than 180 years ago Cournot (1838) provided the first formal theory of oligopoly. Despite a rather negative perception given to Cournot's work, it remains the benchmark model of oligopoly.[2]

Although oligopoly fits conceptually between the extremes monopoly and perfect competition, its study requires a rather different set of tools, namely those of game theory. The important characteristic of oligopoly is the presence of strategic interactions among rivals. The oligopolists are aware that the success of their ac-

[1] The introduction of Vives (1999) gives an excellent historical perspective on oligopoly theory.

[2] Seminal reviews of Cournot's book including not only important developments in theory but also fundamental criticism are provided by Bertrand (1883) and Edgeworth (1925).

tions depend on the reactions of their competitors. The adoption of game theory allows for explicitly modeling this oligopolistic interdependency and to analytically describe the underlying strategic game.[3]

On this basis we rely on oligopoly theory to provide predictions regarding the behavior in and performance of markets. We also would like the theory to provide a way of thinking about strategic behavior more generally. By strategic behavior, usually not only production and pricing policies are meant but also decisions regarding investments, inventories, product choice, marketing, distribution, cooperation, and others. Oligopoly theory can help to understand many aspects of strategic rivalry.

Unlike perfect competition or monopoly, there is no single theory of oligopoly. What we have instead is a collection of different oligopoly models. Every theory seems to have its appropriate application. Only by making special assumptions about the oligopolistic environment, each of which will be appropriate in only a limited set of research questions, we can await to properly predict specific oligopoly behavior. We should not expect oligopoly theory to give tight universal predictions.

Various models give a relatively large array of prospects regarding a wide scope of issues. Such variety of predictions simply reflects that a great deal of variety also exists among and within industries and even firms. The different theories of oligopoly should not be seen as competing, but rather as complementary and relevant in different circumstances.

Static oligopoly models are convenient for many matters of study, however, their limitations have been evident at least since Stigler's (1964) classic paper. Stigler identified and stressed the importance of factors such as speed with which competitors learn a rival's move. To study reactions we require explicit dynamic models of oligopoly. This allows to explore strategic rivalry. In addition to nonstationary industries also mature industries where the behavior and performance depends crucially on the history of that industry call for dynamic frameworks. The natural

[3]For early comprehensive surveys on oligopoly theory see for example Schmalensee (1988), Shapiro (1989) and Vickers (1985, 1995). In addition to the monograph by Fudenberg and Tirole (1986), Maskin and Tirole (1987, 1988a, 1988b) provide an overview of topics in dynamic oligopoly. More recently standard textbooks like Tirole (2007), Carlton and Perloff (2005), Lipczynski, Wilson, and Goddard (2009), Belleflamme and Peitz (2009), and many more offer good overviews on the widely accepted canon of field. However, industrial economics is a very dynamic research field and the body of knowledge expands and evolves rapidly.

way to begin studying strategic behavior is in the context of two-period models of oligopoly. Truly dynamic models, i.e., models of many periods in which the economic environment changes with time, hold out the most hope of advancing our understanding of oligopolistic rivalry.

Sunk costs play an essential role in a dynamic, strategic environment. All of the analysis in the following chapters considers situations where firms can make commitments that influence the competition to follow. The sunkness of these commitments qualifies them as strategic decisions. The following studies focus on identifying strategic effects that influence strategic behavior and on characterizing the resulting strategic rivalry.

This thesis deals with three selected dimensions of strategic behavior, namely investment in R&D, mergers and acquisitions, and inventory decisions in dynamic oligopolies.

The question the first essay addresses is how the market structure evolves due to innovative activities when firms' level of technological competence is valuable for more than one project. The focus of the work is the analysis of the effect of learning-by-doing and organizational forgetting in R&D on firms' incentives to innovate. A dynamic step-by-step innovation model with history dependency is developed. Firms can accumulate knowledge by investing in R&D. As a benchmark without knowledge accumulation it is shown that relaxing the usual assumption of imposed imitation yields additional strategic effects. Therefore, the leader's R&D effort increases with the gap as she is trying to avoid competition in the future. When firms gain experience by performing R&D, the resulting effect of knowledge induces technological leaders to rest on their laurels which allows followers to catch up. Contrary to the benchmark case the leader's innovation effort declines with the lead. This causes an equilibrium where the incentives to innovate are highest when competition is most intense.

Using a model of oligopoly in general equilibrium the second essay analyzes the integration of economies that might be accompanied by cross-border merger waves. Studying economies which prior to trade were in stable equilibrium where mergers were not profitable, we show that globalization can trigger cross-border merger waves for a sufficiently large heterogeneity in marginal cost. In partial equilibrium, consumers benefit from integration even when a merger wave is triggered which considerably lowers intensity of competition. Welfare increases. In contrast, in general equilibrium where interactions between markets and therefore effects on factor prices are considered, gains from trade can only be realized by reallocation of resources. The higher the technological dissimilarity between countries the better can efficiency gains be realized in integrated general equilibrium. The overall welfare effect of integration is positive when all firms remain active but indeterminate when firms exit or are absorbed due to a merger wave. It is possible for decreasing competition to dominate the welfare gain from more efficient resource allocation across sectors. Allowing for firms' entry alters results as in an integrated world coexistence of firms of different countries is never possible. Comparative advantages with respect to entry and production are important for realizing efficiency gains from trade.

The third essay analyzes the interaction between price and inventory decisions in an oligopoly industry and its implications for the dynamics of prices. The work extends existing literature and especially the work of Hall and Rust (2007) to endogenous prices and strategic oligopoly competition. We show that the optimal decision rule is an (S, s) order policy and prices and inventories are strategic substitutes. Fixed ordering costs generate infrequent orders. Additionally, with strategic competition in prices, (S, s) inventory behavior together with demand uncertainty generates cyclical pattern in prices.

The last chapter presents some concluding remarks on the results of the essays.

Chapter 2

Competition, Innovation, and the Effect of Knowledge Accumulation

2.1 Introduction

Innovation is an instrument for competitive advantage and often seen as one or even the engine for growth. Therefore, it is crucial to understand its determinants. Competition and innovation are intimately connected. This relation is twofold. On the one hand incentives to innovate are driven by the competitive situation. On the other hand successful innovations affect and thus change the market structure. Due to this interdependency the impact of market structure on innovation can only be assessed if the converse direction – i.e., the changes in market structure caused by innovations – is accounted for. Hence, an analysis of the evolution of market structure due to innovations is best done by means of a dynamic framework.

The link between product market competition and innovation has been studied for a long time. The classic contributions of Schumpeter and Arrow shaped the corresponding polar positions of competition hindering innovation (often attributed to Schumpeter) on the one hand and competition spurring innovation (often attributed to Arrow) on the other hand. Closely connected to the question whether incentives to innovate are increasing or decreasing with more intense market competition is the question on the endogenous evolution of the market. Leaving aside changes in the number of firms (due to entry, exit or mergers and acquisitions), this reduces to the question on the evolution of differences between incumbent firms. Is one firm becoming more and more efficient leaving other firms behind or do we see neck-and-neck competition? Casual observations and empirical evidence suggest a process of action-reaction in markets, i.e., market leadership is constantly changing hands. In theoretical analyses different modeling strategies lead to widely differing conclusions.[1]

However, most of this literature seems to leave out some important aspects. It neglects that past experience in R&D usually has an impact on current success. Considering only one innovation project or several projects separately omits the fact that a level of technological competence may be valuable for following projects. On the one hand, the success in preceding projects helps in securing income. Beyond that, the pure experience of these projects improves performance in other projects. This is due to experience, learning-by-doing, users' feedback etc.

Our approach tries to identify the effect of experience in R&D in a stylized model designed to capture the essentials of the problem.

We develop a dynamic model with history dependency. History affects market opportunities, i.e., previous actions and outcomes determine the range of available actions and outcomes. This is modeled in a way that a firm's investment in R&D does not only increase the chance of making a discovery, but additionally increases the knowledge stock.[2] This knowledge stock is a measure of firm's past R&D effort and allows to model learning, i.e., firm's past experiences add to its current capabilities, and organizational forgetting. Learning-by-doing in R&D has been observed in many empirical works. In practice learning may occur when the innovation activities of a firm are adjusted due to past experiences or when innovation projects are cumulative, i.e., sequential and building on each other. An example would be an investment in laboratory equipment which could be used for other than the current project or gained experience of the researchers. Organizational forgetting in R&D on the other hand is a phenomenon that has been shown in more recent studies.[3] Sticking to the idea of knowledge capturing the experience of the workers, organizational forgetting would be the result of turnover and layoffs.[4]

[1] A brief review of literature is provided at the end of this section.

[2] This way of modeling is based upon the work of Doraszelski (2003).

[3] See for example Argote, Beckman, and Epple (1990) and Benkard (2000).

[4] Note that learning-by-doing and organizational forgetting in R&D is not the same as learning-by-doing and organizational forgetting in production which is a well established approach in literature (see e.g. Cabral & Riordan, 1994, 1997). In contrast to the latter, in this framework firms learn by doing research and development not by producing.

As described, firms' continuous investment in R&D creates the permanent possibility of a successful innovation. These innovations come in successive steps, i.e., a step has to be completed to proceed. Due to the "step-by-step" innovations a technological laggard must first catch up with the leading-edge technology before battling for technological leadership in the future. This in turn implies that if we do not see a process of increasing dominance then every once in a while competition will be neck-and-neck and therefore the escape competition effect will be strongest.[5] With respect to the product market, we assume the industry to be characterized by a duopoly where firms are competing in prices. The incumbent firms simultaneously engage in R&D in order to decrease their relative costs.

The main research focus of the model where history and dynamics are essential is the effect of experience on the firms' incentives to invest in innovation activities. How does this effect influence the evolution of market structure over time? What are the effects of competition in innovation on market structure? Does one firm become increasingly dominant by being more successful in R&D, i.e., do we see a process of increasing dominance, or is there a process of action reaction, in which market leadership is constantly changing hands? Above all we wish to discover when competition in innovation is most intense.

Starting with the benchmark case without learning we show that relaxing the usual assumption of imposing imitation adds strategic effects. Therefore, without the exogenous possibility of immediate imitation leader's R&D effort is increasing with the lead while laggard's effort is decreasing. The industry's leader is trying to avoid competition in the future while the reduced prospect of moving ahead diminishes incentives for the follower. Nevertheless, leaders always invest less and hence a process of action reaction results.

Allowing for knowledge accumulation adds another effect. If one firm has accumulated enough knowledge, its chances to successfully innovate are increased and therefore further R&D effort is less rewarding. The leading firm can afford to rest on its laurels and hence, in steady state the firm invests less the higher the technological lead. The knowledge effect outweighs the increased incentive for the leader to innovate in order to avoid competition. This may induce the follower to catch up. With respect to product market competition our findings are in line with Arrow's

[5] This escape competition motive has been pointed out in previous theoretical work on innovation, for example by Mookherjee and Ray (1991).

position of competition spurring innovation. In our framework we clearly find that due to the effect of knowledge accumulation the incentives to perform R&D are increasing with the intensity of competition.

As mentioned earlier, the relation between competition and innovation has been studied for decades and hence literature on this issue is very extensive. However, neither theoretical nor empirical research has led to a clear result on this link.

In empirical literature some support for the theory that predictions are highly specific to characteristics of innovations and the mechanism to protect the value created by new technologies can be found. One reason why empirical studies have not generated clear conclusions about the relationship might be a failure of many of these studies to account for different market and technological conditions (see for example Cohen & Levin, 1989, and Gilbert, 2006, for surveys). We will discuss this issue in the lights of the results of our model in the respective sections.

Regarding economic theory, in addition to Sutton's work on industrial market structure (Sutton, 1991, 1998, 2007), the voluminous literature dealing with static models (see for example Belleflamme & Vergari, 2006, and Vives, 2008, for important recent advances), this essays is especially related to the literature on dynamic evolution of oligopoly.

Budd, Harris, and Vickers (1993) present a work that analyzes whether the gap between two firms in a model of dynamic competition tends to increase or decrease. While modeling the gap in terms of an abstract (bounded) state of competition parameter without modeling the product market explicitly they find that the gap tends to evolve into the direction where joint payoffs are greater. This most often results in a process of increasing dominance. Cabral and Riordan (1994) provide further indications of increasing dominance. Segal and Whinston (2005) study the effects of antitrust in a dynamic R&D model based on "winner-take-all" competition. Ericson and Pakes (1995) develop a comprehensive model of industry behavior with firm specific sources of uncertainty. The work is intended to be a framework for empirical analysis and more considered as a model for industry dynamics due to entry, exit and mergers.

Articles analyzing industry evolution when there is learning-by-doing like Dasgupta and Stiglitz (1988) and Cabral and Riordan (1994, 1997) usually simply model cost reduction as a function of output decisions. Basically, firms learn by producing not by researching and developing. That means it becomes less costly for the leader to gain higher profits as the lead widens. With this way of modeling R&D is complementary with production. Besides, organizational forgetting can not be modeled in these frameworks.

Our work is also related to the literature on patent races (see Reinganum, 1989, for an early summery). Due to the endpoint that players are aiming for, usually the property of increasing dominance results. This characteristic remains in multistage race models, where several stages are introduced into a patent race, as there is still a definite end.[6] To the best of our knowledge Doraszelski (2003) was the first to introducing knowledge accumulation into patent races. However, he does not model product market competition.

Regarding dynamic step-by-step innovation, our work is most closely related to and extends the works of Aghion, Harris, Howitt, and Vickers (2001) and Acemoglu and Akcigit (2006). Although our model builds on these papers, it also differs from them in significant ways. Most importantly, our main research regards the effect of experience in R&D. Therefore, we extend the model to learning-by-doing and organizational forgetting. Besides, we do not imply the strong assumptions on imitation as Aghion et al. (2001) and Acemoglu and Akcigit (2006). These authors assume the follower at least catches up with the frontier technology with one successful innovation.[7] Thus, in contrast to our model strategic incentives to perform R&D are absent. Acemoglu and Akcigit (2006) show numerically, based on the model of Aghion et al. (2001), that optimal intellectual property rights policy provides more protection to firms that are technologically more advanced as this policy strengthens the escape competition effect. Obviously, R&D by firms that are sufficiently ahead is encouraged just as well as effort by companies with a limited lead because of their prospect of reaching levels of gaps associated with higher protection. That is to say, the effect of avoiding competition that is absent in the basic model is introduced by means of intellectual property rights policy.

Based on the work of Aghion et al. (2001), Aghion, Bloom, Blundell, Griffith, and Howitt (2005) analyze the relationship between product market competition

[6] See for example Fudenberg, Gilbert, Stiglitz, and Tirole (1983), Harris and Vickers (1985), Grossman and Shapiro (1987), Harris and Vickers (1987), and Lippman and McCardle (1988).

[7] Acemoglu and Akcigit (2006) also consider the case where the follower might even be able to improve over the frontier technology.

and innovation. However, they only allow for two possible states (one step behind and neck-to-neck). In a related work Hörner (2004) develops a model allowing a firm to be an arbitrary number of steps ahead or behind. His contribution and the effect of a firm being sufficiently far ahead suggests that an analysis à la Aghion et al. (2005) with only two possible states leaves out some aspects. Unfortunately, Hörner does not model product market competition.

Our work differs from all of the above papers as we consider the effects of learning-by-doing and organizational forgetting in R&D with firms competing on the product market.

The rest of the chapter is organized as follows. Section 2.2 presents the basic model. Section 2.3 provides an analysis of optimal R&D when accumulation of knowledge is not possible. In this section we also compare our result to the one of the related framework of Aghion et al. (2001). Section 2.4 analyzes the equilibrium R&D investment when the effect of knowledge accumulation is at place and compares it with the benchmark case without knowledge. Section 2.5 concludes while the appendix contains the proofs of the results stated in the text.

2.2 The Model

We consider an industry with two ex-ante symmetric firms $i = 1, 2$ producing homogeneous goods.[8] Firms' costs of production depend on their technologies. A firm's technology is given by x_i, and a firm produces output quantity y at cost $c_i(y) = ye^{-x_i}$. Each firm can continuously engage in R&D in order to improve its technology and thereby decrease its relative cost. Innovative investment is denoted by z_i. Innovations are uncertain and come in successive steps. Hence, a step has to be completed to proceed. When a firm moves one technical step ahead its technology increases by one.[9]

[8]Extending the derived results to the more general case of differentiated goods would be possible at the cost of additional notation and a considerably higher complexity in derivation. As only minor additional insights can be gained by such an extension as long as the degree of substitution is exogenous we restrict attention to the case of perfect substitutes.

[9]The stepsize is arbitrarily set equal to one. As long as the size is exogenous and constant all results remain unchanged with a different increment. However, allowing for different possible stepsizes of innovations may alter the outcome considerably.

Investments in R&D increase the chance of a successful innovation, i.e., the chance of moving one step ahead. Besides, there is another effect of R&D. Firms accumulate knowledge. A firm's gathered knowledge is denoted by k_i and evolves according to

$$\frac{dk_i}{dt} = \dot{k}_i = u(z_i) - \delta k_i. \tag{2.1}$$

Here, u_i is firm i's rate of knowledge acquisition. We assume the rate of knowledge acquisition to be a function of investment in R&D given by $u_i = u(z_i) = (\eta z_i)^{\frac{1}{\eta}}$ so that the cost incurring to acquire knowledge at rate u_i is $z(u_i) = \frac{1}{\eta} u_i^{\eta}$. The elasticity of the cost function is measured by $\eta > 1$. Hence, the R&D-cost function is an increasing and convex function. The depreciation rate of the knowledge stock is given by $\delta \geq 0$.

The more knowledge a firm has accumulated, the more successful – in expectation – is the firm's R&D. Hence, the distribution of a firm's success times, given by the firm's hazard rate h_i, does not only depend on the current investment z_i but also on past effort measured by the knowledge stock k_i.[10] A firm moves one technical step ahead with hazard rate

$$h_i = \lambda u(z_i) + \gamma k_i^{\alpha}. \tag{2.2}$$

A firm's hazard rate of successful innovation is the rate at which the discovery is made at a certain point in time given that it has not been made before. The parameter λ measures the effectiveness of current effort while γ measures the effectiveness of past effort. The marginal impact of past R&D efforts is determined by α. To exclude increasing returns to scale, we assume that $\alpha \leq \eta$ holds. A firm's technology follows a Poisson process $dx_i(t) = 1 \cdot dq_i(t)$ where $q_i(t)$ is the underlying process with the non-constant hazard rate $h_i(t)$.[11]

If $\gamma > 0$, the model allows for history dependency. Hence, R&D effort for one project is – by means of the gathered knowledge – valuable for the following projects. This allows to model learning and organizational forgetting. In general, learning means a firm's past experiences add to its current capabilities. Organizational forgetting is modeled as depreciation of knowledge. This implies that a firm's recent

[10] Note that due to this way of modeling we cannot interpret knowledge as capital in the usual way since knowledge is not an input factor in production and knowledge as such does not directly influence the production technology.

[11] For detailed information on Poisson processes see Ross (2007).

experiences are more important and valuable than older know-how. Organizational forgetting is captured in the model by setting $\delta > 0$.

Firms are assumed to be Bertrand competitors and maximize expected discounted profits. Demand at price p is given by $y(p) = \frac{1}{p}$. The instantaneous profit in Bertrand equilibrium then only depends on the technology gap leaving the laggard j with nothing while the leader i earns $1 - e^{-x_i(t)+x_j(t)}$.[12] With the technology gap $\Delta_i(t) \equiv x_i(t) - x_j(t)$ instantaneous profits are

$$\pi_i(\Delta_i, t) = \begin{cases} 1 - e^{-\Delta_i(t)} & \text{for } \Delta_i(t) > 0, \\ 0 & \text{for } \Delta_i(t) \leq 0. \end{cases}$$

On top of these profits both firms have to pay their investment z_i in R&D. Note that even if the industry is leveled, i.e., $\Delta(t) \equiv |\Delta_i(t)| = |\Delta_j(t)| = 0$, the situation is not necessarily symmetric since firms may (and most often will) dispose of different knowledge stocks.

Figure 2.1 shows how the firm's market profit varies with the size of the lead Δ. It shows that profit increases slower the higher the lead already is, i.e., $\frac{\partial \pi_i(\cdot)}{\partial \Delta_i} > 0$

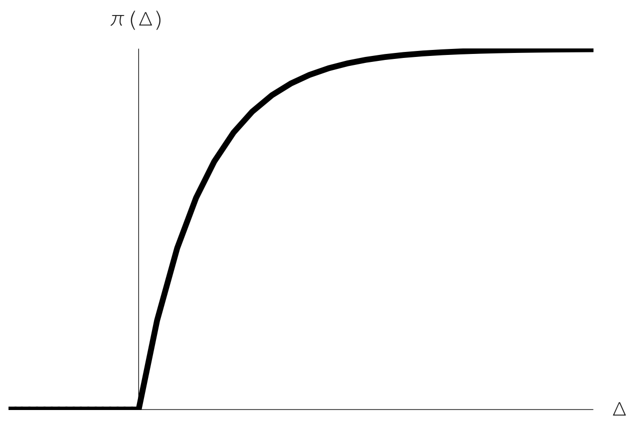

Figure 2.1: A firm's profit π as a function of its technological lead Δ.

and $\frac{\partial^2 \pi_i(\cdot)}{\partial \Delta_i^2} < 0$ for $\Delta_i > 0$. Thus, the motive of escape competition is potentially more important for firms in the neck-and-neck state. On the other hand in an industry with a large technological gap neither firm makes much immediate gain

[12]For the sake of readability throughout the rest of the chapter we will denote firm i's competitor by j, i.e., $j \neq i$ will always hold. Besides, we suppress the indication of time where not necessary.

from innovating; the leader is already earning almost the maximum possible profit and the follower will still earn nothing even if he catches up.

Firms are assumed to maximize expected discounted profits with time preference rate $\rho \in [0, 1)$. As firm i's instantaneous profit is $\pi_i(\Delta_i, t)$, the firm's objective function to be maximized over z_i is

$$\Pi_i(t) \equiv E_t \int_t^\infty \left(\pi_i(\Delta_i, \tau) - z_i(\tau)\right) e^{-\rho(\tau - t)} d\tau. \tag{2.3}$$

We next analyze the equilibrium research intensities. We assume that these equilibrium innovation rates are determined by the necessary conditions for a Markov-stationary equilibrium (steady state fraction of states) in which each firm seeks to maximize expected discounted profits. Hence, firm i maximizes its objective function (2.3) subject to the evolution of the knowledge stocks (2.1) and the technologies (2.2).

2.3 Equilibrium without Acquisition of Knowledge

As a benchmark and starting point we analyze the extreme case, when there is no effect of knowledge and only current effort counts. In this case the state variables are x_1 and x_2. Due to the modeling approach we can use $\Delta_i = x_i - x_j$ as the only state variable. As we do not have to distinguish the impact of past and current effort we can set λ arbitrarily equal to one.

With this exclusion of knowledge our benchmark model is very similar to the one of Aghion et al. (2001), but there is one crucial difference. Aghion et al. (2001) and also Acemoglu and Akcigit (2006) assume that the laggard always catches up with the industry's leader with only one successful innovation no matter how big the gap is. That means the R&D cost function of catching up is independent of the gap that has to be bridged. Therefore, there is no strategic motive for performing R&D. The only incentive for leaders to increase the industry's gap results from the immediate increase in profit. On the long run being sufficiently far ahead does not provide any competitive advantage in future R&D. The converse is true for the follower, i.e., being far behind is not disadvantageous for future competition. In fact, for followers the current gap is irrelevant as it even does not influence product market profits.

Contrarily, we assume the laggard has to catch up step-by-step.[13] Hence, being sufficiently far ahead provides advantages in future technological competition and strategic effects are at place to invest in R&D. This is the case for the laggard and the leader.

Besides, in the framework of Aghion et al. (2001), the assumption on imitation is expected to have similar effects as knowledge accumulation, namely reducing innovation incentives for leaders. When the follower has no possibility of imitating the leader's technology we are able to disentangle the knowledge effect in the next section.

2.3.1 Optimal R&D Effort

In this section we analyze some properties of the firms' optimal effort in R&D. For the sake of simplification we assume firms maximize over u instead of z. To solve for the Markov-stationary equilibrium we use dynamic programming methods. This yields the maximized Bellman equations:[14]

$$\begin{aligned}\rho V_i(\Delta_i) =& \pi_i(\Delta_i) - z(u_i) + \left[V_i(\Delta_i + 1) - V_i(\Delta_i)\right] h_i(u_i(\Delta_i)) \\ &+ \left[V_i(\Delta_i - 1) - V_i(s)\right] h_j(u_j(\Delta_i)). \end{aligned} \tag{2.4}$$

These equations state that the annuity value $\rho V_i(\Delta_i)$ of each firm i in industry state Δ_i at any date t equals the current profit flow $\pi_i(\Delta_i) - z(u_i)$ plus the expected capital gain $\left[V_i(\Delta_i + 1) - V_i(\Delta_i)\right] h_i(u_i(\Delta_i))$ from moving one technological step forward plus the expected capital loss $\left[V_i(\Delta_i - 1) - V_i(\Delta_i)\right] h_j(u_j(\Delta_i))$ from having the competitor stepping forward.

With $\lambda = 1$ and $\eta = 2$ we get the following relations of optimal R&D effort:[15]

Lemma 2.1. *Assuming $\eta = 2$, the optimal R&D effort satisfies the following equations:*

1. *When firms are neck-and-neck, i.e., $\Delta = 0$:*

$$u(0) = \sqrt{2 - \tfrac{2}{e} + \rho^2 + u(1)^2} - \rho; \tag{2.5}$$

[13] A discussion on how realistic these assumptions are is given at the end of this section.

[14] This result is derived in Appendix 2.6.1.

[15] For the sake of readability we will suppress the identity of the firm where not necessary.

2. *For the industry's leader with $\Delta > 0$:*

$$u(\Delta) = u(-\Delta-1) - \rho + \sqrt{e^{-\Delta}\left(2-\tfrac{2}{e}\right) + \left(u(-\Delta-1)-\rho\right)^2 - 2u(\Delta-1)u(-\Delta) + u(\Delta+1)^2}; \tag{2.6}$$

3. *For the follower with $-\Delta < 0$:*

$$u(-\Delta) = \tfrac{1}{2}u(\Delta-1) - \tfrac{\rho}{2} + \sqrt{\tfrac{1}{4}\left(u(\Delta-1)-\rho\right)^2 - u(-\Delta-1)u(\Delta) + u(-\Delta+1)^2}. \tag{2.7}$$

Proof. See Appendix 2.6.2.

From Lemma 2.1 we cannot find a closed form solution for optimal R&D effort as a function of the gap but under the assumption of $\rho = 0$[16] we can derive a pattern regarding the optimal R&D investment:

Proposition 2.1. *Assuming $\eta = 2$ and a time preference rate $\rho = 0$, firms' optimal behavior satisfies the following conditions:*

- *R&D investment is highest for a firm being one step behind and the effort of the laggard decreases with the gap, i.e., $\forall\Delta > 0 : z(-\Delta) > z(-\Delta-1)$.*
- *Investment of a laggard is always higher than that of a neck-and-neck firm, i.e., $\forall\Delta > 0 : z(-\Delta) > z(0)$.*
- *Investment of a leader increases with the gap, i.e., $\forall\Delta > 0 : z(\Delta) < z(\Delta+1)$.*
- *Investment of a leader is always smaller than that of a neck-and-neck firm, i.e., $\forall\Delta > 0 : z(\Delta) < z(0)$.*

Proof. See Appendix 2.6.2.[17]

The pattern resulting from the statements of Proposition 2.1 is illustrated in Figure 2.2.

[16] As shown by Dutta (1991), in a model like ours the assumption of zero discounting is not crucial for the results. Besides, we were able to show numerically that the results basically hold with $\rho > 0$ in quality (with the additional feature that R&D investment eventually falls to zero). However, the analytical derivation for the discounted problem is excessively more complex.

[17] Note, however, that this result is not necessarily the only possible pattern, i.e., it is not possible to show uniqueness (see Appendix).

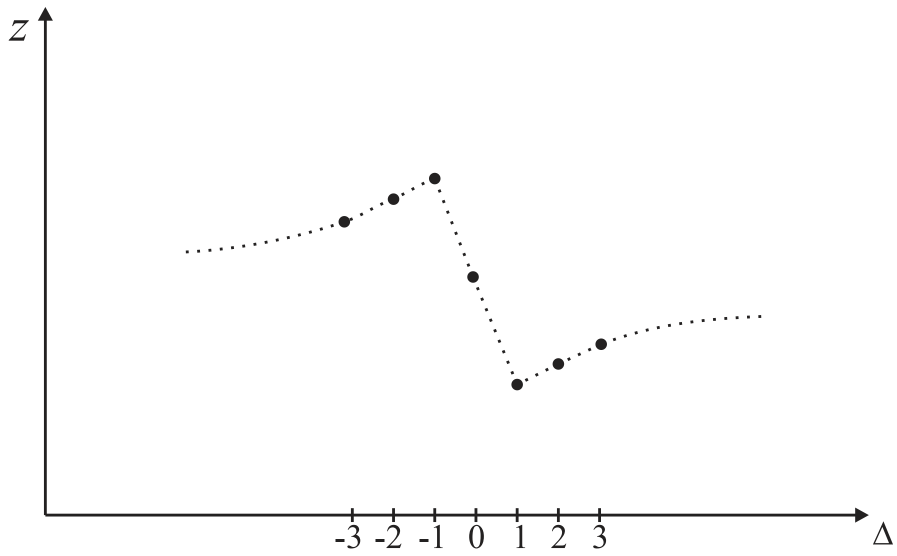

Figure 2.2: Optimal R&D effort subject to the firm's gap.

We see that the R&D effort of a firm being exactly one step behind provides the highest incentive to perform R&D. When the laggard falls further behind, the usual Schumpeterian effect of a reduced prospect of moving ahead diminishes incentives.

The opposite is true for the leader. The incentive is lowest when being one step ahead and increases when moving further ahead. The motive for this increasing effort is not the raise in immediate profit (as this decreases with the lead) but the raise in expected future profit. When the leader moves ahead she decreases the probability that the laggard catches up within a certain time and hence she increases the expected duration of maintaining positive profits.

The greatest R&D effort to enhance the leading edge technology is made when both companies dispose of this technology, i.e., in a neck-and-neck industry. This result is due to the escape-competition effect. It is clear that neck-and-neck firms innovate to escape the strong competition on the product market.

Overall, the laggard is trying hard to catch up and always invests more than the leader which induces a process of action-reaction. This result is in line with empirical evidence especially found in high-tech industries. For example in a market for computer disk drives Lerner (1997) obtained that the market leader was less likely and took longer to introduce a better drive than did firms whose technologies lagged the market leader. Khanna (1995) finds similar results for the high-end computer industry. Czarnitzki and Kraft (2004) find as well that the status of being a challenger has a positive and significant impact while being a defensive firm has a negative impact on the incentives to innovate.

Interestingly, our result quite differs from the result in Aghion et al. (2001). In their model the leader's effort decreases while the follower's effort increases with the industry's gap. This difference in outcome is due to the mentioned difference in modeling: Aghion et al. (2001) assume that the R&D cost function of catching up is independent of the technological gap to be made up. Due to this strong assumption the only incentive for the industry's leader to innovate is a further increase in profit while in our model strategic effects are at place. By widening the technological gap the industry's leader makes it more difficult for the follower to catch up. As a result, we see two characteristics of the escape-competition effect. On the one hand firms in a neck-and-neck industry perform R&D to escape the competition while firms that are technologically advanced innovate to avoid competition in the future. In this framework without imitation we can subdivide the escape competition effect into the basic effect at work in a neck-and-neck state and the *avoid competition effect* at work in a staggered industry.[18]

A similar incentive scheme holds for the follower. In the model à la Aghion et al. (2001) the follower can always catch up immediately and battle for industry leadership. Therefore, the described Schumpeterian effect is almost absent. Here in contrast, a follower being sufficiently far behind has to invest a large amount into R&D to get into the position of being able to battle for market leadership. Hence, the incentive to invest is decreasing with the gap for the follower and increasing for the leader and the described incentive scheme seems to be exactly opposite to Aghion et al. (2001).

However, this is not the entire story. In fact, our model does to some extent incorporate the model of Aghion et al. (2001). As in their model there is no strategic effect of competition in innovation, our states $\Delta_i \in \{-1, 0, 1\}$ contain the basic features of their framework. Considering only these states leaves out the effect of changes in the leader's profit, of course, but can still be used to illustrate the basic result. With a gap not bigger than one, the leader can catch up immediately as it is

[18] It is exactly the strengthening of this avoid competition effect that drives the results of Acemoglu and Akcigit (2006).

the case in the model of Aghion et al. (2001). Taking only these states into account, we find the same result, namely a decrease in R&D with the gap. Strategic effects come into place when the lead widens.

Thus, we can conclude that the incentive scheme resulting in Aghion et al. (2001) is mainly a result of the strong imitation assumption which even Aghion et al. consider as not very realistic and a point for extension. Obviously, our extreme case is not the most realistic scenario either as this would be in between the two extreme cases. However, we were able to show the additional effects when strategic motives to perform R&D come into place. The outcome of a more realistic framework where imitation is possible to some extent would still be driven by the displayed strategic incentives. The impact of these incentives, however, would depend on how catching up with the leading edge technology is possible. This is the realm of intellectual property rights policy.

2.3.2 Steady State Industry Structure

With the results derived so far we will now analyze the industry's structure in steady state. As the leader's R&D effort is always smaller than the laggard's effort the firms will not drift apart in expectation and a steady state exists.[19]

Let μ_Δ denote the steady-state probability of the industry showing a technological gap Δ. As we do not consider knowledge acquisition, a firm's effort $u(\Delta)$ equals the transition rate. Stationarity implies that for any state Δ the flow of industries into this state Δ must equal the flow out. Consider first state 0 (neck-and-neck). During time interval dt, in $\mu_1 u(-1)dt$ in industries with technological gap 1 the follower catches up with the leader, hence, the total flow of industries into state 0 is $\mu_1 u(-1)dt$. On the other hand, in $\mu_0 2u(0)dt$ in neck-and-neck industries one of the two firms acquires a lead, hence the total flow of industries out of state 0 is $2\mu_0 u(0)dt$. Thus in steady state

$$2\mu_0 u(0) = \mu_1 u(-1).$$

Replicating the same reasoning for all states yields

$$\mu_1(u(1) + u(-1)) = 2\mu_0 u(0) + \mu_2 u(-2),$$
$$\mu_2(u(2) + u(-2)) = \mu_1 u(1) + \mu_3 u(-3),$$

[19] See Acemoglu and Akcigit (2006) for a formal proof on the existence of a steady state.

and in general

$$\mu_\Delta(u(\Delta) + u(-\Delta)) = \mu_{\Delta-1}u(\Delta - 1) + \mu_{\Delta+1}u(-\Delta - 1) \text{ for all } \Delta > 1. \tag{2.8}$$

Using these conditions, it is easy to see, that

$$\mu_\Delta u(\Delta) = \mu_{\Delta+1}u(-\Delta - 1) \text{ for all } \Delta \geq 1 \tag{2.9}$$

has to hold.

With the derived stationary conditions it is possible to determine the steady state growth rate. The growth rate of the industry is asymptotically given as $g = \lim_{\Delta t \to \infty} \frac{\Delta \ln y}{\Delta t}$ with y as industry's output.[20]

The quantity sold by the industry as a whole grows at rate e with every step the follower catches up. Thus, over any long time interval, the logarithmic change in output can be approximated by the number of times the follower catches up one step over the time interval. The asymptotic frequency of a catch up equals the steady state flow of the industry from state Δ to state $\Delta - 1$, which in turn equals the fraction μ_Δ of industries in state Δ times the transition rate that the follower catches up one step. This is given by $u(-\Delta)$. Hence, $g = \sum_{\Delta \geq 1} \mu_\Delta u(-\Delta)$ can be written as

$$g = 2\mu_0 u(0) + \sum_{\Delta \geq 1} \mu_\Delta u(\Delta). \tag{2.10}$$

using the stationary conditions (2.9). Equation (2.10) states the following proposition:

Proposition 2.2. *The steady state growth rate in a step-by-step innovation model equals the frequency of frontier innovation, i.e., innovations by industry leaders and neck-and-neck firms, which advance the industry's frontier technology.*

This proposition shows how neck-and-neck rivalry promotes growth. When an industry is neck-and-neck there are two firms trying to advance the industry's frontier technology, whereas in all other states just one firm is trying. Thus, even if

[20] Although we do not model an entire closed economy and cannot provide a general equilibrium analysis, our model can easily be transferred into such a framework. Thus, we can draw conclusions on the economy's growth rate from the growth rate of the industry or sector. In a general equilibrium framework with an economy consisting of a mass of 1 identical industries, the defined industry growth rate g equals the growth rate of the economy $\frac{d \ln Y}{dt}$ with Y as the economy's aggregate output.

all the efforts were the same, technology would advance in average twice as fast in neck-and-neck industries as in any other.

Moreover, as the R&D effort of a neck-and-neck firm is always greater than that of a leader such an industry grows more than twice as fast as other industries. Note that the described characteristic of the steady state growth rate is not a consequence of the no-knowledge assumption but rather the result of any similar step-by-step model showing a steady state.

2.4 The Effect of Knowledge

Now, we analyze the situation where knowledge is introduced, i.e., the market is modeled as described in Section 2.2. The industry's state will be denoted as $s \equiv (\Delta_1, k_1, k_2)$.

Using dynamic programming methods for the problem given by (2.3) subject to (2.1) and (2.2) yields the maximized Bellman equations for the firms:

$$\begin{aligned} \rho V_i(s) =& \pi_i(\Delta_i) - z_i(s) + \left[V_i(x_i+1,\cdot) - V_i(s)\right] h_i(z_i(s)) \\ &+ \left[V_i(x_j+1,\cdot) - V_i(s)\right] h_j(z_j(s)) \\ &+ \frac{\partial V_i(s)}{\partial k_i}\left(u(z_i(s)) - \delta k_i\right) + \frac{\partial V_i(s)}{\partial k_j}\left(u(z_j(s)) - \delta k_j\right). \end{aligned} \tag{2.11}$$

Again, the annuity value $\rho V_i(s)$ of firm i in industry state s at date t equals the current profit flow $\pi_i(\Delta_i) - z_i(s)$ plus the expected capital gain $\left[V_i(x_i+1,\cdot) - V_i(s)\right] h_i(z_i(s))$ from moving one technological step forward plus the expected capital loss $\left[V_i(x_j+1,\cdot) - V_i(s)\right] h_j(z_j(s))$ from having the competitor stepping forward. Now, two other terms are added, namely the capital gain from increased knowledge $\frac{\partial V_i(s)}{\partial k_i}\left(u(z_i(s)) - \delta k_i\right)$ and the capital loss $\frac{\partial V_i(s)}{\partial k_j}\left(u(z_j(s)) - \delta k_j\right)$ from the competitor's acquired knowledge.

To apply the dynamic programming methods we make another simplifying assumption, namely that investment in R&D does not immediately influence a firm's probability of success. Hence, the parameter λ is assumed to be zero. This yields the following proposition:

Proposition 2.3. *If investment in R&D has no immediate influence on the chances of a successful innovation, i.e., when $\lambda = 0$, firms do not immediately react when* *a jump in the firm's own or the competitor's technology occurs, i.e., firm's optimal investment does not immediately change.*

Proof. See Appendix 2.6.3.

This result is not very surprising and a direct consequence of the assumption of $\lambda = 0$. Since firms cannot react directly on technology jumps they do not and hence investment in R&D does not jump when technology does.

And there is another consequence of assuming $\lambda = 0$. A steady state fails to exist. A stationary Markov chain would imply that for any state s the flow of industries into state s must equal the flow out. This again implies hazard rates and hence knowledge stock to immediately react on technological jumps which cannot be the case in this framework. However, we can still determine the optimal rule describing the evolution of investment under firms' optimal behavior. Using the result of Proposition 2.3 in the dynamic programming approach yields:

Lemma 2.2. *If investment in R&D has no immediate influence on the chances of a successful innovation, optimal investment evolves according to*

$$\frac{dz_i(s)}{dt} = z_i(s)\frac{\eta}{\eta-1}\Big(\rho + \delta - (\eta z_i(s))^{\frac{1-\eta}{\eta}}\Phi(\Delta_i)\alpha\gamma k_i^{\alpha-1}\Big), \tag{2.12}$$

with $\Phi(\Delta) > 0$, $\frac{\partial\Phi(\Delta)}{\partial\Delta} < 0$ *for* $\Delta > 0$ *and* $\frac{\partial\Phi(\Delta)}{\partial\Delta} > 0$ *for* $\Delta < 0$.

Proof. See Appendix 2.6.3.

We can immediately and clearly see from (2.12) what the direct effect of knowledge is: The more knowledge a firm has acquired the smaller is the growth rate of optimal investment. This illustrates the effect of resting on one's laurels. Firms acquire knowledge by investing in R&D. Hence, the knowledge stock grows and the more it grows the less do firms invest since they can afford to be based on this stock. Although knowledge as such does not enter in the production function, knowledge is productive in terms of expectations and therefore valuable for firms.

The effect of knowledge in the long run is more difficult to assess. As $z_i(s)$ on the right hand side depends both on a firm's own and the competitor's knowledge, equation (2.12) does not immediately tell the long run effects of knowledge on the evolution of the market.[21]

[21] Note that different to technology levels there is no direct effect of the competitor's knowledge on optimal investment.

Besides, there is another effect of investment, namely technological progress. R&D effort leads to firms moving technologically ahead. Thus, by investing in R&D and thereby accumulating knowledge in expectation firms also increase their technology. We know that $\frac{\partial \Phi(\Delta)}{\partial \Delta} < 0$ for $\Delta > 0$. Therefore, from (2.12) it is clear that investment grows faster the higher the technological lead. For the follower we know that $\frac{\partial \Phi(\Delta)}{\partial \Delta} > 0$ for $\Delta < 0$. Hence, the firm that is behind invests more the closer it gets. This shows again the effects described for the benchmark case (cf. Proposition 2.1).

To see how these effects influence each other in the long run we would need to assess the overall dynamic properties of the model in terms of steady states. Unfortunately, as Δ follows a stochastic process a steady state does not exist. However, we can for the moment assume Δ to be constant to get an idea of the dynamics. Using equation (2.12) we are able to analyze ”temporary steady states”. This approach is closely related to literature on natural volatility.[22]

From (2.12) it is clear that besides the trivial temporary steady state O with $z_i = 0$ and $k_i = 0$, there is the locus $\frac{dz_i}{dt} = \dot{z}_i = 0$ at

$$z_i \quad = \quad \frac{1}{\eta} \left(\frac{\delta + \rho}{\alpha \gamma \Phi(\Delta_i)} \right)^{\frac{\eta}{1-\eta}} k_i^{\frac{(1-\alpha)\eta}{1-\eta}}. \tag{2.13}$$

From (2.1) we have the locus $\frac{dk_i}{dt} = \dot{k}_i = u(z_i) - \delta k_i = 0$. These two loci partition the space $\{k, z\}$[23] into different regions. Temporary steady states are identified by intersections between loci. The properties of steady states depend on the shape of the locus $\dot{z} = 0$ and this again on the marginal impact of knowledge determined by α and the elasticity of the cost function η. In either case the loci partition the space $\{k, z\}$ into four regions. We obtain one intersection of loci (2.1) and (2.13) for positive values of z and k and hence one nontrivial steady state point P. The situation is illustrated in Figure 2.3. The graph on the left shows the phase diagram

[22] This relatively new, mainly macroeconomic approach jointly analyzes short-run instability and long-run growth due to innovations. Important papers in this strand of literature include for example Bental and Peled (1996), Matsuyama (1999), P. Francois and Lloyd-Ellis (2003), Maliar and Maliar (2004), Gabaix (2005), and Haruyama (2009). In these macroeconomic models, however, the motivation for fluctuations in aggregate growth and the link to long run growth are important issues that are irrelevant to our model.

[23] Again, we suppress the identification of the identity of the firm.

for the hazard rate being a concave function of knowledge ($\alpha < 1$) while the right diagram shows the case where the hazard rate is a convex function of knowledge ($\eta > \alpha > 1$). The dynamics are summarized by vertical and horizontal arrows.

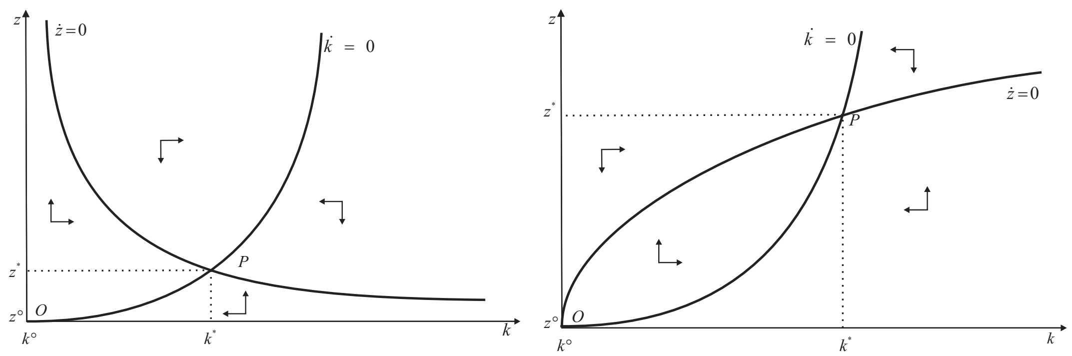

Figure 2.3: Convergence to the temporary steady state P for $\alpha < 1$ and $\eta > \alpha > 1$.

For $\alpha < 1$ the function described by (2.13) is decreasing for all $k > 0$ while the function given by $\dot{k} = 0$ has a positive and increasing slope. Therefore, we obviously obtain one intersection and hence one nontrivial steady state point P.

If $\eta > \alpha > 1$, the function given by (2.13) is increasing with a decreasing slope while the function given by $\dot{k} = 0$ has an increasing slope.[24] Furthermore it is easy to show that the function given by $\dot{z} = 0$ is steeper for sufficiently small values of k. Therefore, we obtain two intersections and hence two steady state points O and P, where O is again the trivial steady state. Only for the special case of $\alpha = \eta$ just the trivial steady state exists.[25]

It is easy to see that in all cases points converge towards the stationary equilibrium P at (k^*, z^*). Hence, the equilibrium P is always reached and stable as long as Δ does not jump. Note that the described results simultaneously hold for both firms in the economy, i.e., as long as Δ does not jump, knowledge and R&D investment for leader and follower converge to two different steady states.

If one firm successfully implements an innovation, Δ jumps. If the leader in-

[24]The hazard rate being a linear ($\alpha = 1$) function of knowledge is a special case where the function described by (2.13) is a horizontal line. The results are similar to the described cases and therefore not given in detail.

[25]For the very special case of $\alpha = \eta$ and $\left(\frac{\delta+\rho}{\alpha\gamma\Phi(\Delta)}\right)^{\frac{\eta}{1-\eta}} = \delta^{\eta}$ the two functions are identical and we have an infinite number of steady states.

novates, the technological gap increases. This causes the line $\dot{z} = 0$ for the leader to decrease while it increases for the follower. Hence, the new temporary steady state towards points converge is left and below the old one for the leader and right and above for the follower. The converse is true if the follower innovates. These dynamics are illustrated for $\eta > \alpha > 1$ in Figure 2.4. The new loci are shown by the dashed lines. We see that both steady state investment and knowledge decrease for the leader when being successful. By contrast, they increase for the follower.

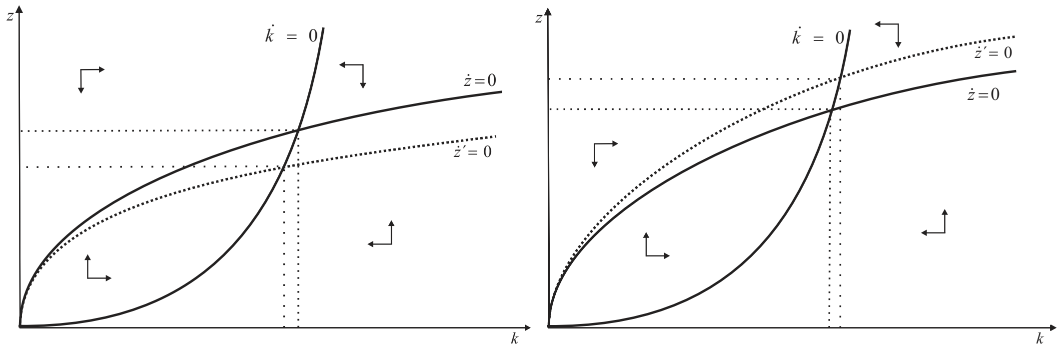

Figure 2.4: Temporary steady states before and after a successful innovation by the leader when $\eta > \alpha > 1$. The leader's fluctuation can be seen on the left, the follower's on the right phase diagram.

Subsequently, the economy approaches towards the new steady states until another jump in technology occurs which might move firms towards a former steady state again. These cyclical equilibria are described by a "Sisyphus-type" behavior. Investment and knowledge approach the steady state but are thrown back due to the implementation of a new successful innovation.

The resulting investment z_i^* in the temporary steady state P is given as

$$z_i^* = \frac{1}{\eta} \left(\frac{\delta + \rho}{\alpha \gamma \Phi(\Delta_i)} \right)^{\frac{\eta}{\alpha - \eta}} \delta^{\frac{(\alpha-1)\eta}{\alpha-\eta}}. \tag{2.14}$$

We now proceed to the comparative statics on z_i^* w.r.t. the technological gap. As long as $\alpha < \eta$ it is verified from (2.14) that for the leading firm

$$\frac{\partial z_i^*}{\partial \Delta_i} < 0$$

holds true. The result is opposite for the follower, i.e., when $\Delta_i < 0$:

$$\frac{\partial z_i^*}{\partial \Delta_i} > 0.$$

This can be summarized by the following proposition:

Proposition 2.4. *The leader's and follower's temporary steady state investment in R&D is decreasing with the technological gap.*

In contrast to the benchmark case, now ceteris paribus steady state investment for the leader is decreasing with the technological lead. She has accumulated enough knowledge such that her chances to successfully innovate and to maintain positive profits are sufficiently high and further R&D effort is less rewarding.

On the other hand, the follower invests more the closer he gets. His incentive scheme is basically the same as in the benchmark case, i.e., the reduced prospect of moving ahead diminishes incentives to innovate when the gap increases.

Comparing this result with the one of the benchmark case we can clearly identify the effect of knowledge: As investment in R&D is never lost the leading firm can afford to scale back its R&D effort. These results might cause the described process of action reaction. The leader rests on her laurels which allows the follower to catch up. The result is a market where leadership is constantly changing hands.

Obviously, struggle is fiercest when firms are shoulder to shoulder and the intensity of competition is higher the closer the technologies of the firms. Thus, the incentive to invest in innovation is increasing with the intensity of competition. This result can be interpreted in the light of the debate between the polar positions attributed to Schumpeter and Arrow, concerning the relationship between the intensity of market competition and the incentives to invest in R&D. Here, the position of Arrow of competition spurring innovation finds support.

This result is supported by empirical work. Especially in other than high-tech industries, studies show some tendency for increased R&D investment when competition is intense (see for example Culbertson & Mueller, 1985, Lunn, 1986, Lunn & Martin, 1986, MacDonald, 1994, Nickell, 1996, and Tang, 2006). In his survey, Gilbert (2006) concludes that "there is some evidence that competition promotes innovation when the measure of competition is an index of proximity of firms to a technological frontier." This is the case in our theoretical framework.

Hence, we have found that the result in the pure knowledge case of this section is in line with empirical evidence from other than high-tech industries while our result in the benchmark case of the previous section is supported by findings from high-tech industries. This suggests that experience is of less importance in high-tech R&D, which might be true as in these industries methods and processes change rapidly such that technology and thus knowledge of this technology is outdated very fast. Casual observations support this view as constant changes of leadership – even involving new entrants – are very common. This could not be the case if experience was essential.

2.5 Conclusion

In this essay, we developed a dynamic step-by-step innovation framework where firms' level of innovative competence is valuable for more than one R&D project in order to investigate the impact of knowledge on firms' optimal innovative effort and the evolution of industrial market structure.

The focus has been the general questions of whether the firm that is currently in the lead tends to increase its advantage over its rival, or whether there is a tendency for the rival to catch-up. We attempted to determine the effect of learning-by-doing and organizational forgetting in R&D on firms' incentives to innovate.

In order to address these questions we analyzed a model where the state of competition is represented in one dimension. In the model firms engage in step-by-step innovation. The leader can innovate in order to widen the technological gap between herself and the follower. This does not only increase her profit but also decreases the probability of getting caught up by the follower. The follower on the other hand innovates to first catch up step-by-step with and then to surpass the leader. Firms acquire knowledge by engaging in R&D projects. This knowledge is valuable not only for the current but also for future projects. Hence, successful projects provide a competitive advantage on the product market and in innovation activities.

In order to assess the effect of knowledge, we first analyzed the case where knowledge is worthless for R&D. As the possibility of imitation for the follower as well as the effect of knowledge accumulation induce the leader to invest less in R&D the higher the gap, we exclude the possibility of imitation so to disentangle these two effects. Besides, the exclusion of imitation adds strategic motives to competition in innovation. We found that a leader in an economy without the possibility of imitation increases her innovative effort the further away she moves as she is trying to avoid competition in the future.

Introducing the possibility of gaining experience by innovative activities adds the knowledge effect which outweighs the avoid competition effect and the leader's R&D effort decreases with the lead. She rests on her laurels which in turn might induce the follower to catch up. Besides, we see that when knowledge is at place the incentives to innovate are higher the higher the intensity of competition. Hence, competition spurs innovation.

The main aim of the essay has been to understand the incentives generated by learning-by-doing and organizational forgetting and how these incentives influence the evolution of the market.

Nevertheless, our findings are based on two extreme cases of an analytical model. It would be interesting to see whether the results of the model in general are in line with our special case results. This could be done by means of a numerical analysis. Intuitively, one would assume that results of such a more general analysis would be a mixture of the two given scenarios. Depending on the parameters determining the impact of past and current R&D effort, the result would either tend more into the direction of the benchmark case of Section 2.3 or the pure knowledge case of Section 2.4.

Furthermore, investigating the impact of intellectual property rights policy could be a revealing task. This could even be done without considering knowledge. On the one hand, a less protective policy would make catching up easier and the industry would more often show a neck-and-neck state in which the growth rate is highest. On the other hand, such a policy would diminish the Schumpeterian effect for the follower and the avoid competition effect for the leader. This would decrease their incentives to invest in R&D and decrease growth rates in other than neck-and-neck states. Hence, the overall outcome is not clear.

It would also be interesting to check the robustness of the results with respect to different models of industry dynamics, i.e., different sources of firms' variety like the degree of substitution, extent of fixed costs etc.

Another natural extension would be to allow for entry and exit. Exit would bound the industry's gap and encourages predatory behavior. This would be a kind of endpoint effect and rise the incentives to move ahead for leaders. Allowing for (re-) entry by making it possible to copy the incumbent's technology at certain cost might promote efforts by the incumbent to gain so much experience that relative high R&D cost for a new firm deter entry. In such an extension the modeling of imitation and licensing would be crucial. Thus, the concept of knowledge accumulation introduces a new strategic aspect to competition.

2.6 Appendix

2.6.1 Dynamic Programming and the General Model

To solve for the Markov-stationary equilibrium we use dynamic programming methods and therefore derive the Bellman equations. Defining the optimal programs for the firms $i = 1, 2$ as $V_i(s) \equiv \max_{\{z_i(\tau)\}} \Pi_i(s(t))$ s.t. the evolutions of the state variables $s \equiv (k_1, k_2, x_1, x_2)$, the Bellman equations are given by

$$\rho V_i(s(t)) = \max_{z_i(t)} \left\{ \pi_i(s(t)) - z_i(t) + \frac{1}{dt} E_t dV_i(s(t)) \right\}, \tag{2.15}$$

where the R&D effort of the competitor is taken as given. Given this general form we compute the differential $dV_i(s(t))$ given the evolutions of the state variables and form expectations. This yields

$$\begin{aligned} E_t dV_i(s(t)) = & \left[\frac{\partial V_i(s)}{\partial k_i} (u(z_i) - \delta k_i) + \frac{\partial V_i(s)}{\partial k_j} (u(z_j) - \delta k_j) \right] dt \\ & + [V_i(x_i + 1, \cdot) - V_i(s)]\, h_i(z_i) dt + [V_i(x_j + 1, \cdot) - V_i(s)]\, h_j(z_j) dt. \end{aligned}$$

The Bellman equations therefore read

$$\begin{aligned} \rho V_i(s(t)) = \max_{z_i(t)} \Bigg\{ & \pi_i(s(t)) - z_i(t) + \left[\frac{\partial V_i(s)}{\partial k_i} (u(z_i) - \delta k_i) + \frac{\partial V_i(s)}{\partial k_j} (u(z_j) - \delta k_j) \right] \\ & + [V_i(x_i + 1, \cdot) - V_i(s)]\, h_i(z_i) + [V_i(x_j + 1, \cdot) - V_i(s)]\, h_j(z_j) \Bigg\}. \end{aligned} \tag{2.16}$$

Then, the first-order conditions are

$$\begin{aligned} & -1 + \frac{\partial V_i(s)}{\partial k_i} u'(z_i) + [V_i(x_i + 1, \cdot) - V_i(s)]\, h_i'(z_i) \overset{!}{=} 0 \\ \Leftrightarrow\ & \frac{\partial V_i(s)}{\partial k_i} = \frac{1 - \lambda u'(z_i) [V_i(x_i + 1, \cdot) - V_i(s)]}{u'(z_i)}. \end{aligned} \tag{2.17}$$

Current gain from not investing an additional unit, i.e., -1 , must equal future gain from an additional unit of investment which is influenced by the change of knowledge stock (through the increase in effort) and the probability of a successful innovation.

This yields

$$\begin{aligned} d\frac{\partial V_i(s)}{\partial k_i} = & d\frac{1 - \lambda u'(z_i) [V_i(x_i + 1, \cdot) - V_i(s)]}{u'(z_i)} \\ = & d\frac{1}{u'(z_i)} - \lambda d [V_i(x_i + 1, \cdot) - V_i(s)]. \end{aligned} \tag{2.18}$$

In the next step, we state the maximized Bellman equations from (2.16) as the Bellman equations where controls are replaced by their optimal values:

$$\begin{aligned}\rho V_i(s) =&\pi_i(s) - z_i(s) + \frac{\partial V_i(s)}{\partial k_i}\left(u(z_i(s)) - \delta k_i\right) + \frac{\partial V_i(s)}{\partial k_j}\left(u(z_j(s)) - \delta k_j\right)\\ &+ \left[V_i(x_i+1,\cdot) - V_i(s)\right] h_i(z_i(s)) + \left[V_i(x_j+1,\cdot) - V_i(s)\right] h_j(z_j(s)). \end{aligned} \tag{2.19}$$

We now compute the derivatives with respect to the state variables k_i using the envelope theorem on the Bellman equation (2.16). This gives expressions for the shadow prices $\frac{\partial V_i(s)}{\partial k_i}$:

$$\begin{aligned}\rho\frac{\partial V_i(s)}{\partial k_i} =&\frac{\partial^2 V_i(s)}{\partial k_i^2}\left(u(z_i(s)) - \delta k_i\right) - \delta\frac{\partial V_i(s)}{\partial k_i} + \frac{\partial^2 V_i(s)}{\partial k_i \partial k_j}\left(u(z_j(s)) - \delta k_j\right)\\ &+ \left[\frac{\partial V_i(x_i+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_i}\right] h_i(z_i(s)) + \left[V_i(x_i+1,\cdot) - V_i(s)\right]\alpha\gamma k_i^{\alpha-1}\\ &+ \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_i}\right] h_j(z_j(s)). \end{aligned} \tag{2.20}$$

Furthermore,

$$\begin{aligned}\rho\frac{\partial V_i(s)}{\partial k_j} =&\frac{\partial^2 V_i(s)}{\partial k_i \partial k_j}\left(u(z_i(s)) - \delta k_i\right) + \frac{\partial^2 V_i(s)}{\partial k_j^2}\left(u(z_j(s)) - \delta k_j\right) - \delta\frac{\partial V_i(s)}{\partial k_j}\\ &+ \left[\frac{\partial V_i(x_i+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] h_i(z_i(s))\\ &+ \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] h_j(z_j(s)) + \left[V_i(x_j+1,\cdot) - V_i(s)\right]\alpha\gamma k_j^{\alpha-1}. \end{aligned} \tag{2.21}$$

Given the evolutions of the state variables we can compute the differentials of the shadow prices:

$$\begin{aligned}d\frac{\partial V_i(s)}{\partial k_i} =& \left[\frac{\partial^2 V_i(s)}{\partial k_i^2}\left(u(z_i) - \delta k_i\right) + \frac{\partial^2 V_i(s)}{\partial k_i \partial k_j}\left(u(z_j) - \delta k_j\right)\right] dt\\ &+ \left[\frac{\partial V_i(x_i+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_i}\right] dq_i(s) + \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_i}\right] dq_j(s), \end{aligned} \tag{2.22}$$

and

$$\begin{aligned}d\frac{\partial V_i(s)}{\partial k_j} =& \left[\frac{\partial^2 V_i(s)}{\partial k_i \partial k_j}\left(u(z_i) - \delta k_i\right) + \frac{\partial^2 V_i(s)}{\partial k_j^2}\left(u(z_j) - \delta k_j\right)\right] dt\\ &+ \left[\frac{\partial V_i(x_i+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] dq_i(s) + \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] dq_j(s), \end{aligned} \tag{2.23}$$

Replacing $\frac{\partial^2 V_i(s)}{\partial k_i^2}\left(u(z_i)-\delta k_i\right)+\frac{\partial^2 V_i(s)}{\partial k_i \partial k_i}\left(u(z_j)-\delta k_j\right)$ in (2.22) by the same expressions from (2.20) gives

$$
\begin{aligned}
d\frac{\partial V_i(s)}{\partial k_i} =& \Bigg\{ (\rho+\delta)\frac{\partial V_i(s)}{\partial k_i} - \left[\frac{\partial V_1(x_i+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_i}\right] h_i(z_i(s)) \\
& - \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_1}\right] h_j(z_j(s)) - \left[V_i(x_i+1,\cdot) - V_i(s)\right]\alpha\gamma k_i^{\alpha-1}\Bigg\} dt \\
& + \left[\frac{\partial V_i(x_i+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_i}\right] dq_i(s) + \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_i} - \frac{\partial V_i(s)}{\partial k_i}\right] dq_j(s),
\end{aligned}
\tag{2.24}
$$

and

$$
\begin{aligned}
d\frac{\partial V_i(s)}{\partial k_j} =& \Bigg\{ (\rho+\delta)\frac{\partial V_i(s)}{\partial k_j} + \left[\frac{\partial V_i(x_i+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] h_i(z_i(s)) \\
& + \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] h_j(z_j(s)) + \left[V_i(x_j+1,\cdot) - V_i(s)\right]\alpha\gamma k_j^{\alpha-1}\Bigg\} dt \\
& + \left[\frac{\partial V_i(x_i+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] dq_i(s) + \left[\frac{\partial V_i(x_j+1,\cdot)}{\partial k_j} - \frac{\partial V_i(s)}{\partial k_j}\right] dq_j(s),
\end{aligned}
\tag{2.25}
$$

Finally, we replace the marginal values by marginal profits from the first order conditions (2.17). This yields

$$
\begin{aligned}
& d\frac{1}{u'(z_i)} - \lambda d\left[V_i(x_i+1,\cdot) - V_i(s)\right] \\
=& \Bigg\{ (\rho+\delta)\frac{1-\lambda u'(z_i)\left[V_i(x_i+1,\cdot)-V_i(s)\right]}{u'(z_i)} \\
& - \Bigg[\frac{1-\lambda u'(z_i(x_i+1))\left[V_i(x_i+2,\cdot)-V_i(x_i+1)\right]}{u'(z_i(x_i+1))} \\
& \quad - \frac{1-\lambda u'(z_i)\left[V_i(x_i+1,\cdot)-V_i(s)\right]}{u'(z_i)}\Bigg] h_i(z_i(s)) \\
& - \Bigg[\frac{1-\lambda u'(z_i(x_{-i}+1))\left[V_i(s)-V_i(x_{-i}+1)\right]}{u'(z_i(x_{-i}+1))} \\
& - \frac{1-\lambda u'(z_i)\left[V_i(x_i+1,\cdot)-V_i(s)\right]}{u'(z_i)}\Bigg] h_j(z_j(s)) \\
& - \left[V_i(x_i+1,\cdot)-V_i(s)\right]\alpha\gamma k_i^{\alpha-1}\Bigg\} dt
\end{aligned}
$$

$$
\begin{aligned}
&+ \left[\frac{1 - \lambda u'(z_i(x_i+1)) \left[V_i(x_i+2,\cdot) - V_i(x_i+1) \right]}{u'(z_i(x_i+1))} \right. \\
&\left. - \frac{1 - \lambda u'(z_i) \left[V_i(x_i+1,\cdot) - V_i(s) \right]}{u'(z_i)} \right] dq_i(s) \\
&+ \left[\frac{1 - \lambda u'(z_i(x_{-i}+1)) \left[V_i(s) - V_i(x_{-i}+1) \right]}{u'(z_i(x_{-i}+1))} \right. \\
&\left. - \frac{1 - \lambda u'(z_i) \left[V_i(x_i+1,\cdot) - V_i(s) \right]}{u'(z_i)} \right] dq_j(s).
\end{aligned} \tag{2.26}
$$

This simplifies to

$$
\begin{aligned}
& d\frac{1}{u'(z_i)} - \lambda d \left[V_i(x_i+1,\cdot) - V_i(s) \right] \\
=& \Bigg\{ (\rho + \delta) \left(\frac{1}{u'(z_i)} - \lambda \left[V_i(x_i+1,\cdot) - V_i(s) \right] \right) \\
& - \left[\frac{1}{u'(z_i(x_i+1))} - \frac{1}{u'(z_i)} - \lambda \left[V_i(x_i+2,\cdot) + V_i(s) \right] \right] h_i(z_i(s)) \\
& - \left[\frac{1}{u'(z_i(x_{-i}+1))} - \frac{1}{u'(z_i)} + \lambda \left[V_i(x_{-i}+1) + V_i(x_i+1,\cdot) \right] \right] h_j(z_j(s)) \\
& - \left[V_i(x_i+1,\cdot) - V_i(s) \right] \alpha\gamma k_i^{\alpha-1} \Bigg\} dt \\
& + \left[\frac{1}{u'(z_i(x_i+1))} - \frac{1}{u'(z_i)} - \lambda \left[V_i(x_i+2,\cdot) + V_i(s) \right] \right] dq_i(s) \\
& + \left[\frac{1}{u'(z_i(x_{-i}+1))} - \frac{1}{u'(z_i)} + \lambda \left[V_i(x_{-i}+1) + V_i(x_i+1,\cdot) \right] \right] dq_j(s).
\end{aligned} \tag{2.27}
$$

On the other hand we can use the maximized Bellman equations together with the first order condition and get

$$
\begin{aligned}
\rho V_i(s) =& \pi_i(s) - z_i(s) + \frac{1 - \lambda \left[V_i(x_i+1,\cdot) - V_i(s) \right]}{u'(z_i)} \left(u(z_i(s)) - \delta k_i \right) \\
& + \frac{\partial V_i(s)}{\partial k_j} \left(u(z_j(s)) - \delta k_j \right) \\
& + \left[V_i(x_i+1,\cdot) - V_i(s) \right] h_i(z_i(s)) + \left[V_i(x_j+1,\cdot) - V_i(s) \right] h_j(z_j(s)).
\end{aligned} \tag{2.28}
$$

We make use of the derivation above in the next subsections analyzing special cases of the general model.

2.6.2 No Acquisition of Knowledge

Without knowledge acquisition, i.e., with $\gamma = 0$ and $\Delta_i = x_i - x_j$ as the only state variable, the maximized Bellman equations (2.28), (2.19) respectively, simplify to:

$$\begin{aligned}\rho V_i(\Delta_i) =& \pi_i(\Delta_i) - z(u_i(\Delta_i)) \\ &+ [V_i(\Delta_i + 1) - V_i(\Delta_i)]\, h_i(u_i(\Delta_i)) + [V_i(\Delta_i - 1) - V_i(s)]\, h_j(u_j(\Delta_i)).\end{aligned} \tag{2.29}$$

Proof of Lemma 2.1. Using the envelope theorem and $\lambda = 1$ the first order condition for firm i yields

$$V_i(\Delta_i + 1) - V_i(\Delta_i) = u_i(\Delta_i)^{\eta - 1}. \tag{2.30}$$

Note that each R&D effort is proportional to the incremental value that would result from innovating.[26] Inserting this in (2.29) gives

$$V_i(\Delta_i) = \frac{1}{\rho}\Big(\pi_i(\Delta_i) - z(u_i(\Delta_i)) - z'(u_i(\Delta_i))u_i(\Delta) - z'(u_i(\Delta_i - 1))u_j(\Delta_i)\Big) \tag{2.31}$$

and

$$\begin{aligned}V_i(\Delta_i + 1) = \frac{1}{\rho}\Big(&\pi_i(\Delta_i + 1) - z(u_i(\Delta_i + 1)) - z'(u_i(\Delta_i + 1))u_i(\Delta_i + 1) \\ &- z'(u_1(\Delta_i))u_j(\Delta_i + 1)\Big).\end{aligned} \tag{2.32}$$

Using this in the first order condition yields

$$\begin{aligned}&u_i(\Delta_i)^{\eta-1}\left(\rho + u_i(\Delta_i) - u_j(\Delta_i + 1)\right) \\ =&\pi_i(\Delta_i + 1) - z(u_i(\Delta_i + 1)) - \pi_i(\Delta_i) + z(u_i(\Delta_i)) \\ &+ u_i(\Delta_i + 1)^{\eta} - (u_i(\Delta_i - 1))^{\eta-1}u_j(\Delta_i).\end{aligned} \tag{2.33}$$

As the firms are ex ante symmetric, $u_i(\Delta_i) = u_j(-\Delta_i)$ holds. This yields the reduced form R&D equations

$$\begin{aligned}&u_i(\Delta_i)^{\eta-1}\left(\rho + u_i(\Delta_i) - u_i(-\Delta_i - 1)\right) \\ =&\pi_i(\Delta_i + 1) - z(u_i(\Delta_i + 1)) - \pi_i(\Delta_i) + z(u_i(\Delta_i)) \\ &+ u_i(\Delta_i + 1)^{\eta} - (u_i(\Delta_i - 1))^{\eta-1}u_i(-\Delta_i).\end{aligned} \tag{2.34}$$

[26] For the special case $\eta = 2$ effort is even strictly proportional to the incremental value.

For the special case of $\Delta_i = 0$, this simplifies to

$$\begin{aligned} &u_i(0)^{\eta-1}\left(\rho + u_i(0) - u_1(-1)\right) \\ =&1 - e^{-1} - \frac{1}{\eta}\left(u_i(1)^\eta - u_i(0)^\eta\right) + u_i(1)^\eta - (u_i(-1))^{\eta-1}u_i(0). \end{aligned} \tag{2.35}$$

Solving this for $u(0)$[27] and assuming $\eta = 2$ yields

$$u(0) = \sqrt{2 - \frac{2}{e} + \rho^2 + u(1)^2} - \rho. \tag{2.36}$$

For $\Delta > 0$ we have $\pi(\Delta+1) - z(u(\Delta+1)) - \pi(\Delta) + z(u(\Delta)) = e^{-\Delta-1}(1-e) + \frac{1}{\eta}\left(u(\Delta)^\eta - u(\Delta+1)^\eta\right)$ while when $\Delta < 0$ the increase in profit when moving one step ahead is zero, i.e., $\pi(u(\Delta+1)) - \pi(u(\Delta)) = 0$. Using this we get the relations of optimal R&D stated in Lemma 2.1. □

Proof of Proposition 2.1. We restrict attention to non-fluctuating pattern. That means for all $\Delta > 0$: if $u(\Delta) > u(\Delta+1)$ then $u(\Delta+1) > u(\Delta+2)$ and vice versa. The same has to be true for the follower, i.e., for all $-\Delta < 0$: if $u(-\Delta) > u(-\Delta-1)$ then $u(-\Delta-1) > u(-\Delta-2)$ and vice versa. Hence, we are not able to rigorously prove the uniqueness of the equilibrium we derive. However, economic intuitions for fluctuating effort do not exist when time preference rate is set to zero.

From equation (2.5) in the first part of Lemma 2.1, obviously $u(0) > u(1)$ results.

To see how the effort of the industry's follower reacts on technological jumps we firstly analyze equation (2.7) given in the third part of Lemma 2.1.

This equation gives for $-\Delta < 0$

$$\begin{aligned} \left(u(-\Delta) - \tfrac{1}{2}u(\Delta-1)\right)^2 = &\left(u(-\Delta+1) - \tfrac{1}{2}u(\Delta-1)\right)^2 \\ &+ u(-\Delta+1)u(\Delta-1) - u(-\Delta-1)u(\Delta) \end{aligned}$$

and hence

$$\begin{aligned} &\text{sign}\{(u(-\Delta) - \tfrac{1}{2}u(\Delta-1))^2 - (u(-\Delta+1) - \tfrac{1}{2}u(\Delta-1))^2\} \\ &\quad = \text{sign}\{u(-\Delta+1)u(\Delta-1) - u(-\Delta-1)u(\Delta)\}. \end{aligned}$$

[27]For the sake of readability we will suppress the identity of the firm where not necessary.

Here, we have to distinct different cases. Let us first assume that $u(-\Delta) > \frac{1}{2}u(\Delta)$ $\forall \Delta > 0$ (implying $u(-\Delta+1) > \frac{1}{2}u(\Delta-1)$ $\forall\Delta > 0$). Then, we get

$$\begin{aligned} &\operatorname{sign}\{u(-\Delta) - u(-\Delta+1)\} \\ &\qquad = \operatorname{sign}\{u(-\Delta+1)u(\Delta-1) - u(-\Delta-1)u(\Delta)\}. \end{aligned} \tag{2.37}$$

This means efforts for the follower are decreasing with the gap when $u(-\Delta+1)u(\Delta-1) < u(-\Delta-1)u(\Delta)$ holds. The opposite is true for $u(-\Delta) < \frac{1}{2}u(\Delta)$ $\forall\Delta > 0$, i.e., follower's effort increases if $u(-\Delta+1)u(\Delta-1) > u(-\Delta-1)u(\Delta)$.

For $\Delta = 1$, these relations also compare the follower's effort with the effort of a neck-and-neck firm. Analyzing the situation for $-\Delta = -1$ yields

$$\operatorname{sign}\{u(-1) - u(0)\} = \operatorname{sign}\{u(0)^2 - u(-2)u(1)\}.$$

We already know that investment in neck-and-neck state is higher than effort of a firm being one step ahead, i.e., $u(0) > u(1)$. This indicates that $u(0)^2 > u(-2)u(1)$ might hold, implicating $u(-1) > u(0)$. This is true as long as $u(-2)$ is not too large, i.e., $u(-1) > u(0) > u(1)$ iff $u(-2) < \frac{u(0)^2}{u(1)} > u(0)$. If $u(-2)$ is large enough to outweigh the difference between $u(0)$ and $u(1)$ we have $u(-1) > u(0)$. In that case the relation $u(-2) > u(0) > u(-1)$ holds. That means the optimal patterns shows some kind of fluctuation.

We can illustrate the characteristics of the general equation (2.37) by means of the example of $\Delta = 2$. Equation (2.37) yields

$$\operatorname{sign}\{u(-2) - u(-1)\} = \operatorname{sign}\{u(-1)u(1) - u(-3)u(2)\}.$$

As we are not looking for fluctuating patterns, we either have $u(-3) > u(-2) > u(-1)$ or $u(-3) < u(-2) < u(-1)$. In the first case, $u(2)$ would have to be sufficiently small to ensure $u(-1)u(1) > u(-3)u(2)$. In the case of $u(-3) < u(-2) < u(-1)$, $u(2)$ needs to be sufficiently large. Obviously, in either case do R&D efforts for leader and follower go into opposite directions when the gap increases. This clearly also holds true for the general case of $\Delta > 2$ and equation (2.37). More precisely, $u(-\Delta) \gtreqless u(-\Delta+1)$ if $\frac{u(-\Delta+1)}{u(-\Delta-1)} \gtreqless \frac{u(\Delta)}{u(\Delta-1)}$. If now $u(\Delta) > u(\Delta-1)$, $u(-\Delta+1) > u(-\Delta) > u(-\Delta-1)$ has to hold and vice versa.

Keeping in mind that leader's and follower's effort move into opposite directions when the gap increases, let's now analyze the leader's optimal effort given by equation (2.6) in the second part of Lemma 2.1. We can directly see that for $\Delta > 0$

$$\begin{aligned}&(u(\Delta) - u(-\Delta - 1))^2\\ &= (u(\Delta + 1) - u(-\Delta - 1))^2\\ &\quad + 2\left(u(-\Delta - 1)u(\Delta + 1) - u(\Delta - 1)u(-\Delta) + e^{-\Delta}\left(1 - \tfrac{1}{e}\right)\right),\end{aligned}$$

and hence

$$\begin{aligned}&\operatorname{sign}\{(u(\Delta) - u(-\Delta - 1))^2 - (u(\Delta + 1) - u(-\Delta - 1))^2\}\\ &\quad = \operatorname{sign}\{u(-\Delta - 1)u(\Delta + 1) - u(\Delta - 1)u(-\Delta) + e^{-\Delta}\left(1 - \tfrac{1}{e}\right)\}\end{aligned}$$

holds. Again, we have to distinct different cases. Let us first assume $u(\Delta) < u(-\Delta-1)$ and $u(\Delta) < u(-\Delta)\ \forall \Delta > 0$. In that case the assumption $u(-\Delta) > \frac{1}{2}u(\Delta)\ \forall \Delta > 0$ holds as well. Besides, as we know leader's and follower's effort move into opposite directions, the assumptions implicate $u(\Delta') < u(-\Delta)\ \forall \Delta, \Delta' > 0$.

Then, we get

$$\begin{aligned}&\operatorname{sign}\{u(\Delta + 1) - u(\Delta)\}\\ &\quad = \operatorname{sign}\{u(-\Delta - 1)u(\Delta + 1) - u(\Delta - 1)u(-\Delta) + e^{-\Delta}\left(1 - \tfrac{1}{e}\right)\}.\end{aligned} \tag{2.38}$$

That means, we see increasing efforts for leaders when $u(-\Delta - 1)u(\Delta + 1) > u(-\Delta)u(\Delta - 1) - e^{-\Delta}\left(1 - \frac{1}{e}\right)$.

In the case of $u(\Delta') < u(-\Delta)\ \forall \Delta, \Delta' > 0$ the leader's effort can only be increasing if the follower's effort is decreasing and furthermore if $u(-\Delta - 1)u(\Delta + 1) > u(-\Delta)u(\Delta - 1) - e^{-\Delta}\left(1 - \frac{1}{e}\right)$ holds. As in this case $u(\Delta + 1) > u(\Delta - 1)$ holds, we have found an equilibrium where the leader's increase in effort with an increase in gap is not too large. Besides, it is clear that beyond this $u(-1) > u(0) > u(1)$ holds, since effort is decreasing for the follower and therefore $u(-2) > u(0) > u(-1)$ cannot hold. The resulting pattern is that summarized in terms of investment by Proposition 2.1. Hence, we have shown that the described optimal behavior is indeed an equilibrium. □

To show that no other equilibria exist is a very comprehensive task and needs quantifying analysis. Unfortunately, we are not able to analytically show the uniqueness of the equilibrium.

2.6.3 The Effect of Knowledge

Proof of Proposition 2.3. With the simplifying assumption that investment in R&D does not influence a firm's probability of success immediately, i.e., with $\lambda = 0$, the first order conditions read

$$-1 + \frac{\partial V_i(s)}{\partial k_i} u'(z_i(s)) = 0, \tag{2.39}$$

yielding

$$d\frac{\partial V_i(s)}{\partial k_i} = d\frac{1}{u'(z_i(s))}. \tag{2.40}$$

Therefore, we get the optimal rule describing the evolution of marginal profits:

$$\begin{aligned} -d\frac{-1}{u'(z_i(s))} = &\Bigg\{ -(\rho+\delta)\frac{-1}{u'(z_i(s))} + \left[\frac{-1}{u'(z_i(x_i+1,\cdot))} - \frac{-1}{u'(z_i(s))}\right]\gamma k_i^{\alpha} \\ &+ \left[\frac{-1)}{u'(z_i(x_j+1,\cdot))} - \frac{-1}{u'(z_i(s))}\right]\gamma k_j^{\alpha} \\ &- \left[V_i(x_i+1,\cdot) - V_i(s)\right]\alpha\gamma k_i^{\alpha-1} \Bigg\} dt \\ &+ \left[\frac{-1}{u'(z_i(s))} - \frac{-1}{u'(z_i(x_i+1,\cdot))}\right] dq_i(s) \\ &+ \left[\frac{-1}{u'(z_i(s))} - \frac{-1}{u'(z_i(x_j+1,\cdot))}\right] dq_j(s). \end{aligned} \tag{2.41}$$

The rule shows how marginal profit changes in a deterministic and stochastic way. While there is a one-to-one mapping from marginal profit to investment which allows some inferences about investment from (2.41), it would be more useful to have a rule for optimal investment itself. With firms' instantaneous profits (2.3) and rate of knowledge acquisition $u(z_i) = (\eta z_i)^{\frac{1}{\eta}}$ we get

$$d\frac{-1}{u'(z_i(s))} = -\frac{\partial \frac{1}{u'(z_i(s))}}{\partial z_i(s)} dz_i(s) = \frac{u''(z_i(s))}{u'(z_i(s))^2} dz_i(s) = (1-\eta)(\eta z_i(s))^{-\frac{1}{\eta}} dz_i(s). \tag{2.42}$$

Due to the modeling approach only the technological gap and not the technological levels as such matters for firms' values, i.e., the effect of the competitor moving one step forward is the same as moving one step backwards. Thus, using (2.42) we can write

$$\begin{aligned} dz_i(s) =& \frac{(\eta z_i(s))^{\frac{1}{\eta}}}{\eta - 1} \Bigg[\Bigg\{ \frac{(\rho+\delta)}{u'(z_i(s))} + \left[\frac{1}{u'(z_i(s))} - \frac{1}{u'(z_i(x_i+1,\cdot))} \right] \gamma k_i^{\alpha} \\ &+ \left[\frac{1}{u'(z_i(s))} - \frac{1}{u'(z_i(x_i-1,\cdot))} \right] \gamma k_j^{\alpha} - [V_i(x_i+1,\cdot) - V_i(s)]\, \alpha \gamma k_i^{\alpha-1} \Bigg\} dt \\ &+ \left[\frac{1}{u'(z_i(x_i+1,\cdot))} - \frac{1}{u'(z_i(s))} \right] dq_i(s) \\ &+ \left[\frac{1}{u'(z_i(x_i-1,\cdot))} - \frac{1}{u'(z_i(s))} \right] dq_j(s) \Bigg]. \end{aligned} \tag{2.43}$$

These rules describe the evolution of investment under optimal behavior for the firms. Growth of investment depends on the right-hand side in a deterministic way on the typical sum of the depreciation and time preference rate per marginal rate of knowledge acquisition plus the "k-terms" which capture the impact of uncertainty. To understand the meaning of these terms we analyze whether investment jumps up or down, following a jump of the own or competitor's technology. Since $\eta > 1$ the term $\frac{1}{u'(z_1(s))} - \frac{1}{u'(z_1(x_1+1,\cdot))} = \eta^{\frac{\eta-1}{\eta}} \left(z_1(s)^{\frac{\eta-1}{\eta}} - z_1(x_1+1,\cdot)^{\frac{\eta-1}{\eta}} \right)$ is negative if $z_1(s) < z_1(x_1+1,\cdot)$. If this is the case, investment increases slower (or decreases even faster) if the probability of a jump of the own technology due to a higher knowledge stock is high. On the other hand investment increases faster (or decreases slower) if the probability of a jump of the competitor's technology due to his higher knowledge stock is high.

The dq_{xi}-terms give discrete changes in the case of a jump in x_i. When x_i jumps and $dq_{x_i(s)} = 1$ ($dq_{x_j(s)} = 0$, i.e., there is no contemporaneous jump in x_j) and $dt = 0$ for this small instant of the jump, equation (2.43) says that $dz_i(s)$ on the left hand side is given by

$$\frac{\eta z_i(s)}{\eta - 1} \left(z_i(s)^{\frac{1-\eta}{\eta}} z_i(x_i+1,\cdot)^{\frac{\eta-1}{\eta}} - 1 \right) \tag{2.44}$$

on the right hand side. This is positive as long as $z_i(s) < z_i(x_i+1,\cdot)$ which is consistent with the definition of $dz_i(s)$ given by $z_i(x_i+1,\cdot) - z_i(s)$. Solving this for $z_i(s)$ interestingly yields $z_i(s) = z_i(x_i+1,\cdot)$. Hence, optimal investment does not immediately react to a jump in the industry's state. This is the result stated in Proposition 2.3. □

Proof of Lemma 2.2. Using the derived fact, from (2.43) we can determine the evolution of optimal investment:

$$\frac{dz_i(s)}{dt} = z_i(s)\frac{\eta}{\eta-1}\Big(\rho+\delta-(\eta z_i(s))^{\frac{1-\eta}{\eta}}\underbrace{\left[V_i(x_i+1,\cdot)-V_i(s)\right]}_{\equiv\Phi_i(s)}\alpha\gamma k_i^{\alpha-1}\Big). \quad (2.45)$$

We can not determine the value of $\Phi_i(s)$, but we know it is always positive and from the first order condition we get $\frac{\partial\Phi_i(s)}{\partial k_i}=0$. Thus, $\Phi(\cdot)$ is only a function of the technological gap Δ and the same function for both firms, i.e $\Phi_i(\Delta)=\Phi_j(-\Delta)$. Hence, we can write $\Phi(\Delta_i)\equiv\Phi_i(\Delta)$. With this result it can directly be seen that the more knowledge a firm has acquired the smaller is the growth rate of optimal investment.

Furthermore, from (2.28) and with $\lambda=0$ we can analyze the shape of $\Phi(\Delta)$. As the value function inherits its shape from the profit function, the value function will be bounded from below and above and will converge to these bound for $\Delta\to-\infty$ and $\Delta\to\infty$ respectively. Hence, $\frac{\partial(V_i(\Delta+1)-V_i(\Delta))}{\partial\Delta}$ is negative for high values of Δ and positive for small (negative) values.

As the slope of the value function measures a leader's incentive to innovate, this slope is maximal around the neck and neck point since neck-and-neck firms perform R&D at a higher intensity than industry leaders.

The maximized Bellman equations (2.28) hold for all optimal efforts, especially at steady state as well, i.e., when $u(z_j)-\delta k_j=0$. In this case, for $\Delta<0$ we have:

$$\rho V_i(s) = -z_i(s)+\left[\Phi(\Delta)\right]\gamma k_i^{\alpha}-\left[\Phi(\Delta-1)\right]\gamma k_j^{\alpha}.$$

The right hand side can only be positive for positive $\Phi(\Delta)$ if $\Phi(\Delta)>\Phi(\Delta-1)$ and therefore we have $\frac{\partial\Phi(\Delta)}{\partial\Delta}>0$ for $\Delta<0$.

The derivative of the maximized Bellman equation (2.28) with respect to Δ for $\Delta<0$ using the envelope theorem gives

$$\begin{aligned}\rho\frac{\partial V_i(\Delta,k_i,k_j)}{\partial\Delta} =& \frac{\partial^2 V_i(\Delta,\cdot)}{\partial\Delta\partial k_j}\left(u(z_j(s))-\delta k_j\right)\\ &+\left[\frac{\partial V_i(\Delta+1,\cdot)}{\partial\Delta}-\frac{\partial V_i(\Delta,\cdot)}{\partial\Delta}\right]\gamma k_i^{\alpha}\\ &+\left[\frac{\partial V_i(\Delta-1,\cdot)}{\partial\Delta}-\frac{\partial V_i(\Delta,\cdot)}{\partial\Delta}\right]\gamma k_j^{\alpha}. \quad (2.46)\end{aligned}$$

For $\Delta > 0$ the derivative yields

$$\begin{aligned}\rho\frac{\partial V_i(\Delta,k_i,k_j)}{\partial\Delta} =& e^{-\Delta}+\frac{\partial^2 V_i(\Delta,\cdot)}{\partial\Delta\partial k_j}\left(u(z_j(s))-\delta k_j\right)\\ &+\left[\frac{\partial V_i(\Delta+1,\cdot)}{\partial\Delta}-\frac{\partial V_i(\Delta,\cdot)}{\partial\Delta}\right]\gamma k_i^{\alpha}\\ &+\left[\frac{\partial V_i(\Delta-1,\cdot)}{\partial\Delta}-\frac{\partial V_i(\Delta,\cdot)}{\partial\Delta}\right]\gamma k_j^{\alpha}. \qquad (2.47)\end{aligned}$$

Here, we can see that the only critical value of Δ, i.e., a value where signs could possibly change, is indeed at $\Delta = 0$.

We can now determine the derivative of the value effect of a technological step ahead for $\Delta > 0$:

$$\begin{aligned}&\rho\frac{\partial\left(V_i(\Delta+1,k_i,k_j)-V_i(\Delta,k_i,k_j)\right)}{\partial\Delta}\\ &= e^{-\Delta-1}(1-e)+\left(\frac{\partial^2 V_i(\Delta+1,\cdot)}{\partial\Delta\partial k_{-i}}-\frac{\partial^2 V_i(\Delta,\cdot)}{\partial\Delta\partial k_{-i}}\right)\left(u(z_j)-\delta k_j\right)\\ &+\left[\frac{\partial V_i(\Delta+2,\cdot)}{\partial\Delta}-2\frac{\partial V_i(\Delta+1,\cdot)}{\partial\Delta}+\frac{\partial V_i(\Delta,\cdot)}{\partial\Delta}\right]\gamma k_i^{\alpha}\\ &+\left[\frac{\partial V_i(\Delta,\cdot)}{\partial\Delta}-\frac{\partial V_i(\Delta+1,\cdot)}{\partial\Delta}-\frac{\partial V_i(\Delta-1,\cdot)}{\partial\Delta}+\frac{\partial V_i(\Delta,\cdot)}{\partial\Delta}\right]\gamma k_j^{\alpha}. \qquad (2.48)\end{aligned}$$

We know $V(\cdot)$ is approaching the upper bound for $\Delta \to \infty$. Hence, for sufficiently large values of Δ, the second derivative of the value function and thus the first derivative of $\Phi(\Delta)$ will be negative. But even more, we see that the right hand side of equation (2.48) is negative for all $\Delta > 0$ in steady state with sufficiently low knowledge stocks as $e^{-\Delta-1}(1-e) < 0$. Hence, the value function's inflection point has to be at $\Delta = 0$ and we have $\frac{\partial\Phi(\Delta)}{\partial\Delta} < 0$ for $\Delta > 0$.

□

Chapter 3

Integration of Economies Triggering Cross-Border Merger Waves: A General Equilibrium Analysis

3.1 Introduction

During past decades the world is moving closer to free trade. In North America, the Canada-US Free Trade Agreement (1989) was followed by NAFTA (1994). Europe is moving gradually towards full economic integration and tariff levels across many other countries have been declining (Baggs, 2005).

At the same time there is a tendency inherent in markets for increasing concentration. During the period 1980-99 worldwide mergers grew at annual rate of 42% (UNCTAD, 2000). The merger wave in late 1990's was five times larger than in late 1980's (Hijzen, Görg, & Manchin, 2008). Gugler, Mueller, and Weichselbaumer (2008) present empirical evidence that merger waves occurred in the USA, UK, and Continental Europe at the end of the 20th century. The development of global steel production that attracted some attention in media can be seen as a prime example (see for example Shah, 2004; Kloepfer, 2005; Hargreaves, 2006). Especially market leader ArcelorMittal increased its share of worldwide steel production from 2% in 2000 to 10% in 2007 due to numerous mergers and acquisitions (see ArcelorMittal, 2008).

This work exploits the tendency of dissolving barriers to trade and to enter new markets. We analyze welfare effects of globalization, i.e., the integration of economies, using a general equilibrium framework. To exploit this development we need to ask whether dissolving barriers to trade and to enter new markets increase incentives for mergers and acquisitions and so enforce the tendency for increasing concentration. If so, this accompanied phenomenon has to be taken into account.

Various explanations have been offered for the rise of mergers in recent years. Merger waves as such have been explained by Harford (2005) as a result of different shocks, some technical, some regulatory, and some taking on other forms. Qiu and Zhou (2007) further exploit the phenomenon of merger waves due to firm heterogeneity. Several studies investigate whether trade liberalization actively encourage mergers (e.g. Gaudet & Kanouni, 2004; Faria & Yildiz, 2005). Bjorvatn (2004) attributes the rising number of mergers to closer economic integration of markets as increasing competition may increase merger incentives. Qiu and Zhou (2006) explain the phenomenon by information asymmetry about domestic demand that exists between domestic and foreign firms. Additionally, in the presence of trade costs, cross-border mergers provide an alternative access to a foreign market that avoids these costs. Amongst others Horn and Persson (2001) and Salvo (2009) explore this ”tariff jumping” effect that rises merger incentives. Salvo provides a very detailed analysis of effects of trade costs and international differences in quality of goods.

The described development lends itself to analyzing the interaction of competition policy and trade policy. Studies like Horn and Levinsohn (2001) and De Stefano and Rysman (2009) examine the optimal competition policy taking trade liberalization into account.[1] This strand of literature is still growing fast and the work of Horn and Levinsohn (2001) has been extended into several directions. For example, Motta and Ruta (2008) develop a model of merger policy and lobbying in international markets, while Chaudhuri and Benchekroun (2008) analyze welfare effects of equal bilateral tariff reduction taking merger incentives into account.

Thus, partial equilibrium analysis of liberalization of trade and cross-border mergers is very comprehensive and has provided numerous diverse results. In con-

[1] The seminal paper by Horn and Levinsohn (2001) also provides a detailed overview of important literature up to this point.

trast, general equilibrium frameworks are still scarce.

We analyze effects of integration in oligopolistic industries with a theoretical general equilibrium model. The main goal of this essay is to analyze how the integration of (maybe more efficient) markets and the accompanying mergers and acquisitions affect consumer surplus and welfare.

Only very recently general equilibrium trade models have been used to study foreign market access. J. Francois and Horn (2007) argue that these frameworks are more appropriate than the established partial equilibrium models to study matters like the integration of economies and international merger waves that have strong cross-sectoral ramifications. Empirical studies such as Resende (1999) and Gärtner and Halbheer (2009) indeed find that industries seem to have been rather uniformly affected by merger waves. However, in contrast to our work Francois and Horn focus on antitrust with foreign direct investment. They do not assume imperfect competition and do not analyze mergers. Mergers in a general equilibrium model are studied by Nocke and Yeaple (2007). The scope of their study significantly differs from our framework as different modes of foreign market access are compared for different kinds of firm heterogeneity in a model of monopolistic competition. They find that mergers are in many cases preferred by the most efficient firms such that our concentration on mergers seems to not exclude important effects. Closest to our work is the paper of Neary (2007) who studies merger incentives in oligopoly in general equilibrium. Nevertheless, integration of economies is not analyzed. In that framework integration rather serves as a motivation for the starting point of two groups of firms. In contrast we focus on the effect of integration that may be accompanied by mergers. Additionally, we extend the study to endogenous market structure and free entry.

For illustrating firms' incentives and price effects initially we consider a partial equilibrium sector model. We show that integration of economies as such yields decreasing market prices even when all firms of one country are so inefficient that they have to exit. Overall welfare rises as possibly declining producer surplus is overcompensated by consumer gains. We further show that merger waves might indeed be triggered by globalization. These waves take place only for a sufficiently large heterogeneity in marginal cost. Prices increase with merger waves, of course. Nevertheless, consumer surplus determined by prices increases with the integration of economies even if this triggers a merger wave as the latter only appears when the acquiring firms are sufficiently efficient. As profitable mergers increase firms' profits, overall welfare rises with integration of economies.

However, this result may be misleading. Partial equilibrium frameworks implicitly assume that the industry under study is the only industry that is affected by integration such that it takes factor prices and aggregate income as given, and pays no attention to interactions between markets. The reliance on a partial equilibrium framework may be appropriate when the focus is on a matter in a particular sector. But when the analysis concerns a development affecting the entire economy, it is less adequate. In particular the integration of economies and accompanying merger waves in many industries will affect broad swathes of the countries involved and is therefore likely to have strong cross-sectoral ramifications. The purpose of this essay is to examine some aspects of globalization in a framework that takes account of such ramifications.

Therefore, the main analysis of this chapter is provided using an appropriate general equilibrium model as proposed by Neary (2002). A continuum of sectors is considered and countries dispose of different comparative advantages in every sector such that integrated equilibrium market structure varies across sectors. In contrast to partial equilibrium, due to bounded supply of production factors welfare gains can only be realized by reallocation of resources. Comparative advantage is thus important for gains from trade. The higher the technological dissimilarity between countries the better can efficiency gains due to resource reallocation be realized in integrated general equilibrium. Evaluating integration yields that in case of full diversification, i.e., neither exit nor acquisitions of firms in any sector, welfare always increases with free trade and gains are higher the higher the degree of comparative advantages. However, specialization in sectors due to exit or merger waves let intensity of competition decrease which raises prices and lowers welfare. On the other hand wages decline as output does with absorption of firms. This tends to lower prices and increases welfare. The overall effect is indeterminate. These results suggest that general equilibrium matters as efficiency gains can be realized by resource reallocation across sectors only.

So far it is assumed that firms can strategically leave the market or take over other firms. We further endogenize the market structure by allowing for free entry so that mergers incentives vanish. Nevertheless, coexistence of firms from different countries in an integrated world is no longer possible. In partial equilibrium welfare always increases. In contrast, in general equilibrium where full specialization results, i.e., in each sector only firms of one country are active, the overall welfare effect is ambiguous. As resources for entry are bounded as well technological dissimilarities of sectors with respect to entry and production are important for realizing welfare gains.

The rest of the chapter is organized as follows. Section 3.2 presents the sector model and benchmark analysis of integration in the absence of mergers. Section 3.3 provides the study of merger incentives and resulting welfare effects. Results with free entry are displayed in Section 3.4. Section 3.5 presents the general equilibrium model while integration in general equilibrium is then analyzed in Section 3.6. Again, the subsequent Section 3.7 shows how results change when allowing for free entry. Eventually, Section 3.8 concludes while the appendix contains the proofs of the results stated in the text.

3.2 Integration of Economies in the Absence of Mergers

As a benchmark and for illustration of basic incentives and effects, we consider a single sector in partial equilibrium and leave the rest of the economy out of the analysis. We show how integration can lead to less competition due to rising merger incentives. We will analyze the effects on consumer and producer surplus and on welfare.

3.2.1 The Sector Model

We consider two different economies, called home and foreign. In both economies the same homogeneous commodity is produced by a small number of firms. Each firm is only active in one economy, i.e., there are no multinational firms, such that each firm is allocated at a unique location only. In each location firms face the same infrastructure and it is assumed that firms dispose of the same technologies such that production costs are identical. Nevertheless, technologies and infrastructure and thus unit cost differ across countries. We define the cost functions as $C_i(x) = cx$

and $C_i'(x) = c'x$ for home and foreign firms, respectively.[2] Marginal costs are thus constant for all firms.[3]

Firms are assumed to be Cournot competitors and firm i's output is denoted as $x_i > 0$ when it is a domestic firm and $x_i' > 0$ when it is a foreign firm. To begin with, we consider the economies to be separated, i.e., in autarky. Initially, there are $n > 2$ firms at home and $n' > 2$ firms competing abroad. In each economy, firms are facing a linear decreasing inverse demand function[4]

$$p(X) = a - bX, \tag{3.1}$$

with X as an economy's total output, e.g., $X = \sum_{i=1}^{n} x_i$ as domestic output. We assume that tastes and market size are identical in both countries so that $p'(X) = a - bX$.[5]

Economies are symmetric and we will now focus on the home country. Assuming that $c < a$, i.e., firms are active in the market, in equilibrium each firm sells quantity

$$x = \frac{a-c}{(n+1)b}. \tag{3.2}$$

Total quantity produced is $X = n\frac{a-c}{(n+1)b}$ and the corresponding market price is

$$p = \frac{a+nc}{n+1} \tag{3.3}$$

such that each firm's profit in equilibrium is given by $\pi = \frac{(a-c)^2}{(n+1)^2 b} = bx^2$.[6] Similar results hold for the foreign economy, of course.

[2]Henceforth, we will indicate variables associated with the foreign economy by prime.

[3]We abstract from fixed costs as they provide a trivial incentive for mergers.

[4]An explanation of why quadratic utility and thus linear demand is very reasonable to assume from general equilibrium perspective is provided in footnote 29. Additionally, linear demand provides the desirable feature that equilibrium price does not depend on the market size determined by b, but only on the maximum willingness to pay a. Hence, when analyzing the integration of economies we can leave the effect of a larger market unconsidered and concentrate on competition effects through number and efficiency of firms only.

[5]As only minor additional insights can be gained by the general setting with different market characteristics we think that the gain in clarity outweighs the loss in generality such that we restrict attention to the case of identical markets.

[6]For a general derivation of equilibrium see Appendix 3.9.1.1.

3.2.2 Integration of Economies

Let us now consider an economy wide shock that eliminates restrictions on trade and to engage in the foreign economy. Thus, economies get fully integrated and integration of the economies lead to one demand function of the new bigger economy. This is given as $\widehat{D}(p) = D(p)+D'(p) = 2\frac{a-p}{b}$ and the inverse demand function simply is

$$\widehat{p}(X) = a - \frac{b}{2}X.^{7} \tag{3.4}$$

The reservation price a in the integrated economy is the same as in autarky while the saturation quantity is now doubled to $2\frac{a}{b}$.

Firms may now exit as they might no longer be able to earn positive profits.[8] If no firm exits, firms' new equilibrium quantities are

$$\hat{x}(n, n') = \frac{a-(n'+1)c+n'c'}{(n+n'+1)\frac{b}{2}} = 2\frac{a-c+n'(c'-c)}{(n+n'+1)b}; \tag{3.5}$$

$$\hat{x}'(n, n') = \frac{a-(n+1)c'+nc}{(n+n'+1)\frac{b}{2}}. \tag{3.6}$$

where $\hat{x}(n, n')$ and $\hat{x}'(n, n')$ denote the output of the domestic firm and foreign firm, respectively, with n domestic and n' foreign firms in the economy.

These quantities induce the market price

$$\hat{p}(n, n') = \frac{a+nc+n'c'}{n+n'+1} \tag{3.7}$$

and firms' equilibrium profits

$$\hat{\pi}(n, n') = \frac{(a-c+n'(c'-c))^2}{(n+n'+1)^2\frac{b}{2}} = \frac{b}{2}\hat{x}(n, n')^2;$$

$$\hat{\pi}'(n, n') = \frac{(a-c'+n(c-c'))^2}{(n+n'+1)^2\frac{b}{2}} = \frac{b}{2}\hat{x}'(n, n')^2.$$

All firms remaining active and producing positive quantities in the new market requires

$$c \leq \frac{a+n'c'}{n'+1} \text{ and } c' \leq \frac{a+nc}{n+1}. \tag{3.8}$$

[7]Henceforth, we will indicate variables associated with the integrated economy by hat.

[8]For now we assume that trade liberalization cannot induce entry of new firms.

Firms which are not able to gain positive profits will exit. With $\theta_0 \equiv \frac{1}{n'+1}$, the number of firms in the new global economy will decrease due to exit of home firms if and only if

$$c > \theta_0 a + (1 - \theta_0)c'. \tag{3.9}$$

Here, $\theta_0 \in [0, \frac{1}{2}]$ and is decreasing in n'. On the other hand, foreign firms will exit if and only if

$$c' > \theta_0' a + (1 - \theta_0')c$$

with $\theta_0' \equiv \frac{1}{n+1}$. Thus, profits of a firm will be positive if and only if its unit costs are less than or equal to a weighted average of the demand intercept a and the unit cost of the other country's firms. As θ_0 is independent of n, either all or no firms of one country exit and the situation is independent of considering sequential or simultaneous exit such that condition (3.9) states that all domestic firms exit when they are not efficient enough. The condition continues to hold with $n' = 0$ (yielding $\theta_0 = 1$) as in that case firms earn positiv profits when $c < a$.

3.2.3 Welfare Effects of Integration

We will now turn to considering gains from trade. We wish to determine the effect of the integration of economies on consumers, producers and overall welfare.

Consumer's utility is solely determined by market prices so that we compare the respective prices to analyze whether consumers of the former separated economies are better or worse off through integration. The following proposition can easily be derived:

Proposition 3.1. *Prices in both countries decrease such that consumer surplus increases with the integration of economies.*

Proof: See Appendix 3.9.1.2.

With identical linear demand consumers in both economies benefit from integration as with all firms remaining active the increase in competition let prices fall. In case of firms' exit the gain in efficiency outweighs the possible loss due to less intense competition. Integration reallocates production to more efficient firms. After integration inefficient firms produce less (maybe even nothing) while efficient firms more than substitute this fall in output. As average production cost decline with reallocation overall output increases.[9] With firms' exit, this effect is even more obvious. In that case efficient firms double their production as market size is doubled while their initial production was higher than that of inefficient firms so that overall output rises.

Although prices decline with the integration of economies total producer surplus might nevertheless rise as the average unit production costs fall.

Lemma 3.1. *Producer surplus increases when*

(i) all firms remain active and countries are not too similar with respect to size and technologies such that

$$
\begin{aligned}
&(a-c)^2(n'+1)^2 n\left((n-n'+1)^2-2n'^2\right)\\
&\quad+(a-c')^2(n+1)^2 n'\left((n'-n+1)^2-2n^2\right)+2nn'\,(n+n'+2)\,(c-c')^2>0.
\end{aligned}
\tag{3.10}
$$

(ii) one group of firms exit and competition in the less efficient country was sufficiently intense, such that there were at most two firms less active than in the efficient country, i.e., for inefficient domestic firms $n \geq n'-2$.

Proof: See Appendix 3.9.1.5.

Total profit increases when more efficient firms to some extend crowd out less efficient ones. Thus, in total firms benefit from more efficient resource allocation. Only when inefficient firms were able to gain sufficiently high profits before integration, this can not be outweighed by competitive reallocation.

However, it is exactly in these cases of very inefficient firms gaining relatively large profits in autarky that consumers benefit most from integration and increasing competition. This rise in consumer surplus dominates the possible loss in producer surplus.

Proposition 3.2. *Overall welfare rises with the integration of economies.*

Proof: See Appendix 3.9.1.6.

[9]The fact that total output increases with integration of economies can be deduced from Proposition 3.1, of course. However, for later reference output effects are formally analyzed in Appendix 3.9.1.3.

Welfare increases with integration as only the decline in prices facilitates a fall in producer surplus and this is outweighed by rising consumer surplus due to more efficient production on average.

3.3 Integration of Economies and Triggered Mergers

So far we assumed that all firms that are sufficiently efficient to gain profits continue to produce. However, opening markets to international competition may generate incentives to merge.[10] This section shows how results change when cross-border mergers and acquisitions are taken into account.[11]

3.3.1 Merger Incentives in the Integrated Economy

Our way of modeling an acquisition is based upon Salant, Switzer, and Reynolds (1983). We assume firms are able to extend capacities without costs. Thus, in case of an acquisition of a less efficient firm the acquiring firm will extend her own production and stop inefficient production such that the acquired firm is completely absorbed in the acquiring company.[12] It is necessary to impose some further structure on how exactly firms decide to merge. First, it is convenient to assume that only one firm is able to acquire other firms at one point in time. This assumption does not preclude sequential mergers, but implies that they must consist of a sequence

[10] We will use the terms mergers and acquisitions interchangeably as the way they are modeled can be interpreted both as a merger or as a takeover.

[11] As already described, in contrast to the work of Neary (2007) we wish to analyze effects of globalization, not merger waves per se. However, for analyzing the integration of economies, the resulting mergers have to be taken into account since merger waves are a direct and inevitable consequence of the integration of economies. Therefore, the results in the following subsection analyzing merger incentives inevitably replicate some results of Neary (cf. Neary, 2007, Section 3).

[12] In the present model where firms produce identical products and there are no tariffs or transport costs and without scope for internal competition as introduced by Creane and Davidson (2004), there is no incentive for a firm to operate more than one plant. A merger implies that the less efficient of the merging firms is closed down.

of bilateral mergers, each of which is desired by both parties.[13]

The merger must yield a surplus which is sufficient to compensate both participating firms. The surplus following the merger equals the profits of the surviving firm less the initial profits of both firms such that the acquired firm must be paid at least its initial profits plus an infinitesimal amount if it is to agree to the merger.

To fix ideas, let's assume a foreign firm is considering acquiring a domestic firm. The foreign firm will buy a domestic firm only if the increase in profit exceeds the domestic firm's profit. Buying is profitable if

$$\hat{\pi}'(n-1, n') - \hat{\pi}'(n, n') - \hat{\pi}(n, n') > 0. \tag{3.11}$$

A merger will not take place if (3.11) is not fulfilled. Additionally, we assume that a profitable merger will always take place such that (3.11) is a necessary and sufficient condition for a merger to take place. The equation shows that with two merging firms with the same cost and thus the same initial output and profit, i.e., with $\hat{\pi}'(n, n') = \hat{\pi}(n, n')$, the profit of the resulting firm would have to double for a merger to be profitable. This illustrates the famous result derived by Salant et al. (1983).[14]

With mergers defined as above, it is straightforward to show that a merger between two firms with the same unit cost is never profitable provided it is not a merger to monopoly.[15] Thus, there are no merger incentives in autarky and the starting point of n and n' firms in each economy is an equilibrium industrial structure in autarky.

[13] If simultaneous mergers between any number of firms were possible, then there would always be an incentive for all firms to merge, and the model could not capture the empirical phenomenon of merger waves that stop short of monopoly. However, the assumption can be relaxed to the case of simultaneous acquisitions of up to $m < \frac{1}{2}\min\{n, n'\}$ firms. Most of the following results hold with this more general setting that extends Neary's (2007) results. However assuming that only bilateral mergers take place is the most natural way to proceed. We derive results for the general case in Appendix 3.9.3.

[14] It is shown by Falvey (1998) that this result basically holds for an arbitrary distribution of marginal costs of non-participating firms. In contrast, once additional factors such as cost synergies (Farrell & Shapiro, 1990), fixed stock of production factors (Perry & Porter, 1985), and spatial competition (Levy & Reitzes, 1992) are introduced or a differentiated Bertrand oligopoly (Deneckere & Davidson, 1985) is considered, merger incentives arise even with a lower output share of the merged firm.

[15] See Appendix 3.9.2.1.

The incentive for mergers is intuitive. The acquisition and subsequently closing down of one firm raises output and therefore profits of the remaining firms as quantities are strategic substitutes. Competitors not involved in the acquisition benefit as well. With the difference in cost being sufficiently high profits of low cost foreign firms increase sufficiently to justify the takeover of one high-cost home firm.

We can now derive a condition on the difference between marginal cost of the low-cost and the ”to-be-merged” high-cost firm such that a merger is in principle profitable:

Proposition 3.3. *An acquisition of a firm with marginal cost c by a (more efficient) firm with cost c' is profitable if and only if*

$$c > \theta_1 a + (1 - \theta_1)c' \tag{3.12}$$

with

$$\theta_1 \equiv \frac{(n+n')^2 - 2(n+n') - 1}{2n(n+n') + (n'+1)\left((n+n')^2 - 1\right)}$$

holds.

Proof: See Appendix 3.9.2.2.

This proposition states that a takeover of a home firm will be profitable when its unit cost exceeds a weighted average of the demand intercept and unit cost of the acquiring firm. This condition has obviously the same form as the condition for firms' exit (3.9).

Note that θ_1 is positive for $n + n' \geq 3$ which is fulfilled by assumption. Though θ_1 is non-linear in n and n', equation (3.12) (as well as equation (3.9)) defines a linear upward sloping locus in the $\{c, c'\}$ space. The slopes of these conditions intersect at $c = c' = a$. Production in at least one economy is only profitable up to this point and thus before curves intersect. Considering only the intersections with the axis $a\theta_0$ and $a\theta_1$, respectively, is therefore sufficient to compare the strength of the conditions. It is now easy to see that the merger condition is weaker than the condition for firms' exit:[16]

$$\theta_0 > \theta_1. \tag{3.13}$$

Thus, the linear upward sloping locus in the $\{c, c'\}$ space defined by inequality (3.12) lies strictly below the one given by (3.9). This is depicted in Figure 3.1.

[16]This and the following properties of merger condition (3.12) are derived in Appendix 3.9.2.3.

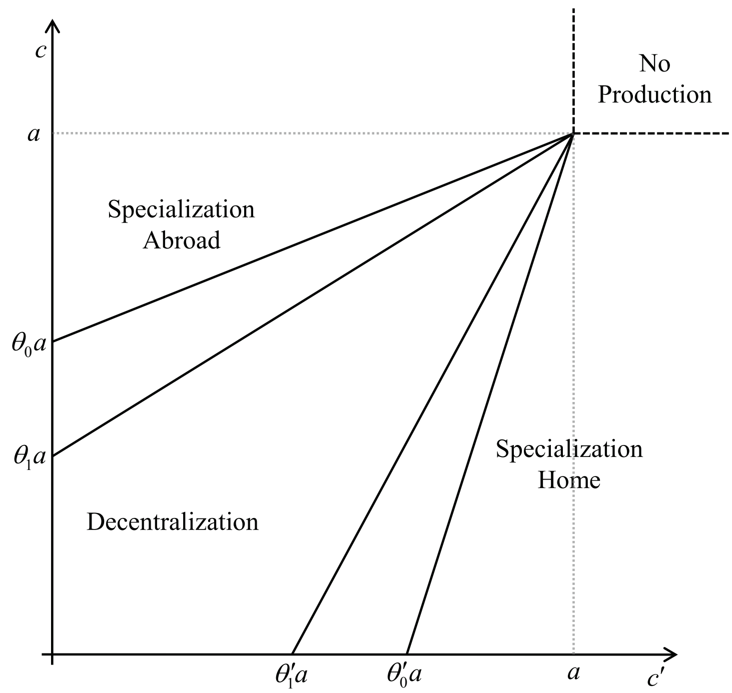

Figure 3.1: Merger incentives and resulting market structure in the $\{c, c'\}$ space.

From the effect on θ_1 it is obvious, that not only a large cost differential but also the number of firms of both types determine merger incentives. To show the effect of the number of firms on incentives, it is convenient to consider the difference $\theta_0 - \theta_1$, which in essence measures the size of the merger region when $c' = 0$ and a is normalized to one. In this case a merger takes place if and only if $c \in [\theta_1, \theta_0]$.

Taking derivatives of this difference yields

$$\frac{\partial(\theta_0(n, n') - \theta_1(n, n'))}{\partial n} < 0,$$
$$\frac{\partial(\theta_0(n, n') - \theta_1(n, n'))}{\partial n'} < 0,$$

such that the difference is decreasing in both n and n'. Therefore, for given cost and demand parameters, mergers are more likely, the higher the economy's concentration both in the home and foreign country.

With this result, it is obvious what the effect of such a bilateral takeover on the incentives for further takeovers is. With a merger the number of home firms decreases such that another merger is even more likely to be profitable. A more intuitive explanation can be provided as follows. As described above, since outputs are strategic substitutes, all outputs and thus all profits of the remaining firms increase. This increase in output is the same for all remaining firms.[17] However, foreign firms' output is larger to begin with such that the increase in profit that is proportional to the squared output is higher. Thus, a further acquisition increases profit by more such that the surplus following a merger increases with a decreasing number of less efficient firms.

Therefore, a profitable acquisition of a home firm by a foreign one and the resulting fall in the number of home firms rises the incentive for another acquisition of a remaining home firm by a foreign firm. If one acquisition is profitable, the gain of the following one will be even higher. As all home firms have identical cost, this implies that if one firm is taken over, then all of them will be. Hence, a merger rises the incentive for another merger such that a merger wave results.[18] This can be summarized by the following proposition:

Proposition 3.4. *A profitable acquisition rises the incentive for another acquisition such that if one takeover is profitable a merger wave results and all less efficient firms are absorbed.*

This means that with mergers we have a another threshold for specification, given by equation (3.12). Whenever home firms are sufficiently efficient no firms will be acquired and full diversification results. On the other hand, if home firms' cost are higher than the threshold, a merger wave occurs such that all home firms are taken over and full specialization results. The profitability of the first merger is crucial for the merger wave to be initiated.

To sum up, introducing the possibility of mergers has the effect of expanding the cost-combinations where only foreign firms are producing as combinations exist in which high cost home firms can earn positive profits, but are vulnerable to a bilateral takeover by low-cost foreign rivals.[19] These regions in the $\{c, c'\}$ space are

[17] The fact that increase in output due to an acquisition is the same for all remaining firms is stated in Lemma 3.2 in Appendix 3.9.1.4.

[18] One implication of this result is that forced domestic mergers to create national champions in the less efficient country raises the probability of a merger wave to take place resulting in disappearance of all domestic firms.

[19] The entire analysis has been focused on acquisitions of home firms by foreign firms. For the reversed case of home firms acquiring foreign firms similar derivations yield analogous results, of course. Thus, there is a similar threshold θ_1' determining the region of specialization of the home

illustrated in Figure 3.1. Obviously, the cone of diversification shrinks with mergers and acquisitions.

3.3.2 Welfare Effects of Integration of Economies Triggering Merger Waves

The aim of this essay is not to analyze the effect of mergers but the effect of the integration of economies. Integration does not serve for motivating the starting point of two groups of firms giving rise to merger incentives. Integration is rather the focus of this study.[20] Nevertheless, as integration might trigger merger waves, these are a direct consequence of globalization and, thus, have to be taken into account.

It is instructive to begin with a separated analysis of mergers and acquisitions leaving the integration of economies aside. Obviously, a merger eliminates firms such that prices rise which in turn lowers consumer surplus. On the other hand profitable mergers let aggregate profits increase such that producer surplus rises while the net effect on total welfare is ambiguous. However, the ambiguity can be resolved in the present case. As only high cost firms are eliminated, the increase in production efficiency ensures that the rise in profits overcompensates the fall in consumer surplus. Hence, welfare measured by the sum of profits and consumer surplus rises with profitable mergers.[21]

Eventually, we wish to determine the effect of integration and accompanying mergers on consumer surplus and welfare.

We have shown that integration as well as merger waves let welfare rise such that the overall welfare effect is unambiguously positive. Producers benefit from profitable mergers and might benefit from integration. The overall effect is indeterminate depending on profits that inefficient firms were able to gain in autarky and loose by integration.[22]

With regard to consumer surplus we have shown that integration increases consumer surplus while mergers let consumer surplus fall so that the overall effect is

country as a result of a merger wave absorbing all foreign less efficient firms.

[20]Therefore, the following analysis (as well as the analysis of Section 3.2, of course) extends Neary's work.

[21]This result is described by Lahiri and Ono (1988) and proven by Neary (2007).

[22]This and the following results are derived in Appendix 3.9.2.4.

unclear. The question is whether it is possible for consumers to be better off in the new global economy that might have been affected by a merger wave in comparison to the situation of the closed economies. As explained before we assume that M&A are a direct and inevitable consequence of the integration of the economies.

As consumer surplus is determined by prices we now compare the new with the former market price in case of a merger wave and thereby focus again on acquisitions of domestic firms by foreign firms.

We have shown that if a merger wave is triggered by integration, all firms of the less efficient domestic economy are absorbed. In that case the new market price is $\hat{p} = \frac{a+n'c'}{n'+1}$. As equilibrium price is independent of parameter b which is essentially halved due to doubled market size and only the number and efficiency of firms matter the new price after a merger wave is the same as the price in the closed foreign economy. Thus, the market price foreign consumers face does not change.

In the domestic economy the situation is different as two opposed effects are at place. On the one hand, the number of firms might decrease when $n' < n$ which weakens competition and let prices increase. On the other hand, firms get more efficient in average which tends to decrease prices. The overall effect is unclear. However, it is shown in the appendix that consumer surplus increases when integration triggers a merger wave such that all home firms are absorbed if

$$c > \phi a + (1-\phi)c', \tag{3.14}$$

where $\phi \equiv \frac{n-n'}{nn'+n}$. When the absorbed firms are inefficient enough, i.e., when their marginal costs are above a weighted average of the demand intercept and and the remaining firms' unit costs, consumers' gain from increased efficiency outweighs the loss due to decreasing competition.

It is easy to see that this condition is weaker than the exit condition, i.e.,

$$\phi < \theta_0. \tag{3.15}$$

Thus, there is a region where less efficient home firms produce after integration although these firms are so inefficient that domestic surplus after integration would still be higher even if these firms are absorbed.[23] This result is of course not very surprising as it has been already derived in Proposition 3.1.

[23]Note, that the exit of these firms as such still decreases consumer surplus.

The relevant question is now what the effect of integration on prices is when we take the generated incentives for mergers and acquisitions into account. Hence, we compare the strength of the derived condition (3.14) with the one of the merger condition (3.11).

As shown in the appendix

$$\phi < \theta_1 \tag{3.16}$$

if

$$\frac{n'(n'+1)}{n(n+1)}\left((n+n')^2-1\right)-2(n+n')>0. \tag{3.17}$$

For $n' \geq n$ this is obviously always fulfilled. In that case competition as well as efficiency at home rises so that prices unambiguously decrease.

For $n' < n$ the reduction in the number of firms consumers face can not be too extreme to rise their surplus. The respective regions in the $\{n', n\}$ space are depicted in Figure 3.2.

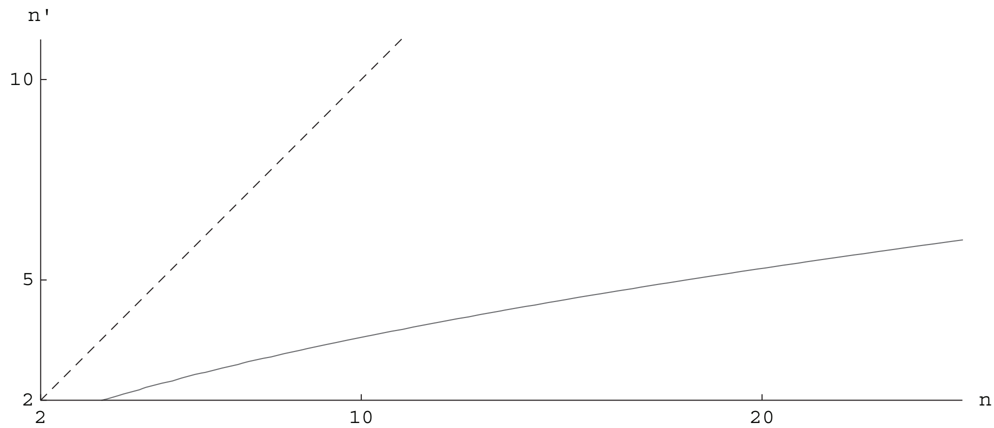

Figure 3.2: Regions in the $\{n', n\}$ space such that domestic price decreases when integration results in wipeout of all domestic firms.

The dashed line illustrates the situation of economies of the same size, i.e., $n = n'$. This is the case considered in general equilibrium in the next section. In the region above the solid curve prices decrease whereas for combinations of numbers of firms below this line, i.e., when the number of foreign firms is extremely small compared to number of domestic firms, prices at home might increase for certain cost combinations. This is the case as with very few foreign firms intensity of competition extremely declines with a merger wave. The lower the number of firms in total the more crucial is the restriction above as with relatively few firms active the price effect of a further disappearance of firms is more drastic.[24]

Thus, given the reduction in competition is not too extreme, consumers of the less efficient home economy are better off by integration of economies triggering merger waves even when the number of firms that domestic consumers face strongly decreases as a result of acquisitions.

The situation with (3.17) being fulfilled with a merger wave is depicted in Figure 3.3. Outside the cone of diversification a merger wave (or voluntary exit) possibly

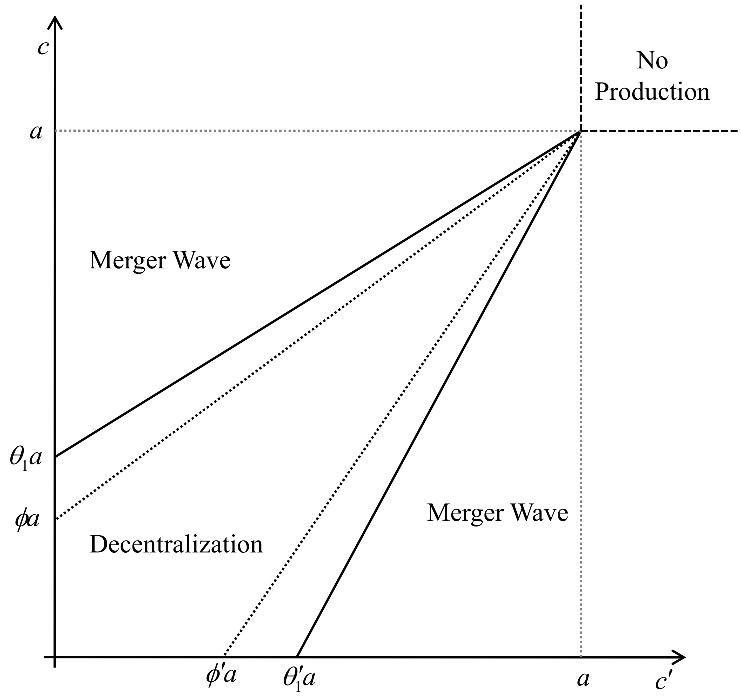

Figure 3.3: The effect of integration and accompanying merger waves on consumer surplus.

reduces competition in one country. The dashed lines determine regions where even if integration is accompanied by a merger wave consumer surplus increases. These lay inside the cone of diversification. Thus, as these merger waves only take place when

[24]For example with 10 foreign firms absorbing more than 60 domestic firms consumers still benefit, whereas with 4 domestic firms being acquired by 2 foreign firms consumers at home might suffer.

the foreign firms are sufficiently efficient consumers are better off. The efficiency gain outweighs the loss by decreasing competition.

Keeping in mind that prices in the more efficient country do not change with a merger wave the derived result can be summarized by the following proposition:

Proposition 3.5. *Integration of economies raises consumer surplus (in both economies) even if acquisitions are possible given intensity in competition does not decrease too drastically due to the merger wave, i.e., given* $\frac{n'(n'+1)}{n(n+1)}((n+n')^2-1)-2(n+n')>0$*. Welfare always increases with integration.*

To sum up: Integration yields a market outcome where consumers are usually better off and overall welfare unambiguously increases.

However, the partial equilibrium approach is misleading as we will show in Sections 3.5 and 3.6 when considering general equilibrium where effects of integration of economies on factor prices and aggregate income are taken into account. Then, it is possible for the reduced competition and the associated raise in prices in the sectors where mergers take place to dominate the welfare gain from lower prices induced by the fall of wages due to mergers on the one hand and increasing competition due to integration on the other hand. This might be the case when efficiency gains from trade can not be realized as a result of a low degree of comparative advantage across sectors yielding little scope for resource reallocation.

3.3.3 The Effect of a Cost Shock

Before turning to general equilibrium with the results derived so far eventually it is worth having a closer look on the effects of changes in firms' respective cost. We do not consider incentives to decrease these cost, i.e., incentives to innovate, as our framework is not the most appropriate one for this kind of incentive analysis. Thus, we focus on price effects of an exogenous cost shock affecting one entire economy.[25] This shock could be caused by an investment into a country's infrastructure aiming at supporting domestic firms. This might appear reasonable when globalization increases intensity of competition yielding the imminent risk of merger waves wiping

[25] We do not aim at rigourously determining welfare and producer surplus effects. A very comprehensive analysis of these effects without considering mergers is provided by Février and Linnemer (2004).

out less efficient domestic firms or the imminent opportunity of acquisitions of less efficient foreign firms. We focus on a change in domestic firms' cost and denote their new marginal cost by $\tilde{c} < c$.

There are several different scenarios for cost shocks. First, the cost shock could affect firms which nevertheless are active in the market such that only cost of some of the decentralized firms decrease. In that case the shock does not trigger mergers and will only alter firms' profits and decrease market price. With a domestic shock, market price falls by $\Delta p \equiv p - \tilde{p}$ with $\tilde{p}$ as the new price. The price decrease

$$\Delta p = \frac{a + nc + n'c'}{n + n' + 1} - \frac{a + n\tilde{c} + n'c'}{n + n' + 1} = \frac{n}{n + n' + 1}(c - \tilde{c})$$

is proportional to the weighted cost shock $c - \tilde{c}$.

For the case of specialization similar results hold. With no home firms active after integration a cost shock that does not prevent these firms from disappearing (whether it is through exit or takeovers) has no effect on market prices, of course. On the other hand, a domestic cost shock in case of foreign firms being nevertheless absorbed affects all remaining active firms. The fall in market price is again proportional to the weighted cost change:

$$\Delta p = \frac{a + nc}{n + 1} - \frac{a + n\tilde{c}}{n + 1} = \frac{n}{n + 1}(c - \tilde{c}).$$

As only home firms are active a cost decrease affecting these and so all firms, of course, has a bigger impact on market price than in the case of decentralization.

The most interesting scenario is a cost shock effectively changing the resulting market structure, i.e., the cost shock either makes less efficient firms sufficiently efficient to remain active in the market or makes more efficient firms sufficiently efficient to profitable taking over other firms and so triggering a merger wave.

We will first consider the case of a cost shock preventing home firms from being acquired, i.e., $c > \theta_1 a + (1 - \theta_1)c'$ and $\tilde{c} < \theta_1 a + (1 - \theta_1)c'$:[26]

$$\Delta p = \frac{a + n'c'}{n' + 1} - \frac{a + n\tilde{c} + n'c'}{n + n' + 1} = n\frac{a + n'c' - (n' + 1)\tilde{c}}{(n' + 1)(n + n' + 1)} = \frac{n}{n' + 1}b\tilde{x}(n, n').$$

Here, $\tilde{x}(n, n')$ denotes home firms' output with decreased unit cost. Now, the fall in market price is no longer linear in the size of the cost shock but depends on the respective cost of all active firms.

[26]Note that if the shock just prevents firms from voluntary exiting, these firms are going to be acquired and still full specialization results.

A cost shock that triggers the absorption of foreign firms as it allows home firms to profitably taking over foreign firms has two effects. In that case

$$\Delta p = \frac{a+nc+n'c'}{n+n'+1} - \frac{a+n\tilde{c}}{n+1} = \frac{n'(-a-n\tilde{c}+(n+1)) + n(n+1)(c-\tilde{c})}{(n+1)(n+n'+1)}$$
$$= \frac{n}{n+n'+1}(c-\tilde{c}) - \frac{n'}{n+1} b\tilde{x}'(n,n').$$

Thus, the price shock consists of two elements. On the one hand, the product market effect $\frac{n}{n+n'+1}(c-\tilde{c})$ is identical to the price decrease without exit. On the other hand a second effect resulting from the imposed reduction in competition has a negative impact on the market price. Depending on which of the effects dominates the price could decrease or increase.

To sum up, a change in unit production cost for example resulting from improved infrastructure might even increase market price when it allows inefficient firms to defend themselves against being acquired and to remain active in the market.

3.4 Endogenous Market Structure with Free Entry

In the described framework strategic behavior is essential for firms' exit and absorbtion. Nevertheless, entry is not considered. A richer and maybe more convenient theory of firm behavior should allow for entry when considering exit as the assumption that the number of firms is exogenous and can never increase is rather artificial. We will show in this section that results change considerably when allowing for entry.

We consider the described framework where now domestic firms have to carry a fixed cost of size F, foreign firms of size F', respectively, upon entry. After they have entered they produce with constant unit cost. Equilibrium market structure is therefore a result of an economy's technology given by entry and production cost.

With free entry there is obviously no incentive for mergers as their profitability is a result of the disappearance of the acquired firm.

As before, we start with two economies in autarky focussing on the domestic country. Ignoring the integer constraint allows us to determine the number of home and foreign firms entering the market by the zero profit condition

$$\pi = \frac{(a-c)^2}{(n+1)^2 b} - F = 0. \tag{3.18}$$

With free entry

$$n = \frac{a-c}{\sqrt{bF}} - 1$$

firms will produce, each $x = \sqrt{\frac{F}{b}}$ units such that aggregate output is $\frac{a-c}{b} - \sqrt{\frac{F}{b}}$.[27] These units are sold at equilibrium price

$$p = c + \sqrt{bF}.$$

Integration now yields two groups of firms. With a given number of foreign firms domestic firms enter until

$$\hat{\pi}(n, n') = \frac{(a - c + n'(c' - c))^2}{(n + n' + 1)^2 \frac{b}{2}} - F = 0$$

so that

$$\hat{n}(n') = \frac{a - c + n'(c' - c)}{\sqrt{\frac{b}{2}F}} - (n' + 1)$$

home firms will be active as long as the number of foreign firms is exogenous. The following proposition shows equilibrium structure in an integrated world with free entry for both type of firms:

Proposition 3.6. *In equilibrium there is no coexistence of domestic and foreign firms.*

Proof: See Appendix 3.9.4.1.

With this proposition it is clear that the effect of free entry is full specialization. With free entry, either home or foreign firms dominate but decentralization is never possible.

When domestic firms dominate, each of

$$\hat{n}(0) = \frac{a-c}{\sqrt{\frac{b}{2}F}} - 1$$

firms produces $\hat{x}(\hat{n}(0), 0) = \sqrt{2\frac{F}{b}}$ units so that aggregate output is $2\frac{a-c}{b} - \sqrt{2\frac{F}{b}}$ and the resulting market price is

$$\hat{p}(\hat{n}(0), 0) = c + \sqrt{\frac{b}{2}F}.$$

[27] For entry and production taking place we assume that $c < a - \sqrt{bF}$ ignoring the integer constraint.

Home firms dominate if price in case of their domination is lower than that of foreign firms, i.e., if

$$c + \sqrt{\tfrac{b}{2}F} < c' + \sqrt{\tfrac{b}{2}F'}$$
$$\Leftrightarrow c < c' + \sqrt{\tfrac{b}{2}}\left(\sqrt{F'} - \sqrt{F}\right). \quad (3.19)$$

Hence, which group of firms dominates depends of course on the respective cost, i.e., unit cost and fixed entry cost. This result is depicted in Figure 3.4. The 45° line shifted due to the difference in fixed entry cost determines regions for specialization.

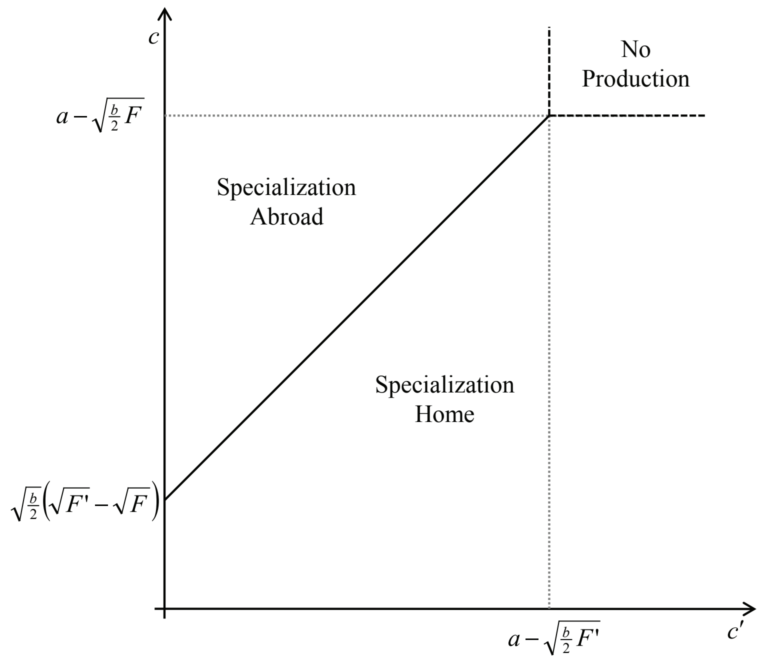

Figure 3.4: Domination due to free entry and resulting market structure in the $\{c, c'\}$ space for an arbitrary entry cost combination (with $F < F'$).

With domestic firms dominating the new domestic price is obviously smaller than the price in autarky. An interesting question is now whether it is possible that in the integrated world home firms dominate although domestic price initially was higher than foreign price in autarky. A higher market price in autarky suggests that this country is less efficient such that intuitively firms in this country should not be able to dominate a technology allowing a lower price. However, it can be shown that this is indeed possible.

Proposition 3.7. *A technology given by unit and fixed entry cost can dominate another technology in an integrated world although it yields a higher market price in autarky.*

Proof: See Appendix 3.9.4.2.

We show in the appendix that the described scenario with domestic firms to dominate although home price was higher in autarky is an equilibrium outcome if $c \in \left[c' - \sqrt{b}\left(\sqrt{F} - \sqrt{F'}\right); c' - \sqrt{\frac{b}{2}}\left(\sqrt{F} - \sqrt{F'}\right)\right]$ with $F > F'$. In that case lower entry cost abroad allows more firms to enter such that market price is lower in the closed economy abroad while with integration lower domestic unit costs facilitate home firms to dominate such that all foreign firms exit. Thus, home firms can exploit their economies of scale.

To analyze welfare effects it is sufficient to concentrate on prices as producer surplus is zero such that welfare is determined by consumer surplus only. Prices unambiguously decrease. This can easily be illustrated as prices clearly fall in the dominating country. However, prices might rise in the dominated country. Consider foreign firms to be dominated. A rise in foreign market price could only be the case in the scenario described by Proposition 3.7 where the resulting equilibrium price $\hat{p}$ is higher than it would have been with foreign firms dominating, i.e., $\hat{p} > \hat{p}'$. Therefore, $\hat{p}$ might also be higher than p' in autarky. However, this can not be the case as in autarky $p < p'$ and $\hat{p} < p$ such that $\hat{p} < p'$. This proves the following proposition:

Proposition 3.8. *Consumer surplus and so welfare rise with integration of economies.*

It can directly be deduced that total output increases as well.

To sum up, the result with free entry is very clear. Integration yields always full specialization and increasing consumer surplus and welfare. However, the country with lower prices in autarky might not dispose of the more efficient technology in the sense of dominance with globalization. Free entry prevents that a group of firms that competes relatively moderate (due to high entry cost in this section or a low exogenous number in Section 3.3) in autarky can dominate in an integrated economy. Therefore, intensity of competition can never decrease drastically enough to increase prices for one group of consumers.

3.5 The General Equilibrium Model

In recent literature it is argued that a general equilibrium framework is more adequate to analyze developments that cut across the general run of industries and sectors which is certainly the case for integration of economies (cf. J. Francois & Horn, 2007). Additionally, empirical studies like Gärtner and Halbheer (2009) find that recent merger waves also occurred in many industries. Partial equilibrium models take factor prices and aggregate income as given, and disregard interactions between markets. However, satisfactory analytic general equilibrium frameworks are still scarce. In this section we will provide a convenient general equilibrium analysis of integration of economies that might trigger merger waves.

To model oligopoly in general equilibrium we build on a framework developed by Neary (2002). We allow firms to be large in their own sector z but require them to be small in the economy as a whole. Therefore, we assume a continuum of sectors, each with a small number of firms. Thus, firms cannot influence national income or factor prices.

More specifically, on the one hand, firms face a small number of local competitors, so they have an incentive to compete strategically against them in the manner familiar from the theory of industrial organization. On the other hand, firms cannot influence economy-wide variables. Furthermore, provided all profits are returned costlessly to the aggregate household, the implications of profit maximization are independent of tastes and of the normalization rule for prices.[28]

3.5.1 A Closed Economy

To begin with, we will again focus on one economy in autarky before we assume that economies integrate. In each country a continuum of goods indexed by $z \in \{0, 1\}$ can potentially be produced and consumed.

[28] This approach avoids all problems appearing in other ways of modeling oligopoly in general equilibrium (for a detailed characterization of disadvantages of alternative frameworks see Neary, 2002). The approach usually used in industrial organization where it is assumed that preferences are quasi-linear (where goods are produced under perfect competition in one sector and under oligopolistic competition in the other yielding the inverse demand curve to be independent of income) yields that all income effects fall on the good produced in the perfectly competitive sector. Thus, quasi-linear preferences just provide a secure foundation for partial equilibrium analysis.

Let preferences of households in each country be given by $U = U[\{y(z)\}]$, where $y(z)$ is the demand of good z. Utility is assumed to be an additively separable function of a continuum of goods, i.e., $U[\{y(z)\}] = g\left(\int_0^1 u(y(z))dz\right)$, where $g(h) = h$ and the subutility functions are quadratic:[29]

$$U[\{y(z)\}] = \int_0^1 ay(z) - \frac{1}{2}by(z)^2 dz. \tag{3.20}$$

A representative consumer maximizes $U[\{y(z)\}]$ subject to the budget constraint

$$\int_0^1 p(z)y(z)dz \leq I, \tag{3.21}$$

where I is the aggregated income. This yields the inverse demand functions

$$p(z) = \frac{1}{\lambda}u'(y(z)) = \frac{1}{\lambda}(a - by(z)), \tag{3.22}$$

where λ is the usual Lagrange multiplier attached to the budget constraint and thus marginal utility of income. Apart from λ, the demand price of good z depends only on variables pertaining to the sector z itself. From the perspective of firms in each sector z, λ is an exogenous variable over which they have no control so that the perceived demand curves are linear and similar to equation (3.1).

Although firms take λ as given it is determined endogenously in general equilibrium so that it can not be eliminated and the value is not constant. It depends on equilibrium prices and income. To solve for λ multiplying $p(z) = \frac{1}{\lambda}(a - by(z))$ by $p(z)$ and integrating yields $\lambda = \frac{1}{\sigma_p^2}\int_0^1 p(z)(a - by(z))dz$, where $\sigma_p^2 \equiv \int_0^1 p(z)^2 dz$. So λ equals a price-weighted mean of the marginal utilities of individual goods, divided by the uncentred variance of prices. With $\mu_p \equiv \int_0^1 p(z)dz$ as the mean price,

[29] In principle any form of additively separable preferences could be used, as they share the convenient property that demand facing each sector depends only on variables pertaining to that sector and on the economy-wide marginal utility of income. The simplest case is where preferences are Cobb-Douglas: $u(y(z)) = \beta(z)lny(z)$, $\int_0^1 \beta(z)dz = 1, g(h) = \exp h$. In this case inverse demand functions are unit-elastic: $p(z) = \frac{\beta(z)I}{y(z)}$. As an alternative one could take the constant-elasticity-of-substitution form as in Dixit and Stiglitz (1977): $u(y(z)) = y(z)^\eta$, $0 < \eta < 1$. Then the inverse demand functions would be iso-elastic. These functions allow convenient solutions in models of monopolistic competition but have unattractive implications in oligopoly. Outputs are often strategic complements in Cournot competition, and reaction functions may be non monotonic. Hence, quadratic subutility is the most convenient assumption. Continuum-quadratic preferences implying linear perceived demand functions guarantee existence and uniqueness of equilibrium at the sectoral level and allow convenient aggregation across sectors.

marginal utility of income can be written as

$$\lambda[\{p(z)\}, I] = \frac{a\mu_p - bI}{\sigma_p^2}.$$

A rise in income, a rise in the variance of prices, or a fall in the mean of prices, all reduce λ and therefore shift the demand function for each good outwards.

To close the model, we need to explain how costs are determined in general equilibrium. We adopt Ricardian assumptions about technology and factor markets. Labor is the only production factor and is intersectorally but not internationally mobile.

In each sector, n firms produce the homogenous consumption good by using a technology $x_i(z) = \frac{1}{\tau_i(z)} l_i(z)$. Here, the productivity parameter $\tau_i(z)$ gives firm's labour requirement per unit output and $l_i(z)$ is employment in firm i. Profit of a firm is given by $\pi_i(z) = p(z)x_i(z) - w\tau_i(z)x_i(z)$. As the economy-wide wage rate w is fix from the perspective of firms, we can talk about $w\tau_i(z)$ as marginal or unit cost and write $c_i(z) \equiv w\tau_i(z)$. We assume that τ_i are continuous in z.

The equilibrium condition in the labour market requires the exogenous labour supply L to be equal to the total demand for labour from all sectors, i.e., $L = \int_0^1 \sum_{i=1}^n \tau_i(z)x_i(z)dz$. Hence, there is full employment.

3.5.2 Equilibrium in Autarky

Let us consider two economies in autarky without any relations and trade. The home country can be described by the preferences of households and the supply side of the economy specified in Section 3.5.1. We assume that all firms in one sector dispose of the same technology. The foreign economy is alike.

We assume goods market clearing in each sector, i.e., total demand equals total supply $X(z) = \sum_{i=1}^n x_i(z) = nx(z)$. Equilibrium in each sector is similar to the one described in Section 3.2.1 with demand given by equation (3.22).

Under Cournot competition each firms sells

$$x(z) = \frac{\frac{a}{\lambda} - c(z)}{(n+1)\frac{b}{\lambda}} \tag{3.23}$$

units such that the corresponding market price is

$$p(z) = \frac{\frac{a}{\lambda} + nc(z)}{n+1} \tag{3.24}$$

and each firm's profit

$$\pi(z) = \tfrac{b}{\lambda}x(z)^2. \tag{3.25}$$

A general equilibrium approach offers a different perspective on welfare effects than partial equilibrium. The focus are no longer surpluses of consumers and producers but real incomes and welfare. Thus, the general equilibrium framework provides a richer modeling of determinants of income distribution.

To solve the model for general equilibrium, we need to evaluate $L = n\int_0^1 \tau(z)x(z)dz$ using the equilibrium output (3.23).[30]

We denote μ_τ and σ_τ as the first and second moments of the technology distribution, i.e., average labor requirements and average squared labor requirements, respectively:

$$\mu_\tau \equiv \int_0^1 \tau(z)dz, \quad \sigma_\tau^2 \equiv \int_0^1 \tau(z)^2 dz.$$

Note that the variance v^2 of the technology distribution is given by $v^2 = \sigma_\tau^2 - \mu_\tau^2$.

This yields the equilibrium labor market condition

$$L = \frac{n}{b(n+1)}\left(a\mu_\tau - \lambda w\sigma_\tau^2\right) \tag{3.26}$$

and so equilibrium wage:

$$w = \left(a\mu_\tau - \frac{n+1}{n}bL\right)\frac{1}{\lambda\sigma_\tau^2}. \tag{3.27}$$

On the right hand side of this expression appears one unknown parameter, the marginal utility of income λ. Inspecting equations (3.1), (3.22), (3.23), and (3.24) it can be seen that the values of real variables are homogeneous of degree zero in the nominal variables wage rate w and the inverse of the marginal utility of income λ^{-1}. This means, the absolute values of these and all other nominal variables like prices and costs are indeterminate. This is the standard property of real models, and it implies that we can choose an arbitrary numeraire or normalization of nominal variables. So, it is convenient to work with wages normalized by marginal utility: λw.[31] Changes in units in which values are measured yield equal and opposite

[30] The following results are derived in Appendix 3.9.5.1.

[31] These are not the same as real wages in the conventional sense as tastes are not homothetic, so real wages cannot be defined independently of the level of utility. Additionally, as absolute values of variables are uninteresting in the following for comparative statics we might want to choose utility as a numeraire so that λ is unity by choice of units.

changes in λ and w such that λw does not change. A good interpretation for this term is marginal real wages, as these equal nominal wages deflated by marginal cost of utility. Solving (3.27) for this real wage at the margin now yields

$$\lambda w = \left(a\mu_\tau - \frac{n+1}{n} bL \right) \frac{1}{\sigma_\tau^2}. \tag{3.28}$$

To evaluate the effect of changes in competition (due to integration and in the sense of a decrease in the number of firms in all sectors due to mergers) on the functional distribution of income we specify the national income I. This equals the sum of wages and total profits:

$$I = wL + \Pi,$$

where $\Pi = \int_0^1 n\pi(z)dz$. Hence, all profits are returned costlessly to the aggregate household.

Total profit in each sector is

$$n\pi(z) = \frac{\lambda n}{(n+1)^2 b} \left((\tfrac{a}{\lambda})^2 - 2\tfrac{a}{\lambda} w\tau(z) + w^2\tau(z)^2 \right)$$

such that total profit of the economy simply is

$$\Pi = \frac{\lambda n}{(n+1)^2 b} \left((\tfrac{a}{\lambda})^2 - 2\tfrac{a}{\lambda} w\mu_\tau + w^2\sigma_\tau^2 \right). \tag{3.29}$$

From equilibrium wage and total profits we can determine the effects of changes in competition, i.e., the number of firms, on wages and national income.

Using (3.27) in (3.29) yields

$$\Pi = \left(\frac{bL^2}{n} + \frac{na^2}{(n+1)^2 b} v^2 \right) \frac{1}{\lambda \sigma_\tau^2}. \tag{3.30}$$

Using utility as a numeraire so that λ is unity we find that wage rate is strictly increasing in n: $\frac{\partial w}{\partial n} = bL \frac{1}{n^2 \sigma_\tau^2} > 0$; while total profits are strictly decreasing in n: $\frac{\partial \Pi}{\partial n} = -\left(\frac{bL^2}{n^2} + \frac{(n-1)a^2}{(n+1)^3 b} v^2 \right) \frac{1}{\sigma_\tau^2} < 0$. Thus, in such an economy the share of wages in national income increases with the number of firms.

To determine the net effect on income we calculate national income which is

$$I = \left(a\mu_\tau L - bL^2 + \frac{na^2}{(n+1)^2 b} v^2 \right) \frac{1}{\lambda \sigma_\tau^2}. \tag{3.31}$$

This is weakly decreasing in n for λ being unity:

$$\frac{\partial I}{\partial n} = -\frac{(n-1)av^2}{(n+1)^3 b \sigma_\tau^2} \leq 0, \tag{3.32}$$

such that wages increase while national income declines with the number of firms. Note that in the special case of a featureless economy where all sectors have identical cost so that $\sigma_\tau = \mu_\tau = \tau$ such that the variance of the technology distribution is zero the derivative above is zero. Then, the number of firms does not influence national income. Nevertheless, the share of wages in national income still rises with more firms.

Now, we want to determine economy's welfare and analyze the effects of a change in competition on welfare. Using (3.22) we can rewrite the utility function and get

$$U = \frac{a^2}{2b} - \frac{\lambda^2}{2b}\sigma_p^2. \tag{3.33}$$

Hence, ignoring constants, the indirect utility function is simply given by the variance of the distribution of prices deflated by marginal utility of income: $\tilde{U} = -\lambda^2\sigma_p^2$.

To calculate utility, we make use of the Cournot equilibrium prices $p(z) = \frac{\frac{a}{\lambda}+nc(z)}{n+1}$. Squaring and integrating yields

$$\begin{aligned} U &= \frac{a^2}{2b} - \frac{\lambda^2}{2b}\int_0^1 p(z)^2 dz \\ &= \frac{a^2}{2b} - \frac{1}{2b(n+1)^2}\left(a^2 + 2na\lambda w\mu_\tau + n^2(\lambda w)^2\sigma_\tau^2\right). \end{aligned} \tag{3.34}$$

Substituting equilibrium wage (3.27) yields

$$U = \frac{a^2}{2b} - \frac{1}{2b}\left(\frac{1}{(n+1)^2}\frac{a^2 v^2}{\sigma_\tau^2} + \frac{(a\mu_\tau - bL)^2}{\sigma_\tau^2}\right). \tag{3.35}$$

Note that this is independent of the marginal utility of income λ.

This indirect utility is obviously increasing in n such that a rise in the number of firms raises welfare. If $v^2 > 0$, the effect is as expected and welfare strictly increases. However, in a featureless economy, the number of firms has no effect on welfare.

Thus, when sectors are rather similar an increase in competition merely redistributes income from profits to wages without gains in efficiency. The increase in the number of firms raises the demand for labor, but as the aggregate labor supply is constant and the corresponding constraint binding wages increase and profits decrease such that the overall effect might be insignificant. In general equilibrium increased competition can only raise output if labor is reallocated from less to more efficient sectors. When all sectors are identical this is not possible and so welfare costs of imperfect competition vanish.[32]

A mean preserving spread in the distribution of technologies also raises welfare as U is increasing in v^2 for a given μ_τ.[33] This shows two general equilibrium effects. On the one hand, consumers prefer homogeneous consumption levels, and therefore they dislike heterogeneous prices (see equation (3.33)). This effect tends to lower welfare at given wages. On the other hand, (3.27) shows that heterogeneous technologies let wages decrease. So aggregate welfare and wages do not move together. Falling wages reduce the uncentred variance of prices and tend to raise welfare. The second effect obviously dominates as welfare is increasing in v^2.

3.6 Integration of Economies and Merger Incentives in General Equilibrium

We now study globalization and conditions under which mergers can arise. The analysis is similar to the one of Sections 3.2 and 3.3. Thus, analysis again starts from an equilibrium with two separated economies without any merger incentives. The home country can be described by the preferences of households and supply side of the economy specified before. The foreign economy is alike and we assume the same tastes, i.e., the same demand slope b and intercept a. Hence, starting point for each economy is the equilibrium in autarky as derived in Section 3.5. We now assume liberalization of trade and analyze which equilibria arise if mergers are allowed.

3.6.1 Integration of Product Markets

Consider now the two economies in a free-trade world. The new economy can be described by the specified parameters. The behavior of each of the two countries

[32]This result is not an argument against activism in competition policy. The model is far too stylized to provide a basis for policy making. In the realistic case where sectors are heterogeneous, the welfare cost of oligopoly implied by σ_p^2 may be greater than those implied by partial equilibrium calculations. Rather, the result should be seen as an extreme case which brings out the importance of a general equilibrium perspective.

[33]Note that not only v^2 but also $\sigma_\tau^2 = v^2 + \mu_\tau^2$ increases with a mean preserving spread. It is proven in Appendix 3.9.5.1 that U is indeed increasing in v^2 for given μ_τ.

can be characterized by an aggregate utility function like (3.20). The world's inverse demand curve for each good comes from equation (3.22) for the home country and the corresponding equation for the foreign country. Assuming that the demand of the foreign country has the same slope b and intercept a yields the world's inverse demand curve $\hat{p}(z) = \frac{2}{\widehat{\lambda}+\widehat{\lambda}'}\left(a - \frac{1}{2}by(z)\right)$. Then, $\lambda^W \equiv \widehat{\lambda} + \widehat{\lambda}'$ is the world marginal utility of income.

To complete the model we assume that labor, the sole factor of production, is intersectorally but not internationally mobile. Sectors differ in their unit labor requirements, denoted by $\tau(z)$ and $\tau'(z)$ at home and abroad, respectively. Thus, unit costs of firms in sector z are given as follows:

$$c(z) = \widehat{w}\tau(z), \quad c'(z) = \widehat{w}'\tau'(z)$$

where $\widehat{w}$ and $\widehat{w}'$ denote home and foreign wages.

It is convenient to assume that $\tau(z)$ and $\tau'(z)$ are continuous in z and that sectors are ordered such that $\tau(z)$ is increasing and $\tau'(z)$ is decreasing in z. Thus, z can be interpreted as an index of foreign competitive advantage.

The equilibrium in free trade in the absence of mergers is now inter alia determined by threshold sectors $\bar{z}$ and $\bar{z}'$ defining sectors where after integration only firms in one country are producing. These thresholds are implicitly determined by setting (3.9) and the corresponding equation for the foreign economy to equalities. This is illustrated in Figure 3.5. The downward sloping lines depict two arbitrary distributions of cost or rather technologies, conditional on wages in the two economies.[34] In case of a cost distribution yielding the upper line the home country produces positive outputs in all sectors for which z is below $\bar{z}$ while the foreign country produces in sectors where z is above $\bar{z}'$ as long as mergers are absent. In case of a distribution given by the lower line there is diversification in all sectors.

Obviously, the effect of cross-border mergers is a shifting of the thresholds of specialization which in turn affects the outcome of the model. However, the approach to determining the equilibrium is independent of how the thresholds of specialization are determined.

To complete the model, we evaluate again the full employment conditions using the equilibrium outputs of firms. With thresholds of specialization these outputs do

[34] Note, that at home a higher sector denotes higher labor requirements and thus higher cost while abroad a higher sector denotes lower labor requirements and lower cost.

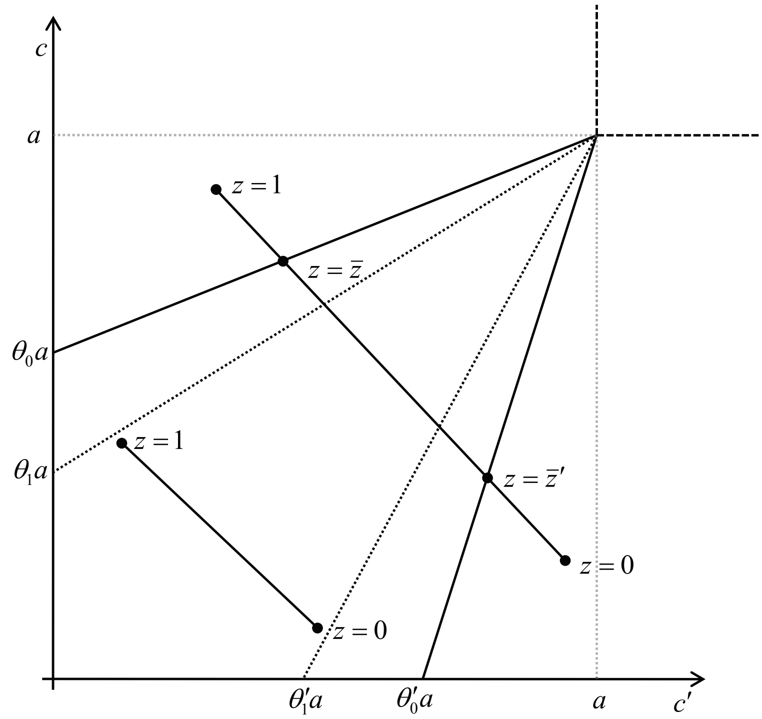

Figure 3.5: Two arbitrary distributions of technologies across sectors and resulting threshold sectors.

not only depend on the world's demand curve but also on the activities of firms. For the home country, this condition is as follows:

$$L = \int_0^{\bar{z}'} n\tau(z)\hat{x}(\widehat{w}, z, n, 0)dz + \int_{\bar{z}'}^{\bar{z}} n\tau(z)\hat{x}(\widehat{w}, \widehat{w}', z, n, n')dz \equiv L(\widehat{w}, \widehat{w}', \bar{z}, \bar{z}', n, n') \tag{3.36}$$

with the levels of output in sectors that do not and that do face foreign competition

$$\hat{x}(\widehat{w}, z, n, 0) = \frac{\frac{2a}{\lambda^W} - \widehat{w}\tau(z)}{(n+1)\frac{b}{\lambda^W}},$$
$$\hat{x}(\widehat{w}, \widehat{w}', z, n, n') = \frac{\frac{2a}{\lambda^W} - (n'+1)\widehat{w}\tau(z) + n'\widehat{w}'\tau'(z)}{(n+n'+1)\frac{b}{\lambda^W}}.$$

The analogous condition for the foreign economy completes the model.

To solve the model explicitly, it is convenient to assume that countries are the same size and exhibit symmetric intersectoral differences. Hence, it is assumed that countries possess the same endowments $L = L'$ and industrial structure $n = n'$. Additionally, technology distributions are mirror images of each other, i.e., $\tau(z) =$

$\tau'(1-z)\forall z$. Due to these assumptions in equilibrium, therefore, equal wages $\widehat{w} = \widehat{w}'$, national incomes $\widehat{I} = \widehat{I}'$, and marginal utilities of income $\lambda^W = 2\widehat{\lambda} = 2\widehat{\lambda}'$ and symmetric threshold sectors $\bar{z} = 1 - \bar{z}'$ result. Thus, the global inverse demand function is

$$\hat{p}(z) = \frac{2}{\lambda^W}\left(a - \frac{b}{2}y(z)\right) = \frac{1}{\widehat{\lambda}}\left(a - \frac{b}{2}y(z)\right). \tag{3.37}$$

3.6.2 Integration with Full Diversification

To analyze the effect of integration it is convenient to initially consider the case of a technology distribution where diversification in all sectors results, e.g. a distribution yielding the lower line in Figure 3.5.

The two countries are symmetric but they are not identical. An important measure of the technological difference of the countries is the uncentred covariance of technology distributions

$$\rho^2 \equiv \int_0^1 \tau(z)\tau'(z)dz. \tag{3.38}$$

Note that the true covariance is $\int_0^1 (\tau(z) - \mu_\tau)(\tau'(z) - \mu_\tau)dz = \rho^2 - \mu_\tau^2$. We can now define the technological dissimilarity between countries:

$$\gamma \equiv \sigma_\tau^2 - \rho^2. \tag{3.39}$$

With complete diversification labor market equilibrium is simply given by equation (3.36) with $\widehat{w} = \widehat{w}'$, $n = n'$, $\widehat{\lambda} = \widehat{\lambda}'$, $\bar{z} = 1$, and $\bar{z}' = 0$. Solving this for marginal wage yields[35]

$$\widehat{\lambda}\widehat{w} = \left(a\mu_\tau - \frac{2n+1}{2n}bL\right)\frac{1}{\sigma_\tau^2 + n\gamma}. \tag{3.40}$$

Comparing this with the result in autarky (3.28) reflects two conflicting effects of trade liberalization. On the one hand, the increasing market size raises the numerator. This is represented by the increase in term $-\frac{1}{2n}$ in (3.40) from $-\frac{1}{n}$ in autarky. A doubled market size raises output and thus labor demand and wages. On the other hand, increasing competition tends to reduce the numerator and raises the denominator. Obviously, the decline in the numerator due to the competition effect dominates the raise by increasing market size so that the numerator increases. The

[35] This and the following results are derived in Appendix 3.9.5.2. To help the exposition we refer to this and the following equations for the home country. Similar equations apply for the foreign country, with symmetry these are even identical.

magnitude of the competition effect in the denominator depends on the technological dissimilarity between the countries γ. The higher the degree of comparative advantage of sectors, the more output is produced by firms with relatively low labor requirements as these firms are competing against firms with relatively high cost. Therefore, a high technological dissimilarity reduces labor demand and so wages. Overall, the effect of free trade on wages is indeterminate. Additionally, although marginal wage in (3.28) and (3.40) is both evaluated by a country's marginal utility of income, it is still not directly comparable as only wage at the margin is measured. Nevertheless, the analysis above shows important labor demand effects. Additionally, as wage rate is an important determinant for welfare, this analysis suggests that comparative advantage of sectors is important for welfare effects of trade liberalization.

We will now turn to consider welfare implications. As before welfare is determined by the uncentred variance of prices weighted by the marginal utility of income. Using equilibrium prices yields welfare of one of the two countries:

$$\widehat{U} = \frac{a^2}{2b} - \frac{\widehat{\lambda}^2}{2b}\widehat{\sigma}_p^2 \tag{3.41}$$

$$= \frac{a^2}{2b} - \frac{1}{2b(2n+1)^2}\left(a^2 + 4na\mu_\tau\widehat{\lambda}\widehat{w} + (2n)^2(\widehat{\lambda}\widehat{w})^2\left(\sigma_\tau^2 - \frac{\gamma}{2}\right)\right). \tag{3.42}$$

Comparing (3.34) with (3.42) illustrates gains from trade apart from those induced by changes in labor demand. Similar to the impact on marginal wage there are two conflicting effects influencing welfare in three ways. First, doubled market size doubles impact of marginal wage which lowers welfare since welfare is decreasing in $\widehat{\lambda}\widehat{w}$. Second, due to increasing competition there is an increase in the denominator which tends to raise welfare. This is a result of falling prices and so their variability. Third, again the magnitude of the competition effect depends on the degree of comparative advantage of sectors, i.e., on γ. The higher the technological dissimilarity of countries the lower is the variation in prices which lowers welfare. With given mean and variance of technological distributions a high dissimilarity truncates variation of prices as in each sector low cost firms tend to dominate more strongly.

The first effect is directly connected to changes in wage rate which depend on the same factors as welfare. Taking now changes in wage into account, i.e., inserting equilibrium wage at the margin, yields

$$\widehat{U} = \frac{a^2}{2b} - \frac{1}{2b(2n+1)^2}\left(a^2\left(\frac{2v^2-\gamma}{2\sigma_\tau^2-\gamma}\right)\right.$$
$$\left. + (2n+1)^2 \frac{2\sigma_\tau^2-\gamma}{2\left(\sigma_\tau^2+n\gamma\right)^2}\left(a\mu_\tau \frac{2\sigma_\tau^2}{2\sigma_\tau^2-\gamma} - bL\right)^2\right). \quad (3.43)$$

We can now compare this expression with the corresponding expression in autarky (3.35). Recall that in a featureless closed economy, the number of firms has no effect on welfare. It is convenient to initially evaluate possible gains from trade for this special case where $\mu_\tau^2 = \sigma_\tau^2 = \rho^2$ so that $v^2 = \gamma = 0$. Then, welfare under free trade is simply $\widehat{U} = \frac{a^2}{2b} - \frac{1}{2b\sigma_\tau^2}\left(a\mu_\tau - bL\right)^2$ which is the same as in autarky. This extends the result of no welfare effects of the number of firms in autarky. The reason now is similar to the earlier finding. When all sectors are identical, efficiency gains cannot be realized.

Another and less restrictive special case is where only countries but not all sectors are identical, i.e., $\sigma_\tau^2 = \rho^2 > \mu_\tau^2$ so that $\gamma = 0$. Welfare is now given by

$$\widehat{U} = \frac{a^2}{2b} - \frac{1}{2b}\left(\frac{1}{(2n+1)^2}\frac{a^2v^2}{\sigma_\tau^2} + \frac{\left(a\mu_\tau - bL\right)^2}{\sigma_\tau^2}\right). \quad (3.44)$$

This expression is very similar to (3.35). Only one of the described effects remains. The increase in competition lets welfare rise. Therefore, integration of identical countries with full diversification always raises welfare.

Eventually, we determine the effect of technological dissimilarity on gains from trade. It is straightforward to show that with fixed μ_τ and σ_τ welfare under free trade is increasing in γ, i.e.,

$$\frac{\partial \widehat{U}}{\partial \gamma} > 0. \quad (3.45)$$

Hence, the higher the degree of comparative advantages, the higher the gains from trade. Although even for the case of identical countries welfare is increasing with integration, welfare increases even more with countries being different. This is summarized in the following proposition:

Proposition 3.9. *Welfare always increases with the integration of economies in case of full diversification and gains from trade are higher the higher technological dissimilarity between countries.*

The reason behind this result is similar to already illustrated effects. With a high degree of comparative advantage of sectors efficiency gains can best be realized. These gains affect welfare in two ways. First, as discussed before, wages decrease with technological dissimilarity as production is allocated more efficiently so that labor demand declines. Second, at given wages gains from trade rise with γ as the variability of prices falls. This highlights general equilibrium effects that are left unconsidered when studying integration in a partial equilibrium model. To sum up, competition effects of integration raise welfare when firms do not exit.

3.6.3 Integration with Specialization: The Effect of Mergers and Acquisitions

The effect of specialization due to exit or mergers on wages can intuitively be explained by considering the effect on labor demand. With thresholds for specialization as for example given by a technological distribution according to the upper line in Figure 3.5, in some sectors high cost firms of the home country exit or are bought out by more efficient foreign firms, while in other sectors the converse happens such that there are expanding and contracting sectors. However, specialized firms only increase their output by a fraction of the output of the disappeared firms such that the world's sector output declines although the country's sector output rises. Thus, aggregating over all sectors yields a declining country's aggregated output. Additionally, expanding sectors of an economy have lower labor requirements per unit output than contracting ones as these are the more efficient sectors. Therefore, aggregating over all sectors in one country, the total labor demand declines such that with perfectly flexible labor markets wages must fall in general equilibrium.

Two equations now define the equilibrium.[36] The first of these equations is the equation for the equilibrium home threshold sector or the "extensive margin". This threshold could be the zero profit threshold given by (3.9) or the threshold for profitable takeovers given by (3.12). Both equations take the same form, highly non-linear in $n = n'$ but linear in τ and τ'.

[36] The analysis in this specific subsection is closely related to Neary's work of mergers in an integrated economy and to some extent replicates his results (cf. Neary, 2007, p. 1241–1243).

This threshold can be written as

$$T_k(\widehat{w}, \bar{z}, n, \lambda) = \widehat{w}\tau(\bar{z}) - (1 - \theta_k(n))\widehat{w}\tau'(\bar{z}) - \theta_k(n)\frac{a}{\widehat{\lambda}} = 0. \tag{3.46}$$

Here, $\theta_k(n)$ only depends on n and takes different values depending on the context.[37] In threshold sector $\bar{z}$ home firms earn zero profits and barely stay in the market such that in all sectors where domestic costs are higher than in this threshold sector, home firms are no longer active. In this threshold sector $\tau(\bar{z}) > \tau'(\bar{z})$ holds, of course.[38] It is now easy to see that this threshold function T is increasing in the economy's wage $\widehat{w}$ and the threshold sector $\bar{z}$. Thus, high values of $\widehat{w}$ and $\bar{z}$ make specialization, i.e., goods being only produced abroad due to exit or takeovers, more likely. Hence, the loci corresponding to $T = 0$ are downward sloping in the $\{\widehat{w}, \bar{z}\}$ space as a higher value of one parameter has to be compensated by a lower value of the other parameter to restore equilibrium. In addition, T is decreasing in θ_k as $\widehat{w}\tau'(z) < \frac{a}{\widehat{\lambda}}$ by assumption such that higher values of θ make specialization less likely. Therefore, loci corresponding to $T = 0$ in the $\{\widehat{w}, \bar{z}\}$ space shift downwards with a lower θ_k.

The second equation defining the equilibrium is the labor market equilibrium condition. With symmetric countries this is simply (3.36) with $\widehat{w} = \widehat{w}'$, $n = n'$, $\widehat{\lambda} = \widehat{\lambda}'$, and $\bar{z} = 1 - \bar{z}'$. Labor demand at symmetric equilibrium is therefore given as

$$L(w, \bar{z}, n) = \frac{2n}{b} \cdot \frac{a\mu_S - \widehat{\lambda}\widehat{w}\sigma_S^2}{n+1} + \frac{2n}{b} \cdot \frac{a\mu_D - (n+1)\widehat{\lambda}\widehat{w}\sigma_D^2 + n\widehat{\lambda}\widehat{w}\rho^2}{2n+1} \tag{3.47}$$

with

$$\mu_S \equiv \int_0^{\bar{z}'} \tau(z)dz, \quad \mu_D \equiv \int_{\bar{z}'}^{\bar{z}} \tau(z)dz,$$
$$\sigma_S^2 \equiv \int_0^{\bar{z}'} \tau(z)^2 dz, \quad \sigma_D^2 \equiv \int_{\bar{z}'}^{\bar{z}} \tau(z)^2 dz,$$
$$\rho_S^2 \equiv \int_{\bar{z}'}^{\bar{z}} \tau(z)\tau'(z)dz,$$

[37] Here, the threshold for profitable takeovers could be given by (3.12) assuming only bilateral takeovers are possible or by (3.68) in Appendix 3.9.3 assuming that the acquisition of more than one firm is possible at one point in time. However, with the more general assumption of the possibility of acquiring m firms $\theta_k(\cdot)$ would naturally be a function of m as well.

[38] Additionally, with symmetry $\bar{z}$ is never smaller than $\frac{1}{2}$.

where $\bar{z}' = 1-\bar{z}$, $\tau'(z) = \tau(1-z)$, and S and D denote the specialized and diversified sectors, respectively.[39] To show the properties of the labor market equilibrium we calculate the derivatives of (3.47) and find

$$\frac{\partial L(\cdot)}{\partial \widehat{w}} < 0, \quad \frac{\partial L(\cdot)}{\partial \bar{z}} > 0. \tag{3.48}$$

The labor demand function $L(\widehat{w}, \bar{z}, n)$ is decreasing in wage rate as labor demand falls with higher labor costs. Additionally, $L(\cdot)$ is increasing in $\bar{z}$ as labor demand increases in the region below full employment when the extensive margin expands.

These properties are intuitively plausible and the locus given by the labor market equilibrium $L = L(\cdot)$ is upward sloping to the left of the locus given by (3.46). This is illustrated for two possible scenarios in Figure 3.6. On the left hand side the

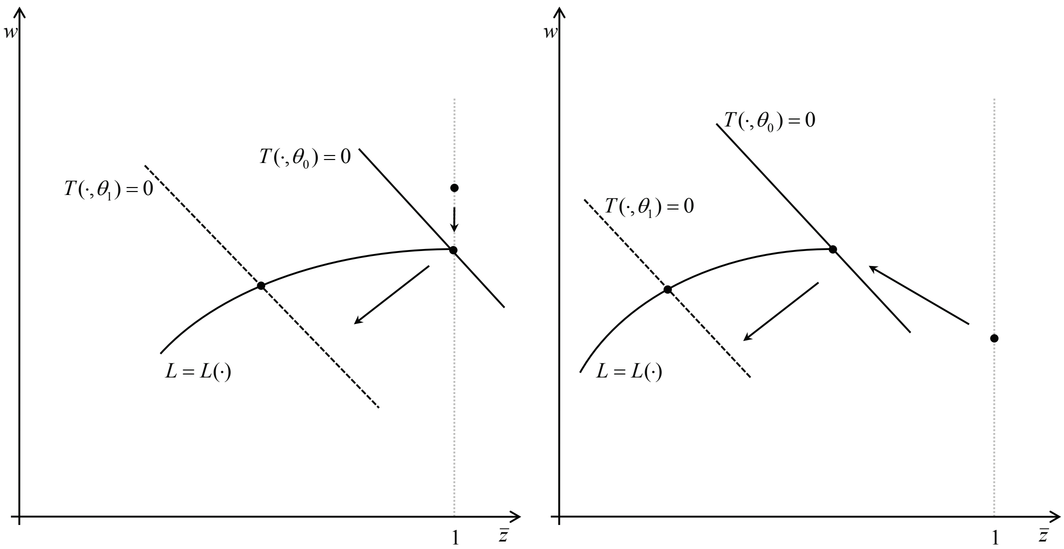

Figure 3.6: Changes in wages and thresholds after integration of economies with mergers.

situation of full diversification before merger considerations and falling wages as a result of integration is depicted whereas on the right hand side integration without mergers yields firms' exit and an increase in wage rate. Note that these scenarios are arbitrary and integration could as well result in rising wages and full diversification (even when mergers are considered) or falling wages and specialization.

[39]The labor market equilibrium and its properties are derived in Appendix 3.9.5.3.

The effects of integration and cross-border mergers on wages can now be illustrated at this figure. Before integration, in all sectors firms in both countries were active such that with the notation of this section $\bar{z} = 1$. As described before, integration might induce wages to rise or fall. The equilibrium at free trade at the absence of mergers and acquisitions is at the intersection of loci. When mergers take place the number of specialized sectors increases such that the locus given by $T = 0$ shifts downwards (to the dashed line) as the new θ_1 is lower than θ_0. The L-locus does not depend on θ and is thus unaffected. The demand for labor decreases in both countries and the wage rate decreases as output does.

As a result falling wage rates raise profitability of marginal high cost firms. This is the case for both countries. However, to analyze effects let us again focus on the home country. From (3.46) home firms will disappear, either by exit or due to acquisitions, if

$$(1+\theta_k)\tau' < \tau - \theta_k \frac{a}{\widehat{\lambda}\widehat{w}}$$

holds. The right hand side of this incentive constraint is increasing in $\widehat{\lambda}\widehat{w}$ such that with increasing wages exit and mergers are more likely. That means that integration of economies raising wages in general equilibrium makes exit more likely while with falling wages mergers are less likely. Hence, general equilibrium considerations of labor markets enforce market exit and dampen the tendency towards merger waves.

However, as the effect of integration is indeterminate the overall effect on wage rate with perfectly flexible labor markets in general equilibrium is ambiguous as well and this ambiguity cannot be resolved.

3.6.4 Welfare Effects

Now, we wish to establish the effects of integration of economies and mergers on welfare.

With thresholds of specialization welfare determined by the variance of prices is

$$\begin{aligned}\widehat{U} &= \frac{a^2}{2b} - \frac{\widehat{\lambda}^2}{2b}\widehat{\sigma}_p^2 \\ &= \frac{a^2}{2b} - \frac{\widehat{\lambda}^2}{b}\left(\int_0^{1-\bar{z}} p(\widehat{w}, n, 0, z)^2 dz - \int_{\frac{1}{2}}^{\bar{z}} p(\widehat{w}, n, n, z)^2 dz\right)\end{aligned} \tag{3.49}$$

as due to symmetry $\int_0^{1-\bar{z}} p(\widehat{w}, n, 0, z)^2 dz = \int_{\bar{z}}^1 p(\widehat{w}, 0, n, z)^2 dz$ and $\int_{\frac{1}{2}}^{\bar{z}} p(\widehat{w}, n, n, z)^2 dz = \int_{1-\bar{z}}^{\frac{1}{2}} p(\widehat{w}, n, n, z)^2 dz$.

Since prices are increasing in wages, it follows that welfare is decreasing in $\widehat{w}$. To establish how welfare varies with the extensive margins, partial equilibrium considerations are adequate. With constant wages, mergers always raise prices in sectors where they occur such that welfare is increasing in $\bar{z}$ (and $1 - \bar{z}'$). In general, the effect of mergers on welfare is ambiguous. On the one hand, higher concentration as the result of mergers raise prices which tends to lower welfare. On the other hand, mergers let competition and thus wages and so prices decline which raises welfare. A very detailed analysis by Neary (2007) shows that this ambiguity cannot be resolved, even in a less general setting as long as the cost distribution $\tau(z)$ is relatively unrestricted.

This ambiguity remains when considering the effect of the integration of economies possibly triggering merger waves. The possibility of decreasing welfare due to specialization is not always overcompensated by gains from integration as the former depends on comparative advantages only for specializing sectors whereas the magnitude of the latter depends on average dissimilarity.

However, if the distribution of sectoral unit requirements $\tau(z)$ is linear in z, an infinitesimal merger wave raises welfare such that the following proposition can be deduced:

Proposition 3.10. *Integration of economies possibly triggering merger waves raises welfare when the technology distribution $\tau(z)$ is linear in z.*

This case ensures that for merger waves the reduction in labor demand from the contracting sectors is sufficiently large relatively to the increase in labor demand resulting from a decrease in wages to ensure that the fall in labor demand overcompensates the reduction in competition.

Thus, gains from trade depend on the distribution of sectoral unit requirements. On the one hand, with a high degree of average comparative advantage of sectors efficiency welfare gains from integration can be realized best. On the other hand, large heterogeneity of sectors triggers merger waves and these merger waves raise welfare only if comparative advantage for expanding sectors is sufficiently high. This again illustrates the importance of a general equilibrium framework for matters like globalization affecting entire economies.

3.7 Free Entry in General Equilibrium

In the general equilibrium framework building on the work of Neary (2007) strategic exit and absorbtion is fundamental. Thus, it is again convenient to extend the model by allowing for entry.

In general equilibrium with free entry now even in autarky the number of active firms differ across sectors depending on the respective cost. With labor as the only factor of production firms enter by using a technology $\kappa(z)$. This technology is defined analogously to production technology such that $F(z) \equiv w\kappa(z)^2$.[40] To solve the model, we need to evaluate $L = \int_0^1 n(w,z)(\tau(z)x(w,z) + \kappa(z)^2)dz$ using the equilibrium output from Section 3.4.[41]

Similar to μ_τ and σ_τ we denote μ_κ and σ_κ as the first and second moments of the entry technology distribution, i.e., average labor requirements and average squared labor requirements for entering the market, respectively:

$$\mu_\kappa \equiv \int_0^1 \kappa(z)dz, \quad \sigma_\kappa^2 \equiv \int_0^1 \kappa(z)^2 dz.$$

Additionally, with

$$\varrho_{\tau\kappa}^2 \equiv \int_0^1 \tau(z)\kappa(z)dz$$

as the uncentred covariance of distributions of technologies, the labor market condition reads

$$L = \frac{1}{b}\left(a\mu_\tau - \lambda w \sigma_\tau^2\right) + \frac{1}{\sqrt{b\lambda w}}\left(a\mu_\kappa - 2\lambda w \varrho_{\tau\kappa}^2\right) - \sigma_\kappa^2. \tag{3.50}$$

Labor demand in case of endogenous entry substantially differs from demand given by (3.26) in Section 3.5 since now labor is also required for entering the market such that there are two sources of labor demand, namely entry and production. The first term of the above equation corresponds to demand from production, the other terms illustrate free entry effects. The wage rate is affected by labor requirement for entry κ in two conflicting ways. On the one hand, a higher labor requirement for entering the market increases labor demand and thus wages. On the other hand, due to

[40] For modeling entry in general equilibrium see for example Konishi, Okuno-Fujiwara, and Suzumura (1990). Note that since we do not impose any assumption on technology distributions squaring of labor requirement for entry is for mathematically convenience only.

[41] The following results are derived in Appendix 3.9.6.1.

higher entry cost less firms enter and labor demand for entry as well for production decreases.

Which of the effects dominates depends on the relative cost for entry. With relatively high labor requirement for entry marginal wage is decreasing in labor requirement whereas wage increases with labor requirement for entry if this is relatively low. Thus, an inverted-U relationship results.

As firms make no profits free entry redistributes income from profits to wages so that national income is simply given as

$$I = wL.$$

Therefore, income is determined by wages only and the described effects apply for national income as well.

To determine welfare in the model with entry recall that utility is determined by the variance of the distribution of prices deflated by marginal utility of income: $U = \frac{a^2}{2b} - \frac{\lambda^2}{2b}\sigma_p^2$. To calculate this indirect utility function, we make use of the Cournot equilibrium prices with free entry $p(z) = w\tau(z) + \sqrt{\frac{b}{\lambda} w\kappa(z)}$ and get[42]

$$U = \frac{a^2}{2b} - \frac{\lambda^2}{2b}\int_0^1 p(z)^2 dz = \frac{a^2}{2b} - \frac{1}{2b}\lambda w \left(\lambda w \sigma_\tau^2 + 2\sqrt{\lambda w b}\varrho_{\tau\kappa}^2 + b\sigma_\kappa^2\right). \tag{3.51}$$

Labor requirement for entry influences welfare in two ways. First, as illustrated above, wages might decrease with entry cost which raises welfare. On the other hand, higher entry cost induce less entry and competition and so higher prices which reduces welfare. This effect can be seen from the equation above when neglecting marginal wage effects. The net effect of entry requirements is indeterminate and depends on the distribution of labor requirement for entry.

Let us again assume an economy wide shock eliminating barriers to trade according to Section 3.6. It is convenient to assume that $\kappa(z)$ and $\kappa'(z)$ are continuous in z and that additionally $\kappa(z)$ is increasing while $\kappa'(z)$ is decreasing so that if for any given identical wages at home and abroad $w\tau(\tilde{z}) + \sqrt{\frac{bw}{2\lambda}}\kappa(\tilde{z}) < w\tau'(\tilde{z}) + \sqrt{\frac{bw}{2\lambda}}\kappa'(\tilde{z})$ holds for any $\tilde{z}$, then it also holds for all $z < \tilde{z}$. The equilibrium is then determined by just one threshold sector $\bar{z}$. The home country produces in all sectors where z is below $\bar{z}$ while in all other sectors foreign firms produce.

[42] The following results are derived in Appendix 3.9.6.2.

To complete the model, we evaluate again the labor market conditions using the equilibrium output of sectors and threshold of specialization $\bar{z}$. Since there is specialization in every sector, for the home country this condition reads:

$$L = \int_0^{\bar{z}} \hat{n}(\widehat{w}, z) \left(\tau(z)\hat{x}(\widehat{w}, z, 0) + \kappa(z)^2\right) dz \equiv L(\widehat{w}, \bar{z}). \tag{3.52}$$

Similar conditions for the foreign country complete the model. In contrast to the results in the previous framework without entry, now home firms do never face foreign competition. However, there is an indirect influence of the foreign economy. The foreign general equilibrium outcome, i.e., foreign wages and thus costs, determine the threshold sector $\bar{z}$ and so domestic wages and utility. Thus, despite full specialization and a clear separation every domestic sector's outcome is influenced by the foreign technologies in general equilibrium.

For explicitly solving the model, it is convenient to assume that countries are identical with respect to technologies, i.e., entry technology distributions are mirror images of each other as well, i.e., $\kappa(z) = \kappa'(1-z)\forall z$. Due to this additional assumption it is guaranteed that in equilibrium wage rates, national incomes, and marginal utilities of income are identical in both countries. This is the case as the threshold halves the global economy, i.e., $\bar{z} = \frac{1}{2}$.

We define again

$$\begin{aligned} \mu_{\tau S} &\equiv \int_0^{\frac{1}{2}} \tau(z)dz, \quad \mu_{\kappa S} \equiv \int_0^{\frac{1}{2}} \kappa(z)dz, \\ \sigma^2_{\tau S} &\equiv \int_0^{\frac{1}{2}} \tau(z)^2 dz, \quad \sigma^2_{\kappa S} \equiv \int_0^{\frac{1}{2}} \kappa(z)^2 dz, \\ \varrho^2_S &\equiv \int_0^{\frac{1}{2}} \tau(z)\kappa(z)dz \end{aligned}$$

as truncated moments. Note that these values are all smaller than half the respective non-truncated values as τ and κ are increasing. Equilibrium wage is now given by

$$L = \frac{2}{b}\left(a\mu_{\tau S} - \widehat{\lambda}\widehat{w}\sigma^2_{\tau S}\right) + \sqrt{\frac{2}{b\widehat{\lambda}\widehat{w}}}\left(a\mu_{\kappa S} - 2\widehat{\lambda}\widehat{w}\varrho^2_S\right) - \sigma^2_{\kappa S}. \tag{3.53}$$

Comparing this with the corresponding expression for autarky (3.50) shows two effects of integration resulting in full specialization. Due to resulting full specialization in each country only in half of the sectors, the more efficient ones, home firms continue to produce. This effect is reflected in (3.53) by the truncated moments.

On the other hand, integration as such doubles market size in all sectors such that more firms enter and each firm produces more. This is reflected by the multiplier 2. Thus, production in these expanding sectors rises. This increase in output overcompensates drop out of foreign firms such that total sector output rises. From the perspective of a country, the output increase in expanding sectors dominates the reduction of contracting sectors such that a country's output rises. However, expanding sectors dispose of more efficient technologies such that the effect on labor demand is indeterminate for the case of general technology distributions.

Due to symmetry, welfare for one of the countries is

$$\widehat{U} = \frac{a^2}{2b} - \frac{\widehat{\lambda}^2}{2b}\widehat{\sigma}_p^2 \tag{3.54}$$

$$= \frac{a^2}{2b} - \frac{1}{2b}\widehat{\lambda}\widehat{w}\left(2\widehat{\lambda}\widehat{w}\sigma_{\tau S}^2 + 2\sqrt{2\widehat{\lambda}\widehat{w}b}\varrho_S^2 + b\sigma_{\kappa S}^2\right). \tag{3.55}$$

Neglecting wage effects, gains from trade can be identified when comparing (3.55) and (3.51). Obviously, without any wage effects welfare would always increase with integration. Partial equilibrium analysis is sufficient to see that prices decline in every sector due to increased market size inducing more firms to enter. Additionally, integration allows more efficient resource allocation as in all sectors only the more efficient firms produce. Truncated moments of technology distributions formalize this effect.

Whether gains from trade can be realized in general equilibrium crucially depends on the wage effect. Wages might increase or decrease and in case of an increase gains from trade may not be possible.

However, for the special case of negligible small entry cost yielding perfect competition and when the distribution of unit requirements $\tau(z)$ belongs to the class of exponentiation functions, i.e., $\tau(z) = \tau z^{\alpha}$ with $\alpha > 0$, including the special case of a linear distribution, the following proposition can be deduced:

Proposition 3.11. *Integration of economies with free market entry and negligible small entry costs raises welfare when the technology distribution $\tau(z)$ is given by an exponentiation function.*

Proof: See Appendix 3.9.6.3.

Perfect competition assures that efficient resource reallocation dominates the relatively moderate increase in total output to the effect that labor demand declines. Thus, gains from trade are realized with perfect competition and a smooth distribution function.

In case of free entry welfare gains depend on the distribution of sectoral unit requirements $\tau(z)$ as well as entry requirements $\kappa(z)$. As specialization in every sector results, comparative advantages of countries with respect to production and entry is important for gains from trade.

3.8 Conclusion

This essay has provided further integration of industrial organization with international trade. Especially with respect to general equilibrium analysis existing literature is rather scarce. Our framework has important implications for the understanding of integration of economies affecting all industries of the involved countries. Partial equilibrium frameworks implicitly assume that the impact is only on the industry under study. Thus, a general equilibrium framework is more adequate to analyze the integration of economies affecting all industries of the involved economies and additionally triggering international mergers that occur in many industries and sectors.

We make use of a model offering convenient aggregation over a continuum of sectors allowing for oligopolistic competition in every sector where firms behave strategically. In this framework, the integration of economies is analyzed. Our main finding has been that in general equilibrium technological dissimilarity of countries is important to realize gains from trade as production factors are restricted.

However, partial equilibrium analysis suggests that integration of economies possibly triggering merger waves most likely raises consumer surplus as efficiency gains outweigh losses by decreasing competition so that overall welfare increases. In contrast, in general equilibrium, it is possible for raising concentration and thus prices due to specialization to dominate the welfare gains from more efficient resource allocation when there is no scope for reallocation due to a low degree of comparative advantage. This highlights the importance of a general equilibrium perspective. Globalization and accompanied mergers and acquisitions have a direct effect on the nature of firms supplying in a country and so influence aggregate industry efficiency. Changes in the characteristics of an industry and country alter supply and demand and so factor requirements. When competition is increased not only will prices tend to fall but marginal cost will also tend to rise. The competition effect of integration is most effective when there is the possibility for reallocating production factors.

Nevertheless, this model is far too stylized to provide as a basis for policy making or even deduce explicit policy implications.

The results of this essay lend themselves to exploring robustness of conclusions. This has been done to some extend with regard to endogenous market structure. In partial equilibrium endogenous industry structure prevents decreasing intensity of competition such that welfare gains are always positive. In general equilibrium the possibility for reallocating resources has been found as even more important. Since full specialization always results in that case, technological dissimilarity with respect to entry and production is crucial for realizing gains from trade.

A wide range of alternative assumptions about market structure, firm behavior, and factor markets is possible. For example, a richer theory of firm behavior should allow for entry deterrence. Relaxing these restrictions would complicate the analysis, but would be an important advance which is left for future research.

3.9 Appendix

3.9.1 Integration of Economies in the Absence of Mergers

3.9.1.1 Cournot Competition with Linear Demand

For later references, we will derive the Cournot-Nash equilibrium with n firms facing different unit costs $c_i < a \,\forall\, i \in \{1, \dots, n\}$. Then, a firm's best response is given as $x_i(X_{-i}) = \frac{a-c_i-bX_{-i}}{2b}$. Hence, the other firms' aggregate best response is given as

$$X_{-i} = \sum_{j\neq i}\left[\frac{a-c_j-bX_{-j}}{2b}\right] = \frac{n-1}{2b}a - \frac{\sum_{j\neq i} c_j}{2b} - \frac{n-2}{2}X_{-i} - \frac{n-1}{2}x_i$$
$$\Leftrightarrow X_{-i} = \frac{(n-1)a - \sum_{j\neq i} c_j - (n-1)bx_i}{nb}$$

such that firm i's optimal quantity is

$$x_i = \frac{a - nc_i + \sum_{j\neq i} c_j}{(n+1)b} = \frac{a-(n+1)c_i + \sum_{j=1}^{n} c_j}{(n+1)b}. \tag{3.56}$$

Firm i's optimal quantity is positive if

$$c_i \leq \frac{a + \sum_{j\neq i} c_j}{n}. \tag{3.57}$$

With all n firms being active equilibrium price is given as

$$p = \frac{a + \sum_i c_i}{n+1}, \tag{3.58}$$

and a firm's profit by

$$\pi_i = \frac{1}{b}\left(\frac{a - nc_i + \sum_{j\neq i} c_j}{n+1}\right)^2 = bx_i^2. \tag{3.59}$$

3.9.1.2 Proof of Proposition 3.1

Proof. Comparing prices for consumers at home assuming that no firms exit yields

$$\hat{p}(n,n') - p(n) = \frac{(n+1)c' - nc - a}{(n+1)(n+n'+1)}.$$

Hence, consumers in the domestic country benefit from integration if $c' < \theta_0' a + (1-\theta_0')c$, i.e., if firms from abroad are at least efficient enough to stay in the market. If they are too inefficient and exit, the new price is the same as the former.

The situation is different if home firms are too inefficient to earn positive profits and exit. Consumers at home benefit as the now active firms are more efficient but their number could be smaller and hence competition could be less intense so that the overall effect is unclear. The change in price when home firms exit is

$$\hat{p}(0,n') - p(n) = \frac{a+n'c'}{n'+1} - \frac{a+nc}{n+1} = \frac{(n-n')a + n'(n+1)c' - n(n'+1)c}{(n'+1)(n+1)},$$

given $c > \frac{a+n'c'}{n'+1}$. Hence, inserting the smallest possible value for c yields that this price change is always negative if

$$(n-n')a + n'(n+1)c' - n(n'+1)\frac{a+n'c'}{n'+1} = -n'(a-c') < 0,$$

which is always fulfilled by assumption. Thus, the market price consumers at home face decreases in all possible cases.[43] Due to symmetry the same is true for consumers abroad. □

3.9.1.3 Changes in Firms' Output with Integration of Economies

Before analyzing total output, it is helpful to determine a firm's increase in output without exit:

$$\begin{aligned}\hat{x}(n,n') - x(n) &= 2\frac{a-(n'+1)c+n'c'}{(n+n'+1)b} - \frac{a-c}{(n+1)b}\\ &= \frac{2(n+1)((a-c)+n'(c'-c)) - (n+n'+1)(a-c)}{(n+n'+1)(n+1)b}\\ &= \frac{(n-n'+1)(a-c) + 2(n+1)n'(c'-c)}{(n+n'+1)(n+1)b}.\end{aligned}$$

With firm's increase in output we can determine total change in output given firms do not exit:

$$\begin{aligned}&n(\hat{x}(n,n') - x(n)) + n'(\hat{x}'(n,n') - x'(n'))\\ &= \frac{n(n'+1)(n-n'+1)(a-c) + n'(n+1)(n'-n+1)(a-c')}{(n+n'+1)(n+1)(n'+1)b}\\ &= \frac{a\left(n(n+1) + n'(n'+1)\right) - n(n'+1)(n-n'+1)c - n'(n+1)(n'-n+1)c'}{(n+n'+1)(n+1)(n'+1)b}\end{aligned}$$

[43] This is basically a result of the assumption of linear demand as the market price does not change with the expansion of economies. Considering market demand effects may alter the analysis.

$$= \frac{1}{(n+n'+1)(n+1)(n'+1)b}\big(a\,(n(n+1)+n'(n'+1))$$
$$- n(n+1)c + nn'(n-n')c - n'(n'+1)c' + nn'(n'-n)c'\big)$$
$$= \frac{n(n+1)(a-c)+n'(n'+1)(a-c')+nn'(n-n')(c-c')}{(n+n'+1)(n+1)(n'+1)b}.$$

With $(n-n')(c-c')$ being nonnegative, output unambiguously rises. With $(n-n')(c-c')$ being negative there is a lower bound such that exit does not take place. This is $(n-n')(c-\frac{a+nc}{n+1}) = \frac{n'-n}{n+1}(a-c)$. Additionally, the first part of this expression is decreasing in c' such that showing that it is positive for the largest possible value of c' is sufficient to prove positivity. Inserting this in the numerator above yields the lower bound

$$n(n+1)(a-c)+n'(n'+1)\left(a-\frac{a+nc}{n+1}\right)+nn'\frac{n'-n}{n+1}(a-c)$$
$$=n(n+1)(a-c)+nn'\frac{n'+1}{n+1}(a-c)+nn'\frac{n'-n}{n+1}(a-c)$$
$$=\left[n(n+1)^2+nn'(n'+1)+nn'(n'-n)\right]\frac{a-c}{n+1}.$$

Only the last term in squared brackets is negative in case of $n>n'$. However, then $nn'(n-n')<n^3<n(n+1)^2$ holds such that the entire term is always positive.

Hence, total output rises when all firms remain active in the market.

With one group of firms, say home firms, exiting the remaining firms double their output:

$$\hat{x}'(0,n')-x'(n')=\frac{a-c'}{(n'+1)b}=x'(n').$$

Thus, the overall change in output is

$$n'\hat{x}'(0,n')-n'x'(n')-nx(n)=n'x'(n')-nx(n)$$
$$=\frac{n'(n+1)(a-c')-n(n'+1)(a-c)}{(n+1)(n'+1)b}.$$

This expression is increasing in c such that showing that it is positive for the smallest possible value of c is sufficient. Inserting the smallest possible value for $c\le\frac{a+n'c'}{n'+1}$ such that exit takes place yields that the change in output is always positive when $n'(n+1)(a-c')-n(n'+1)\left(a-\frac{a+n'c'}{n'+1}\right)$ is. This can be simplified to

$$n'(n+1)(a-c')-n\left((n'+1)a-a-n'c'\right)=n'(n+1)(a-c')-nn'(a-c')$$
$$=n'(a-c')>0.$$

To sum up, total output increases with the integration of economies both with and without firms' exit.

3.9.1.4 Effect of Disappearance of Firms on Remaining Firms' Outputs

It is now convenient to state a key lemma, that determines the effects on the remaining firms' outputs after an acquisition of m home firms.

Lemma 3.2. *The increase in output due to an acquisition of m home firms is the same for all remaining firms:*

$$\hat{x}(n-m,n') - \hat{x}(n,n') = \hat{x}'(n-m,n') - \hat{x}'(n,n') = \hat{x}(n,n')\frac{m}{n-m+n'+1}. \quad (3.60)$$

Proof.

$$\hat{x}(n-m,n') - \hat{x}(n,n') = 2\frac{a-(n'+1)c+n'c'}{(n+n'+1-m)b} - 2\frac{a-(n'+1)c+n'c'}{(n+n'+1)b}$$
$$= 2m\frac{a-(n'+1)c+n'c'}{(n+n'+1-m)(n+n'+1)b}$$
$$= \hat{x}(n,n')\frac{m}{n-m+n'+1} = \hat{x}(n-m,n')\frac{m}{n+n'+1}.$$

and interestingly,

$$\hat{x}'(n-m,n') - \hat{x}'(n,n')$$
$$= 2\frac{a-(n-m+1)c'+(n-m)c}{(n-m+n'+1)b} - 2\frac{a-(n+1)c'+nc}{(n+n'+1)b}$$
$$= \frac{2}{(n-m+n'+1)(n+n'+1)b}\Big(am + c'\left((n+1)(n-m+n'+1)-(n+n'+1)(n-m+1)\right) + c\left((n-m)(n+n'+1)-n(n-m+n'+1)\right)\Big)$$
$$= \frac{2}{(n-m+n'+1)(n+n'+1)b}\Big(am + c'\left((n+1)n'-n'(n-m+1)\right) + c\left(n(n+n'+1-m-n+m-n'-1)-m(n'+1)\right)\Big)$$
$$= 2m\frac{a+n'c'-(n'+1)c}{(n-m+n'+1)(n+n'+1)b} = \hat{x}(n,n')\frac{m}{n-m+n'+1}$$
$$= \hat{x}(n-m,n')\frac{m}{n+n'+1}.$$

□

This result of course is a consequence of the assumption of linear demand.

3.9.1.5 Proof of Lemma 3.1

Proof. Assuming that all firms are efficient enough to stay in the market yield for a firm's profit

$$\hat{\pi}(n,n') - \pi(n) = \frac{b}{2}\hat{x}(n,n')^2 - bx(n)^2 = \frac{b}{2}\left(\hat{x}(n,n')^2 - 2x(n)^2\right).$$

We know that $x(n) = \frac{1}{2}\hat{x}(n,0)$ such that

$$\begin{aligned}
\hat{\pi}(n,n') - \pi(n) &= \frac{b}{2}\left(\hat{x}(n,n')^2 - \frac{1}{2}\hat{x}(n,0)^2\right) = \frac{b}{4}\left(2\hat{x}(n,n')^2 - \hat{x}(n,0)^2\right) \\
&= \frac{b}{4}\left(\left(\hat{x}(n,n') - \hat{x}(n,0)\right)\left(x(n,n') + \hat{x}(n,0)\right) + x(n,n')^2\right).
\end{aligned}$$

Lemma 3.2 yields $\hat{x}(n,n') - \hat{x}(n,0) = -\hat{x}'(n,n')\frac{n'}{n+1}$ such that

$$\begin{aligned}
\hat{\pi}(n,n') - \pi(n) &= \frac{b}{4}\left(-\hat{x}'(n,n')\frac{n'}{n+1}\left(\hat{x}'(n,n')\frac{n'}{n+1} + 2\hat{x}(n,n')\right) + \hat{x}(n,n')^2\right) \\
&= \frac{b}{4}\left(-\frac{n'^2}{(n+1)^2}\hat{x}'(n,n')^2 - \frac{2n'}{n+1}\hat{x}(n,n')x'(n,n') + \hat{x}(n,n')^2\right) \\
&= \frac{b}{4}\left(-\left(\frac{n'}{n+1}\hat{x}'(n,n') + \hat{x}(n,n')\right)^2 + 2\hat{x}(n,n')^2\right).
\end{aligned}$$

This is positive if

$$\begin{aligned}
&\frac{n'}{n+1}\hat{x}'(n,n') + \hat{x}(n,n') < \sqrt{2}\hat{x}(n,n') \\
&\Leftrightarrow n'\hat{x}'(n,n') < (\sqrt{2}-1)(n+1)\hat{x}(n,n').
\end{aligned}$$

This is the case if total foreign output is smaller than a certain fraction of domestic output which might be the case when either the total number of foreign firms or each foreign firm's output is relatively small. This also gives some intuition of when total profits increase. This is obviously the case if the number of firms in one country, say abroad, is relatively small so that home firms can increase their output considerably. In autarky relatively many domestic firms competed for half the market while by integration these firms get just a slightly smaller share of a doubled market so that each domestic firm's output and so profit increases and this overcompensates the loss of foreign firms.

To calculate the total difference in producer surplus we determine surplus before and after integration.

We start with the case of no exit. In a integrated world without exit, total profit is

$$\begin{aligned}
&n\hat{\pi}(n,n') + n'\hat{\pi}'(n,n') \\
=&\frac{2}{(n+n'+1)^2 b}\big[n\left((a-c)^2 + 2n'(a-c)(c'-c) + n'^2(c'-c)^2\right) \\
&\quad + n'\left((a-c')^2 + 2n(a-c')(c-c') + n^2(c-c')^2\right)\big] \\
=&\frac{2}{(n+n'+1)^2 b}\big[n(a-c)^2 + n'(a-c')^2 + 2nn'(c'-c)\left((a-c)-(a-c')\right) \\
&\quad + nn'^2(c'-c)^2 + n'n^2(c-c')^2\big] \\
=&\frac{2}{(n+n'+1)^2 b}\big[n(a-c)^2 + n'(a-c')^2 + 2nn'(c'-c)^2 \\
&\quad + nn'^2(c'-c)^2 + n'n^2(c-c')^2\big] \\
=&\frac{2}{(n+n'+1)^2 b}\left[n(a-c)^2 + n'(a-c')^2 + nn'\left(n+n'+2\right)(c-c')^2\right].
\end{aligned}$$

Before integration, producer surplus was

$$\begin{aligned}
&n\pi(n) + n'\pi'(n') \\
=&\frac{1}{(n+1)^2(n'+1)^2 b}\left(n(n'+1)^2(a-c)^2 + n'(n+1)^2(a-c')^2\right).
\end{aligned}$$

Hence, the increase in total profit through integration is

$$\begin{aligned}
&n\hat{\pi}(n,n') + n'\hat{\pi}'(n,n') - n\pi(n) - n'\pi'(n') \\
=&\frac{1}{(n+1)^2(n'+1)^2(n+n'+1)^2 b}\Big[2(n+1)^2(n'+1)^2\big(n(a-c)^2 + n'(a-c')^2 \\
&\quad + nn'\left(n+n'+2\right)(c-c')^2\big) \\
&\quad - (n+n'+1)^2\left(n'(n+1)^2(a-c')^2 + n(n'+1)^2(a-c)^2\right)\Big] \\
=&\frac{1}{(n+1)^2(n'+1)^2(n+n'+1)^2 b}\Big[(a-c)^2(n'+1)^2 n\left(2(n+1)^2 - (n+n'+1)^2\right) \\
&\quad + (a-c')^2(n+1)^2 n'\left(2(n'+1)^2 - (n+n'+1)^2\right) \\
&\quad + 2nn'\left(n+n'+2\right)(c-c')^2\Big]
\end{aligned}$$

$$
\begin{aligned}
=&\frac{1}{(n+1)^2(n'+1)^2(n+n'+1)^2b}\Big[(a-c)^2(n'+1)^2n\left((n+1)^2-n'(2n+n'+2)\right)\\
&+(a-c')^2(n+1)^2n'\left((n'+1)^2-n(n+2n'+2)\right)\\
&+2nn'\left(n+n'+2\right)\left(c-c'\right)^2\Big]\\
=&\frac{1}{(n+1)^2(n'+1)^2(n+n'+1)^2b}\Big[(a-c)^2(n'+1)^2n\left((n-n'+1)^2-2n'^2\right)\\
&+(a-c')^2(n+1)^2n'\left((n'-n+1)^2-2n^2\right)+2nn'\left(n+n'+2\right)\left(c-c'\right)^2\Big].
\end{aligned}
$$

This is negative when countries are similar with respect to size and technologies. This confirms the intuition above. Total surplus decrease for example if $n = n'$ and $c = c'$ as prices decline due to more intense competition.

When home firms have to exit the profit of the remaining firms is $n'\hat{\pi}'(0, n') = n'\frac{b}{2}\hat{x}'(0, n')^2 = 2n'\frac{(a-c')^2}{(n'+1)^2b} = 2n'\pi'(n')$. This is larger than the profit of all firms in autarky $n'\pi'(n') + n\pi(n)$ when $n'\pi'(n')$ is larger than $n\pi(n)$ given that home firms are so inefficient that they exit after integration, i.e., $c > \frac{a+n'c'}{n'+1}$:

$$
\begin{aligned}
n'\pi'(n')-n\pi(n)=&n'\frac{(a-c')^2}{(n'+1)^2b}-n\frac{(a-c)^2}{(n+1)^2b}\\
=&\frac{1}{(n+1)^2(n'+1)^2b}\left(n'(n+1)^2(a-c')^2-n(n'+1)^2(a-c)^2\right).
\end{aligned}
$$

This is positive when $n'(n+1)^2(a-c')^2 - n(n'+1)^2(a-c)^2$ is. The entire term is of course increasing in c (as $\pi(n)$ is decreasing) so that for proving the positivity inserting the smallest possible value for c is sufficient. Inserting $c = \frac{a+n'c'}{n'+1}$ in $a - c$ yields $a-c = \frac{a(n'+1)-a-n'c'}{n'+1} = \frac{n'}{n'+1}(a-c')$ so that $n'(n+1)^2(a-c')^2-n(n'+1)^2(a-c)^2$ can never be smaller than

$$
(a-c')^2\left(n'(n+1)^2-nn'^2\right)=n'\left(n^2+2n+1-nn'\right)(a-c')^2. \tag{3.61}
$$

This is positive as long as $n' < n + 2 + \frac{1}{n}$ (with number of firms as integers, the term is positive when $n \geq n' - 2$) so that integration increases producer surplus even when firms have to exit as long as the competition at home was intense enough compared to abroad. □

3.9.1.6 Proof of Proposition 3.2

Proof. Welfare as a function of total output X is determined by $W(X) = aX - \frac{b}{2}X^2 - C(X)$. In autarky, total welfare is

$$W(X)+W'(X') = a(nx(n)+n'x'(n'))-\frac{b}{2}(n^2x(n)^2+n'^2x'(n')^2)-(cnx(n)+c'n'x(n'))$$

while after integration, this is given as

$$\begin{aligned}\widehat{W}(\widehat{X}) &= a\widehat{X} - \frac{b}{4}\widehat{X}^2 - C(\widehat{X})\\ &= a(n\hat{x}(n,n') + n'\hat{x}'(n,n')) - \frac{b}{4}(n\hat{x}(n,n') + n'\hat{x}'(n,n'))^2\\ &\quad - (cn\hat{x}(n,n') + c'n'\hat{x}'(n,n')).\end{aligned}$$

Hence, not only the total change in output but also the distribution matters for evaluating the welfare effect. We know from Section 3.9.1.3 that output always increases with integration such that $\widehat{X} \geq X + X'$. As welfare is increasing with output as long as market price is below the constant marginal cost (which is the case with oligopoly competition) we know that

$$\begin{aligned}\widehat{W}(\widehat{X}) \geq \widehat{W}(X+X') &\geq a(X+X') - \frac{b}{4}(X+X')^2 - cX - c'X'\\ &= a(X+X') - \frac{b}{4}(X^2 + X'^2 + 2XX') - cX - cX'\\ &= W(X) + W'(X') + \frac{b}{4}(X-X')^2.\end{aligned}$$

As $(X - X')^2$ is obviously always positive, welfare increases with the integration of economies. □

3.9.2 Integration of Economies and Triggered Mergers

3.9.2.1 Merger between two Firms with Same Unit Cost is never Profitable

Proof. A merger between two firms with the same unit cost, i.e., w.l.o.g. two domestic firms, yields the following surplus:

$$\begin{aligned}\hat{\pi}(n-1,n') - \hat{\pi}(n,n') - \hat{\pi}(n,n') &= \frac{b}{2}(\hat{x}(n-1,n')^2 - 2\hat{x}(n,n')^2)\\ = \tfrac{b}{2}&\left((\hat{x}(n-1,n') - \hat{x}(n,n'))(\hat{x}(n-1,n') + \hat{x}(n,n')) - \hat{x}(n,n')^2\right)\\ = \tfrac{b}{2}&\left((\hat{x}(n-1,n') - \hat{x}(n,n'))(\hat{x}(n-1,n') - \hat{x}(n,n') + 2\hat{x}(n,n')) - \hat{x}(n,n')^2\right).\end{aligned}$$

Using Lemma 3.2 for $m = 1$ yields

$$\begin{aligned}
\hat{\pi}(n-1,n') - \hat{\pi}(n,n') - \hat{\pi}(n,n') \\
&= \tfrac{b}{2}\left(\hat{x}(n,n')\frac{1}{n+n'}\left(\hat{x}(n,n')\frac{1}{n+n'} + 2\hat{x}(n,n')\right) - \hat{x}(n,n')^2\right) \\
&= \tfrac{b}{2}\left(\hat{x}(n,n')\frac{1}{n+n'}\hat{x}(n,n')\left(\frac{1+2n+2n'}{n+n'}\right) - \hat{x}(n,n')^2\right) \\
&= \tfrac{b}{2}\left(\hat{x}(n,n')^2\frac{1+2n+2n'}{(n+n')^2} - \hat{x}(n,n')^2\right) \\
&= \tfrac{b}{2}\hat{x}(n,n')^2\frac{1+2(n+n')-(n+n')^2}{(n+n')^2} \\
&= -\tfrac{b}{2}\hat{x}(n,n')^2\frac{(n+n'-1)^2-2}{(n+n')^2}.
\end{aligned}$$

This can only be positive for $n + n' \leq 2$ and as we are considering the case of one domestic firm acquiring another domestic firm, this merger is only profitable if $n = 2$ and $n' = 0$, i.e., a merger to monopoly. □

3.9.2.2 Proof of Proposition 3.3

Proof. The surplus following a merger is given by

$$\begin{aligned}
\Delta(n,n') \equiv& \hat{\pi}'(n-1,n') - \hat{\pi}'(n,n') - \hat{\pi}(n,n') \\
=& \frac{b}{2}\left(\hat{x}'(n-1,n')^2 - \hat{x}'(n,n')^2 - \hat{x}(n,n')^2\right) \\
=& \frac{b}{2}\left(\left(\hat{x}'(n-1,n') + \hat{x}'(n,n')\right)\left(\hat{x}'(n-1,n') - \hat{x}'(n,n')\right) - \hat{x}(n,n')^2\right) \\
=& \frac{b}{2}\Big(\left(\hat{x}'(n-1,n') - \hat{x}'(n,n') + 2\hat{x}'(n,n')\right) \\
& \cdot \left(\hat{x}'(n-1,n') - \hat{x}'(n,n')\right) - \hat{x}(n,n')^2\Big).
\end{aligned}$$

Using Lemma 3.2 with $m = 1$, we can write

$$\begin{aligned}
\Delta(n,n')\frac{2}{b} &= \left(\hat{x}'(n,n') + \hat{x}'(n-1,n') - \hat{x}'(n,n') + \hat{x}'(n,n')\right)\hat{x}(n,n')\frac{1}{n+n'} \\
&\quad - \hat{x}(n,n')^2 \\
&= \left(2\hat{x}'(n,n') + \frac{1}{n+n'}\hat{x}(n,n')\right)\hat{x}(n,n')\frac{1}{n+n'} - \hat{x}(n,n')^2 \\
&= \frac{1}{(n+n')^2}\hat{x}(n,n')\left[2(n+n')\hat{x}'(n,n') - \left((n+n')^2 - 1)\hat{x}(n,n')\right)\right].
\end{aligned}$$

Inserting the respective quantities in the term in squared brackets that determines the sign of $\Delta(n,n')$, yields

$$\begin{aligned}\Delta(n,n') =&\frac{\hat{x}(n,n')}{(n+n'+1)(n+n')^2}\big[2(n+n')(a-(n+1)c'+nc)-((n+n')^2-1)\\&\cdot(a-(n'+1)c+n'c')\big]\\=&\frac{\hat{x}(n,n')}{(n+n'+1)(n+n')^2}\Big[a\left(2(n+n')-\left((n+n')^2-1\right)\right)\\&+c\left(2n(n+n')+(n'+1)\left((n+n')^2-1\right)\right)\\&-c'\left(2(n+1)(n+n')+n'\left((n+n')^2-1\right)\right)\Big]\\=&\frac{2n(n+n')+(n'+1)\left((n+n')^2-1\right)}{(n+n'+1)(n+n')^2}\hat{x}(n,n')\\&\cdot\Bigg[a\frac{2(n+n')-((n+n')^2-1)}{2n(n+n')+(n'+1)\left((n+n')^2-1\right)}+c\\&-c'\frac{2n(n+n')+(n'+1)\left((n+n')^2-1\right)+2(n+n')-((n+n')^2-1)}{2n(n+n')+(n'+1)\left((n+n')^2-1\right)}\Bigg].\end{aligned}$$

With

$$\theta_1 \equiv \frac{((n+n')^2-1)-2(n+n')}{2n(n+n')+(n'+1)\left((n+n')^2-1\right)} \tag{3.62}$$

this simplifies to

$$\Delta(n,n') =\frac{2n(n+n')+(n'+1)\left((n+n')^2-1\right)}{(n+n'+1)(n+n')^2}\hat{x}(n,n')\Big[-\theta_1 a+c-c'(1-\theta_1)\Big].$$

The term θ_1 is positive for $n+n'\geq 3$ which is fulfilled by assumption.

To sum up, a merger is profitable if and only if

$$c>\theta_1 a+(1-\theta_1)c' \tag{3.63}$$

holds. □

3.9.2.3 Properties of Merger Profitability Condition

To compare the strength of the conditions (3.9) and (3.12) it is sufficient to compare the parameters θ_1 and θ_0:

$$\begin{aligned}\theta_0 - \theta_1 =& \frac{1}{(n'+1)\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)} \\ &\cdot \Big(2n(n+n') + (n'+1)\left((n+n')^2-1\right) \\ &- (n'+1)\left((n+n')^2-1\right) + 2(n'+1)(n+n')\Big) \\ =& \frac{2(n+n')(n+n'+1)}{(n'+1)\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)} > 0.\end{aligned}$$

To calculate the derivatives of this difference we first differentiate θ_0 and θ_1.

$$\begin{aligned}\frac{\partial\theta_0(n,n')}{\partial n} &= 0, \\ \frac{\partial\theta_0(n,n')}{\partial n'} &= -\frac{1}{(n'+1)^2} < 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial\theta_1(n,n')}{\partial n} =& \frac{1}{\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)^2} \\ &\cdot \Big((2(n+n')-2)\left(2n + (n'+1)\left((n+n')^2-1\right)\right) \\ &- ((n+n')^2 - 2(n+n') - 1)(2n' + 4n + (n'+1)2(n+n'))\Big) \\ =& \frac{1}{\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)^2} \\ &\cdot \Big(2(n+n')\Big(2n(n+n') + (n'+1)\left((n+n')^2-1\right) \\ &- (n'+1)((n+n')^2 - 2(n+n') - 1)\Big) \\ &- 2\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right) + n((n+n')^2 - 2(n+n') - 1)\right) \\ &- 2(n+n')((n+n')^2 - 2(n+n') - 1)\Big) \\ =& \frac{1}{\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)^2} \\ &\cdot \Big(2(n+n')\Big(2(n+n')(n+n'+1)\Big) \\ &- 2\left((n+n'+1)\left((n+n')^2-1\right)\right) - 2(n+n')((n+n')^2 - 2(n+n') - 1)\Big)\end{aligned}$$

$$= \frac{1}{(2n(n+n') + (n'+1)\left((n+n')^2-1\right))^2}$$
$$\cdot \Big(2(n+n'+1)\left((n+n')^2+1\right) - 2(n+n')((n+n')^2 - 2(n+n') - 1) \Big)$$
$$= \frac{6(n+n')^2 + 4(n+n') + 2}{(2n(n+n') + (n'+1)\left((n+n')^2-1\right))^2} > 0.$$

Hence, taking derivatives of the difference $\theta_0 - \theta_1$ yields

$$\frac{\partial(\theta_0(n,n') - \theta_m(n,n'))}{\partial n} < 0.$$

Differentiation with respect to n' yields

$$\frac{\partial \theta_1(n,n')}{\partial n'} = \frac{1}{\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)^2}$$
$$\cdot \Big((2(n+n') - 2)\left(2n + (n'+1)\left((n+n')^2-1\right)\right)$$
$$- ((n+n')^2 - 2(n+n') - 1)(2n + (n+n')^2 - 1 + 2(n'+1)(n+n')) \Big)$$

so that

$$\frac{\partial \theta_1(n,n')}{\partial n'}\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)^2$$
$$= 2\left((n+n')^2-1\right)((n+n')(n'+1) - (n'+1)) + 4n(n+n'-1)$$
$$- ((n+n')^2-1)(2(n'+1)(n+n') + 2n) + 4(n+n')(n + (n'+1)(n+n'))$$
$$- \left((n+n')^2 - 2(n+n') - 1\right)((n+n')^2-1)$$
$$= -2\left((n+n')^2-1\right)(n'+1+n) + 4(n + (n+n')^2(n'+1))$$
$$- \left((n+n')^2 - 2(n+n') - 1\right)((n+n')^2-1)$$
$$= 2(n+n')^2(n'+1) + 4n - 2n\left((n+n')^2-1\right) + 2(n+n'+1)$$
$$- \left((n+n')^2 - 2(n+n') - 1\right)((n+n')^2-1)$$
$$= 2(n+n')^2(n'+1-n) + 2(4n+n'+1) - ((n+n')^2-1)^2$$
$$+ 2(n+n')^3 - 2(n+n')$$
$$= 2(n+n')^2(2n'+1) + 2(3n+1) - ((n+n')^2-1)^2.$$

Hence, taking derivatives of the difference $\theta_0 - \theta_1$ yields

$$\begin{aligned}
&\frac{\partial(\theta_0(n,n') - \theta_1(n,n'))}{\partial n'} \\
&= -\frac{1}{(n'+1)^2} - \frac{2(n+n')^2(2n'+1) + 2(3n+1) - ((n+n')^2-1)^2}{(2n(n+n') + (n'+1)\left((n+n')^2-1\right))^2} \\
&= \frac{1}{(n'+1)^2\left(2n(n+n') + (n'+1)\left((n+n')^2-1\right)\right)^2} \\
&\quad \cdot \Big(-4n^2(n+n')^2 - 4n(n+n')(n'+1)\left((n+n')^2-1\right) \\
&\qquad - (n'+1)^2\left(2(n+n')^2(2n'+1) + 2(3n+1)\right)\Big).
\end{aligned}$$

This is obviously negative such that the difference $\theta_0 - \theta_1$ is decreasing in both n and n'.

3.9.2.4 Welfare Effects of the Integration of Economies Triggering Merger Waves

As shown in Section 3.9.1.5 producer surplus might increase with integration even if all firms remain active.

Introducing the possibility of mergers expands cost combinations where only one group of firms is producing. In case of specialization, say abroad, change in producer surplus is increasing in c. Now, with mergers the smallest possible value for c is larger than in case of pure exit yielding the following positivity constraint:

$$\left(n'(n+1)^2 - n(n'+1)^2(1-\theta_1)\right)(a-c')^2 > 0.$$

The term on the left hand side is larger than expression (3.61) but not always positive.

We now turn to consumer surplus. When domestic firms are absorbed consumers of the less efficient domestic economy are better off after integration if

$$\begin{aligned}
\hat{p} - p' &= \frac{a+nc}{n+1} - \frac{a+n'c'}{n'+1} \\
&= \frac{a(n'-n) + n(n'+1)c - n'(n+1)c'}{(n+1)(n'+1)} > 0.
\end{aligned}$$

This is the case when

$$\begin{aligned}&a\frac{n'-n}{nn'+n}+c-c'\frac{nn'+n'+n-n}{nn'+n}\\=&a\frac{n'-n}{nn'+n}+c-c'\left(1-\frac{n'-n}{nn'+n}\right)>0.\end{aligned}$$

With $\phi \equiv \frac{n-n'}{nn'+n}(>0)$, this condition can again be written as c to be above a weighted average of a and c':

$$c > \phi a + (1-\phi)c'.$$

It is easy to see that less values for c fulfil this condition compared to the exit condition:

$$\theta_0 - \phi = \frac{1}{n'+1} - \frac{n-n'}{n(n'+1)} = \frac{n'}{n(n'+1)} > 0.$$

Comparing the strength of the derived condition (3.14) with that of the merger condition (3.11) yields

$$\begin{aligned}\theta_1-\phi=&\frac{1}{\left(2n(n+n')+(n'+1)\left((n+n')^2-1\right)\right)n(n'+1)}\\&\cdot\Bigg(n(n'+1)\left(\left((n+n')^2-1\right)-2(n+n')\right)\\&-(n-n')\left(2n(n+n')+(n'+1)\left((n+n')^2-1\right)\right)\Bigg)\\=&\frac{n'(n'+1)\left((n+n')^2-1\right)-2n(n'+1)(n+n')-2n(n+n')(n-n')}{2n^2(n'+1)(n+n')+n(n'+1)^2\left((n+n')^2-1\right)}\\=&\frac{n'(n'+1)\left((n+n')^2-1\right)-2n(n+1)(n+n')}{2n^2(n'+1)(n+n')+n(n'+1)^2\left((n+n')^2-1\right)}.\end{aligned}$$

This term is positive if

$$\frac{n'(n'+1)}{n(n+1)}\left((n+n')^2-1\right)-2(n+n')$$

is.

3.9.3 Merger Incentives in a More General Case – Simultaneous Acquisitions of More Than One Firm

We abstract from the assumption of bilateral mergers and instead assume that a firm is able to acquire more than one firm at one point in time. With m as the number of firms that can be acquired simultaneously the following proposition holds:

Proposition 3.12. *With mergers defined as above, a merger between firms with the same unit cost is never profitable provided* $m \leq \frac{1}{2}\min\{n, n'\}$ *holds.*

Proof. Taking over m firms dipsosing of the same technologies, say home firms, is profitable if $\Delta(n, n', m) = \hat{\pi}(n-m, n') - \hat{\pi}(n, n') - m\hat{\pi}(n, n')$ is positive. This yields

$$\begin{aligned}
&(\hat{\pi}(n-m, n') - (m+1)\hat{\pi}(n, n'))\frac{2}{b} \\
&= (\hat{x}(n-m, n') - \hat{x}(n, n'))\,(\hat{x}(n-m, n') + \hat{x}(n, n')) - m\hat{x}(n, n')^2 \\
&= \left(\frac{m}{n-m+n'+1}\hat{x}(n, n')\right)(2\hat{x}(n, n') + \hat{x}(n-m, n') - \hat{x}(n, n')) - m\hat{x}(n, n')^2 \\
&= \left(\frac{m}{n-m+n'+1}\hat{x}(n, n')\right)\left(2\hat{x}(n, n') + \frac{m}{n-m+n'+1}\hat{x}(n, n')\right) - m\hat{x}(n, n')^2 \\
&=\frac{m}{n-m+n'+1}\hat{x}(n, n')\frac{m+2(n-m+n'+1)}{n-m+n'+1}\hat{x}(n, n') - m\hat{x}(n, n')^2 \\
&=\frac{m\hat{x}(n, n')^2}{(n-m+n'+1)^2}\left(m + 2(n-m+n'+1) - (n-m+n'+1)^2\right).
\end{aligned}$$

This is positive when $2(n+n'+1) - m - (n-m+n'+1)^2$ is. This can be written as

$$\begin{aligned}
&2(n+n'+1) - m - \left((n+n'+1)^2 - 2m(n+n'+1) + m^2\right) \\
&=(n+n'+1)\,(2 - (n+n'+1) + 2m) - m(m+1) \\
&=(n+n'+1)\,(2m - (n+n'-1)) - m(m+1).
\end{aligned}$$

With $2m \leq min\{n, n'\} \leq n + n'$, this can never be positive. □

Hence, an acquisition of equally efficient firms is never profitable. Additionally, there are no merger incentives in autarky as the above term is even negative in case of $n' = 0$. Hence, the starting point of n and n' firms in each economy is an equilibrium industrial structure in autarky.

To fix ideas, let us assume that one efficient foreign firm takes over an arbitrary number of $m \leq \frac{1}{2}\min\{n, n'\}$ home firms.

Hence, buying is profitable if

$$\Delta(n, n', m) \equiv \hat{\pi}'(n-m, n') - \hat{\pi}'(n, n') - m\hat{\pi}(n, n') > 0. \tag{3.64}$$

We now derive a condition similar to the one of Proposition 3.3 on the difference between marginal cost of the low-cost and the ”to-be-merged” high-cost firm such that merger is in principle profitable.

Proposition 3.13. *An acquisition of $m \in [1, \frac{1}{2}\min\{n, n'\}]$ firms with marginal cost c by a (more efficient) firm with cost c' is profitable if and only if*

$$c > \theta_m \alpha + (1 - \theta_m)c' \tag{3.65}$$

with

$$\theta_m \equiv \frac{\left(\frac{n-m+n'+1}{m}\right)^2 - 2\frac{n-m+n'+1}{m} - m}{2n\frac{n-m+n'+1}{m} + (n'+1)\left(\left(\frac{n-m+n'+1}{m}\right)^2 - m\right)} \tag{3.66}$$

holds.

Proof.

$$\begin{aligned}\Delta(n, n', m) =& \hat{\pi}'(n-m, n') - \hat{\pi}'(n, n') - m\hat{\pi}(n, n')\\ =& \frac{b}{2}\left(\hat{x}'(n-m, n')^2 - \hat{x}'(n, n')^2 - m\hat{x}(n, n')^2\right)\\ =& \frac{b}{2}\left((\hat{x}'(n-m, n') + \hat{x}'(n, n'))(\hat{x}'(n-m, n') - \hat{x}'(n, n')) - m\hat{x}(n, n')^2\right)\\ =& \frac{b}{2}\Big((\hat{x}'(n, n') + \hat{x}'(n-m, n') - \hat{x}'(n, n') + \hat{x}'(n, n'))\\ & \cdot (\hat{x}'(n-m, n') - \hat{x}'(n, n')) - m\hat{x}(n, n')^2\Big).\end{aligned}$$

Using Lemma 3.2, we can write

$$\begin{aligned}\frac{2}{b}\Delta(n, n', m) =& (\hat{x}'(n, n') + \hat{x}'(n-m, n') - \hat{x}'(n, n') + \hat{x}'(n, n'))\\ & \cdot \hat{x}(n, n')\frac{m}{n-m+n'+1} - m\hat{x}(n, n')^2\\ =& \left(2\hat{x}'(n, n') + \frac{m}{n-m+n'+m}\hat{x}(n, n')\right)\hat{x}(n, n')\frac{m}{n-m+n'+1}\\ & - m\hat{x}(n, n')^2\\ =& \left(\frac{m}{n-m+n'+1}\right)^2 \hat{x}(n, n')\\ & \cdot \left[2\left(\frac{n-m+n'+1}{m}\right)\hat{x}'(n, n')\right.\\ & \left. - \left(\left(\frac{n-m+n'+1}{m}\right)^2 - m\right)\hat{x}(n, n')\right].\end{aligned}$$

For the sake of clarity we will call home firms' inverse relative increase in output $\xi \equiv \frac{n-m+n'+1}{m}$. Inserting now the respective quantities in the term in squared brackets which determines the sign of Δ, yields

$$\begin{aligned}
\frac{2}{b}\Delta(n,n',m) =& \frac{\hat{x}(n,n')}{\xi^2}\left[2\xi\hat{x}'(n,n') - (\xi^2-m)\hat{x}(n,n')\right] \\
=& \frac{2\hat{x}(n,n')}{(n+n'+1)\xi^2 b}\big[2\xi(a-(n+1)c'+nc) \\
& - (\xi^2-m)(a-(n'+1)c+n'c')\big] \\
=& \frac{2\hat{x}(n,n')}{(n+n'+1)(n+n')^2 b}\Big[a\left(2\xi - \left(\xi^2-m\right)\right) \\
& + c\left(2n\xi + (n'+1)\left(\xi^2-m\right)\right) - c'\left((n+1)2\xi + n'\left(\xi^2-m\right)\right)\Big] \\
=& 2\frac{2n\xi+(n'+1)\left(\xi^2-m\right)}{(n+n'+1)(n+n')^2 b}\hat{x}(n,n')\Big[a\frac{2\xi-\left(\xi^2-m\right)}{2n\xi+(n'+1)\left(\xi^2-m\right)} + c \\
& - c'\frac{2n\xi+(n'+1)\left(\xi^2-m\right)+2\xi-\left(\xi^2-m\right)}{2n\xi+(n'+1)\left(\xi^2-m\right)}\Big].
\end{aligned}$$

With

$$\theta_m \equiv \frac{\xi^2-2\xi-m}{2n\xi+(n'+1)\left(\xi^2-m\right)} = \frac{\left(\frac{n-m+n'+1}{m}\right)^2 - 2\frac{n-m+n'+1}{m} - m}{2n\frac{n-m+n'+1}{m} + (n'+1)\left(\left(\frac{n-m+n'+1}{m}\right)^2 - m\right)} \tag{3.67}$$

this simplifies to

$$\Delta(n,n',m) = \frac{2n\xi+(n'+1)\left(\xi^2-m\right)}{(n+n'+1)(n+n')^2}\hat{x}(n,n')\Big[c - \theta_m\alpha - c'(1-\theta_m)\Big].$$

Hence, an acquisition of $m \geq 1$ less efficient firms is profitable if and only if

$$c > \theta_m\alpha + (1-\theta_m)c' \tag{3.68}$$

holds, which completes the proof. □

It is easy to see that this condition is indeed weaker than the condition for firms' exit (3.8):

$$\begin{aligned}
\theta_0 - \theta_m =& \frac{2n\xi+(n'+1)\left(\xi^2-m\right)-(n'+1)\left(\xi^2-2\xi-m\right)}{(n'+1)\left(2n\xi+(n'+1)\left(\xi^2-m\right)\right)} \\
=& \frac{2\xi(n+n'+1)}{(n'+1)\left(2n\xi+(n'+1)\left(\xi^2-m\right)\right)} > 0.
\end{aligned}$$

Thus, similar to equation (3.12), equation (3.68) defines a linear upward sloping locus in the $\{c, c'\}$ space which lies strictly below that given by (3.9).

Differentiation of θ_m with respect to m illustrates that the simultaneous acquisition of more firms provides indeed higher merger incentives. We will write $\xi_m = \frac{\partial \xi}{\partial m} = -\frac{n+n'+1}{m^2} < 0$ as the first derivative of ξ with respect to m and get:

$$\begin{aligned}
\frac{\partial \theta_m(n,n',m)}{\partial m} &= \frac{1}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&\quad \cdot \Big((2\xi\xi_m - 2\xi_m - 1)(2n\xi + (n'+1)\big(\xi^2 - m\big)) \\
&\quad - (\xi^2 - 2\xi - m)(2n\xi_+(n'+1)(2\xi\xi_m - 1))\Big) \\
&= \frac{1}{(2n\xi + (n'+1)(\xi^2 - m))^2}\Bigg(2\xi\xi_m \\
&\quad \cdot \big(2n\xi + (n'+1)\big(\xi^2 - m\big) - (n'+1)(\xi^2 - 2\xi - m)\big) \\
&\quad - 2\xi_m\big(2n\xi + (n'+1)\big(\xi^2 - m\big) + n(\xi^2 - 2\xi - m)\big) \\
&\quad - 2n\xi - (n'+1)(\xi^2 - m) + (n'+1)(\xi^2 - 2\xi - m)\Bigg) \\
&= \frac{2\xi\xi_m(2n\xi + 2n'\xi + 2\xi) - 2\xi_m(n+n'+1)(\xi^2 - m) - 2\xi(n+n'+1)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= \frac{2(n+n'+1)(2\xi^2\xi_m - \xi_m(\xi^2 - m) - \xi)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= \frac{2(n+n'+1)(\xi^2\xi_m + m\xi_m - \xi)}{(2n\xi + (n'+1)(\xi^2 - m))^2} < 0.
\end{aligned}$$

Hence, the merger condition in the $\{c, c'\}$ space is lower the higher m is such that a takeover of more firms is more profitable and the condition for a merger wave to be triggered is weaker the more firms can be acquired at once.

It is now possible to show that with one merger being profitable a merger wave results:

Proposition 3.14. *The derivatives of the difference $\theta_0 - \theta_m$ is decreasing in both n and n' so that a profitable acquisition of m firms rises the incentive for another acquisition such that if one takeover is profitable a merger wave results and all less efficient firms are absorbed.*

Proof. To calculate the derivatives of this difference we first differentiate θ_0 and θ_m. This yields

$$\frac{\partial \theta_0(n,n',m)}{\partial n} = 0,$$
$$\frac{\partial \theta_0(n,n',m)}{\partial n'} = -\frac{1}{(n'+1)^2} < 0.$$

To simplify expressions we henceforth write ξ' for $\frac{\partial\xi}{\partial n} = \frac{\partial\xi}{\partial n'} = \frac{1}{m}$.

$$\begin{aligned}
\frac{\partial\theta_m(n,n',m)}{\partial n} &= \frac{1}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&\cdot \Big((2\xi\xi' - 2\xi')\left(2n\xi + (n'+1)\left(\xi^2 - m\right)\right) \\
&\quad - (\xi^2 - 2\xi - m)(2\xi + 2n\xi' + (n'+1)2\xi\xi')\Big) \\
&= \frac{2\xi\xi'\left(2n\xi + (n'+1)(\xi^2 - m) - (n'+1)(\xi^2 - 2\xi - m)\right)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&\quad - \frac{2\xi'\left(2n\xi + (n'+1)(\xi^2 - m) + n(\xi^2 - 2\xi - m)\right) - 2\xi(\xi^2 - 2\xi - m)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= \frac{2\xi\xi'\left(2n\xi + 2n'\xi + 2\xi\right) - 2\xi'(n+n'+1)(\xi^2 - m) - 2\xi(\xi^2 - 2\xi - m)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= \frac{(n+n'+1)(2\xi^2\xi' + 2\xi' m) - 2\xi(\xi^2 - 2\xi - m)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= \frac{2\xi'(n+n'+1)(\xi^2 + m) - 2\xi(\xi^2 - 2\xi - m)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= 2\frac{\frac{n+n'+1}{m}(\xi^2 + m) - \frac{n+n'+1-m}{m}(\xi^2 - 2\xi - m)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= 2\frac{2(n+n'+1) + 2\xi\frac{n+n'+1-m}{m} + \xi^2}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= 2\frac{2(n+n'+1) + 3\left(\frac{n+n'+1-m}{m}\right)^2}{(2n\xi + (n'+1)(\xi^2 - m))^2} > 0.
\end{aligned}$$

Hence, taking derivatives of the difference $\theta_0 - \theta_m$ yields

$$\frac{\partial(\theta_0(n,n') - \theta_m(n,n'))}{\partial n} < 0.$$

Differentiation with respect to n' yields

$$\begin{aligned}
\frac{\partial\theta_m(n,n',m)}{\partial n'} &= \frac{1}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&\cdot \Big((2\xi\xi' - 2\xi')\left(2n\xi + (n'+1)\left(\xi^2 - m\right)\right) \\
&\quad - \left(\xi^2 - 2\xi - m\right)\left(2n\xi' + (\xi^2 - m) + (n'+1)2\xi\xi'\right)\Big) \\
&= \frac{2\xi'(n+n'+1)(\xi^2 + m) - (\xi^2 - m)(\xi^2 - 2\xi - m)}{(2n\xi + (n'+1)(\xi^2 - m))^2} \\
&= \frac{2\xi'(n+n'+1)(\xi^2 + m) - (\xi^2 - m)^2 - 2\xi(\xi^2 - m)}{(2n\xi + (n'+1)(\xi^2 - m))^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2\frac{n+n'+1}{m}(\xi^2+m) - 2\frac{n+n'+1-m}{m}(\xi^2-m) - (\xi^2-m)^2}{(2n\xi + (n'+1)(\xi^2-m))^2} \\
&= \frac{4(n+n'+1) + 2(\xi^2-m) - (\xi^2-m)^2}{(2n\xi + (n'+1)(\xi^2-m))^2} \\
&= \frac{4(n+n'+1) + (\xi^2-m)(2-(\xi^2-m))}{(2n\xi + (n'+1)(\xi^2-m))^2}.
\end{aligned}
$$

Hence, taking the derivative of the difference $\theta_0 - \theta_m$ yields

$$
\begin{aligned}
&\frac{\partial(\theta_0(n,n') - \theta_m(n,n'))}{\partial n'} = -\frac{1}{(n'+1)^2} - \frac{4(n+n'+1) + (\xi^2-m)(2-(\xi^2-m))}{(2n\xi + (n'+1)(\xi^2-m))^2} \\
&= \frac{1}{(n'+1)^2(2n\xi + (n'+1)(\xi^2-m))^2} \\
&\quad \cdot \Bigg(- \Big((2n\xi)^2 + (n'+1)^2 \left(\xi^2-m\right)^2 + 4n\xi(n'+1)\left(\xi^2-m\right) \Big) \\
&\quad - 4(n+n'+1)(n'+1)^2 - 2(n'+1)^2(\xi^2-m) + (n'+1)^2(\xi^2-m)^2 \Bigg) \\
&= \frac{1}{(n'+1)^2(2n\xi + (n'+1)(\xi^2-m))^2} \\
&\quad \cdot \Bigg(-(2n\xi)^2 - 4n\xi(n'+1)\left(\xi^2-m\right) - 4(n+n'+1)(n'+1)^2 \\
&\quad - 2(n'+1)^2(\xi^2-m) \Bigg).
\end{aligned}
$$

This is obviously negative such that the difference $\theta_0 - \theta_m$ is decreasing in both n and n'.

Hence, the same reasoning as before applies to this more general case. A profitable acquisition of m home firm increases the gain of another acquisition. If one acquisition is profitable, the gain of the following one will be even higher. This implies that if one firm is taken over, then all of this country will be and a merger wave results. □

3.9.4 Free Entry in Partial Equilibrium

3.9.4.1 Proof of Proposition 3.6

Proof. To prove that there can never be any coexistence in equilibrium we consider the determinant of Jacobian matrix of firms' profit with respect to their number:

$$
|J| = \frac{\partial\hat{\pi}(n,n')}{\partial n}\frac{\partial\hat{\pi}'(n,n')}{\partial n'} - \frac{\partial\hat{\pi}(n,n')}{\partial n'}\frac{\partial\hat{\pi}'(n,n')}{\partial n}.
$$

The respective partial derivatives are as following:

$$
\begin{aligned}
\frac{\partial \hat{\pi}(n,n')}{\partial n} &= -\frac{4(a-(n'+1)c+n'c')^2}{b(n+n'+1)^3},\\
\frac{\partial \hat{\pi}(n,n')}{\partial n'} &= \frac{\partial \hat{\pi}'(n,n')}{\partial n} = -\frac{4(a-(n'+1)c+n'c')(a-(n+1)c'+nc)}{b(n+n'+1)^3},\\
\frac{\partial \hat{\pi}'(n,n')}{\partial n'} &= -\frac{4(a-(n+1)c'+nc)^2}{b(n+n'+1)^3}.
\end{aligned}
$$

The Jacobian determinant is obviously zero. This implies that $\hat{\pi}(n,n')$ and $\hat{\pi}'(n,n')$ are linearly dependent and no equilibrium solution with positive n and n' exists. Therefore, the equilibrium in a symmetric oligopoly can only be a corner solution such that with free entry there can not be coexistence of both groups of firms. □

3.9.4.2 Proof of Proposition 3.7

Proof. To prove Proposition 3.7 we show that $p > p'$ and at the same time $\hat{p}(\hat{n}(0),0) < \hat{p}'(0,\hat{n}'(0))$ is possible. This yields that on the one hand

$$c > c' + \sqrt{b}\left(\sqrt{F'} - \sqrt{F}\right)$$

while on the other hand

$$c < c' + \sqrt{\tfrac{b}{2}}\left(\sqrt{F'} - \sqrt{F}\right)$$

such that the question is whether a value for c fulfilling both equations exists. This is possible if and only if

$$
\begin{aligned}
c' + \sqrt{b}\left(\sqrt{F'} - \sqrt{F}\right) &< c' + \sqrt{\tfrac{b}{2}}\left(\sqrt{F'} - \sqrt{F}\right)\\
\Leftrightarrow \sqrt{b}\left(\sqrt{F'} - \sqrt{F}\right) &< \sqrt{\tfrac{b}{2}}\left(\sqrt{F'} - \sqrt{F}\right)\\
\Leftrightarrow \sqrt{2}\left(\sqrt{F'} - \sqrt{F}\right) &< \left(\sqrt{F'} - \sqrt{F}\right).
\end{aligned}
$$

This is obviously always fulfilled for $F' < F$. □

To sum up, to fulfill $p > p'$ and at the same time $\hat{p}(\hat{n}(0),0) < \hat{p}'(0,\hat{n}'(0))$, fixed entry cost have to be higher in the dominating country do allow for a higher price in that country in autarky while to dominate afterwards unit cost must be below a certain threshold such that for $c \in \left[c' - \sqrt{b}\left(\sqrt{F} - \sqrt{F'}\right); c' - \sqrt{\tfrac{b}{2}}\left(\sqrt{F} - \sqrt{F'}\right)\right]$ the described scenario is true.

3.9.5 General Equilibrium

3.9.5.1 Equilibrium in Autarky

To solve the model, we need to evaluate $L = n\int_0^1 \tau(z)x(z)dz$ using the equilibrium output $x = \frac{\frac{a}{\lambda}-c}{(n+1)\frac{b}{\lambda}}$:

$$\begin{aligned} L =& n\int_0^1 \tau(z)x(z)dz = n\int_0^1 \tau(z)\frac{\frac{a}{\lambda}-w\tau(z)}{(n+1)\frac{b}{\lambda}}dz \\ =& \frac{n}{(n+1)\frac{b}{\lambda}}\left(\frac{a}{\lambda}\int_0^1 \tau(z)dz - w\int_0^1 \tau(z)^2 dz\right) \\ \Leftrightarrow w =& \frac{an\int_0^1 \tau(z)dz - (n+1)bL}{\lambda n\int_0^1 \tau(z)^2 dz}. \end{aligned}$$

We denote μ_τ and σ_τ as the first and second moments of the technology distribution:

$$\mu_\tau \equiv \int_0^1 \tau(z)dz, \quad \sigma_\tau^2 \equiv \int_0^1 \tau(z)^2 dz.$$

This yields

$$w = \left(a\mu_\tau - \frac{n+1}{n}bL\right)\frac{1}{\lambda\sigma_\tau^2}. \tag{3.69}$$

National income equals the sum of wages and total profits, measured in utility units:

$$I = wL + \Pi$$

with $\Pi = \int_0^1 n\pi(z)dz$.

Total profit in each sector is

$$n\pi(z) = n\frac{b}{\lambda}x^2 = n\frac{\lambda}{b}\frac{(\frac{a}{\lambda}-c)^2}{(n+1)^2} = \frac{\lambda n}{(n+1)^2 b}\left((\tfrac{a}{\lambda})^2 - 2\tfrac{a}{\lambda}w\tau(z) + w^2\tau(z)^2\right).$$

Now, we can calculate total profit of the economy:

$$\begin{aligned} \Pi =& \frac{\lambda n}{(n+1)^2 b}\left((\tfrac{a}{\lambda})^2 - 2\tfrac{a}{\lambda}w\int_0^1 \tau(z)dz + w^2\int_0^1 \tau(z)^2 dz\right) \\ =& \frac{\lambda n}{b(n+1)^2}\left((\tfrac{a}{\lambda})^2 - 2\tfrac{a}{\lambda}w\mu_\tau + w^2\sigma_\tau^2\right). \end{aligned} \tag{3.70}$$

Using (3.69) in (3.70) yields

$$\begin{aligned}
\Pi =& \frac{n\lambda}{b(n+1)^2}\left((\tfrac{a}{\lambda})^2 - 2\tfrac{a}{\lambda}\mu_\tau\left(\tfrac{a}{\lambda}\mu_\tau - \frac{n+1}{n}\tfrac{b}{\lambda}L\right)\frac{1}{\sigma_\tau^2} + \left(\tfrac{a}{\lambda}\mu_\tau - \frac{n+1}{n}\tfrac{b}{\lambda}L\right)^2\frac{1}{\sigma_\tau^2}\right)\\
=& \frac{n\lambda}{b(n+1)^2}\Bigg((\tfrac{a}{\lambda})^2 - 2(\tfrac{a}{\lambda})^2\frac{\mu_\tau^2}{\sigma_\tau^2} + 2\tfrac{a}{\lambda}\frac{n+1}{n}\tfrac{b}{\lambda}L\frac{\mu_\tau}{\sigma_\tau^2} \qquad (3.71)\\
&+ (\tfrac{a}{\lambda})^2\frac{\mu_\tau^2}{\sigma_\tau^2} - 2\tfrac{a}{\lambda}\frac{n+1}{n}\tfrac{b}{\lambda}L\frac{\mu_\tau}{\sigma_\tau^2} + \left(\frac{n+1}{n}\right)^2(\tfrac{b}{\lambda})^2L^2\frac{1}{\sigma_\tau^2}\Bigg)\\
=& \frac{n\lambda}{b(n+1)^2}\left((\tfrac{a}{\lambda})^2 - (\tfrac{a}{\lambda})^2\frac{\mu_\tau^2}{\sigma_\tau^2} + \left(\frac{n+1}{n}\right)^2(\tfrac{b}{\lambda})^2L^2\frac{1}{\sigma_\tau^2}\right)\\
=& \frac{1}{\sigma_\tau^2}\left((\tfrac{a}{\lambda})^2\frac{n\lambda}{b(n+1)^2}\left(\sigma_\tau^2 - \mu_\tau^2\right) + \frac{1}{n}\tfrac{b}{\lambda}L^2\right)\\
=& \left(\frac{bL^2}{n} + \frac{na^2}{(n+1)^2 b}v^2\right)\frac{1}{\lambda\sigma_\tau^2}. \qquad (3.72)
\end{aligned}$$

For $\lambda = 1$ wage rate is strictly increasing in n:

$$\frac{\partial w}{\partial n} = bL\frac{1}{n^2\sigma_\tau^2} > 0;$$

while total profits are strictly decreasing in n:

$$\frac{\partial \Pi}{\partial n} = -\left(\frac{bL^2}{n^2} + \frac{(n-1)a^2}{(n+1)^3 b}v^2\right)\frac{1}{\sigma_\tau^2} < 0.$$

For national income we get

$$\begin{aligned}
I &= \left(a\mu_\tau - \frac{n+1}{n}bL\right)\frac{1}{\lambda\sigma_\tau^2}L + \left(\frac{bL^2}{n} + \frac{na^2}{(n+1)^2 b}v^2\right)\frac{1}{\lambda\sigma_\tau^2}\\
&= \left(a\mu_\tau L - bL^2 + \frac{na^2}{(n+1)^2 b}v^2\right)\frac{1}{\lambda\sigma_\tau^2} \qquad (3.73)
\end{aligned}$$

such that for $\lambda = 1$ national income is decreasing in n:

$$\frac{\partial I}{\partial n} = -\frac{(n-1)av^2}{(n+1)^3 b\sigma_\tau^2} < 0. \qquad (3.74)$$

Welfare. To determine welfare we can rewrite the utility function using (3.22):

$$\begin{aligned}
U &= \int_0^1 ay(z) - \frac{b}{2}y(z)^2 dz \\
&= \int_0^1 \left(-a^2 + 2aby(z) - b^2 y(z)^2\right)\frac{1}{2b} + \frac{a^2}{2b} dz \\
&= \frac{a^2}{2b} - \frac{\lambda^2}{2b}\int_0^1 p(z)^2 dz \\
&= \frac{a^2}{2b} - \frac{\lambda^2}{2b}\sigma_p^2. \qquad (3.75)
\end{aligned}$$

To calculate this indirect utility function, we make use of the Cournot equilibrium prices $p(z) = \frac{\frac{a}{\lambda}+nc(z)}{n+1}$. Squaring and integrating yields

$$\lambda^2\sigma_p^2 = \lambda^2 \int_0^1 p(z)^2 dz = \frac{1}{(n+1)^2}\left(a^2 + 2na\lambda w\mu_\tau + n^2(\lambda w)^2\sigma_\tau^2\right).$$

Substituting (3.27) this becomes

$$\begin{aligned}
\lambda^2\sigma_p^2 =& \frac{1}{(n+1)^2}\left(a^2 + 2na\left(a\mu_\tau - \frac{n+1}{n}bL\right)\frac{\mu_\tau}{\sigma_\tau^2} + \left(a\mu_\tau - \frac{n+1}{n}bL\right)^2\frac{n^2}{\sigma_\tau^2}\right) \\
=& \frac{1}{(n+1)^2}\left(a^2 + \frac{2na^2\mu_\tau^2}{\sigma_\tau^2} - \frac{2(n+1)a\mu_\tau bL}{\sigma_\tau^2}\right) + \frac{\left(\frac{n}{n+1}a\mu_\tau - bL\right)^2}{\sigma_\tau^2} \\
=& \frac{1}{(n+1)^2}\left(a^2 + \frac{2na^2\mu_\tau^2 - 2(n+1)a\mu_\tau bL}{\sigma_\tau^2}\right) \\
& - \frac{2n+1}{(n+1)^2}\frac{a^2\mu_\tau^2}{\sigma_\tau^2} + \frac{2}{(n+1)}\frac{a\mu_\tau bL}{\sigma_\tau^2} + \frac{(a\mu_\tau - bL)^2}{\sigma_\tau^2} \\
=& \frac{1}{(n+1)^2}\left(a^2 - \frac{a^2\mu_\tau^2}{\sigma_\tau^2}\right) + \frac{(a\mu_\tau - bL)^2}{\sigma_\tau^2} \\
=& \frac{1}{(n+1)^2}\left(\frac{a^2 v^2}{\sigma_\tau^2}\right) + \frac{(a\mu_\tau - bL)^2}{\sigma_\tau^2}. \qquad (3.76)
\end{aligned}$$

Finally, we consider the effects of an in increase in the technology variance v^2 at constant μ_τ. It is possible to show that a mean preserving spread in the technology distribution raises aggregate welfare but lowers the wage rate.

With $\sigma_\tau^2 = v^2 + \mu_\tau^2$ in (3.27), (3.29) and σ_p^2 we get $\lambda w = \left(a\mu_\tau - \frac{n+1}{n}bL\right)\frac{1}{v^2+\mu_\tau^2}$ which is strictly decreasing in v such that the effect of v^2 on wages is negative.

Additionally, $\sigma_p^2 = \frac{1}{(n+1)^2}\left(\frac{a^2v^2}{v^2+\mu_\tau^2}\right) + \frac{(a\mu_\tau - bL)^2}{v^2+\mu_\tau^2}$ and

$$\begin{aligned}\frac{\partial \sigma_p^2}{\partial v^2} &= -\frac{(na\mu_\tau - (n+1)bL)((n+2)a\mu_\tau - (n+1)bL)}{(n+1)^2((v^2+\mu_\tau^2)^2)}\\ &= -\frac{n(n+2)}{(n+1)^2(v^2+\mu_\tau^2)^2}\left(a\mu_\tau - \frac{n+1}{n}bL\right)\left(a\mu_\tau - \frac{n+1}{n+2}bL\right).\end{aligned}$$

As $\left(a\mu_\tau - \frac{n+1}{n}bL\right) > 0$, so is $\left(a\mu_\tau - \frac{n+1}{n+2}bL\right)$ and thus $\frac{\partial \sigma_p^2}{\partial v^2} < 0$. That means that a raise in variance increases aggregate welfare.

3.9.5.2 Integration with Full Diversification

Labor demand at symmetric equilibrium with full diversification is given as

$$\begin{aligned}L = L(w,n) &= \int_0^1 n\tau(z)\frac{\frac{2a}{\lambda^W} - (n+1)w\tau(z) + nw\tau'(z)}{(2n+1)\frac{b}{\lambda^W}}dz\\ &= \int_0^1 n\tau(z)\frac{2a - (n+1)\lambda^W w\tau(z) + n\lambda^W w\tau'(z)}{(2n+1)b}dz\\ &= \frac{n}{(2n+1)b}\left(2a\mu_\tau - (n+1)\lambda^W w\sigma_\tau^2 + n\lambda^W w\rho^2\right).\end{aligned}$$

Solving this for marginal real wage yields

$$\begin{aligned}\lambda^W w &= 2\frac{a\mu_\tau - \frac{2n+1}{2n}bL}{(n+1)\sigma_\tau^2 - n\rho^2}\\ &= 2\frac{a\mu_\tau - \frac{2n+1}{2n}bL}{\sigma_\tau^2 + n\gamma}.\end{aligned}$$

With $\lambda^W = 2\widehat{\lambda}$, wage in both countries is

$$\widehat{\lambda} w = \frac{a\mu_\tau - \frac{2n+1}{2n}bL}{\sigma_\tau^2 + n\gamma}.$$

Welfare. Using (3.37) we can rewrite the utility function (3.20):

$$\begin{aligned}U^W = \widehat{U} + \widehat{U}' &= \int_0^1 ay(z) - \frac{b}{2}y(z)^2 dz\\ &= \int_0^1 \left(-a^2 + aby(z) - \left(\frac{b}{2}\right)^2 y(z)^2\right)\frac{1}{b} + \frac{a^2}{b}dz\\ &= \frac{a^2}{b} - \frac{\widehat{\lambda}^2}{b}\int_0^1 p(z)^2 dz\\ &= \frac{a^2}{b} - \frac{\widehat{\lambda}^2}{b}\sigma_p^2. \qquad (3.77)\end{aligned}$$

Hence, ignoring constants the indirect utility function for one country in the underlying symmetric situation depends inversely on the variance of the distribution of prices weighted by the marginal utility of income, $\widehat{U} = \frac{a^2}{2b} - \frac{\widehat{\lambda}^2}{2b}\sigma_p^2$ such that for comparing welfare it is sufficient so compare $-\widehat{\lambda}^2\widehat{\sigma}_p^2$ and $-\lambda^2\sigma_p^2$.

This second moment of price distribution using Cournot equilibrium price (3.7) yields

$$\begin{aligned}
-\widehat{\lambda}^2\widehat{\sigma}_p^2 =& -\frac{\widehat{\lambda}^2}{(2n+1)^2}\left(\int_0^1 \left(\frac{a}{\widehat{\lambda}}\right)^2 + 2n\frac{a}{\widehat{\lambda}}\widehat{w}(\tau(z)+\tau'(z))\right.\\
&\left. + n^2w^2\left(\tau(z)^2 + 2\tau(z)\tau'(z) + \tau'(z)^2\right)dz\right)\\
=& -\frac{\widehat{\lambda}^2}{(2n+1)^2}\left(\left(\frac{a}{\widehat{\lambda}}\right)^2 + 4n\frac{a}{\widehat{\lambda}}\widehat{w}\mu_\tau + 2n^2\widehat{w}^2\left(\sigma_\tau^2+\rho^2\right)\right)\\
=& -\frac{1}{(2n+1)^2}\left(a^2 + 4na\widehat{\lambda}\widehat{w}\mu_\tau + 2n^2(\widehat{\lambda}\widehat{w})^2\left(\sigma_\tau^2+\rho^2\right)\right)\\
=& -\frac{1}{(2n+1)^2}\left(a^2 + 2n\widehat{\lambda}\widehat{w}\left(2a\mu_\tau + n\widehat{\lambda}\widehat{w}\left(\sigma_\tau^2+\rho^2\right)\right)\right).
\end{aligned}$$

Inserting equilibrium marginal wage (3.40) yields

$$\begin{aligned}
&-\widehat{\lambda}^2\widehat{\sigma}_p^2\\
&= -\frac{1}{(2n+1)^2}\left(a^2 + \left(a\mu_\tau - \frac{2n+1}{2n}bL\right)\frac{2n}{\sigma_\tau^2+n\gamma}\right.\\
&\quad\left.\cdot\left(2a\mu_\tau + \left(a\mu_\tau - \frac{2n+1}{2n}bL\right)\frac{n\left(\sigma_\tau^2+\rho^2\right)}{\sigma_\tau^2+n\gamma}\right)\right)\\
&= -\frac{1}{(2n+1)^2}\left(a^2 + 2\left(an\mu_\tau - \frac{2n+1}{2}bL\right)\frac{\sigma_\tau^2+\rho^2}{\sigma_\tau^2+n\gamma}\right.\\
&\quad\left.\cdot\left(\frac{2a\mu_\tau}{\sigma_\tau^2+\rho^2} + \left(an\mu_\tau - \frac{2n+1}{2}bL\right)\frac{1}{\sigma_\tau^2+n\gamma}\right)\right)\\
&= -\frac{1}{(2n+1)^2}\left(a^2\right.\\
&\quad\left. + 2(\sigma_\tau^2+\rho^2)\left(\left(\frac{an\mu_\tau - (2n+1)\frac{b}{2}L}{\sigma_\tau^2+n\gamma} + \frac{a\mu_\tau}{\sigma_\tau^2+\rho^2}\right)^2 - \left(\frac{a\mu_\tau}{\sigma_\tau^2+\rho^2}\right)^2\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{(2n+1)^2}\Bigg(a^2\left(\frac{\sigma_\tau^2+\rho^2-2\mu_\tau^2}{2\sigma_\tau^2-\gamma}\right) \\
&\quad + 2\frac{\sigma_\tau^2+\rho^2}{\left(\sigma_\tau^2+n\gamma\right)^2}\left(a\mu_\tau\left(\frac{n(\sigma_\tau^2+\rho^2)-\sigma_\tau^2+n\gamma}{\sigma_\tau^2+\rho^2}\right)-(2n+1)\tfrac{b}{2}L\right)^2\Bigg).
\end{aligned}
$$

As $\sigma_\tau^2+\rho^2 = 2\sigma_\tau^2-\sigma_\tau^2+\rho^2 = 2\sigma_\tau^2-\gamma$ this can be written as

$$
\begin{aligned}
&-\widehat{\lambda}^2\widehat{\sigma}_p^2 \\
&= -\frac{1}{(2n+1)^2}\left(a^2\left(\frac{2v^2-\gamma}{2\sigma_\tau^2-\gamma}\right)+(2n+1)^2\frac{2\sigma_\tau^2-\gamma}{2\left(\sigma_\tau^2+n\gamma\right)^2}\left(a\mu_\tau\frac{2\sigma_\tau^2}{2\sigma_\tau^2-\gamma}-bL\right)^2\right).
\end{aligned}
$$

The first derivative of $\widehat{U}$ given in (3.42) with respect to γ leaving μ_τ and σ_τ fix is

$$
\begin{aligned}
\frac{\partial\widehat{U}}{\partial\gamma} &= -\frac{1}{2b(2n+1)^2}\left(4na\frac{\partial(\widehat{\lambda}\widehat{w})}{\partial\gamma}\mu_\tau+4n^2\widehat{\lambda}\widehat{w}\frac{\partial(\widehat{\lambda}\widehat{w})}{\partial\gamma}\left(\sigma_\tau^2-\gamma\right)-2n^2\left(\widehat{\lambda}\widehat{w}\right)^2\right) \\
&= \frac{2n}{2b(2n+1)^2}\left(-2\frac{\partial(\widehat{\lambda}\widehat{w})}{\partial\gamma}\left(a\mu_\tau+n\widehat{\lambda}\widehat{w}\left(\sigma_\tau^2-\gamma\right)\right)+n\left(\widehat{\lambda}\widehat{w}\right)^2\right).
\end{aligned}
$$

As marginal wage (3.40) is obviously decreasing in γ, i.e., $\frac{\partial(\widehat{\lambda}\widehat{w})}{\partial\gamma}<0$, the derivative above is positive.[44]

3.9.5.3 Integration with Specialization: Properties of Labor Market Equilibrium

Labor demand at symmetric equilibrium is given as

$$
\begin{aligned}
L(w,\bar{z},n) &= \int_0^{\bar{z}'} n\tau(z)\frac{\frac{a}{\widehat{\lambda}}-\widehat{w}\tau(z)}{(n+1)\frac{b}{2\widehat{\lambda}}}dz+\int_{\bar{z}'}^{\bar{z}} n\tau(z)\frac{\frac{a}{\widehat{\lambda}}-(n+1)\widehat{w}\tau(z)+n\widehat{w}\tau'(z)}{(2n+1)\frac{b}{2\widehat{\lambda}}}dz \\
&= \frac{2n\widehat{\lambda}}{(n+1)b}\left(\frac{a}{\widehat{\lambda}}\int_0^{\bar{z}'}\tau(z)dz-\widehat{w}\int_0^{\bar{z}'}\tau(z)^2dz\right) \\
&\quad + \frac{2n\widehat{\lambda}}{(2n+1)b}\Bigg(\frac{a}{\widehat{\lambda}}\int_{\bar{z}'}^{\bar{z}}\tau(z)dz-(n+1)\widehat{w}\int_{\bar{z}'}^{\bar{z}}\tau(z)^2dz \\
&\quad + n\widehat{w}\int_{\bar{z}'}^{\bar{z}}\tau'(z)\tau(z)dz\Bigg) \\
&= \frac{2n}{b}\frac{a\mu_S-\widehat{\lambda}\widehat{w}\sigma_S^2}{n+1}+\frac{2n}{b}\frac{a\mu_D-(n+1)\widehat{\lambda}\widehat{w}\sigma_D^2+n\widehat{\lambda}\widehat{w}\rho^2}{2n+1}
\end{aligned}
\tag{3.78}
$$

[44]Recently, after we wrote this essay we became aware that in parallel a derivation similar to the one above has been done (maybe not coincidentally but rather inevitably) by Neary, although the respective paper is not completed yet (cf. Neary, 2009).

with

$$\mu_S \equiv \int_0^{\bar{z}'} \tau(z)dz, \quad \mu_D \equiv \int_{\bar{z}'}^{\bar{z}} \tau(z)dz,$$
$$\sigma_S^2 \equiv \int_0^{\bar{z}'} \tau(z)^2 dz, \quad \sigma_D^2 \equiv \int_{\bar{z}'}^{\bar{z}} \tau(z)^2 dz, \quad \rho^2 \equiv \int_{\bar{z}'}^{\bar{z}} \tau(z)\tau'(z)dz.$$

To show the properties of the labor market equilibrium we calculate the derivatives of (3.47). Consider the derivative with respect to $\widehat{w}$:

$$\frac{\partial L(\cdot)}{\partial \widehat{w}} = -\frac{2\widehat{\lambda} n}{b}\left(\frac{\sigma_S^2}{n+1} + \frac{(n+1)\sigma_D^2 - n\rho^2}{2n+1}\right).$$

From symmetry we get

$$\begin{aligned}\sigma_S^2 - \rho^2 &= \int_{1-\bar{z}}^{\bar{z}} \tau(z)^2 - 2\tau(z)\tau'(z) + \tau'(z)^2 + \tau(z)\tau'(z) - \tau'(z)^2 dz \\ &= \int_{1-\bar{z}}^{\bar{z}} (\tau(z) - \tau(1-z))^2 + \tau(z)\tau(1-z) - \tau(1-z)^2 dz \\ &= \int_{1-\bar{z}}^{\bar{z}} (\tau(z) - \tau(1-z))^2 dz + \rho^2 - \sigma_S^2.\end{aligned}$$

Hence,

$$\sigma_S^2 - \rho^2 = \frac{1}{2}\int_{1-\bar{z}}^{\bar{z}} (\tau(z) - \tau'(z))^2 dz > 0.$$

From this it follows that $\frac{\partial L(\cdot)}{\partial \widehat{w}}$ is negative.

Next consider the effect of a change in the extensive margin and differentiate (3.36) with respect to $\bar{z}$ using $\bar{z}' = 1 - \bar{z}$:

$$\begin{aligned}\frac{\partial L(\cdot)}{\partial \bar{z}} =& - n\tau(1-\bar{z})\hat{x}(\widehat{w}, 1-\bar{z}, n, 0) + n\tau(1-\bar{z})\hat{x}(\widehat{w}, \widehat{w}, 1-\bar{z}, n, n) \\ &+ n\tau(\bar{z})\hat{x}(\widehat{w}, \widehat{w}, \bar{z}, n, n) \\ =& n\tau(1-\bar{z})\left(\hat{x}'(\widehat{w}, \widehat{w}, \bar{z}, n, n) - \hat{x}'(\widehat{w}, \bar{z}, n, 0)\right) + n\tau(\bar{z})\hat{x}(\widehat{w}, \widehat{w}, \bar{z}, n, n).\end{aligned}$$

Using Lemma 3.2 for $m = n$ we know that $\hat{x}'(\widehat{w}, \bar{z}, n, 0) - \hat{x}'(\widehat{w}, \widehat{w}, \bar{z}, n, n) = \frac{n}{n+1}\hat{x}(\widehat{w}, \widehat{w}, \bar{z}, n, n)$ so that

$$\frac{\partial L(\cdot)}{\partial \bar{z}} = n\hat{x}(\widehat{w}, \widehat{w}, \bar{z}, n, n)\left(\tau(\bar{z}) - \tau(1-\bar{z})\frac{n}{n+1}\right).$$

As $\bar{z} \geq \frac{1}{2}$, the above expression is positive.

3.9.6 Free Entry in General Equilibrium

3.9.6.1 Free Entry with Autarky

To solve the model, we need to evaluate $L = \int_0^1 n(z)(\tau(z)x(z) + \kappa(z)^2)dz$ using the equilibrium output from Section 3.4:

$$\begin{aligned}
L =& \int_0^1 n(z)(\tau(z)x(z) + \kappa(z)^2)dz \\
=& \int_0^1 \frac{\lambda}{b}\tau(z)\left(\frac{a}{\lambda} - w\tau(z) - \sqrt{\frac{b}{\lambda}F(z)}\right) + \sqrt{\frac{\lambda}{b}}\frac{\kappa(z)}{\sqrt{w}}\left(\frac{a}{\lambda} - w\tau(z)\right) - \kappa(z)^2 dz \\
=& \frac{a}{b}\int_0^1 \tau(z)dz - \frac{\lambda w}{b}\int_0^1 \tau(z)^2 dz - \sqrt{\frac{\lambda w}{b}}\int_0^1 \kappa(z)\tau(z)dz \\
&+ \frac{a}{\sqrt{b\lambda w}}\int_0^1 \kappa(z)dz - \sqrt{\frac{\lambda w}{b}}\int_0^1 \tau(z)\kappa(z)dz - \int_0^1 \kappa(z)^2 dz.
\end{aligned}$$

With μ_κ, σ_κ, and $\varrho^2_{\tau\kappa}$ as defined this can be written as

$$\begin{aligned}
L =& \frac{a}{b}\mu_\tau - \lambda w\frac{\sigma_\tau^2}{b} - 2\sqrt{\frac{\lambda w}{b}}\varrho^2_{\tau\kappa} + \frac{a}{\sqrt{b\lambda w}}\mu_\kappa - \sigma_\kappa^2 \\
=& \frac{1}{b}\left(a\mu_\tau - \lambda w\sigma_\tau^2\right) + \frac{1}{\sqrt{b\lambda w}}\left(a\mu_\kappa - 2\lambda w\varrho^2_{\tau\kappa}\right) - \sigma_\kappa^2.
\end{aligned}$$

Assuming entry technology to be the same in all sectors, i.e., $\kappa(z) = k$, yields for labor demand:

$$L = \frac{1}{b}\left(a\mu_\tau - \lambda w\sigma_\tau^2\right) + \frac{k}{\sqrt{b\lambda w}}\left(a - 2\lambda w\mu_\tau\right) - k^2.$$

Using the implicit function theorem it is obvious that partial derivative of $L(\cdot)$ with respect to w is negative. In contrast partial derivative with respect to k might be positive or negative, depending on the parameters and especially on labor requirement for entry. With relatively high labor requirement for entry the derivative is negative such that in equilibrium marginal wage is decreasing in k. Conversely, with relatively low k wage increases with labor requirement for entry.

Indirect utility is given by the variance of the distribution of prices deflated by marginal utility of income:

$$\lambda^2\sigma_p^2 = \lambda^2\int_0^1 p(z)^2 dz = \lambda w\left(\lambda w\sigma_\tau^2 + 2\sqrt{\lambda w b}\varrho^2_{\tau\kappa} + b\sigma_\kappa^2\right).$$

3.9.6.2 Free Entry with Integration

The labor market conditions using the equilibrium output of sectors and threshold of specialization $\bar{z}$ with specialization in every sector for the home country this condition reads:

$$\begin{aligned}
L =& \int_0^{\bar{z}} \hat{n}(\widehat{w}, z) \left(\tau(z)\hat{x}(\widehat{w}, z, 0) + \kappa(z)^2\right) dz \\
=& \int_0^{\bar{z}} \tau(z) \left(2\frac{a - \widehat{\lambda}\widehat{w}\tau(z)}{b} - \sqrt{2\frac{\widehat{\lambda}\widehat{w}}{b}}\kappa(z)\right) \\
&+ \sqrt{\frac{2}{b}}\kappa(z) \left(\frac{a}{\sqrt{\widehat{\lambda}\widehat{w}}} - \sqrt{\widehat{\lambda}\widehat{w}}\tau(z)\right) - \kappa(z)^2 dz \\
=& 2\frac{a}{b}\mu_{\tau S} - 2\widehat{\lambda}\widehat{w}\frac{\sigma_{\tau S}^2}{b} - 2\sqrt{2\frac{\widehat{\lambda}\widehat{w}}{b}}\varrho_S^2 + \sqrt{\frac{2}{b\widehat{\lambda}\widehat{w}}}a\mu_{\kappa S} - \sigma_{\kappa S}^2 \\
=& \frac{2}{b}\left(a\mu_{\tau S} - \widehat{\lambda}\widehat{w}\sigma_{\tau S}^2\right) + \sqrt{\frac{2}{b\widehat{\lambda}\widehat{w}}}\left(a\mu_{\kappa S} - 2\widehat{\lambda}\widehat{w}\varrho_S^2\right) - \sigma_{\kappa S}^2.
\end{aligned}$$

Indirect utility is again given by the variance of the distribution of prices deflated by marginal utility of income:

$$\begin{aligned}
\widehat{\lambda}^2\widehat{\sigma}_p^2 =& \widehat{\lambda}^2 \int_0^1 p(w, z)^2 dz \\
=& \widehat{\lambda}^2 \int_0^{\frac{1}{2}} \widehat{w}^2\tau(z)^2 + 2\widehat{w}\sqrt{w}\tau(z)\kappa(z)\sqrt{\frac{b}{2\widehat{\lambda}}} + \frac{b}{2\widehat{\lambda}}\widehat{w}\kappa(z)^2 dz \\
&+ \widehat{\lambda}^2 \int_{\frac{1}{2}}^1 \widehat{w}^2\tau'(z)^2 + 2\widehat{w}\sqrt{w}\tau'(z)\kappa'(z)\sqrt{\frac{b}{2\widehat{\lambda}}} + \frac{b}{2\widehat{\lambda}}\widehat{w}\kappa'(z)^2 dz.
\end{aligned}$$

Due to symmetry this is

$$\begin{aligned}
\widehat{\lambda}^2\widehat{\sigma}_p^2 =& 2\widehat{\lambda}^2 \int_0^{\frac{1}{2}} \widehat{w}^2\tau(z)^2 + 2\widehat{w}\sqrt{w}\tau(z)\kappa(z)\sqrt{\frac{b}{2\widehat{\lambda}}} + \frac{b}{2\widehat{\lambda}}\widehat{w}\kappa(z)^2 dz \\
=& 2\widehat{\lambda}\widehat{w}\left(\widehat{\lambda}\widehat{w}\sigma_{\tau S}^2 + \sqrt{2\widehat{\lambda}\widehat{w}b}\varrho_S^2 + \frac{b}{2}\sigma_{\kappa S}^2\right).
\end{aligned}$$

3.9.6.3 Proof of Proposition 3.11

Proof. For the case of negligible small entry costs in every sector we can solve (3.53) and (3.50) for marginal wages and get

$$\lambda w = \frac{a\mu_\tau - bL}{\sigma_\tau^2} \text{ and}$$
$$\widehat{\lambda}\widehat{w} = \frac{a\mu_{\tau S} - \frac{b}{2}L}{\sigma_{\tau S}^2}.$$

Note that these wage rates under perfect competition are obviously identical to wage rates in Section 3.5 and 3.6 with the number of firms approaching infinity.

If the distribution of unit requirements $\tau(z)$ belongs to the convenient class of exponentiation functions, i.e., $\tau(z) = \tau z^\alpha$ with $\alpha > 0$, we have that

$$\mu_\tau = \frac{\tau}{\alpha+1},$$
$$\sigma_\tau^2 = \frac{\tau^2}{2\alpha+1},$$
$$\mu_{\tau S} = \frac{\tau}{\alpha+1}\frac{1}{2^{\alpha+1}} = \frac{1}{2^{\alpha+1}}\mu_\tau,$$
$$\sigma_{\tau S}^2 = \frac{\tau^2}{2\alpha+1}\frac{1}{2^{2\alpha+1}} = \frac{1}{2^{2\alpha+1}}\sigma_\tau^2.$$

Note that for an interior solution for labor market equilibrium in case of integration $a > 2^\alpha(\alpha+1)\frac{bL}{\tau}$ has to be fulfilled and is therefore assumed.

Thus, the increase in marginal wage is

$$\begin{aligned}\widehat{\lambda}\widehat{w} - \lambda w =& \frac{1}{2^{2\alpha+1}\sigma_\tau^2}\left(\frac{a}{2^{\alpha+1}}\mu_\tau - \frac{b}{2}L - 2^{2\alpha+1}\left(a\mu_\tau - bL\right)\right)\\ =& \frac{1}{2^{2\alpha+1}\sigma_\tau^2}\left(a\mu_\tau\left(2^{-\alpha-1} - 2^{\alpha+1}\right) - bL\left(2^{-1} - 2^{2\alpha+1}\right)\right).\end{aligned}$$

Since $0 > 2^{-\alpha-1} - 2^{\alpha+1} > 2^{-1} - 2^{2\alpha+1}$, for the above term to be positive,

$$a\mu_\tau = a\frac{\tau}{\alpha+1} < bL\frac{2^{-1} - 2^{2\alpha+1}}{2^{-\alpha-1} - 2^{\alpha+1}} = 2^\alpha$$

has to hold which cannot be the case such that wages decrease with integration. Note that the dependency of extent of the fall in wages on the shape of technology distribution is not straight forward.

However, as wages decrease with integration, welfare always increases. □

Chapter 4

Price and Inventory Dynamics in an Oligopoly Industry

4.1 Introduction

This essay analyzes the interaction between price and inventory decisions in an oligopoly industry and its implications for the dynamics of prices such as price dispersion. Cross-sectional price dispersion is a common feature in many retail markets. Since Stigler's (1961) seminal work price dispersion has usually been explained by consumer search costs. In contrast, Aguirregabiria (1999) shows that retail inventories can generate (S, s) dynamics of inventories which in turn can explain time variability of prices of supermarket chains.[1] However, as in his model monopolistic competition is analyzed price dispersion between different firms can not be observed.

Extending the described work, this chapter addresses the question how oligopolistic competition affects these dynamics.[2]

Previous papers have characterized the optimal decision rules of similar dynamic models. In addition to Aguirregabiria (1999) who analyzes price and inventories

[1]Under an (S, s) rule inventory moves between the target inventory level, S, and the order threshold, s, with $s < S$. Whenever the firm's inventory level falls below the order threshold, a new order is placed such that the target inventory level S is attained.

[2]Additionally, the main focus of the paper by Aguirregabiria is an empirical analysis building on a numeric simulation. The formal theoretical proof of the optimality of the considered inventory decision is therefore not rigourously done and incomplete. Thus, our essay is the first to formally prove the optimality of (S, s) policy with endogenous prices.

with lump-sum costs under monopolistic competition Hall and Rust (2007) study optimal inventory decisions with lump-sum costs under perfect competition. Their paper extends the framework of Aguirregabiria (1999) in some ways but is otherwise limited to one decision variable as prices are taken as given. Hall and Rust (2007) show that in their perfect competition model the (S, s) policy is an optimal order strategy.[3] To the best of our knowledge, these two works studying extreme cases of competition are by far the most elaborated papers investigating these decision problems.[4] The analysis of optimal decision rules under oligopolistic competition forms an obvious gap in the literature.

However, related studies of oligopolistic competition exists. Dutta and Sundaram (1992) and Dutta and Rustichini (1995) analyze a discrete choice stochastic duopoly game with lump-sum costs. In these frameworks the one abstract decision variable affecting both firms' payoffs cannot be interpreted as being related to inventory. Nevertheless, the optimality of an (S, s) policy can also be shown. More recently, Besanko and Doraszelski (2004) study decisions about prices and capacity. However, the main and important difference between inventory and capacity is that excess capacity is worthless while keeping inventory affects future competition. Hence, additional strategic effects due to kept stock are at place. This is especially important when investigating oligopolistic competition.

This essay extends the literature by characterizing an equilibrium in a model of price and inventory competition in oligopoly. We allow oligopolistic firms to interact strategically. This allows for studying price dispersion between firms.

Besides, such a model that is incorporating inventory and oligopoly in dynamic competition provides the most plausible framework for retail industries. Retail industries have become highly concentrated, i.e., in most categories like grocery, supermarkets, and office supplies just a handful of rivals compete locally. In the supermarket industry for example a small number of firms capture the majority of sales as supermarkets compete in tight regional oligopolies. Thus, this industry is

[3]Thereby Hall and Rust (2007) extend earlier work like Sethi and Cheng (1997) and Cheng and Sethi (1999) to a more general specification of the Markov process.

[4]There exist also some papers analyzing dynamic oligopoly with inventories without considering lump-sum ordering cost, like Kirman and Sobel (1974) or more recently Bernstein and Federgruen (2004). However, without ordering cost stationary optimal strategies result which are in essence identical to those of the corresponding static single period game.

a prime example of oligopoly. Besides, inventory costs are of major importance. Supermarkets invest in state of the art distribution systems to minimize storage and transportation costs (see e.g. Beresteanu & Ellickson, 2006; Ellickson, 2007). Hence, deciding the optimal inventory and store offer forms an important optimization problem for supermarket chains.

In this work we study the decision problem of a central store, i.e., its decision about retail prices and orders to suppliers, facing oligopolistic competition and taking into account the existence of lump-sum ordering cost. We develop a model of retail competition in which the impact of inventories on competition and prices can be evaluated. We analyze the characteristics of the optimal decision rule.

The main findings of our theoretical model of oligopoly support the simulation results of Aguirregabiria (1999) studying monopoly. Key factors for price fluctuations are lump-sum ordering costs and demand uncertainty. Lump-sum ordering cost generate (S, s) inventory behavior. Demand uncertainty creates a positive probability of excess demand, i.e., stockouts. The positive stockout probability has a negative effect on expected sales which in turn creates substitutability between prices and inventories in the profit function such that in equilibrium prices depend negatively and very significantly on the level of inventories. This results in a cyclical pattern of inventories and prices where prices decline significantly when an order is placed and consequently inventory reduction generates price increase. The pricing behavior in this model can generate cross-sectional price dispersion with cyclical patterns even without menu costs.

The rest of the chapter is organized as follows. Section 4.2 introduces the model and shows important characteristics of firms' expected sales. Section 4.3 characterizes the optimal decision rules. Section 4.4 concludes while the appendix contains the proofs of the results stated in the text.

4.2 The Model

Consider an oligopoly market where risk neutral firms, indexed by $i \in \{1, 2, ..., N\}$, sell differentiated storable products. Each firm sells a variety of the product. Firms compete in prices and they have uncertainty about temporary demand shocks. In the short run, firms cannot respond to these temporary shocks neither by changing prices nor by increasing supply, in case of excess demand. Firms do not face any delivery lags and cannot backlog unfilled orders. Thus, whenever demand exceeds quantity on hand, the residual unfilled demand is lost. Therefore, the quantity sold by firm i at period t is the minimum of supply and demand:

$$y_{it} = \min\left\{s_{it} + q_{it}, d_{it}\right\}, \tag{4.1}$$

where y_{it} is the quantity sold; s_{it} is the level of inventories at the beginning of period t; q_{it} represents new orders to wholesalers during period t; and d_{it} is consumers' demand. Every period t a firm knows the levels of inventories of all the firms in the market, i.e., the vector $\mathbf{s}_t \equiv \{s_{1t}, s_{2t}, ..., s_{Nt}\}$.[5] Given this information, the firm decides on prices and new orders (p_{it}, q_{it}) to maximize its expected value $E_t(\sum_{r=1}^{\infty} \beta^r \Pi_{i,t+r})$, where $\beta \in (0,1)$ is the discount factor and Π_{it} is the current profit of firm i at period t.

Firms have uncertainty about current demand. The demand of product i at period t is

$$d_{it} = \exp\{\varepsilon_{it}\} d^e_{it}.$$

Here, ε_{it} is a temporary and idiosyncratic demand shock that is independently and identically distributed over time with cumulative distribution function $F(\cdot)$ that is continuously differentiable on the Lebesgue measure. These shocks are unknown to firms when they decide prices and orders. Furthermore, d^e_{it} is the expected demand that depends on the endogenous prices and the exogenous qualities of all products. The expected demand d^e_{it} is a function of the prices of all firms such that it is strictly increasing in the own price, strictly decreasing in the prices of competitors, and the revenue function $p_i d^e_i$ is strictly concave in p_i. By definition of expected demand, we have that $E(\exp\{\varepsilon_{it}\}) = 1$. For technical reasons it is useful to assume that $F(\cdot)$ is such that the respective hazard rate $h(\cdot) = \frac{f(\cdot)}{1-F(\cdot)}$ is smaller than one.[6] For examples and numerical exercises it may be useful to consider a logit demand model for the expected demand:

$$d^e_{it} = \frac{\exp\{w_i - \alpha p_{it}\}}{1 + \sum_{j=1}^{N} \exp\{w_j - \alpha p_{jt}\}}, \tag{4.2}$$

[5]This is a very reasonable assumption as firms can observe prices and are therefore able to learn and deduce stock levels.

[6]This assumption is especially helpful for proving Lemma 4.2, although it is only a sufficient but not necessary condition.

where $\{w_i : i = 1, 2, ..., N\}$ are exogenous parameters that represent product qualities, and α is a parameter that represents the marginal utility of income. The logit demand model is convenient for the derivation and illustration of some future results, but it can be relaxed for all our results.[7]

A firm's current profit is equal to revenue minus ordering cost and inventory holding cost:

$$\Pi_{it} = p_{it} y_{it} - c_i q_{it} - k_i I\{q_{it} > 0\} - h_i s_{it}, \tag{4.3}$$

where c_i is the unit ordering cost; k_i is the fixed or lump-sum ordering cost; and h_i is the inventory holding cost.

The transition rule of inventories, i.e., state variables, is:

$$s_{it+1} = s_{it} + q_{it} - y_{it} = \max\{0, s_{it} + q_{it} - d_{it}\}. \tag{4.4}$$

4.2.1 Implications of Demand Uncertainty for Expected Sales

A firm does not know the temporary demand shock ε_{it}, and therefore it does not know actual sales y_{it}. Expected profits are $\Pi^e_{it} = p_{it}\ y^e_{it} - c_i\ q_{it} - k_i\ I\{q_{it} > 0\} - h_i s_{it}$, where y^e_{it} represents expected sales, i.e., $y^e_{it} = E[\min\{d_{it}, s_{it} + q_{it}\}]$. Demand uncertainty has important implications for the relationship between prices and inventories.

Lemma 4.1. *Expected sales y^e_{it} is equal to expected demand d^e_{it} times a function $\lambda\left(\frac{s_{it}+q_{it}}{d^e_{it}}\right)$, i.e.,*

$$y^e_{it} = d^e_{it} \lambda \left(\frac{s_{it} + q_{it}}{d^e_{it}}\right). \tag{4.5}$$

The function $\lambda(x)$ is defined as $\int \min\{x, \exp(\varepsilon)\} dF(\varepsilon)$ and it has the following properties:

(i) It is continuously differentiable;

(ii) it is strictly increasing;

(iii) $\lambda(0) = 0$;

[7] See Aguirregabiria (2007) for a derivation of this demand model from a model of consumer behavior under possible excess demand.

(iv) $\lambda(\infty) = E(\exp(\varepsilon)) = 1$*; and*

(v) for $x > 0$*,* $\lambda'(x) = \int_{-\infty}^{\ln(x)} dF(\varepsilon) = 1 - F(\ln(x)) \in (0, 1)$*.*

Proof: See Appendix 4.5.1.

In case of a very small (close to zero) supply-to-expected-demand-ratio $\frac{s_{it}+q_{it}}{d^e_{it}}$ stockout probability is very large such that expected sales are much lower than expected demand (approaching zero). On the other hand, a high ratio (approaching infinity) yields low probability for stockouts such that expected sales are almost equal to expected demand. The higher the supply-to-expected-demand-ratio the lower gets the probability of stockout and the more do expected sales converge to expected demand. This is formalized in properties *(ii)* - *(iv)*. From property *(v)* yielding $\lambda''(x) < 0$ it is now clear that the gain of a higher supply-to-expected-demand-ratio for expected sales is higher the lower the ratio. For low ratios the gain is almost equal to the increase of stock as one unit more in stock in essence is a unit more sold. For high ratios the probability of selling an additional unit in stock decreases to zero.

Therefore, variability over time in the supply-to-expected-demand-ratio can generate significant fluctuations in expected sales and thus in optimal prices.

4.2.2 Markov Perfect Equilibrium

The model has a Markov structure and we assume that firms play Markov strategies. That is, a firm's strategy depends only on payoff relevant state variables, which in this model is the vector of inventories $\mathbf{s}_t$. Therefore, a strategy for firm i is a function $\sigma_i(\mathbf{s}_t)$ from the space of the vector of inventories, $\mathbb{R}^N_+$, into the space of the decision variables (p_{it}, q_{it}), $\mathbb{R}^2_+$, i.e., $\sigma_i(\mathbf{s}_t)$ is a function from $\mathbb{R}^N_+$ into $\mathbb{R}^2_+$. Let $\sigma \equiv \{\sigma_i : i = 1, 2, ..., N\}$ be a set of strategy functions, one for each firm. Suppose that firm i considers the rest of the firms to behave according to their respective strategies in σ. Under this condition, other firms' inventories, $\mathbf{s}_{-it}$, follow a Markov transition probability function $F^\sigma_{\mathbf{s}_{-i}}(\mathbf{s}_{-it+1}|\mathbf{s}_{-it})$. Note that this transition probability function depends on the other firms' strategies in σ. Taking $F^\sigma_{\mathbf{s}_{-i}}$ as given, firm i's decision problem can be represented using the Bellman equation:

$$V^\sigma_i(\mathbf{s}_t) = \max_{\{p_i, q_i\}} \left\{ \Pi^\sigma_i(p_i, s_{it} + q_i) + \beta \int V^\sigma_i(s_{i,t+1}, \mathbf{s}_{-it+1}) dF(\varepsilon_{it}) dF^\sigma_{\mathbf{s}_{-i}}(\mathbf{s}_{-it+1}|\mathbf{s}_{-it}) \right\}. \tag{4.6}$$

The (expected) profit function is continuously differentiable and the standard regularity conditions apply such that the value function V_i^σ is uniquely determined as the fixed point of a contraction mapping. Note that this value function is conditional to the other firms' strategies. A Markov perfect equilibrium (MPE) is a set of equilibrium strategies σ such that for every firm i and for every vector $\mathbf{s}_t \in \mathbb{R}_+^N$ we have that

$$\sigma_i(\mathbf{s}_t) = \arg\max_{\{p_i,q_i\}} \Bigg\{ \Pi_i^\sigma\left(p_i, s_{it}+q_i\right) + \beta \int V_i^\sigma(s_{i,t+1}, \mathbf{s}_{-it+1}) dF(\varepsilon_{it}) dF_{\mathbf{s}_{-i}}^\sigma(\mathbf{s}_{-it+1}|\mathbf{s}_{-it}) \Bigg\}. \quad (4.7)$$

4.3 Optimal Decision Rule

Let us now characterize the optimal decision rule for a firm in this game of oligopolistic competition.

In this section we will show that the (S,s) rule is indeed the best response not only to an (S,s) rule but to any given strategy of the opponents. This, of course, implies that the equilibrium resulting from (S,s) strategies by all players is a MPE.

In order to represent the optimal decision rule of the oligopolists, it is convenient to represent the decision problem in terms of the variables p_{it} and $z_{it} \equiv s_{it} + q_{it}$. The variable z_{it} represents the total supply of the product during period t. It is also useful to define the following "value" function which is independent of the firm's own current inventory, i.e., the only state variable the firm can influence (however, it is not independent of the current state per se), and taking the other firms' strategies in σ and so $F_{\mathbf{s}_{-i}}^\sigma$ as given:

$$Q_i^\sigma(z_{it}, p_{it}; \mathbf{s}_{-it}) \equiv -cz_{it} + p_{it} \int \min\left\{z_{it}; e^{\varepsilon_{it}} d_{it}^{e\sigma}(p_{it})\right\} dF(\varepsilon_{it}) + \beta \int V_i^\sigma\left(\max\left\{0; z_{it} - e^{\varepsilon_{it}} d_{it}^{e\sigma}(p_{it})\right\}; \mathbf{s}_{-it+1}\right) dF(\varepsilon_{it}) dF_{\mathbf{s}_{-i}}^\sigma(\mathbf{s}_{-it+1}|\mathbf{s}_{-it}) \quad (4.8)$$

such that

$$V_i^\sigma(\mathbf{s}_t) = \max_{\{p_i,q_i\}} \left\{ Q_i^\sigma(s_{it}+q_i, p_i; \mathbf{s}_{-it}) - (h_i - c_i)s_{it} - k_i I_{\{q_i>0\}} \right\}.$$

Given the function Q_i^σ, it is clear that an oligopolist chooses (z_{it}, p_{it}) as a best response to the other firms' strategies in σ, i.e., other firms order and pricing decisions, to maximize $Q_i^\sigma(z_{it}, p_{it}; \mathbf{s}_{-it}) - kI\{z_{it} > s_{it}\}$. Making use of this "value" function Q_i^σ we can derive important characteristics of competition in prices and inventories:

Lemma 4.2. *The function Q_i^σ is such that:*

(i) Q_i^σ is strictly concave in prices, i.e., $\partial^2 Q_i^\sigma(z_i, p_i)/\partial p_i \partial p_i < 0$.

(ii) Prices and total supply are strategic substitutes, i.e., $\partial^2 Q_i^\sigma(z_i, p_i)/\partial p_i \partial z_i \leq 0$.

Proof: See Appendix 4.5.2.

The positive stockout probability has a negative effect on expected sales which in turn creates substitutability between prices and inventories in the profit function. This is the case as with low inventory optimal expected demand (under given demand uncertainty) is low and thus optimal price is high.

Using $\sigma_p^\sigma(\mathbf{s})$ and $\sigma_z^\sigma(\mathbf{s})$ to represent the optimal response rules for p and z, respectively, we have

$$\{\sigma_{iz}^\sigma(\mathbf{s}), \sigma_{ip}^\sigma(\mathbf{s})\} = \arg \max_{\{z_i \geq s_i, p_i \geq 0\}} \{Q_i^\sigma(z_i, p_i; \mathbf{s}_{-it}) - kI\{z_i > s_i\}\}.$$

We define the optimal price as a function of current supply:

$$\bar{p}_i^\sigma(z_i; \mathbf{s}_{-i}) \equiv \arg \max_{\{p_i\}} Q_i^\sigma(z_i, p_i; \mathbf{s}_{-i}). \tag{4.9}$$

Since Q_i^σ is continuously differentiable and strictly concave in prices, $\bar{p}_i^\sigma(z; \mathbf{s}_{-i})$ is implicitly defined by the first order condition $\frac{\partial Q_i^\sigma(z_i, \bar{p}_i; \mathbf{s}_{-i})}{\partial \bar{p}_i} = 0$.

It is now possible to show that the best response to any strategy is an (S, s) rule:

Proposition 4.1. *Firm i considers the rest of the firms to behave according to their respective strategies in σ. Taking $F_{\mathbf{s}_{-i}}^\sigma$ as given, let firm i's best response rule for total supply and prices be $\sigma_{iz}^\sigma(\mathbf{s})$ and $\sigma_{ip}^\sigma(\mathbf{s})$, respectively. These functions are such that:*

1. *$\sigma_{ip}^\sigma(\mathbf{s}) = \bar{p}^\sigma(\sigma_{iz}(\mathbf{s}); \mathbf{s}_{-i})$, where $\bar{p}^\sigma(z_i; \mathbf{s}_{-i})$ is continuous and strictly decreasing in z_i; and*

2. $\sigma_{iz}^{\sigma}(\mathbf{s})$ *has the following form:*

$$\sigma_{iz}^{\sigma}(\mathbf{s}) = \begin{cases} s_i^{*\sigma}(\mathbf{s}_{-i}) & \text{if } s_{it} \leq \underline{s}_i^{\sigma}(\mathbf{s}_{-i}) \\ s_{it} & \text{if } s_{it} > \underline{s}_i^{\sigma}(\mathbf{s}_{-i}), \end{cases} \tag{4.10}$$

where $s_i^{*\sigma}$ *and* $\underline{s}_i^{\sigma}$ *are scalars, with* $s_i^{*\sigma} > \underline{s}_i^{\sigma} \, \forall \mathbf{s}_{-i}$*, and the following definitions:*

$$s_i^{*\sigma}(\mathbf{s}_{-i}) \equiv \arg\max_{\{z_i\}} Q_i^{\sigma}(z_i, \bar{p}_i(z_i); \mathbf{s}_{-i}), \tag{4.11}$$

$$\underline{s}_i^{\sigma}(\mathbf{s}_{-i}) \equiv \inf\{s_i | Q_i^{\sigma}(s_i^{*\sigma}, \bar{p}_i(s_i^{*\sigma}); \mathbf{s}_{-i}) - k \leq Q_i^{\sigma}(s_i, \bar{p}(s_i); \mathbf{s}_{-i})\}. \tag{4.12}$$

Proof: See Appendix 4.5.3.

The proposition shows that consideration of oligopolistic competition does not affect the optimality of (S, s) inventory rules.[8] Fixed ordering costs generate infrequent orders. The upper band $s_i^{*\sigma}$ is defined as the optimal order quantity when the firm has no inventory on hand, i.e., the optimal inventory level. The lower band $\underline{s}_i^{\sigma}$ is the smallest value of inventory such that the desired order quantity is zero. This order policy might appear to be a very natural and intuitive strategy. However, as shown in the appendix the value function is not concave such that a much more complex decision rule could in principle be optimal. Additionally, oligopolistic competition assures that no additional condition on prices like the "no expected loss condition" of Hall and Rust (2007) is necessary for the optimal trading strategy to be of the (S, s) form.[9]

This (S, s) inventory behavior together with demand uncertainty generates cyclical patterns in prices. The optimal price is a strictly decreasing function of a firm's inventory on hand z_i as the positive probability of stockouts creates strategic substitutability between prices and inventories. Thus, the price increases between two orders when the stock level decreases and it drops down when new orders are placed. This is the case as with low inventories the optimal expected demand is lower and hence the optimal price is higher. When the level of inventories decreases between two orders, the probability of stockout increases and so expected sales decrease and

[8]However, as thresholds depend on the competitors' inventories, we have an $(S(\mathbf{s}_{-i}), s(\mathbf{s}_{-i}))$ decision rule.

[9]The "no expected loss condition" requires that the exogenous nonconstant retail price exceeds a certain (endogenous) nonconstant threshold any time. With endogenous prices, we do not need to impose such a condition.

become more inelastic with respect to the price. Thus, the optimal price increases between two orders, and decreases when the elasticity of sales goes up as the result of positive orders.

The largest price increase occurs just after a positive order and the increments tend to be smaller when we approach to the next positive order. The reason for this behavior is that the cyclical path of prices generates a cyclical behavior in sales. The largest sales and, consequently, the largest stock reductions and price increases, occur just after a positive order.

The interesting result here is that the pricing behavior in this model can generate cross-sectional price dispersion with cyclical patterns even without menu cost. The magnitude of this price dispersion will depend on the magnitude of lump-sum ordering costs, the sensitivity of the price elasticity of sales to changes in the probability of stockout, and the degree of correlation between the demand shocks at individual firms.

4.4 Conclusion

We have shown that the best response not only to (S, s) strategies but to any strategy is an (S, s) rule. This result extends earlier findings of models without price competition (Hall & Rust, 2007) and models without strategic competition (Aguirregabiria, 1999) where fixed ordering costs generate infrequent orders. Thus, the (S, s) policy might appear to be a very robust strategy. However, it is not hard to change assumptions in ways that destroy its optimality.

Additionally, with strategic competition in prices (S, s) inventory behavior together with demand uncertainty generates cyclical pattern in prices.

The model developed in this chapter provides a very promising alternative for studying commodity markets.

Commodity prices are extremely volatile and papers of the respective literature strand are concerned whether theory is capable of explaining the actual behavior of prices. The more recent literature on this topic (see for example Deaton & Laroque, 1992, 1996, and Pindyck, 1994) builds on the supply and demand tradition (see e.g. Ghosh, Gilbert, & Hughes Hallett, 1987, for a review), but with explicit modeling of the behavior of competitive speculators who hold inventories of commodities in the expectation of making profits.[10] However, perfect competition and the absence of lump-sum ordering cost is always assumed in these papers. The studies are trying to explain extremely volatile prices as a result of exogenous shocks by modeling the behavior of competitive speculators holding inventories.

Results are rather unsatisfying: In contrast to the models' predictions, real price fluctuations are not randomly distributed over time and this autocorrelation cannot be explained by these types of models. In addition, some probably important characteristics of commodity markets are not captured in this literature. Studies of these characteristics (e.g. Carter & MacLaren, 1997, and Slade & Thille, 2006) find that commodity markets are best described by oligopoly instead of perfect competition. Besides, lump-sum ordering cost are realistic in some markets (e.g. at London Metal Exchange where orders can result in physical delivery and all contracts assume delivery). Incorporating oligopoly competition and lump sum ordering costs could be important to study the dynamics of some commodity prices. In a model like ours we are able to generate some kind of time dependent pattern which is apparently in line with empirical evidence. This is in contrast to the usual hypothesis that price fluctuations are the result of exogenous shocks and therefore randomly distributed over time.

Making use of the developed model it should now be possible to relate findings to commodity price dynamics and show that lump-sum ordering cost and oligopoly competition can be important to explain extremely volatile prices and especially time dependencies in price fluctuations.

However, due to the relatively high complexity of the framework further research requires numerical experiments. By this means, other topics like precise reactions of firms on competitors' orders provide scope for interesting studies. This important work is left for future research.

[10] As even estimating the models is computational demanding authors mostly use simulations.

4.5 Appendix

4.5.1 Expected Sales: Proof of Lemma 4.1

Proof. For notational simplicity, we omit here the firm and time subindexes. By definition, expected sales y^e are:

$$y^e = \int \min\{s+q, d^e \exp(\varepsilon)\} dF(\varepsilon) = d^e \lambda \left(\frac{s+q}{d^e} \right)$$

where $\lambda(x)$ is defined as $\int \min\{x, \exp(\varepsilon)\} dF(\varepsilon)$. The function $\lambda(x)$ has the following properties:

$$\lim_{x \to 0} \lambda(x) = \int \min\{0, \exp(\varepsilon)\}) dF(\varepsilon) = 0.$$

Also,

$$\lim_{x \to \infty} \lambda(x) = \lim_{x \to \infty} \int \min\{x, \exp(\varepsilon)\}) dF(\varepsilon) = \int \exp(\varepsilon) dF(\varepsilon) = 1.$$

Finally,

$$\lambda'(x) = \int I\{x < \exp(\varepsilon)\} dF(\varepsilon) = 1 - F(\ln x).$$

□

4.5.2 The "Value" Function: Proof of Lemma 4.2

Proof. We use backwards induction and first show that the properties of Lemma 4.2 hold for the finite horizon problem with time horizon equal to T.

Let us consider $Q^{\sigma}_{iT}(\cdot)$ to represent the profit function in the last period, i.e.,

$$\begin{aligned} Q^{\sigma}_{iT}(z_i, p_i; \mathbf{s}_{-i}) &= -cz_i + p_i y_i^{e\sigma}(z_i, p_i) \\ &= -cz_i + p_i d_i^{e\sigma}(p_i) \lambda \left(\frac{z_i}{d_i^{e\sigma}(p_i)} \right) \\ &= -cz_i + p_i \int \min\{z_i; e^{\varepsilon_i} d_i^{e\sigma}(p_i)\} dF(\varepsilon_i). \end{aligned}$$

Therefore,

$$\frac{\partial Q^{\sigma}_{iT}(\cdot)}{\partial p_i} = y_i^{e\sigma}(z_i, p_i) + p_i \frac{\partial y_i^{e\sigma}(z_i, p_i)}{\partial p_i},$$

and

$$\frac{\partial^2 Q_{iT}(\cdot)}{\partial p_i^2} = 2 \frac{\partial y_i^{e\sigma}(z_i, p_i)}{\partial p_i} + p_i \frac{\partial^2 y_i^{e\sigma}(z_i, p_i)}{\partial p_i^2}. \tag{4.13}$$

Given that $y_i^{e\sigma}(z_i, p_i) = d_i^{e\sigma}(p_i)\lambda\left(\frac{z_i}{d_i^{e\sigma}(p_i)}\right)$, we have that

$$\frac{\partial y_i^{e\sigma}(z_i, p_i)}{\partial p_i} = \frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} F(\ln z_i - \ln d_i^{e\sigma}(p_i)),$$

and

$$\frac{\partial^2 y_i^{e\sigma}(z_i, p_i)}{\partial p_i^2} = \frac{\partial^2 d_i^{e\sigma}(p_i)}{\partial p_i^2} F(\ln z_i - \ln d_i^{e\sigma}(p_i)) - \left(\frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i}\right)^2 \frac{f(\ln z_i - \ln d_i^{e\sigma}(p_i))}{d_i^{e\sigma}(p_i)}.$$

Inserting these expressions in equation (4.13), we get:

$$\begin{aligned}\frac{\partial^2 Q_{iT}^{\sigma}(\cdot)}{\partial p_i^2} =& 2\frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} F(\ln z_i - \ln d_i^{e\sigma}(p_i)) \\ &+ p_i \left(\frac{\partial^2 d_i^{e\sigma}(p_i)}{\partial p_i^2} F(\ln z_i - \ln d_i^{e\sigma}(p_i)) \right. \\ &\left. - \left(\frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i}\right)^2 \frac{f(\ln z_i - \ln d_i^{e\sigma}(p_i))}{d_i^{e\sigma}(p_i)}\right) \\ =& F(\ln z_i - \ln d_i^{e\sigma}(p_i)) \left(2\frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} + p_i \left(\frac{\partial^2 d_i^{e\sigma}(p_i)}{\partial p_i^2}\right)\right) \\ &- \left(\frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i}\right)^2 \frac{f(\ln z_i - \ln d_i^{e\sigma}(p_i))}{d_i^{e\sigma}(p_i)}.\end{aligned}$$

The first term is negative because $\left(2\frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} + p_i\left(\frac{\partial^2 d_i^{e\sigma}(p_i)}{\partial p_i^2}\right)\right)$ is just the second derivative of the function $p_i d_i^{e\sigma}(p_i)$, that is strictly concave by assumption. It is clear that the second term is also negative. Therefore, $\frac{\partial^2 Q_{iT}(\cdot)}{\partial p_i^2} < 0$.

Furthermore, since $\frac{\partial Q_{iT}^{\sigma}(\cdot)}{\partial p_i} = y_i^{e\sigma}(z_i, p_i) + p_i \frac{\partial y_i^{e\sigma}(z_i,p_i)}{\partial p_i}$, we have that

$$\frac{\partial^2 Q_{iT}^{\sigma}(\cdot)}{\partial p_i \partial z_i} = \frac{\partial y_i^{e\sigma}(z_i, p_i)}{\partial p_i} + p_i \frac{\partial^2 y_i^{e\sigma}(z_i, p_i)}{\partial p_i \partial z_i}. \tag{4.14}$$

As we have shown above, $\frac{\partial y_i^{e\sigma}(z_i,p_i)}{\partial z_i} = \lambda'\left(\frac{z_i}{d_i^{e\sigma}(p_i)}\right) = 1 - F(\ln z_i - \ln d_i^{e\sigma}(p_i))$. We have also shown that $\frac{\partial y_i^{e\sigma}(z_i,p_i)}{\partial p_i} = \frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} F(\ln z_i - \ln d_i^{e\sigma}(p_i))$, and therefore

$$\frac{\partial^2 y_i^{e\sigma}(z_i, p_i)}{\partial p_i \partial z_i} = \frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} \frac{f(\ln z_i - \ln d_i^{e\sigma}(p_i))}{z_i}.$$

Inserting these expressions into the equation (4.14), we get:

$$\frac{\partial^2 Q_{iT}^{\sigma}(\cdot)}{\partial p_i \partial z_i} = 1 - F(\ln z_i - \ln d_i^{e\sigma}(p_i)) + \frac{p_i}{z_i} \frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} f(\ln z_i - \ln d_i^{e\sigma}(p_i)).$$

With $\eta_d(p_i) \equiv -\frac{\partial d_i^{e\sigma}(p_i)}{\partial p_i} \frac{p_i}{d_i^{e\sigma}(p_i)} > 0$ as the elasticity of expected demand, and $\eta_\lambda \left(\frac{z_i}{d_i^{e\sigma}(p_i)} \right) \equiv -\lambda' \left(\frac{z_i}{d_i^{e\sigma}(p_i)} \right) \frac{z_i}{\lambda(\cdot) d_i^{e\sigma}(p_i)} < 0$ as the elasticity of the $\lambda(\cdot)$-function the above expression can be written as

$$\frac{\partial^2 Q_{iT}^{\sigma}(\cdot)}{\partial p_i \partial z_i} = \lambda'(\cdot)(1 - \eta_d(\cdot)(1 - \eta_\lambda(\cdot))) + \lambda(\cdot)\eta_d(\cdot)\eta_\lambda'(\cdot) \tag{4.15}$$

with

$$\eta_\lambda'(\cdot) = -\frac{\frac{z_i}{d_i^{e\sigma}(p_i)} \lambda'(\cdot)^2 + \lambda(\cdot)(\lambda'(\cdot) + \frac{z_i}{d_i^{e\sigma}(p_i)} \lambda''(\cdot))}{\lambda(\cdot)^2}.$$

The term $\eta_\lambda'(\cdot)$ is negative as $\lambda'(\cdot) + \frac{z_i}{d_i^{e\sigma}(p_i)} \lambda''(\cdot) = 1 - F(\cdot) - f(\cdot)$ is positive for $1 - F(\cdot) > f(\cdot)$ which is fulfilled by assumption. Thus, the second term of equation (4.15) is negative.

Now, let's particularize expression (4.15) at $(z, \bar{p}_T(z))$. We can write

$$\frac{\partial Q_{iT}^{\sigma}(\cdot)}{\partial p_i} = y^{e\sigma}(z_i, p_i) \left(1 - \eta_d(\cdot) \left(1 - \eta_\lambda(\cdot)\right)\right)$$

such that $1 - \eta_d(\cdot) \left(1 - \eta_\lambda(\cdot)\right)$ can never be positive at the optimal decision and therefore $\frac{\partial^2 Q_{iT}^{\sigma}(\cdot)}{\partial p_i \partial z_i} < 0$ holds.

We will now show that if $Q_{it+1}^{\sigma}(\cdot)$ is strictly concave in prices and prices and supply are strategic substitutes in $t+1$, then $Q_{it}^{\sigma}(\cdot)$ is strictly concave in prices and prices and supply are strategic substitutes in t as well.

We make use of the fact that the profit function is bounded from above. More specifically,

$$\max_{s_i \geq 0} \max_{\{z_i \geq s_i, p_i \geq 0\}} \left\{ p_i d_i^{e\sigma}(p_i) \lambda \left(\frac{z_i}{d_i^{e\sigma}(p_i)} \right) - c_i z_i - k_i I_{\{z_i > s_i\}} \right\}$$

is smaller than some constant $\tau < \infty$. This property guarantees that for any values of z_i and p_i

$$Q_i^{\sigma}(z_i, p_i) = \lim_{T \to \infty} Q_{iT}^{\sigma}(z_i, p_i).$$

Thus, as in $t+1$ the "value" function given as

$$\begin{aligned} Q_i^{\sigma}(z_{it+1}, p_{it+1}; \cdot) \equiv & -c z_{it+1} + p_{it+1} \int \min \left\{ z_{it+1}; e^{\varepsilon_{it+1}} d_{it+1}^{e\sigma}(p_{it+1}) \right\} dF(\varepsilon_{it+1}) \\ & + \beta \int V_i^{\sigma} \left(\max \left\{ 0; z_{it+1} - e^{\varepsilon_{it+1}} d_{it+1}^{e\sigma}(p_{it+1}) \right\}; \cdot \right) dF(\varepsilon_{it+1}) dF_{\mathbf{s}_{-i}}^{\sigma}(\mathbf{s}_{-it+2} | \mathbf{s}_{-it+1}) \end{aligned}$$

is strictly concave in prices and prices and supply are strategic substitutes, so is the function in t. This completes the proof. □

4.5.3 Optimal Decision Rule: Proof of Proposition 4.1.

Following Scarf (1960), the key to proving that the optimal strategy is of the (S, s) form is to show that the value function V is k-concave. Our proof exploits several properties of k-concave functions.

A real-valued function $f(s)$ is a k-concave function if and only if for every s_0 and s_1 such that $s_0 \leq s_1$ and every scalar $\delta \in (0, 1)$:

$$\delta f(s_0) + (1-\delta)f(s_1) \leq (1-\delta)k + f(\delta s_0 + (1-\delta)s_1). \tag{4.16}$$

Consider the following properties of $k-$concave functions.

1. If f is strictly k-concave it has a unique global maximum.
2. If f is strictly k-concave, and s^* is the global maximum, then the equation $f(z) = f(s^*) - k$ has two solutions, s^L and s^H with $s^L < s^H$. Furthermore, $f(s) > f(s^*) - k$ if and only if $s \in (s^L, s^H)$.
3. If $f(x, y)$ is k-concave in x for any value of y, and k-concave in y for any value of x, and $y^*(x) \equiv \arg\max_y f(x, y)$, then $g(x) \equiv f(x, y^*(x))$ is k-concave.
4. If $f_1(\cdot)$ is k_1-concave, $f_2(\cdot)$ is k_2-concave, and α_1, α_2 are two positive scalars, then $\alpha_1 f_1 + \alpha_2 f_2$ is $(\alpha_1 k_1 + \alpha_2 k_2)$-concave.

Before starting with the formal proof, we will briefly illustrate the main idea of why k-concavity is important.

Consider the k-concave function $V(s)$ to be a firm's value function. If V is a continuous differentiable function from k-concavity $V(s_1) - k - V(s_0) - (s_1 - s_0)V'(s_0) \leq 0$ directly follows. Thus for each local extremum s' with $V'(s') = 0$, it is the case that $V(s') \geq V(s) - k \quad \forall s \geq s'$. This means that each local extremum (minimum) s' is at most k units below a function's maximum right of this local minimum. This property is illustrated in Figure 4.1. The function on the left hand side is an arbitrary value function that is not k-concave, while the function on the right graph fulfills the condition above.

With lump-sum ordering cost of k and a firm's value function like the one depicted on the left hand side a complex optimal order policy results where the firm

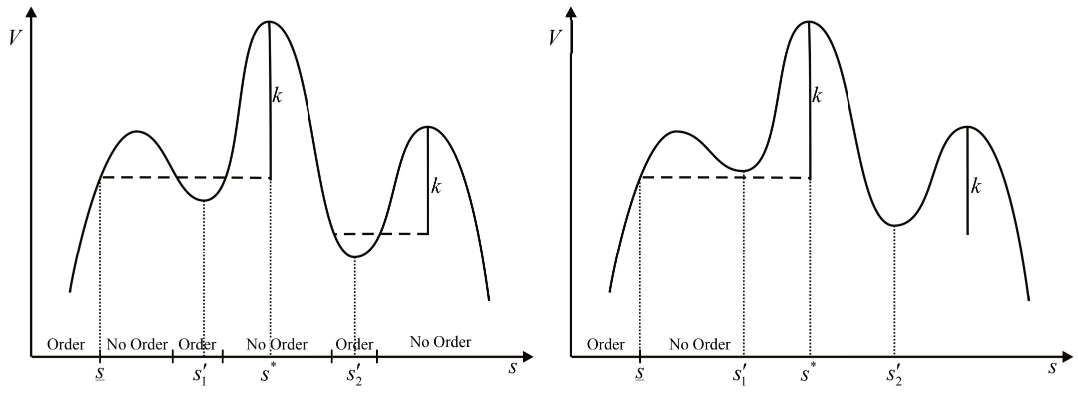

Figure 4.1: Non-concave value function and respective order decisions when the value function is not k-concave (left) and when it is k-concave (right).

orders when inventory is below $\underline{s}$ or around s'_1 such that inventory level s^* is attained. Additionally, the firm orders such that an even higher target level is reached when inventory is around s'_2 (which is even above s^*).

With the value function being k-concave like the one depicted on the right hand side, it is easy to see that the optimal strategy is of (S, s) type. In that case firms never order with inventory above s^* and firms never order around a local minimum in between the inventory threshold $\underline{s}$ and the optimal inventory level s^*.

In the following we will make use of this idea with regard to the decision problem of our model.

Proof. Suppose that Q_i^σ is strictly k-concave in z_i for any value of p_i and strictly k-concave in p_i for any value of z_i for all values of $\mathbf{s}_{-it}$.

The optimal price decision can be written as

$$\sigma_{ip}^\sigma(\mathbf{s}) \equiv \bar{p}_i^\sigma(z_i; \mathbf{s}_{-i}).$$

That means, giving the optimal pricing function $\bar{p}_i^\sigma(z; \mathbf{s}_{-i})$ the firm chooses inventory level $\sigma_{iz}^\sigma(\mathbf{s})$ which results in pricing $\sigma_{ip}^\sigma(\mathbf{s})$ as a function of the pre-order inventory.

As $Q_i^\sigma(\cdot)$ is strictly k-concave, $s_i^{*\sigma}(\mathbf{s}_{-i})$ and $\bar{p}_i^\sigma(s_i^{*\sigma}(\mathbf{s}_{-i}), \mathbf{s}_{-i})$ are unique and $\bar{p}_i^\sigma(\cdot,\cdot)$ is a real function. Furthermore, $Q_i^\sigma(z_i, \bar{p}_i^\sigma(z_i); \mathbf{s}_{-i})$ is also strictly k-concave.

By definition of $\sigma_{iz}^{\sigma}(\mathbf{s})$, $s_i^{*\sigma}(\mathbf{s}_{-i})$, and $\bar{p}_i^{\sigma}(s_i^{*\sigma}(\cdot),\cdot)$, it is clear that

$$\sigma_{iz}^{\sigma}(\mathbf{s}) = \begin{cases} s_i^{*\sigma}(\mathbf{s}_{-i}) & \text{if } Q_i^{\sigma}(s_i^{*\sigma},\bar{p}_i^{\sigma}(s_i^{*\sigma});\cdot) - k > Q_i^{\sigma}(s_i,\bar{p}_i^{\sigma}(s_i);\cdot) \\ s_i & \text{if } Q_i^{\sigma}(s_i^{*\sigma},\bar{p}_i^{\sigma}(s_i^{*\sigma});\cdot) - k \leq Q_i^{\sigma}(s_i,\bar{p}_i^{\sigma}(s_i);\cdot). \end{cases}$$

Due to the k-concavity of $Q_i^{\sigma}(z_i,\bar{p}_i^{\sigma}(z_i);\cdot)$ the equation $Q_i^{\sigma}(s_i^{*\sigma},\bar{p}_i^{\sigma}(s_i^{*\sigma});\cdot) - k = Q_i^{\sigma}(s_i,\bar{p}_i^{\sigma}(s_i);\cdot)$ has only two solutions.

Let these two solutions be $s_i^L(\cdot)$ and $s_i^H(\cdot)$, where $s_i^L(\cdot) \leq s_i^{*\sigma}(\cdot) \leq s_i^H(\cdot)$. Then, k-concavity implies

$$Q_i^{\sigma}(s_i^{*\sigma},\bar{p}_i^{\sigma}(s_i^{*\sigma});\cdot) - k \leq Q_i^{\sigma}(s_i,\bar{p}_i^{\sigma}(s_i);\cdot) \Leftrightarrow s_i^L(\cdot) \leq s_i(\cdot) \leq s_i^H(\cdot).$$

It is clear that the conditions $s_i > s_i^H$ and $s_i \leq s_i^H$ do not play any role because the stock level is always lower or equal to $s_i^{*\sigma}$. With $\underline{s}_i^{\sigma}$ as the smaller of the two solutions by definition we can write the optimal decision as

$$\sigma_{iz}^{\sigma} = \begin{cases} s_i^{*\sigma} & \text{if } s_i \leq \underline{s}_i^{\sigma}, \\ s_i & \text{if } s_i > \underline{s}_i^{\sigma}. \end{cases}$$

The according optimal pricing decision for the inventory before ordering is

$$\sigma_{ip}^{\sigma}(\mathbf{s}) = \bar{p}^{\sigma}(\sigma_{iz}(\mathbf{s})) = \begin{cases} \bar{p}^{\sigma}(s_i^{*\sigma}) & \text{if } s_i \leq \underline{s}_i^{\sigma}, \\ \bar{p}^{\sigma}(s_i) & \text{otherwise.} \end{cases}$$

It further remains to show that Q_i^{σ} is indeed k-concave.
We proceed in three steps:

(a) If $V_i^{\sigma}(\mathbf{s})$ is strictly k-concave in s_i, then $Q_i^{\sigma}(\cdot)$ is strictly k-concave in z_i for any value of p_i.

(b) If $V_i^{\sigma}(\mathbf{s})$ is strictly k-concave in s_i, then $Q_i^{\sigma}(\cdot)$ is strictly k-concave in p_i for any value of z_i.

(c) $V_i^{\sigma}(\mathbf{s})$ is strictly k-concave in s_i.

(a) We will now show that if $V_i^\sigma(\mathbf{s})$ is strictly k-concave in s_i, then $Q_i^\sigma(z_i, p_i; \mathbf{s}_{-i})$ is strictly k-concave in z_i for any value of p_i.

By the first part of the proof, there exist $s_i^{*\sigma}$ and $\underline{s}_i^\sigma$ satisfying $0 \leq \underline{s}_i^\sigma \leq s_i^{*\sigma}$ for which V_i^σ can be represented as

$$V_i^\sigma(\mathbf{s}) = V(Q)_i^\sigma(\mathbf{s}, \sigma_{ip}^\sigma(\mathbf{s})) = \begin{cases} Q_i^\sigma(s_i^{*\sigma}, \bar{p}_i(s_i^{*\sigma}); \mathbf{s}_{-i}) + cs_i - h(s_i) - k & \text{if } s_i \in [0, \underline{s}_i^\sigma), \\ Q_i^\sigma(s_i, \sigma_p(s_i); \mathbf{s}_{-i}) + cs_i - h(s_i) & \text{if } s \geq \underline{s}. \end{cases} \tag{4.17}$$

$V_i^\sigma(\mathbf{s})$ can be extended to be a function defined on $\mathbb{R} \times \mathbb{R}_+^{n-1}$:

$$V_i^\sigma(\mathbf{s}) = \begin{cases} V_i^\sigma(0, \mathbf{s}_{-i}) + cs_i & \text{if } s_i \leq 0, \\ V_i^\sigma(\mathbf{s}) & \text{else,} \end{cases}$$

which is needed as the proof of (c) implies that V_i is k-concave in s_i over $\mathbb{R}$.

We can write Q_i^σ as

$$Q_i^\sigma(\cdot) = Q_i^{\sigma R}(\cdot) + \beta Q_i^{\sigma V}(\cdot),$$

where

$$\begin{aligned} Q_i^{\sigma R}(\cdot) &\equiv -cz_i + p_i \int \min\{z_i; e^{\varepsilon_i} d_i^{e\sigma}(p_i)\}\, dF(\varepsilon_i) \\ &= -cz_i + p_i d_i^{e\sigma}(p_i) \lambda \left(\frac{z_i}{d_i^{e\sigma}(p_i)} \right) \end{aligned}$$

and

$$Q_i^{\sigma V}(\cdot) \equiv \int V_i^\sigma \left(\max\{0; z_{it} - e^{\varepsilon_{it}} d_{it}^{e\sigma}(p_{it})\}; \mathbf{s}_{-it+1} \right) dF(\varepsilon_{it}) dF_{\mathbf{s}_{-i}}^\sigma(\mathbf{s}_{-it+1} | \mathbf{s}_{-it}).$$

Let us now consider the function $\int V_i^\sigma \left(s_i - \exp(\varepsilon) d_i^{e\sigma}(\cdot); \cdot \right) dF(\varepsilon_i) dF_{\mathbf{s}_{-i}}^\sigma$. Since each $V_i^\sigma(\cdot)$ is k-concave in s_i over $\mathbb{R}$, and since positive linear combinations of pointwise limits of k-concave functions are k-concave, it follows that $\int V_i^\sigma \left(s_i - \exp(\varepsilon) d_i^{e\sigma}(\cdot); \cdot \right) dF(\varepsilon_i) dF_{\mathbf{s}_{-i}}^\sigma$ is k-concave in s_i on $\mathbb{R}$. With $\bar{\varepsilon}_i(\cdot)$ as the value of ε_i for which demand is equal to supply z_i, i.e. $z_i = \exp(\bar{\varepsilon}_i(z_i)) d^{e\sigma}(\cdot)$, we have

$$\begin{aligned} &\int V_i^\sigma \left(s_i - \exp(\varepsilon) d_i^{e\sigma}(\cdot); \cdot \right) dF(\varepsilon_i) dF_{\mathbf{s}_{-i}}^\sigma \\ &= \int_{-\infty}^{\bar{\varepsilon}_i(s_i)} V_i^\sigma \left(s_i - \exp(\varepsilon) d_i^{e\sigma}(\cdot); \cdot \right) dF(\varepsilon_i) dF_{\mathbf{s}_{-i}}^\sigma \\ &\quad + \int_{\bar{\varepsilon}_i(s_i)}^{\infty} V_i^\sigma \left(s_i - \exp(\varepsilon) d_i^{e\sigma}(\cdot); \cdot \right) dF(\varepsilon_i) dF_{\mathbf{s}_{-i}}^\sigma \end{aligned}$$

$$\begin{aligned}
=&\int_{-\infty}^{\bar{\varepsilon}_i(s_i)} V_i^\sigma\left(s_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot);\cdot\right)dF(\varepsilon_i)dF_{\mathbf{s}_{-i}}^\sigma+V_i^\sigma(0;\cdot)\int_{\bar{\varepsilon}_i(s_i)}^{\infty}dF(\varepsilon_i)dF_{\mathbf{s}_{-i}}^\sigma\\
&+c\int_{\bar{\varepsilon}_i(s_i)}^{\infty}\left(s_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot)\right)dF(\varepsilon_i)\\
=&Q_i^{\sigma V}(s_i,p_i,\cdot)+c\int_{\bar{\varepsilon}_i(s_i)}^{\infty}\left(s_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot)\right)dF(\varepsilon_i).
\end{aligned}$$

Using the definition of Q_i^σ, we have

$$\begin{aligned}
Q_i^\sigma(\cdot)=&Q_i^{\sigma R}(\cdot)+\beta Q_i^{\sigma V}(\cdot)\\
=&p_i\int\min\left\{z_i;e^{\varepsilon_i}d_i^{e\sigma}(p_i)\right\}dF(\varepsilon_i)-cz_i\\
&+\beta\int_{-\infty}^{\bar{\varepsilon}_i(z_i)}V_i^\sigma\left(z_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot);\cdot\right)dF(\varepsilon_i)dF_{\mathbf{s}_{-i}}^\sigma\\
&+\beta\int_{\bar{\varepsilon}_i(z_i)}^{\infty}V_i^\sigma\left(z_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot);\cdot\right)dF(\varepsilon_i)dF_{\mathbf{s}_{-i}}^\sigma\\
&-\beta c\int_{\bar{\varepsilon}_i(z_i)}^{\infty}\left(z_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot)\right)dF(\varepsilon_i).
\end{aligned}$$

The sum of the third and fourth terms in the last equation is k-concave since $\int V_i^\sigma\left(s_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot);\cdot\right)dF(\varepsilon_i)dF_{\mathbf{s}_{-i}}^\sigma$ is k-concave. Since cz_i is a linear and hence convex function of z_i, a sufficient condition for the k-concavity of $Q_i^\sigma(\cdot)$ is that the function

$$p_i d_i^{e\sigma}(p_i)\lambda\left(\frac{z_i}{d_i^{e\sigma}(p_i)}\right)-\beta c\int_{\bar{\varepsilon}_i(z_i)}^{\infty}\left(z_i-\exp(\varepsilon)d_i^{e\sigma}(\cdot)\right)dF(\varepsilon_i)$$

is concave in z_i. The function is continuously differentiable in z_i with second derivatives

$$(p_i-\beta c)(1-F(\ln z_i-\ln d_i^{e\sigma}(\cdot))).$$

As $F(\cdot)<1$, this expression is non-positive and hence Q_i^σ is k -concave as long as $p_i\geq\beta c_i$. (Obviously, a weaker condition for that result exists.)

For proving that Q_i^σ is indeed k-concave we need to show that $\sigma_{ip}^\sigma(\mathbf{s})-\beta c\geq 0$ holds. Recall

$$\begin{aligned}
Q_i^\sigma(z_i,p_i;\mathbf{s}_{-i})\equiv&-cz_i+p_i d_i^{e\sigma}(p_i)\lambda\left(\frac{z_i}{d_i^{e\sigma}(p_i)}\right)\\
&+\beta\int V_i^\sigma\left(\max\left\{0;z_i-e^{\varepsilon_i}d_i^{e\sigma}(p_i)\right\};\mathbf{s}_{-it+1}\right)dF(\varepsilon_i)dF_{\mathbf{s}_{-i}}^\sigma(\mathbf{s}_{-it+1}|\mathbf{s}_{-i})
\end{aligned}$$

and

$$V_i^\sigma(\mathbf{s}_t) = \max_{\{p_i,q_i\}} \left\{ Q_i^\sigma(s_{it} + q_i, p_i; \mathbf{s}_{-it}) - (h_i - c_i)s_{it} - k_i I_{\{q_i>0\}} \right\}.$$

where the expected sales $d_i^{e\sigma}(p_i)\lambda\left(\frac{z_i}{d_i^{e\sigma}(p_i)}\right)$ are always smaller than or equal to total supply z_i. Let's suppose to the contrary that there is an optimal price $\sigma_{ip}^\sigma < \beta c < c$. In that case $-cz_i + p_i d_i^{e\sigma}(p_i)\lambda\left(\frac{z_i}{d_i^{e\sigma}(p_i)}\right)$ would be negative. Thus, without a new order the current value $V_i^\sigma(\mathbf{s}_t)$ would be smaller than the expected value $V_i^\sigma(\mathbf{s}_{t+1})$ after selling the goods at price $\sigma_p(s_t)$ although the inventory is larger, i.e., $s_{it} > s_{it+1}$. This cannot be the case in equilibrium. The same is true in the case with ordering. Ordering goods and simultaneously selling them for a price lower than the purchase price cannot be an optimal strategy. Thus, the optimal price σ_{ip}^σ is always greater c.

(b) We will show that if $V_i^\sigma(\mathbf{s})$ is strictly k-concave in s_i, then $Q_i^\sigma(\cdot)$ is strictly k-concave in p_i for any value of z_i.

We can represent the function $Q_i^{\sigma R}(\cdot)$ as $-cz_i + p_i y^{\sigma e}(z_i, p_i; \mathbf{s}_{-i})$, where $y^{\sigma e}(\cdot)$ is the expected sales function. The function $Q_i^{\sigma R}(\cdot)$ is the same as the function Q_i^σ at the last period Q_{iT}^σ. We have shown in the proof of Lemma 4.2 that this function is convex.

Therefore, $\frac{\partial^2 Q_i^{\sigma R}(\cdot)}{\partial p_i^2} < 0$.

An argumentation analogous to part (a) yields a similar sufficient condition for the k-concavity of $Q_i^\sigma(\cdot)$ in p_i, namely that the function

$$p_i d_i^{e\sigma}(p_i)\lambda\left(\frac{z_i}{d_i^{e\sigma}(p_i)}\right) - \beta c \int_{\bar{\varepsilon}_i(z_i)}^{\infty} (z_i - \exp(\varepsilon) d_i^{e\sigma}(\cdot))\, dF(\varepsilon_i)$$

is concave in p_i. The function is continuously differentiable in p_i with a second derivative that is negative. Therefore, $Q_i^\sigma(\cdot)$ is k-concave in p_i.

(c) Finally, we show that $V_i^\sigma(\mathbf{s})$ is strictly k-concave in s_i.

Like in proof of Lemma 4.2 we make use of the fact that the profit function is bounded from above. This property guarantees that for any value of s_i

$$V_i^\sigma(s_i; \cdot) = \lim_{T\to\infty} V_{iT}^\sigma(s_i; \cdot)$$

with $V_{iT}^\sigma(s_i)$ as the value function for the finite horizon problem with time horizon equal to T. We prove k-concavity by induction.

For $T = 1$ we have $Q^{\sigma}_{i1}(\cdot)$ is strictly concave in z_i and p_i due to (a) and (b). Using the result of the first part of the proof, the optimal decision for this one-period problem has the form of equations (4.9) and (4.10). Hence, the value function of this one period problem is

$$V^{\sigma}_{i1}(s_i,\cdot) = I(s_i < \underline{s}^{\sigma}_{i1})\left(Q^{\sigma}_{i1}(s^{*\sigma}_{i1}, \bar{p}^{\sigma}_{i1}(s^{*\sigma}_{i1})) - k\right) + I(s_i \geq \underline{s}^{\sigma}_{i1})Q^{\sigma}_{i1}(s_i, \bar{p}^{\sigma}_{i1}(s_i,\cdot)) - (h_i - c_i)s_i.$$

With $Q^{\sigma}_{i1}(\cdot)$ being concave, it is simple to verify that $V^{\sigma}_{i1}(s_i,\cdot)$ fulfills the definition of strict k-concavity.

Assume now that for any $t \geq 1$, $V^{\sigma}_{it}(s_i,\cdot)$ is strictly k-concave. Then,

$$\begin{aligned} Q^{\sigma}_{it+1}(z_i, \bar{p}^{\sigma}_{it+1}(\cdot); \mathbf{s}_{-it+1}) &= -cz_i + \bar{p}^{\sigma}_{it+1}(\cdot) d^{e\sigma}_i(\bar{p}^{\sigma}_{it+1}(\cdot))\lambda\left(\frac{z_i}{d^{e\sigma}_i(\bar{p}^{\sigma}_{it+1}(\cdot))}\right) \\ &+ \beta \int V^{\sigma}_i\left(\max\left\{0; z_i - e^{\varepsilon_i} d^{e\sigma}_i(\bar{p}^{\sigma}_{it+1}(\cdot))\right\};\cdot\right) dF(\varepsilon_i) dF^{\sigma}_{\mathbf{s}_{-i}}(\mathbf{s}_{-it+2}|\mathbf{s}_{-it+1}). \end{aligned}$$

As $\bar{p}^{\sigma}_{it+1}(\cdot) d^{e\sigma}_i(\bar{p}^{\sigma}_{it+1}(\cdot))\lambda\left(\frac{z_i}{d^{e\sigma}_i(\bar{p}^{\sigma}_{it+1}(\cdot))}\right) - cz_i$ is again strictly concave and $V^{\sigma}_{it}(s_i,\cdot)$ is strictly k-concave, due to property (iv) of k-concave functions, $Q^{\sigma}_{it+1}(z_i, \bar{p}^{\sigma}_{it+1}(\cdot);\cdot)$ is also strictly k-concave. Hence, the optimal decision has again the form of equations (4.9) and (4.10) and the value function of this finite-horizon problem is

$$\begin{aligned} V^{\sigma}_{it+1}(s_i,\cdot) = &I(s_i < \underline{s}^{\sigma}_{it+1})\left(Q^{\sigma}_{it+1}(s^{*\sigma}_{it+1}, \bar{p}^{\sigma}_{it+1}(s^{*\sigma}_{it+1})) - k\right) \\ &+ I(s_i \geq \underline{s}^{\sigma}_{it+1}) Q^{\sigma}_{it+1}(s_i, \bar{p}^{\sigma}_{it+1}(s_i,\cdot)) - (h_i - c_i)s_i. \end{aligned}$$

Similar to $V^{\sigma}_{i1}(s_i,\cdot)$, this value function is strictly k-concave which completes the proof by induction. Therefore, $V^{\sigma}_i(s_i;\cdot) = \lim_{T\to\infty} V^{\sigma}_{iT}(s_i;\cdot)$ is strictly k-concave.

This completes the proof of the optimality of the described ordering strategy.

Properties of the optimal price. We complete the proof of Proposition 1 by showing that $\bar{p}(\cdot)$ is a continuous and strictly decreasing function.

The function $\bar{p}^{\sigma}_i$ is the value of p_i that maximizes Q^{σ}_i in p_i for a given z_i. Since Q^{σ}_i is continuously differentiable and strictly concave in prices, $\bar{p}^{\sigma}_i(z; \mathbf{s}_{-i})$ is implicitly defined by the first order condition $\frac{\partial Q^{\sigma}_i(z_i, \bar{p}; \mathbf{s}_{-i})}{\partial p_i} = 0$. By the implicit function theorem, we have that $\frac{d\bar{p}_i(z_i)^{\sigma}}{dz_i} = -\frac{\partial^2 Q^{\sigma}_i(z_i,\bar{p}_i)/\partial p_i \partial z_i}{\partial^2 Q^{\sigma}_i(z_i,\bar{p}_i)/\partial p_i \partial p_i}$, that by Lemma 4.2 is negative.

This completes the proof. □

Chapter 5

Concluding Remarks

In this thesis, we identified various strategic considerations that come into play when there are oligopolists' sunk costs in a dynamic context.

One class of strategic behavior that we understand better on the basis of these models is strategic investment in R&D. Here we have learned about factors such as learning-by-doing in R&D that tend to facilitate constantly changing market leadership and to increase incentives to invest with increasing intensity of competition. This helps us to understand the strategic role of many business practices.

The second class of behavior revolves around firms' efforts to merge. We have seen that globalization can trigger cross-border merger waves. This process has important implications not only for the good but also for factor markets and so for the economy as a whole.

The third class of strategic behavior concerns inventory decisions. We have learned that inventory can be a strategic decision variable in a dynamic context. Moreover, we have seen how strategic inventory ordering affects the dynamics of prices.

These essays have shed light on selected dimensions of strategic effects that influence strategic behavior in dynamic oligopolies. This facilitated characterization of the resulting dynamic rivalry and gives a better understanding of the identified strategies, behavior, and performance.

The oligopoly models give a relatively large array of predictions regarding firm behavior in and performance of markets. Such variety of predictions reflects that a great deal of variety also exists in industrial competition. The different models of oligopoly provide insights that can be used to structure the analysis of particular issues and give us a multitude of possibilities.

This way of looking at things is important for this thesis and, more generally, for the whole of oligopoly theory. There is no single model that is better than all other models. Each model is a better formalization of a certain type of industry and more appropriate to analyze a certain matter. The art and science is then to determine what is the appropriate model for each situation.

There are many theories of oligopoly and the variety of models of oligopolistic interactions is a virtue.

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