

**DESIGN, DEVELOPMENT AND EVALUATION OF  
A VIRTUAL CLASSROOM AND TEACHING  
CONTENTS FOR BERNOULLI STOCHASTICS**

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# Preface

This thesis is devoted to Bernoulli Stochastics, which was initiated by Jakob Bernoulli more than 300 years ago by his master piece *Ars conjectandi*, which can be translated as *Science of Prediction*. Thus, Jakob Bernoulli's Stochastics focus on prediction in contrast to the later emerging disciplines probability theory, statistics and mathematical statistics. Only recently Jakob Bernoulli's focus was taken up in [49] and [50], in which a unified theory of uncertainty is developed aiming at making reliable and accurate predictions. In this thesis, teaching material as well as a virtual classroom are developed for fostering ideas and techniques initiated by Jakob Bernoulli and developed in [49] and [50].

The thesis is part of an extensively construed project called *Stochastikon* aiming at introducing Bernoulli Stochastics as a unified science of prediction and measurement under uncertainty. This ambitious aim shall be reached by the development of an internet-based comprehensive system offering the science of Bernoulli Stochastics on any level of application. So far it is planned that the *Stochastikon* system<sup>1</sup> will consist of five subsystems. Two of them are developed and introduced in this thesis. The first one is the e-learning programme *Stochastikon Magister* and the second one *Stochastikon Graphics* that provides the entire *Stochastikon* system with graphical illustrations.

E-learning is the outcome of merging education and internet techniques. E-learning is characterized by the facts that teaching and learning are independent of place and time and of the availability of specially trained teachers. Knowledge offering as well as knowledge transferring are realized by using modern information technologies. Nowadays more and more e-learning environments are based on the internet as the primary tool for communication and presentation. E-learning presentation tools are for instance text-files, pictures, graphics, audio and videos, which can be networked with each other. There could be no limit as to the access to teaching contents. Moreover, the students can adapt the speed of learning to their individual abilities.

E-learning is particularly appropriate for newly arising scientific and technical disciplines, which generally cannot be presented by traditional learning methods sufficiently well, because neither trained teachers nor textbooks are available.

The first part of this dissertation introduces the state of the art of e-learning in statistics, since statistics and Bernoulli Stochastics are both based on probability theory and exhibit many similar features. Since *Stochastikon Magister* is the first e-learning programme for Bernoulli Stochastics, the educational statistics systems is selected for the purpose of comparison and evaluation. This makes sense as both disciplines are an attempt to handle uncertainty and use methods that often can be directly compared.

The second part of this dissertation is devoted to Bernoulli Stochastics. This part aims at outlining the content of two courses, which have been developed for the anticipated e-learning programme *Stochastikon Magister* in order to show the difficulties in teaching, understanding and applying Bernoulli Stochastics.

The third part discusses the realization of the e-learning programme *Stochastikon Magister*, its design and implementation, which aims at offering a systematic learning of principles and techniques developed in Bernoulli Stochastics. The resulting e-learning programme differs from the commonly developed e-learning programmes as it is an attempt to provide a virtual classroom that simulates all the functions of real classroom teaching. This is in general not necessary, since most of the e-learning programmes aim at supporting existing classroom teaching.

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<sup>1</sup><http://www.stochastikon.com/>



The fourth part presents two empirical evaluations of *Stochastikon Magister*. The evaluations are performed by means of comparisons between traditional classroom learning in statistics and e-learning of Bernoulli Stochastics. The aim is to assess the usability and learnability of *Stochastikon Magister*.

Finally, the fifth part of this dissertation is added as an appendix. It refers to *Stochastikon Graphics*, the fifth component of the entire *Stochastikon* system. *Stochastikon Graphics* provides the other components with graphical representations of concepts, procedures and results obtained or used in the framework of Bernoulli Stochastics.

The primary aim of this thesis is the development of an appropriate software for the anticipated e-learning environment meant for Bernoulli Stochastics, while the preparation of the necessary teaching material constitutes only a secondary aim used for demonstrating the functionality of the e-learning platform and the scientific novelty of Bernoulli Stochastics. To this end, a first version of two teaching courses are developed, implemented and offered on-line in order to collect practical experiences. The two courses, which were developed as part of this projects are submitted as a supplement to this dissertation.

For the time being the first experience with the e-learning programme *Stochastikon Magister* has been made. Students of different faculties of the University of Würzburg, as well as researchers and engineers, who are involved in the *Stochastikon* project have obtained access to *Stochastikon Magister* via internet. They have registered for *Stochastikon Magister* and participated in the course programme. This thesis reports on two assessments of these first experiences and the results will lead to further improvements with respect to content and organization of *Stochastikon Magister*.

**Part I**

**E-Learning for Statistics and Bernoulli  
Stochastics**



# Chapter 1

## The Role of E-Learning for Bernoulli Stochastics

### 1.1 Introductory Remarks

This dissertation project refers to the development of a web-based e-learning programme for Bernoulli Stochastics addressing graduates of secondary education and undergraduates of tertiary education. Therefore, this overview highlights existing e-learning on the one hand and ‘stochastics’ on the other.

Both e-learning and ‘stochastics’ are ambiguous terms with many meanings and, therefore, it is of utmost importance to explain their meaning as used in this dissertation. Actually, there are no standard models or templates with respect to e-learning and its products. Consequently there is an almost unlimited imagining, inventing and developing space for them. This is expressed by Jay Cross<sup>1</sup> with the following words (2004): “Nowadays, the word e-learning is whatever anybody says it is”. Stochastics on the other hand is an extremely obscured term, and one can find various definitions in publications and via internet.

### 1.2 Bernoulli Stochastics

The term ‘stochastics’ was introduced by Jakob Bernoulli about 300 years ago as the name for a new branch of science, which should deal with making reliable and accurate predictions. However, after Jakob Bernoulli had passed away in 1705, the term fell soon into oblivion and with the term also Bernoulli’s aim of developing a science of prediction.

The modern re-birth of the term was due to Ladislaus Bortkiewicz (1868-1931), who quoted Jakob Bernoulli in 1917 in his monograph “Die Iterationem”. On page 3 he notes<sup>2</sup>

*The consideration of empirical quantities, oriented to probability theory but based on the “law of large numbers”, may be called Stochastik ( $\sigma\tau\omicron\chi\alpha\zeta\epsilon\theta\alpha\iota$ ) = aim, conjecture. Stochastik is not merely probability theory in its application . . . ‘Stochastik’ also signified applied probability theory for Bernoulli; only he represented an anti-quoted viewpoint with respect to the application considered.*

Obviously, this is not the same interpretation as that of Jakob Bernoulli. In fact, for Bortkiewicz stochastics served for investigating the performance of large quantities, and not for making

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<sup>1</sup>One version said Jay Cross was the coiner of the word ‘e-learning’ (1998).

<sup>2</sup>Translation taken from: “The origins and legacy of Kolmogorov’s *Grundbegriffe*” by Glenn Shafer and Vladimir Vovk, <http://www.probabiliyand finance.com> (December 2007)

predictions. Finally, A.A. Chuprov (1922) and Du Pasquier (1926) adopted the term. Chuprov put the term into English and explained<sup>3</sup>:

*I use the word ‘stochastical’ as synonymous to ‘based on the theory of probability’.*

The revival did not take long, Shafer and Vovk state:

*In Stalin’s time, the Russians were not interested in suggesting that the applications of probability require a separate science. So stochastic became merely a mysterious but international synonym for random, [...].*

In Germany the term *Stochastik* became popular only at the beginning of the 1980s. For example, the well-known journal *Stochastik in der Schule*, started in 1978 under the name *Statistik in der Schule*, but changed to the actual name with the second issue published in 1982. Nowadays, the term ‘stochastics’ is used in Germany for the branch of mathematics, which covers probability theory and mathematical statistics. Note that, this commonly accepted meaning of stochastics as a special branch of mathematics differs much from Jakob Bernoulli’s intention to initiate a *Science of Prediction* or *Ars conjectandi*.

About twenty years ago, Bernoulli’s plan was taken up again by a research project called *Stochastikon* aiming at developing a Science of Prediction, i. e., stochastics in its original meaning. The plan was to realize a Science of Prediction based on modern information technology. In 2004, the research results were published in two seminal papers, which define a new branch of science called here *Bernoulli Stochastics*. An establishment of Bernoulli Stochastics would have far-reaching consequences for all branches of science, because of the new definition of uncertainty in Bernoulli Stochastics. Moreover, it would also have an impact on mathematics, as the fundamental model in Bernoulli Stochastics differs from the probability space being the foundation of probability theory. This implies that for Bernoulli Stochastics new terms and mathematical methods have to be developed.

Actually, Bernoulli Stochastics constitutes a new approach to uncertainty, which is not part of general education and, therefore, teaching cannot be based on existing textbooks, on available courses in the secondary or tertiary educational systems and on skilled teachers.

### 1.3 Modern Information Technology and Bernoulli Stochastics

Modern information technology is a very powerful and efficient tool to improve and disseminate a newly developed subject.

In order to meet the mathematical challenges raised by Bernoulli Stochastics, a research project was established about fifteen years ago aiming at developing a special computer algebra system (CAS) capable for solving the new problems. This stochastic computer algebra system called *Stochastikon Calculator* has been designed and operates although so far with a limited range. Besides the computer algebra system and the e-learning programme *Stochastikon Magister* - the topic of this dissertation, there are three other systems, which are developed in the framework of *Stochastikon*. The first one is called *Stochastikon Graphics*, which in cooperation with *Stochastikon Calculator* provides graphical representations of the calculated results. Moreover, there is *Stochastikon Encyclopedia* representing a cross-linked work of reference in

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<sup>3</sup><http://members.aol.com/jeff570/s.html> (December 2007)

Bernoulli Stochastics and science, while the last one is named *Stochastikon Mentor* representing a web-based diagnosis and consultancy tool for application.

Web-based e-learning is the best way to educate Bernoulli Stochastics, since without internet, establishment and dissemination of such newly developed areas would take decades.

There are many types of web-based e-learning products, and almost all of them are developed and used as supplements for traditional classroom teaching in well established fields of teaching. This holds especially for e-learning programmes in mathematical subjects.

Things look different for Bernoulli Stochastics, which so far is not established and, thus, is not a part of the curricula of general education. For a successful dissemination of a new scientific area, two conditions must be met:

- The new scientific area has to be prepared in a didactically appropriate way. Otherwise a successful establishment would probably be impossible.
- The e-learning programme must offer an as complete as possible virtual classroom, making available the entire list of possibilities within traditional classroom teaching that are designed for getting individual supports.

These two items must be the focus of an e-learning programme developed for Bernoulli Stochastics.

In other words, an e-learning programme for a newly developed teaching area does not aim at supporting an already existing education system by modern information technology based on blended learning, but to substitute traditional educational features. The two aims are different and, therefore, the requirements to be met for the two types of e-learning programmes must be different, too.

# Chapter 2

## The State of the Art of E-Learning in Statistics Education

### 2.1 Status of Education in Statistics

The status is described by Larreamendy-Joerns et al. [139] as follows:

*In 1987, George W. Cobb examined 16 introductory textbooks in statistics in the Journal of the American Statistical Association. In the 17 years since Cobb's evaluative framework was published, two major changes have occurred in the landscape of statistics education: First, there has been growing recognition of statistical knowledge as a crucial component of core scientific literacy. As a result, we now see the teaching and learning of statistics in elementary, secondary, and higher education. Second, there has been a flowering of online technologies and courses that both support and teach statistics. The use of online technologies is often predicated under the assumptions that the Internet can contribute to making statistical knowledge accessible to vast audiences and that online multimedia environments can make learning more interactive and meaningful.*

The present situation in the U.S. is given by Rubin [193]:

*Since 1992, data analysis and statistics have enjoyed more attention and visibility in educational circles. The National Council of Teachers of Mathematics' Standards (NCTM 2000) include data analysis and probability as a strand parallel to algebra and geometry, and elementary math curricula have been designed to give students more experience reasoning about data. ETS introduced the AP Statistics test in 1997, and the number of students taking the test has increased every year. Both beginning data analysis and sophisticated statistics courses are more generally available in high school, and statistics is sometimes considered a suitable capstone high school math class for those who are not going on in science or engineering.*

Cramer [56] depicts the scenario in Germany:

*In German school curricula, statistics is already equally mentioned to algebra, geometry and analysis but owing to a small range of teaching materials and a lack in the statistical education on the teacher's side this subject often is put in the second place when organizing the syllabus. Therefore, it is necessary that statistics and data analysis, theoretically and practically, have to become part of teachers' studies at university and at in-service training courses. Moreover, new teaching material is*

*needed. Because of the learning goal “media competence” which is also demanded by the school curricular these materials should also train the ability to use the so called New Media. Especially for teaching statistics these technologies offer many didactical advantages.*

It is clear that modern information technology and online teaching are expected to be powerful supports to the more and more important statistical education.

## 2.2 Modern Information Technologies and Statistics Education

There is an abundance of computer-based e-learning products developed for supporting statistical teaching and learning, and it is also easy to find lists of these resources from internet. For instance:

- Statistical Software: [http://www.prm.nau.edu/prm447/statistical\\_software.htm](http://www.prm.nau.edu/prm447/statistical_software.htm), Northern Arizona University, 167 items.
- Math Forum - Math Library - Internet Mathematics Library - Statistics: [http://mathforum.org/library/results.html?textsearch=Statistics&textsearch\\_bool\\_type=and&textsearch\\_whole\\_words=no&topics=&ed\\_topics=&resource\\_types=&levels=](http://mathforum.org/library/results.html?textsearch=Statistics&textsearch_bool_type=and&textsearch_whole_words=no&topics=&ed_topics=&resource_types=&levels=), The Math Forum Drexel, 640 items.
- E-Learning-Angebote zur Methodenlehre und Statistik: <http://mathforum.org/library/results.html?textsearch=Statis>, Free University Berlin, 24 items, a lot of them are educational statistics systems developed by German universities.
- Statistical Commercial Software: <http://www.statistics.com/resources/software/commercial/fulllist.php3>, Statistics.com, 166 items.
- Consortium for the Advancement of Undergraduate Statistics Education: <http://www.causeweb.org/resources/>, including Lecture Examples 561 items, Laboratories 377 items, Out-of-class 252 items, Teaching Methods 40 items, Datasets 234 items, Analysis Tools 271 items, Curriculum 345 items, Fun, Building Blocks 30 items and Multimedia 40 items.

Besides the lists of e-learning products, there are also many articles with reviews on these statistical technology resources.

In [44] the types of technologies used in statistics and probability instruction is broken into seven categories: statistical software packages, educational software, spreadsheets, applets/stand-alone applications, graphing calculators, multimedia materials, and data repositories.

### 1. Statistical Software Packages

*Statistical packages are software designed for the explicit purpose of performing statistical analysis. Several packages have been used by statisticians for many years, including SPSS, S-plus, R, SAS, and Minitab. While development of these packages has focused on uses by industry, they have also evolved into more menu-driven packages that are more user friendly for students.*



(see [44])

Besides the special statistical software packages, the mathematical software package Mathematica is also a very popular tool used in Germany for research and education in statistics.

Strictly speaking, R and SAS are more than statistical software packages in common sense like SPSS and Minitab. They are languages and environments for statistical computing and graphics. (<http://www.r-project.org/about.html>). Therefore, we propose to call this kind of technology “statistical language and environment software packages”.

## 2. Educational Software

“Different kinds of statistical software programs have been developed exclusively for helping students learn statistics.” [44] The author lists Fathom (for grade 11 - college students), TinkerPlots (for grades 4-8), and InspireData (a commercial extended version of TableTop for grade 4-8 students) as examples.

## 3. Spreadsheets

*Spreadsheets such as Excel are widely available on many personal computers. However, care must be exercised in using Excel as a statistical educational package.*

(see [44])

## 4. Applets/Stand-alone Applications

“Over the last decade there has been extraordinary growth in the development of on-line applets that can help students explore concepts in a visual, interactive and dynamic environment.” [44] These sources can be found, for instance, in the following websites: <http://www.shodor.org/interactivate/activities/AdvancedMontyHall> and <http://www.rossmanchance.com/applets/Reeses/ReesesPieces.html>

Chance et al. also note [44]: “In addition, a large number of computer programs can be (freely) downloaded from the Internet and run without an Internet connection that allow students to explore a particular concept.” While these tools are too numerous to be listed here, and they can be found from the websites:

[http://www.tc.umn.edu/~delma001/stat\\_tools/software.htm](http://www.tc.umn.edu/~delma001/stat_tools/software.htm),

Principles and Standards for School Mathematics

(<http://standards.nctm.org/document/eexamples/>, for pre-12 grade students), and

Consortium for the Advancement of Undergraduate Statistics Education

(<http://www.causeweb.org/resources/>).

## 5. Graphing Calculators

*Perhaps the most portable tool and one that is being increasingly used in lower grade levels is the graphing calculator [...] Advancements in technology have made the graphing calculator a powerful tool for analysing and exploring data.*

(see [44])

## 6. Multimedia Materials

“These materials often seek to combine several different types of technology, like ActivStats and CyberStats.” [44] These two exemplary systems are used for undergraduate statistical education, which is also the focus of our discussion.

## 7. Data and Materials Repositories

“Another popular and important use of the World Wide Web in statistics instruction is in locating and using pedagogically rich data sets and exploratory activities for use with students.” [44] For instances: The Data and Story Library<sup>1</sup>, Journal of Statistics Education (JSE) Dataset and Stories<sup>2</sup>, and CAUSE<sup>3</sup>.

## 8. Language and Environment

Within the category ‘statistical software packages’, we introduced a further category of statistical technology namely the statistical language and environment software packages, like R and SAS. This kind of software is first of all a complex statistical software package, then a statistical programming language together with the supporting environment. As described in <http://www.r-project.org/about.html>, R “is a well-developed, simple and effective programming language which includes conditionals, loops, user-defined recursive functions and input and output facilities [...]. The term “environment” is intended to characterize it as a fully planned and coherent system, rather than an incremental accretion of very specific and inflexible tools, as is frequently the case with other data analysis software.”

## 9. Integrated Statistical Systems

There is a ninth category of educational statistical software - the integrated statistical system, which also does not belong to the seven categories listed in [44]. Examples are MD\*REX and REXCEL. These systems aim at integrating statistical environments with other systems.

Aydylnly et al. [13] describe the two integrated system MD\*REX and REXCEL:

*Both MD\*REX and REXCEL integrate statistical languages with a completely different system, a standard spreadsheet application via well defined interfaces: MD\*Crypt in case of the former and (D)COM in the latter. The rationale behind these efforts is straight-forward. It is useful in our opinion to combine matrix oriented procedural statistical languages (XploRe and R) with a spreadsheet application (Microsoft Excel).*

Rubin [193] reviewed five categories of software, which serve mainly for the statistical education in middle and high school: software that uses video as data, Geographical Information Systems, graph construction tools, systems with distribution and data manipulation capabilities, and probability generation tools.

- VIDEO AS DATA: TapeMeasure, CamMotion, Alberti’s Window, Measurement in Motion
- GEOGRAPHICAL INFORMATION SYSTEMS: ESRI, GLOBE, MyWorld, GOOGLE Earth,

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<sup>1</sup><http://lib.stat.cmu.edu/DASL>

<sup>2</sup>[http://www.amstat.org/publications/jse/jse\\_data\\_archive.html](http://www.amstat.org/publications/jse/jse_data_archive.html)

<sup>3</sup><http://www.causeweb.org/resources/>

- GRAPH CONSTRUCTION SOFTWARE: TableTop, InspireData, TinkerPlots,
- PLAYING AROUND WITH DATA: Statistics Workshop (ELASTIC), Fathom, Logger Pro,
- PROBABILITY AND SAMPLING: TinkerPlots, Model Chance, and
- REFLECTIONS AND REACTIONS: LenoxSoftWorks

There are many articles, for example [13, 55, 103, 104, 105, 134, 139, 218, 219, 233], that focus on the effects of statistical software on the undergraduate education in the U.S. and in Germany by comparing the software functions and the teaching results of different programs, and even by comparing the results obtained by traditional instruction. In [55] and [139] eight instructional systems (in English) are cited to exemplify the diversity of introductory statistics materials available in the internet or by CD. In these two reviews the authors focus on:

- a) *examples as core components of instructional explanations;*
- b) *exercises or problem solving venues as environments that support and foster learning by doing and skill acquisition; and*
- c) *interactive learning environments as distinctive features of online courseware.*

For analysing the third point of these systems, the authors use a table, which summarizes the online resources available in each course. This table will be cited in our discussion (see Table 2.7) and extended by adding corresponding information from four statistical systems used in German universities: MM\*Stat, EMILeA-stat, Statistics and New Statistics together with *Stochastikon Magister*.

## 2.3 Characteristics of Educational Statistics Systems and Stochastikon Magister

The characteristics of the four statistical systems developed and used in German universities will be listed according to four major aspects:

1. Project Information, which may include the project name, funding, developer, web address, project aim, language, developing stage, releasing, project cooperator, cost, and award;
2. System Information, which may include the running mode, access, software dependence, major development work, programming work, major programming languages, cooperative system, and software reusability;
3. Pedagogical Features, which may include the applied field of the system, target group, role in teaching, usage, teaching content, content structure, teaching style, communication and support, learning track, teaching track, learning organization and leaning process management, learner / user management, and knowledge accumulation;
4. System Highlight.

For comparison, *Stochastikon Magister* is also looked at from these four aspects. However, in contrast to [55] and [139], the emphasis of our comparison is on the technical implementation of courseware and instructional environment, so CyberStats, one of the most important educational statistics systems used in the U.S. universities, is analysed again here from our point of view.

In the following tables, the *italics* are original names of items used in each system.

### 2.3.1 MM\*Stat

#### Project Information:

Developer	Institute for Statistics and Econometrics & Centre for Applied Statistics and Economics, Humboldt University Berlin.
Web Address	<a href="http://www.mm-stat.de/">http://www.mm-stat.de/</a> (2000) <a href="http://www.md-stat.com/">http://www.md-stat.com/</a> (2001, 2003) <a href="http://www.quantlet.com/mdstat/mmstat.html">http://www.quantlet.com/mdstat/mmstat.html</a> (2003, 2006)
Aim	To be a flexible tool to support teaching and learning statistics in introductory courses via the internet or from a CD.
Language	German, English, etc. more than 12 languages now.
Releasing	Web version: since 2000. CD version: since 2001.
Project Cooperator	MD*Tech ( <a href="http://www.mdtech.de/">http://www.mdtech.de/</a> ), Springer-Verlag, and Multimedia Hochschulservice Berlin GmbH (MHSG, <a href="http://www.mhsg.de/">http://www.mhsg.de/</a> ).
Cost	Web version with all languages except German needs a registration and an access fee. CD version costs 17.2 Euro/CD accompanied with a short manual booklet.

#### System Information:

Running Mode	Web version: a pure web application CD version: stand-alone software.
Access	Web version: Available via internet connection and will be automatically updated and extended. CD version: buy, install, configure the computer, and update along with different publishing versions.
Software Dependence	XploRe-Software ( <a href="http://fedc.wiwi.hu-berlin.de/xplore.php">http://fedc.wiwi.hu-berlin.de/xplore.php</a> ) is the calculating and plotting engine.
Development Work	The whole system including the software and the teaching content.
Programming Work	Different dynamic or interactive teaching materials used in <i>interactive examples</i> and <i>multiple choice questions</i> . The website structure and the interface of MM*Stat. MD*Book ( <a href="http://www.md-book.com/">http://www.md-book.com/</a> , in co-operation with Springer-Verlag, and on the basis of LaTeX2HTML) converts the Latex source file to the desired format: PDF, HTML, Postscript, MM*Stat format or e-stat format.
Programming Language	HTML 4.0 combined with JavaScript and Cascading Style-Sheets (CSS), XploRe Quantlet technology (XQC is Java Applet, XQS is written in C++).
Cooperative System	Electronic Books ( <a href="http://fedc.wiwi.hu-berlin.de/xplore/ebooks/html/">http://fedc.wiwi.hu-berlin.de/xplore/ebooks/html/</a> ).

Software Reusability	The programming work in this project is mainly for the statistical teaching contents. Only the technical skill used for MD*Book can be reused by other e-learning systems or for teaching other disciplines.
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**Pedagogical Features:**

Applied Field	Statistics (Introductory Course)
Target Group	Originally for undergraduate students at the Faculty of Economics and Business Administration, Humboldt University Berlin, Germany.
Role in teaching	An electronic lecture script intended as an add-on to traditional classroom lecture, which aims at supplementing traditional lectures by creating a framework to repeat the content of lectures by students at their own place.
Usage	<p>For the teacher:</p> <ul style="list-style-type: none"> <li>• Move to more elaborate statistical data analysis,</li> <li>• Handle more data and examples in the instruction,</li> <li>• Emphasize more statistical critical thinking,</li> <li>• Demonstrate the assumptions connected with various statistical methods and models,</li> <li>• Concentrate more on the students' difficulties.</li> </ul> <p>For the student:</p> <ul style="list-style-type: none"> <li>• Review the lectures at his/her own pace,</li> <li>• Perform examples with several data sets and view the results,</li> <li>• Discover relations between the course subjects.</li> </ul>
Teaching Content	Containing 12 statistic topics. Each topic is divided into several subtopics that are presented as separate lectures.
Content Structure	<p>The sequence of lectures is canonical in statistical theory.</p> <p>The main elements of a lecture are:</p> <ul style="list-style-type: none"> <li>• <i>lecture</i> (content),</li> <li>• (further) <i>information</i>, which is not necessary for each lecture,</li> <li>• (three kinds of) <i>examples</i>, but not every of these three types is available for each lecture,</li> <li>• <i>multiple choice questions</i> at the end of paragraphs.</li> </ul> <p>There is a <i>glossary</i> for all lectures.</p> <p>The user interface is a composition of filing cards, which contain the various elements of MM*Stat. Up to 10 cards can be opened at the same time. A special filing card called bookmark contains a list of the filing cards last viewed.</p>
Teaching Style	<p>All teaching contents are pre-settled.</p> <p>The <i>lecture</i> (content), (further) <i>information</i> and <i>glossary</i> are HTML-based text information for basic knowledge, broader and deeper knowledge and standard definition respectively.</p> <p>A few lecture units contain audio explanations for statistical facts.</p> <p>Major attention in designing MM*Stat was paid to <i>examples</i>.</p> <p>The three kinds of <i>examples</i> are fully <i>explained</i> examples, <i>enhanced</i> examples and <i>interactive</i> examples.</p> <p>The <i>multiple choice questions</i> are for self-assessment.</p> <p>Teaching elements are hyperlinked with each other.</p>
Communication and Support	with (from) program-based <i>interactive examples</i> and <i>multiple choice questions</i> .

**System Highlight:**

Highlight	<p>1. The learning process is assumed to be based on a three-dimensional structure:</p> <ul style="list-style-type: none"> <li>• The first dimension is the sequence of the lesson, which presents and explains the important methods and models with their assumptions.</li> <li>• The second dimension are examples and further information, which shall deepen the subjects.</li> </ul> <p>The third dimension is jumping between frames of the sequence, which reinforce previous ideas within the process of presenting new ones.</p> <p>2. The interactive examples are based on the XploRe Quantlet technology. This is a client/server (XQC/XQS) concept, which was especially developed for statistical computing over the internet.</p> <p>3. A multilingual internationalized system with so far 13 different languages.</p>
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Table 2.1: Characteristics of MM\*Stat system.

The information listed in Table 2.1 is collected from [103] (2007), [104] (2006), [105] (2005), [13] (2003), [218] (2003), [219] (2002), [135] (2001), [191] (2001), [192] (2000), [65] (1999), MM\*Stat (CD),

[http://stirner.wiwi.hu-berlin.de/mediawiki/mmstat\\_de/index.php/Hauptseite](http://stirner.wiwi.hu-berlin.de/mediawiki/mmstat_de/index.php/Hauptseite)

<http://fedc.wiwi.hu-berlin.de/xplore/ebooks/html/>

<http://www.quantlet.com/index.php?p=start>

**2.3.2 EMILeA-stat**

EMILeA-stat (a multimedia-, web-based and interactive teaching and learning environment in applied statistics):

**Project Information:**

Project	“e-stat” project within the “New Media in Education Funding Programme”
Funding	German Federal Ministry of Education and Research (BMBF), with 2.9 Mio. Euro grant.
Developer	Set up by 13 partners at 7 German universities, about 70 people are co-working in developing and realizing the learning and teaching environment. Led and sustained by the RWTH Aachen.
Aim	Development of one learning and teaching environment suitable for the statistical education in all branches. Development of one system suitable for teaching statistics at schools, universities, and in further vocational training which supports both supervised and self-directed learning.
Language	German
Developing Period	April 2001 - December 2004.
Project Cooperator	The project is also supported by further partners in advice and it cooperates with economic partners such as SPSS Software, Science + Business Media (Springer Verlag), Bertelsmann Springer and MD*Tech Method & Data Technologies (XploRe-Software). Up to 2003 more than 15 associated partners working at universities, schools, and business have joined the team.
Cost	Free of charge.

Web Address	Version 1.0: <a href="http://emilea-stat.rwth-aachen.de/">http://emilea-stat.rwth-aachen.de/</a> Version 2.0: <a href="http://emilea-stat.stochastik.rwth-aachen.de/">http://emilea-stat.stochastik.rwth-aachen.de/</a> . . . . .
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### System Information:

Running Mode	A pure web application
Access	Available via internet connection; will be automatically updated and extended.
Software Dependence	XploRe-Software (MD*Tech - Method and Data Technologies - mdtech@mdtech.de) and R ( <a href="http://www.r-project.org/index.html">http://www.r-project.org/index.html</a> ) are the calculating and plotting engines.
Development Work	The whole system including the software and the teaching content except the calculating engines.
Programming Work	Different dynamic or interactive teaching materials used in the <i>Courses</i> , <i>Practice</i> and the <i>Tree</i> - a graphical representation for the concept hierarchy. The website structure and the interface of EMILeA-stat. Authoring tools for modules (implemented in the version 2.0).
Software Reusability	The programming work in this project focuses on statistical teaching contents. Only the technical skill used for MD*Book can be reused by other e-learning systems or for teaching other disciplines.

### Pedagogical Features:

Applied Field	Statistics (Descriptive and Inductive Statistics)
Target Group	Pupils, students, teachers, technicians, practitioners and all those interested in statistics.
Role in Teaching	Supporting both supervised and self-directed learning for statistics education at schools, universities, and in further vocational training.
Usage	<ul style="list-style-type: none"> <li>• Teaching, selective support of teaching (examples, visualizations,...),</li> <li>• (Online) collection of slides,</li> <li>• Self-directed learning before/after lectures,</li> <li>• Self-directed learning (eLearning),</li> <li>• Supervised learning (blended eLearning),</li> <li>• Online script with add-on's (visualizations, self-assessments,...).</li> </ul>
Teaching Content	Containing an encyclopedia with 23 categories, 25 courses, and 4 case studies for 10 statistic topics.
Teaching Style	<p>All teaching contents are pre-settled.</p> <p><i>Browse EMILeA-stat</i>: the module world acts as a statistical encyclopedia. Each basic concept is a module and categorized into three knowledge levels for different users.</p> <p><i>Visualization of the concept of world</i>: a graphical representation of the concept hierarchy. The leaf of the concept-tree links to the corresponding concept in the module world.</p> <p><i>Browse a EMILeA-stat-view</i>: categorizing the concept modules from the perspective of a specialized group of users.</p> <p><i>Public Courses</i>: theory, examples, exercises...represented by text, graphics, animation and applets.</p> <p><i>Statistics in Practice</i>: practicing statistics by case studies through the simulations of 'authentic' scenarios.</p>

Content Structure	Content is strictly modular. The screen-sized <i>module</i> is the smallest element, which can be definition, remark, theorem, proof, example, exercise, discretization, classifications, etc. Modules may be combined to form course units or courses. Course can be divided into sub-courses, chapters or units.
Communication and Support	with (from) applets in <i>Course</i> and interactive programs in <i>Practice</i> .

**System Highlight:**

Highlight	<ol style="list-style-type: none"> <li>1. A user-oriented product: one system for all users e. g., three levels of abstraction: A: elementary B: basic C: advanced</li> <li>2. The teaching content is trying to serve statistical education in all branches (10 topics) and for different groups of users from secondary school students students to statisticians.</li> <li>3. A ‘long tail’ web application, which continuously expanding, improving and sustaining its product especially the teaching content.</li> <li>4. Organizing the teaching content through different views e. g., method-led, problem-led, view-led and scenario-led in supporting different teaching concepts and instructional designs.</li> </ol>
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Table 2.2: Characteristics of EMILeA-stat system.

‘e-stat’ is an important project for enhancing statistical education in Germany. The developer of MM\*Stat is one of the thirteen main participants of this project. Therefore these two systems have some similarities such as the layout, the usage of MD\*Book and XploRe technologies, but EMILeA-stat is far more complex and comprehensive. EMILeA-stat is an ambitious system. When we first visited it in 2005, it was still under construction, some parts and contents were unavailable and some were evolving. Being a pure web application, the internet provides EMILeA-stat an unlimited expandable memory and developing period. Now, not only the teaching content is almost completed but also the second version has been established. According to the main developer Professor Uwe Kamps, a new version has been prepared, but at present there are no resources to put it in the internet. The possibilities for revising and extending the system, is one of the main advantages of a long tail web application compared with CD-Room software.

The information listed in Table 2.2 is collected from [33] (2008), [34] (2007), [134] (2006), [104] (2006), [57] (2004), [58] (2003), [13] (2003), [56] (2003),

<http://www.emilea.de/> (EMILeA-stat)

<http://emilea-stat.rwth-aachen.de/> (EMILeA-stat 1.0)

<http://emilea-stat.stochastik.rwth-aachen.de/> (EMILeA-stat 2.0)



### 2.3.3 Statistics

#### Project Information:

Developer	FernUniversität in Hagen
Web Address	<a href="http://www.fernuni-hagen.de/e-statistik/inhalt.htm">http://www.fernuni-hagen.de/e-statistik/inhalt.htm</a>
Aim	Intended for the individual learner (self-study at the home PC) as supplement to statistical textbooks and lectures. Designed for teachers to be used to accompany lessons, lectures and seminars in schools, universities or in vocational training.
Project Cooperator	Fachbuchverlag Leipzig im Carl Hanser Verlag. Kompetenznetzwerk Universitätsverbund Multimedia NRW.
Cost	34 Euro/CD for the 5th edition with a booklet in German.
Award	The second edition was nominated for the European Academic Software Award.

#### System Information:

Running Mode	A stand-alone software (CD-ROM) running in a PC web browser.
Access	Buy, install, configure the computer, and update along with different publishing versions.
Development Work	The whole system including the software and the teaching content.
Programming Work	Animations in <i>theory</i> and <i>example</i> , and applets in <i>exercises</i> . The website structure and the interface of Statistics.
Cooperative System	Links to the interesting websites such as <a href="http://www.fernuni-hagen.de/">http://www.fernuni-hagen.de/</a> .
Software Reusability	Only for statistics. Some animations and applets may be reused in the project New Statistics.

#### Pedagogical Features:

Applied Field	Statistics (Descriptive Statistics and Exploratory Data Analysis Project Information)
Target Group	Individual learner and teacher of universities, polytechnics, colleges, secondary schools and in further education (training of scientists and social and educational economists, engineers, researchers) at the shop-floor.
Role in Teaching	To supplement traditional personal teaching and static textbooks in different environments.
Usage	An animated and interactive (supplement) textbook.
Releasing	The first edition, December 1999. The second edition, January 2001. The third edition, October 2001. The fourth edition, September 2002. The fifth edition, February 2004.
Teaching Content	Containing 52 chapters within 9 statistic courses.
Content Structure	Navigation for the <i>content</i> , <i>literature</i> , <i>note-sheet</i> , and <i>communication</i> . In <i>content</i> for each chapter there are 3 learning levels: <ul style="list-style-type: none"> <li>• the first learning level: <i>summary</i>,</li> <li>• the second learning level: <i>theory</i>, <i>examples</i>, <i>exercises</i> and <i>expertise</i>,</li> <li>• the third learning level: <i>glossary</i> and <i>internet connection</i>.</li> </ul>

Teaching Style	The <i>summary</i> : static text and graphics. The <i>theory</i> and <i>examples</i> : text, sound (optional), and animation. The <i>exercises</i> : applet. Almost all teaching contents are pre-settled, except the <i>communication</i> between the user and the content author by email.
Communication and Support	with (from) interactive <i>exercises</i> by applets, the author of content by e-mail, and other users by newsgroup.

**System Highlight:**

Highlight	1. Using animations to dynamically exhibit background theories and examples. 2. Collecting plenty of real-life examples from different disciplines such as medicine, psychology, economics and social sciences, and engineering.
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Table 2.3: Characteristics of Statistics system.

The information listed in Table 2.3 is collected from [155] (2007), [156] (2006), [134] (2006), [100] (2006), [157] (2005), [158] (2004),  
Beschreibende Statistik und explorative Datenanalyse (CD)  
<http://www.fernuni-hagen.de/e-statistik/>  
<http://www.fernuni-hagen.de/e-statistik/Begleitheft.pdf> (booklet for the 5th edition of Statistics)

**2.3.4 New Statistics****Project Information:**

Developer	CeDiS of Free University Berlin in cooperation with 13 academic institutes and 10 German universities.
Funding	German Federal Ministry of Education and Research (BMBF).
Web Address	<a href="http://www.neuestatistik.de/">http://www.neuestatistik.de/</a> and <a href="http://www.statistiklabor.de/">http://www.statistiklabor.de/</a> .
Aim	Creating a multimedia-based and web-supported virtual learning environment for basic studies in statistics to enrich the traditional instructional methods with new multimedia-based, interactive material, and in particular to improve the quality of the statistical education by the blended learning approach.
Language	German
Developing Stage	1998-2000 Interactive Statistics, the precursor of New Statistics developed by the Free University Berlin. 2001-2004 New Statistics
Cost	Free of charge.
Award	The Statistical Lab won the Medida Prix 2003.

**System Information:**

Components	1. Total Multimedia New Statistics Curriculum, and 2. Statistical Lab.
Running Mode	1. Total Multimedia New Statistics Curriculum: a pure web application. 2. Statistical Lab: stand-alone software.
Software Dependence	R software package ( <a href="http://www.r-project.org/index.html">http://www.r-project.org/index.html</a> ) as the core of the Statistical Lab.
Language	HTML, JavaScript, Java (applet), Flash (animation), C++, C.

Development Work	Teaching content, and Statistical Lab.
Programming Work	Animations and applets for the dynamic interactive teaching materials. The ‘Learning module editor’ based on MS-Word for the static teaching content. The website structure and the interface of New Statistics. A GUI shell for using R software package.
Access	Total Multimedia New Statistics Curriculum: available via internet connection and will be automatically updated and extended. Statistical Lab: free download, install, configure the computer, and updated along with different published versions.
Software Reusability	Only for teaching statistics.

**Pedagogical Features:**

Applied Field	Statistics (Basic and Elementary Studies)
Target Group	Undergraduates for elementary study in statistics. Students from 10 German universities have used it. These 10 universities are participants of this project.
Role in Teaching	Elements for blended learning.
Usage	Online textbook
Teaching Content and Teaching Style	Containing more than 70 learning modules for 6 statistics topics, <i>case studies</i> , <i>glossary</i> , <i>formula</i> , 40 short <i>lectures</i> in form of text (in HTML format), around 50 flash animations and more than 60 java applets. All teaching contents are pre-settled.
Communication and Support	with (from) interactive program - applets and the <i>Statistical Lab</i>

**System Highlight:**

Highlight	To replace the formal and mathematical way in the education of statistics by a problem-oriented and practical approach. This task is mainly undertaken by means of the <i>Statistical Lab</i> .
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Table 2.4: Characteristics of New Statistics system.

‘New Statistics’ is another big important project funded by BMBF and developed by a co-operation of a dozen universities and institutes. The CeDiS of Free University Berlin gave contribution to the Statistical Lab, the team of Statistics project in Hagen University developed most of the applets and animations, and other developers concentrated on the content development under the environment of its ancestor ‘Interactive Statistics’ developed by the CeDiS of Free University Berlin before New Statistics project.

The information listed in Table 2.4 is collected from [134] (2006), [159] (2004), [188] (2003), [92] (2003), [160] (2003), [38],

<http://www.fernuni-hagen.de/neuestatistik/>

<http://www.neuestatistik.de/>

<http://www.fernuni-hagen.de/e-statistik/>

<http://www.statistiklabor.de/en/index.html>

<http://www.cedis.fu-berlin.de/en/projekte/index.html>

### 2.3.5 CyberStats

#### Project Information:

Developer	CyberGnostics, Inc. The teaching content is written by over two dozen experienced statistics professors in the U.S.
Funding	The U.S. National Science Foundation (NSF).
Web Address	<a href="http://www.cyberk.com/">http://www.cyberk.com/</a> (2003). <a href="http://statistics.cyberk.com/splash/">http://statistics.cyberk.com/splash/</a> (2005).
Aim	An empowering, interactive and easy to use online learning system to help students to build conceptual understanding, increase statistical literacy, and improve performance in the relevant courses.
Language	English
Releasing	Frequent updates. The free test version, 1996. Version 1.0, first release, 1999. Version 2.0, a commercial version, Fall 2001. Version 2.1, a pure web application, mentioned in [218] 2003. Version 2.2, a pure web application, mentioned in "CyberStats: An introduction to statistics" ( <a href="http://www.ellipse.ch/SousGroupe.aspx?SousGroupe=1804">http://www.ellipse.ch/SousGroupe.aspx?SousGroupe=1804</a> ) 2003.
Project Cooperator	Duxbury published the manual of CyberStats.
Cost	\$15 for Web site access + \$ 5 base charge + \$ 1 per unit selected.

#### System Information:

Running Mode	A pure web application
Access	Available via internet connection and will be automatically updated and extended.
Software Dependence	WebStat 3.0 ( <a href="http://www.stat.sc.edu/%7Ewest/webstat/">http://www.stat.sc.edu/%7Ewest/webstat/</a> 1998, <a href="http://www.webstatsoftware.com/2003">http://www.webstatsoftware.com/2003</a> ) or now called StatCrunch 3.0 ( <a href="http://www.statcrunch.com/">http://www.statcrunch.com/</a> ) is the calculating and plotting engine.
Development Work	The whole system including the software (except WebStat 3.0) and the teaching content.
Programming Work	Different dynamic or interactive teaching materials like applets, interactive exercises in the units. Interactive programs in the system are produced by either the project itself or by WebStat 3.0. The online calculator. The website structure and the interface of CyberStats. The Course Management System.
Software Reusability	The technical skill used for CyberStats course management system is reusable by other e-learning systems or for teaching other disciplines.

**Pedagogical Features:**

Applied Field	Statistics (a traditional general introductory course)
Target Group	Undergraduate students
Role in Teaching	A web tool to support instructors in classroom or distance learning setting, and students to learn by doing and discovery in achieving the project aims.
Usage	<ul style="list-style-type: none"> <li>• Replacing or complementing a textbook in a traditional classroom,</li> <li>• as lab or homework assignments,</li> <li>• as an exclusively online course, with an instructor, but with students working from remote locations.</li> </ul>
Teaching Content	Containing 41 units and are grouped into 7 statistics courses.
Content Structure	Each unit has <i>Unit Home</i> , <i>Think First</i> , Three Keys ( <i>Basics</i> , <i>Uses</i> , and <i>Warnings</i> ), <i>Examples</i> , <i>Exercises</i> , <i>Self-Assessment</i> and <i>Unit Review</i> . A <i>Calculator</i> , a <i>Glossary</i> and a <i>Data Tool</i> (WebStat 3.0) work for all units.
Teaching Style	<p><i>Unit Home</i> is a one-paragraph text summary outlining the concepts and the real-world significance of the topic of the Unit.</p> <p><i>Think First</i> gives the purpose and usefulness by Q &amp; A and interactive simulations.</p> <p>Three Keys (<i>Basics</i>, <i>Uses</i>, and <i>Warnings</i>) define the terms and concepts by (interactive) examples, which introduce the techniques and methods pertinent usually by real data, and address possible false assumptions and pitfalls in reasoning.</p> <p>Each of the three pages has a “<i>Practice</i>” in form of Questions with automatic Answers.</p> <p><i>Examples</i> use real data examples and interactive simulations.</p> <p><i>Exercises</i> are home assignments in Q &amp; A style, which are submitted by students then corrected and graded by instructors.</p> <p><i>Self-Assessment</i> is in style of multiple choices questions with automatic corrections.</p> <p><i>Unit Review</i> collects all important contents in a page.</p> <p><i>Glossary</i> likes a textbook and is available at the click of a button.</p> <p><i>Calculator</i> has the usual basic scientific functions.</p> <p><i>Data Tool</i> is a Java applet implementing a rudimentary statistics package and including a reasonable set of graphics.</p> <p>The instructors can add their own teaching material and test questions. Other teaching contents are pre-settled.</p>
Learning Track	Grades to students for the assigned <i>Exercises</i> and <i>Self-Assessment</i> are recorded and supervised.
Teaching Track	Instructors customize the learning <i>Unit</i> and add own teaching material, assign <i>Exercises</i> as homework and give grade to students’ work, add own <i>Test</i> questions, set up <i>Test</i> and correct <i>Test</i> .
Communication and Support	<p>with (from)</p> <ul style="list-style-type: none"> <li>• interactive simulations by applets, and interactive practices and tests in the style of multiple choices questions;</li> <li>• grades given by the instructors to assignments and tests;</li> <li>• email, message board with e-note, and chat room.</li> </ul>

Learning Organization and Learning Process Management	To a specific group of students, the course management system allows the instructors to customize the course by selecting units for the course content, providing their own “syllabus”, adding their own teaching contents, assigning their selected homework, giving grades to the assignments, writing electronic exams that can include their own questions, correcting the tests with grades and comments, reviewing the students’ grade, and creating reports about the students’ work.
Learner/ User Management	Containing some simple and basic functionality and acting as a background part invisible to the users.
Knowledge Accumulation	Instructors can add their own test questions.

**System Highlight:**

Highlight	<ol style="list-style-type: none"> <li>The core of CyberStats is “Interactive”, therefore it <ul style="list-style-type: none"> <li>contains more than 600 interactive simulations and data-analytic calculations by applets, more than 1000 interactive exercises with immediate feedback, and uses real data in real-world settings;</li> <li>has a sophisticated analytic software (WebStat 3.0);</li> <li>has an off-the-shelf calculator.</li> </ul> </li> <li>A course management system, which allows the instructors to apparently customize their courses for a specific group of students, and take care the students’ work during the period of their learning.</li> </ol>
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Table 2.5: Characteristics of CyberStats system.

CyberStats is an important statistical project funded by the U.S. government and authored by two dozens of university professors. Its pedagogical features provide its teacher customers more opportunities to implement their own ideas and tailor the big system for their specific usage. These characteristics make it easier to be used as an independent teaching environment. Like EMILeA-stat, CyberStats is also a ‘long tail’ web application, which is continuously improved according to the customer feedbacks. [62, 219]

The information listed in Table 2.5 is collected from [44] (2007), [220] (2006), [221] (2006), [55] (2006), [139] (2005), [218] (2003), [233] (2003), [3] (2002), [219] (2002), [62] (2001), Statistical Tools on the World Wide Web

(<http://www.csm.ornl.gov/asasie/west.html>, 1998)

CyberStats Demo: [http://media.duxbury.com/duxbury/cyberstats/cyberstats\\_demo.html](http://media.duxbury.com/duxbury/cyberstats/cyberstats_demo.html)

Mathforum - CyberStats (<http://mathforum.org/library/view/65247.html>)

**2.3.6 Stochastikon Magister****Project Information:**

Project	Stochastikon
Developer	Stochastikon GmbH
Web Address	<a href="http://www.magister.stochastikon.com/">http://www.magister.stochastikon.com/</a>
Aim	To educate a newly developed sophisticated mathematics-based scientific discipline completely within a web-based virtual learning environment.
Language	English, German
Project Cooperator	University of Würzburg

Developing and Releasing	Frequent updates. February 2004 Graphical Lab. December 2004 e-learning Stochastics, which is the precursor of Magister. June 2005 Magister Learning Platform (software). January 2007 Magister teaching content.
Cost	Free of charge

### System Information:

Running Mode	A pure web application
Access	Available via internet connection and will be automatically updated and extended.
Software Dependence	/
Development Work	The whole system including the software and the teaching content.
Programming Work	Different dynamic or interactive teaching materials used in <i>Exercise</i> and for the <i>Graphical Lab</i> . The website structure and the interface of Magister. A PDF creator for Magister teaching content for <i>Introduction</i> , <i>Target</i> , <i>Content</i> , and <i>Example</i> . Course Management System. Learning Management System User Management System
Language	JSP, MySQL, JavaScript, HTML, Java (for the <i>Graphical Lab</i> )
Cooperative System	Stochastikon Calculator Stochastikon Graphics Stochastikon Encyclopedia Stochastikon Mentor
Software Reusability	The virtual learning platform i. e., the virtual learning-teaching-managing environment is reusable for educating different disciplines completely via internet. The Stochastikon Graphics, when serving for different calculating engines, is extendable to construct different Graphical Laboratory visualizing different computational conceptions and procedures for online.

### Pedagogical Features:

Applied Field	Bernoulli Stochastics
Target Group	Engineers, scientists, university students, etc. those who have to solve problems taking into account the inherent uncertainty or persons who are just interested in the aspect of uncertainty.
Role in Teaching	A web-based virtual classroom, which can accomplish the complete learning process for Bernoulli Stochastics - a newly developed scientific discipline that has no traditional educational background.
Usage	Online classroom
Teaching Content	Containing 69 learning units in 8 modules for 2 Bernoulli Stochastics courses.

Content Structure	<p>There are <i>Introductions</i> for each course and module.</p> <p>Each unit has 9 activities:  <i>Target, Content, Example, Exercise</i> and <i>Test</i> are mandatory, <i>Question, Discussion, Literature, FAQ</i> are recommended.</p> <p>All teaching contents are displayed in a fixed panel and navigated by a two level index, one for the learning units and the other for the nine learning activities in each unit.</p>
Teaching Style	<p><i>Target, Content, Example</i> are PDF documents.</p> <p><i>Exercise</i> is dynamic and/or interactive program-based content together with the <i>Graphical Lab</i>.</p> <p>The teaching content of these four activities are pre-settled and they can be implemented in any multimedia style available via internet according to the content requirement.</p> <p>The teaching content in <i>Question, Discussion, Literature, FAQ</i> and <i>Test</i> of each unit can be dynamically extended along with the learning process executed in the Magister environment.</p> <p>The teaching process is also supported by Stochastikon Encyclopedia, Calculator and Mentor by hyperlinks.</p>
Learning Track	Learner's personal profile and study records made in learning activity <i>Question, Discussion, Test</i> and <i>Learner Progress Management</i> package are tracked and supervised for teaching and learning.
Teaching Track	Instructor and administrator's personal profile and working records made in learning activity <i>Question, Discussion, Test, FAQ, Literature, Learner Progress Management</i> package and <i>Learner (User) Management</i> package are tracked for teaching and managing.
Communication and Support	<p>with (from)</p> <ul style="list-style-type: none"> <li>• Different interactive programs including the <i>Graphical Lab</i>,</li> <li>• Human beings (instructors and other learners) by simulated face-to-face supports and social interactions.</li> </ul>
Learning Organization and Learning Process Management	For each learner the system provides a systematically organized, supervised, supported and learner-oriented learning process and the corresponding environment.
Learner/ User Management	<p>Defining, regulating and adjusting behaviors of different roles <i>Master, Administrator, Teacher</i> and <i>Learner</i> in the virtual learning society.</p> <p>Taking care of the learning lifecycle and improvements for each learner.</p>
Knowledge Accumulation	Knowledge and teaching content in the <i>FAQ, Test, Literature, etc.</i> can dynamically grow-up along with the teaching-learning processes executed in the system. This growth relies on the co-development and the collective intelligence of both the instructors and learners.

### System Highlight:

Highlight	<ol style="list-style-type: none"> <li>1. Accomplishing a complete learning process within a purely web-based virtual teaching-learning society without any classroom background.</li> <li>2. Not only taking the advantage of being a 'long tail' web application, but also supporting 'users add value' i. e., building a learning environment in which both learning and teaching collaborate and stimulate each other.</li> <li>3. Establishing an online dynamic interactive graphical laboratory, which visualizes the highly sophisticated-mathematics-based stochastic procedures.</li> </ol>
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Highlight (continue)	<p>4. Seamlessly cooperating with Stochastikon Encyclopedia, Calculator and Mentor in order to achieve the pedagogical goal from different perspectives by different approaches.</p> <p>5. Multilingual (so far Magister has only two languages but it will be extended) and internationalized.</p>
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Table 2.6: Characteristics of *Stochastikon Magister* system.

## 2.4 Analysis and Comparison

Table 2.7 is a combination of two tables named ‘Available online resources per course’ displayed in [55] and [139] for analyzing “the interactive learning environments as distinctive features of online courseware”, which are extended here to also cover (in shadow) the above introduced four German educational statistics systems and *Stochastikon Magister*. The compared resources in Table 2.7 include the technologies used for the representation of teaching materials, online note taking facilities, integrated statistical software, course management systems for instructors, links to external internet sources, the use of electronic forums to support online discussions and group work, virtual labs, and the type of knowledge assessment questions.

Based on the information listed in Table 2.7, similarities and differences between educational statistics systems and *Stochastikon Magister* will be discussed from four aspects: range of use and reusability, technologies and resources, statistical software and virtual labs, and CMS, LMS and UMS.

### 2.4.1 Range of Use and Reusability

The situation of software development for statistical education at universities may be described by “self-produced – self-consumed”. This becomes obvious when looking at the statistics courses offered in the internet [55, 139]:

- Psychology 3000 Online is an online course developed at the University of Utah.
- Seeing Statistics is a web-book authored by Gary McClelland of University of Colorado at Boulder and published by Duxbury Press.
- SurfStat is an online course developed by Keith Dear, Rod Smith, Jonathon Coombes, and Robert Brennan at the University of Newcastle, Australia.
- Investigating Statistics is a course website developed by Robert Hale of Penn State University and supported by Wadsworth Corporation.
- Introductory Statistics is an electronic textbook authored by David Stockburger, Southwest Missouri State University, published and web-supported by Atomic Dog publishing.
- OLI statistics course is one of several courses included in the Online Learning Initiative authored by Carnegie Mellon University faculty and available for free in the internet.

	CyberStat	SurfStat	Psychology 3000 Online	Introductory Statistics	Seeing Statistics	Investigating Statistics	OLI	ActivStats (CD)	New Statistics	Statistics (CD)	MM *stat (CD/Web)	EMIL.eA -stat	Magister
Applets	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Videos						✓	✓	✓					
Statistical software	✓		✓			✓	✓	✓	✓		✓	✓	✓
Virtual labs			✓					✓	✓			✓	✓
Note-taking facilities	✓			✓			✓			✓	✓		
Course map	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Glossary	✓	✓		✓	✓	✓						✓	✓
Search engine													
Course management system	✓		✓				✓	✓					✓
Links to external sources	✓	✓				✓	✓			✓	✓		✓
Electronic forums	✓		✓						✓				✓
Multiple-choice questions	✓	✓	✓	✓			✓	✓	✓		✓	✓	✓
Short answer questions	✓		✓		✓	✓	✓	✓				✓	✓
Feedback	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Table 2.7: Available resources of Magister and some statistical e-learning systems.

The same happened in Germany. A lot of universities developed their own statistical software for supporting classroom teaching, such as Humboldt University Berlin, University Freiburg, University Konstanz, University Hamburg, University Saarland, University Munster, etc. Some universities developed more than one statistical system for different purposes like the Hagen University.

EMILeA-stat was set up by thirteen partners from seven German universities, under the direction of the RWTH Aachen<sup>4</sup>. New Statistics was developed by CeDiS in Free University Berlin in cooperation with thirteen academic institutes and ten German universities. Even for these two big government-funded cooperation projects, the users are mainly students of the project participants as reported by one of the major developers.

In [13] the authors pointed out three limitations of the MM\*Stat technology. Two of them are:

1. *The specific course orientation might be considered as a disadvantage, because there is little flexibility for the teachers to include other statistical topics.*
2. *The examples have mostly an economics (specific discipline) flavor.*

These limitations are characteristics of most university-produced statistical systems, which prevent them to be reused by others.

ActivStats is developed by Paul Velleman, a highly respected statistician and statistics educator, and released by Addison Wesley, Inc. and Data Description, Inc. CyberStats was developed by Alexander Kugushev, and authored by a team of twenty one faculty members of eighteen different colleges and universities, and offered online by CyberGnostics, Inc. These two systems are commercial products, thus their teaching contents are more general, their teaching styles are more flexible, and their customers are much broader than university-produced systems.

The customer profiles of ActivStats (<http://www.datddesk.com/company/profiles/>) list users from different fields: accounting, biotechnology, earth sciences, education, engineering, environmental sciences, healthcare, marketing and nonprofit organizations. Teachers at University of California, Utah State University, Washington State University, University of Pittsburgh, Australian National University, Gonzaga University, Humboldt State University, University of South-central Los Angeles, etc. report their teaching experiences with CyberStats. These two educational statistics systems seem to be well known in the U.S. and they are usually mentioned together and compared with each other. [1, 3, 139, 193, 218, 219, 221, 233]

It is clear that reusability of either the content or the software, is not carefully considered by most of the statistical systems. Only in [58] it is said that EMILeA-stat uses future standards like XML and MathML to ensure the compatibility of its huge comprehensive teaching content to future platforms. *Stochastikon Magister* teaches Bernoulli Stochastics and the educational statistics systems teach statistics, therefore the content reusability of the statistical courseware does not belong to the topic discussed here. The reusable software and technical skills of five statistical systems have been listed in their introducing tables (Table 2.1 to Table 2.5). One can see that most of the programs developed for these systems are tightly content-dependent, therefore the reusability of these programs are restricted to statistics, a specific branch of statistics or a special application field of statistics.

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<sup>4</sup>EMILeA-stat was initiated by Professor Uwe Kamps from University Oldenburg. Professor Kamps later changed to the RWTH Aachen.

In contrast, Magister software framework is not subject-specific but can be used for teaching any other scientific discipline, i. e., the course management system, learning management system and user management system of Magister framework are independent of the teaching content. Besides, the Graphical Lab used for teaching Bernoulli Stochastics procedures is also available for other mathematical or mathematics-based disciplines when it is connected with corresponding calculating engines.

In the following content, information technologies used for representing the courseware and for implementing the learning environment will be discussed and compared.

### 2.4.2 Commonly Used Technologies and Resources

As shown in Table 2.7, Magister contains most of the “resources” used in educational statistics systems for undergraduates.

Glossary, course map and search engine are common scaffolding in e-learning systems. Multiple-choice questions are usually used for exercises and tests. Applets with dynamic interactive graphical representations are powerful tools in statistical e-learning system for illustrating, explaining, practicing and distinguishing the abstract concepts and procedures. Videos used in educational statistics system are gradually replaced by Applet- or Flash-based animations.

The provision of a note-taking facility, though not belonging to a fashion resource among the educational statistics systems, is a very good idea for supporting learning. This resource is not used so far in Magister, but may be added in a future version.

Feedback in Table 2.7 refers to the automatic feedback from the programs such as judgments to the multiple-choice questions and to the short answer questions, or responses from the Applets to different input parameter values. Automatic feedback is the main interactive style used in statistical e-learning system. This holds especially for CD-Rom based programs. Human communications are undertaken by emails. Few of the listed systems have electronic forums. In [220] the author feels confused why it is “almost impossible to motivate students to participate in discussion via message board”. Actually, online discussions seem to be unnecessary when a real classroom exists.

Except Magister, only CyberStats supports the human instructors to give grades to their homework assignments and to correct the students’ electronic tests.

Magister has a sound human communication system, which simulates face-to-face supports and social interactions, and, thus, accounts for the fact that communication is an important part of the students’ learning process. The importance of human communication and support to an e-learning system will be discussed in the later section CMS, LMS and UMS.

### 2.4.3 Statistical Software and Virtual Labs

If the statistical software shall not be restricted to descriptive methods, but shall also include inductive statistics, it is necessary to be “tightly linked to a data analysis software package that can be used from inside the system for an immediate presentation of examples and interactive exercises, but also for analyzing one’s own data.” [218]

“The data analysis package integrated into ActivStats is the award winning Data Desk 6.1 (Data Description, Inc.). ActivStats is also available in versions that teach JMP, Minitab, SPSS, and Excel. CyberStats is equipped with Data Tools, a clone of the popular WebStat software. MM\*Stat incorporates the well known XploRe software.” [218] The OLI statistics

course provides three activity formats for Microsoft Excel, Minitab and R Package, respectively. Users need to have their own copy of at least one of the specified softwares installed on their own computers. The interface of the EMILeA-stat displays three selectable online statistical packages XploRe, SPSS and R for handling examples, but only XploRe is available. The New Statistics has a stand-alone program called Statistical Lab for exploring data, which is developed based on the R software package.

Both, statistics and Bernoulli Stochastics need a complex calculating engine for analyzing real data especially when the system includes a virtual laboratory to explain and handle real world examples and problems. To the CD-Room based system, the calculating engine is installed as part of the learning environment like in the case of the ActivStats. To the online learning environment especially when dynamic interactive graphical representations are offered, the size of the calculating engine becomes an obstacle. Java applet is the most commonly used technology to realize the online dynamic interactive graphical representation. Unfortunately it is impossible to integrate any of the calculating engines into an applet.

There are two ways to solve this problem. One is asking the users to install the calculating engine in their own computers like the OLI statistics course and the New Statistics. The other is separating the graphical representation from the calculating and plotting engine like the XploRe Quantlet technology used by MM\*Stat and EMILeA-stat. In *Stochastikon Magister* the second possibility is selected.

The first solution is easier to be implemented but the online system may lose some advantages of a pure web application. The first way imposes additional requirements and restrictions on the users' resources and configuration, and it implies inconvenience during the installation, usage and updating, etc. In this context, it is worth to be noted that pure web applications are the trend of current web-based (e-learning) program development.[47, 59, 69, 70, 118, 130, 150, 175]

The basic idea of the second solution is letting the calculating engine and plotting engine run on the server, implying that the graphical user interface needs only to accept parameters and display graphs. So far, only three solutions for this kind of technologies can be found in published articles.

1. XploRe is the name of a commercial statistics software, developed by the German software company MD\*Tech, one of the partners in the EMILeA-stat and MM\*Stat projects. XploRe is also the calculating and plotting engine of these two systems. However, XploRe is not sold anymore. The last version is available for free download.

Rönz et al. [192] describe the use of XploRe as follows:

*A special feature of MM\*STAT is its interaction capability. This allows the user to study statistical methods under varying application conditions or by using different data sets without any additional software overhead. To include the interactive examples, the XploRe Quantlet technology was used. This is a client/server concept which was especially developed for statistical computing over the internet. In this architecture the graphical user interface of the program, the XploRe Quantlet Client (XQC), is separated from the computational part, the XploRe Quantlet Server (XQS).*

*The XQC is a Java Applet and can therefore be integrated into HTML documents. On one hand its flexible configuration makes it possible to provide the inexperienced user with a menu driven software by allowing only certain changes of parameters, the selection of variables or data sets and executing the*

*statistical method. On the other hand it permits the experienced user to edit the program and to execute the modification.*

*The XQS is a statistical software environment which provides a variety of statistical methods as well a matrix oriented programming language. The communication between client and server is realized by a special protocol (MD\*Crypt), using TCP/IP which is available on each internet capable computer.*

In [13], Wolfgang Härdle points out that “A specific statistical engine is addressed for the interactive examples” which represents the third limitation of the MM\*Stat technology. The author does not explain the kind of limitation. From the related technical reports we guess that the limitation might be the incompatible between the XQS (written in C++) and the XQC (written in Java).

Symanzik and Vukasinovic [219] look at CyberStats and MM\*Stat from the costumer’s point of view, they note: “CyberStats and MM\*Stat lack a direct integration of the data analysis software. The accompanying software must be loaded from the server, which is very time demanding and might be disturbing for a student when the internet connection is slow.” Although the reason of the delay is a little bit doubtful as the loading of the accompanying software from server to client seems no necessary, but the result is clear that the online interactive graphical representation based on XploRe Quantlet technology is not very fast and sometimes instable. Besides, the applets of MM\*Stat (CD version) cannot run in the Firefox web browser.

2. The second method is described in [239] and [240] published in 2006, more than two years after *Stochastikon Graphics* was put in use:

*As part of a Maths, Stats and OR Network mini-project at the University of Southampton, a statistics e-learning website on the design and analysis of experiments is under development. The website has a particular focus on the application of the methods in chemistry research, although it is suitable for use in a variety of applied statistics courses.*

*The latest version of the website is available at <http://www.doe.soton.ac.uk/elearning>.*

*To embed output from the R system into the webpages requires a fast and secure interface between the webbrowser and R. This is achieved using the JavaServer Page (JSP) system, which allows Java code to be embedded within the HTML of a website. JSP allows the development of custom tags, which resemble HTML or XML tags. When the webpage is retrieved by a browser, the custom tag is replaced with the output of the Java code associated with the tag. This code is invoked on the server and so no Java software is required on the client machine.*

*A library of custom JSP tags has been developed to allow the embedding of R code within the website. Through tags, input boxes can be added to pages to allow values to be submitted to scripts, and the output from the scripts presented to the user. All R computations are done on the server, giving the advantage that the R software is not required on the client machine. For security reasons, users can only run the embedded R code and not upload their own.*

The following figure shows schematically the interface between the web browser and R in [239].

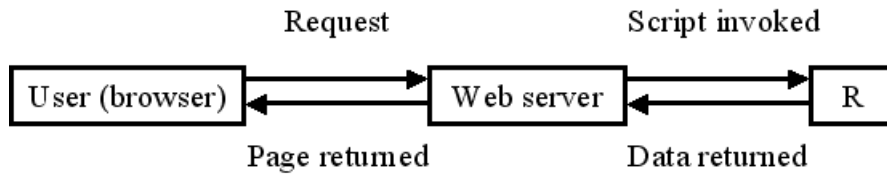


Figure: Schematic of the interface between the web browser and R.<sup>5</sup>

This method is simple and clever, but for representing each of the results graphically, all information within the whole page has to be redundantly transferred forth and back between the server and the client, and the updated JSP program has to be recompiled for running.

- The third technology is realized by *Stochastikon Graphics*, one application of which is the Graphical Lab for the *Stochastikon Magister*. The client/server idea of the *Stochastikon Graphics* is implemented by the normal Java Socket technology. Figure 16.18 is the schematic representation of the workflow of the Graphical Lab used in 16.3.3 ‘Realization of the Stochastic Graph Systems’.

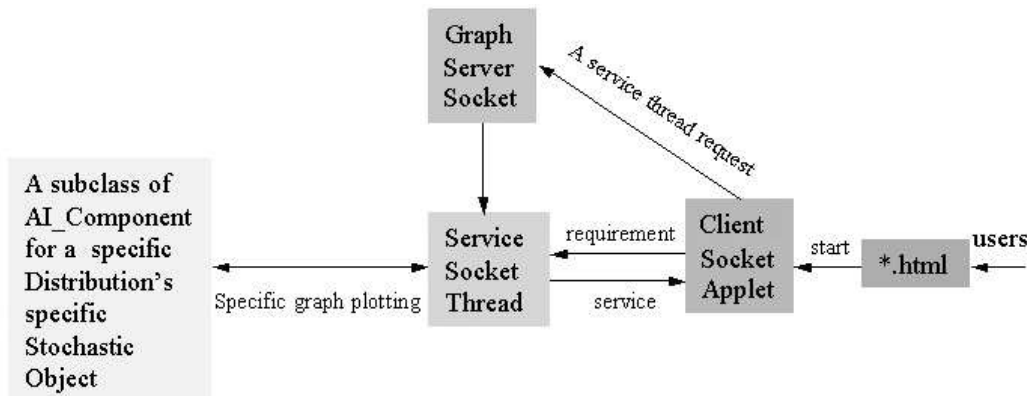


Figure 16.18: Schematic representation of the workflow of the Graphical Lab.

The idea of XQC/XQS used for XploRe Quantlet technology is similar to the technology used for the Magister Graphical Lab, but the implementing technologies are different. XploRe Quantlet technology is impossible to be used by the Graphical Lab because:

- the calculating engine XQS is a statistical package, which cannot solve the problems occurring in Bernoulli Stochastics,
- the XploRe Quantlet technology is designed to connect a C++ based calculating engine with the Java based graphical representation, but the situation for the Graphical Lab is completely different,
- the results of the XploRe Quantlet technology are not impeccable and (at least) not better than the Graphical Lab.

<sup>5</sup>Taken from [239].

Both the calculating engine - *Stochastikon Calculator* and the plotting engine - *Stochastikon Graphics* of the Graphical Lab are components of the *Stochastikon* system and implemented by Java. They harmonize with all other components, and do not lead to any limitation to *Magister* and the other components. And, the client/server technology implemented by *Stochastikon Graphics* has no web browser limitation.

Besides, the technical solution of *Magister Graphical Lab* restricts the information transmitted between the client and server to parameter values determined by the user and the resulting graphs created by the calculating and plotting engines. Therefore, no transmission redundancy or repeated compiling is necessary.

Table 2.8 provides a comparison between the award-winning Statistical Lab used in *New Statistics* and the Graphical Lab used in *Stochastikon Magister*. As these two programs serve for two different disciplines, the comparison refers mainly to the technical features and functional abilities.

The Statistical Lab belongs to the ninth category of statistical software - the integrated statistical system. (see Section 2.1) The major programming work for the stand-alone software Statistical Lab is to build a convenient GUI for using the calculating and plotting engine R software package. Because the mathematical problems in Bernoulli Stochastics are different from those in statistics, the statistical calculation packages cannot be used as calculating engine for Bernoulli Stochastics. Therefore, the development of the Graphical Lab in comparison with the Statistical Lab included the development of the calculating engine and the plotting engine. Moreover, the difficulty to implement a sophisticated plotting engine for dynamic interactive graphical representations is much larger for a pure web application than for a stand-alone software.

By comparing one can see that the range, load and difficulty of the software development work for the Graphical Lab and the plotting abilities of the Graphical Lab are much higher than those of the Statistical Lab.

#### 2.4.4 Course Management, Learning Management and User Management

Course management system (CMS), learning management system (LMS) and user management system (UMS) are not the focus of e-learning researches as most of the e-learning products are intended to be a supplement for classroom teaching. The same holds for studies of educational statistics systems. This situation is discussed in [134]:

*Zunächst stellt sich die Frage, welche Anforderungen ein Lernender an eine E-learning Anwendung hat. Hier sind wohl vor allem fünf Punkte zu nennen:*

1. *Einfache Bedienbarkeit durch Verwendung einer eingängigen grafischen Benutzeroberfläche,*
2. *Wiederholungsmöglichkeiten zur gezielten Aufarbeitung einzelner Passagen,*
3. *Lernfortschrittskontrolle,*
4. *Internes Informations- und/order Kommunikationssystem zur Kontaktaufnahme mit anderen Lernenden oder dem Lehrer,*
5. *Als eher technische Forderung: Kompatibilität mit verschiedenen Systemen. Hier geht es vor allem um die verschiedenen auf dem Markt üblichen Betriebssysteme oder (falls benötigt) Browsertypen.*



Name	Statistical Lab	Graphical Lab
<b>Developer</b>	CeDiS of Free University Berlin	Stochastikon GmbH
<b>Usage</b>	Statistical software intends to be an exploratory tool kit.	A graphical tool for visualizing and exploring Bernoulli Stochastic procedures.
<b>Release</b>	A part of the Interactive Statistics (the precursor of the New Statistics, 1998-2000) <i>Statistical Lab V 2.0.1.6</i> and <i>R-Engine V 1.4.1</i> , 04.11.2004 <i>Statistical Lab V 3.7</i> (22.01.2008) and <i>R-Engine V 2.0.1</i> (31.10.2005)	Stochastikon <i>Calculator</i> (11.2003) Stochastikon <i>Graphics</i> (02.2004)
<b>Running Mode</b>	Free downloadable standalone software <a href="http://www.statistiklabor.de/">http://www.statistiklabor.de/</a>	A pure web application
<b>Access</b>	Download, install, configure the computer, and update along with different publishing versions.	Available via Internet connection and will be automatically updated and extended.
<b>Interactivity</b>	real-time interactive	real-time interactive
<b>Software Dependency</b>	R software package as the core.	/
<b>Programming Work</b>	Developing a GUI shell for using R software package. R software package ( <a href="http://www.r-project.org/index.html">http://www.r-project.org/index.html</a> ) is both the calculating engine and the plotting engine.	Developing the whole software including: Calculating engine: <i>Stochastikon Calculator</i> , and Plotting engine: <i>Stochastikon Graphics</i>
<b>Language</b>	R (C, C++, Fortran), C++, C	Java
<b>Working Model</b>	input Data → use or build Model → get Conclusion	select Model → input Data → get Conclusion
<b>Data Collection</b>	by worksheet or data importer.	A GUI for determining the values of parameters.
<b>Data Analysis</b>	by R software package	by <i>Stochastikon Calculator</i>
<b>Data Interpretation</b>	A report editor, in which texts, graphs and objects can be implemented. The report can be save in *.RTF format.	In <i>Stochastikon Calculator</i> there exists an extendable report template bank. <i>Calculator</i> provides a PDF report for each calculation with texts, graphs (by <i>Graphics</i> ), procedures and interpretations.
<b>Visualization</b>	plain static graphs by R software package.	2- or 3- dimensional static graphs, and 2- or 3- dimensional dynamic interactive graphical representation by <i>Stochastikon Graphics</i>
<b>Tutorial Features</b>	The various objects can be linked using 'connector', which supports a step-by-step analysis of complex procedures and even improves comprehension of complex phenomena. Thus, the students can determine the logic and an individual order of processing every assignment. Creates assignments as sample solutions, incentives towards a solution and eventually the entire solution are provided on a step-by-step basis.	Corresponding functions in the <i>Stochastikon</i> system: <i>Stochastikon Magister</i> provides the possibility for systematic learning the principles and techniques of Bernoulli Stochastics. <i>Stochastikon Mentor</i> is designed to step-by-step support the formulation of the problem and the envisaged goal.
<b>Model</b>	Authentic models are supported by R software package. Offers interface to build new models by extending R.	All stochastic models will be built in <i>Calculator</i> , and users only need to formulate a problem and select a correct model.
<b>Award</b>	Medida Prix 2003	/

Table 2.8: Comparisons between the Statistical Lab used in New Statistics and the Graphical Lab used in *Stochastikon Magister*.

*Während die Punkte Bedienbarkeit (mit leichten Abstrichen bei “Statistik Interaktiv” bzw. “Neue Statik”, auf die wir noch eingehen werden), Wiederholungsmöglichkeiten und Kompatibilität bei allen hier besprochenen Programmen weitgehend erfüllt sind, sind vor allem die Möglichkeiten der Lernerfolgskontrolle bei einigen Anwendungen überhaupt nicht vorhanden.*

*Der Forderung nach einem internen Informations- und/oder Kommunikationssystem darf nicht unbedingt dahingehend verstanden werden, dass der/die Autor(en) des Programms erreichbar sein sollen, sondern eher so, dass die meisten Nutzer ihre Statistikausbildung im Rahmen einer Schulausbildung, eines Studiums oder einer Weiterbildung erhalten. Es gibt in diesen Fällen immer einen verantwortlichen Lehrer bzw. andere Lernende, die über ein entsprechendes internes System erreichbar sein sollen. Solche Systeme sind in den von uns untersuchten Lernplattformen aber bisher nur rudimentär integriert.*

In Table 2.7, four of the twelve listed educational statistics systems have an entry ‘Course management’. However, the meaning of the implemented course management system is unclear.

ActivStats provides functions to “adjusts the lesson order to match the two most common subject orders in statistics texts” and to maintain the users’ progress by recording activities as they are completed. “ActivStats has no built-in student tracking features.”<sup>6</sup>

Psychology 3000 keeps the students’ grades of their homework, playing games, practices in the Virtual Lab and exams. All the grades are given automatically by the programs.<sup>7</sup>

In OLI the course management system means that “the academic courses track student’s learning of key concepts and give the student and the instructor formative feedback to improve learning outcomes”, and its tutoring system “StatTutor will use scaffolding and immediate feedback flexibly, tracking and responding to individual students as they navigate the learning environment.”<sup>8</sup>

None of the four selected German systems Statistics, New Statistics, MM\*stat and EMILeA-stat provides functions for Learning Track, Teaching Track, Learning Organization and Learning Process Management, Learner/ User Management and Knowledge Accumulation.

Only CyberStats provides a kind of mixture of content management functions and learning management functions. Details are listed in Table 2.5.

Next, the management functions of CyberStats shall be compared with the learning-teaching-content management system in Magister:

1. CyberStats is designed to be used either in an electronic (Web-enhanced face-to-face) classroom or as a supplement for classroom teaching. CyberStats does not aim at supporting a complete learning process within a purely virtual environment, therefore its course management system therefore exhibits following features:

- There are no learning process management and no user management system.

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<sup>6</sup><http://www.datadesk.com/products/mediadx/activstats/>

<sup>7</sup><http://www.psych.utah.edu/stat/>

<sup>8</sup><http://oli.web.cmu.edu/openlearning/forstudents/freecourses/statistics>

- Online communication is rather limited and not recorded as part of the learning process.
- The implemented functions aim primarily at customizing the course for an individual instructor to teach a specific group of students. No function serves for organizing, supervising and supporting the students' learning process.
- The learning activities are not integrated as a whole, thus cannot be interactive with each other. That is why they cannot be systematically used by an instructor to guide, evaluate and supervise the processes of students' learning.

For example, in CyberStats if a student has questions to ask the instructor, he sends an email to the instructor and gets answers from the instructor. The information is not recorded so it cannot be used for evaluating the students' work or for developing new teaching content by considering the questions.

2. Statistics is taught in schools, universities and further educational institutions, thus references and learning supports of any kind can be found easily. Maybe this is the reason that instructors, who reported about the use of CyberStats paid little attention to the course management system. Only one of the instructors mentioned that the students like the electronic submission of homework assignments. He also talked about the electronic tests available in CyberStats. In brief, except the functions, which allow the instructor to customize his course by selecting some units within the provided units, the course management system in CyberStats is not really essential for its teacher customers.

Again we use the analysis in [134], which describes the desirable features that realizations of e-learning platforms for statistics should have:

*Eine entsprechende Umsetzung für den E-learning Bereich nimmt Salmon (2000), S.30-50 vor:*

1. *Zugang und Motivation: Diese Phase ist eher technisch zu sehen und umschreibt die Bereitstellung der entsprechenden Hard- und Software.*
2. *Online-Sozialisierung: Erste Bekanntschaft mit dem neuen Medium, sozusagen der Versuch sich dort "heimisch" zu fühlen.*

*Der eigentliche (zunehmende) Lernerfolg ist erst in den Stufen drei bis fünf zu erkennen:*

3. *Informationsaustausch*
4. *Wissenskonstruktion*
5. *Selbstorganisation.*

*Auffällig ist dabei wiederum, dass sehr viel Wert auf den Austausch mit anderen Lernenden gelegt wird.*

*Inwiefern der einzelne Autor einer Lernplattform im Fach Statistik seine eigenen Ideen in diesem (beispielhaften!) Konzept wieder findet, ist natürlich eine ganz andere Frage. Genauso ist für jeden Autor selber zu entscheiden, welche dieser doch sehr weit gehenden Schritte er überhaupt nachvollziehen will. Entscheidend dafür dürfte vor allem sein, in welchem Kontext eine Lernplattform entsteht. Natürlich sind die Anforderungen an einen kompletten virtuellen Studiengang andere als wenn E-learning als Ergänzung zu den Präsenzveranstaltungen in einem herkömmlichen Studiengang betrachtet wird.*

*Stochastikon Magister* aims at building a virtual classroom, which accomplishes the whole teaching-learning process in a pure online environment. Therefore the two missing parts in most of the educational statistics systems, i.e., monitoring of the learning progress and the internal information and/or communication system for communicating with other learners or teachers are necessary elements of *Magister*. In order to reach this ambitious aim, the course management system, learning management system and user management system have been designed, implemented and integrated to build a complete effective learning environment according to the pedagogical theories. It follows that with respect to CMS, LMS and UMS, none of the above discussed educational statistics systems is comparable with *Stochastikon Magister*.

# Chapter 3

## Assessment of Educational Statistics Systems

### 3.1 Types, Purposes and Techniques

There are many reasons for wanting to evaluate e-learning (system), and these are reflected in different types of evaluation that are used, i. e., formative evaluation, summative (experimental) evaluation, illuminative evaluation, integrative evaluation, evaluation for quality assurance (auditive evaluation), etc. [173]

Another way of considering evaluation, probably in a more useful manner, is to look at the purpose of evaluation. “Harland [106] argues that there are three purposes for evaluation [88]. They are evaluation for action (decision making), evaluation for understanding (enlightenment) and evaluation for control. While it is clear that any evaluation may have elements of all three, what we are concerned with is the emphasis given to each in the evaluation” [90]. The JISC<sup>1</sup> Committee for Awareness, Liaison and Training (JCALT) has issued an ‘Evaluation Toolkit for Practitioners’ for evaluation e-learning materials, which contains the list of possible methods displayed in Table 3.1.

Techniques are used to collect and analyse data for different types or purposes of evaluation. “...once data are gathered, analysis requirements are assessed in terms of the need for textual, visual or numerical representation, subjective or objective analysis and whether all or some of the data will be presented. Finally, recommendations on means of presenting the data to audiences are made in terms of the speed, level of detail and tone of the report.” [54] Table 3.1 is a summary of the data collection, analysis and representation techniques for e-learning (system) evaluation by JISC<sup>2</sup>. [90]

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<sup>1</sup>Joint Information Systems Committee

<sup>2</sup><http://www.ltss.bris.ac.uk/jcalt/>

Data collection phase	<p>Methods</p> <p>System log data</p> <p>Checklists</p> <p>Cognitive walk through/think aloud protocols</p> <p>Concept maps</p> <p>Confidence logs</p> <p>Controlled experiment</p> <p>Cost-benefit</p> <p>Cost-effectiveness</p> <p>Designing experiments</p> <p>pre- and post testing</p> <p>Focus Groups</p> <p>In-course experiment</p> <p>Nominal group techniques</p> <p>Observation</p> <p>Performance test</p> <p>Questionnaires</p> <p>Reflective logs/studentdiaries</p> <p>Resource Questionnaires</p> <p>Split screen video</p> <p>Structured interviews</p> <p>Trials</p> <p>Unstructured interviews</p> <p>Video log</p>
Data analysis phase	<p>Bar or pie charts</p> <p>Categories of quotes</p> <p>Correlation</p> <p>Designing experiments</p> <p>pre- and post testing</p> <p>Emergent themes</p> <p>Factor analysis</p> <p>Illustrative quotes</p> <p>Narrative case study</p> <p>Percentages</p> <p>Performance test</p> <p>Pre-coded categories</p>
Presentation methods	<p>Conference presentation</p> <p>Email</p> <p>Executive summary/press release</p> <p>Journal article</p> <p>Leaflet</p> <p>Poster</p> <p>Presentation</p> <p>Report</p> <p>Web site</p> <p>Word of mouth</p> <p>Workshop</p>

Table 3.1: Techniques of evaluation described in the JCALT Evaluation Toolkit.

## 3.2 Empirical Evaluation of Educational Statistics System

The search engines Google and Google Scholar show a considerable literature dealing with the evaluation of e-learning and e-learning products.

Some articles propose an evaluation process for (the critical points i.e., different phases or different aspects of) the whole development life-cycle of the e-learning (system) [16, 90, 177], some provide quantifiable quality metrics by adaptation and adoption of ISO<sup>3</sup> or other standards for evaluating the usability or quality of e-learning (system) [10, 97, 176, 213], some center the e-learning (outcomes) on instructional design (ID) [63, 120], some apply the evaluation on the base of the Technology Acceptance Model (TAM), IS (Information and System Quality) Success Model, Iterative Evaluation Model (IEM), etc. or model combinations [174, 178, 179], some judge the effectiveness of an e-learning system by comprising students' test results [3, 99], etc.

With regard to the systems listed in Table 2.7, especially to system MM\*stat, EMILeA-stat, Statistics, New Statistics and CyberStats, there are only two papers concerning quantitative evaluation of the usability and learnability written by J. Richard Alldredge and his colleagues. They use questionnaires and examination scores to compare the usability and learnability of ActivStats and CyberStats [3], and explore the relationship between gender and the instructional software environment [4]. According to private communication with Professor Uwe Kamps (EMILeA-stat) and Professor Hans-Joachim Mittag (New Statistics), there are no published evaluations of EMILeA-stat and New Statistics. Thus, it seems that the big German e-learning systems in statistics have not been thoroughly evaluated so far.

Many articles deal with the empirical assessment of educational statistics system [3, 4, 136, 174, 179]. Empirical means that samples are drawn and data are collected which subsequently are analyzed in order to assess the quality of the e-learning system.

## 3.3 Empirical Evaluation of Stochastikon Magister Programme

The e-learning system *Stochastikon Magister* refers to a new branch based on new concepts, new models and new methods, for which there are no traditional classroom teaching. However, because Bernoulli Stochastics aims to replace in some sense statistics, the difficulty that there is no classroom teaching of Bernoulli Stochastics is overcome by taking a traditional statistics course for comparison. This has the advantage that not only Magister is evaluated, but in some sense also Bernoulli Stochastics.

The evaluation of the Magister e-learning programme is performed by means of two very different empirical evaluations. One is based on the subjective outcome of a questionnaire, while the other uses the more objective results of final examinations. The purpose of the evaluation is to assess the usability and learnability of the Magister programme. The analysis is performed by using tools provided by Bernoulli Stochastics illustrating the distinct difference between statistics and Bernoulli Stochastics.

The two evaluation of Magister programme will be presented in Part IV: 'Empirical Evaluation of *Stochastikon Magister* Programme'.

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<sup>3</sup>International Organization for Standardization.

## 3.4 Conclusion

The subject matter (viewpoints, concepts, models, methods, problems, etc) in statistics and in Bernoulli Stochastics is different. Therefore, the technologies used to support the development and application of the subject matter, for instance the software packages, which are usually integrated as calculating and plotting engines within the educational statistics systems, are different, and therefore had to be newly developed for Stochastikon project. The implementation of Magister Graphical Lab not only successfully achieves the goal to establish a virtual graphical laboratory for educating Bernoulli Stochastics but it also overcomes a technical bottleneck, which all the advanced online statistical software face.

The development stage and the educational situation of statistics and of Bernoulli Stochastics are different, and shall therefore briefly outlined below.

1. The role of e-learning products in the education of statistics and Bernoulli Stochastics is different, and therefore the requirements for the e-learning system are also different. In the case of statistics, online products are primarily used as a supplement to traditional classroom teaching, while in the case of Bernoulli Stochastics, Magister is expected to fill the gap of non-existing classroom teaching. For this aim Magister contains a complete learning management system, user management system and course management system, and emphasis is laid on the learning process management and the simulation of face-to-face support, of social communication, and of an educational community. For this aim the system design and software development of Magister is different in structure and functions in comparison with most of the commonly used educational statistics systems.
2. The running context<sup>4</sup> of e-learning programs in statistics and in Bernoulli Stochastics is different. Teaching statistics requires e-learning tools like stand-alone courses, datasets and applets. These tools are combined with a standard teaching content and aim at enhancing the understanding of statistical concepts by highlighting them from different aspects. *Stochastikon Magister* is part of the super ordinate software package *Stochastikon* and thus depends on the entire system. Without *Stochastikon* framework and the tight cooperation among its components, an e-learning program in Bernoulli Stochastics cannot be powerful enough to achieve its pedagogical goal, i. e., to educate the new, unfamiliar and sophisticated mathematics-based discipline via internet.
3. The foundational difference in principles between the newly developed Bernoulli Stochastics and the traditional statistics determines that all teaching materials and course ware used in Magister have to be newly developed, continuously improved and gradually accumulated.

In order to illustrate the above indicated differences between the teaching contents of statistics and Bernoulli Stochastics, the latter are briefly outlined in the following chapter.

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<sup>4</sup>The entry point in the application and the data bases to use.



**Part II**

**Brief Introduction to Bernoulli  
Stochastics**



# Chapter 4

## Uncertainty and Predictions

### 4.1 Making Decisions

Starting even before birth everybody makes permanently consciously and unconsciously decisions. Any physical movement assumes numerous unconscious decisions which and how muscles and other parts of the body are to be deployed. A baby having no or only little experience is not able to foresee the consequences of the unconscious decisions and, therefore, fails in making proper decisions. With growing experience or knowledge, the results of decisions can be better foreseen and, thus, decisions become gradually better. Therefore, we may conclude that the ability of making appropriate decisions depends on the capability to foresee the corresponding future consequences. In other words, each decision is followed by an action producing a new situation. What is needed is to know the relation between the newly produced situation and the subsequent future development.

This holds for everyday decisions like “whether I should take an umbrella along with me today” or other more important ones. The higher the position of the decision-maker in the social hierarchy the more important decisions he has to make which may affect directly or indirectly a larger number of persons. The capability of making reliable and accurate predictions gets more and more important with the increasing impact of a decision.

The problem of making a good decision is heavily dependent on making a good prediction, and to make a good prediction becomes a problem, because of the inherent uncertainty of any future development. Therefore, the ability of making good decisions refers directly to the ability of handling ‘uncertainty’, which is an omnipresent aspect of real world.

### 4.2 Handling Uncertainty

Whether or not a decision is appropriate depends on the envisaged aim and how the prevailing uncertainty is taken into account. The latter is often determined by rather subjective issues of the decision maker like for instance preference, bravery and fear. But these individual issues are generally misleading. The most promising support for improving the decision-making process is to enhance the ability to handle uncertainty, which means to develop objective methods for describing uncertainty in obtaining reliable and accurate ‘predictions’ or ‘forecasts’ and, in fact, mankind has arrived at this conclusion from its very beginning.

The demand for predictions reminds of the eight trigrams or 64 hexagrams of the Chinese canonical book I Ching, landscape geomancy, divination slips, astrology, crystal ball, fortuneteller, sorcerer, Gypsy, and many other well-known and still used methods. There are controversial attitudes towards these methods: worship, sneer, superstition, etc.

In [48] we read: “The written history of mankind starts with the Chinese canonical book I Ching, in which random experiments are described used for overcoming uncertainty about past and future events.” After thousands of years the I Ching is still the overwhelming tool for ‘deriving’ statements about private future and fate in Asia. Similar ‘tools’ exist all over the world, mystical, inconceivable, old and forever lasting and proving that mankind has confided its fortune to methods of prediction. One characteristic property of these time-honored methods is their independence of human subjectivity. This independence is warranted e. g., by consulting celestial bodies in the sky or random experiments. The underlying idea for consulting stars and random experiments have been the perception of universal connectivity. Originally, the use of the methods were reserved to wise women and men. But this changed. Again in [48] it is said: “It is not surprising that it did not take long that random experiments like throwing bones, drawing cards or lots were not anymore exclusively used for divination, but on a more playful side, for diversion. They became known and popular as games of chance and some millenniums later in the 17th century the mathematical theory of games of chance was launched by the famous correspondence between Blaise Pascal and Pierre de Fermat.” Especially over the last 25 years, the mathematical theory of games is used extensively in economics, as illustrated for instance by the Nobel Prize in Economics for 2005<sup>1</sup>, and in many areas of social sciences.

Moreover, some sort of model-based predictions founded on factor or sample analysis are frequently used in medical and different natural sciences in order to support the decision making process. These methods are also popular in the area of prevention of disasters, diseases, and pollution. Besides, they are in an ever increasing extent used in the areas of social studies, management, business, manufacturing, and some microeconomic areas like stock, fund, bonds, interest rate, etc. Thus the great importance of predictions not only in the small frame of individuals and families but also in a larger frame of organizations, states and societies is obvious. In war times, a decision based on a reliable and accurate prediction can save thousands of lives or a whole country.

Modern science and technology have changed the earth to a ‘village’, the international situations are intricate, fast changing and increasingly nested. Scientific predictions, which can help to control the more or less blind expansion, to seize chances and to avoid risks, would be of immense importance for a safer construction and development of nations and societies. The only problem to overcome is uncertainty about future developments which materialize in variability, i. e., different things can occur in future. Large uncertainty means large variability and small uncertainty means small variability. Thus, studying uncertainty is tantamount to study the properties of variability.

## 4.3 Types of Prediction

There are many possibilities to classify predictions. One possibility is to distinguish between “qualitative” and “quantitative” predictions.

### 4.3.1 Qualitative Prediction

Qualitative predictions are based on methods which do not use mathematical concepts as main tools. They ‘deduce’ the results according to some rules which often are secret and handed over from master to disciple for generations. The predictions are usually made verbally but sometimes especially in their modern forms also numerically.

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<sup>1</sup>The prize was awarded to Robert J. Aumann, and Thomas C. Schelling “for having enhanced our understanding of conflict and cooperation through game-theory analysis”.

Qualitative methods, used by fortunetellers are sometimes looked upon as profound, mysterious, idealistic and supersensible. They exist for ages and nobody is able to explain how and why these methods can, to some extent, predict the future. Hence, they are attributed to secret and supernatural doctrines. It is believed that these methods can be applied successfully only by persons having special talents, which let them feel consciously the universal connectivity and enable them to anticipate the future evolution. Consequently any failure is attributed to missing talents, which is considered as fraud.

The ancient qualitative prediction methods yielding non-numerical results have proved to be rather flexible, as the predicted events comprise many possible outcomes of the future development in accordance with the given starting situation and, therefore, guarantee a certain reliability.

Nowadays, reports about national and international economics are often based on qualitative predictions made by the United Nations Organization (UNO), national agencies, big consultant companies and policy research organizations. Although, these predictions are generally given numerically, the applied methods are mainly qualitative.

More recently, the word ‘prediction’ appears frequently in reports for a country’s future economic development - GDP. These predictions are basically of qualitative nature although they are announced in numbers and supported by certain mathematical formulas and powerful computer systems. However, from experience of many years, it is known that the predicted value will hardly ever occur. Therefore, these pseudo-quantitative prediction methods constitute rather a step backwards than a step forward when compared with the ancient qualitative prediction methods. What is needed are methods which yield predictions which actually occur as a necessary condition of making an appropriate decision.

### 4.3.2 Quantitative Prediction

Quantitative prediction methods must be based on a mathematical model describing quantitatively the starting situation and the transition into the future terminal situation. By inserting the numerical values reflecting the starting situation, the future terminal situation can be determined yielding the desired prediction.

Probably due to the scientific and technological revolution mankind has firmly accepted the idea that anything develops according to strict (deterministic) laws. It is believed that once these laws are discovered, uncertainty about the future development would disappear. Deterministic laws are based on cause-effect chains, where each chain-link represents an isolated system. This perception leads to rely on deductions assuming the existence of isolated systems, which is totally contradictory to the ancient view of universal connectivity. The ancient idea of one nested universe is replaced by a set of chains of isolated systems developing in time according to strictly deterministic laws. Actually, this approach has been successfully applied in some areas of physics and astronomy. However, in almost any other area of human activities the assumption of a deterministic universe yields more or less useless results.

Based on the assumption of cause-effect chains, the future development is mathematically described by functions, i. e., one-to-one relations between past and future. In order to discover the functions, which are often called ‘Laws of Nature’, samples are distinguished and numerical models are generated by data analysis or statistical methods. These functional models are seemingly objective and rational when compared with the obscure ancient qualitative methods. The data are obtained by controlled experiments and the formulas are derived by careful and sophisticated mathematical deductions. The numerical results are unambiguous compared

with any verbal prediction which generally admits many different interpretations and possible outcomes. Therefore, the quantitative models are considered as a revolutionary breakthrough in science. However, the limitations of these methods became clear latest when investigating radioactive decay, where any prediction based on a deterministic model is meaningless.

Obviously, the problem refers to the question how to describe uncertainty in a given real world situation sufficiently well that the model can be used to predict in a reliable and accurate way the future development. Any model constitutes a simplified image. Simplification refers to the number of aspects and relations included in the model. The causality perception leads to considering isolated systems which leads to oversimplification as it does not allow to take into consideration of the inherent variability exhibited by reality. The connectivity perception on the other hand demands strictly speaking to include infinite many aspects and relation into the model, which is simply impossible. The only way out of this dilemma is to identify the 'key' factors, which have to be included necessarily into the model for obtaining meaningful predictions and neglect all those factors which have only an marginal effect on the future development of interest.

Secondly, studies in any specific science focus on a rather concrete and small part of real world. Each factor related to the development of a process has its own developing laws, but definitely it is not possible to predict the future development of the whole object by simply 'piling up' these particular laws. From a macroscopic view, the synthetic result of a large quantity of factors with rather different laws may be described by very few and simple laws. Macroscopic laws are generally not the same as any of the microscopic laws.

Thirdly, any model has to be built on knowledge. If no or not sufficient knowledge about an object is available, a meaningful description of the object is impossible and, clearly, the more complex an object is, the more knowledge is necessary for describing it. The problem with knowledge is that it is complete only in rare cases, generally only partial knowledge is available. Hence, the question arises how to deal with incomplete knowledge. For example, when according to the available knowledge, a quantity may adopt a value between 0.3 and 0.4, then for constructing a model, often the mean value 0.35 is selected. However, the small miss may be as bad as thousand miles.

Finally, consider a prediction made by using a deterministic 'Law of Nature'. The prediction consists of one single value, obtained by inserting the 'known' starting conditions into the 'law' and calculating the value of the future terminal situation. It is widely known that generally the only founded statement which can be made about a prediction consisting of one point is that it will not occur almost with certainty. Evidently, this is the worst judgment about a prediction one can think of.

The last two points illustrate that deterministic models, which are based on a functional relationship between past and future do not include the aspect of uncertainty. Therefore, they are not suitable to deal or describe uncertainty. Of course, the shortcomings of traditional deterministic prediction models were recognized and theoretical models were proposed, which allow for a certain variability. Models founded on samples reflecting to some extent the inherent variability of a given situations are used in many areas replacing purely deterministic descriptions. However, a majority of methods used in these areas do not aim at investigating variability, but to reduce or even to eliminate variability. Therefore, their contribution to handle uncertainty for making appropriate predictions is limited.

Many so-called statistical models are basically deterministic with an additive or multiplicative term called 'error' which provides the desired variability. The assumed properties of the resulting variability are not case-related but often derived from a purely theoretical 'universal error

law'. Proceeding like this makes a thorough investigation of the characteristic variability, which is the manifestation of uncertainty, impossible. Therefore, statistical models are in general not suitable for describing uncertainty, similar as deterministic models.

To sum up, it can be stated that neither the deterministic nor the statistical approach is appropriate for handling uncertainty in order to arrive at reliable and accurate predictions. The deterministic approach denies completely uncertainty, and the statistical approach attempts to eliminate its characteristic feature, i. e., variability. Hence for the quantification of uncertainty a novel approach has to be developed.

## 4.4 Bernoulli Stochastics

More than 300 years ago, Jakob Bernoulli a Swiss theologian and mathematician began the study of scientific prediction. Starting from the mathematical analysis of games of chance, which did not include predictions, but was more or less restricted to the calculation of the expected profit of games of chance in order to define 'fair games', he realized the necessity and the possibility to quantify uncertainty. To this end he inaugurated the "science of prediction", which he named "Ars conjectandi" in Latin or "Stochastice" in Greek. In a German encyclopedia of the 18th century [244], we read:

*Stochastice = Art to Conjecture, Ars conjectandi, is a science to determine the probability of something. For example, who has more hope to win in a game than the other; the extent of hope to hit a target, which is set at a given distance; how much one should arrange for the proceeding of a project, and similar things more. This art has not been investigated so far. Jakob Bernoulli has started to build up the theory, however, it is to be deplored that he could not complete it, as there is no application in moral and politics in his Ars conjectandi, which was published by his brother's son Nicolaus Bernoulli after his death, instead there are only examples of various games which were already stated by Pascal and Fermat in France. Huygens was the first to bring forward in a clear and cumbersome way these elementary teachings which were published by Frans van Schooten with his permission in his book "Exercitationibus Mathematicis", and which Bernoulli has again published with scholarly annotations instead of an introduction. Rémond de Monmort's 'Analyse sur les jeux de hasard' being increased in another edition also belongs to this area.*

Probably, the greatest achievement of Jakob Bernoulli is the introduction of the concept of probability. Bernoulli recognized that in a given situation each future event is marked by a specified degree of certainty of occurrence. He called this degree 'probability' of the corresponding event, and considered the probability as a property of an event, just as the temperature is a property of an object. Moreover, he quantified it by assigning the value 1 to an event which will occur with certainty, and the value 0 to an impossible event.

However, Jakob Bernoulli passed away 1705, leaving an uncompleted manuscript on Stochastics. His widow, knowing of the epochal importance of the manuscript and fearing that someone would appropriate Jakob's ideas, tried to keep it back. Finally, it was published by Jakob's nephew Niklaus Bernoulli in 1713 under the programmatic title "Ars conjectandi." Unfortunately, Jakob Bernoulli's Stochastics did not take over the role it was designed for. Particularly, the concept of probability as an inherent property of future events, which constitutes the first but decisive step for quantifying uncertainty, was dismissed in favor of several contradictory "definitions".

In [49] it is stated:

*“In place of Stochastics, the science of prediction, came two other disciplines to seemingly fill the gap. These disciplines were probability theory and statistics. The word ‘Stochastics’ survived in the term ‘stochastic processes’, which is used to name a special branch of probability theory. Nowadays, the word ‘Stochastics’ is experiencing a revival, as it is used by an ever-increasing number of publishing companies and academics as an umbrella term for probability theory and statistics. This development is unfortunate, as it adds to existing confusion about the nature, aims and objectives of probability theory, of mathematical statistics and of statistics. Worse still, it severs the link between the term and Bernoulli’s intentions when he originally coined it, thereby reducing the prospect of reviving and advancing research in the area.”*

and:

*“... mathematics cannot replace Stochastics similar as it cannot replace physics and it appears to be imperative to revive Jakob Bernoulli’s Stochastics in view of the ever increasing need for a scientific treatment of the real aspect uncertainty. Therefore, ... define the Science of Stochastics as an independent, exact science, which investigates the inherent variability exhibited by any real phenomenon.”*

In [49] a paradigm change is proposed in science and in our perception of the world and the way of thinking. ‘Variability’ as the manifestation of uncertainty is set in the center of research, science and thinking. In [49] it is explained:

*“Bernoulli had identified the omnipresent ‘uncertainty’ as a real-world aspect, causing serious problems for mankind. When he succeeded in showing that ‘uncertainty’ could be quantified, just as physical quantities such as weight, temperature or strength can be quantified, he was convinced that this new branch of science could and would change the world for the better.”*

Thus, Bernoulli Stochastics was born as a quantitative science, which “aims at investigating and quantifying uncertainty, i. e., variability, in order to make reliable and accurate predictions in real world situations.” [49]

As an independent part of science, Bernoulli Stochastics provides a robust and complete methodological foundation and a unified approach to investigate the elements of uncertainty which become visible by variability exhibited by any real world process. Modeling and investigating variability instead of ‘stable laws’ yields reliable and accurate predictions and overcomes the problems of traditionally used quantitative prediction methods outlined above.

The main difference between statistical models and stochastic models is that the latter include explicitly all sources of uncertainty and build a realistic quantitative image of the transition from past to future. Uncertainty creates variability which is looked at in Bernoulli Stochastics from a macroscopic, holistic and synthetic view rather than from a microscopic view which might cause simplification problems.

Bernoulli Stochastics keeps the advantages of the quantitative approach, which are founded in using mathematics, and at the same time absorbs and combines nutrition from the qualitative prediction approach by adopting the connectivity perception of world. Thus, it acknowledges the scientific nature of the ancient Chinese I Ching and shows that it is not based on mysterious doctrines but on an advanced understanding of real world. By considering uncertainty as an inherent aspect of real world, Bernoulli Stochastics becomes as flexible as the qualitative prediction approach. However, Bernoulli Stochastics enables to accurately transform any available qualitative knowledge into a mathematical model.

Finally, despite its high mathematical and numerical complexity, Bernoulli Stochastics is simple, comprehensible and operable, at least, if the corresponding technical devices are made



available. In the following chapter, the basic ideas of Stochastics are briefly reviewed following the pioneering works [49] and [50] contained in the book ‘Defining the Science of Stochastics’, which was published in 2004.

# Chapter 5

## Modelling Uncertainty

The first question which must be answered refers to the aspects which have to be included necessarily into a mathematical model which shall describe the transition from past to future. As explained above, the answer depends on the perception of real world. There are two contrary perceptions, the deterministic one which assumes that everything is predetermined and any development can be represented by a cause-effect chain and the stochastic one which is based on the realization that everything is connected with everything over time and space, which excludes the existence of isolated cause-effect chains. Without wanting to start a philosophical discussion here, it is to be noted that there are overwhelming hints starting with the classical law of gravitation which back the stochastic view and there are sound arguments such as radioactivity that the deterministic view does not hold.

Any transition from past to future begins with a starting situation and ends with a future, terminal situation. The starting situation is fixed by the initial conditions which are facts determined by a past development. In contrast, the future terminal situation is not determined so far and, therefore, indeterminate. The task is to mathematically describe the intrinsic relations between the determinate initial state and the indeterminate future state.

The fixed initial conditions may lead to different terminal states. However, randomness as manifestation of universal connectivity prevents chaos which would allow everything. Randomness associates past and future by probabilities. The probability of occurrence of any final state depends on the degree of its conformance with the initial conditions. Those with higher degree of conformance are more likely to occur than those with smaller degree. And those which are in contradiction to the initial conditions cannot occur at all. Thus, randomness does not allow chaos, but creates and maintains order in the world by admitting only a finite number of events (= extent of variability) and assigning different probabilities to each of the possible events (= random structure).

As to the initial conditions which are represented by facts, they are only in rare cases completely known. Generally, the limitation of human perception allows only for partial knowledge, i. e., in a given situation, one can only specify a set of potential initial conditions containing the actual initial conditions. The size of this set reflects the available knowledge or equivalently the existing ignorance. A large size means large ignorance or little knowledge, and a small set means small ignorance and much knowledge. Note that the range of what had happened in the past as well as the range what will happen in the future is always finite. In order to get a good model one should use the entire, available knowledge to make these sets as small as possible.

Below the set of potential initial conditions (ignorance), the set of future outcomes (size of variability) and the corresponding probabilities (random structure) are illustrated.

In Figure 5.1, the knowledge about the past is represented on the left hand side by a set of four potential initial conditions each represented by a circle. The actual but unknown initial condition is given by the black circle. The future is represented by the set of six possible outcomes which are the only outcomes compatible with the true initial condition. The degrees of conformance between the initial conditions and compatible outcomes are given by the corresponding probabilities represented by the horizontal bars set up in each of the future outcomes.

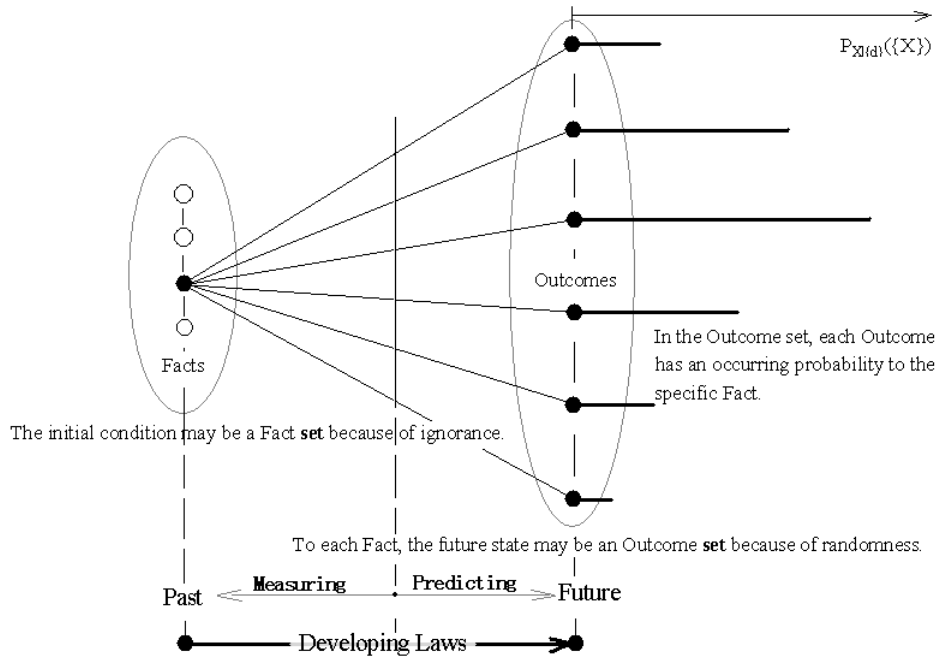


Figure 5.1: A graphical illustration of a transition from past to future.

A prediction refers to a transition from past to future. Therefore, any mathematical model used for generating predictions should include the elements illustrated in Figure 5.1. These are the set of potential initial conditions, the set of possible future outcomes and the random structure over the set of possible outcomes. Once these aspects have been described sufficiently well, procedures for making reliable predictions can be derived to be used as basis of the decision making process.

## 5.1 The Mathematical Model

In the preceding section the elements have been described verbally and graphically, which are needed for developing a mathematical model. A mathematical model assumes quantification of the relevant characteristics, which, therefore, have to be identified.

The first step consists of identifying the aspect of interest of the future, final state, which is represented by a variable denoted by  $X$  and called 'random variable'. Quantification means to assign a real-valued range of variability to  $X$ .

However, as illustrated in Figure 5.1, the set of possible future outcomes may be different for different initial conditions. Thus, the second step consists of identifying those aspects of the starting state which are relevant for  $X$ . Again quantification means to represent these aspects by a variable and assigning a suitable range of variability to it. The variable which describes the relevant past facts is denoted  $D$  and called 'deterministic variable'.

The transition of interest refers to the pair of variables  $(X, D)$ , and a mathematical description of the situation means to mirror the relations between the determinate past represented by  $D$  and the indeterminate future represented by  $X$ .

### 5.1.1 Modelling Ignorance

Knowledge refers necessarily to facts, i. e., to the deterministic variable  $D$  representing the initial conditions. If the true value say  $d_0$  of  $D$  is known then the range of variability of  $D$  is given by the singleton  $\{d_0\}$ . However, in general the initial conditions are not known exactly. In this case it is only possible to specify a set of potential initial conditions which include the unknown true one. This set, which will serve as range of variability for  $D$  is called ‘ignorance space’ denoted by  $\mathcal{D}$ .

$$\mathcal{D} = \text{set of potential values of } D \quad (5.1)$$

The determinate past represented by the variable  $D$  and the indeterminate future represented by  $X$  are related to each other and the ultimate aim is to describe this relation mathematically. Of course, the description depends on the state of available knowledge or equivalently on the existing ignorance. In order to describe different states of ignorance, a suitable system of subsets of  $\mathcal{D}$  denoted by  $\mathcal{T}_D(\mathcal{D})$  is introduced <sup>1</sup>.

$$\mathcal{T}_D(\mathcal{D}) = \text{suitable system of subsets of } \mathcal{D} \quad (5.2)$$

Each element of  $\mathcal{T}_D(\mathcal{D})$  stands for a certain state of knowledge which determines the nature of the system. In particular  $\mathcal{T}_D(\mathcal{D})$  contains often the singletons of  $\mathcal{D}$  as elements.

Let  $\mathcal{D}_0 \in \mathcal{T}_D(\mathcal{D})$  be a specified state of ignorance, then the random variable  $X|\mathcal{D}_0$  is introduced:

$$\begin{aligned} X|\mathcal{D}_0 &= \text{random variable } X \text{ for given} \\ &\text{state of ignorance } \mathcal{D}_0 \end{aligned} \quad (5.3)$$

In particular we have

$$\begin{aligned} X|\{d\} &= \text{random variable } X \text{ assuming complete knowledge} \\ &\text{about the true initial conditions} \end{aligned} \quad (5.4)$$

The representation of the deterministic variable  $D$  is by no means unique. In fact any bijective transformation of  $D$  yields a new representation with different factors. Approaching the problem of describing the initial conditions from a given special branch of science like physics or economics results almost necessarily in a microscopic representation with a multitude of factors. In contrast, the stochastic approach offers a macroscopic representation with only a few relevant key factors describing the random structure.

### 5.1.2 Modelling the Extent of Variability

Each potential initial condition  $d \in \mathcal{D}$  determines the set of compatible future outcomes of  $X$ , which describes the extent of future variability and, thus, has to be taken into account. Clearly, the larger the ignorance about the true initial condition, the larger is the variability of  $X$  to be considered. In order to describe the extent of variability of  $X$  for different states

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<sup>1</sup>Note that the system of subsets  $\mathcal{T}_D(\mathcal{D})$  does not need to meet mathematical properties with respect to certain set-theoretic operations.

of ignorance, given by the elements of  $\mathcal{T}_D(\mathcal{D})$ , the variability function  $\mathcal{X}$  is introduced which assigns to each state of ignorance a corresponding range of variability. Let  $X$  be a  $s$ -dimensional random variable, then

$$\mathcal{X} : \mathcal{T}_D(\mathcal{D}) \rightarrow \mathcal{T}_X(\mathbb{R}^s) \quad (5.5)$$

$$\mathcal{X}(\mathcal{D}) = \cup_{d \in \mathcal{D}} \mathcal{X}(\{d\}) \quad (5.6)$$

where  $\mathcal{T}_X(\mathbb{R}^s)$  denotes the system of sets representing the ranges of variability.

The relation (5.6) implies that the variability function  $\mathcal{X}$  is completely determined by the images  $\mathcal{X}(\{d\})$  of the singletons  $\{d\}$ . Note that the variability function may also be subject of ignorance.

### 5.1.3 Modelling the Structure of Variability

The degree of conformance between initial condition and future outcome determines the probability of occurrence of any event with respect to  $X$ . Thus, randomness which is determined by the initial condition generates a structure on the range of variability of  $X$ , which is adequately modelled by a probability measure. This situation is described by a function denoted  $\mathcal{P}$ , called ‘random structure function’ and defined as follows<sup>2</sup>:

$$\mathcal{P} : \mathcal{T}_D(\mathcal{D}) \rightarrow \mathbb{P} \quad (5.7)$$

$$\mathcal{P}(\{d\}) = P_{X|\{d\}} \text{ for } d \in \mathcal{D} \quad (5.8)$$

$$\mathcal{P}(\mathcal{D}_0) = P_{X|\mathcal{D}_0} \text{ with} \quad (5.9)$$

$$P_{X|\mathcal{D}_0}(\{x\}) = \frac{1}{|\mathcal{D}_0|} \sum_{d \in \mathcal{D}_0} P_{X|\{d\}}(\{x\}) \quad (5.10)$$

$$\text{for } \mathcal{D}_0 \in \mathcal{T}_D(\mathcal{D}) \text{ and } x \in \mathcal{X}(\mathcal{D}_0)$$

where

$$\mathbb{P} = \text{set of probability measure over } \mathcal{X}(\mathcal{D}) \quad (5.11)$$

$$P_{X|\{d\}} = \text{probability measure of } X|\{d\} \text{ with} \\ \text{support } \mathcal{X}(\{d\}) \quad (5.12)$$

Similar as in the case of the variability function  $\mathcal{X}$ , the random structure function  $\mathcal{P}$  is generally subject to ignorance.

### 5.1.4 The Bernoulli Space

The pair of variable  $(X, D)$  identifies the aspects of future and past which are of interest or relevance. The ignorance set  $\mathcal{D}$  describes what is known about the starting situation, while the variability function  $\mathcal{X}$  provides the extent of future variability and the random structure function  $\mathcal{P}$  the structure within the variability created by randomness.

In commemoration of Jakob Bernoulli the hyperspace defined by the three quantities  $\mathcal{D}$ ,  $\mathcal{X}$  and  $\mathcal{P}$  has been named ‘Bernoulli Space’ in [49] denoted  $\mathbb{B}_{X,D}$ :

$$\mathbb{B}_{X,D} = (\mathcal{D}, \mathcal{X}, \mathcal{P}) \quad (5.13)$$

---

<sup>2</sup>The definition of the random structure function used here, is slightly different from the one given in [49]. However, (5.7) seems to be more consistent with (5.5)

The Bernoulli Space is a fully quantified description of a transition from past to future. It is based on the entire available knowledge and contains all elements which are necessary and sufficient to arrive at a useful model.

## 5.2 Graphical Image of a Bernoulli Space

There are two situations to be distinguished concerning the knowledge about the initial condition: complete and incomplete knowledge.

### 5.2.1 Complete Knowledge

Complete knowledge means that the ignorance space  $\mathcal{D}$  contains only the true value denoted  $d_0$ . In this case domain and codomain of the variability function  $\mathcal{X}$  and random structure function  $\mathcal{P}$  are singletons. Thus, the Bernoulli Space for the pair of variable  $(X, D)$  can be represented by two sets and one probability measure.

$$\mathbb{B}_{X,D} = \left( \{d_0\}, \mathcal{X}(\{d_0\}), P_{X|\{d_0\}} \right) \quad (5.14)$$

In the rare case of complete knowledge, randomness is the exclusive source of uncertainty. Game Theory is the most prominent example, where complete knowledge about initial conditions is assumed.

Figure 5.2<sup>3</sup> illustrates a notional example of complete knowledge for the univariate case (with respect to  $X$  and  $D$ ), where the singleton  $\{d_0\}$  stands for the completely known past. The marked set of points on the  $x$ -axis is the set of the possible future values, i. e.,  $\mathcal{X}(\{d_0\})$ . Each pair  $(d_0, x)$  with  $x \in \mathcal{X}(\{d_0\})$  represents one possible transition from past to future. The probabilities of occurrence of each of the possible transitions are depicted by the corresponding vertical bars.

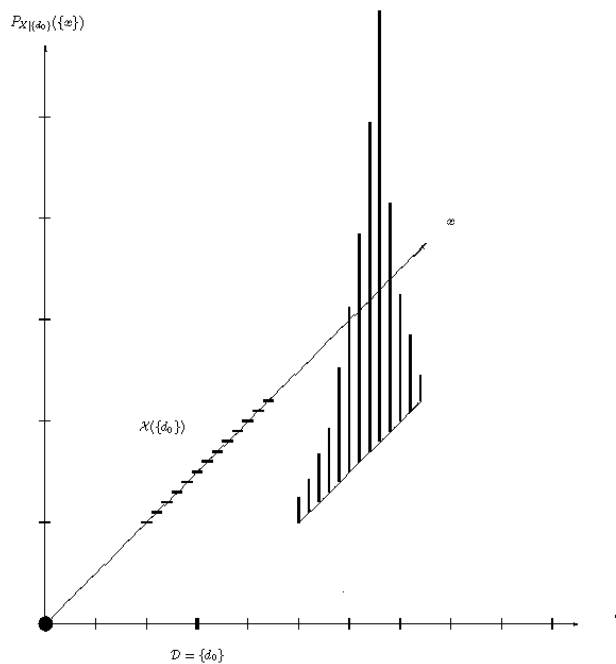


Figure 5.2: A graphical illustration of a Bernoulli Space in case of complete knowledge.

<sup>3</sup>Taken from [49]

### 5.2.2 Incomplete Knowledge

Generally, there is only incomplete knowledge about the initial conditions, because the true initial conditions cannot be identified. However, it is always possible to specify a set which contains with certainty the true initial conditions. Let  $\mathcal{D}$  be the smallest set according to the available knowledge containing the true initial conditions. Then  $\mathcal{D}$  is taken as the Ignorance Space. Assume for example that everything can be excluded with respect to the true initial conditions except for four values:

$$\mathcal{D} = \{d_1, d_2, d_3, d_4\} \quad (5.15)$$

In Figure 5.3<sup>4</sup> the resulting Bernoulli Space is illustrated. The ignorance space contains four elements, one of them is the true value, while three of them are wrong. Because of ignorance, the true initial conditions are unknown. Each of potential initial conditions leads to a different range of variability of the corresponding random variables  $X|\{d_i\}$ ,  $i = 1, 2, 3, 4$  and to different probability distributions  $P_{X|\{d_i\}}$ ,  $i = 1, 2, 3, 4$ . Thus the total range of variability which has to be taken into account is given by

$$\mathcal{X}(\mathcal{D}) = \cup_{d \in \mathcal{D}} \mathcal{X}(\{d\}) \supset \mathcal{X}(\{d_0\}) \quad (5.16)$$

where  $d_0 \in \mathcal{D}$  is the true but unknown value representing the initial conditions.

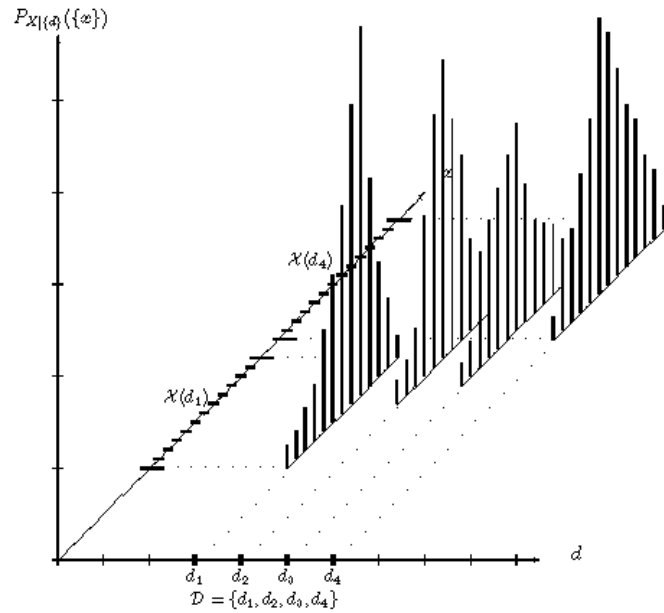


Figure 5.3: A graphical illustration of a Bernoulli Space in case of incomplete knowledge.

## 5.3 The Uncertainty Space

Uncertainty is nurtured by two sources ‘ignorance’ and ‘randomness’ and this fact can clearly be seen from Figure 5.3.

There is the seemingly variability produced by ignorance and represented by the four elements of  $\mathcal{D}$ . The available knowledge does not admit to exclude any of these values and, thus, each of

<sup>4</sup>Taken from [49]

them could be the true one. Therefore, each of them has to be considered likewise, and none of them can be preferred.<sup>5</sup>

In contrast to the seemingly variability produced by ignorance, the variability generated by randomness is structured, i. e., exhibits order, which is illustrated by the corresponding probabilities in Figure 5.3.

The entire variability which has to be considered extends over a two-dimensional area, where the first dimension refers to the deterministic variable  $D$  and the second dimension to the random variable  $X$ . The area of the entire variability reflects the total uncertainty and, therefore, is called Uncertainty Space of  $(X, D)$  denoted by  $\mathcal{U}_{X,D}$  and defined by:

$$\mathcal{U}_{X,D} = \left\{ (x, d) \mid x \in \mathcal{X}(\{d\}), d \in \mathcal{D} \right\} \quad (5.17)$$

The Uncertainty Space can be regarded as the static part of the Bernoulli Space, containing all transitions which have to be considered at least in principle.

The random structure function constitutes the dynamic part, which contains the rules how the future transition will be selected by nature. These rules make up randomness and have the effect that everything remains in order and nothing can slip off into chaos.

## 5.4 Selecting the Random Structure Function

The random structure function  $\mathcal{P}$  constitutes the dynamic part of a Bernoulli Space and, therefore, it is of particular importance. It contains the rules of randomness, where randomness personifies the natural principle of order. Describing an object assumes that sufficient knowledge is available, otherwise a meaningful description is impossible. Moreover, the more complex the object is, the more knowledge is necessary to arrive at a useful description.

Thus, the question arises which and how much knowledge must be available for describing the random structure by a mathematical function in a given situation. The question is answered in [49] and the answer shall be briefly outlined here.

As indicated above high complexity with respect to the random structure of a random variable  $X$  necessitates more knowledge for a useful description than low complexity. Therefore, the possible situations are divided into disjoint families according to their complexity with respect to the resulting random structure. By means of such a partition it becomes possible to assign the appropriate family of probability measures to a given situation, as soon as its complexity is known.

The complexity of a probability measure  $P_X$  is defined in [49] by the number of extremes of the corresponding probability mass function  $f_X$ . The most important cases are those with only very few extremes. These are listed below.

1. No extreme in  $\mathcal{X}(\{d\}) \Leftrightarrow$  Family of constant distributions denoted  $\mathbb{P}_0$ .
2. Two boundary extremes, no inner extreme in  $\mathcal{X}(\{d\}) \Leftrightarrow$  Family of monotonic distributions denoted  $\mathbb{P}_1$  with two subfamilies.
  - (a) Family of monotonic decreasing distributions denoted  $\mathbb{P}_{1,1}$

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<sup>5</sup>Assigning a so-called prior probability distribution over  $\mathcal{D}$  means to rely on subjective belief, as each of the  $d_i$  is either the true one or not. If science should be objective, then a prior probability distribution must be avoided.



- (b) Family of monotonic increasing distributions denoted  $\mathbb{P}_{1,2}$
3. Two boundary extremes, one inner extreme in  $\mathcal{X}(\{d\}) \Leftrightarrow$  Family of uni-extreme distributions denoted  $\mathbb{P}_2$  with two subfamilies.
    - (a) Family of uni-modal distributions denoted  $\mathbb{P}_{2,1}$
    - (b) Family of uni-bathtub distributions denoted  $\mathbb{P}_{2,2}$
  4. Two boundary extremes, two inner extremes in  $\mathcal{X}(\{d\}) \Leftrightarrow$  Family of bi-extreme distributions denoted  $\mathbb{P}_3$  with two subfamilies.
    - (a) Family of bi-modal distributions denoted  $\mathbb{P}_{3,1}$
    - (b) Family of bi-bathtub distributions denoted  $\mathbb{P}_{3,2}$

The most important families are the family of uni-modal distributions and the family of constant distributions. The family of uni-modal distributions is appropriate for numerous random variables representing the outcome of natural processes, while the family of constant distribution is appropriate for many man-made processes, which are constructed deliberately so that each outcome has the same probability of occurrence and, therefore has to be taken into account likewise, similar as the elements of the ignorance space. Thus, in the latter case there is no order within the range of variability and, therefore, it could be looked upon as “chaos”.

The probability mass functions having not more than one maximum stand for simple processes, while those with more than one maximum indicate compound processes, which consist of several future stages, where the preceding stage determines the probability measure in the subsequent stage.

In order to determine the random structure function for a given random variable  $X$  in a meaningful way, the true family of probability distributions should necessarily be known. This knowledge is available, if number and type of extremes of the corresponding probability mass function have been identified, which is possible by sufficient empirical experience or by theoretical insight in the process of interest. If an identification of number and type of extremes is impossible, then a meaningful description of the random structure cannot be warranted and, therefore, should be postponed until enough information have been distinguished.

In Figure 5.4 the four first families of probability distributions with simplest complexity including the subfamilies are displayed.

For identifying the family on hand, some qualitative properties of the process have to be known. Once the family has been selected, an appropriate family member must be determined, based on quantitative information. The question, what kind and how much quantitative knowledge is necessary to select an appropriate family member is answered again in [49], by deriving and using the ‘minimum information principle’.

According to the minimum information principle, each family necessitates a certain minimum amount of quantitative knowledge for a meaningful selection of a family member, i. e., of a probability distribution.

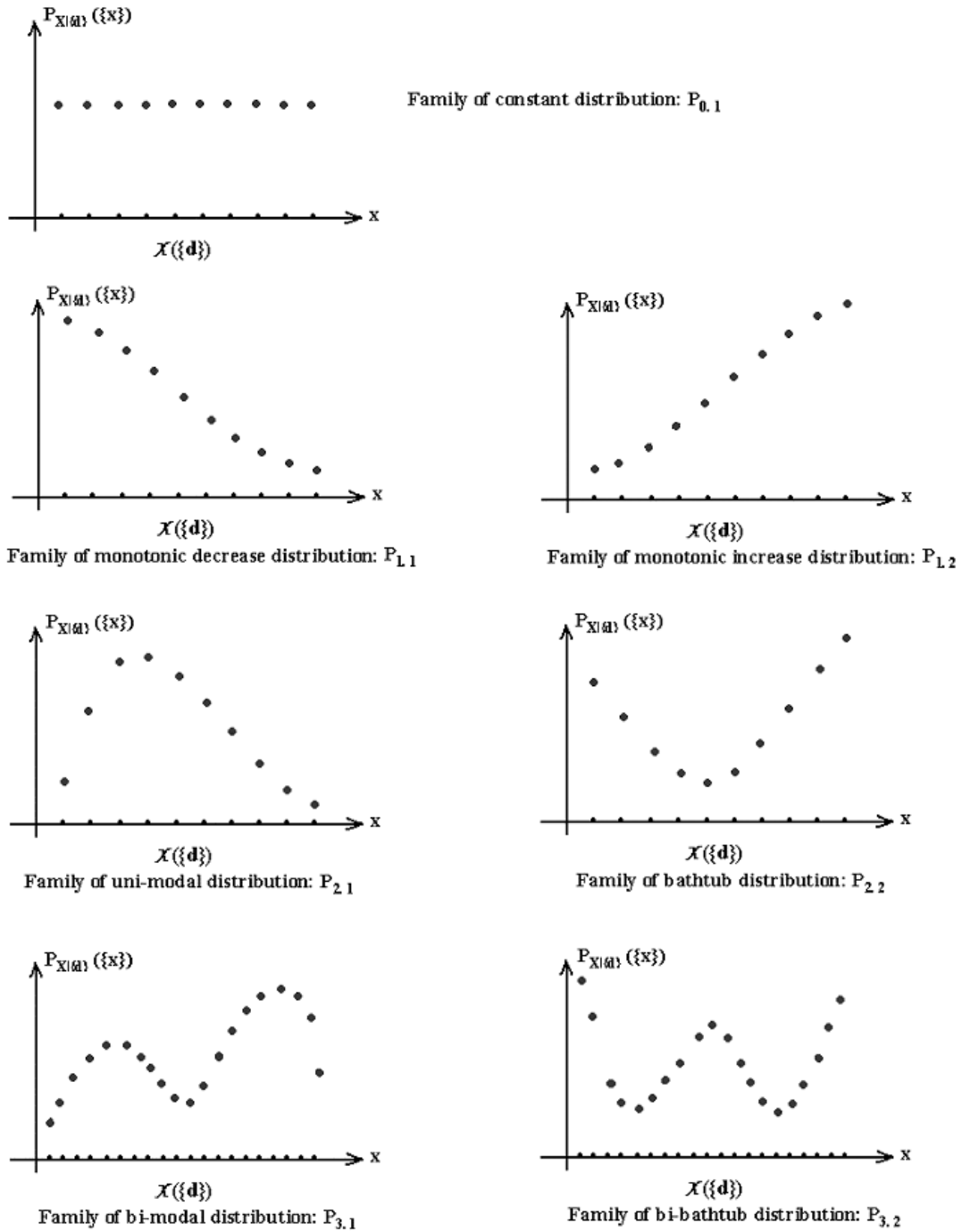


Figure 5.4: Illustration of the first four families of probability distributions over  $\mathcal{X}(\{d\})$ .

Family	Minimum amount of necessary quantitative knowledge
$\mathbb{P}_0$	$\Rightarrow \mathcal{X}(\{d\})$
$\mathbb{P}_1$	$\Rightarrow \mathcal{X}(\{d\})$ and $E[X \{d\}]$
$\mathbb{P}_2$	$\Rightarrow \mathcal{X}(\{d\})$ , $E[X \{d\}]$ and $E[(X \{d\})^2]$
$\mathbb{P}_3$	$\Rightarrow \mathcal{X}(\{d\})$ , $E[X \{d\}]$ , $E[(X \{d\})^2]$ and $E[(X \{d\})^3]$

The above list shows, how increasing complexity of a process necessitates more and more

quantitative knowledge.

Note that the actual values of the moments of  $X|\{d\}$  are uniquely determined by the actual values  $d$  of the deterministic variable. Therefore, the moments constitute a special representation of the deterministic variable denoted by  $D_\mu$  with values  $d_\mu$ :

$$d_\mu(d) = (\mu_1(d), \mu_2(d), \dots) \quad (5.18)$$

with

$$\mu_i(d) = E \left[ (X|\{d\})^i \right] \text{ for } i = 1, 2, \dots \quad (5.19)$$

For any family  $\mathbb{P}_k$  and range of variability  $\mathcal{X}(\{d_\mu\})$  and given value of  $d_\mu$ , the probability distribution  $P_{X|\{d_\mu\}}$  can uniquely determined by solving a corresponding system of equations given in [49]

# Chapter 6

## Utilizing Predictions

### 6.1 Stochastic Procedures

As argued earlier, any decision is based explicitly or implicitly on a prediction. The possibility of predicting future events has always been in the center of human dreams and activities. The Bernoulli Space opens the way for deriving ‘scientific’ prediction procedures and the question arises how shall prediction procedures be determined and how can they be exploited for solving problems. This questions leads from the stochastic model to consider and investigate stochastic procedures.

A Bernoulli Space is utilized for deriving ‘stochastic procedures’ which support making decisions and solving problems. The Bernoulli Space describes mathematically the relations between past and future and, therefore, the Bernoulli Space enables either to look into the future or to look into the past. Accordingly, there are two types of problems which define two classes of stochastic procedures. In [50] these are defined as follows:

- The prediction class, by means of which statements are made about the future development.
- The measurement class, by means of which unknown facts are determined.

A procedure of the prediction class is completely based on the given Bernoulli Space, while a procedure of the measurement class requires additional experiments.

Consider a pair of variables  $(X, D)$ , a corresponding Bernoulli Space  $\mathbb{B}_{X,D}$  and a stochastic procedure which shall be used to find the answer for a specified question. Possible questions and corresponding procedures introduced in detail in [50], shall be outlined here.

Before specific stochastic procedures are described, general properties of stochastic procedures are listed in order to be able to define the quality of stochastic procedures as a prerequisite for comparing different stochastic procedures and for defining optimal ones.

There are three quality characteristics which may be looked at when dealing with stochastic procedures. These are

- reliability,
- precision, and
- expense.

### 6.1.1 Reliability

The more frequently a procedure yields correct results, the more it may be regarded as being reliable. The observed frequency of correct results reflects the probability of a correct result. Hence, the probability of a correct result when applying a stochastic procedure is called its “reliability”.

The reliability or at least a lower bound for the reliability must be known before the procedure may be applied, as otherwise the risk is not known and, hence, the results obtained are more or less meaningless. The controlled reliability of stochastic procedures distinguish them from unscientific procedures.

It seems to be clear that it makes no sense to use a procedure which yields wrong results with probability 1. However, statistical estimators are commonly used in many fields of application without considering that the obtained results are generally wrong with certainty no matter whether the estimator is biased or unbiased. Implying that the isolated use of a statistical estimator must be called careless.

Because of the fundamental importance of reliability, it will generally be imposed on the procedure as a specification, implying that it is only reasonable to compare different procedures if they meet the same reliability specification.

### 6.1.2 Precision

A result may be correct, but completely useless. For instance consider that one wants to decide whether or not to take an umbrella, then the forecast “there will be either no rain or rain.” is with certainty correct, i. e., has reliability 1, but is totally useless as it does not add to the trivially available knowledge.

Therefore, the precision of a procedure is defined as a measure how well the underlying aim is reached to get additional knowledge by applying the procedure for supporting the decision making process.

Explaining quality of something as the degree of meeting its purpose suggests that the precision of a procedure may serve as optimization criterion for procedures meeting the same reliability specification.

### 6.1.3 Expense

The expense of a stochastic procedure measures the efforts which have to be made in order to perform it. For a procedure of the prediction class the efforts refer to the calculations needed to obtain the desired result based on the Bernoulli Space. If adequate tools are available, the expense should be marginal. In the case of a procedure of the measurement class, however, additional experiments are needed. In case of limited resources, the expense could become a decisive quality characteristic in addition to the procedure’s precision.

## 6.2 Prediction Class

As mentioned earlier, any procedure of the prediction class yields statements about the future. The most common statements are forecasts of future events, which are called predictions here. The quality of any prediction depends on the amount of available knowledge with respect to the initial conditions referring to the past development, and about randomness referring to the

future development. Both are specified by the Bernoulli Space, which, therefore, is the basis for developing prediction procedures.

The prediction class contains the following procedures:

- Procedures for predicting the future outcome.
- Procedures for predicting an upper bound for the future outcome (in the univariate case).
- Procedures for predicting a lower bound for the future outcome (in the univariate case).
- Excluding a future event.
- Guaranteeing the occurrence of future event.
- Comparing different future events with respect to the probability of occurrence.

In this thesis only the first three procedures are discussed in detail and referred to as prediction procedures. The other procedures start with fixed future events and, therefore, can be reduced in principle to the calculation of probabilities, which is a comparatively simple task. Therefore, they are not included in this brief introduction to Bernoulli Stochastics.

## 6.3 Prediction Procedures

A Bernoulli Space  $\mathbb{B}_{X,D}$  constitutes the basis for predicting the future outcome of  $X$  taking into account ignorance and randomness. A prediction procedure maps the past onto a future event. The past is represented by the ignorance space  $\mathcal{D}$  or more generally by a subset  $\mathcal{D}_0 \subset \mathcal{D}$  and any meaningful predicted event is a subset of the corresponding range of variability  $\mathcal{X}(\mathcal{D}_0)$ .

Let  $\mathcal{T}_D(\mathcal{D})$  be a system of suitable subsets over the ignorance space  $\mathcal{D}$ , where each element represents a state of knowledge with respect to the initial conditions, and let  $\mathcal{T}_X(\mathcal{X}(\mathcal{D}))$  be a system of suitable subsets, where each element represents a possible prediction. Then, following [50] a mapping

$$A_X : \mathcal{T}_D(\mathcal{D}) \rightarrow \mathcal{T}_X(\mathcal{X}(\mathcal{D})) \quad (6.1)$$

is called a prediction procedure for  $X$ .

In order to be meaningful the reliability of a prediction procedure must be controlled. To this end, so-called  $\beta$ -prediction procedures denoted  $A_X^{(\beta)}$  are introduced in [50]. A prediction procedure  $A_X$  given by (6.1) is called  $\beta$ -prediction procedure, if

$$P_{X|\{d\}}(A_X(\mathcal{D}_0)) \geq \beta \text{ for } d \in \mathcal{D}_0 \text{ and } \mathcal{D}_0 \in \mathcal{T}_D(\mathcal{D}) \quad (6.2)$$

A  $\beta$ -prediction procedure is denoted  $A_X^{(\beta)}$ . The value  $\beta$  is called the reliability level of the prediction procedure  $A_X^{(\beta)}$ . The reliability level is a lower bound for the probability of occurrence of the predicted event for the given level of ignorance.

### 6.3.1 Prediction Procedure in Case of Complete Knowledge

Let the system of subsets over the ignorance space  $\mathcal{D}$  be given as the system of singletons:

$$\mathcal{T}_D(\mathcal{D}) = \left\{ \{d\} \mid d \in \mathcal{D} \right\} \quad (6.3)$$

and the system of subsets over  $\mathcal{X}(\mathcal{D})$  as the intervals:

$$\mathcal{T}_X(\mathcal{X}(\mathcal{D})) = \left\{ \{x | x_1 \leq x \leq x_2\} | x_1 \leq x_2, x, x_1, x_2 \in \mathcal{X}(\mathcal{D}) \right\} \quad (6.4)$$

A corresponding  $\beta$ -prediction procedure  $A_X^{(\beta)}$  is defined on the singletons, i. e., it assumes complete knowledge about the initial conditions and the predicted events are intervals occurring with a probability of at least  $\beta$ .

The precision of a prediction is given by the size of the predicted event. The smaller the size, the better serves the prediction its purpose. Therefore, the size of the predicted event is an appropriate quality criterion enabling to define optimal  $\beta$ -predictions and, thus, optimal  $\beta$ -prediction procedures.

- A  $\beta$ -prediction procedure  $A_X^{(\beta)}$  with

$$A_X^{(\beta)} : \mathcal{T}_D(\mathcal{D}) \rightarrow \mathcal{T}_X(\mathcal{X}(\mathcal{D})) \quad (6.5)$$

is called a minimum  $\beta$ -prediction procedure denoted by  $*A_X^{(\beta)}$ , if

$$\left| *A_X^{(\beta)}(\{d\}) \right| = \min_{A_X^{(\beta)}} \left| A_X^{(\beta)}(\{d\}) \right| \text{ for } d \in \mathcal{D} \quad (6.6)$$

A prediction obtained by a minimum  $\beta$ -prediction procedure is called analogously, a minimum  $\beta$ -prediction.

**Example:** In Figure 6.1 a minimum  $\beta$ -prediction is displayed. The range of variability  $\mathcal{X}(\{d_0\})$  has 13 elements. The reliability level is given as  $\beta = 0.80$  and the resulting  $\beta$ -prediction contains only seven elements, which means a considerable gain of information.

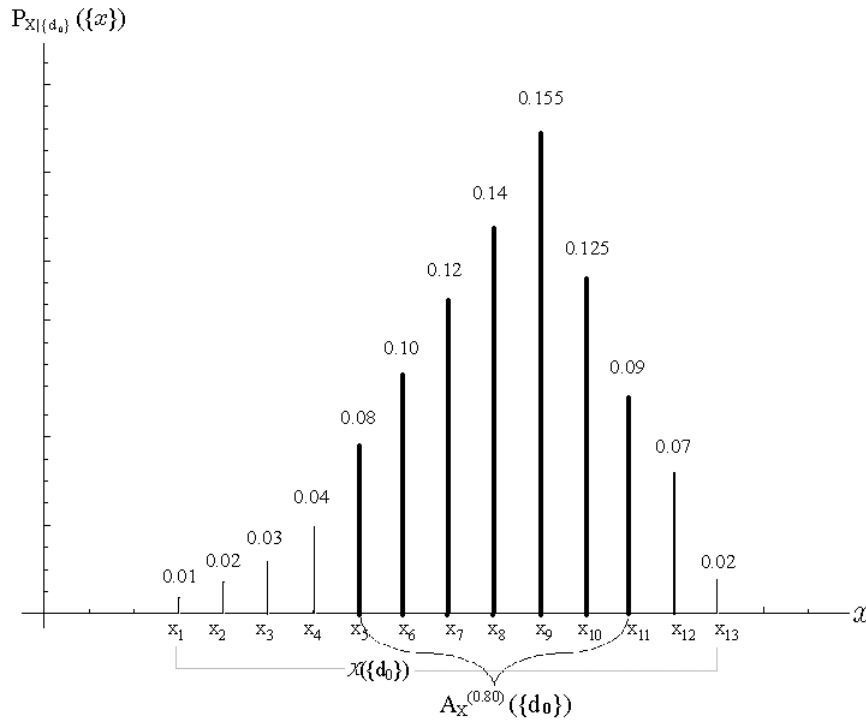


Figure 6.1: Minimum  $\beta$ -prediction for  $\beta = 0.80$  in case of complete knowledge.

A minimum  $\beta$ -prediction procedure in the case of complete knowledge is constructed straightforward by stepwise determining each  $\beta$ -minimum prediction. For example to obtain the minimum  $\beta$ -prediction in Figure 6.1, outcome  $x_9$  which occurs with largest probability 15.5% is firstly selected for the predicted event, next  $x_8$  with the second largest probability 14%, then the third  $x_{10}$  with 12.5%, the fourth  $x_7$  with 12%, the fifth  $x_6$  with 10%, the sixth  $x_{11}$  with 9%, and with the seventh  $x_5$  having a probability of 8% the probabilities of the resulting event exceeds for the first time the required reliability of  $\beta = 0.80$  and falls below the required reliability level if anyone of the outcomes is abandoned from the predicted event. Thus, the event  $\{x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}\}$  is a desired minimum  $\beta$ -prediction.

A minimum  $\beta$ -prediction is in general not unique. If  $x_5$  is substituted by  $x_{12}$ , the event

$$\{x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$$

also meets the reliability requirement given by  $\beta = 0.80$  and is most accurate. However, the actual reliability of the prediction

$$\{x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}\}$$

is larger and, therefore, it is selected.

The totality of the minimum  $\beta$ -predictions for  $d \in \mathcal{D}$  represents the minimum  $\beta$ -prediction procedure. Moreover, it forms a subset of the uncertainty space  $\mathcal{U}_{X,D}$ . This subset is built up by the potential initial conditions and the corresponding events occurring with a probability of at least  $\beta$ . This subset of  $\mathcal{U}_{X,D}$  is called  $\beta$ -Uncertainty Space for  $X$  denoted by  $\mathcal{U}_X^{(\beta)}$ :

$$\mathcal{U}_X^{(\beta)} = \left\{ (x, d) \mid x \in {}^*A_X^{(\beta)}(\{d\}), d \in \mathcal{D} \right\} \quad (6.7)$$

The  $\beta$ -Uncertainty Space for  $X$  is the graphical representation of a minimum  $\beta$ -prediction procedure.

Besides the problem of predicting a future outcome, which is solved by the minimum  $\beta$ -prediction procedure (6.6), there are the problems of predicting an upper or a lower bound for the future outcome. The solutions to these problems are called ‘minimum upper bound  $\beta$ -prediction’ and ‘maximum lower bound  $\beta$ -prediction’, respectively and defined as follows:

- A  $\beta$ -prediction for  $\{d\}$  is called a minimum upper bound  $\beta$ -prediction denoted  ${}^*\bar{A}_X^{(\beta)}(\{d\})$ , if the following conditions are met:

$${}^*\bar{A}_X^{(\beta)}(\{d\}) = \left\{ x \mid x \leq x_u, x, x_u \in \mathcal{X}(\{d\}) \right\} \quad (6.8)$$

$$P_{X|\{d\}}\left({}^*\bar{A}_X^{(\beta)}(\{d\})\right) \geq \beta \quad (6.9)$$

$$P_{X|\{d\}}\left({}^*\bar{A}_X^{(\beta)}(\{d\}) \setminus \{x_u\}\right) < \beta \quad (6.10)$$

**Example:** Suppose that, due to some unavoidable reasons given by the value  $d_0$  of the deterministic variable, a company expects to suffer a loss during the next reporting period. The probability distribution of this loss is shown in Figure 6.2. The company wants to predict the possibly largest loss and require the reliability of this prediction to be at least 80%. The predicted event is given by

$${}^*\bar{A}_X^{(0.80)}(\{d_0\}) = \{x \mid x \leq 800,000\} \quad (6.11)$$



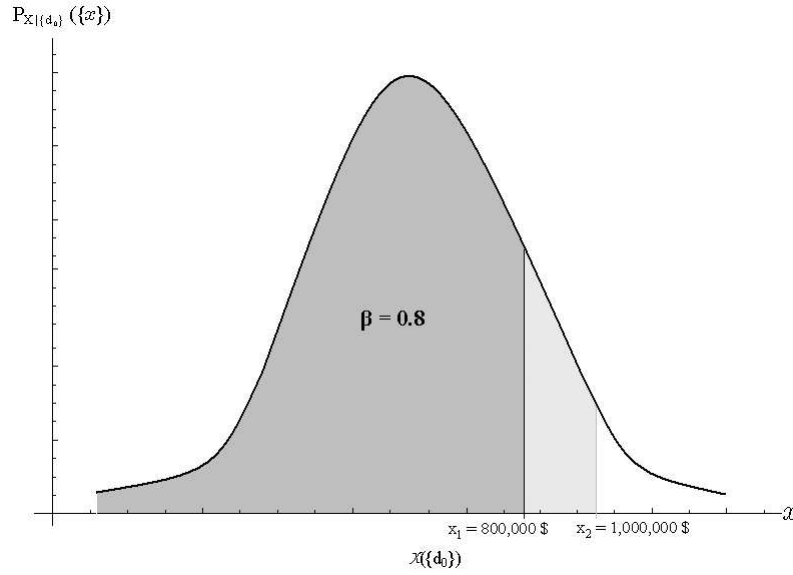


Figure 6.2: Minimum upper bound  $\beta$ -predictions for  $\beta = 0.80$  and  $\beta = 0.90$  in case of complete knowledge.

If the reliability requirement is set to be  $\beta = 0.90$  instead of  $\beta = 0.80$ , then the predicted largest loss increases to  $x_2 = 1,000,000$ . In Figure 6.2 the two situations are displayed.

- A  $\beta$ -prediction is called a maximum lower bound  $\beta$ -prediction denoted  $^* \underline{A}_X^{(\beta)}(\{d\})$ , if the following conditions are met:

$$^* \underline{A}_X^{(\beta)}(\{d\}) = \{x \mid x \geq x_\ell, x, x_\ell \in \mathcal{X}(\{d\})\} \quad (6.12)$$

$$P_{X|\{d\}}(^* \underline{A}_X^{(\beta)}(\{d\})) \geq \beta \quad (6.13)$$

$$P_{X|\{d\}}(^* \underline{A}_X^{(\beta)}(\{d\}) \setminus \{x_\ell\}) < \beta \quad (6.14)$$

**Example:** Assume that a company wants to predict the possibly smallest profit of the next year and sets the reliability requirement to  $\beta = 0.80$ . Let the maximum lower bound  $\beta$ -prediction be given by

$$^* \underline{A}_X^{(0.80)}(\{d_0\}) = \{x \mid x \geq 600,000,000\} \quad (6.15)$$

then the result is similar as above, increasing the reliability requirement to  $\beta = 0.90$ , yields a smaller upper bound. If, for instance, the reliability level is set to  $\beta = 0.90$  the following maximum lower bound  $\beta$ -prediction is obtained:

$$^* \underline{A}_X^{(0.90)}(\{d_0\}) = \{x \mid x \geq 400,000,000\} \quad (6.16)$$

The situation is illustrated in Figure 6.3.

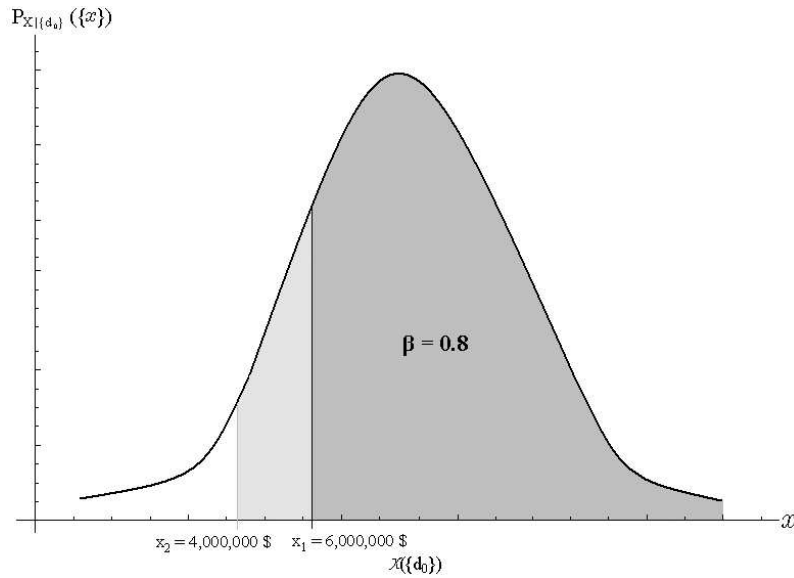


Figure 6.3: Maximum lower bound  $\beta$ -predictions for  $\beta = 0.80$  and  $\beta = 0.90$  in case of complete knowledge.

### 6.3.2 Prediction Procedure in Case of Ignorance

In case of ignorance and assuming a one-dimensional deterministic variable  $D$ , the system of intervals in  $\mathcal{D}$  may be selected to represent different degrees of ignorance

$$\mathcal{T}_D(\mathcal{D}) = \left\{ \{d \mid d_1 \leq d \leq d_2\} \mid d_1 < d_2, d, d_1, d_2 \in \mathcal{D} \right\} \quad (6.17)$$

together with system of intervals over  $\mathcal{X}(\mathcal{D})$  as potential predictions

$$\mathcal{T}_X(\mathcal{X}(\mathcal{D})) = \left\{ \{x \mid x_1 \leq x \leq x_2\} \mid x_1 \leq x_2, x, x_1, x_2 \in \mathcal{X}(\mathcal{D}) \right\} \quad (6.18)$$

A corresponding minimum  $\beta$ -prediction procedure  $*A_X^{(\beta)}$  is given by the following condition:

$$\left| *A_X^{(\beta)}(\mathcal{D}_0) \right| = \min_{A_X^{(\beta)}} \left| A_X^{(\beta)}(\mathcal{D}_0) \right| \text{ for any } \mathcal{D}_0 \in \mathcal{T}_D(\mathcal{D}) \quad (6.19)$$

In order to derive a minimum  $\beta$ -prediction  $*A_X^{(\beta)}(\mathcal{D}_0)$ , one could start from a  $\beta$ -Uncertainty Space  $\mathcal{U}_X^{(\beta)}$ , which immediately yields a  $\beta$ -prediction  $A_X^{(\beta)}(\mathcal{D}_0)$ :

$$A_X^{(\beta)}(\mathcal{D}_0) = \bigcup_{d \in \mathcal{D}_0} *A_X^{(\beta)}(\{d\}) \quad (6.20)$$

with a reliability exceeding the required reliability level for each  $d \in \mathcal{D}_0$ :

$$P_{X|\{d\}} \left( A_X^{(\beta)}(\mathcal{D}_0) \right) > \beta \text{ for } d \in \mathcal{D}_0 \quad (6.21)$$

Therefore, for obtaining a minimum  $\beta$ -prediction the interval must be shortened either from below or above, until any further shortening would let the reliability fall below the required  $\beta$ . This procedure and the obtained result  $*A_X^{(\beta)}(\mathcal{D}_0)$  are illustrated in Figure 6.4.

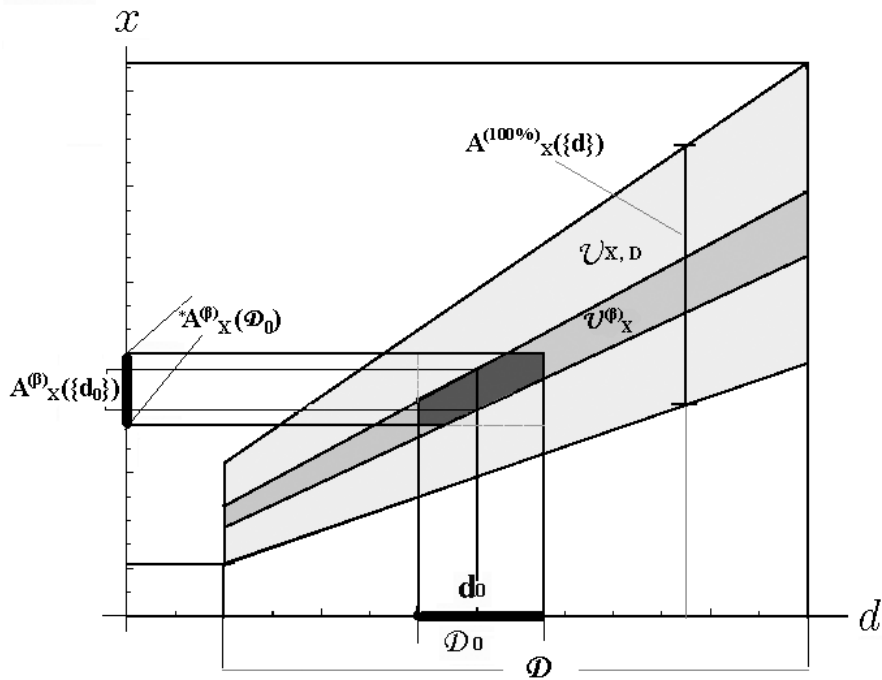


Figure 6.4: The  $\beta$ -Uncertainty Space  $\mathcal{U}_X^{(\beta)}$  for  $X$ , and a minimum  $\beta$ -prediction  $*A_X^{(\beta)}(\mathcal{D}_0)$ .

If  $\mathcal{D}_0 = \mathcal{D}$ , i.e., the entire Ignorance Space  $\mathcal{D}$ , then the procedure and the obtained result  $*A_X^{(\beta)}(\mathcal{D})$  are illustrated in Figure 6.5.

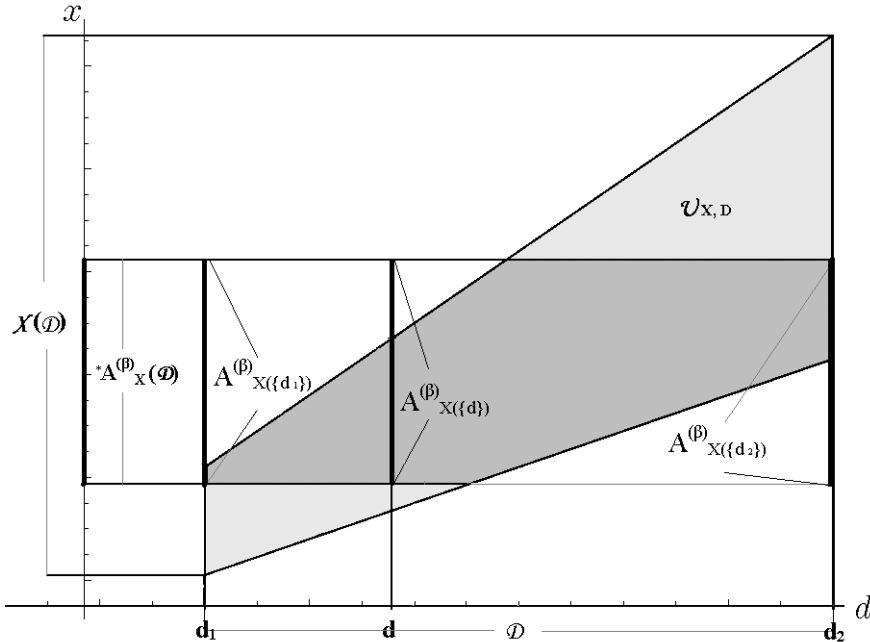


Figure 6.5: Minimum  $\beta$ -prediction for the ignorance space  $\mathcal{D}$ :  $*A_X^{(\beta)}(\mathcal{D})$ .

## 6.4 Measurement Class

Procedures of the measurement class aim at making statements about the past, i.e., about the initial conditions implying that the initial conditions are not known completely, but only

partially which means that necessarily the following holds:

$$|\mathcal{D}| > 1 \quad (6.22)$$

The inequality means that the true value  $d_0$  of the deterministic variable is not known except for

$$d_0 \in \mathcal{D} \quad (6.23)$$

The measurement class contains the following procedures:

- Procedures for determining the true value  $d_0$ .
- Procedures for determining an upper bound for the true value  $d_0$  (in the uni-variate case).
- Procedures for determining a lower bound for the true value  $d_0$  (in the uni-variate case).
- Procedures for excluding  $\mathcal{D}_0 \subset \mathcal{D}$ , i. e., for proving that  $d_0 \notin \mathcal{D}_0$ .
- Procedures for assigning the true value  $d_0$  to one of  $m$  given alternatives  $\mathcal{D}_i$ ,  $i = 1, 2, \dots, m$ .

Any procedure of the measurement class aims at improving the given Bernoulli Space by reducing the Ignorance Space  $\mathcal{D}$  on the basis of additional knowledge. Thus, a measurement class procedure is necessarily based on new information. New information can generally be obtained only by new experiments, which are formally described by a ‘sample for  $X|\{d_0\}$ ’, i. e., a vector of independent copies of  $X|\{d_0\}$  denoted

$$(X_1|\{d_0\}, \dots, X_n|\{d_0\}) \quad (6.24)$$

where the independence of the experiment imply that the following holds:

$$f_{X_1|\{d_0\}, \dots, X_n|\{d_0\}}(x_1, \dots, x_n) = \prod_{i=1}^n f_{X|\{d_0\}}(x_i) \quad (6.25)$$

The experiments are generally costly and, thus, in contrast to the prediction class, the expense of a procedure is an important property with respect to the measurement class. Here, following [50], it is assumed that the expense is fixed and, therefore, the corresponding problems need not to be discussed.

## 6.5 Measurement Procedures

A measurement procedure is used to look into the past aiming at reducing ignorance, which in fact is more difficult than to look into the future. This is because from an objective point of view, ignorance is unstructured and, therefore, can be attacked only indirectly by exploiting the structure of randomness. In other words the structure of randomness is the only means to assess the unknown past.

The development of a measuring tool capable of revealing unknown facts proceeds in several steps:

- Quantification of the unknown fact by means of a (deterministic) variable  $D$  with ignorance space  $\mathcal{D}$ . In this case the ignorance space specifies the range for which the measuring tool shall be admitted.

- Selection of an appropriate random process with a quantitative aspect  $X$  related as close as possible to the unknown fact  $D$ .
- Quantification of the relation between  $D$  and  $X$  by a variability function  $\mathcal{X}$  and a random structure function  $\mathcal{P}$ .
- Calibration of  $X$  with respect to  $D$ , i. e., developing a stochastic measurement procedure.

Here only the last step is considered, i. e., it is assumed that the pair of variables  $(X, D)$  has been identified, and a corresponding Bernoulli Space  $\mathbb{B}_{X,D}$  has been derived. The task remains to develop a stochastic measurement procedure or calibration denoted  $C_D$  for the unknown true value  $d_0$  of  $D$ .

### 6.5.1 Definition of a Calibration $C_D$

To measure the true value  $d_0$  means to perform an experiment with respect to  $X$ , to observe the realization  $x$  and to select the measurement for  $d_0$  denoted  $C_D(\{x\})$  according to a given calibration  $C_D$ , which from a more formal viewpoint is function defined as follows:

$$C_D : \mathcal{T}_X(\mathcal{X}(\mathcal{D})) \rightarrow \mathcal{T}_D(\mathcal{D}) \quad (6.26)$$

where  $\mathcal{T}_D(\mathcal{D})$  is a suitably selected system of subsets of  $\mathcal{D}$ . In the univariate case  $\mathcal{T}_D$  is selected as the system of non-empty intervals of  $\mathcal{D}$ .

A reasonable measurement procedure  $C_D$  must meet the following conditions.

$$1. \quad C_D(\{x\}) \subset \mathcal{D} \text{ for } x \in \mathcal{X}(\mathcal{D}) \quad (6.27)$$

$$2. \quad P_{X|\{d\}}(\{x \mid d \in C_D(\{x\})\}) \geq \beta \text{ for given } \beta \quad (6.28)$$

where a measurement procedure meeting (6.28) is called  $\beta$ -measurement procedure denoted  $C_D^{(\beta)}$

$$3. \quad C_D(\{x\}) \neq \emptyset \text{ for } x \in \mathcal{X}(\mathcal{D}) \quad (6.29)$$

It is easily seen (for details see [50]) that for any given  $\beta$ -prediction procedure (6.5)

$$A_X^{(\beta)} : \mathcal{T}_D(\mathcal{D}) \rightarrow \mathcal{T}_X(\mathcal{X}(\mathcal{D})) \quad (6.30)$$

the following calibration  $C_D$  meets the condition (6.27) and (6.28):

$$C_D^{(\beta)}(\{x\}) = \left\{ d \mid x \in A_X^{(\beta)}(\{d\}) \right\} \quad (6.31)$$

The third condition (6.29) means that for any outcome of the measurement experiment  $x \in \mathcal{X}(\mathcal{D})$  there is at least one  $d \in \mathcal{D}$  so that  $x \in A_X^{(\beta)}(\{d\})$  holds. Condition (6.29) assures that with certainty the measurement procedure yields a non-empty measurement result. Condition (6.29) implies that the  $\beta$ -prediction procedure  $A_X^{(\beta)}$  must meet the following condition, which is not required for a prediction procedure of the prediction class:

$$\bigcup_{d \in \mathcal{D}} A_X^{(\beta)}(\{d\}) = \mathcal{X}(\mathcal{D}) \quad (6.32)$$

Thus, any  $\beta$ -measurement procedure  $C_D^{(\beta)}$  is defined by a  $\beta$ -prediction procedure  $A_X^{(\beta)}$  of type (6.5) which meets additionally condition (6.32).

The totality of measurements  $C_D^{(\beta)}(\{x\})$ ,  $x \in \mathcal{X}(\mathcal{D})$  or equivalently the totality of the corresponding  $\beta$ -predictions  $A_X^{(\beta)}(\{d\})$  represent the  $\beta$ -measurement procedure and form in the  $(d, x)$ -space a subset of the Uncertainty Space  $\mathcal{U}_{X,D}$ , where for  $d \in \mathcal{D}$  the corresponding predicted event  $A_X^{(\beta)}(\{d\})$  occurs with a probability of at least  $\beta$ . This subset is called  $\beta$ -Uncertainty Space for  $D$  denoted  $\mathcal{U}_D^{(\beta)}$ . Although having similar properties as the earlier introduced  $\beta$ -Uncertainty Space  $\mathcal{U}_X^{(\beta)}$  for  $X$ , both sets are in general different, as they serve different purposes. In Figure 6.6 a  $\beta$ -measurement procedure is given by  $\mathcal{U}_D^{(\beta)}$  which is imbedded in the entire Uncertainty Space  $\mathcal{U}_{X,D}$ . Notice the difference with the  $\mathcal{U}_X^{(\beta)}$  displayed in Figure 6.4, which does not meet (6.32).

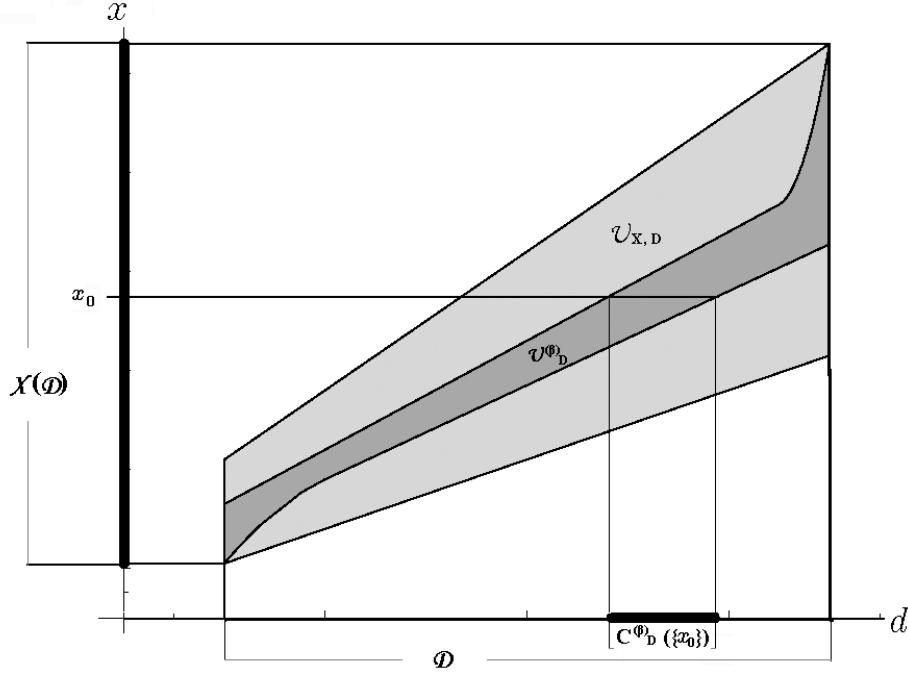


Figure 6.6:  $\beta$ -Measurement procedure  $C_D^{(\beta)}$  graphically represented by the corresponding  $\beta$ -Uncertainty Space  $\mathcal{U}_D^{(\beta)}$  for  $D$  imbedded into  $\mathcal{U}_{X,D}$ .

The smaller a measurement  $C_D^{(\beta)}(\{x\})$  the better it meets the purpose of determining the unknown true value  $d_0$  of  $D$ . However, a  $\beta$ -measurement procedure is subject to randomness and, therefore, an appropriate optimization criterion must refer to the totality of measurements.

Define by

$$S(x) = \left| C_D^{(\beta)}(\{x\}) \right| \text{ for } x \in \mathcal{X}(\mathcal{D}) \quad (6.33)$$

the size of the measurements representing the measurement procedure. Then in [50] the 1st moment  $E_{X|\mathcal{D}}[S]$  is proposed as more or less self-evident optimization criterion (for details see [50]). A measurement procedure with reliability level  $\beta$  is called optimal denoted  $*C_D^{(\beta)}$  if

$$E_{X|\mathcal{D}} \left[ \left| *C_D^{(\beta)}(\{x\}) \right| \right] \leq \min_{C_D^{(\beta)}} E_{X|\mathcal{D}} \left[ \left| C_D^{(\beta)}(\{x\}) \right| \right] \quad (6.34)$$

where the minimum is taken over all measurement procedures for  $D$  with reliability level  $\beta$ .

An optimal  $\beta$ -measurement procedure  $*C_D^{(\beta)}$  is called a “minimum  $\beta$ -measurement procedure”.

Analogously to the minimum upper bound  $\beta$ -prediction procedures and maximum lower bound  $\beta$ -prediction procedures, there are minimum upper bound  $\beta$ -measurement procedures and maximum lower bound  $\beta$ -measurement procedures defined in [50].

The stochastic approach elaborated and taught by *Stochastikon Magister* not only leads to new representations of old methods as shown in the learning units 2.3.9 and 2.3.10, but also to the development of new procedures as illustrated in learning units 2.3.11 and 2.3.12.

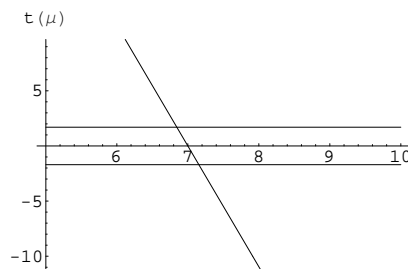
### 6.5.2 Measurement Procedures for $E[X]$ and $V[X]$

The first moment  $E[X]$  and the variance  $V[X]$  of a random variable  $X$  are probably the most important deterministic variables and, therefore, the development of appropriate stochastic measurement procedures is of utmost importance. In fact, since the beginning of statistics, the problem of determining the actual values  $\mu$  and  $\sigma^2$  of  $E[X]$  and  $V[X]$  built a focus of joint efforts. When in 1908 William S. Gosset succeeded in deriving the so-called  $t$ -distribution the problem to determine the value of  $E[X]$  was solved in an approximate but rather general way. In the learning units 3.3.9 and 2.3.10 the two measurement procedures  $C_{E[X]}^{(\beta)}$  and  $C_{V[X]}^{(\beta)}$  are introduced based on an appropriate Bernoulli Space.

In order to illustrate the difference between the traditional approach and the stochastic approach, the resulting measurement procedures or calibrations  $C_{E[X]}^{(\beta)}$  and  $C_{V[X]}^{(\beta)}$  are stated. For details, refer to the learning units 2.3.9 and 2.3.10 of the second course in *Stochastikon Magister*.

$$\begin{aligned} C_{E[X]}^{(\beta)}(\{t(\mu)\}) &= C_{E[X]}^{(\beta)}\left(\left\{\frac{\bar{x}-\mu}{s}\sqrt{n}\right\}\right) \\ &= \left\{\mu \mid \bar{x} + Q_{T|\{\mu\}}^{(\ell)}\left(\frac{1+\beta}{2}\right)\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + Q_{T|\{\mu\}}^{(u)}\left(\frac{1+\beta}{2}\right)\frac{s}{\sqrt{n}}\right\} \end{aligned}$$

Let  $\mathcal{D} = \{\mu \mid 5 \leq \mu \leq 10\}$ ,  $n = 30$  and  $\beta = 0.95$  and consider a realization  $(x_1, x_2, \dots, x_{30})$  with  $\bar{x} = 7$  and  $s^2 = 0.5$ . Then the following graph displays the observation  $t(\mu)$  as function of  $\mu$  given by the straight line running from top left to bottom right and the prediction procedure  $A_T^{(\beta)}(\{\mu\})$  given by the two parallels.



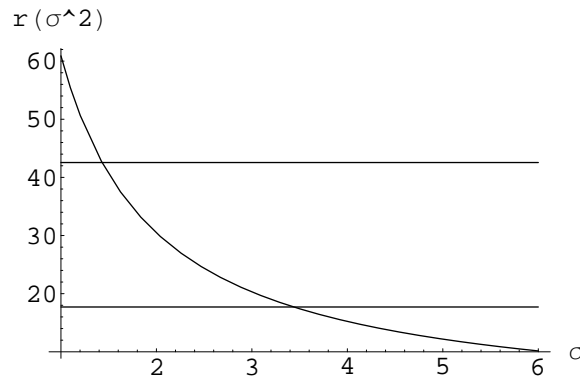
The measurement result contains all those values of  $\mu \in \mathcal{D}$ , for which  $t(\mu)$  is within the prediction given by the parallels. The measurement result for the above given realization of the measurement experiment is as follows:

$$C_{E[X]}^{(0.95)}\left(\frac{7-\mu}{0.5}\sqrt{30}\right) = \{\mu \mid 6.743 \leq \mu \leq 7.257\}$$

Similarly, the measurement procedure  $C_{V[X]}^{(\beta)}$  for the variance of  $X$  takes the following form:

$$\begin{aligned}
C_{V[X]}^{(\beta)}(\{r(\sigma^2)\}) &= C_{V[X]}^{(\beta)}\left(\left\{(n-1)\frac{s^2}{\sigma^2}\right\}\right) \\
&= \left\{\sigma^2 \mid \frac{n-1}{Q_{R(\sigma^2)}^{(u)}\left(\frac{1+\beta}{2}\right)} s^2 \leq \sigma^2 \leq \frac{n-1}{Q_{R(\sigma^2)}^{(\ell)}\left(\frac{1+\beta}{2}\right)} s^2\right\}
\end{aligned}$$

Let  $\mathcal{D} = \{\sigma^2 \mid 0 < \sigma^2 \leq 5\}$ ,  $n = 30$  and  $\beta = 0.95$  and consider a realization  $(x_1, x_2, \dots, x_{30})$  with  $s^2 = 2.1$ . Then the following graph displays the observation  $r(\sigma^2)$  as function of  $\sigma^2$  given by the curve from top left to bottom right, and the prediction  $A_R^{(\beta)}(\{\sigma^2\})$  represented by the two parallels.



The measurement result contains all those values of  $\sigma^2 \in \mathcal{D}$ , for which  $r(\sigma^2)$  is within the prediction  $A_R^{(0.95)}(\{\sigma^2\})$  given by the parallels. The measurement result for the above given realization of the measurement experiment is as follows:

$$C_{V[X]}^{(0.95)}\left(29 \frac{2.1}{\sigma^2}\right) = \{\sigma^2 \mid 1.43 \leq \sigma^2 \leq 3.44\}$$

### 6.5.3 Measurement Procedures for $\min \mathcal{X}$ and $\max \mathcal{X}$

Besides the moments of a random variable  $X$ , the range of variability  $\mathcal{X}$  is of special importance and, therefore, measurement procedures for the minimum value  $\min \mathcal{X}$  and the maximum value  $\max \mathcal{X}$  of the range of variability of  $X$  should be derived and made available. Actually, in many cases only these extreme values are of interest.

Based on a sample  $(X_1, \dots, X_n)$  and the sample functions  $X_{\min} = \min(X_1, \dots, X_n)$  and  $X_{\max} = \max(X_1, \dots, X_n)$  the probability distribution of  $X_{\min}$  and  $X_{\max}$  are determined as functions of  $\min \mathcal{X}$  and  $\max \mathcal{X}$ , respectively.

Then the following measurement procedure for  $\min \mathcal{X}$  is obtained:

$$C_{\min \mathcal{X}}^{(\beta)}(\{x\}) = \left\{a \mid Q_{X_{\min}\{a\}}^{(u)}(\beta) \leq a \leq x\right\}$$

The measurement procedure for  $\max \mathcal{X}$  is obtained analogously. For details see *Stochastikon Magister* learning units 2.3.11 and 2.3.12 or the relevant articles [217, 197, 198].



## 6.6 Exclusion Procedures

A minimum  $\beta$ -measurement procedure starts from a Bernoulli Space  $\mathbb{B}_{X,D}^{(0)} = (\mathcal{D}, \mathcal{X}, \mathcal{P})$  and arrives at a Bernoulli Space  $\mathbb{B}_{X,D}^{(1)} = (C_D^{(\beta)}(\{x\}), \mathcal{X}, \mathcal{P})$ , where the extent of reduction of the ignorance space reflects the learning effect. Using a minimum  $\beta$ -measurement procedure guarantees a maximum learning effect.

Sometimes, there is no interest in determining directly the unknown past fact represented by the true value  $d_0$  of  $D$ , but in proving that a certain subset  $\mathcal{D}_0$  may be excluded from the ignorance space, i. e., from the set of potential values of  $D$ . The aim is to exclude  $\mathcal{D}_0$  from  $\mathcal{D}$  and, therefore, this type of procedure is called “exclusion procedure”.

As the actual value  $d_0$  is not of interest, an exclusion procedure may be compared with a “go/no go gauge”. Consequently, an exclusion procedure is given by a decision function.

Assume that the pair of variables  $(X, D)$  has been identified and the corresponding Bernoulli Space  $\mathbb{B}_{X,D}$  has been derived. Let  $d_0$  be the true, but unknown value of  $D$ . Moreover, let  $\mathcal{D}_0 \subset \mathcal{D}$  and consider the aim of proving the following claim:

$$d_0 \notin \mathcal{D}_0 \quad (6.35)$$

The claim (6.35) refers to a past fact implying that the corresponding procedure belongs to the measurement class. Thus, an experiment (= sample) is necessary, where the sample consists of  $n$  copies of  $X$ . The exclusion procedure is given by a decision function denoted  $\phi_{\mathcal{D}_0}$  and defined by

$$\phi_{\mathcal{D}_0} : \mathcal{X}(\mathcal{D}) \rightarrow \{0, 1\} \quad (6.36)$$

where the value 1 means that  $\mathcal{D}_0$  will be excluded, and the value 0, that it will not be excluded. Thus, the value 1 stands for a success of applying the exclusion procedure, while the value 0 means a failure.

The situation with respect to correct and incorrect decisions may be illustrated in the following way:

initial condition	correct decisions	incorrect decisions
$d_0 \in \mathcal{D}_0$	no exclusion of $\mathcal{D}_0$	exclusion of $\mathcal{D}_0$
$d_0 \notin \mathcal{D}_0$	no exclusion of $\mathcal{D}_0$ or exclusion of $\mathcal{D}_0$	...

The case of no exclusion results in the correct initial Bernoulli Space; the case  $d_0 \notin \mathcal{D}_0$  and exclusion of  $\mathcal{D}_0$  results in a Bernoulli Space with Ignorance Space  $\mathcal{D} \setminus \mathcal{D}_0$ , which is correct too. Only if  $\mathcal{D}_0$  is excluded, but  $d \in \mathcal{D}_0$ , then the exclusion yields a wrong Bernoulli Space and, therefore, constitutes an error.

The reliability of a procedure is defined by the probability that it yields a correct result. Let the reliability requirement be given by the reliability level  $\beta$ . A procedure is called  $\beta$ -exclusion procedure denoted  $\phi_{\mathcal{D}_0}^{(\beta)}$ , if it meets the reliability requirement, i. e., if the following condition holds:

$$P_{X|\{d_0\}} \left( \{x \mid \phi_{\mathcal{D}_0}^{(\beta)}(\{x\}) = 0\} \right) \geq \beta \text{ for } d_0 \in \mathcal{D}_0 \quad (6.37)$$

For deriving a  $\beta$ -exclusion procedure, a  $\beta$ -prediction  $A_X^{(\beta)}(\mathcal{D}_0)$  is needed. Once  $A_X^{(\beta)}(\mathcal{D}_0)$  is available, a  $\beta$ -exclusion procedure for excluding  $\mathcal{D}_0$  is defined as follows:

$$\phi_{\mathcal{D}_0}^{(\beta)}(\{x\}) = \begin{cases} 1 & \text{for } x \notin A_X^{(\beta)}(\mathcal{D}_0) \\ 0 & \text{for } x \in A_X^{(\beta)}(\mathcal{D}_0) \end{cases} \quad (6.38)$$

The graphical representation of a  $\beta$ -exclusion procedure is given in Figure 6.7.

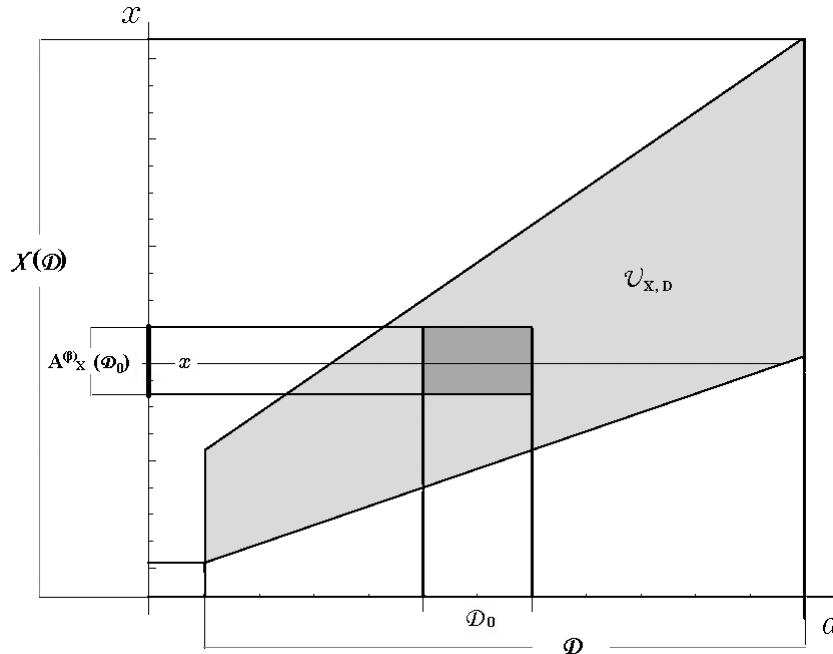


Figure 6.7: Graphical representation of a  $\beta$ -exclusion procedure for  $\mathcal{D}_0$  and  $\beta$ -prediction  $A_X^{(\beta)}(\mathcal{D}_0)$  imbedded into the Uncertainty space  $\mathcal{U}_{X,D}$ .

The purpose of applying  $\phi_{\mathcal{D}_0}^{(\beta)}$  is to exclude  $\mathcal{D}_0$  from the Ignorance Space. Therefore, the larger the probability of exclusion is, the better the procedure serves its purpose. Thus, a  $\beta$ -exclusion procedure is called optimal and denoted  $^*\phi_{\mathcal{D}_0}^{(\beta)}$  if

$$P_{X|D}(\{x | ^*\phi_{\mathcal{D}_0}^{(\beta)}(\{x\}) = 1\}) = \max_{\phi_{\mathcal{D}_0}^{(\beta)}} P_{X|D}(\{x | \phi_{\mathcal{D}_0}^{(\beta)}(\{x\}) = 1\}) \quad (6.39)$$

where the maximum is taken over all  $\beta$ -exclusion procedures for  $\mathcal{D}_0$  based on the same Bernoulli Space, and  $P_{X|D}$  is defined by (5.10). For details see [50].

## 6.7 Classification Procedures

A different problem arises, if a fact must be assigned to one of  $m$  given alternatives with respect to the Ignorance Space. In this case the past situation has to be classified and, therefore, the corresponding procedures are called classification procedures. Similar as in the case of an exclusion procedure, the true value  $d_0$  of the deterministic variable  $D$  is not of interest and, therefore, a classification procedure with  $m$  alternatives is represented by a decision function which may adopt  $m + 1$  values.

Let  $(X, D)$  be the pair of variables of interest and consider the case that there are  $m$  different Bernoulli Spaces

$$\mathbb{B}_{X,D}^{(1)}, \dots, \mathbb{B}_{X,D}^{(m)} \quad (6.40)$$

with

$$\mathbb{B}_{X,D}^{(i)} = (\mathcal{D}_i, \mathcal{X}, \mathcal{P}) \text{ for } i = 1, \dots, m \quad (6.41)$$

where  $\mathcal{D}_i \cap \mathcal{D}_j = \emptyset$  for  $i \neq j$

Relation (6.41) implies that the different alternatives refer only to the knowledge about the past fact, but not to the considered random process represented by  $X$ . The decision function denoted  $\phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}$  is defined as follows:

$$\phi_{\mathcal{D}_1, \dots, \mathcal{D}_m} : \bigcup_{i=1}^m \mathcal{X}(\mathcal{D}_i) \rightarrow \{0, 1, \dots, m\} \quad (6.42)$$

where the value  $i$ , with  $i = 1, \dots, m$  means a decision in favor of the Ignorance Space  $\mathcal{D}_i$  and the value 0 means that no decision in favor of any single alternative can be made. This decision is, of course correct, but constitutes a failure of the procedure.

The possible situations and decisions are displayed in the following table:

initial condition	correct decisions	incorrect decisions
$d_0 \in \mathcal{D}_1$	1.) decision in favor of $\mathcal{D}_1$ 2.) no decision for any alternative	decision in favor of $\mathcal{D}_j$ for $j \neq 1$
...	...	...
$d_0 \in \mathcal{D}_i$	1.) decision in favor of $\mathcal{D}_i$ 2.) no decision for any alternative	decision in favor of $\mathcal{D}_j$ for $j \neq i$
...	...	...
$d_0 \in \mathcal{D}_m$	1.) decision in favor of $\mathcal{D}_m$ 2.) no decision for any alternative	decision in favor of $\mathcal{D}_j$ for $j \neq m$

There are as many incorrect decisions possible when applying a classification procedure as there are alternatives. Therefore, the reliability of a classification procedure with  $m$  alternatives is given by the sequence of probabilities for correct decisions in case of the different alternatives:

$$(p_1, \dots, p_m) \quad (6.43)$$

with

$$p_i = \min_{d \in \mathcal{D}_i} P_{X|\{d\}} \left( \left\{ x \mid \phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}(\{x\}) \in \{0, i\} \right\} \right) \quad (6.44)$$

According to the situation, lower bounds  $\beta_1, \dots, \beta_m$ , for the reliability values  $p_i$ ,  $i = 1, \dots, m$ , are selected. A classification procedure meeting these requirements is called a  $(\beta_1, \dots, \beta_m)$ -classification procedure denoted:

$$\phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)} \quad (6.45)$$

A  $(\beta_1, \dots, \beta_m)$ -classification procedure  $\phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}$  is derived by  $m$  predictions  $A_X^{(\beta_1)}(\mathcal{D}_1), \dots, A_X^{(\beta_m)}(\mathcal{D}_m)$ , which meet the following additional condition (see (6.32)):

$$\bigcup_{i=1}^m A_X^{(\beta_i)}(\mathcal{D}_i) = \bigcup_{i=1}^m \mathcal{X}(\mathcal{D}_i) \quad (6.46)$$

where (6.46) is needed as otherwise the decision function would not be defined for some values  $x$  of  $X$ .

Next define the following sets

$$\begin{aligned} \mathcal{A}_i &= A_X^{(\beta_i)}(\mathcal{D}_i) \setminus \left( \bigcup_{j \neq i}^m A_X^{(\beta_j)}(\mathcal{D}_j) \right) \text{ for } i = 1, \dots, m \\ \mathcal{A}_0 &= \left( \bigcup_{i=1}^m \mathcal{X}(\mathcal{D}_i) \right) \setminus \left( \bigcup_{i=1}^m \mathcal{A}_i \right) \end{aligned} \quad (6.47)$$

The sets  $\mathcal{A}_i$ ,  $i = 0, 1, \dots, m$ , are disjoint and  $\mathcal{A}_i$  is that part of the  $\beta_i$ -prediction, which is not contained in any of the other  $\beta_j$ -predictions,  $j \neq i$ . The set  $\mathcal{A}_0$  is the set of those elements, which are at least in two of the predictions, thus preventing a unique decision. The sets  $\mathcal{A}_i$  define a  $(\beta_1, \dots, \beta_m)$ -classification procedure as follows:

$$\phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}(\{x\}) = \begin{cases} 0 & \text{for } x \in \mathcal{A}_0 \\ 1 & \text{for } x \in \mathcal{A}_1 \\ \dots & \\ m & \text{for } x \in \mathcal{A}_m \end{cases} \quad (6.48)$$

The set  $\mathcal{A}_0$  is called ‘indifference region’, because an observation  $x \in \mathcal{A}_0$  does not allow a positive decision with respect to the alternatives and, thus, is tantamount with a failure. The sets  $\mathcal{A}_i$ ,  $i = 1, \dots, m$  are called ‘acceptance regions’, because an observation  $x \in \mathcal{A}_i$  yields the decision in favor of  $\mathcal{D}_i$ .

Note that classification procedures just as measurement and exclusion procedures are essentially based on predictions, which form the unifying element in Bernoulli Stochastics.

In Figure 6.8, the graphical representation of a  $(\beta_1, \beta_2, \beta_3)$ -classification procedure is displayed. The three acceptance regions  $\mathcal{A}_i$ ,  $i = 1, 2, 3$ , are imbedded into the Uncertainty Space  $\mathcal{U}_{X,D}$ .

The purpose of a classification procedure is to decide in favor of one of the given alternative. Therefore, a  $(\beta_1, \dots, \beta_m)$ -classification procedure  $\phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}$  is called optimal and denoted  $^* \phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}$ , if the probability of a failure to decide in favor of one of the alternatives is minimum, i. e., if following holds

$$\begin{aligned} P_{X|\mathcal{D}} \left( \{x \mid ^* \phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}(\{x\}) = 0\} \right) \\ = \min_{\phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}} P_{X|\mathcal{D}} \left( \{x \mid \phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}(\{x\}) = 0\} \right) \end{aligned} \quad (6.49)$$

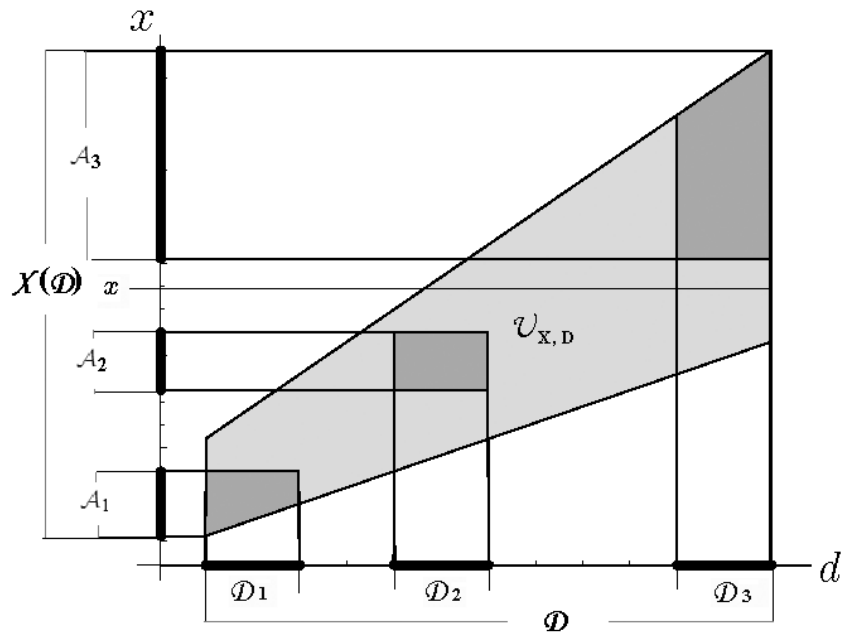


Figure 6.8: Graphical representation of a  $(\beta_1, \beta_2, \beta_3)$ - classification procedure given by the acceptance regions.

With (6.49) an optimization criterion is obtained which directly reflects the capability of the procedure to be successfully applied. The absolute minimum of the failure probability is given by the value 0, which is attained if

$$\left\{ x \mid * \phi_{\mathcal{D}_1, \dots, \mathcal{D}_m}^{(\beta_1, \dots, \beta_m)}(\{x\}) = 0 \right\} = \emptyset \quad (6.50)$$

Clearly, (6.50) follows if the predictions  $A_X^{(\beta_i)}(\mathcal{D}_i)$  for  $i = 1, \dots, m$  are mutually disjoint, which is warranted if the distances between the alternatives are sufficiently large or if the required reliabilities are sufficiently small. If both conditions are not met, then by increasing the expense (i. e., the sample size) the success probability may be improved.

# Chapter 7

## Conclusions and Tasks

### 7.1 Conclusions

There are several features of the science of Bernoulli Stochastics as it is developed in [49] and [50], and outlined above, which distinguish this science from classical quantitative sciences. These particular features refer to the way of modelling on the one hand and the possibilities to utilize the model on the other.

#### 7.1.1 Stochastic Modelling

Unlike in any other branch of science, Bernoulli Stochastics uses a basic model and provides strict rules how to select the mathematical concepts in accordance to real world on the one hand and the available knowledge on the other. The close link of Bernoulli Stochastics to real world constitutes the opposite to the ‘ideal and isolated conditions’ which are the basics for many ‘laws of nature’ in physics. Below some of the characteristic features of stochastic modelling are listed.

1. Bernoulli Stochastics is based on the world view of connectivity in contrast to the prevailing deterministic world view. Thus, Bernoulli Stochastics avoids to divide world in small isolated systems for finding unrealistic deterministic laws.
2. In Bernoulli Stochastics, future and past are fundamentally different and, therefore, have to be described in a different way, in contrast to classical physics where future and past are assumed to be deterministic.
3. Bernoulli Stochastics reveals randomness as nature’s fundamental principle of order, preventing chaotic developments and identifies it as the only means for reducing human ignorance with respect to (past) facts.
4. Bernoulli Stochastics centers on the variability of real world and, therefore, uses as means for describing real world sets and set functions instead of points and point functions as in classical sciences.
5. The two sources ‘ignorance’ and ‘randomness’ are clearly distinguished and directly modelled in Bernoulli Stochastics and, therefore, the models reflect the actual state of knowledge and are immediately improved whenever new knowledge becomes available.
6. Stochastic modelling considers the trivial fact that for describing a complex object one needs more information than for describing a simple object.

The justification of the stochastic approach is the fact that the existence of randomness cannot be excluded and, therefore, randomness has to be incorporated directly into the description.

### 7.1.2 Application of Bernoulli Stochastics

The application of Bernoulli Stochastics is also characterized by some unique features.

1. There is no field of human activities, which could not be benefited by the application of Bernoulli Stochastics procedures.
2. The quality of a stochastic procedure is clearly defined as the degree of meeting its purpose. Thus, in contrast to statistics, Bernoulli Stochastics allows the determination of procedures which serve their purpose best.
3. Any stochastic procedure is based on predictions, allowing a unified approach in developing procedures and supporting an easy understanding.
4. A characteristic feature of stochastic predictions is the known reliability, which distinguishes a scientific prediction from fortunetelling.
5. The result of stochastic procedures are basically sets and, therefore, any stochastic procedures can be represented in principal by a graphical representation making it more transparent and better comprehensible.

### 7.1.3 Barriers to the Adoption of Bernoulli Stochastics

One should think that the above listed features of Bernoulli Stochastics would make sure its quick introduction, at least to the science. However, this is not at all the case. The reasons are manifold.

1. Bernoulli Stochastics requires abandoning of thinking deterministically, i. e., in cause-effect chains. Mankind has cultivated deterministic thinking for hundreds of generations. Children are taught to think in cause-effect relations from the very beginning of their life and this is continued in school, professional training and during their working life. Thus, changing the deeply rooted pattern of thinking is almost impossible for many persons.
2. Science based on deterministic thinking has achieved many seemingly great successes and, clearly, demanding to abandon something which is familiar and has been successful may be looked upon as almost unreasonable.
3. All industries have been developed as a consequence of deterministic science and technology. A change of the basic principles would have dramatic consequences.
4. Modelling situations by Bernoulli Spaces and answering the relevant questions by means of stochastic procedures would lead to transparent and rational decision making and deprive many decision makers of power.

Taking these points into consideration, a general acknowledgement of Bernoulli Stochastics by science and societies could be more a question of centuries than of decades.

## 7.2 Tasks

The entire task of *Stochastikon Magister* is to overcome the obstacles, which hinder a large-scale introduction of Bernoulli Stochastics to science and education and, thereby, support a universal paradigm change. Given the actual situation to the disadvantage of Bernoulli Stochastics as described in the previous section, one would have judged only a few years ago the chances of Bernoulli Stochastics to be negligible, as the traditional ways of disseminating new knowledge would hardly be available for Bernoulli Stochastics and more severe would not suffice to reach people all over the world in foreseeable future.

Fortunately, the last decade saw a revolution of unimaginable dimension in the field of information technology. Nowadays, any knowledge generated at an arbitrary place can be made available globally almost immediately. By means of e-learning self-learning and far-distance-learning have become worldwide a matter of course.

Besides the amazing possibility of spreading knowledge, modern information technology has also led to a revolution in the human capabilities to solve difficult mathematical and numerical problems. Problems which could be solved in earlier times only in years by experts, can now be solved within a split second by anybody.

Evidently, the development of information technology fit extremely well into the situation with respect to the dissemination of Bernoulli Stochastics and may help to overcome the barriers.

However, in order to use the internet as transmitter for material as complex and difficult as Bernoulli Stochastics, it must be appropriately prepared. To this end a research group has been established at the Institute of Economics (Volkswirtschaftliches Institut) of Würzburg University aiming at developing in cooperation with Stochastikon GmbH, Würzburg, the comprehensive information system *Stochastikon*. One part of this project is the development of *Stochastikon Magister*, which is presented in this thesis.

### 7.2.1 Stochastikon and Stochastikon Magister

The entire system *Stochastikon* will consist of a number of on-line components each having certain tasks. The component *Stochastikon-Encyclopedia* may be used as a work of reference, the second component *Stochastikon-Mentor* offers professional support in analyzing situations and problems, and the component *Stochastikon-Calculator* is able to solve the extremely sophisticated mathematical and numerical problems occurring in Bernoulli Stochastics.

These components of the system *Stochastikon* are to be designed so as to represent the core of Bernoulli Stochastics containing all the knowledge and all the expertise for solving problems involving uncertainty. However, just offering knowledge and support is not enough for establishing a new approach in science. At the same time the possibility for systematic learning the principles and techniques must be offered. In case of Bernoulli Stochastics neither teachers nor sufficient textbooks are available and, therefore, the only possibility of disseminating Bernoulli Stochastics is an e-learning programme, which is developed and presented within this thesis and which constitutes a further component of the system *Stochastikon*. It is named *Stochastikon-Magister* and it shall prepare the acceptance of Bernoulli Stochastics and, hence, of the system *Stochastikon* by offering an adequate education for Bernoulli Stochastics.

The basic model in Bernoulli Stochastics is the Bernoulli Space consisting of three components each having an easy to grasp meaning. The first is a set representing ignorance, the second a system of sets representing variability and, therefore, appropriate for graphical representation. The third component describes the random structure related to the transition from past to



future and, thus, seems to be also very qualified for graphical representation. Besides understanding the model, the capability of applying stochastic procedures is of utmost significance for a successful introduction of Bernoulli Stochastics. As shown in the preceding chapters any stochastic procedure is based on predictions, which may be visualized in relation to the available knowledge as subsets of the Uncertainty Space. So far only computer technologies can fully satisfy and qualify these two requirements: dynamically illustrate the abstract concepts which contain high mathematical and numerical complexity, implying that Bernoulli Stochastics and e-learning seem to fit well.

Thus, this thesis aims at developing an e-learning programme for Bernoulli Stochastics, which is called *Stochastikon Magister* and which may be accessed via internet from any part of the world. The operability of the e-learning programme for Bernoulli Stochastics shall be shown by including the first version of two courses covering the stochastic model and the stochastic procedures. Starting with the very elementary but nevertheless novel concepts of ignorance and randomness, it should lead to the ability to understand and to derive Bernoulli Spaces and to apply stochastic procedures for solving problems and for supporting decision-making.

The two courses in Bernoulli Stochastics which have been developed within the dissertation project and which are outlined above are added as a supplement to this dissertation.

**Part III**

**Stochastikon Magister**



# Chapter 8

## E-Learning and Bernoulli Stochastics

### 8.1 Introduction

In January 2005 a search with Google yields the number of approximately 10,700,000 entries for “e-learning” reflecting the popularity of e-learning on the one hand and the fact that e-learning has become a “big business” on the other. Already in 2001 Dichanz and Ernst report in [67] that according to the Financial Times Germany (23/24/25 February 2001) the business volume has been 900 million and that experts expect an increase up to 4 billion until 2004 only in Germany and according to other sources the global e-learning market value has reached in 2003 the huge value of 230 billion US \$.

The question about the reasons for this tremendous financial success arises. Is it true that e-learning will “shape the future” as many protagonists of e-learning claim or is application of the Information and Communication Technology (ICT) in the education sector not as smooth and straightforward as the development seems to suggest.

Traditionally, education has been an extremely profitable business field. However, an access to the traditional education business with schools and teachers requires big investments as for being licensed, many state-controlled restrictions with respect to competence and facilities have to be observed. With e-learning everything is different. For starting a commercial e-learning programme there is only one requirement to meet namely to have the necessary technical equipment and programming competence. Hence, almost anybody can start an e-learning programme and trying to make money with it. From this point of view the surprising global success of e-learning programmes and e-learning firms seems to be not very surprising.

On the other hand, today, traditional education can hardly stand the tremendous pressure of educational demands from the fast changing societies, the incredible knowledge expansion and lifelong education requirements especially in higher lever education. This pressure shows up in all quantity, quality, content and style dimensions. Distance-learning, under the help of rapid developing, versatile and ubiquitous technologies especially internet, which has become dominating among other distance-education media, has arouse huge expectations to be the best candidate for meeting the future education challenge.

Next the questions about the possibilities and difficulties in developing a “good” e-learning programme have to be addressed in order to derive some guidelines for our anticipated e-learning programme meant for Bernoulli Stochastics.

## 8.2 The Success of E-Learning

There are many definitions and descriptions of e-learning. Dichanz [67] cites the following definition which represents the position about e-learning in commerce [234]: “E-learning is an approach which uses different internet and web technologies for enable, evoke, advance and or to moderate learning processes and competence developments. The new internet-based learning systems and architectures can lead everywhere to a qualification ‘just in time’ meeting unified quality standards.” Dichanz also cites the president of Cisco Systems with several definitions:

- “There are two fundamental equalizers in life - the internet and education. E-learning eliminates the barriers of time and distance creating universal, learning-on-demand opportunities for people, companies and countries.” [43]
- “Learning is a transparent event in the context of solving problems. The quicker one learns, the quicker one performs. The ‘if’ of e-learning is over - the relevant question now is who will become the early adopters that will enjoy the benefits and competitive advantage e-learning will yield.” [234]
- “A common belief among education-oriented executives is that learning faster and better may be the only sustainable competitive advantage . . . “

However, these positive opinions could not be unanimously verified by empirical observations. There are many reports on positive experiences, but there are also negative ones. In Ludwig et al. [148], for example, the following report is given: “Weigand (2000) used the program WebCT in a semester course ‘Computers in mathematics education’ for 150 students to provide the students with internet materials, especially internet-based lecture notes, exercises, an electronic discussion and panel and an internal course email communication system. . . . At the end of the semester, the students had to pass a written final test. However, the results of the internet-supported course were not as good as in the previous years when traditional learning methods had been used, and the internet-supported course did not contribute to better results in the written examination.”

Dichanz notes about the many positive appraisals of e-learning [67]: “These positive ‘experiences’, ‘observations’, ‘opinions’, ‘impressions’ are a weak foundation for ensuring the expectations concerning e-learning. They are based rather on hopes and wishes than on clear data, facts, comparisons and controlled experiments.”

As a first conclusion of this unclear situation, it is proposed in [11] to focus much more on the process of learning when developing and organizing an e-learning system. The different roles of ‘teaching’ and ‘learning’ must be more carefully considered.

## 8.3 The Process of Learning

Dichanz and Ernst give in [67] a step-wise description of the learning process as follows:

1. Learning as a process for acquiring information.
2. Learning as a process for acquiring information and processing experience.
3. Learning as a process for acquiring information and processing experience that effects a long-term change in the consciousness of the learner.

4. Learning as a process for acquiring information and processing experience in which the learner integrates new information and experience into his/ her current knowledge base.
5. Learning as a process for acquiring information and processing experience in which the learner perceives, selects and integrates new information and experience into his/her current knowledge base, thereby changing it.
6. Learning as a process for acquiring information and processing experience, in which the learner selects and constructs knowledge that is useful and appropriate for him/herself and in turn uses this to drive and determine his/ her own continuous learning process.
7. Learning that becomes an individual process of interaction between the individual and his/ her environment, in which the subjective reality of the learner is actively constructed.

Accordingly, learning is an individual process of interaction between a person and his environment. It is a self-guided process and, clearly, at least any adult has a rich learning experience and a clear goal of learning in mind. The role of e-learning should be according to Dichanz and Ernst, to rouse curiosity, to maintain motivation, to provide a challenging learning environment and to support individually the learning process. They separate the development and application of ICT for learning to three different stages:

1. Use of ICT for learning was characterized by experimentation and innovation, and driven by pedagogist creators who were enthusiastic amateurs and frequently used games and activity based learning to supplement and build on more traditional forms of learning.
2. The development of ICT for learning constituted a period of entrenchment, i. e., technologies were developed for administrators and managers to control teaching and learning, while pedagogy was subsumed within the doctrine of Instructional Design, and thus they named this phase as driven by technology.
3. As of the widespread disappointment with the results of phase two development (expensive and only works within the limitations of the specific industrial applications), they claim that “easy learning, effective learning, entertaining learning, learning by means of media are promises, maybe programs for the production of learning environments, however, evidence for their superiority are hardly to be found.” They propose to give up the present technological driven development approach in favor of a more pedagogical approach on the grounds that learning processes can at best be promoted only marginally by using media or media programs, they suppose an Open Source Software and standards which allow the accumulation of innovation and facilitate creativity activity and innovation, i. e., the development of rich pedagogical applications, within an administration and learning environment.

Summarizing one can state that despite great expectations, considerable efforts and investments, the use of information and communication technologies for learning remains problematic.

In the case of Bernoulli Stochastics in the sense used here, it is the only possibility for disseminating the knowledge. And moreover, the fact that there is no traditional teaching programme for Bernoulli Stochastics and the circumstance that one of the primary objects is to provoke a fundamental change in thinking makes us intuitively believe that the new media are particularly appropriate for teaching and learning Bernoulli Stochastics.

However, because the above observations concerning a ‘technological driven development approach’ seem to be very justified, the project *Stochastikon Magister* is developed in a way that technological possibilities were utilized to meet pedagogical requirements. In other words the teaching contents played the primary role, while technology took over a solely supporting role.

## 8.4 Requirements for *Stochastikon Magister*

The above described situation at the outset of the project to develop an e-learning programme for Bernoulli Stochastics was insofar unique as there was no basis, neither on the side of e-learning nor on the side of the curriculum, the programme could rest on. As a consequence, the work on *Stochastikon Magister* was started without a blueprint with respect to the curriculum and without a prototype or development toolkit for solving the problem of the IT part.

Any e-learning system should not be developed independently of the knowledge to be imparted. Therefore, both problem areas had to be considered simultaneously, i. e., the thesis had to cover the development of the content of the curriculum in Bernoulli Stochastics and the development of an associated e-learning environment.

Because of the aim of this e-learning project, the conformance between content and IT realization had to be taken into account, and the development of a preliminary content of the curriculum in Bernoulli Stochastics was identified as a crucial prerequisite for the *Stochastikon Magister* programme. As to the curriculum to be developed the following requirements were identified:

1. The content of the curriculum with respect to Bernoulli Stochastics must be based on the two seminal papers [49] and [50] and should cover their contents.
2. The curriculum should be designed for students with a mathematical background as for instance given by an introductory course in mathematics offered in many university faculties.
3. The teaching materials should not adopt the mathematical *Definition-Theorem-Proof* style, because of didactic reasons as the programme should not only address students of mathematics.
4. The curriculum should be structured similar to classroom teaching, because learners are used to it and because there is no supplementing classroom teaching.

The preparation of the content of curriculum was necessary as argued above, but should be considered as preliminary, because of two reasons:

- This project is the first attempt to develop teaching courses in Bernoulli Stochastics and, therefore, the present versions will be improved as soon as relevant experiences are available.
- The courses developed here contain only the most fundamental ideas and concepts of Bernoulli Stochastics. What will be necessary are additional courses focusing on the needs of learners from special fields of application and offering supplementing contents, for instance in mathematics.

As to the e-learning environment itself the following requirements were set at the beginning:

5. Bernoulli Stochastics is not part of the curriculum in traditional education, therefore, *Stochastikon Magister* must be designed in a way not assuming the possibility of any concomitant classroom teaching.
6. *Stochastikon Magister* should be able to exploit the possibilities of the world wide web and admit learners independently of the language, therefore it should be designed multi-lingual.
7. Bernoulli Stochastics is fully mathematized and, therefore, the authoring system of *Stochastikon Magister* must support the production of mathematical texts and must allow for an easy revision and reuse of existing texts.
8. Bernoulli Stochastics requires a new way of stochastic thinking instead of the prevailing deterministic thinking. Therefore, *Stochastikon Magister* must include features supporting the required stochastic thinking.
9. *Stochastikon Magister* shall be part of the master system *Stochastikon*; therefore, it must be able to communicate and take advantage of the existing components and of any future component of the master system *Stochastikon*.

Finally, *Stochastikon Magister* should meet certain technical specifications with respect to its IT realization:

10. *Stochastikon Magister* must not require a special running environment (browser, configuration, etc.), as this would limit the circle of potential users. Contrary, it should run in all commonly existing environments. Besides, the usage of *Stochastikon Magister* should be based on free loadable assistant tools such as Acrobat Reader, Java Virtual Machine, etc.
11. *Stochastikon Magister* should be able to handle any input or output format, as otherwise the capability to produce adequate teaching materials would severely suffer. For instance, there are e-learning products that restrict the input to PowerPoint presentations, which are exported to HTML. Such a restriction would make the development of an appropriate e-learning programme in Bernoulli Stochastics impossible.

Taking into due account all these requirements, we again came to the conclusion that none of the free or commercial shells (platforms) or development toolkits would be of help in developing *Stochastikon Magister*, as it should not be based on special requirements with respect to configuration and browser and should not be restricted to certain types of inputs and outputs, and it should be a substitute but not a supplement of classroom teaching and should be able to interactively communicate with other complex IT components. It seemed to be mandatory to have *Stochastikon Magister* completely self-designed and self-developed by using popular standard programming languages and developing tools.



# Chapter 9

## Designing Stochastikon Magister

### 9.1 Standard E-Learning Shell

The shell of a computer-based system is a program package that presents various system-supporting functions and services. An e-learning shell is a comprehensive software package that builds an electronic (mainly web-based) learning environment for supporting development and operation of an e-learning system. An e-learning shell provides a software channel through which knowledge is disseminated by instructors and acquired by learners. There are many types of e-learning shells with different components and different technology. Some are very simple systems representing just an open environment storing textual contents for any person who has access to web, some others are extremely complex systems supporting dozens of functions for instance: registering, authorizing, tracking, controlling, delivering, collaborating, or a master-system integrating multi-platform components, or a subsystem being integrated with other components within a master system.

### 9.2 Commercial E-Learning Shell

The prospect of high profits derived by the e-learning business has prompted authoritative institutes and committees, big software companies and educational consultants to develop standard e-learning shells. In Figure 9.1 a typical structure of such a ‘business solution’ of an e-learning shell is displayed. These shells consist of three levels, the two interfaces, one for the instructor and the other for the learner, and a management system (MS) in between.

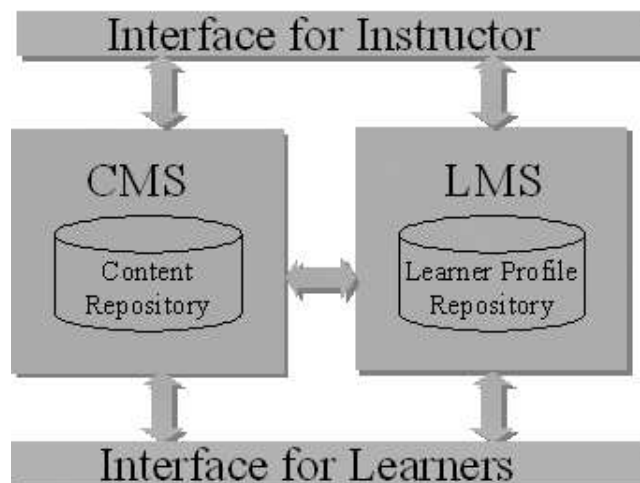


Figure 9.1: Commercial e-learning shell.

The management system is logically separated into two parts. The first one is called Content Management System (CMS) and the second one Learner Management System (LMS).

CMS supports the process of creating, storing and delivering the *teaching content*, i. e., content authoring, installing and the internal cooperation with LMS.

LMS assists the *learning process*, i. e., it contains a learner profile manager, a learning planner, learner registrar, and it manages for example the delivery/participation tracking, assessment and testing tracking.

The above shown structure is typical for what Dichanz and Ernst call ‘technological driven’ e-learning shell for commercial use, in which ‘administration’ and ‘control’ may be very well developed but pedagogical aspects shrink to ‘replicate existing teaching and learning paradigms’ [67] in some specific forms depending on the abilities of CMS, which ‘assembles’ the teaching material into the learning system. In order to become a financial success, a commercial e-learning shell tends to be as general (to satisfy any kind of teaching materials) as possible. Frequently this aim exceeds the available technical abilities, and yields a CMS limited to one or two standard formats and not admitting any extension. These restrictions and limitations cause insurmountable difficulties for developing a pedagogical appropriate e-learning system for any challenging content of teaching. Therefore, most universities and institutes prefer to ‘develop’ their own e-learning shells instead of simply buying a commercial frame.

### 9.3 Development of Magister

The e-learning programme *Stochastikon Magister* to be developed does not pursue any commercial purposes and aims at transferring a new science based on a novel way of thinking. This rather ambitious goal can be reached only, if the shell and the learning system are tailored for teaching Bernoulli Stochastics taking into account its characteristic features:

1. Bernoulli Stochastics is a quantified science using extensively mathematics as language for deriving and formulating the rules for detecting, describing and verifying relations between past and future developments. Thus, the system must be capable of easily generating and displaying mathematical formulas.
2. Bernoulli Stochastics is essentially based on sets, system of sets and relationships between sets. Sets are more difficult than points and, therefore, an appropriate means for visualizing sets is necessary. Thus, the system must be capable of easily generating and displaying graphics of sets and functions.
3. Stochastic procedures have an extremely high mathematical and numerical complexity and, therefore, the e-learning system must have access to powerful mathematical calculation software in order to generate solutions for given problems.

Especially the last requirement necessitates that *Stochastikon Magister* cannot be developed as an isolate system, but must be a subsystem of a master system in Bernoulli Stochastics. This master system is given by *Stochastikon*. Each of the subsystems of *Stochastikon* must be able to communicate, support and get support by each of the other subsystems. Thus, the subsystems have to be harmonized with each other and, clearly, this represents an important requirement for the development of the anticipated e-learning system in Bernoulli Stochastics *Stochastikon Magister*. In Figure 9.2 the master system *Stochastikon* with its subsystems is sketched.



Figure 9.2: Components of *Stochastikon*.

In Figure 9.3 connections between *Stochastikon Magister*, *Stochastikon Graphics* and *Stochastikon Calculator* are displayed. It illustrates the information and functional supports within Magister and other *Stochastikon* components.

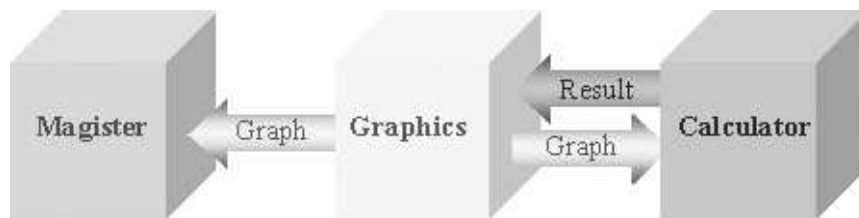


Figure 9.3: Information and functional supports within Magister and other components of *Stochastikon*.

The incorporation of Magister into the master system *Stochastikon* on the one hand and the special requirements of Bernoulli Stochastics on the other, rule out the possibility to use a commercial standard e-learning shell. Therefore, the only way is to develop *Stochastikon Magister* without relying on an existing shell.

Fortunately, the entire development of Magister and other subsystems of *Stochastikon* are carried out within one project and, therefore, the pedagogical and technical expertise of the team members yield a large synergy by cooperating in deciding and realizing the required or requested features of the subsystems. Moreover, this type of development also provides the possibility to experiment with different solutions with respect to certain subsystems.

In the course of experiments it turned out that it would be desirable to represent knowledge by *Stochastikon Magister* in the most coordinate way, by multiple styles and through different multimedia using specifically developed tools. Sets and procedures are appropriately represented by static graphs and by dynamic interactive graphical representations, which are generated under the help of *Stochastikon Graphics*. Examples should be not too simplified and should be taken from real life. It turns out to be desirable that learners can practice their own cases in an interactive graphical ‘laboratory’ based on the computing power of *Stochastikon Calculator* and the visual ability of *Stochastikon Graphics*.

This mode of development gives the chance for *Stochastikon Magister* to reach ‘the third stages of the development and application of ICT for learning’ dreamed of by Dichanz and Ernst [67].

The disadvantage of such a self-development is that it needs to design and implement every detail of a virtual teaching and learning environment, which makes the project much more difficult and time-consuming compared with a development based on ‘buy and install’ system.

In Figure 9.4 the logical structure with its implementing team members of *Stochastikon Magister* is briefly sketched. Each of the components will be described in more detail later.

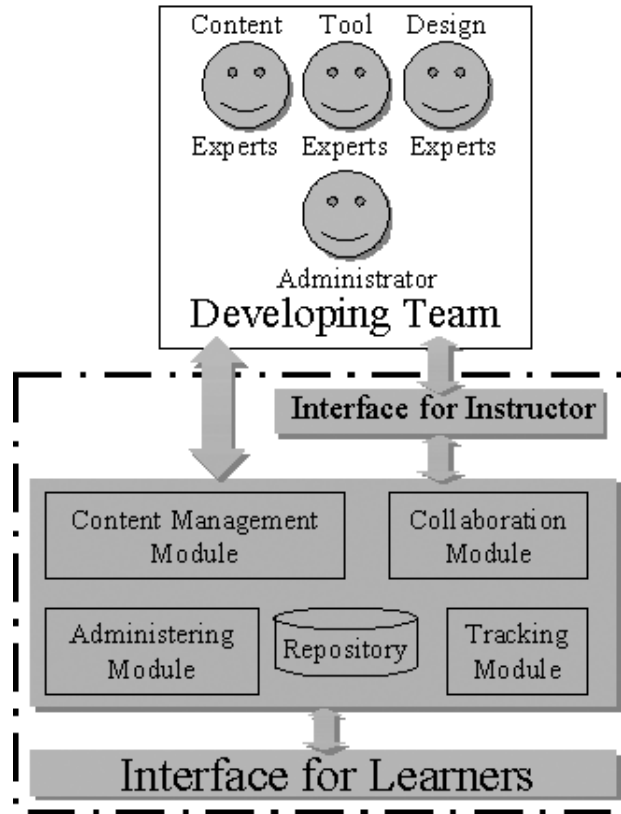


Figure 9.4: Design and Development of *Stochastikon Magister*.

The most significant difference between a development using a commercial e-learning shell and the self-development of *Stochastikon Magister* is that the teaching contents and the technical modules are developed simultaneously supporting harmonization of each other. Content experts and system experts cooperate in the development team and, therefore, the instructors do not have to follow the lines given by a prefabricated Content Management System (CMS), but can base the development of course material solely on the requirement given by the subject matter. Therefore, the actual ‘Content Management Module’ in *Stochastikon Magister* is emphasized on content storing, delivering, managing, maintaining and static content composing.

Besides the ‘Content Management Module’, there are other functional modules in *Stochastikon Magister*, which assure its smooth operation:

- ‘Administering Module’ used for registering, authorizing and profiling the learners.
- ‘Tracking Module’ used for tracking the learning process, i. e., recording, grading, assessing, evaluating the learning process, and so on.
- ‘Collaboration Module’ used managing the communication process between instructor and learners.

Each function module in *Stochastikon Magister* shares information with each other and supports each other.

## 9.4 Organization Design

The organization of learning in *Stochastikon Magister* follows the approved, traditional way of teaching, however, utilizing the specific possibilities made available by modern information technology.

### 9.4.1 Content Structure

As indicated above the preparation of the teaching content represents the most important issue of *Stochastikon Magister*. The material is divided into different courses, each course consists of several modules and, finally, each module is composed of learning units.

Each unit represents one specific learning target and all units are organized identically. The units are arranged in a hierarchical order, i. e., later units assume a learning success with respect to former units.

Figure 9.5 shows the structure of the teaching content. For the time being there are two courses under development. However, *Stochastikon Magister* will be extended parallel to the advancements of Bernoulli Stochastics.

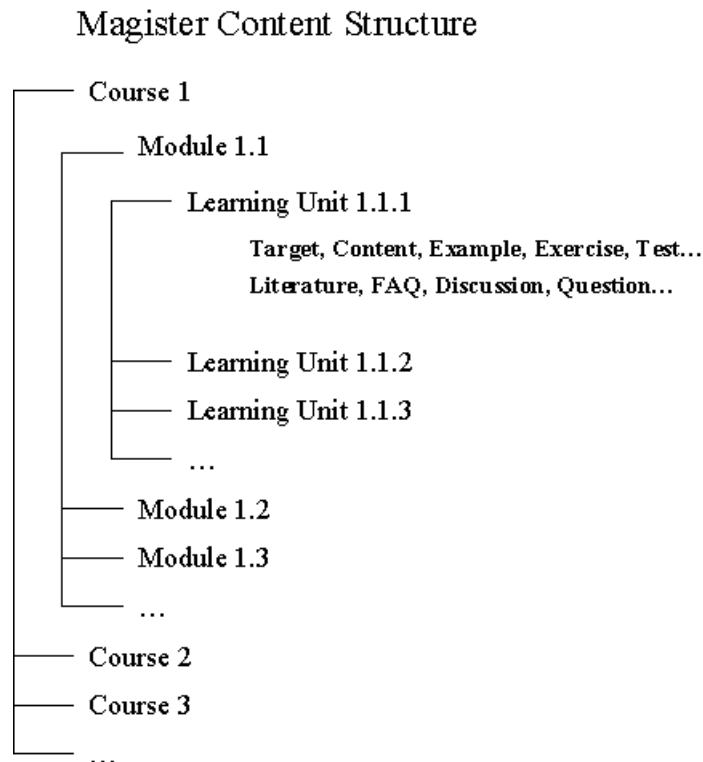


Figure 9.5: Content Structure of *Stochastikon Magister*.

Each unit contains a number of tasks and contains supporting elements called learning activities. The tasks start with the learning content, which is displayed for self-study. The second step consists of examples illustrating the content. The examples shall serve to a better understanding of the content. Once the content is understood, there are exercises for checking and deepening comprehension. Finally, a test is offered for control and grading purposes.

Besides the direct learning issues, there are a number of supporting features offered to the learner. These features start with a proposal of relevant literatures, which is provided on request

by the Content Management Module. A discussion forum allows all instructors and learners to express and publish their opinions about interesting topics. A question panel provides learners a private space to pose own questions to the instructor. Questions and discussion topics provide source material to FAQ - a list of frequently asked questions with answers.

The teaching contents shall be subject of continuous improvements in the course of the further development of Bernoulli Stochastics on the one hand and the growing teaching experiences on the other. Therefore, Version 1, which is presently developed and outlined in this thesis will be followed by further improved versions.

### 9.4.2 Learning Activities

The learning activities refer to an explicitly stated target and are composed by a number of steps, where some of them are mandatory and others only recommended.

- The Target section of each learning unit is explicitly stated in order to motivate the learner and to enable him to control his learning advances.
- The Content section is mainly given as an explanatory text including formulas. As it is to be expected that many learners are not sufficiently familiar with mathematical language the formulas will represent the main obstacle for understanding. Therefore, it is planned in a later state of the project to develop and implement a course in mathematics into *Stochastikon Magister*.
- The Example section consists mainly of verbal examples and static graphics for explaining the formulas and illustrating the meaning of the content. Especially the graphics shall visualize the meaning and, thus, support comprehension.
- The Exercise section reviews and enhances comprehension of the contents by dynamic interactive graphical exercises and practices. They are partly done in a simulation graphical laboratory.
- At the end of the learning process there is the Test section. Any learner having successfully passed the preceding sections is entitled to enter the Test section. When entering a test, a number of examination tasks are posed, which are randomly selected from an examination question bank according to the learner's level or group. The learner has to solve the tasks and will subsequently get the assessment of the test. The marks will be stored and tracked.

The recommended activities in a learning unit provide the possibilities for extending knowledge and for further clarification of difficulties with the contents.

- The Literature section contains a list and links to relevant references and web sources, which can be requested and printed for use besides other materials. As Bernoulli Stochastics itself is in the development phase, the Literature sections are to be filled whenever a new source becomes available.
- The FAQ section will contain a list of selected questions posed by learners or refined from discussions together with their standard answers from the instructor. The answers may contain examples and graphics.

- The Discussion section allows for comments and hints given by the learners and instructors to topics which are of interest with respect to Bernoulli Stochastics. These topics may refer to all activities, to applications, to latest news or to other events related to a unit. The Discussion panel should be continuously organized and regularly updated.
- The Question section enables a direct contact between learner and instructor. If the learner meets problems that cannot be solved by means of the supporting tools, the instructor should be called for help. Moreover, the Question section opens a possibility for personal communications between learner and instructor.

### 9.4.3 Teaching Aims

The e-learning system *Stochastikon Magister* has the general aim to convey a realistic view of the world evolution, i. e., the change from past to future. The learning units aim at identifying concepts and issues being necessary to describe, explain, analyze and learn about the change. The adoption of a stochastic world view and giving up the deterministic one necessitates a change in thinking, which means abandonment of thinking in cause-effect relations in favor of thinking in stochastic relations. The former may be described as thinking in successive fixed points, while the latter means to think in successive sets related to each other in a complex and ever-changing way. This process of learning to view evolution realistically and of solving related problems is separated into steps:

- Evolution is modeled by means of the notion ‘Bernoulli Space’. The needed concepts and issues are subsumed in the learning units of Course 1. These units give an introduction in the concepts of randomness, of ignorance, of the random structure and the fundamental notion of probability. The learning units of Course 1 aim at being a first but decisive step in the process of changing the prevailing causal thinking into stochastic thinking.
- Once a realistic view about the course of evolution has been gained by means of the Bernoulli Space, related problems are to be solved by means of Stochastic Procedures. The needed concepts and issues are subsumed in the learning units of Course 2. These units introduce the procedures, illustrate how they can be used and give the interpretation of the results obtained.
- The above-described two courses contain the basic knowledge and principles concerning Bernoulli Stochastics. Future extensions of *Stochastikon Magister* will provide additional courses addressing specific groups of users and different branches of science.

The two courses mentioned above are entitled ‘Modeling Uncertainty’ and ‘Application’. Their contents can be briefly described as follows:

- ‘Modeling Uncertainty’ introduces the Bernoulli Space as a quantified model describing the change from past to future. Course 1 is divided into three modules: ‘Basic Concepts’ exemplifying the fundamental terms and issues related to uncertainty, ‘Quantification’ for describing the relevant real world issues in a quantified way and ‘Bernoulli Space’ for expressing the existing uncertainty about a future development in a concise and holistic form. The learning units deal with single issues, which are stepwise brought together to form the Bernoulli Space.
- ‘Application’ introduces stochastic procedures for solving all kind of problems occurring in real world. Partial aims are to identify a problem and to select a suitable stochastic procedure to handle it. Course 2 is divided into three modules: ‘Stochastic Procedure’

explaining the basic concepts, ‘Prediction’ treating different types of prediction procedures and ‘Measurement’ containing different types and kinds of measurement procedures. The learning units aim at showing that the solution of any problem can be reduced to the task of developing appropriate prediction procedures.

#### 9.4.4 Teaching Style

The teaching style refers mainly to the used technical tools, which help to present different teaching materials and different activities. For some of the learning activities, which serve for teaching, the teaching style remains unchanged: web pages consisting of downloadable PDF files for Target, Content and Examples. The PDF format seems to be suitable because these pages have stable contents with many mathematical formulas. Exercise, FAQ and Literature are treated differently.

For Exercise, the employed tools vary according to the special requirements with respect to the visualization of complex issues and circumstances.

The exercises of ‘Course 1: Modeling Uncertainty’ are designed in form of interactive explanations. Series instances are used and stepwise extended from learning unit to learning unit. At the beginning the instances are taken from daily life and need no special mathematical or other expert knowledge. The cases should be general and familiar to all kind of learners. Later the instances may be extended to more specific situations for attracting learners with different backgrounds or levels. By tracing back and forward in the series instances, learners may gain a better understanding of the principles in Bernoulli Stochastics. The tools used include multiple choice questions, simulations, dynamic graphics, online calculation and small animations. They aim at giving an immediate feedback to learners, thus raising interest and motivation to further studies.

The exercises of ‘Course 2: Application’ use also interactive illustrations but include extensively a completely different technical tool in accordance with the learning content – namely stochastic procedures. Similar as other scientific procedures, stochastic procedures are based on more or less complex and difficult mathematical operations. However, analogously to technical devices a user must know how to use a stochastic procedure and how to interpret the result, but it is not necessary that the user know the technical details of its derivation. The correct use and interpretation are illustrated by means of graphics, especially by dynamic graphical representations. The exercises in Course 2 aim particularly at showing how to select a proper stochastic procedure for a specific problem, and how to apply the selected procedure to solve the given problem. Exercises for the first aim follow essentially the style used in Course 1. The exercises for the second aim are supported by dynamic interactive graphics from the subsystem *Stochastikon Graphics*, by means of which learners get familiar with the use of the procedures in a more playful style. They can manipulate, detect, feel, think, test, and play around with the procedures and, thus, learn to understand and to accept them. The “learning by doing” teaching concept is realized in the graphical laboratory. Learners can obtain the solutions for a given problem by “scientific reports” which are offered and prepared by another subsystem *Stochastikon Calculator*. These reports support the ability of the learners to formulate a result in a scientific way. The instances used in the exercises are partly inherited from Course 1, which shall assist the ability to distinguish between different problems and, hence, different stochastic procedures on the one hand and at the same time shall consolidate the acquired knowledge on the other.

As to the FAQ, a dynamic generating method is used. Instructors are in charge of producing FAQs according to selected typical questions from learners and representative discussion



contributions. Thus, it is granted that the FAQs focus on real learning conditions.

The same to Literature, each literature item for the Magister teaching content can be referred to by several learning units and several learning activities in one unit, and these connections may be changed irregularly and frequently. Also the types of literature items may vary. So management of literature information has to be centralized and flexible in a dynamic model.

### 9.4.5 Learning Style

Learning is a process of acquiring information and processing experience, in which the learner selects and constructs knowledge that is useful and appropriate for him/herself and in turn uses this to drive and determine his/her own continuous learning process. Learning becomes an individual process of interaction between the individual and his/her environment, in which the subjective reality of the learner is continuously built up.

In *Stochastikon Magister*, the Target, Content, Example, Literature and FAQ activities are used for acquiring information, while Exercise, Question, Discussion and Test are used for gaining and processing experience. Activities in each unit form a small but complete learning process for constructing step by step new knowledge.

Question, Discussion and Test serve and record the whole learning process. They are created and used dynamically by instructors and learners.

Instructors in charge draw a learning plan for each authorized learner at the beginning and evaluate his learning progress regularly during the whole learning process. Learners give feedbacks to all the learning instructions. Learners pose questions and get answers individually from instructors. Learners as well as instructors may generate topics and join discussions. Learners take part in the corresponding tests and get assessments from instructors. All traces made by learners in *Stochastikon Magister* during the learning process are saved and can be traced. This information is supervised by the instructor(s) in charge for evaluating and directing, and by the learner for self-checking her/his level of knowledge.

## 9.5 Interface Design

The interfaces for instructor and learner are as important as the management modules, because the ease to input, extend, modify teaching material for instructors directly determines the quality of the e-learning system, as well as the convenience of the learner to have access to, navigate through and interact with the system. It is important that the user interprets the instructional interface function correctly and the instructional interface function performs according to the user's expectations [146], [114]. As noted in [148], there are two guidelines, which should be observed when deciding about the interface design. In order to feel familiar the design of different web pages should be as akin as possible. Moreover, the internet learning environment should generate a feeling of a classroom.

### 9.5.1 Layout

The frame of *Stochastikon* web pages is the same for all subsystems. Each web page is headed by the logo. Below the logo there is the list of subsystems arranged horizontally. On the left hand side there is the sub-menu tree arranged vertically. The rest of the page displays the corresponding content. In Figure 9.6 the layout of *Stochastikon* is sketched.

Figure 9.6: Layout Frame of *Stochastikon*.

The typical web layouts of *Stochastikon Magister* are shown below. They follow the above given guidelines.

For learning units, an additional vertical right hand side is reserved to call and navigate through the learning activities of the learning unit at hand. The learning units are numbered by three digits, where the first specifies the number of the course, the second the number of the module and the third the respective number of the learning unit. For instance, in Figure 9.7, the third learning unit of Module one Course one is displayed. The heading is the unit number and the unit topic. The actual type of learning activity is displayed in the activity control panel under all control buttons. Below the headline, there is the content frame. The web page for Content, Example and Target are PDF files which are convenient for teachers generating and displaying mathematical formulas in Stochastic teaching material and for learners saving and printing these learning contents.

Figure 9.7: Typical layout of *Stochastikon Magister* learning units.

The main difference between the layout for a learning unit and the layout for a learning/ course management panel is that the function buttons in the vertical right hand side represent the different types of persons involved in the learning process. The actual type of user is shown below these buttons. Figure 9.8 displays an interface in the ‘Discussion’ Management panel for instructors.

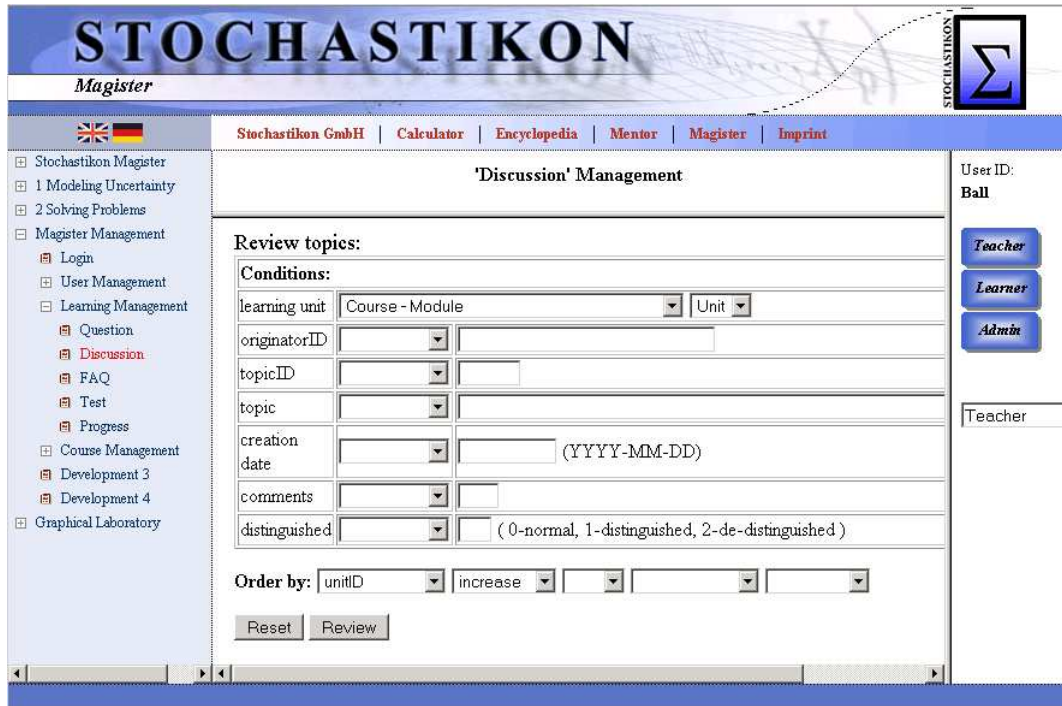


Figure 9.8: Typical layout of *Stochastikon Magister* management panels.

Besides the above two types of layout, there is a third type which is not displayed here. It relates to the ‘user information management panel’. The main difference refers again to the control buttons in the vertical right hand side. The layout of the user management panel contains ‘List’, ‘Update’ and ‘Record’ buttons. Details will be introduced later when the implementation of ‘Magister User Management’ (see Section 10.2) in *Stochastikon Magister* is discussed.

## 9.5.2 Navigation

Navigation is based on the content structure and different learning or managing activities. Hyperlinks to past, related, supplementary or helping information are built in. There are hyperlinks not only to locations within *Stochastikon Magister*, but also - if appropriate - to locations of any of the other subsystems of *Stochastikon*. If the link leads to contents, which are not contained in *Stochastikon Magister* main pages, then the linking materials will be popped by new windows.

## 9.6 Function Design

The issues Target, Content, Example and Exercise are designed, developed and tested as teamwork, then stored, delivered and managed by the ‘Content Management Module’ of *Stochastikon Magister*. The PDF files of Target, Content and Example composed by instructors, are automatically converted from corresponding TeX files. The issue Literature is managed by both the ‘Content Management Module’ and the ‘Administering Module’.

The ‘Content Management Module’, the ‘Collaboration Module’ and the ‘Tracking Module’ are responsible for the issues Question, Discussion and Test. Based on questions and discussions, the ‘Content Management Module’ and the ‘Tracking Module’ help instructors to issue FAQs. The ‘Administering Module’ together with the ‘Tracking Module’ and the ‘Collaboration Module’ supervises the whole learning process.

All activities and cooperation of modules are controlled by and connected with the ‘Administering Module’, sketched in Figure 9.9.

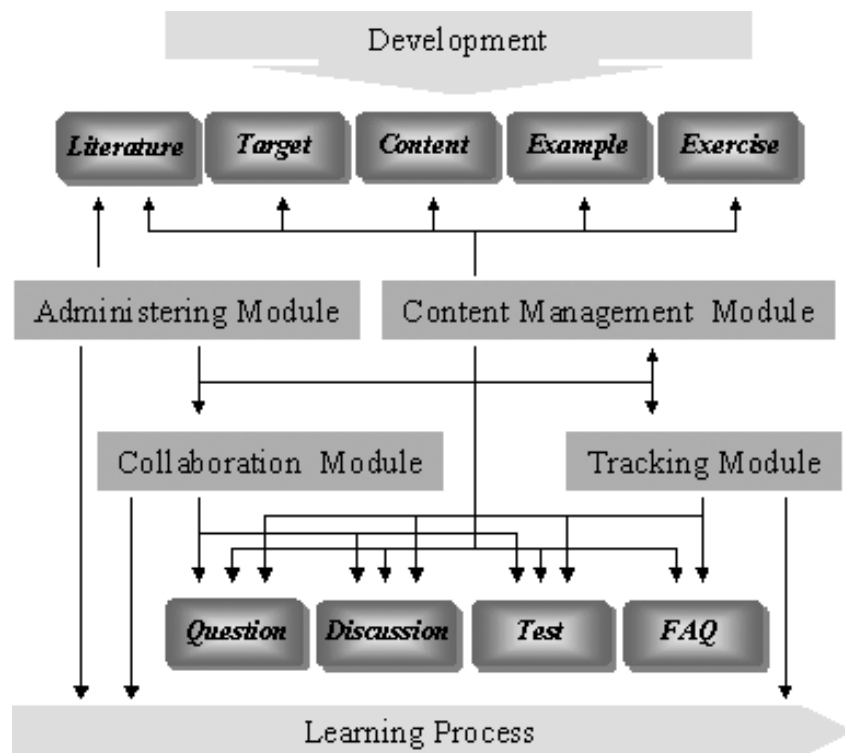


Figure 9.9: Functions of *Stochastikon Magister* Modules.

## 9.7 Repository Design

The e-learning repository is logically divided into content repository and learner profile repository (see Figure 9.1).

Except for the FAQs and some parts of the course data such as: literature index with their connection information and course-module-unit name information that are on-line generated and saved in the database, the content repository is mainly designed and developed according to the teaching material and stored in form of programs and document files. This part of the repository is comparatively invariant and elaborate. All Magister user (instructor and learner) information, learning and corresponding teaching behaviors (question and answer, topic and discussion, test and feedback), test questions, frequently asked questions are dynamically created and stored in the database since most of them are generated, forwarded, updated and used within a learner’s individual learning process.

The database of *Stochastikon Magister* contains two big parts: a user profile repository and a learning repository. The user profile stores users’ private information, their authorities and study levels in Magister. The learning repository refers to the different learning activities:

question-answer, topic-discussion, FAQ-answer, test question bank (question-answer), test-assessment, evaluation-feedback and literature-activity-unit. Correspondingly, each repository has a changing record. Figure 9.10 displays the database structure of *Stochastikon Magister*.

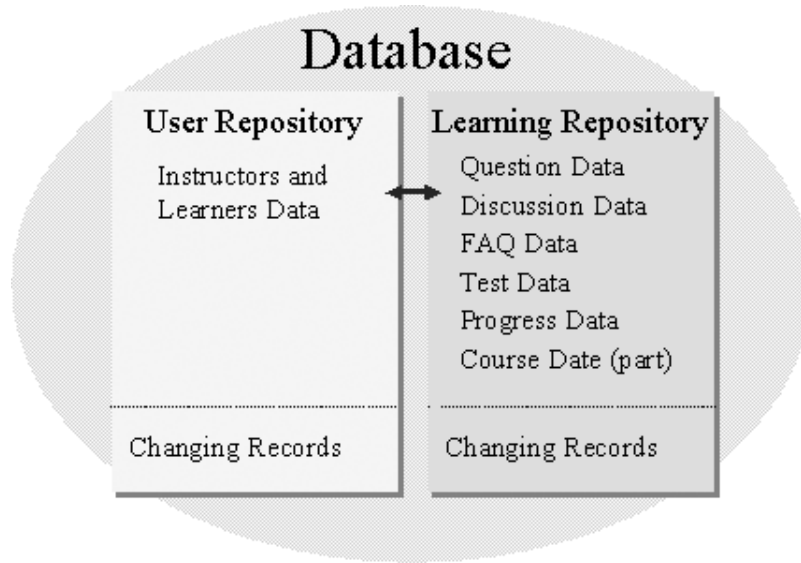


Figure 9.10: *Stochastikon Magister* Database Structure.

## 9.8 System Modeling

Since its standardization by OMG (Object Management Group) in November 1997, the Unified Modeling Language (UML) has quickly become the standard modeling language for software development.[116] The UML consists of nine standard diagram types, which are categorized into four UML models, and each diagram shows a specific static or dynamic aspect of a system. Table 9.1 is a summary of selected UML models and diagrams.

Model Types	Diagrams
Use case model	Use case diagram
Static structure model	Class diagram Object diagram
Behavioral model	State-chart diagram Activity diagram Sequence diagram Collaboration diagram
Implementation model	Component diagram Deployment diagram

Table 9.1: Summary of selected UML models and diagrams.

We select four different UML Diagrams to represent Magister e-learning system from different aspects:

1. Assembly line diagram: “The assembly line diagram is a unique diagram in the Eriksson-Penker Business Extensions. As with the process diagram, it is based to a large extent on the UML activity diagram.”[80] Here the assembly line diagram is used to illustrate the information flow and information combination along with the running process within

Magister e-learning environment. For this aim, we further extend it by adding time series and circulations.

2. Swimlanes in Activity Diagram: This type of diagram describes workflows, actions, and interactions among Magister roles. [111, 112]
3. Use case diagram: This type of diagram pictures roles and use cases together with their relationships in Magister system and subsystems without specifying their internal structure.
4. Class diagram: This type of diagram abstracts the association, hierarchy and other static relationships in Magister e-learning society.

Figure 9.11 is a UML assembly line diagram, which depict the learning - teaching - managing process and knowledge accumulating process in Magister e-learning environment.

Before Magister is put in use, the *Teacher* has to develop the *Primary Teaching Content* such as *Introduction* for different courses and modules, *Target*, *Content*, *Example* and *Exercise* for each *Learning Unit*. Besides, according to the developed *Primary Teaching Content*, the *Teacher* can develop some *FAQ*, *Test* questions and related *Literature*. A Magister user, either informal or formal can learn the pre-settled *Primary Teaching Content* without restriction.

If a user registers and is accepted by Magister, he becomes a (formal) *Learner* and will be informed by email. Henceforward, the newly registered learner is taken care of by the *User Management* process, and the new *Learner's* original learning level will be determined by the *Teacher*.

If a user is accepted as a new *Learner*, his *Learner Progress Management* process is started simultaneously. The *Teacher* should immediately evaluate the new *Learner's* personal profile and formulate a *Learning Plan* for him. After being confirmed, the new *Learner* can *Login* as a formal *Learner* and learning in Magister according to his *Learning Plan*. Besides, the *Learner* can give *Feedback* to the *Teacher* to discuss his private *Learning Plan* until they get the consensus.

A (formal) *Learner* can learn the *Primary Teaching Content*, *FAQ* and related *Literature*, ask *Question* and get answers from the *Teacher*, initiate *Discussion* topics or colorBluedeliver comments to the *Teacher* and the *Administrator*, and finally he can ask for a *Test* to check his understanding level. The *Test* will be corrected and evaluated by the *Teacher*.

The learning trace of a *Learner* will be kept for the *Teacher* and the *Learner* himself to evaluate his learning progress according to the *Learning Plan*, and for Magister knowledge system to accumulate teaching content, such as new or updated *FAQ*, *Test* questions, *Literature* and also the *Primary Teaching Content*.

The *Learner* can also discuss the *Evaluation* with the *Teacher* by *Feedback* within the *Learner Progress Management* process. After a certain period of time, by reviewing the *Learner's* achievement, the *Teacher* may decide whether to adjust the *Learner's* learning level. Finally a *Learner* may get two possible results for his/her e-learning process, either has been successfully accomplished or is abandon. The *User Management* process reacts correspondingly by documenting and confirming the success of the *Learner* or by delete the *Learner* totally from Magister system.

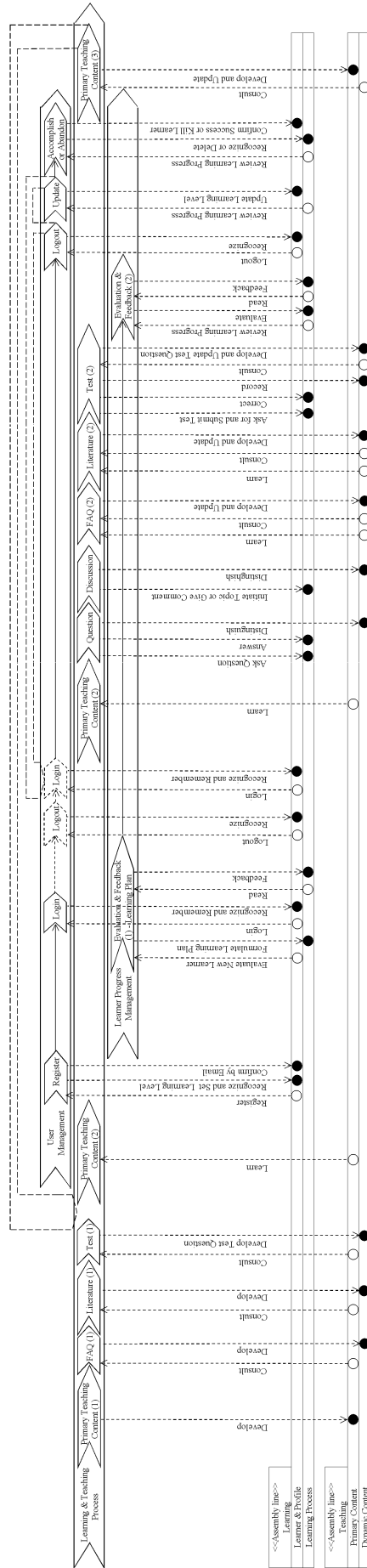


Figure 9.11: Learning - teaching - managing process and knowledge accumulating process in Magister e-learning environment.

In short, the assembly line diagram shows two knowledge accumulation processes along with the teaching-learning-managing process executed in Magister e-learning system, one is the *Learner's* knowledge accumulating processes and the other is the system knowledge accumulating process.

Figure 9.12 is a UML user case diagram, which is an overview of actors and the use cases that are implemented in the Magister e-learning system.

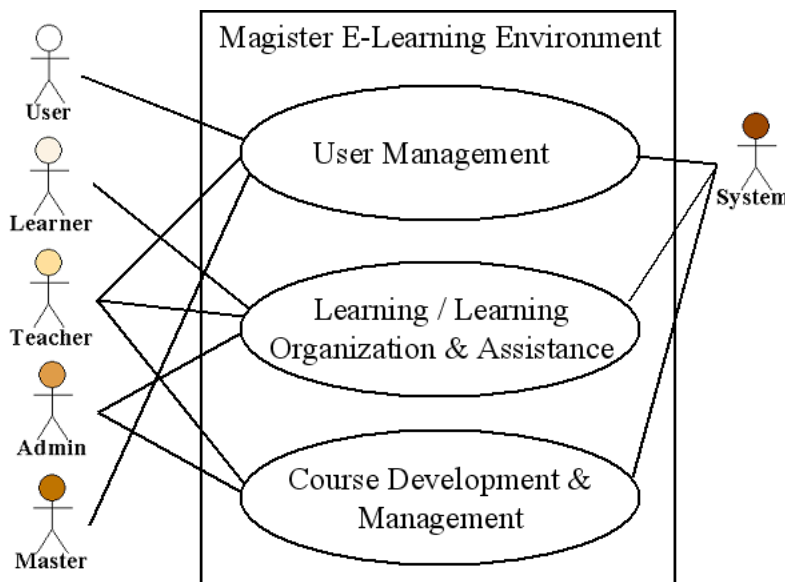


Figure 9.12: Magister e-learning environment.

In Chapter 10 ‘Implementation of *Stochastikon Magister*’, we will use UML to model Magister running processes, different management functions and learning activities, and depict them by the selected UML diagrams.

## 9.9 Technology Solutions

In order to allow for the above-described features the e-learning system *Stochastikon Magister* must meet a variety of advanced technological requirements. It must admit the use of ‘Mathematical Language’ and ‘Graphics’, it must have powerful ‘Computing Abilities’ and it must permit ‘Interactive Discourses’.

Microsoft FrontPage was used for developing the web pages of *Stochastikon Magister* learning units. HTML and PDF files for the text parts, while JavaScript and small Java Applets realize the interactive components. ‘Interactive’ is also embodied in the ‘Graphical Laboratory’ (see Section 10.5) and communications during the learning process. By means of *Stochastikon Calculator*, the ‘Computing Abilities’ is no longer a challenge, and it is powerful enough to handle real world problems. Still two problems need to be discussed for the implementation of *Stochastikon Magister* :

1. Authoring and displaying mathematical notations and formulas.
2. Implementing interactive dynamic graphical representations.



As to the first problem one should note that *Stochastikon Magister* does not aim at teaching how to write and handle actively mathematical formulas. Instead the learner should be able to express problems in mathematical language by means of certain symbols and formulas, i. e., how to state and use the concept Bernoulli Space for describing situations and solving problems taking into account the inherent uncertainty of a given situation. The needed mathematical language abilities are thus reduced to a more passive knowledge of given terms and concepts and, therefore, interactive dynamic mathematical activities are not necessary. As a consequence, it was decided to do without advanced and ambitious technological means as for instance MathML and use rather simple but efficient ways. Two ways are used to handle mathematical language in *Stochastikon Magister* :

1. PDF forms are used for Content and Example which contain most of the Magister's mathematical notations. The PDF forms are automatically generated from TeX files. This transforming program is realized by Java. It is similar as the program used in two other subsystems of *Stochastikon Encyclopedia* and *Stochastikon Mentor*. The advantage of writing complicated mathematical formulas by TeX is well known and the produced PDF files can be displayed in stable quality by different web browsers and by the free software Acrobat Reader. Moreover, PDF files are easy to be saved and printed.
2. For Exercise and other HTML layout pages, formulas and symbols are saved as GIF images and inserted into the HTML files. This ensures a good displaying quality of mathematical formulas with small loading time in any client environment.

The second problem cannot be solved by 'frozen images' but requires 'visualization and graphical representations'. The solution in *Stochastikon Magister* is called 'Graphical Laboratory' (see Section 10.5) which constitutes an interactive and dynamic graphical tool, which reacts immediately to the learner's demands according to the given input. The interactive and dynamic graphic tool helps to make abstract ideas concrete and admits direct observing, detecting, manipulating, investigating, testing, exploring, analyzing procedures and models. The active handling of the abstract concepts shall deepen understanding and facilitate the adoption of new thinking rules. There are several technological tools, which could be used for realizing the 'Graphical Laboratory' of *Stochastikon Magister*. Among them are the following three often-used techniques:

- JavaScript
- ActiveX
- Java and Java applets

JavaScript was designed to add interaction to HTML pages. It is a scripting language, i. e., a lightweight programming language, usually embedded directly in HTML pages. It is an interpreted language, i. e., executes without preliminary compilation, and is supported by all major browsers, like Netscape, Internet Explorer and Firefox. It is simple, but somewhat less powerful compared with Java. JavaScript is effectively used in *Stochastikon Magister* to realize the small interactive programs.

ActiveX is a set of technologies developed by Microsoft that enables interactive content for the World Wide Web. ActiveX provides a standard mechanism to extend any programming language, including Java. However, it currently runs only on computers that use Microsoft operating systems and, therefore, it was decided not to use it within *Stochastikon Magister*.

Java is developed at Sun Microsystems. Java technology is both a programming language and a platform: The Java programming language is a high-level objective-oriented language with superior properties. The Java platform has two components: The Java Virtual Machine (Java VM) and The Java Application Programming Interface (Java API). Java Applets are little programs written in Java language. They are designed to run inside a web browser and to perform some tasks such as animated graphics and interactive tools. Applets are usually embedded in a HTML page on a Web site and can be executed from within a browser.

Java has some specific advantages:

- Powerful language.
- Platform Independence and Portability.
- Development and Maintenance: Main program (does the computations) + graphics (plots the results) + user interface (to the main program and its graphical component) can be integrated into one unified environment
- A lot of Java source code resources can be shared via internet.
- Free of cost for academic use.

The advantages of using Java Applet are the following:

- Encourage asynchronous distance environment.
- Along with desired interaction and multimedia support, provides a representation for better communicating a concept compared with static figures or a written description.

Thus, Java and Java applet were selected for enabling a fully interactive and fully distributed web-based dynamic graphical representations in *Stochastikon Magister*. Another very important argument in favor of Java was the fact that other *Stochastikon* subsystems, especially *Stochastikon Calculator*, are also based on Java.

Besides content design and development, the ‘Administration and Control’ environment for learning execution is another main developing part of *Stochastikon Magister*. It builds on the cooperation of Administering, Tracking, Collaboration and Content Management Modules as shown in Figure 9.4 and Figure 9.9. Among many tools that could be used, Java Server Pages (JSP) in conjunction with JavaScript, HTML together with MySQL database were selected as a cost-effective and promising technological solution.

Summarizing, the web pages of *Stochastikon Magister* learning units are built by HTML and PDF files for the main textual parts, and JavaScript and Java Applets for the interactive parts especially within exercises. Microsoft FrontPage helps to edit the HTML files. Interactive dynamic graphical representations are supported by the subsystems *Stochastikon Calculator* and *Stochastikon Graphics*, which are based on Java, Java applets and Gnuplot (serves for Calculator only). The learning management environment is realized by JSP, JavaScript, HTML and MySQL technologies.

There is no special hardware requirement for the client side, since *Stochastikon Magister* runs together with other *Stochastikon* subsystems on a server.

Learners having access to internet through a modem or ISDN equipped with Internet Explorer, Netscape or Firefox and free downloaded Java Virtual Machine (JVM) and Acrobat Reader can navigate and manipulate all learning contents and practice all learning activities.

# Chapter 10

## Implementation of Stochastikon Magister

According to its aims, the implementation of *Stochastikon Magister* should focus on three aspects:

- User information: documentation and management
- Teaching material: organization and provision
- Learning execution: documentation and management

In the first step, the Magister control system is implemented.

### 10.1 Magister Control System

#### 10.1.1 Control Levels

*Stochastikon Magister* has two control levels. The first level is from the sub-menu tree called ‘structure’ to a learning unit or a user/learning/course management. This control level is determined by the required frame of each *Stochastikon* subsystem (see Section 9.5.1 ‘Layout’ Figure 9.6).

Each leaf of the structure tree calls a file named ‘content’ that sets the Magister panel to three sub-frames: panel titles are the heading, panel contents below and the inner control panel - the second control level - is located in the vertical right hand side. Figure 10.1 sketches this frameset.

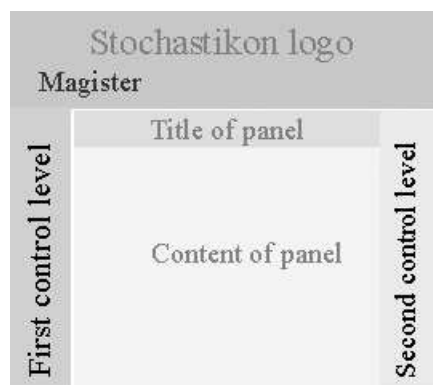


Figure 10.1: *Stochastikon Magister* frameset and control.

By each call from the first control level, i. e., the structure tree, parameters such as: contentID, managementID, operating authority requirements... used in the second control level are also transferred.

The second control level is realized by a file called ‘control’ and constructed by several control buttons. Each button corresponds

- to one learning activity: ‘Content’, ‘Exercise’, ‘Question’, ‘Literature’... for a selected learning unit (see Section 9.5.1 ‘Layout’ Figure 9.7) in the learning units control part, or
- to a series of information manipulations in the user information control part, for example: ‘List’, ‘Update’ and ‘Record’ functions for the ‘User Keys’ management (see Section 10.2.1 ‘User Key Information’ Figure 10.8), or
- to a set of management actions in the learning/course management control part, for instance: functions for ‘Teacher’, ‘Learner’ and ‘Administrator’ in the ‘Discussion’ management panel (see Section 9.5.1 ‘Layout’ Figure 9.8).

Most of these executions are subject to certain restrictions with respect to user’s authority.

*Stochastikon Magister* contains three ‘content’-‘control’ pairs, one for all learning units and learning activities, one for learning/course management, and one for user information management. Figure 10.2 is a UML class diagram which sketches these two control levels.

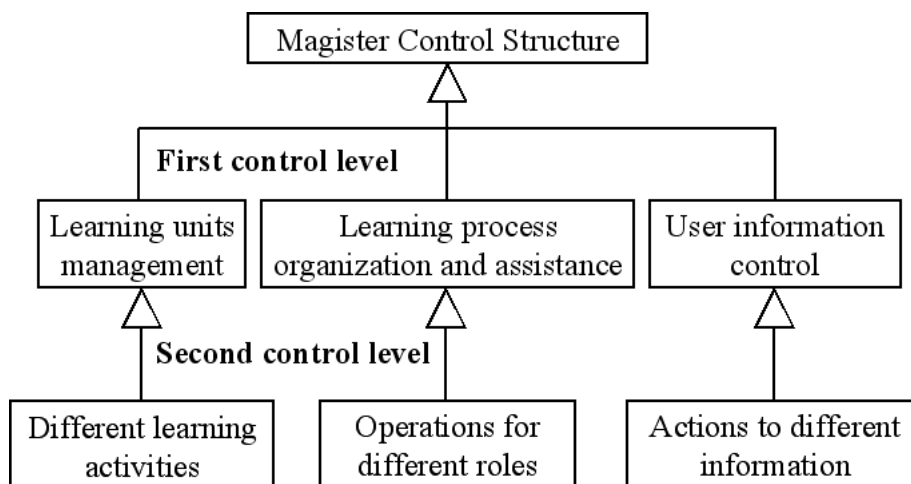


Figure 10.2: *Stochastikon Magister* control hierarchy.

### 10.1.2 Magister User and User Control Authority

#### Magister User

Magister distinguishes between informal and formal users. A user is called informal as long as he/she has not obtained a licence by Magister even after registration. As soon as an informal user got a licence from *Stochastikon Magister*, he/she becomes a formal user. Formal users may be students, teachers, administrators and the Master of Magister.

Informal users have access to the learning units, but must register and become formal users before taking part in the learning programs. Registration is possible online at any time.

If a formal user logs in to Magister successfully, he/she can select either English or German and this ensures that his/her learning records should not be a mix-up of different languages.

It is planned to add more languages to Magister, however, so far Magister offers only the two possibilities English and German.

**User Control Authority**

There are three categories of the formal user of *Stochastikon Magister* : Learners, teachers and administrators. Each of these categories has certain duties and in this context more important certain rights.

Thus, Magister assigns to each formal user a certain ‘authority to use’ called ‘user status’. The user status constitutes another means for control in *Stochastikon Magister*. It takes values from 0 to 9 which entitles the user to certain activities in Magister. The highest user status 9 is reserved to the Master of Magister.

Table 10.1 shows different Magister roles, their authorities (user status) and corresponding rights for executing operations. Where the operations refer to the learning and managing activities.

										Master								
										Admin								
										Teacher								
										low level		high level						
										Informal Learner				(Formal) Learner				
User Status	0					1	2	3	4	5	6	7	8	9				
User \ Behavior	Learning Activities					User Information Management				Learning Management								
	Q	D	T	F	L	Others	List	Record	Update		Learner	Teacher	Admin					
					Δ	✓			Update	Delete User								
Informal Users																		
Formal Users	Learner (1-9)	✓		Δ		✓						✓						
	Teacher (6-9)	✓						✓		> 6			✓					
	Admin (8-9)	✓							✓		9			✓				

Learning activities: Q - Question, D - Discussion, T - Test, F - FAQ, L - Literature.

Learning activities: Others - Introduction, Target, Content, Example, Exercise and Graphical Lab.

Numbers in the table stand for user status.

✓ - available; Δ - partly available.

Admin. - Administrator.

Table 10.1: *Stochastikon Magister* roles, their corresponding user status and control authority.

If an informal user wants to execute operations reserved for formal users, he will get a pop-up alert (see Figure 10.3). In Figure 10.3 an informal user wants to ask and review questions about Unit 1.1.2, his request is rejected together with an information how to become a formal user.

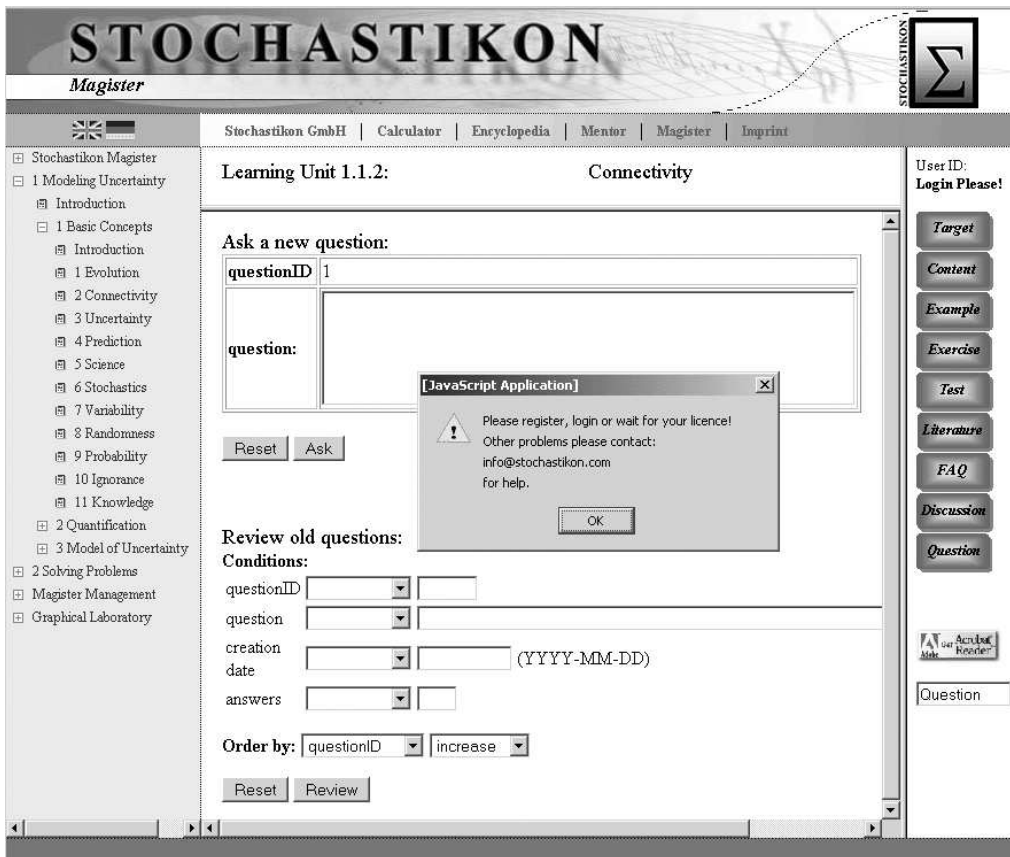


Figure 10.3: The alert for the informal users in Magister.

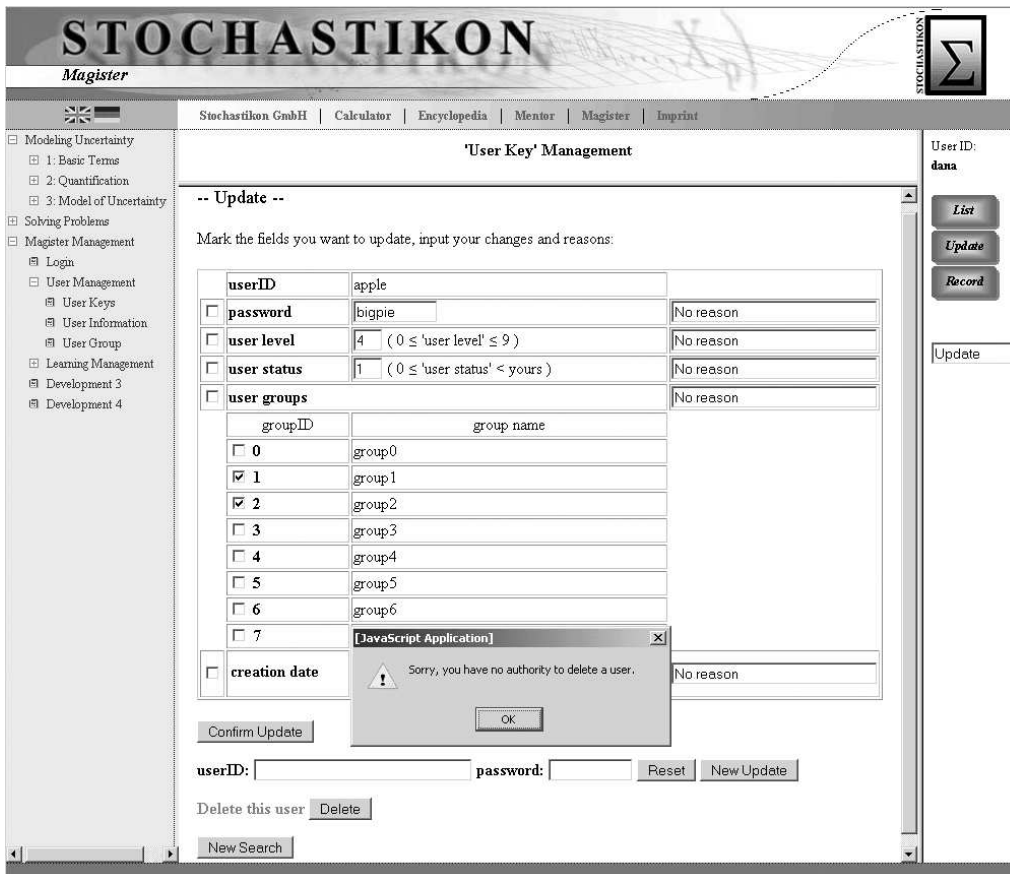


Figure 10.4: Authority alert for different authorities in a Magister panel.

As to learning activity panels or (user/learning/course) management panels, if a user wants to carry out manipulations for which he/she is not authorized, he/she will get a pop-up alert similar as displayed in Figure 10.4. In Figure 10.4, when Dana - an administrator with user status 8 - wants to delete a user, which is reserved to the Master of Magister with the highest user status 9, the deletion is rejected and Dana is informed about the rejection by a pop-up alert.

If a user wants to operate a management panel which exceeds his/her authority, for example, if a learner wants to manipulate the learning management panel for teachers or for administrators, he/she will see a ‘no authority’ page as shown in Figure 10.5. ‘Ball’ is a Magister teacher with user status 6. Ball wants to ‘Update’ Magister user information which requires a user status higher than 6. Thus, Ball gets the ‘no authority’ information as displayed in Figure 10.5.

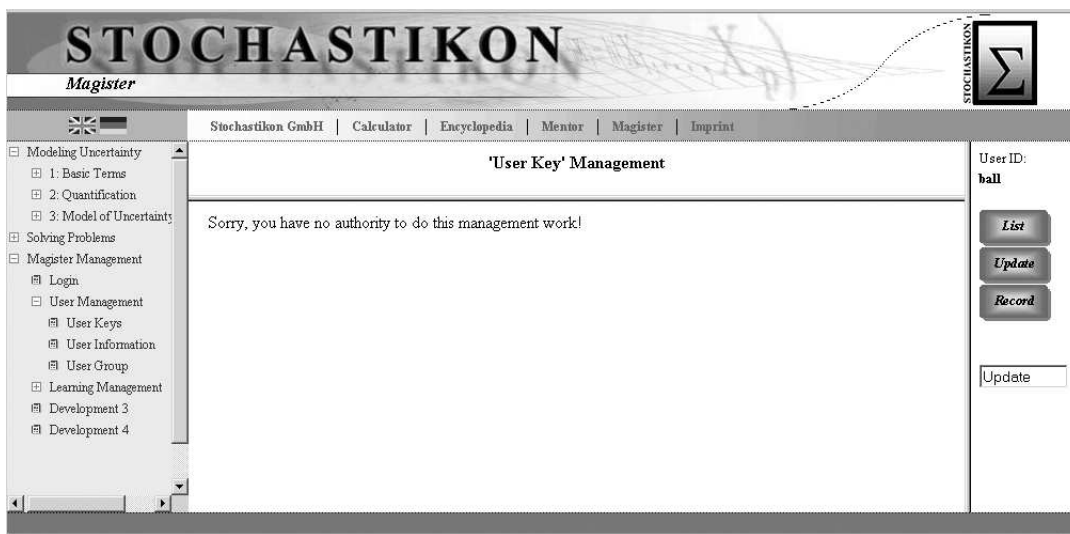


Figure 10.5: The no-authority page for a management panel in Magister.

*Stochastikon Magister* control is realized by JSP, JavaScript, HTML and MySQL database.

In the following sections, the three subparts of Magister are discussed. The first section introduces the user management part. The second deals with the teaching material, its organization and provision, and the third is for learning execution and management part. They are further categorized according to the learning activities and the management behaviors.

## 10.2 Magister User Management

The user management constitutes the basis of learning/course management in *Stochastikon Magister*. It treats any user of the Magister equally no matter whether the user is a teacher or a learner. The only management criterion is the user’s authority given by the user status.

Figure 10.6 is a UML class diagram that displays this static relationship between Magister e-learning process and Magister user.

Figure 10.7 is a UML user case diagram that shows an overview of actors and the use cases present in the Magister User Management system.

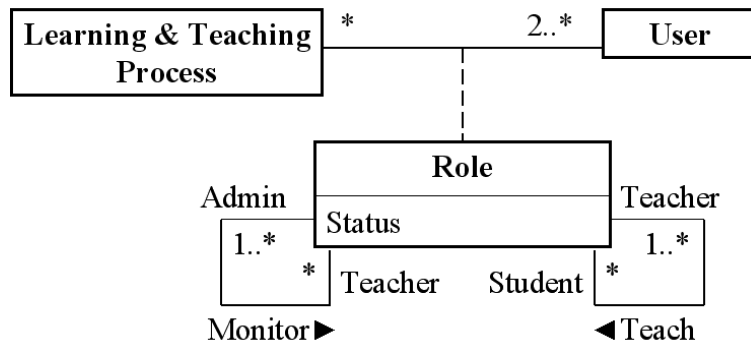


Figure 10.6: Association class of Magister.

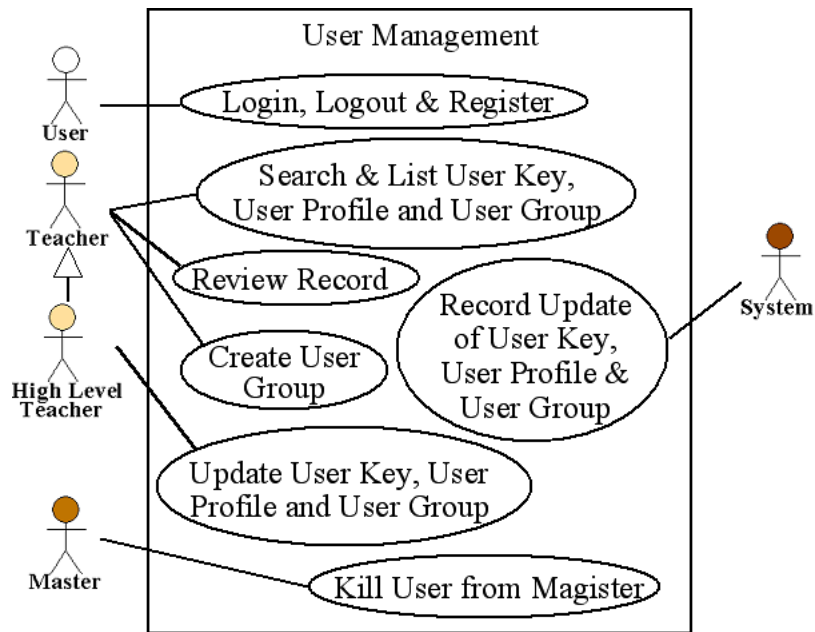


Figure 10.7: User management.

In order to increase the efficiency of the database, Magister user information is physically separated into two tables: key information and other information.

### 10.2.1 User Key Information

Users’ key information includes userID, password, user level, user status, beginning date, and user group. These six items of user information are most critical as they guide the learning process and learning/course management. They represent the key information and are stored in an individual table.

‘UserID’ is the primary key of the ‘user key information’ table and it is also the most important item of all information stored in Magister database. It is unique, stands for a specific user and cannot be changed within a user’s whole ‘life’ in Magister: from registration to being cleared away. A user logs into *Stochastikon Magister* by means of ‘UserID’ and ‘Password’.

User ‘Level’ serves for identifying a learner’s learning level. It is changed according to the learner’s study progress.



User ‘Status’ stands for a user’s authority level. A new enrollee obtains user status 0 before getting final license from *Stochastikon Magister*. User status 0 means that he/she is still not a formal Magister user. Table 10.1 in Section 10.1.2 ‘User and User Control Authority’ lists all formal users’ authorities. Teachers and administrators with different status have different management powers, but possess all powers of learners. Only one person - the Master of *Stochastikon Magister* - has status 9.

In the following chapters three users are taken for illustrating the implementation of ‘learning execution and learning/course management’. Learner ‘Cloud’ has user status 3, Teacher ‘Ball’ has user status 6 and Administrator ‘Dana’ has user status 8.

The item ‘Date’ records the date of a user’s registration. It stands for the starting point of a learner’s learning life in *Stochastikon Magister*. Each activity of a learner after registration is recorded, evaluated and can be checked at any time.

‘User Group’ stores the uses group information. A learner can belong to several groups. Tests for learners are constructed on the basis of the learning level or on group affiliation. Thus, for each group there are special tests which take account of the particular group interests.

The ‘User Key’ Management panel is accessible only for teachers or administrators. It includes three types of functions: ‘List’, ‘Update’ and ‘Record’ (see Figure 10.8). No user can see and update the stored key information of a user with equal or higher status. In order to prevent any malpractice, for ‘Update’ user key information, a user status of at least 7 is necessary and to ‘List’ the user key information or to check their updating by ‘Record’, a user status of at least 6 is required. Thus, Teacher Ball may check the user keys, but is not authorized to update them, while Administrator Dana with user status 8 has access to the update function.

Figure 10.8: *Stochastikon Magister* ‘User Key’ Management - ‘Update’ (a).

Figure 10.8 shows the search interface of the user key update panel. Teachers can input some search conditions and start a ‘Search’. Subsequently Magister provides a list of all those user keys meeting the given conditions. The teacher may select that user key, which shall be updated. Note again that only persons with higher (at least 7) user status have access to the lower status

persons' user keys which represent the most sensible information contained in *Stochastikon Magister*.

Figure 10.9 lists the searching results according to the required input-conditions given in Figure 10.8.

The screenshot displays the 'User Key' Management interface. At the top, the page title is 'User Key' Management. Below the title, there are search conditions: userID, password, user level = 1, user status, user group, and creation date < 2005-04-25. The results are ordered by userID. A table lists users with columns: userID, password, level, status, user group, and creation date. The table contains four rows of data. A pop-up window titled 'User group information' is open, showing details for group0, group1, and group2. Another pop-up window titled '\*\*\* User Information \*\*\*' is open, showing details for user 'Ball', including Surname: Foot, First name: Ball, E-Mail: foot@ball, Birth date: 6, 1978, Place of Birth: Frankfurt, Germany, Gender: Male, and Language: English.

userID	password	level	status	user group	creation date
Apple	bigpie	1	1	1,2.	2005-04-24 19:45:23.0
Ball				0,1,	2005-04-24 19:08:07.0
Cloud				0,1,	2005-04-21 15:08:43.0
Tree				1,2.	2005-04-22 11:18:53.0

groupID	group name	comment	creation date
0	group0	some comments about group 0	2005-04-24 18:39:57.0
1	group1	comments	2005-04-24 17:22:56.0
2	group2	change comment	2005-04-24 19:23:27.0

\*\*\* User Information \*\*\*

Surname: Foot  
 First name: Ball  
 E-Mail: foot@ball  
 Birth date: 6, 1978  
 Place of Birth: Frankfurt, Germany  
 Gender: Male  
 Language: English

Figure 10.9: *Stochastikon Magister* 'User Key' Management - 'Update' (b).

In Figure 10.9, the 'user group' column displays the IDs of the groups the users in question are affiliated to. By a click on the blue title 'user group', one can get detailed information about the user groups. By click on a user's 'userID', the pop-up window will display his/her 'User Information' (see Section 10.2.2). If more than one user satisfy the required conditions, one must mark the user whose information is of interest, e. g., to be 'Updated'. Then the corresponding update page is displayed (see Figure 10.10).

In the update page, the authorized person marks the items to be changed, changes the corresponding information of the selected entries and may additionally record the reasons for each change. 'Confirm Update' creates an alert with a list of items which are to be updated for final confirmation. Marking the changed items and the final confirmation alert help to prevent false updating of user key information.

Also in the updating page, the function 'Delete this user' is displayed with red characters. It is used for deleting a user totally from *Stochastikon Magister*. However, only the Master of Magister has the corresponding authority. When a user is deleted from Magister, all data concerning him/her are erased. If there are data sets which refer simultaneously to the deleted user and some other users, for example by introducing discussions or, in case of a teacher, by answering learners' questions, correcting tests, contributing to test questions and FAQs, supervising learners' progresses, ... the deleted userID is replaced by a 'default' userID, thus, retaining the relevant data for further use.

The ‘List’ panel is similar to the first two steps of the ‘Update’ panel: the search conditions are input, yielding a list of users satisfying the requirements, but no update function follows.

The screenshot shows the 'Update' panel of the 'User Key' Management system. The main heading is 'User Key' Management. Below it, there is a section for updating user information. A table lists fields: userID (cloud), password (colorful), user level (1), user status (3), user groups (group0 to group7), and creation date (2005-04-21 08:43). A JavaScript dialog box is open, displaying 'update 'user status'' with OK and Cancel buttons. The page includes a sidebar with navigation links, a top navigation bar, and a right sidebar with 'List', 'Update', and 'Record' buttons.

Figure 10.10: *Stochastikon Magister* ‘User Key’ Management - ‘Update’ (c).

The screenshot shows the 'Record' panel of the 'User Key' Management system. The main heading is 'User Key' Record Management. The page is divided into two sections: 'Password Record Checking' and 'User-Level Record Checking'. Each section has a 'Conditions' section with dropdown menus for userID, password, changerID, and change date. The 'Password Record Checking' section also has an 'Order by' section with dropdown menus for userID, increase, and change date, decrease. The page includes a sidebar with navigation links, a top navigation bar, and a right sidebar with 'List', 'Update', and 'Record' buttons.

Figure 10.11: *Stochastikon Magister* ‘User Key’ Management - ‘Record’ (a).

The ‘Record’ panel is used to manage the updating records of all user key information. User

registration information is also included in these records. In the interface of this panel, each user key item has a form for inputting the required conditions. Figure 10.11 shows part of the searching page in the ‘Record’ panel, which contains ‘Password’ and ‘User-Level’ Record searching condition input forms.

Figure 10.12 shows the search results according to the required conditions given in Figure 10.11: the records of all changes of users’ passwords prompted by ‘Dana’.

The screenshot displays the 'User Key' Record Management interface. The search conditions are: userID, password, changerID = dana, and change date. The results are ordered by userID, date DESC. The table below shows the search results:

userID	password	changerID	change date	reason
abcde	abcde	dana	2005-05-17 15:13:03.0	test update password

Below the table, there is a link to 'New Listing' and a 'Finish listing' message.

Figure 10.12: *Stochastikon Magister* ‘User Key’ Management - ‘Record’ (b).

Note that:

- Letters are compared according to their alphabetic order. For example if in Figure 10.11 the required condition to the changerID is not ‘equal to’ ‘Dana’ but ‘larger than’ ‘D’, then the program will search for the changed records by changers with the first letter of their userID being ‘E’ to ‘Z’. This search rule is valid within the entire Magister system.
- Magister does not distinguish between capital and small letters, because MySQL database does not. For instance it is the same if Teacher Ball inputs his userID either in form of ‘BALL’ or ‘ball’.

### 10.2.2 User Information

The user information table contains those User Information, such as user’s name, gender, birthday, E-mail, language, address, background and so on, which will not change along with the learning process. Functions, layouts, abilities and manipulations of ‘User Information’ Management panel are similar to those of the ‘User Keys’ Management panel.

### 10.2.3 User Groups

Learners can be categorized to different groups according to their backgrounds, education levels, interests, specialized fields and so on. Group information is managed, i. e., set up, maintained and updated in the ‘User Group’ Management panel, which has similar functions, layouts and abilities as the ‘User Information’ Management.

Learner groups can be generated by and by just as required. The group table contains four items: groupID, name, comment and creation date (see Figure 10.9). ‘groupID’ is generated

automatically in proper order by the program. Teacher managers can only see existing groups and assign the corresponding groupIDs to learners.

*Stochastikon Magister* uses ‘bit’ to manage groupID and User Group information, which is efficient and convenient. There is only one restriction due to MySQL database limiting the total number of groups to 28.

### 10.2.4 Register, Login and Logout

*Stochastikon Magister* includes nine learning activities. Four of them namely Target, Content, Example and Exercise are open for informal users. Thus, anybody can check the learning programme and decide whether or not to register. Only the activities Literature, Question, Discussion, FAQ and Test are reserved to formal users of *Stochastikon Magister*, who want to graduate in Bernoulli Stochastics. Teachers and administrators may participate as learners in any learning activity.

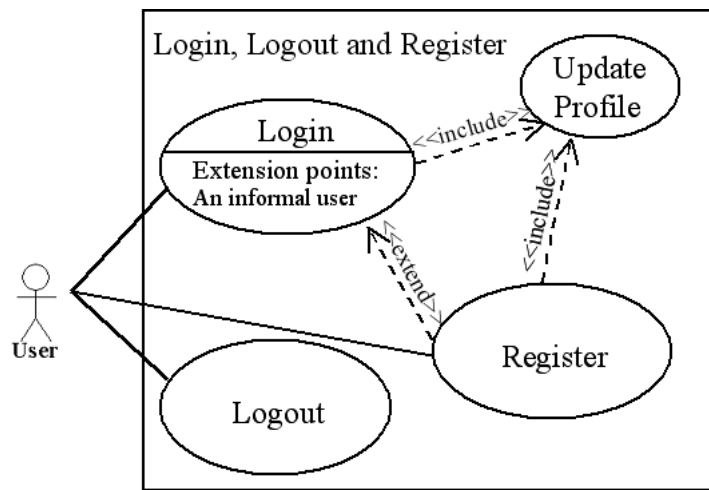


Figure 10.13: Register, Login and Logout.

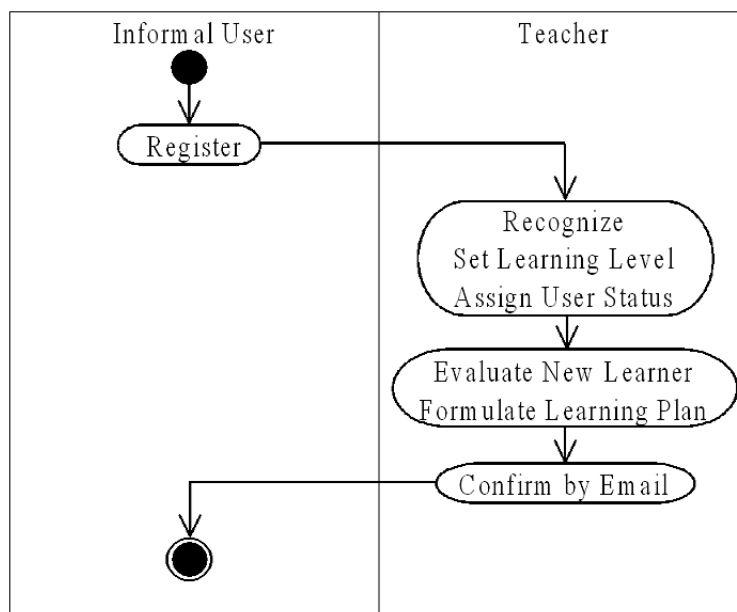


Figure 10.14: Registration process.

Figure 10.13 is a UML user case diagram that shows an overview of Magister user and the use cases present in the register, login and logout process.

Figure 10.14 is a UML swimlanes in activity diagram, which describes the user registration process together with the creation of a learning plan for a new learner.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

Stochastikon Magister Login:

User ID:   
 Password:

If you cannot login, please send E-Mail to [info@stochastikon.com](mailto:info@stochastikon.com) for helps.

Stochastikon Magister Logout:

Stochastikon Magister Register:

User ID:  \*  
 Password:  \*  
 E-Mail:  \*  
 (Note: your final license will be informed by this E-Mail address.)  
 Birth date: Month (01~12)  \* Year (yyyy)  \*  
 Place of birth:   
 Gender:  Male  Female \*  
 Language:  English  German \*  
 \*\*\* Address \*\*\*  
 Country:  \*  
 City:  \*  
 ZIP code:  \*  
 Street:  \*  
 House No.:  \*  
 \*\*\* Background Information \*\*\*  
 Education:   
 Profession:   
 Interests:

[JavaScript Application] Please input your valid 'E-Mail!'

Figure 10.15: Register and Login in *Stochastikon Magister*.

Magister user/learning/course management panels are mainly manipulated by teachers or administrators with user status higher than 5, with different user statuses implying different

management activities. The learning management part also supports communication and cooperation between learners and instructors in ‘learning - learning management’ processes.

To become a formal user of *Stochastikon Magister*, a person must register and obtain a license from Magister. When a formal user wants to work (learn or manage) in Magister, he/she must login, and for leaving Magister, he/she can logout or just close the current Magister window.

Figure 10.15 displays the interface of Magister Register and Login. The register program ensures the uniqueness of the userID. If a person inputs obviously wrong information when register or login, an alert window will pop up with failing and helping information.

After a successful registration or login, Magister will show a welcome address and let the user review or update his/her private profiles as shown in Figure 10.16.

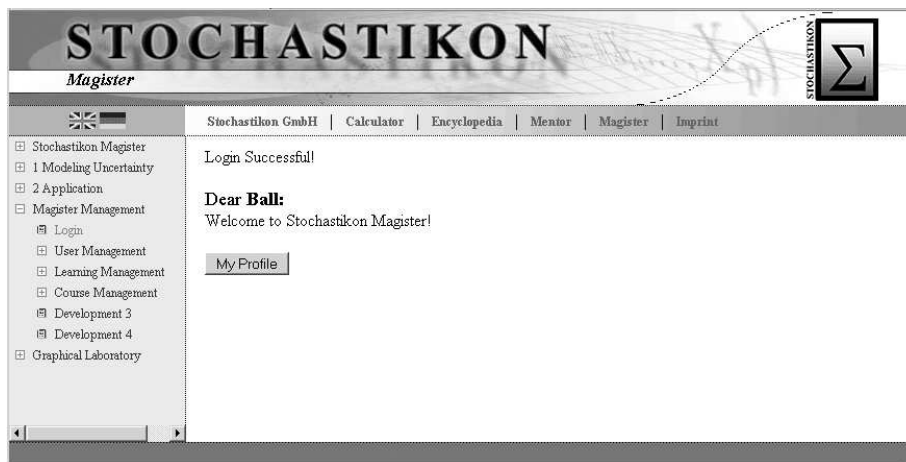


Figure 10.16: A successful login of a user to Magister.

Figure 10.17 displays the interface for users to update their user information and password. Users have to mark the fields they want to change, input the new contents and then ‘Confirm Update’.

When a new person registers, his default user level and status are set to ‘0’, user group is empty and the date is the current time. When the new registrant’s status has been updated to equal or larger than 1, he/she is informed by an E-mail from *Stochastikon Magister* of having been accepted as a formal user. This process is finalized by a teacher in the ‘User Key’ Management panel (see Figure 10.10). After having received the acknowledgement as a formal user, the entire learning programme of *Stochastikon Magister* becomes accessible.

Note that participating in the learning programme will be free of any charge. Thus, it may be hoped that the programme is also accepted and offered in public schools or companies as part of continuing education program.

In case of a successful login to *Stochastikon Magister*, the userID, user status, user level, user group and other key information are remembered and used until the user leaves Magister by logging out or closing the current Magister window.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

- Stochastikon Magister
  - 1 Modeling Uncertainty
  - 2 Application
  - Magister Management
    - Login
    - User Management
    - Learning Management
    - Course Management
    - Development 3
    - Development 4
    - Graphical Laboratory

**\*\*\* Personal Data \*\*\***

userID Cloud

Password Old:  New:

Surname

First name

E-Mail

Birth month

Birth year

Place of birth

Gender  Male  Female

Language  English  German

**\*\*\* Address \*\*\***

Country

City

Zip code

Street

House No.

**\*\*\* Background Information \*\*\***

Education

Profession

Interests

\*\* Reason  No reason

Figure 10.17: The interface for users to update their information after login.

## 10.3 Teaching Contents and their Management

The management of the teaching material in *Stochastikon Magister* follows either a static or a dynamic modes. Static management takes care the ‘Introduction‘ for courses and modules, ‘Target’, ‘Content’, ‘Example’ and ‘Exercise’ in each learning unit. These teaching contents are developed by teachers before Magister is put in use, and they will not be adjusted frequently. Dynamic management controls the name of each course, module and unit, and ‘Literature’ cited within different teaching contents. Though the primary teaching contents are statically developed, the usage of these contents are dynamic, and therefore they need to be managed dynamically.

Figure 10.18 is a UML swimlanes in activity diagram describing the dynamic management process of Magister literature and Magister course, module and unit name.

Figure 10.19 and Figure 10.20 are two UML user case diagrams. Figure 10.19 shows an overview of Teacher and the use cases present in the Magister Course Management system, and Figure 10.20 shows an overview of Administrator and the use cases present in the Magister Course Management system.



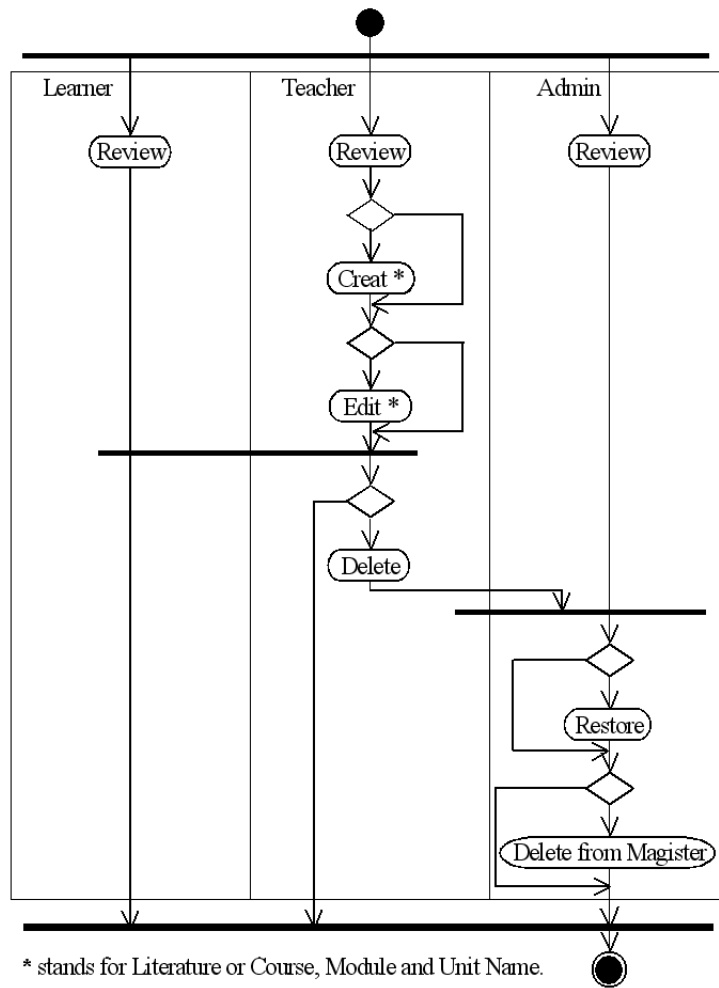


Figure 10.18: Course management process.

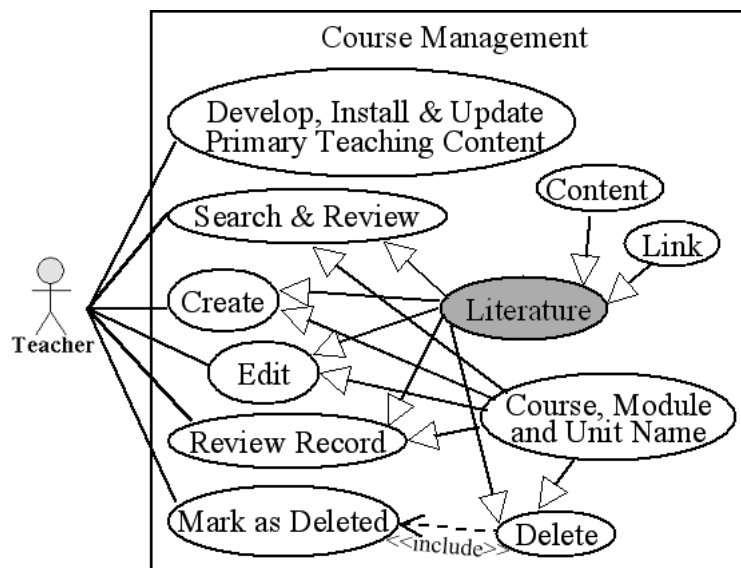


Figure 10.19: Course management for Teacher.

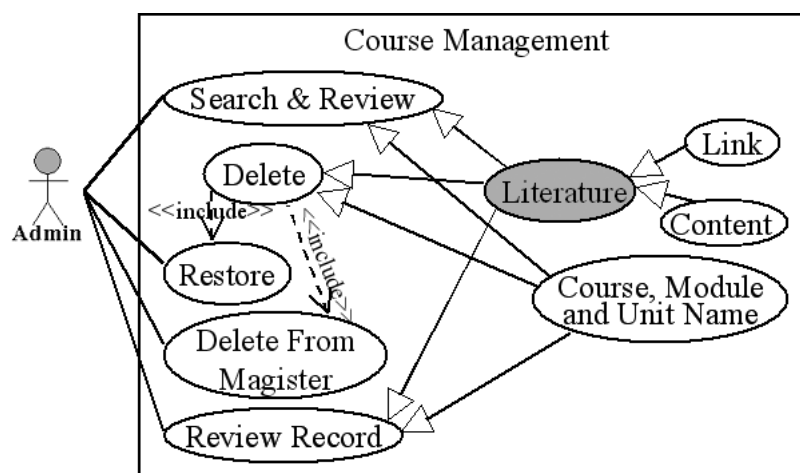


Figure 10.20: Course management for Administrator.

### 10.3.1 Target, Content and Example

The learning activities ‘Target’, ‘Content’ and ‘Example’ are comparatively stable parts. Their subject matter is more or less fixed by the ‘Teaching Aims’ (see Section 9.4.3). They are realized mainly by a combination of plain text and static graphics given in PDF format. A typical layout is shown in Figure 9.7 (see Section 9.5.1 ‘Layout’). As mentioned earlier, PDF files generated from TeX files are suitable for teaching Bernoulli Stochastics, because mathematical formulas can easily be incorporated and displayed, and documents can readily be printed or downloaded.

The TeX files of the teaching materials are authored by the *Stochastikon* project team. The program for transforming TeX files to PDF files and making available in Magister is realized by Java, and not web-based. It is the same program as used in *Stochastikon Encyclopedia* and *Stochastikon Mentor*.

### 10.3.2 Exercise

The characteristic features of the learning activity ‘Exercise’ has been described in detail in Section 9.4.4 ‘Teaching Style’. In the following the technical realization of the requested features in *Stochastikon Magister* is outlined.

#### Exercise Contents

The aim of the exercises is to have the learners deal with simple problems by themselves and, thereby, make them step-by-step familiar with the principles and the methods of stochastic thinking, modelling and stochastic procedures. Therefore, in many exercises the same situation or problem is used for illustrating step-by-step different quantities and posing different questions. By this the learner feels soon familiar with the situations and can concentrate on the problems and questions.

In the present Version 1 of *Stochastikon Magister*, which is in the on-line testing phase, there are only very few Exercises already implemented. However, in a preliminary version, which was developed for internal use only, the different technical possibilities were tested. The following example refers to such an internal advance development.

The exercise had been named ‘Silver Ball Situation’ and is described as following: An urn is filled with silver balls and brown balls and a person takes randomly a number of balls from the

urn.

In Course 1 a series of questions are created around this example aiming at showing how to represent knowledge/ignorance, how to represent variability, how to represent the random structure (different families of probability measures) and how to select the family member in the given situation. In Course 2 the same situation is used for illustrating how to select and apply stochastic procedures and how to interpret the obtained results. Below, the methods used in the exercises are briefly outlined.

## Exercises in Course 1

The exercises of Course 1 include multiple choice questions, simulations, dynamic graphics, online calculation and small animations.

The main teaching style used in Course 1 exercises is ‘Multiple Choice’ questions, as this is an effective method for making learners judge and compare concepts. Figure 10.21 shows a part of the preliminary exercises of Unit 1.2.3: ‘Ignorance Space’.

The screenshot displays the 'Stochastikon Magister' web application. The main content area is titled 'Learning Unit 1.2.3: Ignorance Space'. It contains two multiple-choice questions:

**1. Length**  
*A distance between two places is displayed together with a standard length of e.g. 1 km. The user must specify a lower and an upper bound for the unknown distance (fact). If the bounds are incorrect or too far away from the true length, a new attempt is warranted.*  
 You want to catch the bus and you know the bus stop is:

- A It is exactly  km away from your place,
- B It is about  km from your place,
- C It is at least  and at most  km,
- D you have no idea about the distance between your place and the bus stop.

**2. Number of Elements**  
*An urn is shown with a number of white and black balls. The volume of the urn and the balls is given. The user has to give upper lower bounds for the two numbers. If the bounds are incorrect or too far away from the true numbers, a new attempt is warranted.*  
 At a Christmas party, you see an urn is full filled with silver and brown balls. The silver balls consist of milk chocolates and the brown balls of coffee chocolates. The right-hand side picture shows the volume of the urn and the size of each chocolate ball. How many chocolate balls are there in the urn?

- A There are exactly  balls in the urn,
- B There are about  balls in the urn,
- C There are at least  and at most  balls in the urn,
- D you have no idea about how many balls are there in the urn.

Figure 10.21: ‘Multiple Choice’ questions in *Stochastikon Magister* Exercise.

Each of the choices in a question aims at teaching, explaining, identifying, comparing, distinguishing or correcting one or few small aspects. The whole question shall clarify the more complex idea of the corresponding unit stated in the Target.

Some of the exercises ask for inputs, which are immediately evaluated by the ‘Check’ answer buttons. Learners can input values and check whether their answers are correct obtaining some additional explanations. If a learner inputs inadmissible values, the online calculation will pop-up an alert asking for correction. There are also symbol (animation) pictures used to illustrate

the main idea of these examples. If a learner clicks the symbol picture, a small pop-up window summarizes the corresponding matter.

For implementing the multiple choice questions, JavaScript functions are used to

- support online calculations,
- give interactive answers, and
- control the suitable size of pop-up windows.

The used small animations are based on gif or wmf files.

The example below uses the ‘Silver Ball Situation’ to review the idea of a ‘Deterministic Variable’, the notion of complete/incomplete ‘Knowledge’ and to introduce how to quantify ‘Knowledge and Ignorance’ by means of the ‘Ignorance Space’. (see Figure 10.21)

**Title:** The Silver Ball Situation: Number of Elements

**Question Content:** At a Christmas party, you see an urn completely filled with silver and brown balls. The silver balls consist of milk chocolates and the brown balls of coffee chocolates. The right-hand side picture shows the volume of the urn and the size of each chocolate ball. Please answer the question “how many chocolate balls are there in the urn” as good as possible.

**Multiple Choice Answers:**

A: There are exactly  $\_$  balls in the urn.

B: There are about  $\_$  balls in the urn.

C: There are at least  $\_$  and at most  $\_$  balls in the urn.

D: I have no idea about how many balls there are in the urn.

**Answer A** means that there is ‘Complete Knowledge’ about the number of balls in the urn. In this case any other number of balls can be excluded with certainty for example because the number of balls was counted beforehand.

By clicking ‘A’, the learner claims to have ‘Complete Knowledge’ about the number.

Next, the learner is requested to ‘Check’ his ‘Complete Knowledge’ by inputting the exact number.

If the learner inputs ‘3’ as the exact number of balls, the ‘Check’ button will produce the following comment: *A comparison of the volume of the urn with the size of a ball based on the given picture shows that the **completely filled** urn certainly contains more than 3 balls. Therefore, this number can be excluded and a larger number of balls has to be taken into account.* The comment illustrates the meaning of ‘Complete Knowledge’ as the result of an exclusion procedure.

A similar comment will appear in case of Answer A with a too large number of balls. Additionally, a pop-up reaction will show the urn with the overflowing balls.

If the learner inputs ‘20’ as the exact number of balls and checks his answer, the following pop-up comment appears: *If 20 is the result of a counting process, then you have ‘Complete Knowledge’ as you can exclude any other number. If 20 is the result of an estimation, then you have not complete knowledge, but rely on subjective belief and in this case your answer is wrong.*

**Answer B** means that there is not ‘Complete Knowledge’ or that there is a certain amount of ‘Ignorance’, which, however, is not specified.

By clicking ‘B’, the learner admits that he/she has no ‘Complete Knowledge’ about the number, but the learner does not specify the amount of his ‘Ignorance’, so he gets the following comment: *Expressing ‘Ignorance’ in this way, means to neglect the knowledge about those numbers which can be excluded with certainty based on the given picture of the urn and the balls. The statement ‘about’ can mean almost everything and, therefore, it should not be used as basis for a decision.*

**Answer C** specifies the amount of existing ‘Ignorance’ by stating the too small and too large numbers of balls, which can be excluded with certainty based on the picture of the urn and the balls.

By clicking ‘C’ the learner admits and specifies the existing amount of ‘Ignorance’.

The ‘Check’ of the answer will produce comments corresponding to the given numerical values.

- If the learner specifies the ‘Ignorance’ by the range from ‘6’ to ‘60’ balls, the comment reads as follows: *The range of ‘Ignorance’ is too large! For sure the urn contains more than 6 balls and much less than 60 balls. Thus, the answer is correct, but more or less useless.*
- If the learner specifies the ‘Ignorance’ by the range from ‘2’ to ‘9’ or from 150 to 180 balls, the comment reads: *The given range of ‘Ignorance’ is obviously wrong and, therefore, dangerous as it will lead to wrong decisions.*
- If the learner specifies the range from ‘10’ to ‘30’ balls, the comment reads: *The given range may be the result of a quick glance on the urn and the balls. It is correct and might be useful for making a decision. If the amount of ‘Ignorance’ is too large, a second more intensive glance could reduce the amount of ‘Ignorance’.*

**Answer D** admits complete intellectual failure as it pretends ‘Complete Ignorance’ despite the given evidence of the picture showing the urn and the balls. By clicking ‘D’ the learner will be offered help by the following pop-up comment: *Specifying the existing amount of ‘Ignorance’ is one of the most important task in stochastic modelling. It is done by determining those values which cannot be excluded with certainty in a given situation. The resulting set of values forms the ‘Ignorance Space’. No ignorance i. e., ‘Complete Knowledge’ is a rare event, however ‘Complete Ignorance’ is almost impossible. therefore, knowledge in a real situation generally ranges between these two extremes.*

The concept of ‘Random Structure’ is illustrated by means of online calculations together with small dynamic graphics. Figure 10.22 displays an exercise referring to the minimum information

principle used for selecting the distribution function. The internal example ‘Fishing’ is used for introducing the concept of ‘Random Structure’ and its elements.

The screenshot shows the 'STOCHASTIKON Magister' web application. The main content area is titled 'Learning Unit 1.3.6: Minimum Information Principle' and '2.2. Fishing'. It contains a text block explaining the 'Fish Number' question, a form with input fields for 'Spending Days n' (8), 'Getting Fishes x' (3), and 'Success Fishing Probability p' (0.09), and buttons for 'Probability', 'ShowGraph', and 'Reset'. A bar chart shows the resulting Poisson distribution with  $\lambda = 0.72$  and  $X = 3$ . The chart has a title 'Poisson probability = 0.99366'. To the right of the chart is a cartoon illustration of a fisherman. The right sidebar contains navigation buttons: Target, Content, Example, Exercise, Test, Literature, FAQ, Discussion, Question, and a search box.

Figure 10.22: Online calculation and dynamic graphics in Magister Exercise.

Learners can input values for different parameters and click two ‘Probability’ buttons, which refer to two different events. The corresponding probabilities will appear in the adjacent fields. A click on the ‘ShowGraph’ button yields a graphical illustration of the resulting distribution function with some explanations.

The online calculations of this kind of exercises are realized by JavaScript function, while the small graphics, which change according to the given values, are produced by Java applets.

Besides, the exercises in Course 1 may also incorporate other techniques for demonstration and visualization, for instance more traditional tables, buttons, selecting forms, pictures and other tools for preparing and explaining concepts and methods. There are also assisting links - buttons or (animation) pictures to visit other related places, to refresh the memory or to relax the learning atmospheres.

## Exercises in Course 2

Course 2 has two different aims, namely to teach how to select a proper stochastic procedure and to teach how to apply this procedure. The two different aims necessitate different types of exercises in Course 2.

The first type of exercise shall illustrate the way how to select a stochastic procedure. This type is similar as the exercises of Course 1, i. e., the exercises are generally realized by multiple choice questions with small animations. The contents of these exercises are worked out not only for practicing how to distinguish different problems in order to assign the appropriate stochastic

procedures, but also for reviewing, linking up and enhancing the knowledge delivered in the units of Course 1.

The preliminary exercise referring to the “Prediction in the Univariate Case - Complete Knowledge” shall serve for illustrating the first type of exercise used in Course 2. For a univariate variable and assuming complete knowledge, i. e., the Ignorance Space has only one element, the ‘Silver Ball Situation’ is used, thus, linking exercises of Course 1 on the “Ignorance Space” and the “Minimum Information Principle” with exercises on predictions in Course 2.

After the question is formulated, some related and important concepts taught in Course 1 are put forward: Uncertainty, Complete Knowledge, Ignorance, Randomness, Variability, Random Structure and so on. By clicking the ‘concept’ or ‘symbol’ button, learners can get answers. The ‘information’ buttons give links to relevant definition in the corresponding learning units. The exercise aims at teaching how to select an appropriate prediction procedure to solve the problem posed in the question. Figure 10.23 shows the layout of this exercise.

The screenshot shows the Magister software interface. At the top, the title "STOCHASTIKON" is displayed in a large, bold font, with "Magister" underneath. Below the title, there is a navigation bar with links for "Stochastikon GmbH", "Calculator", "Encyclopedia", "Mentor", "Magister", and "Imprint". The main content area is titled "Learning Unit 2.2.3: Prediction in the Univariate Case - Complete Knowledge". The exercise text reads: "Succeed Drawing a Number of Elements". "In a Christmas party, you see an urn is full of chocolate balls, the silver balls are milk chocolates and the brown balls are coffee chocolates. You know exactly the urn contains 200 chocolates in which 50 are silver milk balls, and you want randomly take 16 balls from the urn." Below this, there is a question: "From former learning you know that Uncertainty is caused by Ignorance and Randomness. Here the Knowledge and Ignorance and Variability and Randomness and it is the same if you change 'Silver' balls to 'Brown' balls. How many silver milk balls you may meet? Which Stochastic Procedure could be used?" The interface includes a sidebar with buttons for "Target", "Content", "Example", "Exercise", "Test", "Literature", "FAQ", "Discussion", and "Question". At the bottom right, there is a "Solve Exercise" button.

Figure 10.23: The first type of Course 2 Exercise in Magister (a).

The problem at hand is to determine the appropriate stochastic procedure. Therefore, the ability to identify the aim of a given problem is of utmost importance. The exercise ‘Minimum Upper Bound  $\beta$ -Prediction Procedure’ is devoted to this task by introducing one special type of stochastic prediction procedure called the ‘minimum upper bound procedure’. The exercise also includes the possibility to review related concepts of Course 1 and Course 2, e. g., Prediction in the Univariate Case - Complete Knowledge, Prediction Procedure,  $\beta$ -Prediction Procedure, Bernoulli Space, Random Variable, Ignorance, Uncertainty Space, Random Structure Function. Figure 10.24 displays the layout of this exercise.

Figure 10.24: The first type of Course 2 Exercise in Magister (b).

Figure 10.25: The second type of Course 2 Exercise (a).

The second part of this Exercise refers to the second aim of Course 2 namely to teach ‘How to Apply Stochastic Procedures’. Reaching this learning target is supported by another compo-



ment of the *Stochastikon* system called *Stochastikon Graphics*, which provides dynamic graphical illustration for each stochastic procedure via internet. The advantages of using dynamic graphical representation in teaching stochastic procedures have been discussed in Section 9.4.4 ‘Teaching Style’. The dynamic interactive graphics are supported by *Stochastikon Calculator* and *Stochastikon Graphics*, and realized by Java programs and Java applets. The connections between the three involved components of the *Stochastikon* system have been outlined in Figure 9.3 (see Section 9.3 ‘Development of Magister’). The implementation of *Stochastikon Graphics* will be described in Chapter 16 ‘Stochastikon Graphics’.

Figure 10.25 is an example of the second type of exercises used in Course 2, which displays the starting part of the exercise page.

The exercise of how to apply a minimum upper bound  $\beta$ -prediction procedure is divided into four steps: Bernoulli Space, Probability Measure, Minimum Upper Bound Prediction and Scientific Report. The first three parts include links, texts, formulas and graphs, which are used for reviewing, visualizing and practicing step-by-step the necessary concepts and result in a minimum upper bound  $\beta$ -prediction procedure.

The third step ‘Minimum Upper Bound Prediction’ is based on the two preceding steps and the prediction answering the posed question is provided by means of a formula and a graph (see Figure 10.26). The learner can also experience how different input values, such as the reliability level  $\beta$ , will change the predictions.

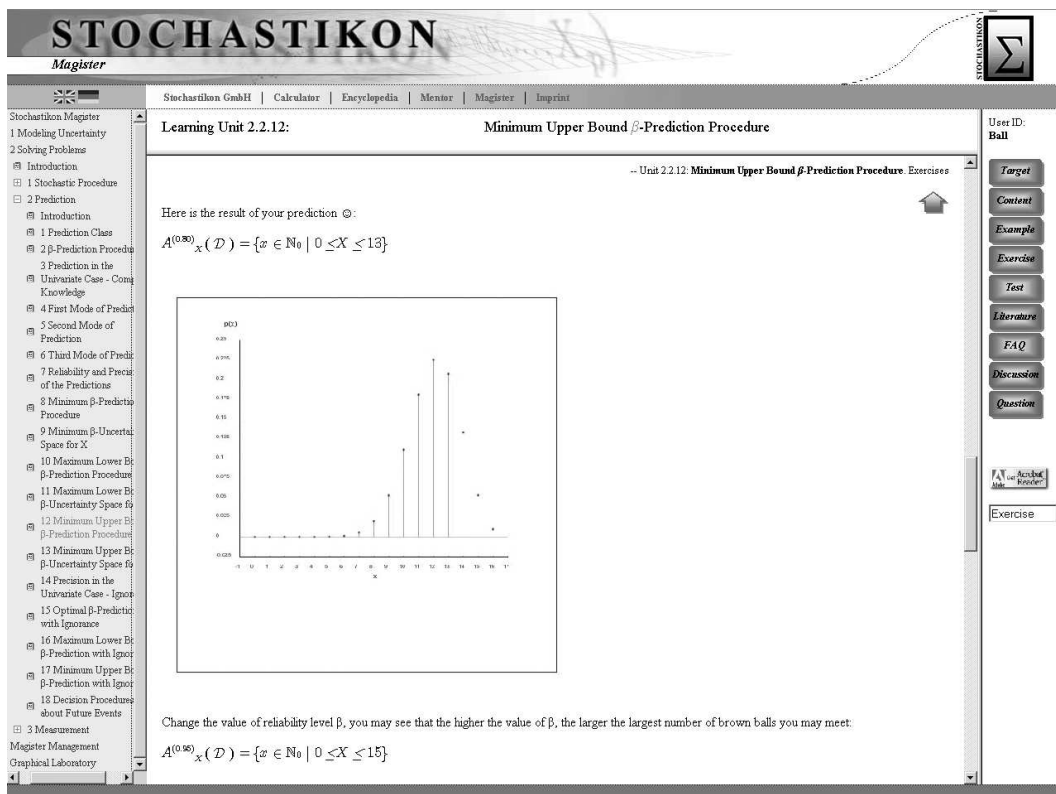


Figure 10.26: The second type of Course 2 Exercise (b).

In addition to the static representations of a prediction procedure by formulas or graphs (see Figure 10.26), there is the possibility to get predictions interactively in the ‘Graphical Laboratory’ with the advantage that one can experience how changes in the past affect the forecast of the future development. This method is illustrated by displaying the corresponding interface together with some explanations in Figure 10.27.

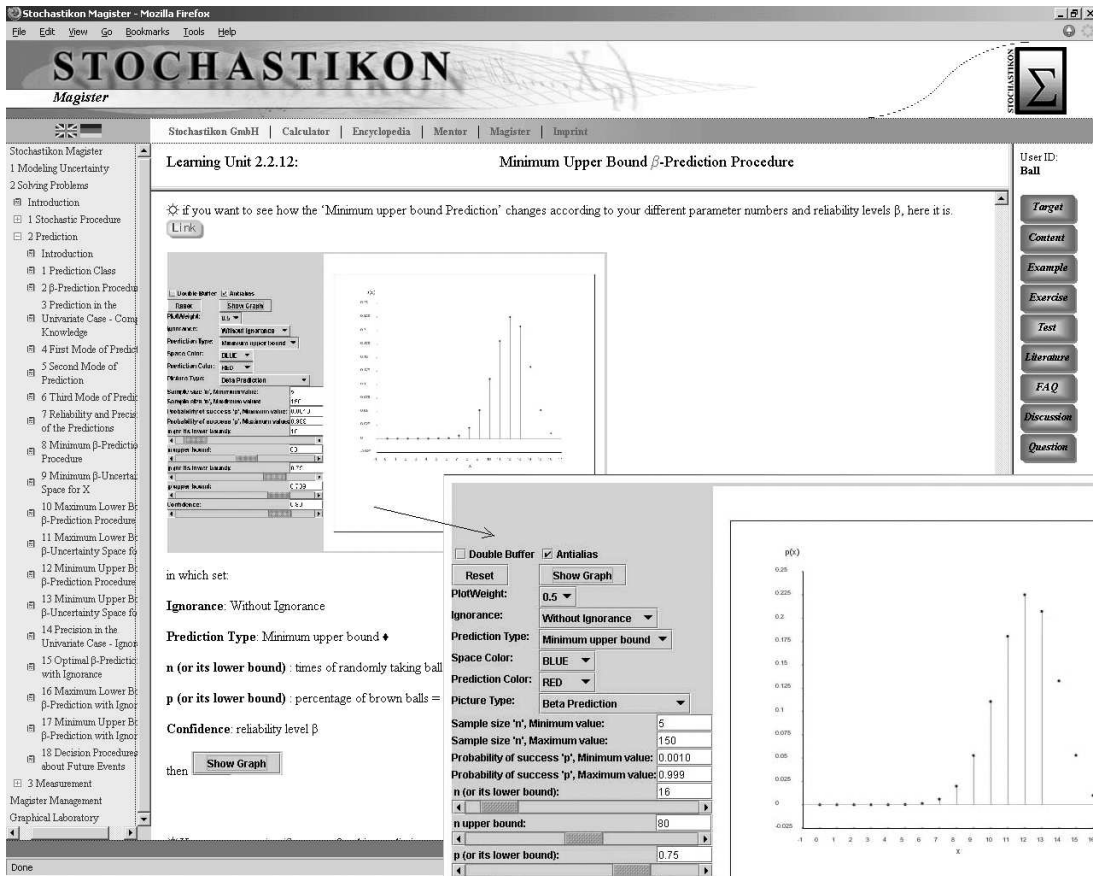


Figure 10.27: One interface of the Graphical Laboratory illustrated in Course 2 Exercise.

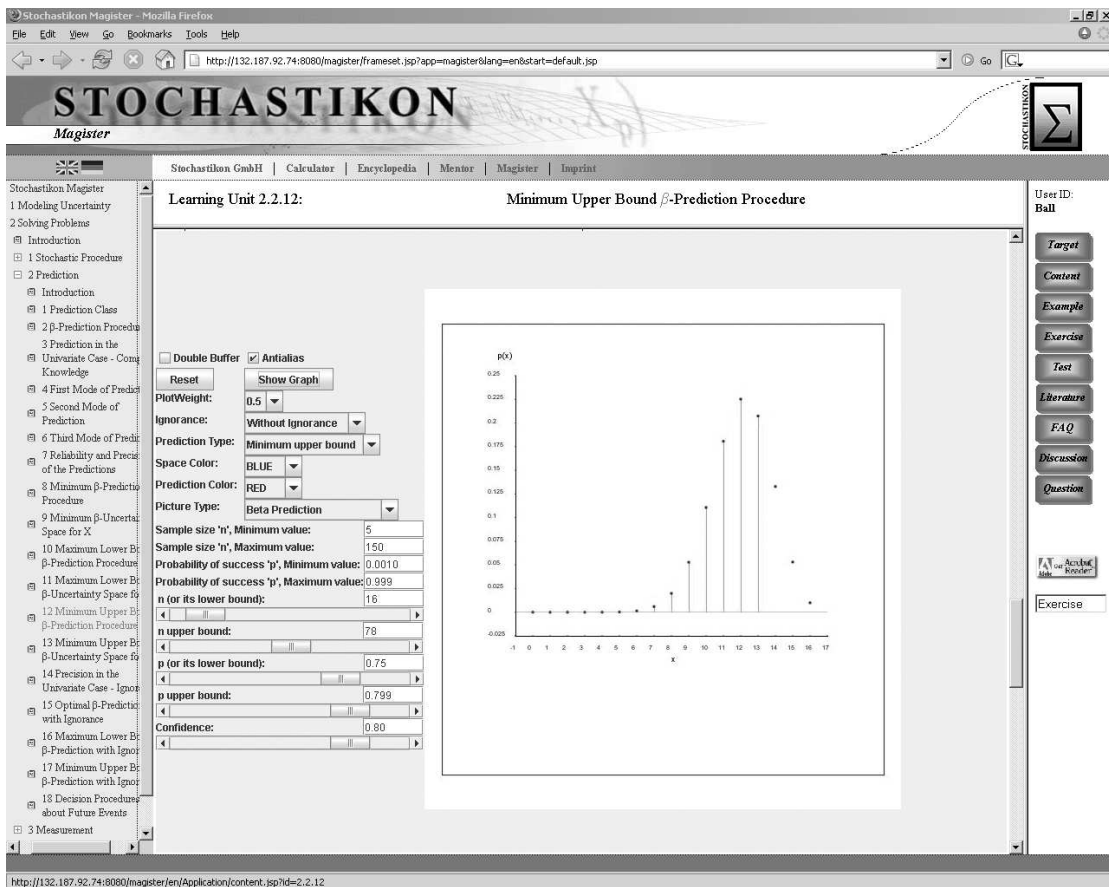


Figure 10.28: The Graphical Laboratory used in Course 2 Exercise.

By clicking a light blue ‘link’ button in Figure 10.27, learners enter the Graphical Laboratory in which they can reproduce e. g., the prediction graph given in Figure 10.26. Moreover, they can vary the input values of parameters and watch the corresponding changes in the graphics (see Figure 10.28). By playing around with the input values, which represent the past, the learner shall get a feeling of stochastic relations in contrast to cause-effect relations.

The second part of the Course 2 Exercise is completed by a ‘Scientific Report’ created by *Stochastikon Calculator*. This report is the formulation of both the problem at hand and its visualized solution by ‘Graphical Laboratory’. In Figure 10.29 the corresponding interface of *Stochastikon Calculator* is displayed as a thumb image. Learners shall become familiar with it, for being able to use the Calculator for solving related problems and for understanding the obtained reports.

The screenshot displays the *Stochastikon* web application interface. The main window is titled "Learning Unit 2.2.12: Minimum Upper Bound". It contains the following text:

**Prediction Type:** Minimum upper bound  $\uparrow$

**n (or its lower bound):** times of randomly taking balls

**p (or its lower bound):** percentage of brown balls = number of brown balls / n

**Confidence:** reliability level  $\beta$

then

✧ You can get a scientific report for this prediction problem from Calculator.

An inset window titled "STOCHASTIKON Calculator" shows the "Prediction of the Maximum Outcome of X" interface. It includes a "Parameter Input" section with the following values:

- Number of experiments: 16
- success probability: 0.75
- Confidence: 0.8

A "Start Calculation" button is visible below the input fields. The main interface also features a sidebar with a list of exercises and a top navigation bar with links to "Stochastikon GmbH", "Calculator", "Encyclopedia", "Mentor", "Magister", and "Impressum".

Figure 10.29: One interface of *Calculator* illustrated in Course 2 Exercise.

A click on the link ‘scientific report’ yields a report (PDF format) containing the exercise problem and the corresponding solution (see Figure 10.30). Note that providing a scientific report is a unique feature of the *Stochastikon* system. The report contains the entire stochastic model for the situation at hand, states the problem and gives the obtained results. By means of these reports the user shall soon feel familiar with the stochastic terminology, the stochastic model conception and the stochastic results including their interpretation. By disseminating these scientific reports Bernoulli Stochastics could be better understood and more widely known.

The screenshot displays the Stochastikon Magister web interface. At the top, the header reads 'STOCHASTIKON Magister'. Below the header, a navigation bar includes links for 'Stochastikon GmbH', 'Calculator', 'Encyclopedia', 'Mentor', 'Magister', and 'Imprint'. The main content area is titled 'Learning Unit 2.2.12: Minimum Upper Bound  $\beta$ -Prediction Procedure'. On the left, a sidebar menu lists various topics, with '12 Minimum Upper Bound  $\beta$ -Prediction Procedure' selected. The central panel shows a scientific report with the following text:

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Prediction of Maximum Outcome of  $X$

1. Description of the Model for  $(X, D)$

Bernoulli-Space  $\mathbb{B}_{n,p} = (D, X, \mathcal{P})$ :  
 The Bernoulli-Space  $\mathbb{B}_{n,p}$  contains the available knowledge about randomness with respect to the random variable  $X$  and ignorance with respect to the deterministic variable  $D$ .

1.) Random variable:  
 $X$  = number of successes in a sequence of independent and identical Bernoulli experiments

2.) Probability structure function:  
 $\mathcal{P}$ : family of Binomial probability distributions

Type of random law:  
 - Exact random law.

3.) Deterministic variable:  
 $D = (D_1, D_2)$   
 -  $D_1$  = number of Bernoulli experiments  
 -  $D_2$  = success probability

4.) Measurement space:  
 $D = \{(16, 0.75)\}$

Degree of ignorance about the initial conditions defined by  $D$ :  
 - Complete knowledge about initial conditions.

5.) Uncertainty function:  
 $\mathcal{X}(\{(16, 0.75)\}) = \{x|x \in \mathbb{N}_0, 0 \leq x \leq 16\}$

On the right side of the interface, there is a 'User ID: Ball' field and a vertical menu with buttons for 'Target', 'Content', 'Example', 'Exercise', 'Test', 'Literature', 'FAQ', 'Discussion', and 'Question'. At the bottom right, there is a 'Exercise' button.

Figure 10.30: An example of a scientific report created by *Stochastikon Calculator* shown in Course 2 Exercise.

As shown in Figure 10.27, Figure 10.29, the interface illustrations for the Graphical Laboratory and *Stochastikon Calculator* given by *Stochastikon Magister* are thumb images, which can be enlarged to their original size in a new pop-up window by clicking on them.

Besides each exercise includes links to related exercises and contents for assisting learners.

### 10.3.3 Literature in Course Management

‘Literature’ is an important supplement especially to the parts ‘Content’, ‘Example’ and ‘FAQ’. Recommendations may be given in the pages of these learning activities when needed which are indicated by citation scripts, and the corresponding information of these bibliographic references are listed in the Literature page of the learning unit at hand or in the ‘Literature’ Management panel. According to the citation scripts, learners can obtain the references either directly in form of digital files (PDF, doc, ppt, jpg...) or through a link to the corresponding web site displaying the recommended reading.

As already mentioned in Section 9.4.4 ‘Teaching Style’, references used in *Stochastikon Magister* should be managed centrally and dynamically. ‘Literature’ Management is, therefore, part of *Stochastikon Magister* Course Management, which is implemented by JSP, JavaScript, HTML and MySQL database.

#### ‘Literature’ in a learning unit

Figure 10.31 displays the Literature interface for a learning unit. It contains a list of references (here only a virtual list) with respect to the learning unit at hand, i. e., to Unit 1.1.1 ‘Evolution’. Only formal users of Magister are entitled to request literature items given in the list. The response to a literature request may consist of a file, a web link, a graph or a text comment.

Each listed reference information contains the referenceID, i. e., the citation script in squared brackets, last name of author, first name of author, date of publication and the reference title.

The screenshot shows the Magister web interface. At the top, the title 'STOCHASTIKON' is displayed in a large, stylized font. Below it, the word 'Magister' is written in a smaller font. The navigation bar includes links for 'Stochastikon GmbH', 'Calculator', 'Encyclopedia', 'Mentor', 'Magister', and 'Imprint'. The main content area is titled 'Learning Unit 1.1.1 Evolution'. It contains a section for 'Literature of current unit' with a list of references. The references are:
 

- [ test4 ] Autumn, G.D., 2003, information about paper written by Autumn in 2003.
- [ test5 ] Max, Data, 2000, information about literature written by MaxData company in 2000.
- [ test2 ] Summer, C.L., 1993, information about literature written by Summer in 1993.
- [ test1 ] Winter, S.N., 1986, literature about Winter's paper written in 1986.

 Below the list, there is a 'Finish listing' button and a 'Reset' button. The 'Edit Literature' button is highlighted, and a 'JavaScript Application' dialog box is open over it, displaying the message: 'Sorry, you have no authority to edit literature.' The dialog box has an 'OK' button. On the right side of the interface, there is a 'User ID: Cloud' section with several buttons: 'Target', 'Content', 'Example', 'Exercise', 'Test', 'Literature', 'FAQ', 'Discussion', and 'Question'. At the bottom right, there is a 'Literature' search box.

Figure 10.31: Literature in a Magister learning unit - List and Edit.

Another function shown in the literature interface (see Figure 10.31) is ‘Edit Literature’, which is reserved to Magister teachers with user status exceeding the value 5, i. e., Teacher Ball with user status 6 has access to the literature edit function.

A learner with user status from 1 to 5 can obtain a reference by performing ‘Select’, but trying to handle literature information by the ‘Edit Literature’ function, makes appear a pop-up alert rejecting the request. See Figure 10.31, the Learner Cloud can request and obtain a literature item, but is refused to execute the ‘Edit Literature’ function because of his insufficient user status.

Note that management functions used in the Literature activity of each learning unit only deal with reference quotations of the current learning unit. For example the ‘Edit Literature’ functions in Figure 10.31 handles literature information concerning Unit 1.1.1. This rule is applied to all activities referring to learning units and containing manageable elements such as Literature, Question, Discussion, FAQ and Test.

Teachers are authorized to edit the information of references cited in the current learning unit. The literature edit page of a learning unit consists of two lists. The first one contains the references that have already been cited in the current learning unit, while the second one lists the references that are not stated so far in the learning unit at hand, but are contained in the whole Magister literature base. These two lists comprise all references offered by Magister for the time being. Figure 10.32 displays an outline of these two lists.

The screenshot shows the 'STOCHASTIKON Magister' interface. At the top, there is a navigation bar with links for 'Stochastikon GmbH', 'Calculator', 'Encyclopedia', 'Mentor', 'Magister', and 'Inprint'. The main content area is titled 'Learning Unit 1.1.1 Evolution'. It contains two lists of literature references. The first list, 'Literature of current unit', includes references [test4] and [test5]. The second list, 'Literatures which have not been referenced in current unit', includes references [test10] and [test6]. A 'Reset' button is located at the bottom of the main content area. On the right side, there is a sidebar with a 'User ID: ball' and several buttons: 'Target', 'Content', 'Example', 'Exercise', 'Test', 'Literature', 'FAQ', 'Discussion', and 'Question'. The 'Literature' button is highlighted.

Figure 10.32: Edit literature in a Magister learning unit - Edit (a).

The two lists as illustrated in Figure 10.32 may be used for editing in the literature panel of Unit 1.1.1 ‘Evolution’ (see Figure 10.33). One can ‘Update’ the references or ‘Add’ a new reference to the current learning unit either by taking it from the second list or by adding a new reference to Magister. Editing the literature information also includes update (add or erase) the places of citation to a reference in the current learning unit. When a new reference is added to Magister, it must be cited at least in one learning activity (location) of the current unit. The place of citation is stored simultaneously with the creation of the new reference as shown in Figure 10.33 ‘Add a Literature’.

Each reference has a unique ‘referenceID’ called ‘citation script’ which allows citations and thus controls all connections, i. e., reference information and places of citation. It is the primary key for any literature information and cannot be ‘Updated’. There are further information which can be used for identifying a reference, these include last and first name of the author, publication date, reference title (or other informative data) called ‘information’, ‘type’ of reference and, finally, the ‘link’ to the reference. The ‘link’ can be the name of a digital file (document or picture) stored in Magister, a web address or some text comments about this literature. The ‘type’ of a reference determines the way how the linked literature is put at the learners’ disposal. Besides the ‘referenceID’, ‘information’ and ‘link’ are also uniquely assigned to each reference.

When updating a reference information, the teacher in charge can input the reasons for updating for later checking. Any change of a reference or the place of its citation is automatically recorded by Magister Management.

By specifying a reference in one of the lists in the literature editing page (see Figure 10.32), a teacher can use the ‘Update learning activity refer to the selected literature’ function (see Figure 10.33) for including a new or erasing an existent place of citation.

Figure 10.33: Edit literature in a Magister learning unit - Edit (b).

There are essentially two functions for placing a citation on the level of the learning units. These two functions are briefly introduced below. Note that the editing work is done on the learning unit level and, therefore, the unitID needs not be specified.

- Deleting an existent place of citation by specifying the name of the corresponding learning activity.

Figure 10.34 refers to the reference '[test1]' of Winter, S.N. and gives a list of the learning activities of its citation in Unit 1.1.1 'Evolution'. Any deletion of a place of citation is preceded by the already well-known demand for final confirmation in order to avoid mistakes.

- Adding a place of citation by specifying the name of the corresponding learning activity.

If the citation to be added already exists, the teacher will be informed and the action is stopped.

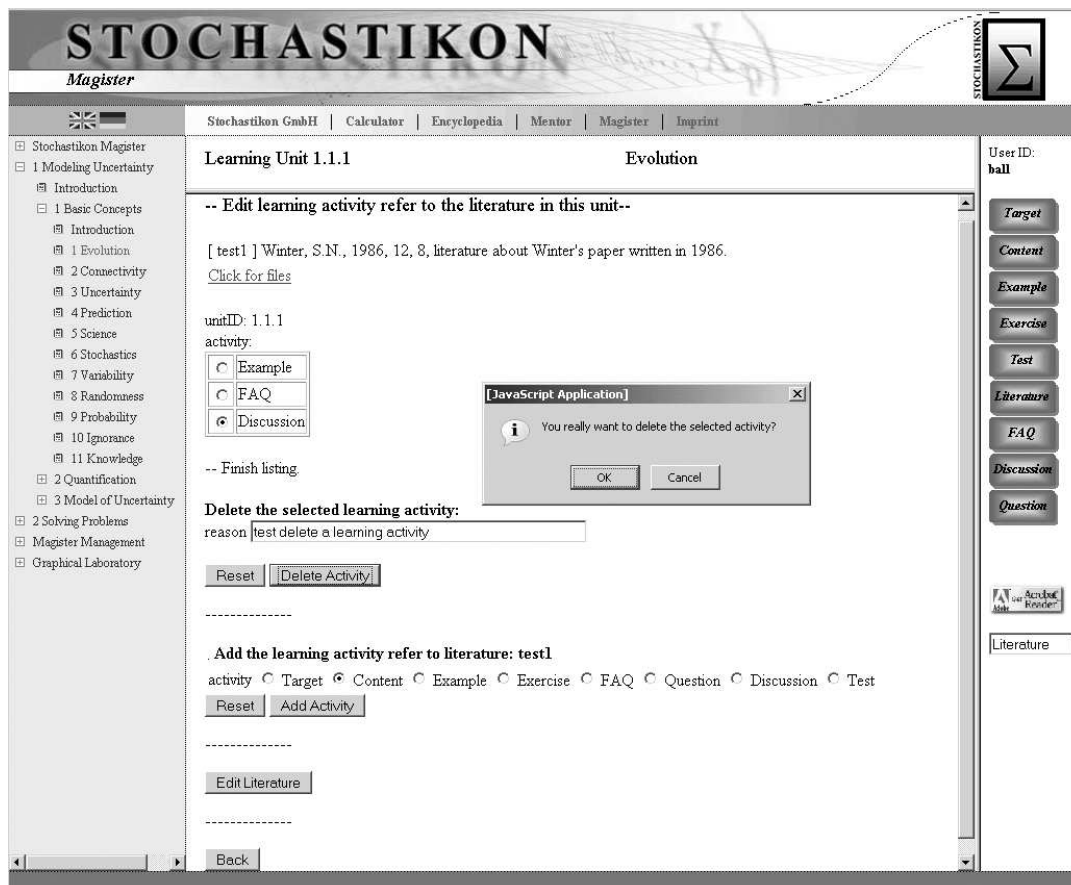


Figure 10.34: Edit literature in a Magister learning unit - Edit (c).

## ‘Literature’ Management

The ‘Literature’ Management panel is part of the ‘Course Management’. The managing functions in this panel refer to the three types of Magister users: learners, teachers and administrators, as discussed in Section 10.1 ‘Magister Control System’.

The ‘Literature’ Management part for teachers supports all functions provided already in the ‘Literature’ panel for the learning units. However, ‘Literature’ Management is the central tool for handling the complete references in Magister and, therefore, provides more powerful handling functions.

In order to get the information about a requested reference quickly, teachers can use different search criteria. Figure 10.35 shows the corresponding input form. The search criteria may refer to any feature of the references, i. e., referenceID, authors’ name, title, publishing date or type. Besides, teachers can search for the references cited in a specific learning activity for instance in the learning activity ‘Content’. Moreover, teachers can restrict the display of the search results by certain order conditions. If no restriction is specified, all references quoted in Magister are displayed in alphabetical order of the authors’ last name and publishing year.

In Figure 10.35 a search request with the following criteria is displayed:



Figure 10.35: ‘Literature’ Management for teachers - Review literature.

1. Learning Unit 1.1.1 Evolution
2. publishing year later than 1980
3. type = 2 (file)
4. learning activity = 2 (Example)
5. order: author’s last name (increasing) and publishing year.

Figure 10.36 shows the results of the search triggered by the request given in Figure 10.35. The search results are preceded by a recapitulation of the required conditions. Then the list of found references is given followed by the ‘Add a new literature’ form. Thus, the teacher may, straight away, add a new reference.

Teachers may edit a reference in the list by marking it and clicking on the ‘Select’ button (see Figure 10.36). At the top of Figure 10.37 the relevant reference information are stated together with a link to the reference itself and the corresponding places of citation. Clicking the unitID opens a pop-up window containing the course-module-unit names and their corresponding unitIDs.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

**'Literature' Management**

User ID: Ball

Teacher  
Learner  
Admin

Teacher

-- Literature List --

**Conditions:**  
course - module - unit = Modeling Uncertainty - Basic 1.1.1: Evolution  
Terms -  
referenceID  
year > 1980  
author lastname  
information  
type = 2 (0 text, 1 image, 2 file, 3 link)  
link  
learning activity = 2 (0 Target, 1 Content, 2 Example, 3 Exercise, 4 FAQ, 5 Question, 6 Discussion, 7 Test)

**Order by:** lastname , year

\*\*\*\*\*

**Mark the reference you want to get or edit:**

[ test2 ] Summer, C.L., 1993, information about literature written by Summer in 1993.  
 [ test1 ] Winter, S.N., 1986, literature about Winter's paper written in 1986.

-- Finish listing

Reset Select

**Add a new literature:**  
referenceID  
author lastname  
publish year  
month (01~12)  
day (01-31)

Figure 10.36: 'Literature' Management for teachers - searching results.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

**'Literature' Management**

User ID: Ball

Teacher  
Learner  
Admin

Get the selected literature:  
[ test1 ] Winter, S.N., 1986, 12, 8, literature about Winter's paper written in 1986.  
[Click for files](#)

learning units and their activities refer to this reference:  
1.1.1: Example, FAQ, Discussion,  
1.1.2: Content

**Edit the selected literature:**

**Update this literature:**  
:  
Reset Update

**Add a new literature:**  
:  
**Add the learning activity refer to the new literature:**  
unitID

**Change records of the selected literature:** Change Record

**Update learning activity refer to the selected literature:** Update Activity of the Literature

Back

**Course-Module-Unit Name List:**

unitID	course name	module name	unit name
1.1.1	Modeling Uncertainty	Basic Terms	Evolution
1.1.2	Modeling Uncertainty	Basic Terms	Connectivity
1.1.3	Modeling Uncertainty	Basic Terms	Uncertainty
1.1.4	Modeling Uncertainty	Basic Terms	Prediction
1.1.5	Modeling Uncertainty	Basic Terms	Science

Figure 10.37: 'Literature' Management for teachers - Edit literature.

Following the citation information are the already described editing functions: ‘Update literature’, ‘Add a new literature’ and ‘Update Literature-Activity Connections’. There is one difference in the ‘Add a new literature’ function, as the teacher must specify a unitID for the place of citation. Finally, there is the additional function button ‘Change Record’ which is explained below.

The ‘Change Record’ contains the history of the reference in question, i. e., the moment of its introduction to the Magister Literature, of updates and finally of its deletion together with the userIDs of the persons executing the changes (see Figure 10.38). The ‘Change Record’ is automatically performed whenever one of the above-mentioned changes has been completed.

The screenshot shows the 'Literature' Management interface for teachers. The page title is 'Literature' Management. The main content area displays the change records for a selected literature entry. The entry is: [ test1 ] Winter, S.N., 1986, 12, 8, literature about Winter's paper written in 1986. Below the entry, there are links for 'Click for files' and 'learning units and their activities refer to literature: test1'. The change record table is as follows:

changerID	type	reason	date
dana	0	add new literature	2005-11-03 17:10:22.0
dana	2	test update.	2005-11-04 15:47:37.0

Below the table, there are buttons for 'Edit activity refer to this Literature', 'Edit Literature', and 'Back'. The right sidebar shows the user ID 'Ball' and buttons for 'Teacher', 'Learner', and 'Admin'.

Figure 10.38: ‘Literature’ Management for teachers - change records.

If a change of a citation of an existing reference or of a new reference refers to a learning activity, the button ‘Update Activity of the Literature’ has to be clicked. Updating is performed in the already known way.

As shown in Figure 10.39, there is again the button ‘Change Record’ available, by means of which the entire ‘citation’ history of the reference in question can be checked. The ‘Change Record’ function is available for all the activities performed in Magister. This function is especially beneficial, if the reasons for the different changes are listed, however, it should be noted that stating the reasons for a change is not mandatory, but voluntary.

The ‘Literature’ Management for administrators provides all functions being available for teachers. Additionally, administrators manage references which are so far not cited in any of the Magister learning units. To this end the literature edit page for administrators contains a ‘Check’ button (see Figure 10.40) by means of which not cited references are listed. An administrator can introduce these references to learning units or delete them.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

**'Literature' Management**

User ID: Ball

Teacher  
Learner  
Admin

Teacher

-- Edit learning activity refer to the literature --

[ test1 ] Winter, S.N., 1986, 12, 8, literature about Winter's paper written in 1986.  
Click for files

	unitID	learning activity
<input type="radio"/>	1.1.1	Example
<input type="radio"/>	1.1.1	FAQ
<input type="radio"/>	1.1.1	Discussion
<input type="radio"/>	1.1.2	Content

-- Finish listing.

**Delete the selected learning activity:**  
reason

**Add the learning activity refer to literature: test1**

unitID

activity  Target  Content  Example  Exercise  FAQ  Question  Discussion  Test

**Change-records of learning activity refer to this literature:**

Figure 10.39: 'Literature' Management for teachers - Edit citations.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

**'Literature' Management**

User ID: Dana

Teacher  
Learner  
Admin

Administrator

**Review literature:**

**Conditions:**

learning unit

referenceID

author

lastname

publish year  (YYYY)

information

type

link

learning activity

Order by:   and

Check references which have not been referred by any learning activity:

Figure 10.40: 'Literature' Management for admin. - Review and Check.

Only administrators have the authority to delete a reference completely from Magister. To this end the 'Edit' page for administrator contains a 'Delete' button (see Figure 10.41).

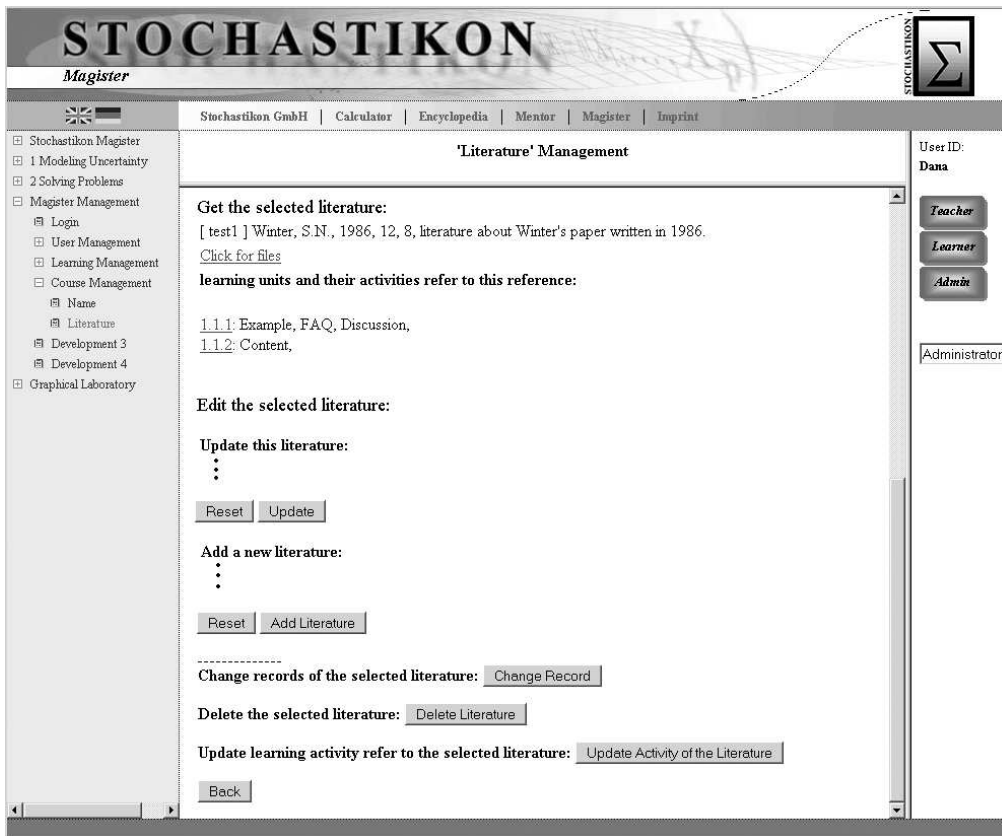


Figure 10.41: 'Literature' Management for administrators - Edit literature.

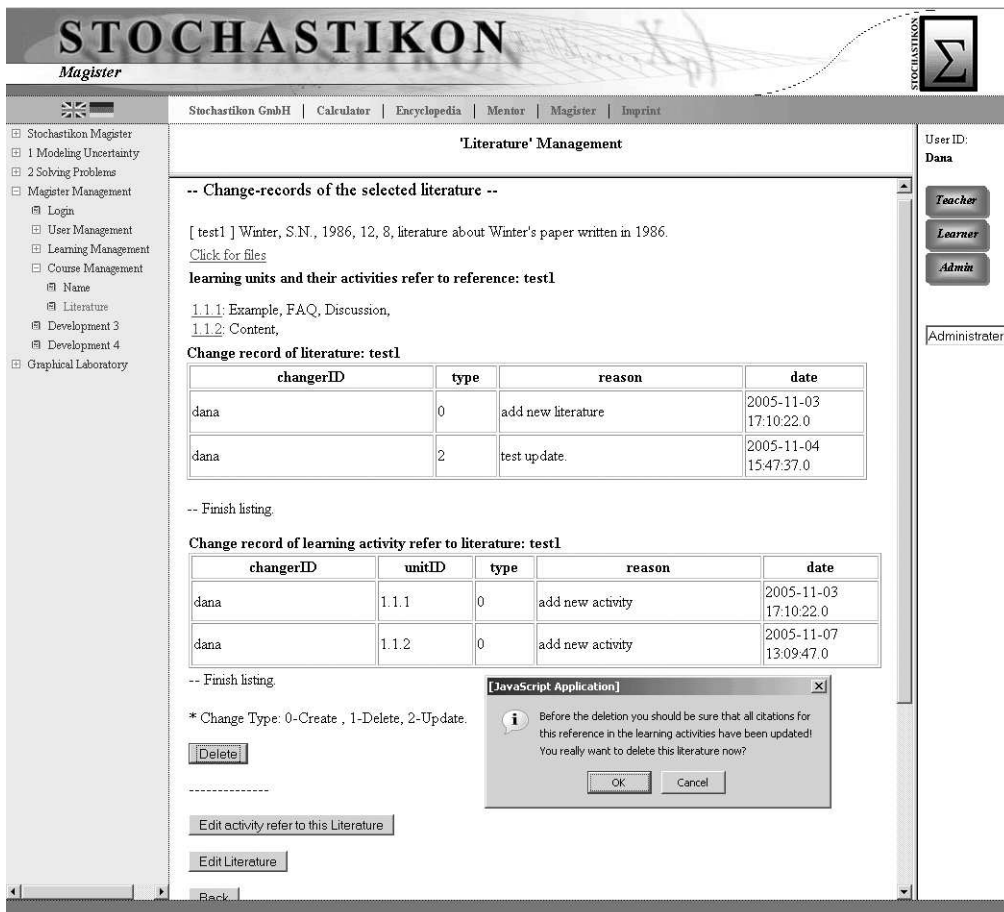


Figure 10.42: 'Literature' Management for administrators - Delete literature.

Before a deletion is executed, all information concerning this reference need to be deleted such as reference information, places of citation, change records, etc. will be displayed in a new page, and the administrator is asked to confirm the deletion as shown in Figure 10.42.

The only authority of learners with respect to ‘Literature’ Management is to activate the search function (see Figure 10.35), to select and obtain a reference and to get a list of learning units and activities where the reference in question is cited (see Figure 10.43).

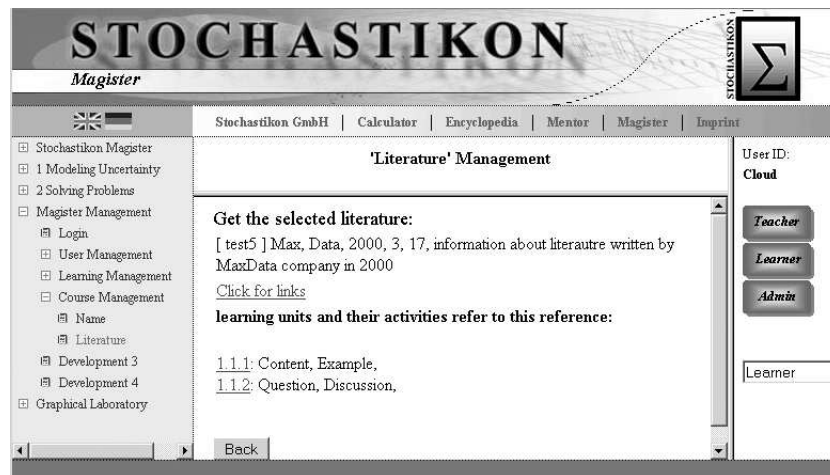


Figure 10.43: ‘Literature’ Management for learners - get a literature.

### 10.3.4 Course Name Management

The next part of the Course Management is the ‘Course-Module-Unit Name’ Management. This part is used for managing the names of courses, modules and learning units. The different users, i. e., learners, teachers and administrators, have access to different managing functions.

The first page of ‘Course-Module-Unit Name’ Management for teachers is shown in Figure 10.44. By clicking the top line, all current ‘Course-Module-Unit Name’ information will be listed in a new window. The management functions with respect to course, module and unit names are only available for teachers with user status larger than 6. Therefore, in Figures 10.44 and Figure 10.45 the Teacher ‘12345’ with user status 7 is used for illustration.

The screenshot shows the 'Stochastikon Magister' web application. The main content area is titled "'Course-Module-Unit Name' Management". It features three buttons: "Course Name Management (Only for teachers whose user status >6)", "Module Name Management (Only for teachers whose user status >6)", and "Unit Name Management (Only for teachers whose user status >6)". A sidebar on the left contains a navigation menu with items like "Stochastikon Magister", "1 Modeling Uncertainty", "2 Solving Problems", "Magister Management", "Login", "User Management", "Learning Management", "Course Management", "Name", "Literature", "Development 3", "Development 4", and "Graphical Laboratory". On the right, there is a "User ID: 12345" field and three role buttons: "Teacher", "Learner", and "Admin". Below the main content area, a browser window displays a table titled "Course-Module-Unit Name List:" with the following data:

unitID	course name	module name	unit name
1.1.1	Modeling Uncertainty	Basic Terms	Evolution
1.1.2	Modeling Uncertainty	Basic Terms	Connectivity
1.1.3	Modeling Uncertainty	Basic Terms	Uncertainty
1.1.4	Modeling Uncertainty	Basic Terms	Prediction

Figure 10.44: Name Management for teachers.

The managing processes for the names of courses, modules and units are similar. When the teacher clicks, for example, the management page for module name, all the currently existing module names are displayed (see Figure 10.45). The available managing functions are: 'Update the selected module name', 'Add a module name', 'Delete the selected module name' and finally 'Check the selected module name's changing records'. A teacher can select one of the listed module names and edit it.

Updating or adding a name on the module level requires that besides the moduleID the corresponding courseID has to be specified. Analogously, on the learning unit level the corresponding moduleID as well as the courseID have to be stated. When updating or deleting a module name, teachers can input the reasons of the change for later revisions.

Deleting a module name requires a further confirmation by a pop-up alert window. If the module, the name of which shall be deleted, contains learning units, it cannot be deleted. For example if module name 1.1 'Basic Terms' is to be deleted, then all names of the corresponding units as 1.1.1 'Evolution', 1.1.2 'Connectivity', 1.1.3 'Uncertainty'... have to be deleted beforehand. The analogous rule holds when deleting a course name. Any name operation will be automatically recorded and can be checked by 'Check the Module Name Change Record'

'Course-Module-Unit Name' Management for administrators is exactly the same as for teachers. 'Course-Module-Unit Name' Management for learners supports only the listing of course names, module names and unit names. There is no editing function admitted to learners.

The screenshot shows the 'STOCHASTIKON Magister' interface. The main content area is titled "'Course-Module-Unit Name' Management". It features a table of modules and several form sections for managing their names.

	moduleID	module name
<input type="checkbox"/>	1.1	Basic Terms
<input type="checkbox"/>	1.2	Quantification
<input type="checkbox"/>	1.3	Model of Uncertainty
<input type="checkbox"/>	2.1	Stochastic Procedure
<input type="checkbox"/>	2.2	Prediction
<input type="checkbox"/>	2.3	Measurement

Below the table, there are three main sections:

- Update the Selected Module Name with:** Includes fields for courseID, moduleID (1~99), module name (with a note: "β please input &#946;"), and reason (with a dropdown set to "No reason"). Buttons: Reset, Update a Module Name.
- Add a Module Name:** Includes fields for courseID, moduleID (1~99), and module name (with a note: "β please input &#946;"). Buttons: Reset, Add a Module Name.
- Delete the Selected Module Name:** Includes a reason dropdown (set to "No reason"). Button: Delete a Module Name.

At the bottom, there is a section for "Check the Module Name Change Record:" with a "Check" button and a "New Operations to Name" button.

Figure 10.45: Name Management for teachers - Module Name Management.

## 10.4 Magister Learning Management

The learning process is controlled and supervised by teachers with the help of the Learning Management panels. The records of a learner's learning activities constitute the learning process. These records cover three learning activities namely Question, Discussion and Test, together with the learner's cooperation with instructors (instructions and feedbacks in the 'Learner Progress' Management panel) during the learning process.

FAQ also belongs to the Learning Management part because frequently asked questions are often created on the basis of the activities Question and Discussion. The advantages have been discussed in Section 9.4.4 'Teaching Style'.

Each of the activities Question, Discussion, FAQ and Test has two interfaces, namely one belonging to each learning unit and one for the management part. Even when referring to the same learning activity, the range, function and focus of the two interfaces are different. The interface of one of these activities within a learning unit refers exclusively to the current unit and focus on learning. In contrast, the corresponding interface of the management part refers to all learning units and focus on teaching and managing.



Figure 10.46 is a UML user case diagram that shows an overview of actors and the use cases present in the Magister Learning and Learning Management system.

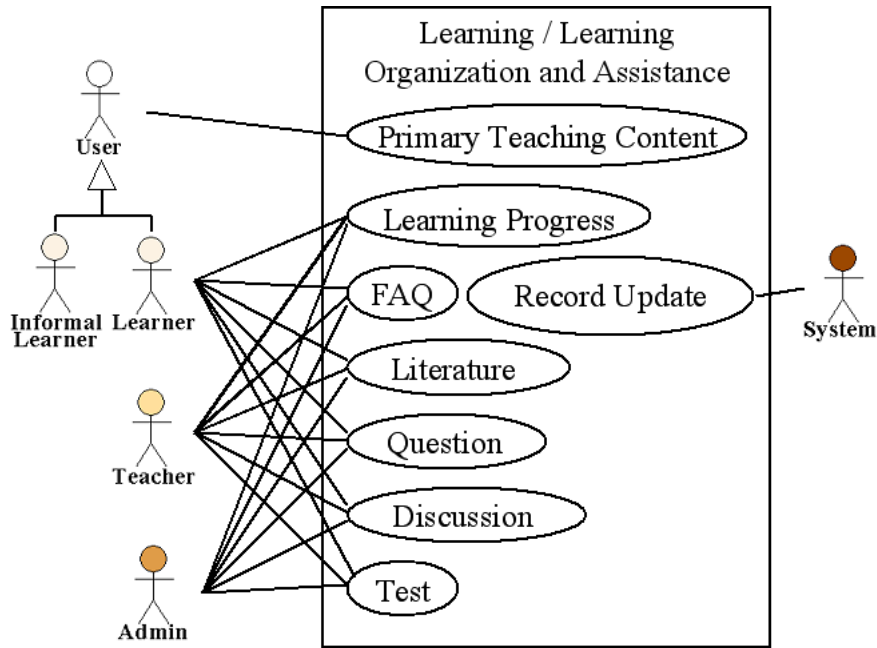


Figure 10.46: Learning, learning organization and assistance in Magister.

Figure 10.47 is a UML user case diagram that shows an overview of Learner and the use cases present in each Magister Learning unit. The main task of each learning unit is to provide sufficient functions to support and accomplish Learner’s learning process.

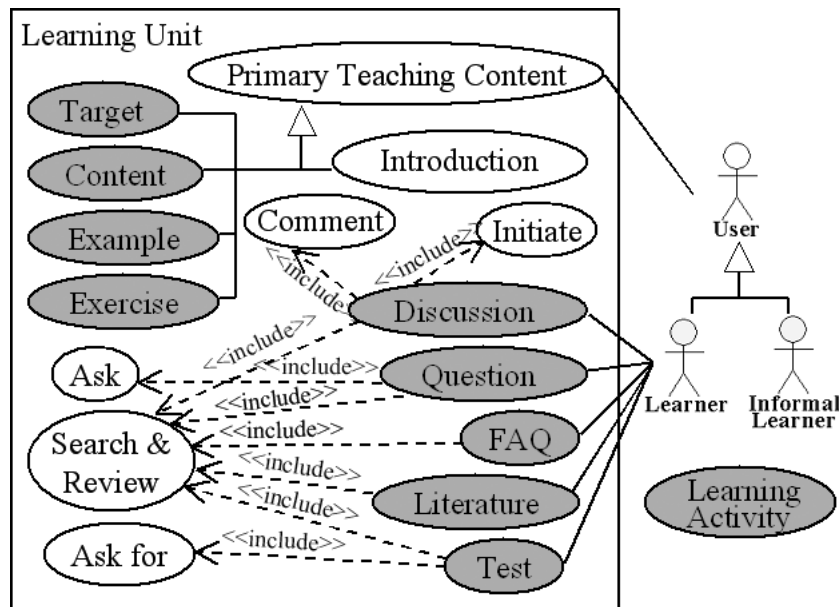


Figure 10.47: Learning activities for Learner in each Learning Unit.

Figure 10.48 is a UML user case diagram that shows an overview of Learner and the use cases present in Magister Course and Learning Management system. The functions for Learner here is reviewing learning records of all Magister learning units.

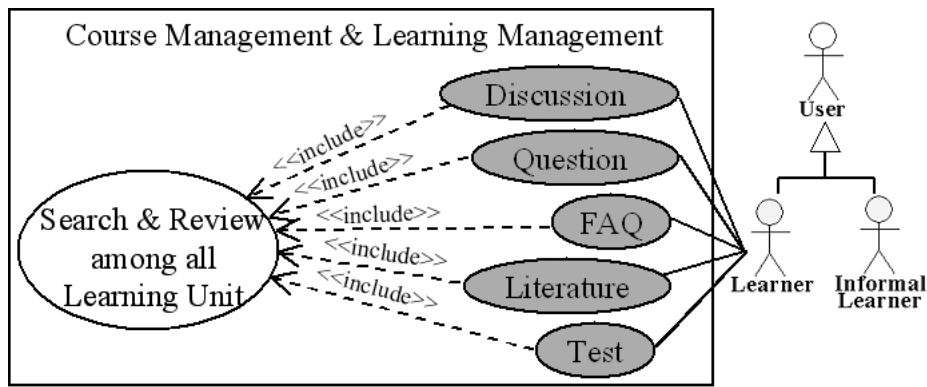


Figure 10.48: Learning assistance provided in course management and learning management packages for Learner.

Figure 10.49 is a UML user case diagram that shows an overview of Administrator and the use cases present in Magister Learning Management system. The functions for Administrator here is mainly to supervise the creation and update made by Teacher during the Learning Management process.

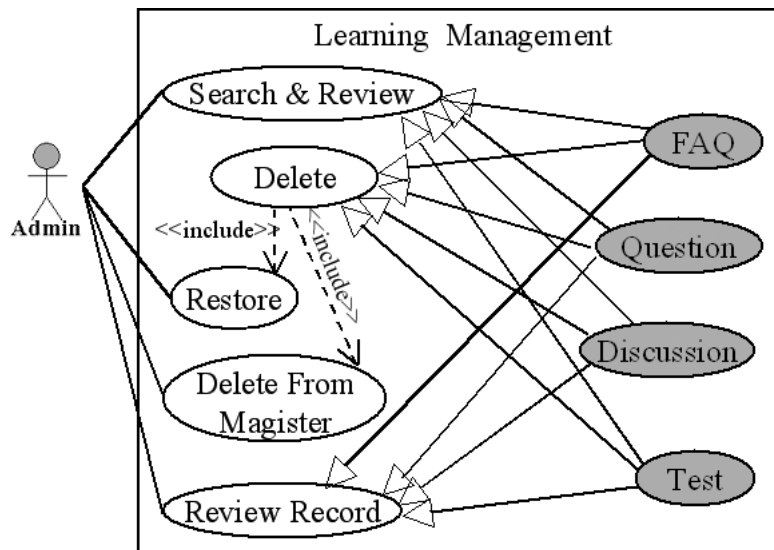


Figure 10.49: Learning Management work for Administrator.

*Stochastikon Magister* Learning Management is implemented by JSP, JavaScript, HTML and MySQL database. In the following sections the management of the different parts is introduced in detail.

### 10.4.1 Question

Figure 10.50 is a UML swimlanes in activity diagram, which describes the Question organization and management process in Magister e-learning environment. This diagram shows that Question is asked by Learner within each learning unit and answered and managed by Teacher within the Question Management package.

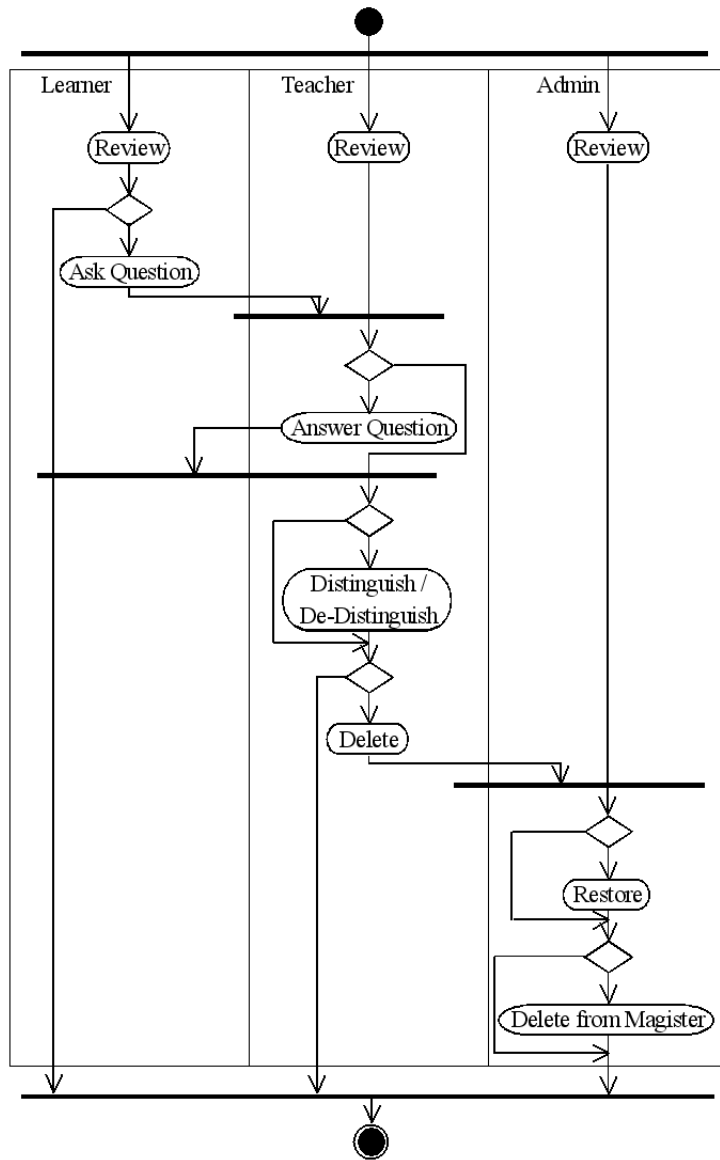


Figure 10.50: Question organization and management process.

**‘Question’ in a learning unit**

The learning activity ‘Question’ enables the learners to ask questions or to review former questions about the current unit. Figure 10.51 displays these two functions.

‘Question’ offers the possibility to learners to communicate directly with teachers, which is important for maintaining and improving motivation. For each question a questionID is generated automatically by the system separately for each learner and each learning unit. In Figure 10.51, Learner Cloud asks his second question about the content of Unit 1.1.2 ‘Connectivity’.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

Learning Unit 1.1.2: **Connectivity**

User ID: cloud

**Ask a new question:**

questionID: 2

question: Cloud gives the second question about learning unit 1.2 'Connectivity'...

Reset Ask

**Review old questions:**

Conditions:

questionID: [dropdown] [input]

question: [input]

creation date: [dropdown] [input] (YYYY-MM-DD)

answers: [dropdown] [input]

Order by: questionID [dropdown] increase [dropdown]

Reset Review

Target  
Content  
Example  
Exercise  
Test  
Literature  
FAQ  
Discussion  
Question

Question

Figure 10.51: Question activity in a Magister learning unit (a).

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

Learning Unit 1.1.2: **Connectivity**

User ID: cloud

-- Searching results --

Conditions:

questionID

question

creation date

answers

Order by: questionID

\*\*\*\*\*

Mark the question you want to check answers:

questionID	question	creation date	answers
1	question 2 about unit 1.2 from cloud, about the connectivity. ask now!@#\$%&^&(**)_+_!+=-.90=-13/<>?gh gfdhdfgdhgf54 9!hg'sfkd fgfgsdfg	2005-04-29 22:27:14.0	5
2	Cloud gives the second question about learning unit 1.2 'Connectivity'...	2005-06-12 17:14:35.0	0

-- Finish listing.

Reset Select

New Question or Review

Target  
Content  
Example  
Exercise  
Test  
Literature  
FAQ  
Discussion  
Question

Question

Figure 10.52: Question activity in a Magister learning unit (b).

Figure 10.52 shows all questions asked by Learner Cloud about Unit 1.1.2 using the 'Review'

button shown in the lower part of Figure 10.51. The final column of the list is ‘Answers’, which records the number of answers to the question by teachers. If a learner wants to see all answers to one question, he/she must mark the question and click the ‘Select’ button.

Figure 10.53 displays some answers to one of Cloud’s questions about Learning Unit 1.1.2. The teacher, who answers a question, is identified by his/her teacherID. By clicking the teacherID learners can directly address him/her by sending an E-mail. In Figure 10.53 Learner Cloud wants to send an E-mail to Teacher Ball, whose E-mail address is foot@Ball. The E-mail function in Magister shall support an easy and informal contact between learner and teacher for improving motivation on either side.

The screenshot shows the 'STOCHASTIKON Magister' interface. The main content area displays 'Learning Unit 1.1.2: Connectivity'. A question is shown with its ID, creation time, and number of answers. Below the question is an 'Answer List' with three entries. Each entry includes the teacher's ID, creation time, and the answer text. An email client window is open over the first answer, showing the email address 'foot@ball' in the 'To:' field.

Figure 10.53: Question activity in a Magister learning unit (c).

## ‘Question’ Management

As discussed in Section 10.1 ‘Magister Control System’, operations in each learning management panel are categorized according to the different users of *Stochastikon Magister*: learner, teacher and administrator.

For learners, ‘Question’ Management can only be used for reviewing their own questions and the corresponding answers by teachers. The only difference between ‘Review old questions’ in the Question activity of a learning unit (see Figure 10.51) is that in the management part learners can review all their questions asked in different learning units. They can also specify a special unit for searching the questions. Figure 10.54 is the interface of the ‘Question’ Management for learners.

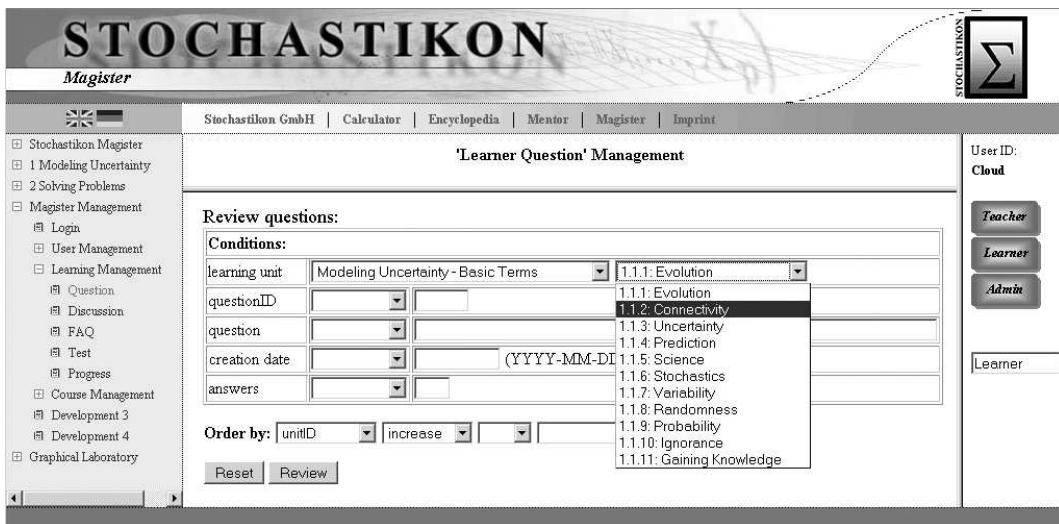


Figure 10.54: ‘Question’ Management for learners in Magister.

Teachers use the ‘Question’ Management panel to answer questions, assign the label ‘distinguished’ to a question, remove the label from questions and delete questions. Figure 10.55 is a UML user case diagram that shows an overview of Teacher and the use cases present in the Magister Question Management system.

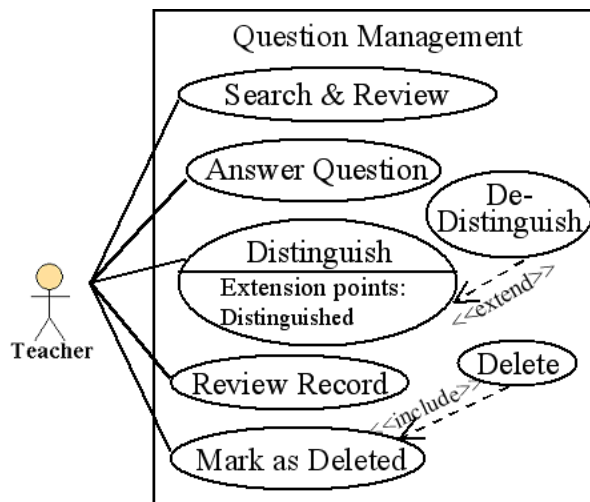


Figure 10.55: Question management for Teacher.

Figure 10.56 displays that Teacher Ball searches for questions asked after ‘2005-04-25’ and still have not been answered. He inputs these search-criteria into the search form and gets the result displayed in Figure 10.57.

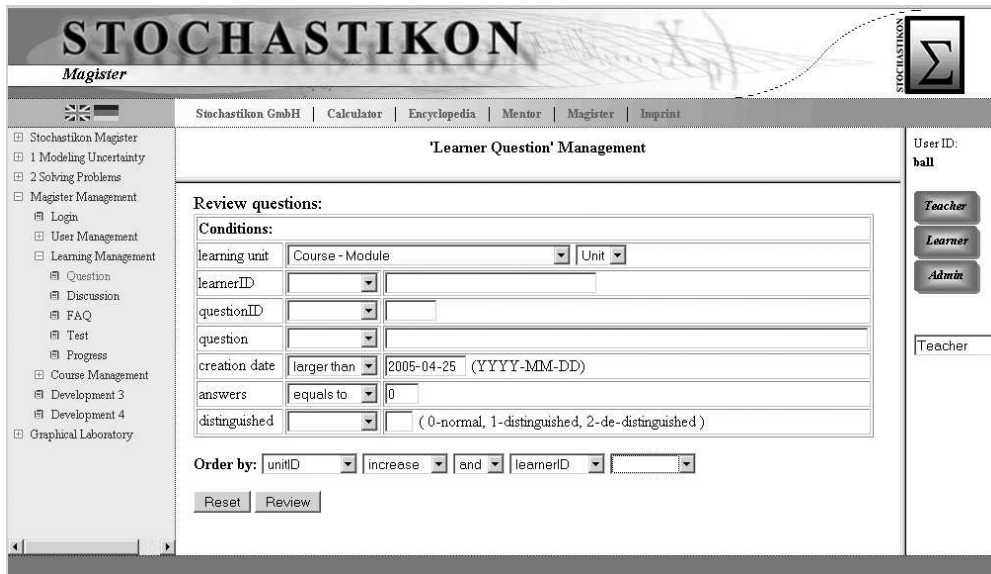


Figure 10.56: 'Question' Management for teachers in Magister (a).

In Figure 10.57 the last column of the table displaying the questions is headed 'distinguished'. A question gets the label 'distinguished' by an authorized teacher, if it is of some general interest and can be used for the FAQ section or Test section. The 'distinguished' information is not visible in the 'Question' Management panel for learners.

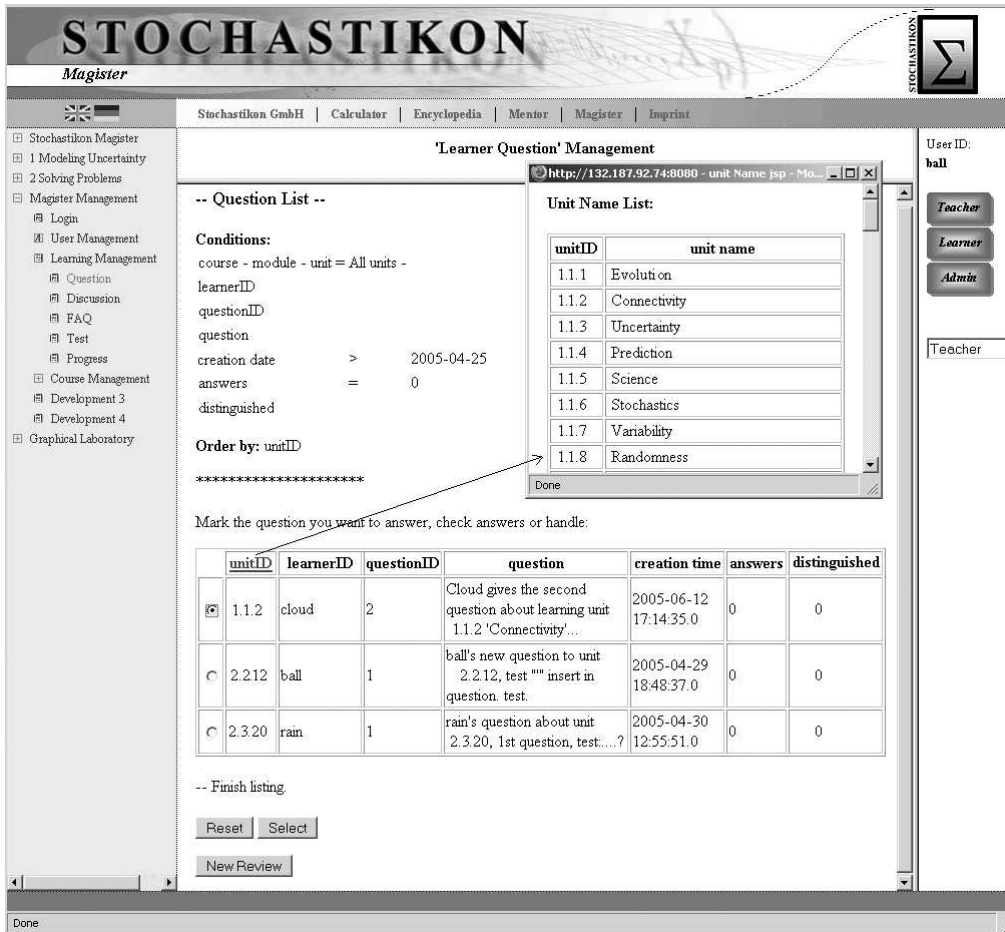


Figure 10.57: 'Question' Management for teachers in Magister (b).

After being distinguished the question's status is changed from 0 to 1. If a distinguished question is included in the activities FAQ or Test, its status can be changed to 2 in order to avoid misunderstandings. The status 2 indicates that the question had been once distinguished. The above-mentioned functions will be discussed again in Section 10.4.3 'FAQ' and Section 10.4.4 'Test'. Besides the 'distinguished' status for questions, there is an analogous status for discussion topics, which will be introduced later in Section 10.4.2 'Discussion'.

The table in Figure 10.57 lists only the 'unitID' for each question, however, by clicking the 'unitID' the corresponding course, module and unit name is displayed. In Figure 10.57, Teacher Ball has marked the question he wants to edit. By clicking the 'Select' button, the answer input interface as shown in Figure 10.58 appears.

The screenshot shows the 'Learner Question' Management interface. The main content area displays the following information for a selected question:

- unitID: 1.1.2: Connectivity
- learnerID: cloud
- questionID: 2
- creation time: 2005-06-12 17:14:35.0
- answers: 0
- Question: Cloud gives the second question about learning unit 1.1.2 'Connectivity'...

Below this information, there is a section for 'Your answer:' with a text input field containing the text: 'Teacher ball gives answer to cloud's second question about Unit 1.1.2 Connectivity....'. There are 'Reset' and 'Submit' buttons below the input field. Further down, there are buttons for 'All Answers', 'Delete this question', 'Distinguish this question', and 'New Review'. A 'Reason' dropdown menu is set to 'No reason'.

The right sidebar shows the user ID 'ball' and buttons for 'Teacher', 'Learner', and 'Admin'. There is also a 'Teacher' label at the bottom of the sidebar.

Figure 10.58: 'Question' Management for teachers in Magister (c).

Teachers can input their answers to a question and then 'Submit' them. An alert will pop-up rejecting a submission, if an empty input form is submitted. The same happens, by the way, throughout *Stochastikon Magister*, any attempt for submitting an 'empty' question, answer, topic, comment or other content through the input form is rejected. As shown in Figure 10.58, a teacher can also check 'All Answers' to a question given by other teachers as shown in Figure 10.53.

Moreover, the teacher can mark a question by 'Distinguish' it for possible use in the FAQ or Test sections. For distinguishing a question, its status is changed from 0 to 1. If the question's status has been set erroneously to 1 (distinguished) or 2 (in the FAQ or Test section), a teacher can change the status back to 0.

Deleted questions are marked as 'deleted' and cannot be displayed anymore by learners and teachers. Teachers can input the reasons for deletion, which are recorded. A distinguished question cannot be deleted unless its 'distinguished' status has been reset to 0. Only administrators can delete completely a question together with its answers from *Stochastikon Magister*.



Figure 10.59 shows the ‘Question’ Management interface for administrators. There is only one difference namely administrators can search for questions that have been ‘deleted’ by teachers.

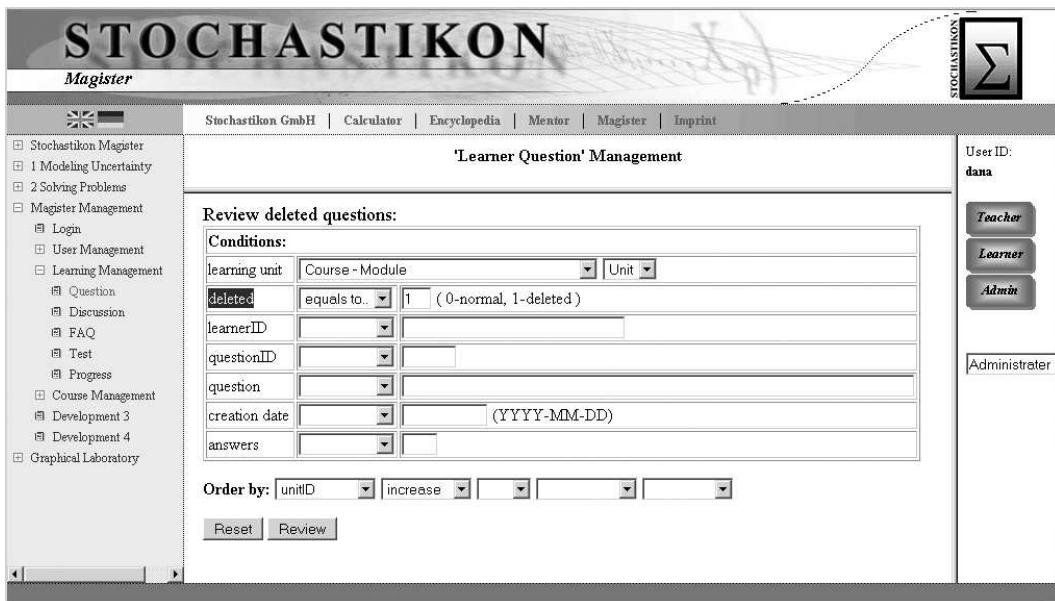


Figure 10.59: ‘Question’ Management for administrators in Magister (a).

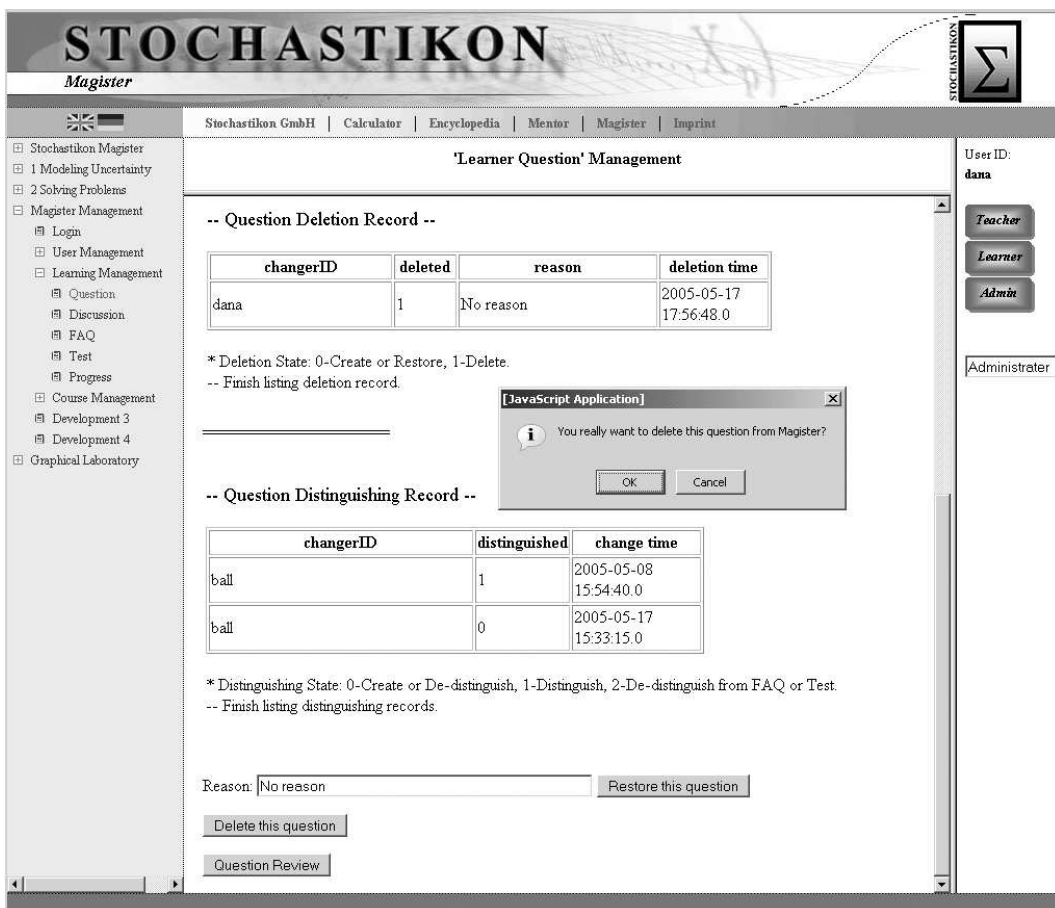


Figure 10.60: ‘Question’ Management for administrators in Magister (b).

As shown in Figure 10.60, the Administrator Dana can ‘Restore’ a deleted question or ‘Delete’ it totally from *Stochastikon Magister*. Before a deletion or a restoration is executed, an alert

will pop-up for a final confirmation. If a question is deleted from Magister by an administrator, its questionID will be assigned to the subsequent question and so on. When reviewing a selected question, an administrator can see not only the question and answer information as shown in Figure 10.53, but also the entire change records of the question: create, change, delete, restore, distinguish, de-distinguish, etc.

### 10.4.2 Discussion

The management of the learning activity Discussion is similar to that of the Question section. The major difference is that Discussion is public and that each formal user of *Stochastikon Magister* has the same discussion authority.

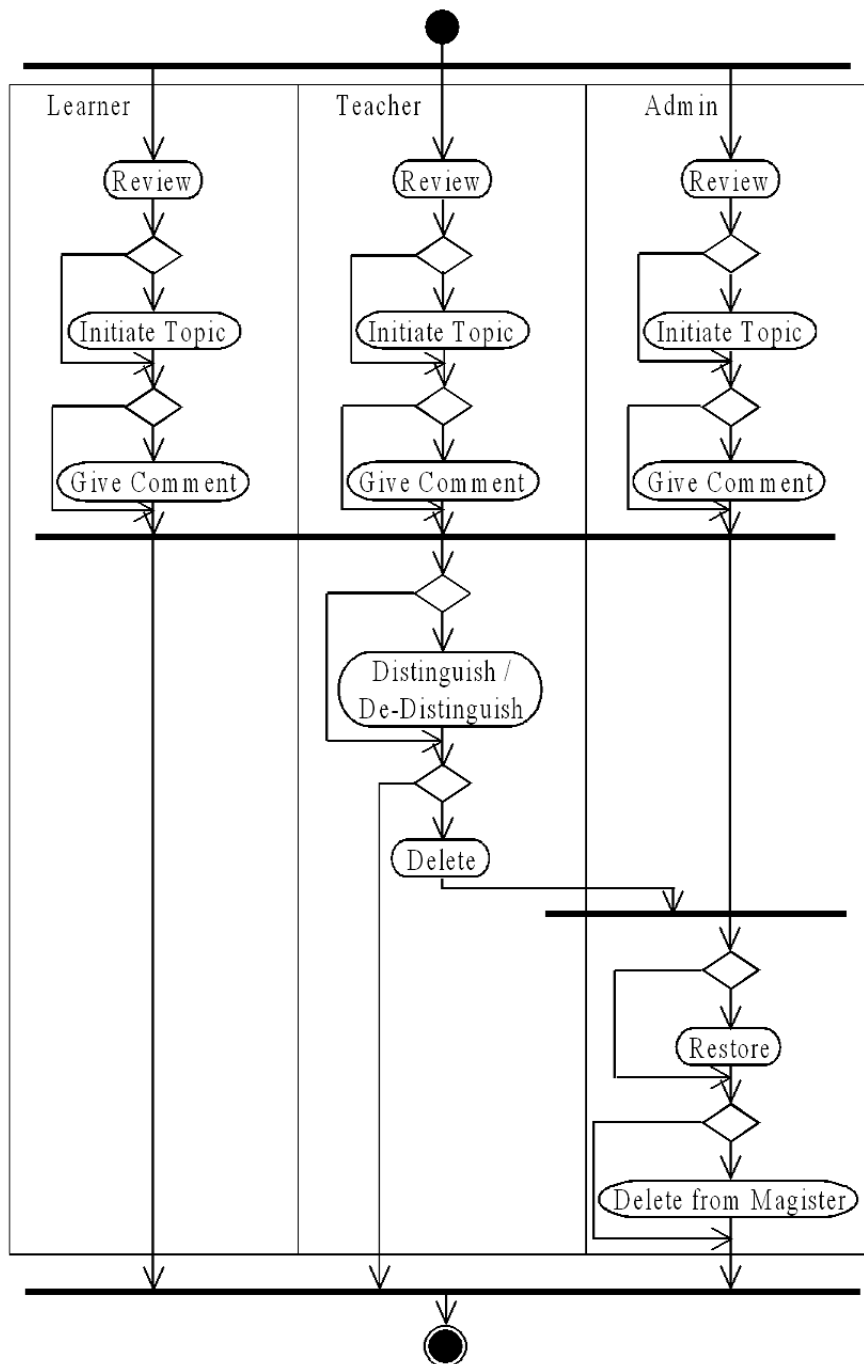


Figure 10.61: Discussion organization and management process.

Figure 10.61 is a UML swimlanes in activity diagram, which describes the Discussion organization and management process in Magister e-learning environment. This diagram shows that Discussion is initiated and commented within each learning unit and managed within Discussion Management package.

### ‘Discussion’ in a learning unit

‘Discussion’ as a learning activity addresses all formal Magister users aiming at clarifying and further explaining the learning content of the current unit. Moreover, the Discussion shall also foster a feeling of togetherness and initiate cooperation among the users of *Stochastikon Magister*. Figure 10.62 shows the interface, which has two functional parts: ‘Introduce’ a new discussion topic and ‘Review’ former discussions.

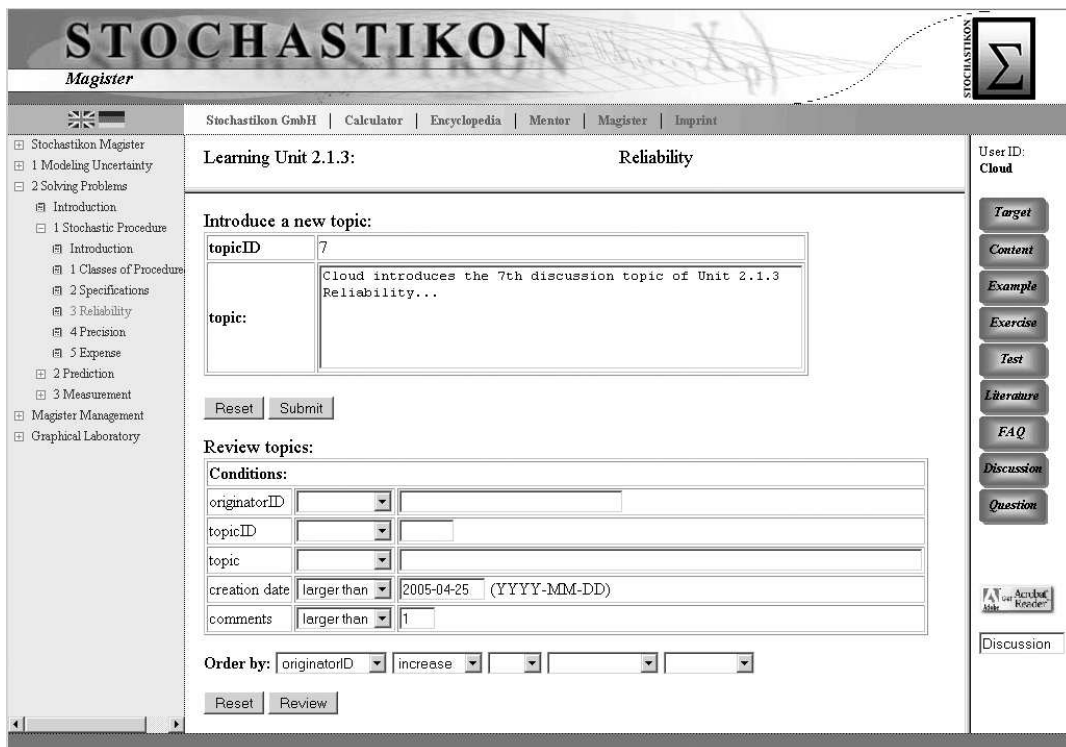


Figure 10.62: Discussion activity in a Magister learning unit (a).

A Magister discussion is composed by a topic proposed by a formal user and the contributions submitted by other formal users. The topicID is generated automatically for any newly introduced topic within the discussion panel of a learning unit. In Figure 10.62, Learner Cloud starts a new discussion topic for Unit 2.1.3 ‘Reliability’. He can also review and join former discussions. For example: Learner Cloud wants to search topics created after 2005-04-25 having more than one contribution.

The result of Cloud’s search that meets his criteria is a list of topics similar to the list of questions given in Figure 10.52. Marking a topic and clicking the ‘Select’ button opens a window as shown in Figure 10.63. In this page one can input comments to the topic at hand or review all contributions to this topic by clicking the ‘All Comments’ button.

Note that any discussion contribution cannot be reviewed without specifying the corresponding topic. No contribution can be updated or deleted individually. However, all contributions to a topic are deleted simultaneously when the topic is deleted. A similar rule holds for answers and the corresponding question.

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User ID: cloud

**Learning Unit 2.1.3: Reliability**

-- Selected Topic --

unitID: 2.1.3: Reliability  
 originatorID: box  
 topicID: 3  
 creation time: 2005-05-17 15:24:05.0  
 comments: 3  
**Topic:** box sponsor a new discussion about unit 2.1.3

**Your comment:**

cloud add some comment about this topic...

Reset Submit

All Comments

New Review

Target  
 Content  
 Example  
 Exercise  
 Test  
 Literature  
 FAQ  
 Discussion  
 Question

Discussion

Figure 10.63: Discussion activity in a Magister learning unit (b).

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User ID: cloud

**Learning Unit 2.1.3: Reliability**

unitID: 2.1.3: Reliability  
 originatorID: box  
 topicID: 3  
 creation time: 2005-05-17 15:24:05.0  
 comments: 4  
**Topic:** box sponsor a new discussion about unit 2.1.

-- Comment List --

-----  
 responderID: ball  
 creation time: 2005-05-17 15:36:57.0  
**Comment:** ball's comments on this topic.  
 -----

responderID: 12345  
 creation time: 2005-05-17 16:27:09.0  
**Comment:** 12345 has some comments about this topic.  
 -----

responderID: snow  
 creation time: 2005-05-17 16:28:43.0  
**Comment:** snow has different opinions about this discussion.  
 -----

responderID: cloud  
 creation time: 2005-06-14 12:57:36.0  
**Comment:** cloud add some comment about this topic...  
 -----

-- Finish listing comment.

Untitled - Message (Plain Text)

File Edit View Insert Format Tools Actions Help

Send Options...

To: three@box

Cc:

Subject:

Target  
 Content  
 Example  
 Exercise  
 Test  
 Literature  
 FAQ  
 Discussion  
 Question

Discussion

Figure 10.64: Discussion activity in a Magister learning unit (c).

The basic Discussion activities, i.e., introducing a topic and submitting contributions are equally open to all formal Magister users. Thus, no separate management panel is necessary and discussions are only executed in each of the learning units. ‘Private’ communications of opinions to originators or responders, are possible for any participant by means of E-mails. In Figure 10.64 Learner Cloud wants to send an E-mail to the originator of the discussion ‘box’ whose E-mail address is three@box.

Figure 10.64 lists the details of an example discussion: originator, topic, responders and comments.

### ‘Discussion’ Management

The ‘Discussion’ Management is similar as ‘Question’ Management. Learners and teachers use this management panel to review discussions of all Magister learning units. Figure 10.65 is a UML user case diagram that shows an overview of Teacher and the use cases present in the Magister Discussion Management system.

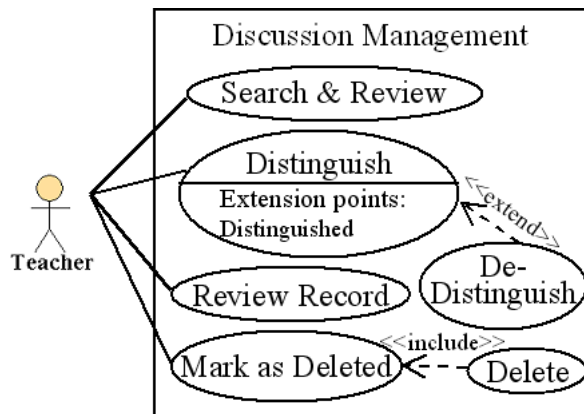


Figure 10.65: Discussion management for Teacher.

The screenshot shows the 'Discussion' Management search interface in the Magister system. The page title is 'STOCHASTIKON Magister'. The main content area is titled ''Discussion' Management' and contains a 'Review topics:' section. This section includes a 'Conditions:' form with the following fields:

- learning unit: Modeling Uncertainty - Basic Terms (dropdown), 1.1.11: Gaining Knowledge (dropdown)
- originatorID: [input field]
- topicID: [input field]
- topic: [input field]
- creation date: equals to (dropdown), 2005-05-04 (YYYY-MM-DD) (input field)
- comments: [input field]
- distinguished: [input field] (0-normal, 1-distinguished, 2-de-distinguished)

Below the form is an 'Order by:' section with dropdowns for 'comments', 'increase', and other options. There are 'Reset' and 'Review' buttons at the bottom of the form. On the right side, there is a 'User ID:' field with the value 'ball' and three buttons: 'Teacher', 'Learner', and 'Admin'. A 'Teacher' button is also visible below the User ID field.

Figure 10.66: Magister ‘Discussion’ Management - search interface.

The search interface of ‘Discussion’ Management for Teacher is shown in Figure 10.66. As an example, Teacher Ball wants to review the ongoing discussion about learning Unit 1.1.11 ‘Gaining Knowledge’, which had been started on 2005-05-04. Figure 10.66 displays the corresponding input form, Figure 10.67 shows the search results satisfying the requirements set by Teacher Ball and being listed in the order of the number of submitted comments.

The screenshot shows the 'Discussion' Management interface in the Stochastikon Magister system. The main content area displays search results for a specific topic. The search criteria are: course - module - unit = Modeling Uncertainty - Basic Terms-1.1.11: Gaining Knowledge, originatorID, topicID, topic, creation date = 2005-05-04, and distinguished. The results are ordered by mark.

The search results table is as follows:

	unitID	originatorID	topicID	topic	creation time	comments	distinguished
<input type="radio"/>	1.1.11	tree	4	topic 6 unit 1.1.10 by tree.	2005-05-04 16:20:11.0	0	0
<input checked="" type="radio"/>	1.1.11	disk	2	topic 3, unit 1.1.11 by disk.	2005-05-04 16:08:31.0	3	1
<input type="radio"/>	1.1.11	cloud	1	topic 2, unit 1.1.11 by cloud.	2005-05-04 16:07:52.0	4	1

Below the table, there are buttons for 'Reset', 'Select', and 'New Review'. A 'Unit Name List' sidebar on the right shows a list of units from 1.1.1 to 1.2.1, with 'Gaining Knowledge' (1.1.11) highlighted.

Figure 10.67: Magister ‘Discussion’ Management - a topic list.

The list of discussion topics in Figure 10.67 is the same as the corresponding list on the learning unit level except for an additional column containing the ‘unitID’. The ‘comments’ column contains the number of contributions to the discussion topic at hand and the ‘distinguished’ column shows whether or not the topic has been distinguished. The meaning of the numbers listed in the ‘distinguished’ column has been introduced in Section 10.4.1 ‘Question’ when explaining the ‘distinguished’ questions. The ‘distinguished’ column is invisible for learners.

Note that in Figure 10.67, the topicIDs in the list are not consecutive. There is no topicID equaling 3, because the corresponding discussion had been deleted by a teacher. However, it has not been completely deleted from *Stochastikon Magister* by an administrator and, therefore, the topicID 3 is still occupied.

By marking a topic, its details can be checked after clicking the ‘Select’ button. In ‘Discussion’ Management there is only one difference between learners and teachers. The latter have the possibilities to see and change the ‘distinguished’ status of a topic or to delete the topic.

The selected topic in Figure 10.68 is a distinguished one, therefore, the available operations are ‘De-distinguish’ or ‘Delete’. However, before a distinguished topic can be deleted, it must

be ‘De-distinguished’. A deletion of a discussion topic by a teacher means that it is marked and cannot be viewed any more by learners and teachers. Teachers may input the reasons of deletion and must confirm the deletion before its execution. Only administrators can delete a discussion (topic with all its comments) completely from *Stochastikon Magister*.



Figure 10.68: Some ‘Discussion’ Management functions for teachers in Magister.

The ‘Discussion’ Management panel for administrators is similar as that for teachers (see Figure 10.66) except that it supports the search for topics that have been ‘deleted’ by teachers similar as for the ‘Question’ Management panel for administrators (see Figure 10.59). An administrator can ‘Restore’ a discussion deleted by a teacher or ‘Delete’ it totally from *Stochastikon Magister*. Before the execution is performed an alert will pop-up for final confirmation. The topicID of a discussion which is totally deleted is automatically assigned to the following topic and so on.

When checking a selected discussion topic, an administrator can see not only details of the discussion as in Figure 10.64, but also the record of all changes, as start, change, delete, restore, history of distinguish and de-distinguish, etc. similar as in the case of ‘Question’ Management for administrators (see Figure 10.60).

### 10.4.3 FAQ

FAQ belongs to those learning activities, which do not involve a direct communication between teachers and learners. It is a part of Magister Learning Management, because it is partly based on the Question and Discussion activities. Figure 10.69 is a UML swimlanes in activity

diagram, which describes the FAQ creation and management process in Magister e-learning environment.

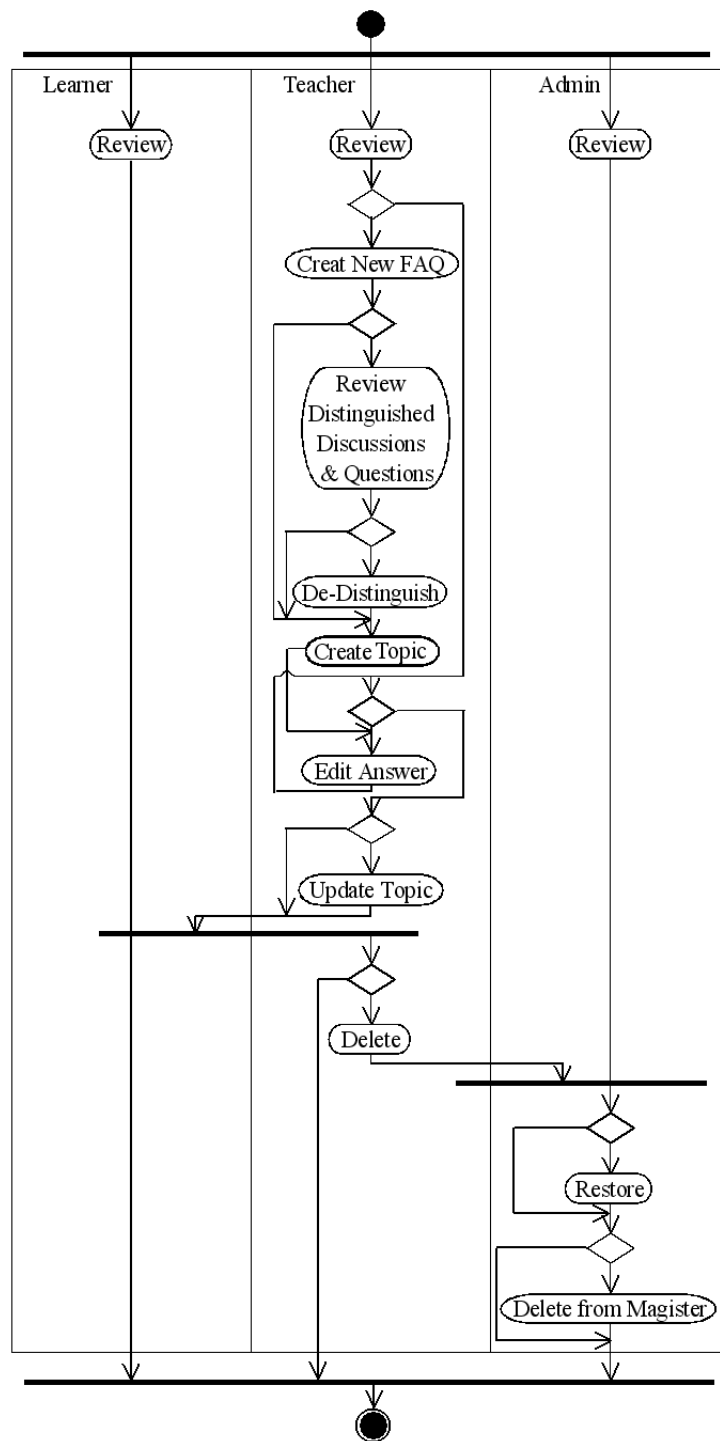


Figure 10.69: FAQ creation and management process.

### ‘FAQ’ in a learning unit

Figure 10.70 shows the ‘Review’ and ‘Create’ interface of FAQ as a learning activity. Any formal user of Magister can search and read frequently asked questions concerning the current learning unit. A search for frequently asked questions can be restricted to certain requirements. Learners are not authorized to create a new FAQ, as shown in Figure 10.70. Learner Cloud wants to create a new FAQ, but an alert pops up and rejects the request. The aim of admitting



the creation of FAQs here is for convenience of teachers. They may formulate FAQs for the current learning unit.

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Learning Unit 2.2.10: Maximum Lower Bound  $\beta$ -Prediction Procedure

User ID: cloud

Review frequently asked questions:

Conditions:

questionID

creatorID

question

creation date  (YYYY-MM-DD)

Order by:

Reset Review FAQ

Create a New FAQ (only for teachers)

[JavaScript Application] Sorry, you have no authority to create a new FAQ. OK

Target Content Example Exercise Test Literature FAQ Discussion Question

FAQ

Figure 10.70: FAQ in a Magister learning unit - Review and Create.

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Learning Unit 2.2.10: Maximum Lower Bound  $\beta$ -Prediction Procedure

User ID: cloud

-- Frequently Asked Question List --

Conditions:

questionID

creatorID

question

creation date

Order by: questionID

\*\*\*\*\*

Mark the question you want to check answers:

	questionID	creatorID	question	creation time
<input type="radio"/>	1	ball	faq 1 unit 2.2.10, try!	2005-05-08 16:04:14.0
<input checked="" type="radio"/>	3	ball	unit 2.2.10, faq 3	2005-05-08 16:13:36.0
<input type="radio"/>	5	12345	frequently asked question 5 of unit 2.2.10. by 12345	2005-05-17 11:52:05.0

-- Finish listing

Reset Select

New Review

Create a New FAQ (only for teachers)

Target Content Example Exercise Test Literature FAQ Discussion Question

FAQ

Figure 10.71: FAQ in a Magister learning unit - searching results.

Figure 10.71 displays the search results for FAQs referring to Unit 2.2.10. By marking a question and clicking the ‘Select’ button, the details of the selected FAQ are shown as in Figure 10.72. Figure 10.72 outlines the details of a FAQ created by Teacher Ball. The answer to a FAQ may contain text, pictures, a hyperlink ‘Click for files’ for an attached file, or another hyperlink ‘Click for links’ to a web site.

The screenshot shows the Magister web interface. At the top, the header reads 'STOCHASTIKON' and 'Magister'. Below the header is a navigation bar with links for 'Stochastikon GmbH', 'Calculator', 'Encyclopedia', 'Mentor', 'Magister', and 'Imprint'. The main content area is titled 'Learning Unit 2.2.10: Maximum Lower Bound  $\beta$ -Prediction Procedure'. It displays a question with ID 3: 'Question: unit 2.2.10, faq 3'. The answer is 'a ball'. Below the answer, there are links for 'Click for files' and 'Click for links'. A 'New Review' button is visible. A JavaScript confirmation dialog is open, asking 'You really want to delete this question?' with 'OK' and 'Cancel' buttons. The right sidebar shows the user ID 'cloud' and a vertical stack of buttons: 'Target', 'Content', 'Example', 'Exercise', 'Test', 'Literature', 'FAQ', 'Discussion', and 'Question'. At the bottom of the sidebar, there is a 'FAQ' search box.

Figure 10.72: FAQ in a Magister learning unit - details of one FAQ.

As shown in Figure 10.72, there are function buttons below the FAQ content, which teachers can select. There are the possibilities of ‘Edit the FAQ’, ‘Create a New FAQ’ or ‘Delete this FAQ’. When a FAQ shall be deleted, the teacher can input the reasons. Before execution, a final confirmation is required. Figure 10.72 displays the additional pop-up confirmation: ‘You really want to delete this question?’ However, even if Learner Cloud selects ‘OK’, the operation will not be performed, because a learner has no authority to handle a FAQ except to read it. So Learner Cloud will get another pop-up alert rejecting his request. Similar as in the cases of ‘Question’ and ‘Discussion’ Management, a FAQ deleted by a teacher only becomes invisible for learners and teachers, while Magister administrators can still see these FAQs and have the authority to completely delete them from Magister in the ‘FAQ’ Management panel for administrator.

Clicking the ‘Edit’ button by a teacher opens the FAQ ‘Edit’ interface as shown in Figure 10.73 and Figure 10.74.

The ‘Edit’ interface for a FAQ answer lists all corresponding sub-answers one-by-one as shown in Figure 10.73, where the answer of the FAQ displayed in Figure 10.72 is composed of 6 sub-answers. To disintegrate the answer of a FAQ into sub-answers is because:

1. each paragraph of the answer should be stored as a sub-answer because a text item in the database cannot contain ‘line breaks’, and
2. the answer may contain different type of contents: text, pictures, files, links, and they have to be stored into different sub-answers for displaying or accessing.

The screenshot shows the 'Stochastikon Magister' interface. The main content area displays the following information:

Learning Unit 2.2.10: Maximum Lower Bound  $\beta$ -Prediction Procedure

questionID: 3  
 creatorID: ball  
 creation time: 2005-05-08 16:13:36.0  
 Question: unit 2.2.10, faq 3

-- FAQ Answer List --

Answer:  
 answerID: 1  
 a ball  
 answerID: 2  
 .

answerID: 3  
 another ball:  
 answerID: 4  
[Click for files](#)  
 answerID: 5  
 add answer  
 answerID: 6  
[Click for links](#)

-- Finish listing.

Add answer:  
 text  image  file  link

add an text answer

Buttons: Reset, Add an answer

Right sidebar: User ID: ball, Target, Content, Example, Exercise, Test, Literature, FAQ, Discussion, Question, User Account, FAQ

Figure 10.73: FAQ in a Magister learning unit - Edit (a).

The FAQ ‘Edit’ functions include: ‘Add’, ‘Insert’, ‘Update’ and ‘Delete a sub-answer’, ‘Update the FAQ question content’ and ‘Delete this FAQ’.

The content of a sub-answer can be represented by four different types with the default type being text. In Figure 10.73 Teacher Ball wants to ‘Add’ a sub-answer to the current FAQ. The answerID of the added sub-answer is 7, which is generated automatically.

The order of answers to posed question is of importance and, therefore, *Stochastikon Magister* makes it possible to renumber the answerIDs for sub-answers by ‘Insert’ sub-answer function, which helps displaying and managing.

As shown in Figure 10.74 Teacher Ball wants to ‘Insert’ the PDF file attachment `faq3.pdf` as a sub-answer with `answerID = 1`. After inserting, all existing answerIDs larger than or equal to 1 will be renumbered automatically. In this case all original answerIDs will be increased by 1. If a new answerID for an inserted sub-answer is larger than the current largest answerID plus

1, the new answerID is set as the current largest answerID plus 1. These rules ensure that the answerIDs are consecutive numbers, which facilitates their displaying, editing and checking.

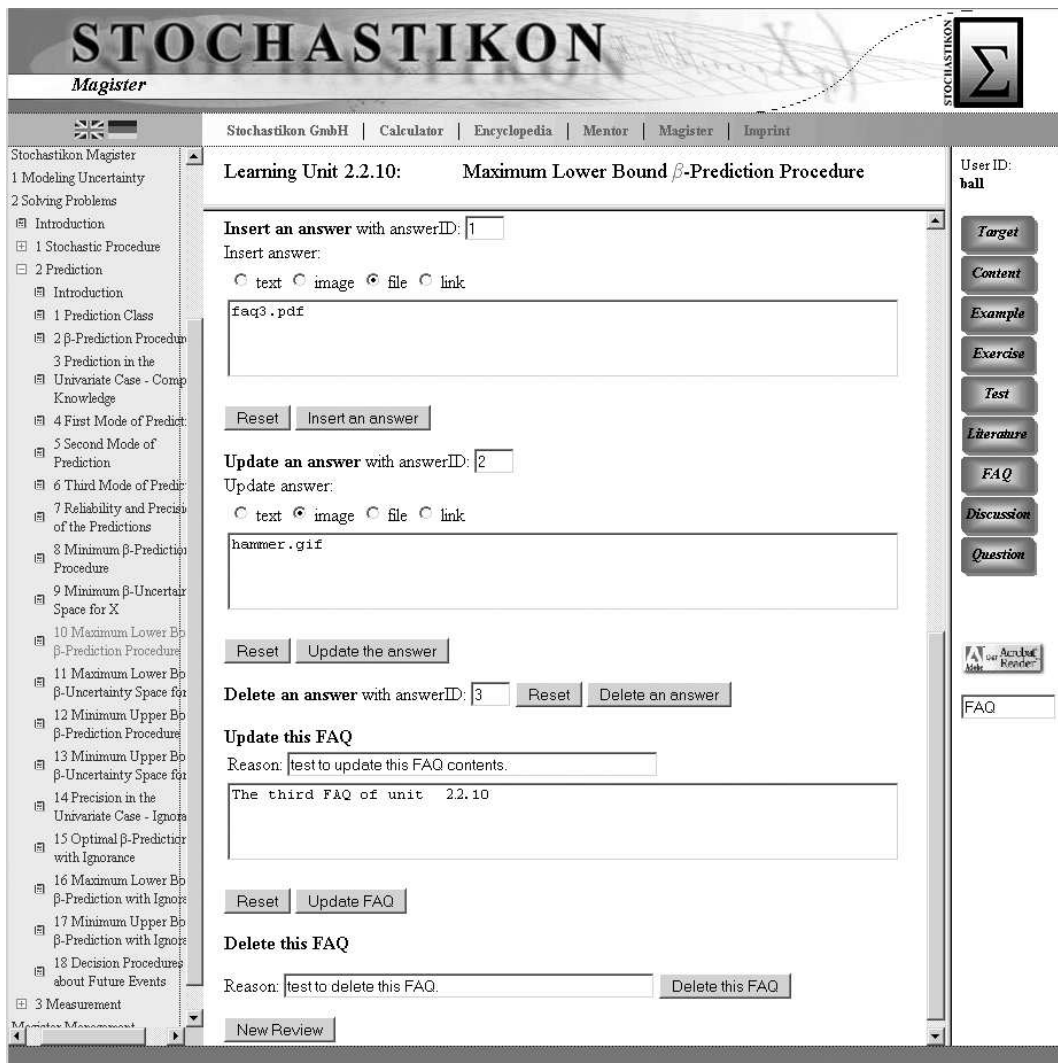


Figure 10.74: FAQ in a Magister learning unit - Edit (b).

A teacher can also 'Update' sub-answers as for instance replacing the picture of a ball in sub-answer 2 by a picture of a hammer (hammer.gif), or even 'Update' the type of a sub-answer.

Moreover, a teacher can delete one of the sub-answers specified by its answerID. After deleting the selected sub-answer, the following sub-answers will be renumbered so that there are no gaps and the numbers of the answerID are still consecutive.

If an inadmissible answerID (containing not numerical characters or does not exist) is input, any operation on the sub-answer such as 'Insert', 'Update' and 'Delete' is rejected.

The content of a FAQ can be replaced by a new content and the reasons for the change may be recorded. Of course, deletion of a FAQ is also possible here.

If a teacher clicks the 'Create a new FAQ' button as shown in Figures 10.70 - Figure 10.72 and Figure 10.76 - Figure 10.78, the FAQ creating interface opens (see Figure 10.75).

Any FAQ questionID referring to a given learning unit is generated automatically. In Figure 10.75 Teacher Ball creates the sixth FAQ for Unit 2.2.10. After giving the new question content

and clicking the ‘Create’ button, the new FAQ question is acknowledged. Subsequently, he can complete the new FAQ by clicking ‘Add answer’ and/or ‘Edit’.

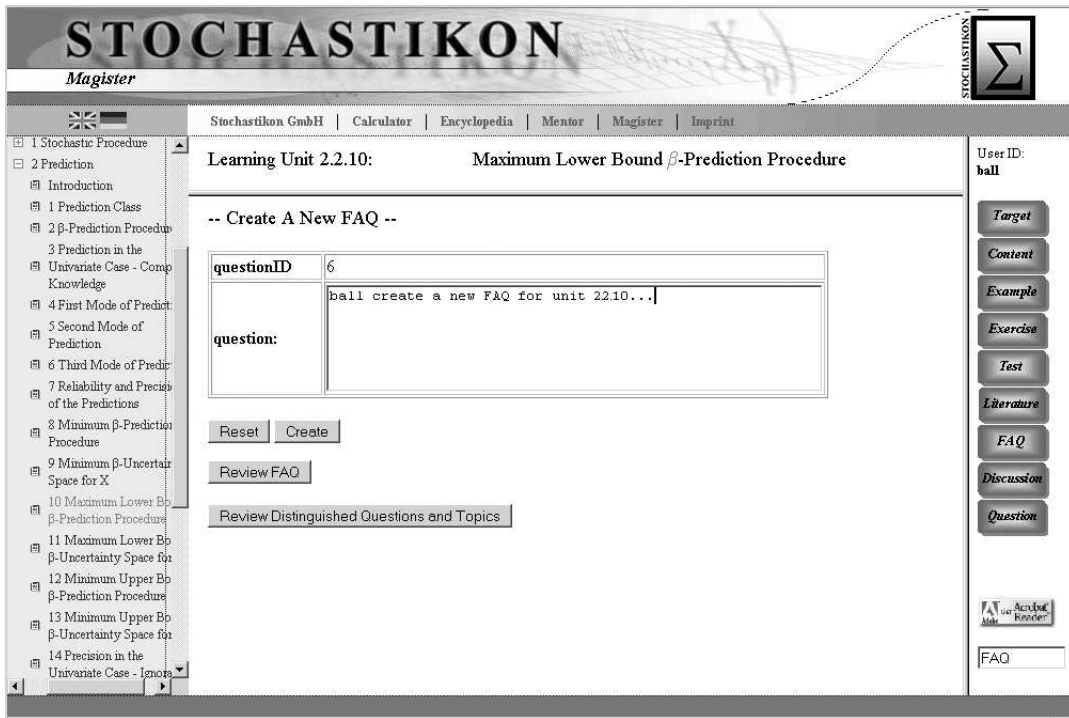


Figure 10.75: FAQ in a Magister learning unit - Create (a).

Figure 10.76 shows the acknowledgement and the related functions for teachers to handle the new generated FAQ.

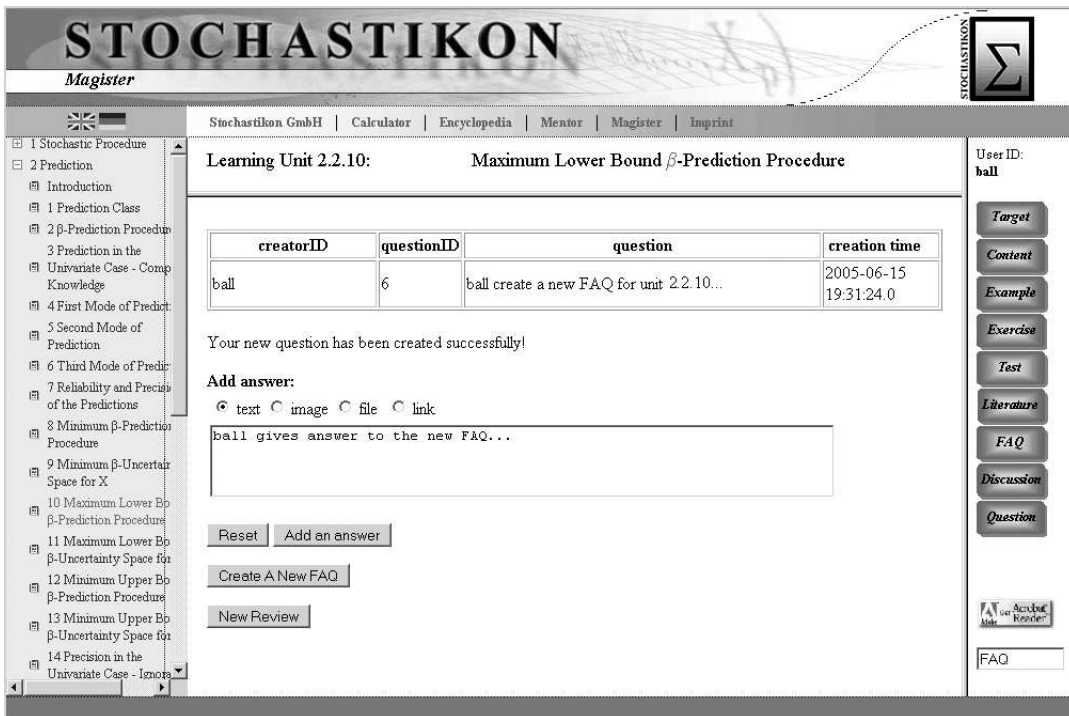


Figure 10.76: FAQ in a Magister learning unit - Create (b).

Before creating a new FAQ, one can ‘Review Distinguished Questions and Topics’ as displayed in Figure 10.75. Figure 10.77 shows the result, i. e., the distinguished questions and discussion topics of the current learning unit. As already stated in ‘Question’ Management and ‘Discussion’ Management, interesting questions and topics are distinguished by teachers in the corresponding management panels (see Figure 10.58 and Figure 10.68) aiming at assisting the creation of FAQs.

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Learning Unit 2.2.10: Maximum Lower Bound  $\beta$ -Prediction Procedure

User ID: Ball

-- Distinguished Question List --

Mark the question you check answers:

	question	creation time	answers
<input type="checkbox"/>	I'm big apple pie, I have a question about unit 5.10. Following is my question:....?	2005-04-30 14:12:47.0	3
<input type="checkbox"/>	question 1 unit 5.10 by ocean	2005-05-04 16:36:51.0	3

-- Finish listing distinguished questions.

-- Distinguished Topic List --

Mark the topic you want to check comments:

	originatorID	topic	creation time	comments
<input type="checkbox"/>	tree	tree gives the first topic about unit 5.10 in discussion.	2005-05-01 16:58:00.0	5
<input type="checkbox"/>	tree	topic 4 by tree.	2005-05-08 15:43:48.0	4

-- Finish listing distinguished topics.

Target  
Content  
Example  
Exercise  
Test  
Literature  
FAQ  
Discussion  
Question

FAQ

Figure 10.77: FAQ in a Magister learning unit - Distinguished Questions and Discussions.

To see details of a distinguished question or discussion, it is necessary to mark it and ‘Select’ it. In Figure 10.77, if Teacher Ball selects the first distinguished question then its details will be displayed as in Figure 10.78.

Teachers can also ‘De-distinguish’ a distinguished question or discussion in this panel (see Figure 10.78) by changing its status from 1 (= distinguished) to 2 (= de-distinguished). However, changing the ‘distinguished’ status has a different meaning from that in the ‘Question’ and ‘Discussion’ Management panels ( $0 \Leftrightarrow 1$ ,  $0 \Leftarrow 2$ ). If a distinguished question or topic has been used for formulating a FAQ, then the ‘distinguished’ status may be set to 2 indicating that it had already been exploited. This helps to prevent the creation of similar or even equal FAQs in a learning unit.

The same rule is also implemented in the Test section (see Section 10.4.4 ‘Test’) where distinguished questions and topics may assist for the generation of new test-questions.

The screenshot shows the STOCHASTIKON Magister interface. The main content area is titled "Learning Unit 2.2.10: Maximum Lower Bound  $\beta$ -Prediction Procedure". It displays a question and its answers. The question is: "I'm big apple pie, I have a question about unit 2.2.10. Following is my question:....?". The answer list includes two entries:

- teacherID: dana, creation time: 2005-04-30 14:14:52.0, Answer: a good question from apple! answer by dana, unit 2.2.10
- teacherID: ball, creation time: 2005-04-30 14:19:04.0, Answer: I agree with dana, this is a good question and dana's answer is also excellent. answered by ball

Below the answers, there is a "Finish listing" section and several buttons: "De-distinguish this question", "Create a New FAQ", "Review FAQ", and "Review Distinguished Questions and Topics". The left sidebar shows a navigation menu with categories like "1 Modeling Uncertainty", "2 Solving Problems", and "3 Measurement". The right sidebar shows the user ID "ball" and buttons for "Target", "Content", "Example", "Exercise", "Test", "Literature", "FAQ", "Discussion", and "Question".

Figure 10.78: FAQ in a Magister learning unit - a distinguished question.

## 'FAQ' Management

The functions in the 'FAQ' Management panels are similar as those in the learning units. The main difference is that FAQs of all learning units can be reviewed and managed here.

The screenshot shows the STOCHASTIKON Magister interface for "Frequently Asked Question" Management. The main content area is titled "'Frequently Asked Question' Management" and contains a "Review frequently asked questions:" section. This section includes a "Conditions:" form with the following fields:

- learning unit: Course - Module (dropdown) | Unit (dropdown)
- questionID: [input field]
- creatorID: [input field]
- question: [input field]
- creation date: [input field] (YYYY-MM-DD)

Below the form, there is an "Order by:" section with dropdowns for "unitID", "increase", and other sorting options. There are also "Reset" and "Review Question" buttons. The left sidebar shows a navigation menu with categories like "1 Modeling Uncertainty", "2 Solving Problems", "Magister Management", "Course Management", and "Graphical Laboratory". The right sidebar shows the user ID "ball" and buttons for "Teacher", "Learner", and "Admin".

Figure 10.79: 'FAQ' Management for learners.

Figure 10.79 shows the 'Review' interface of the 'FAQ' Management for learners.

Figure 10.80 is a UML user case diagram that shows an overview of Teacher and the use cases present in the Magister FAQ Management system.

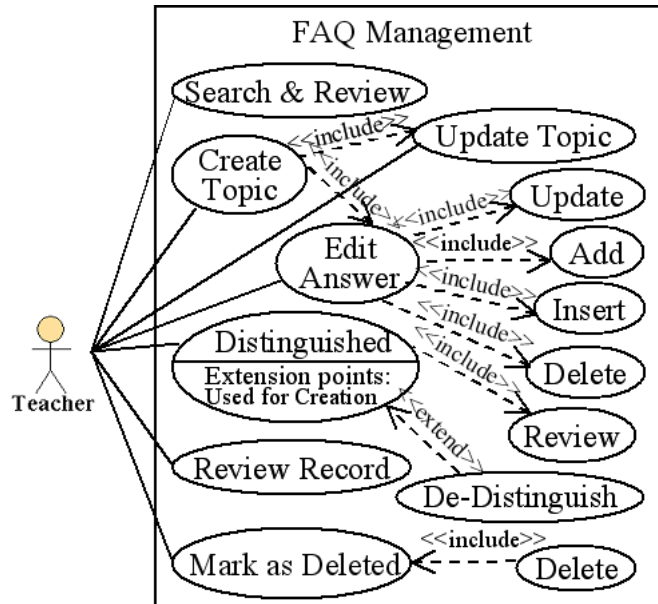


Figure 10.80: FAQ management for Teacher.

Figure 10.81 shows the 'Review' and 'Create' interfaces for teachers.

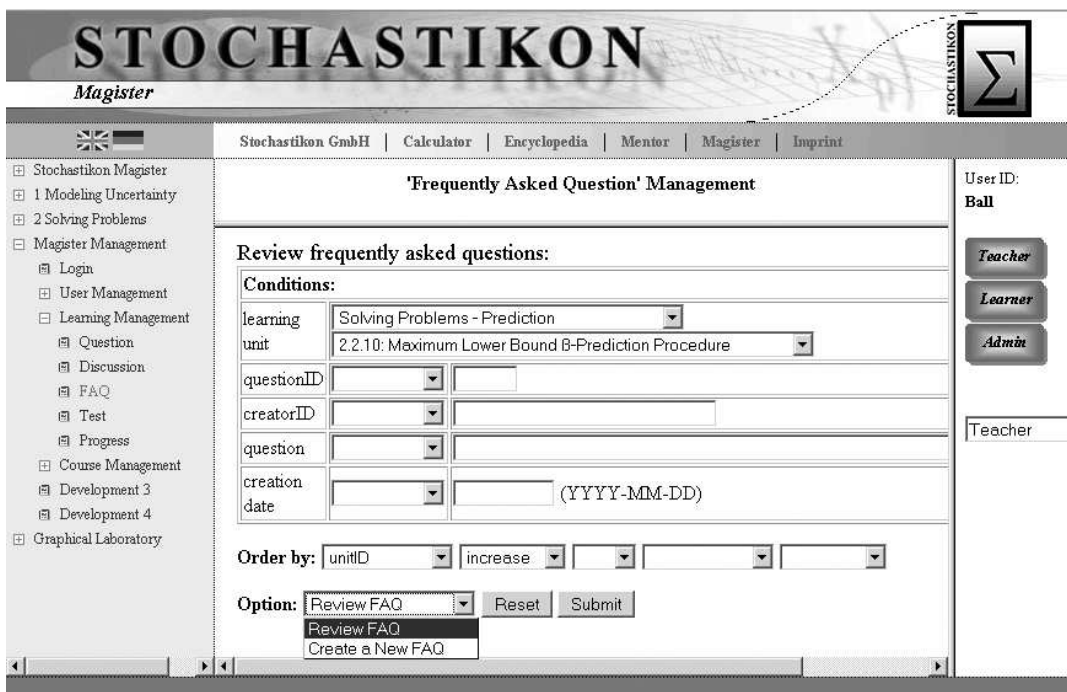


Figure 10.81: 'FAQ' Management for teachers - Review and Create.



For any FAQ to be created, a corresponding learning unit has to be specified. If this is not done, the request will be refused. If a teacher has selected an old FAQ for reviewing and subsequently wants to create a new FAQ, the new one belongs automatically to the same unit as the selected one. For example (see Figure 10.82), after selecting and checking a FAQ of Unit 2.1.3, Teacher Ball wants to create a new FAQ, which is automatically assigned to Unit 2.1.3.

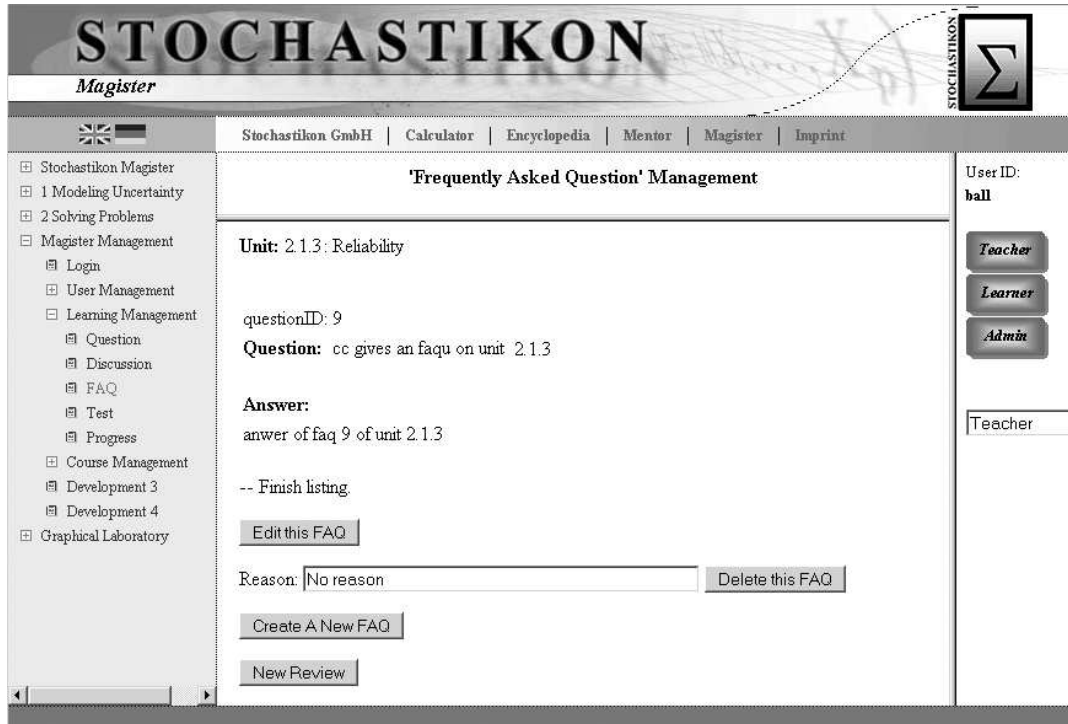


Figure 10.82: ‘FAQ’ Management for teachers - Create a New FAQ.

The functions for administrators in the ‘FAQ’ Management panel are analogous to those in the ‘Question’ and ‘Discussion’ Management panels. Administrators can check all FAQs including those that have been ‘deleted’ by teachers. As to ‘deleted’ FAQs, the administrators can ‘Restore’ them or ‘Delete’ them totally from *Stochastikon Magister* similar as in ‘Question’ and ‘Discussion’ Management.

#### 10.4.4 Test

The ‘Test’ constitutes the most intricate learning activity in *Stochastikon Magister*. The Test section consists of two parts. The first one allows the test-question generation and management and the second one the test construction, execution and management.

Figure 10.83 and Figure 10.84 are UML two swimlanes in activity diagrams. The former one describes the Test Question creation and management process, and the later one displays the Test organization and management process in *Magister* e-learning environment.

A Test as an activity in the learning units focuses on the development of test questions and on controlling the learners’ test-related work. The ‘Test’ Management panel supports the correction of tests and the management of tests and test-questions.

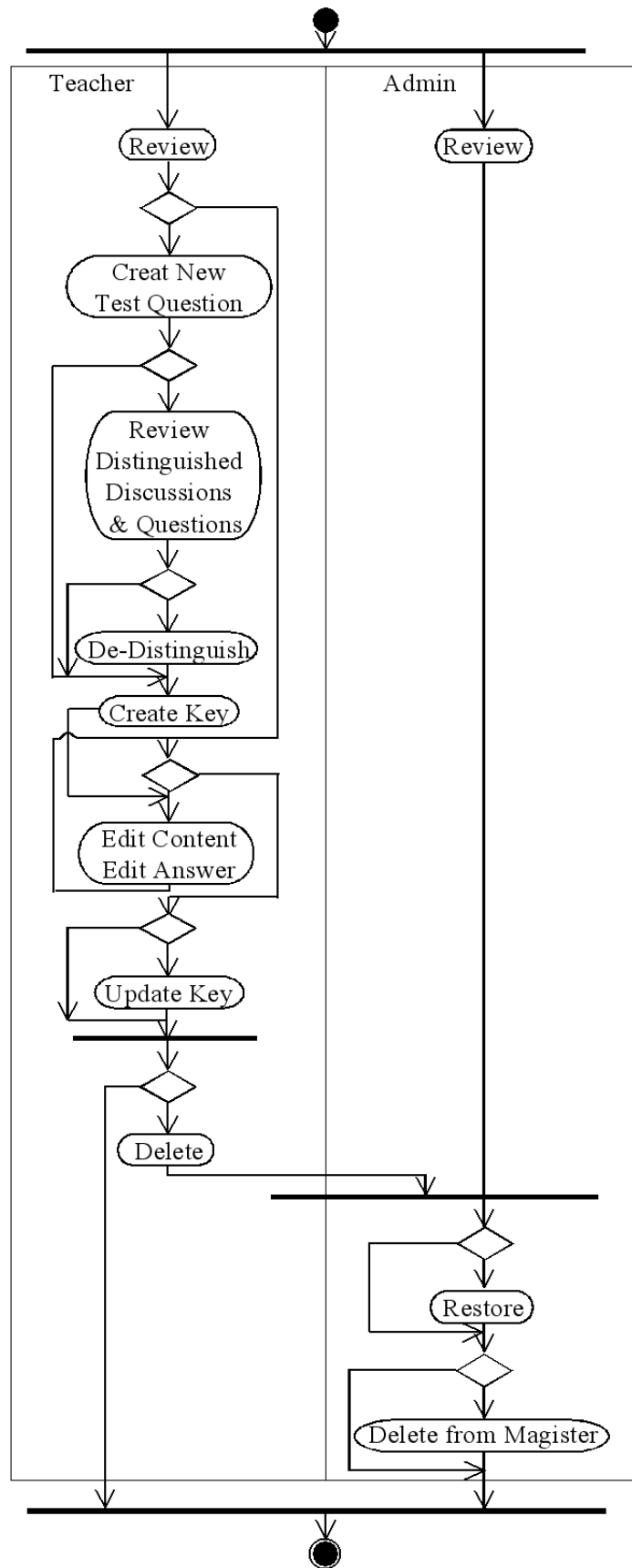


Figure 10.83: Test - Test Question creation and management.

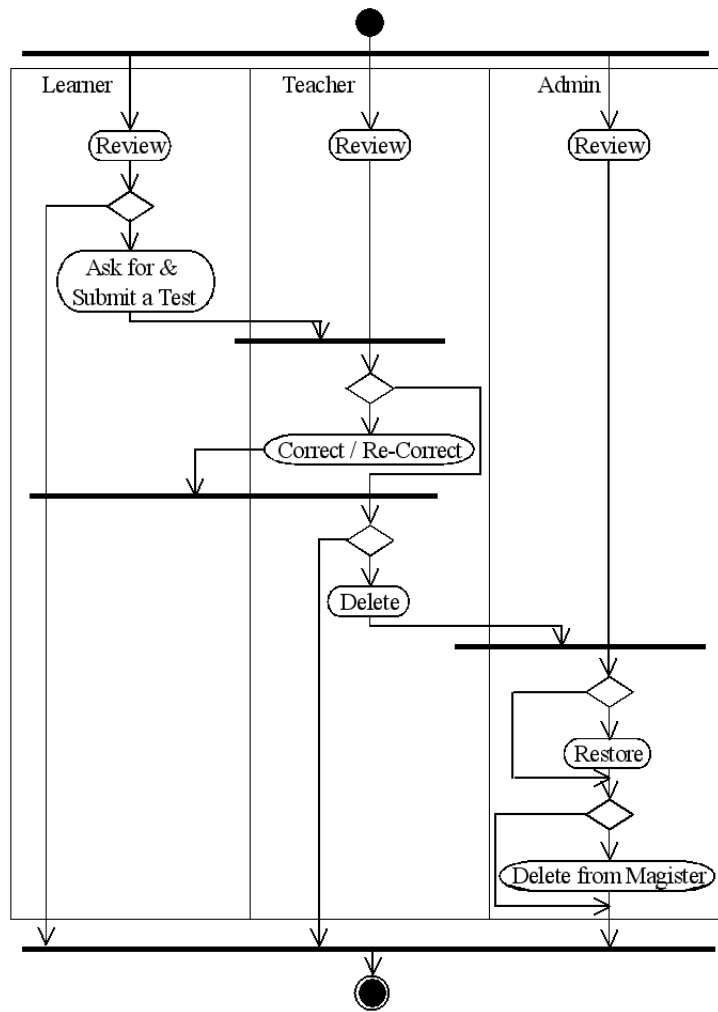


Figure 10.84: Test - Test organization and management process.

**‘Test’ in a learning unit**

Figure 10.85 illustrates the Test-related programs handling tests as a learning activity in one learning unit. The most left branch refers to test-related activities of learners. The second branch describes the possibilities of learners to review their existing tests which have been corrected. Finally, the third and most right branch contains the operations for teachers to generate new test-questions.

Figure 10.86 shows the first Test interface in Learning Unit 1.1.2. It contains three parts: ‘Ask for a new test’, ‘Review former tests’ and ‘Review and Create test-questions’.

A learner can only ask for a test corresponding to his/her user level or to one of his/her user groups. In Figure 10.86, Learner Cloud with user level 1 and group membership 0, 1 and 3 asks for a test meeting the test-criterion Group 1.

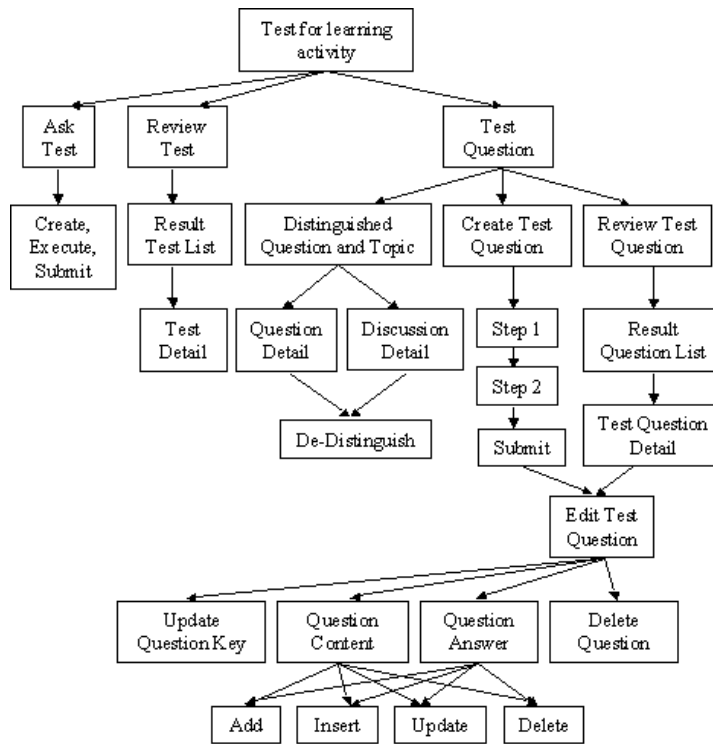


Figure 10.85: Programs for the Test activity in a Magister learning unit.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

Learning Unit 1.1.2: **Connectivity**

User ID: **Cloud**

Ask for a new test according to your:

user level 1

Group: 0 group0

Group: 1 group1

Group: 3 group3

Reset Ask

-- Review Learner's Test --

Conditions:

\* Your study level and group information: level: 1, group: 0,1,3,

test level: [dropdown] (0 ≤ 'test level' ≤ 9)

test group: [dropdown] (0 ≤ 'test group' ≤ largest groupID)

testID: [input]

test date: [dropdown] (YYYY-MM-DD)

teacherID: [dropdown]

score: [dropdown] (0 ≤ 'test score' ≤ 10)

correction date: [dropdown] (YYYY-MM-DD)

Order by: testID increase

Reset Review

-- Review and Create test questions --

Create and Review (only for teachers)

Target  
Content  
Example  
Exercise  
Test  
Literature  
FAQ  
Discussion  
Question

Test

http://132.187.92.76:8080 - usergroup list.jsa - Mozilla Firefox

User group information:

groupID	group name	comment
0	group0	some comments about gro
1	group1	comments

Figure 10.86: Test as a learning activity in a Magister learning unit.

Accordingly, a new test referring to Unit 1.1.2 for Group 1 is compiled (see Figure 10.87 and Figure 10.88). In *Stochastikon Magister* each test is composed by 6 questions. Two questions demanding written answers (Q & A), the next two questions belong to the type judgment, and the final two are multiple choice questions. They are selected randomly from the test-question bank by a test construction program according to the test-criteria given by the unit, the user level/group and the question type.

The screenshot displays the Stochastikon Magister web application. At the top, the logo 'STOCHASTIKON' and 'Magister' are visible. Below the logo is a navigation bar with links for 'Stochastikon GmbH', 'Calculator', 'Encyclopedia', 'Mentor', 'Magister', and 'Imprint'. A sidebar on the left contains a tree view of the course structure, including 'Stochastikon Magister', '1 Modeling Uncertainty', '2 Solving Problems', 'Magister Management', 'Course Management', and 'Graphical Laboratory'. The main content area is titled 'Learning Unit 1.1.2: Connectivity' and shows a test titled '-- A New Test -- Test 4'. It contains three questions:
 

- Question 1:** question 8 of Q & A. Input your answer to this question: Cloud's answer to the first question...
- Question 2:** question 7 of Q & A. Click for question content. Input your answer to this question: Cloud's answer to the second question...
- Question 3:** question 1 of Judgement, correct. The above statement is:  Correct  Incorrect

 On the right side, there is a vertical sidebar with buttons for 'Target', 'Content', 'Example', 'Exercise', 'Test', 'Literature', 'FAQ', 'Discussion', and 'Question'. Below these buttons is a 'User ID: cloud' field and a 'Test' button. At the bottom right, there is a small logo for 'Acrobat Reader'.

Figure 10.87: A test for learner in Magister (a).

For Q & A test questions, learners must input the answer by text. For Judgments and Multiple Choice questions, they only have to make their choice and mark it.

After having completed the test, it has to be submitted. However, before the test is accepted, the learner is asked for a final confirmation of the request to submit the test, as illustrated in Figure 10.88.

The screenshot shows the Magister LMS interface. At the top, the header reads "STOCHASTIKON" and "Magister". Below the header is a navigation bar with links: "Stochastikon GmbH", "Calculator", "Encyclopedia", "Mentor", "Magister", and "Imprint". On the left is a sidebar menu with a tree structure under "Stochastikon Magister", including "1 Modeling Uncertainty" and "2 Solving Problems". The main content area is titled "Learning Unit 1.1.2: Connectivity". It contains three questions:

- Question 4:** "question 2 of Judgement, wrong. The above statement is:  Correct  Incorrect"
- Question 5:** "Select one correct answer from multiple choices:" with three radio buttons and three image icons (a gavel, a stack of books, and a plate of food).
- Question 6:** "question 10 of multiple choice of unit 1.1.2. Select one correct answer from multiple choices:" with two radio buttons labeled "error" (selected) and "correct".

At the bottom of the main area are "Reset" and "Submit" buttons. On the right sidebar, the user ID is "cloud", and there is a vertical stack of buttons: "Target", "Content", "Example", "Exercise", "Test", "Literature", "FAQ", "Discussion", and "Question". A "Test" input field is also present. A JavaScript alert dialog is overlaid on the questions, with the text: "[JavaScript Application] You want to submit your answers to this test?" and "OK" and "Cancel" buttons.

Figure 10.88: A test for learner in Magister (b).

If a learner does not answer one or both Q & A, or does not mark one of the judgements or one of the multiple choice answers, then before the test is accepted an alert pops up indicating the missing answers and asking for final confirmation. For example, if Learner Cloud does not mark an answer for Question 5 and submits the (incomplete) test, he gets the pop up alert 'there is no choice to question 5'. If Cloud indeed does not want to mark an answer, then he confirms by 'OK', otherwise he clicks 'Cancel', marks a choice to Question 5 and submits the test again.

Each successfully submitted test is immediately acknowledged along with the request to wait for the correction and the feedback from teachers.

The second function for learners consists of the possibility to review already completed tests (see Figure 10.86). Learners can review all their tests referring to the current unit which have been corrected so far. Figure 10.89 lists the two tests for Unit 1.1.2 which Learner Cloud has already completed. By marking one of them and clicking the 'Select' button, the details of the test are displayed.

The screenshot shows the Magister web application interface. The header features the 'STOCHASTIKON' logo and navigation links: Stochastikon GmbH, Calculator, Encyclopedia, Mentor, Magister, and Imprint. The user ID is 'cloud'. The main content area is titled 'Learning Unit 1.1.2: Connectivity' and displays a test list. The test list table is as follows:

	testID	level	group	score	teacherID	test time	correction time
<input type="radio"/>	1	1		10	ball	2005-05-17 14:03:24.0	2005-05-17 14:12:32.0
<input checked="" type="radio"/>	3		1,	6	12345	2005-05-17 14:07:08.0	2005-05-17 14:26:49.0

Below the table, there are buttons for 'Reset', 'Select', and 'Review Test'. The right sidebar contains navigation buttons: Target, Content, Example, Exercise, Test, Literature, FAQ, Discussion, and Question.

Figure 10.89: Review former tests in Magister.

Figure 10.90 shows part of an example of a corrected test taken by Learner Cloud and corrected by Teacher '12345'. The first part displays the formal details of the test and the test result, like unitID and title, testID, user group/user level, score, corrector, test date, correction date and an overall evaluation of the test by the corrector. If desired Learner Cloud may contact Teacher '12345' by E-mail for posing further questions.

The next part states the six test questions, the corresponding answers or choices by the learner, and the scores and comments by the corrector. Figure 10.90 only exhibits the first two of the six corrected test questions.

The third function indicated in Figure 10.86 is 'Create and Review Test Question'. This function is restricted to teachers, therefore, Learner Cloud would be rejected if he tries to use it. The test-question creation function is inserted here for enabling teachers to create test questions in close connection to the current learning unit.

For example, Teacher Ball enters the creation-reviewing page for Unit 1.1.2 containing a creation part called 'Create A New Test Question' as shown in Figure 10.91 and a reviewing part called 'Review Test Questions' as shown in Figure 10.94 .

The screenshot displays the Stochastikon Magister web interface. At the top, the header reads "STOCHASTIKON" and "Magister". Below the header is a navigation bar with links for "Stochastikon GmbH", "Calculator", "Encyclopedia", "Mentor", "Magister", and "Imprint".

The main content area is titled "Learning Unit 1.1.2: Connectivity". It features a section for a corrected test with the following details:

- Unit:** 1.1.2: Connectivity
- testID:** 3
- test group:** 1,
- score:** 6
- teacherID:** 12345
- test date:** 2005-05-17 14:07:08.0
- teacher's comments of the whole test:** Cloud: it is a good test, but you should review unit 1.1 and unit 1.2 to clear some concepts, such as...12345,2005-05-17
- correction date:** 2005-05-17 14:26:49.0

Two questions are listed:

**Question 1:**  
question 1 of Q & A.  
**learner's answer to this question:** cloud's answer of question 1 test 3.  
**score:** 2  
**teacher's comments to this question:** a mostly correct answer to question 1.

**Question 2:**  
question 3 of Q & A  
**learner's answer to this question:** cloud's answer of question 2 test 3.  
**score:** 2  
**teacher's comments to this question:** a good answer to question 2.

The right sidebar contains a "User ID: cloud" field, a vertical stack of buttons labeled "Target", "Content", "Example", "Exercise", "Test", "Literature", "FAQ", "Discussion", and "Question", and a search box containing the text "Test".

Figure 10.90: A corrected test in *Stochastikon Magister*.

A new test question is created in two steps. At first the formal features like question type, user level, user groups and number of correct answer in case of judgement and multiple choice questions are fixed. Next the question content has to be formulated together with the corresponding answer.

Step 1: As mentioned earlier there are three types of test-questions: Q & A, Judgment or Multiple Choice. After one of these types has been selected for the new question, the user level and the groups must be stated. Questions with no specially marked groups are used for all groups. Next the number of the correct answer in case of a Judgment or a Multiple Choice question must be fixed.

Assume that Teacher Ball wants to create a Multiple Choice question for Unit 1.1.2 addressing learners with user level 0 or belonging to Group 1 or Group 2. Ball sets the number of the correct answer to be 2. This completes Step 1 and Ball may continue with Step 2 by clicking the 'Continue' button.

Step 2: For formulating the test question, one of four possible types (text, image, file, link) has



to be devised. The creation of a test question is similar as that of a sub-answer for a FAQ, which has been discussed in Section 10.4.3 ‘FAQ in a learning unit’. If the new test question requires more than one content, the other content(s) are added by means of the ‘Edit’ test-question panel.

**STOCHASTIKON**  
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Learning Unit 1.1.2: **Connectivity**

User ID: **ball**

-- Create A New Test Question --

step 1:

question type	<input type="radio"/> Question & Answer <input type="radio"/> Judgment <input checked="" type="radio"/> Multiple Choice	
user level	0 (0 ≤ 'user level' ≤ 9)	
user group	groupID	group name
	<input type="checkbox"/> 0	group0
	<input checked="" type="checkbox"/> 1	group1
	<input checked="" type="checkbox"/> 2	group2
	<input type="checkbox"/> 3	group3
	<input type="checkbox"/> 4	group4
	<input type="checkbox"/> 5	group5
	<input type="checkbox"/> 6	group6
	<input type="checkbox"/> 7	group7
correct answer	2	

question type Q & A: no use; Judgment: 0-false, 1-true; Multiple Choice: 1 ≤ 'correct answerID' ≤ min(answers,9))

Reset Continue

Target  
Content  
Example  
Exercise  
Test  
Literature  
FAQ  
Discussion  
Question

Acrobat Reader  
Test

Figure 10.91: Create a ‘Multiple Choice’ test-question in Magister (a).

Next, the necessary answers for a test question must be input. For each question of type Q & A, a standard answer must be provided, which may be used as a guideline for correction. As to the Judgement the correct answer must be provided and possibly some explanations for the correctors. Each Multiple Choice question must offer at least 2 choices. Moreover, the number of choices must be at least as large as the given number of the correct choice. If the number of the choices is larger than 2 and at the same time larger than the number for the correct choice, additional choices can be added through the ‘Edit’ test-question panel.

Figure 10.92 shows the creation interface for the second step in which Teacher Ball has input a text ‘The picture of a ball’ as question content. Next he has created two choices for this question namely an image file food.gif and an attached file ball.gif.

The screenshot shows the 'Stochastikon Magister' interface. The main content area is titled 'Learning Unit 1.1.2: Connectivity'. The form fields are as follows:

questionID:	11	question type:	Multiple Choice
user level:	0	user group:	1,2,
correct answer:	2		
question content type	<input checked="" type="radio"/> text <input type="radio"/> image <input type="radio"/> file <input type="radio"/> link		
question content:	The picture of a ball.		
Answer 1 :			
answer type	<input type="radio"/> text <input checked="" type="radio"/> image <input type="radio"/> file <input type="radio"/> link		
answer:	food.gif		
Answer 2 :			
answer type	<input type="radio"/> text <input type="radio"/> image <input checked="" type="radio"/> file <input type="radio"/> link		
answer:	ball.gif		

At the bottom of the form are 'Reset' and 'Continue' buttons. On the right sidebar, there are buttons for 'Target', 'Content', 'Example', 'Exercise', 'Test', 'Literature', 'FAQ', 'Discussion', and 'Question'. The user ID is 'ball'.

Figure 10.92: Create a ‘Multiple Choice’ test-question in Magister (b).

Figure 10.93 shows the acknowledgement of a successful generation of a test question. The questionID is generated automatically according to question type and learning unit. After the new question has been acknowledged, the creator can continue and ‘Edit’ the newly generated question, ‘Create’ another new test-question or ‘Review’ the distinguished questions and discussions for help. Note that after a test question has been generated, it can be edited at any time.

In *Stochastikon Magister*, the function ‘Review Distinguished Questions and Topics’ is usually designed in connection with the function ‘Create’ question (test-question or FAQ). Thus, the distinguished questions and discussions may be used straight away for developing new questions. The functions and interfaces for handling the distinguished questions and discussions in Test section are similar to those discussed in the FAQ section (see Section 10.4.3 ‘FAQ in a learning unit’, Figure 10.77 and Figure 10.78).

The development of new test questions has crucial importance for the Test section of *Stochastikon Magister*. A new question will be stored in the database only after all the necessary components have been completely. This is emphasized because an incomplete test question would seriously damage the normal running of the test management programs such as test-question

categorization, test construction and test correction. For the same reason, the same restriction has to be met, whenever a test question is edited or deleted.

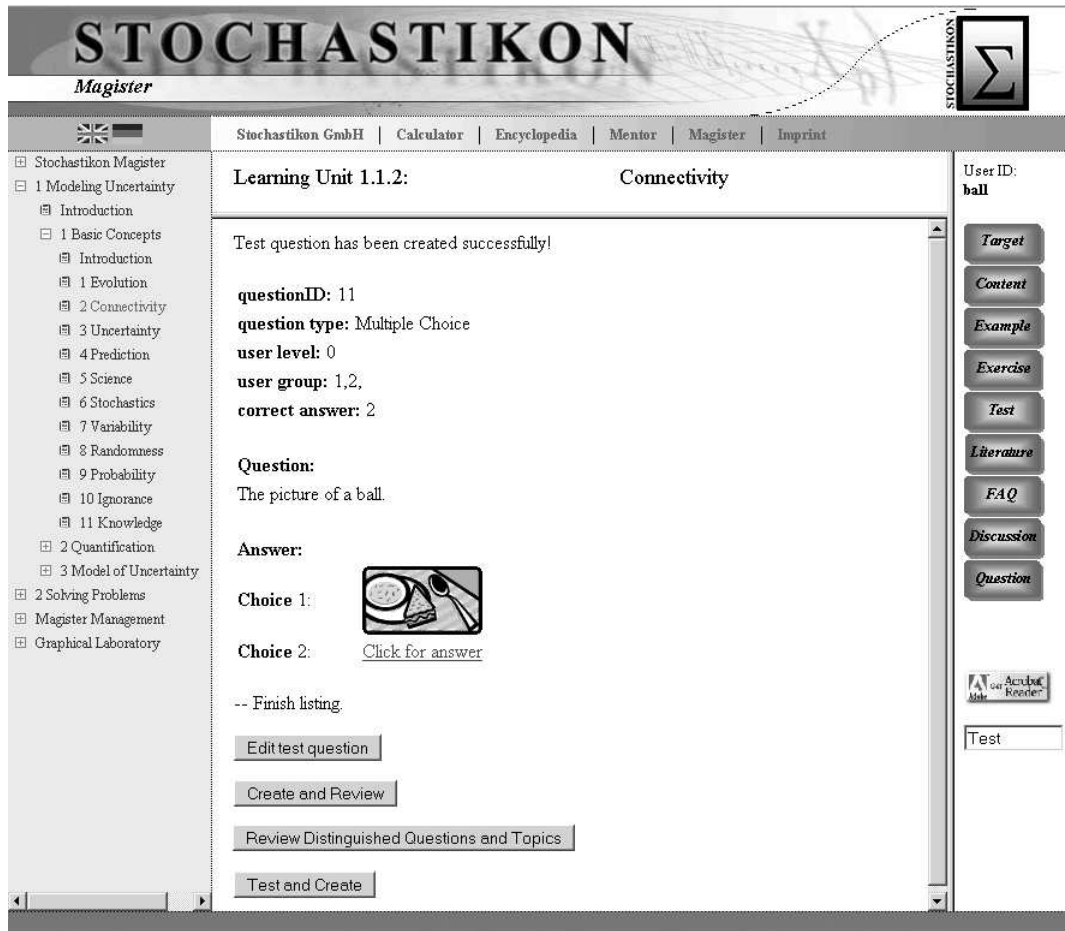


Figure 10.93: Create a ‘Multiple Choice’ test-question in Magister (c).

Figure 10.94 displays the reviewing part of the creation-reviewing page for test questions. The first part, the creation interface, is displayed in Figure 10.91.

Teacher Ball wants to review the Judgment questions for Unit 1.1.2 addressing Group 0 and Group 1. After the list of search results is displayed, Ball marks the question he wants to check and clicks the ‘Select’ button. Then he enters the ‘Edit’ page for test questions (see Figure 10.95).

Figure 10.95 displays a simple example of a Judgment question used only for illustration purpose. The first part of the ‘Edit’ page contains the formal information about the selected test question, followed by the question content and the answers.

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Learning Unit 1.1.2: **Connectivity**

User ID: ball

-- Review Test Questions --

Condition:

question type:  Question & Answer  Judgment  Multiple Choice

user level:  (0 ≤ 'user level' ≤ 9)

user group	groupID	group name
<input checked="" type="checkbox"/>	0	group0
<input checked="" type="checkbox"/>	1	group1
<input type="checkbox"/>	2	group2
<input type="checkbox"/>	3	group3
<input type="checkbox"/>	4	group4
<input type="checkbox"/>	5	group5
<input type="checkbox"/>	6	group6
<input type="checkbox"/>	7	group7

questionID:

creatorID:

creation date:  (YYYY-MM-DD)

Order by:  questionID  increase

Figure 10.94: Review test-questions in Magister.

**STOCHASTIKON**  
Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

Learning Unit 1.1.2: **Connectivity**

User ID: ball

questionID: 4  
question type: Judgment  
user level: 1  
user group: 0,1,2,  
correct answer: 0

**Question:**  
contentID 1  
Judgement of unit 1.2 question 4.

**Answer:**  
answerID 1  
a wrong statement.  
answerID 2

-- Finish listing.  
\*\*\*\*\*

Figure 10.95: A 'Judgment' test-questions in Magister.

The second part is shown in Figure 10.96. It contains the interface for updating the formal information of the test question, i. e., user level, user group and number of the correct answer. Note that the question type cannot be changed, because it constitute the basic criterion for categorizing the test questions. The reason for any change may be stated and recorded for later checking.

Figure 10.96: Edit test-questions in Magister - Formal Information (a).

Figure 10.97 shows the third and fourth part of the ‘Edit’ page, which are used for editing content and answers of the test question by means of the functions ‘Add’, ‘Insert’, ‘Update’ and ‘Delete’. Figure 10.97 contains only the ‘Edit’ part for answers, however, the ‘Edit’ part for the content of a test question has almost the same layout. These two functional parts operate similar as the editing part for FAQ answers described in Section 10.4.3 ‘FAQ in a learning unit’.

However, in the case of test questions there are more conditions and restrictions to be taken into account. For example, deleting an answer from a Multiple Choice question is rejected, if there are only two answers or if the number of answers becomes smaller than the number of the correct answer.

The fifth part of the edit page enables to ‘Delete’ a test-question. It is very similar to deleting a question, discussion or FAQ by a teacher in the corresponding management panels. Any test question which is deleted by a teacher becomes invisible for teachers and cannot be used by the program to compose a test. Nevertheless, it still occupies the test-questionID until it is completely deleted by an administrator from *Stochastikon Magister* or restored as a normal test-question.

The screenshot shows the 'Stochastikon Magister' interface. At the top, the logo 'STOCHASTIKON' is displayed. Below it, the navigation bar includes links for 'Stochastikon GmbH', 'Calculator', 'Encyclopedia', 'Mentor', 'Magister', and 'Imprint'. The left sidebar contains a tree view of the course structure, with '1 Modeling Uncertainty' expanded to show '2 Connectivity'. The main content area is titled 'Learning Unit 1.1.2: Connectivity' and contains three sections for managing test questions:

- Add answer:** Includes radio buttons for 'text', 'image', 'file', and 'link', a text input field, and buttons for 'Reset' and 'Add an answer'.
- Insert an answer with answerID:** Includes a text input field for 'answerID', radio buttons for 'text', 'image', 'file', and 'link', a text input field, and buttons for 'Reset' and 'Insert an answer'.
- Update an answer with answerID:** Includes a text input field for 'answerID', radio buttons for 'text', 'image', 'file', and 'link', a text input field, and buttons for 'Reset' and 'Update the answer'.

At the bottom of the main content area, there is a section for deleting a test question:

**Delete this test question:**  
Reason:

The right sidebar contains a vertical menu with buttons for 'Target', 'Content', 'Example', 'Exercise', 'Test', 'Literature', 'FAQ', 'Discussion', and 'Question'. Below this menu is a 'Test' input field and an 'Acrobat Reader' logo.

Figure 10.97: Edit test-questions in Magister - Question Answers (b).

## ‘Test’ Management

The ‘Test’ Management panel for learners enables them to review all tests they have taken and which have been corrected by teachers of *Stochastikon Magister*.

The interfaces and functions in the ‘Test’ Management panel are similar as ‘Review Learner’s Tests’ in each learning unit (see the middle part of Figure 10.86). Only the search conditions referring to the course and the learning unit are added for selecting a specific learning unit. This is necessary for all searching interfaces used in the learning/course management panels, because the corresponding information covers all existing learning units. In Figure 10.98 the utmost left sequence are programs of ‘Test’ Management for learners.

The ‘Test’ Management panel for administrators has two main tasks namely managing the test-questions and managing the tests. The functions allow to check the test questions and tests including the entire history of their changes, i. e., creation, changing, deletion and restoration. The administrators’ responsibilities are similar as in the cases of ‘Question’, ‘Discussion’ and

‘FAQ’ Management panels. There is one peculiarity for deleting a test question by an administrator. If the test question has been used in some tests, then it can be deleted only if all the tests with the test question to be deleted are also deleted beforehand. The right-hand part in Figure 10.98 shows the functions available in the ‘Test’ Management panel for administrators.

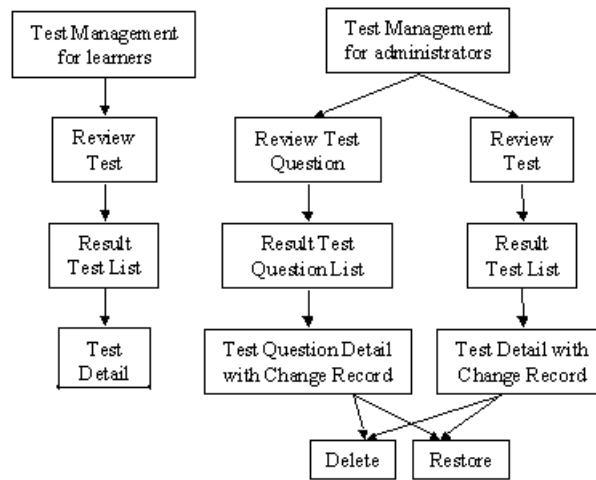


Figure 10.98: Magister ‘Test’ Management programs for learners and for administrators.

More important is the ‘Test’ Management panel for teachers and, in particular the test correction part. Figure 10.99 illustrates programs of the ‘Test’ Management panel for teachers.

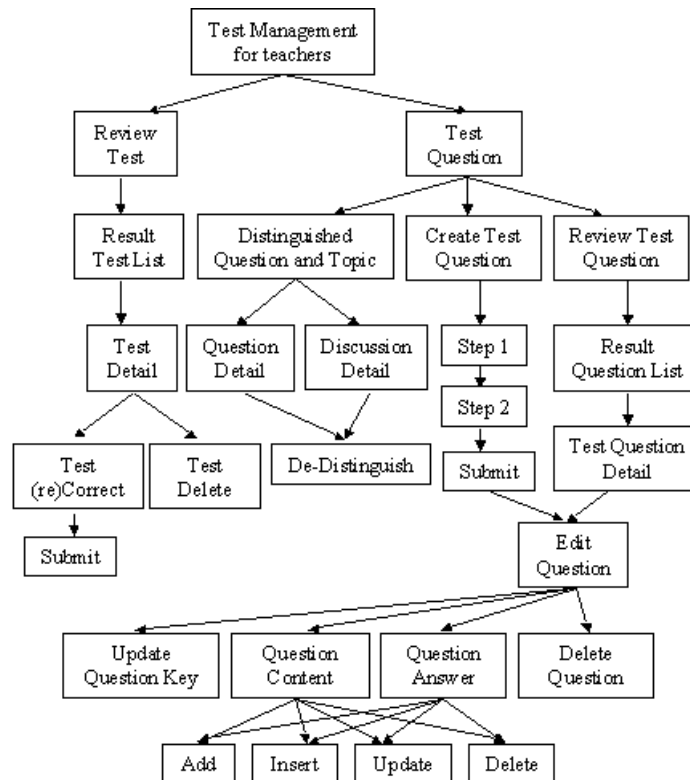


Figure 10.99: Magister ‘Test’ Management programs for teachers.

Figure 10.100 and Figure 10.101 are two UML user case diagrams. The former one shows an overview of Teacher and the use cases present in the Magister Test - Test Question Management

system, and the later one shows an overview of Teacher and the use cases present in the Magister Test - Test Management system.

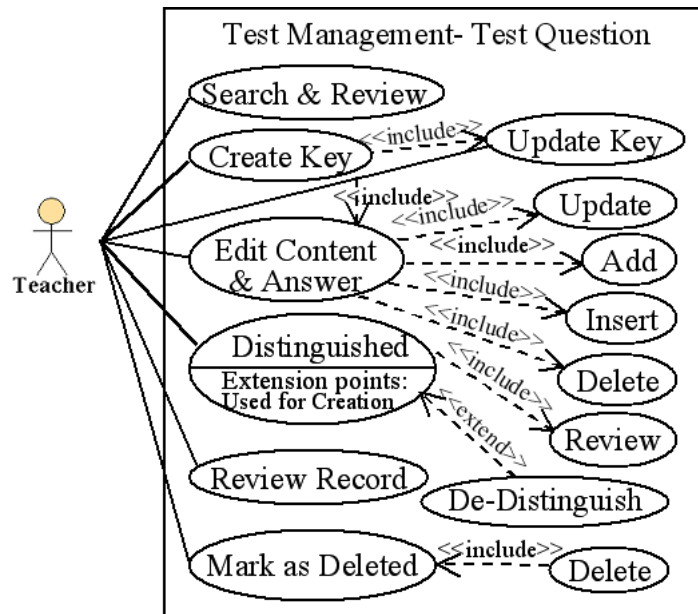


Figure 10.100: Test management - Test Question for Teacher.

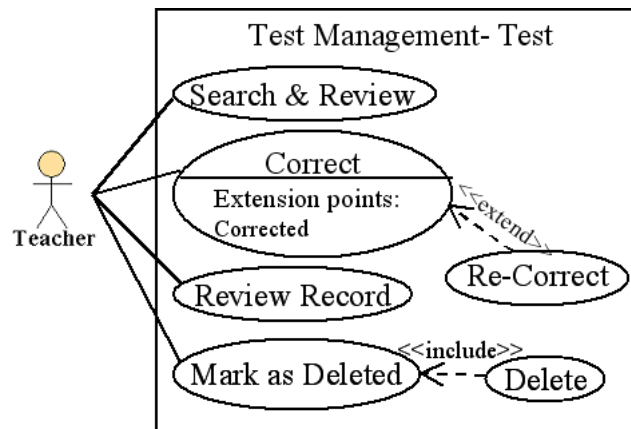


Figure 10.101: Test management - Test for Teacher.

Figure 10.102 shows the starting page of the ‘Test’ Management panel for teachers. It is separated into two parts: ‘Review and Create test-questions’ and ‘Review and Correct learner tests’.

The ‘Review and Create test-questions’ part is similar as the analogous part in Section 10.4.4 ‘Test in a learning unit’ (see Figure 10.91 and Figure 10.94). The main difference is that for creating a new test-question, the corresponding learning unit has to be selected first.



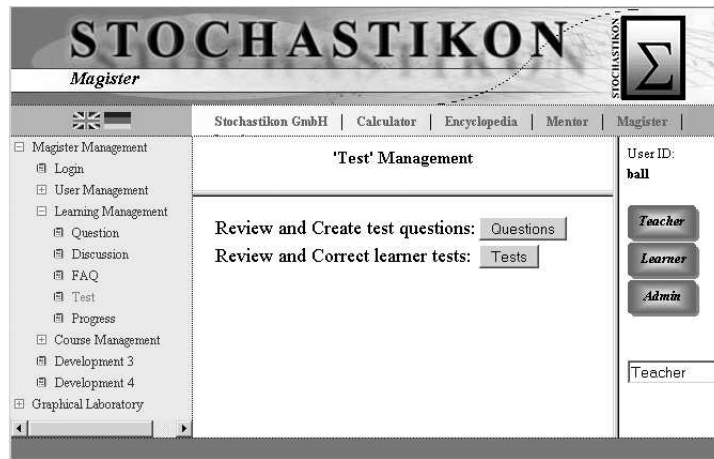


Figure 10.102: Starting page of Magister ‘Test’ Management for teachers.

Figure 10.103 shows the ‘Review and Correct learner tests’ interface for teachers in the ‘Test’ Management panel. To ‘Review’ relevant tests, the corresponding conditions can be input.

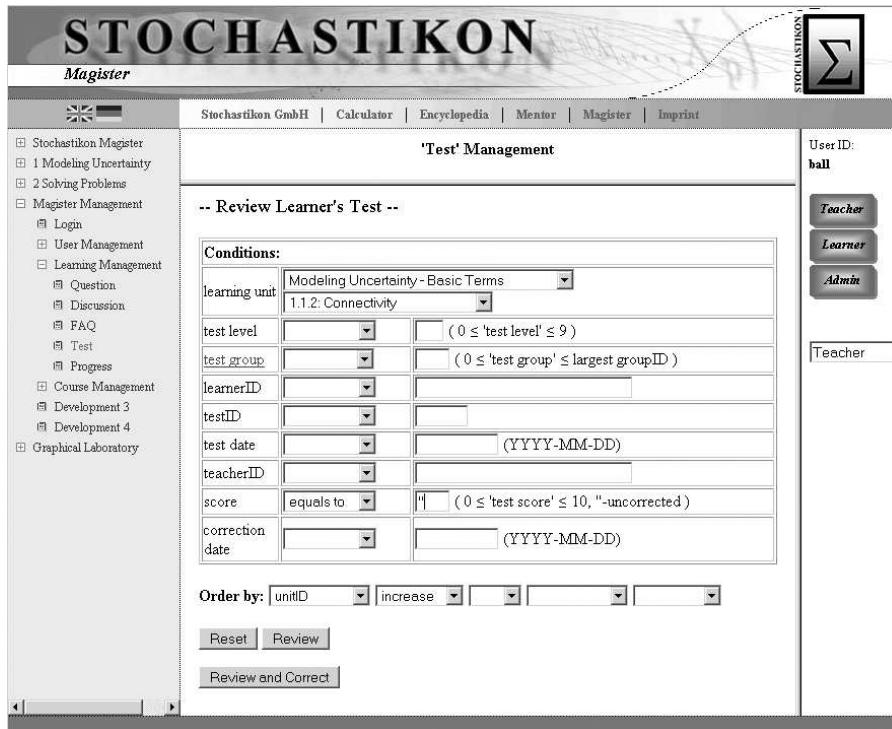


Figure 10.103: Magister ‘Test’ Management for teachers - Review Test (a).

In Figure 10.103 Teacher Ball wants to review the tests referring to Unit 1.1.2, which have not been corrected so far. Note that the relationship between the two conditions ‘test level’ and ‘test group’ is ‘OR’ relation here, i. e., it is possible to set simultaneously ‘test-level’ and ‘test group’ conditions and the result tests are those which satisfy either of the two conditions. The reason has been mentioned earlier, a test serves only for a given user level or for a given user group. All other relations between search conditions are ‘And’ relations.

In Figure 10.104, three tests are found meeting the requirements of Teacher Ball given in Figure 10.103. Teacher Ball marks one of them and enters the test review page shown in Figure 10.105 by clicking the ‘Select’ button.

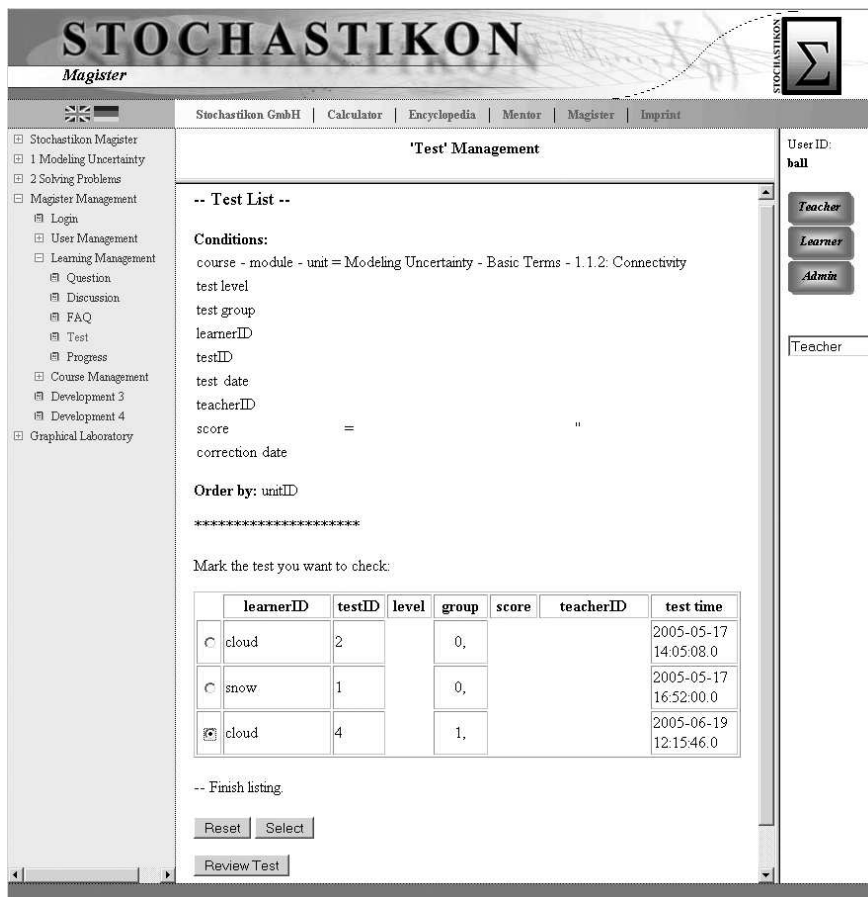


Figure 10.104: Magister 'Test' Management for teachers - Review Test (b).



Figure 10.105: Magister 'Test' Management for teachers - Review Test (c).

In this example, the selected test has not been corrected so far and, thus, only answers of the learner to each test-question are displayed without any corrections, i. e., scores and teacher's comments. The teacher can click the 'Reference Answer' link to see the standard answer to each of the test questions (see Figure 10.105).

At the bottom of the test review page, there are the function buttons '(re)Correct Test', 'Delete Test', 'Review Test' and the button to go back to the entrance page. Teachers can correct an uncorrected test or revise a correction by clicking the '(re)Correct' button. Figure 10.106 shows part of the interface of this test-correction panel.

Figure 10.106: Magister 'Test' Management for teachers - Correct a Test (a).

At the top of the test-correction part the teacher is asked to give an overall assessment of the test. The corrector must give such an assessment although it is possible until after the correction of all the test questions has been completed. If the overall assessment is missing, submission of the test correction is rejected.

The correction of a Q & A question results in a score and a comment. The scores of Judgments and Multiple Choice questions are generated automatically according to the learner's answers. The corrector may (but not necessarily) give comments or further explanations to the judgments and multiple choice questions. The correction is supported by clicking the 'Reference Answer' button or the 'Former Correction' button if the test has been corrected before.

Only complete correction can be submitted. An attempt to submit an incomplete correction for example a correction without overall assessment or without a score to any of the Q & A question, triggers an alert and the submission fails. If the correction does not include a comment to any of the six test-questions, an alert is given indicating the missing comment and asking for confirmation. If the teacher confirms, the correction is acknowledged as displayed in Figure 10.107.

The screenshot displays the 'Test' Management interface for teachers. At the top, a notification states: "Learner cloud's Unit 1.1.2: Connectivity Test 4 has been corrected." Below this, a section titled "-- Review One Test --" provides details for a specific test:

- Unit:** 1.1.2: Connectivity
- learnerID:** [cloud](#)
- testID:** 4
- test group:** 1,
- score:** 7
- teacherID:** [ball](#)
- test date:** 2005-06-19 12:15:46.0
- teacher's comments of the whole test:** A good test...
- correction date:** 2005-06-19 15:33:13.0

The interface then lists two questions:

**Question 1:**  
question 8 of Q & A.  
**learner's answer to this question:** Cloud's answer to the first question...  
**score:** 2      [Reference Answer](#)  
**teacher's comments to this question:** You gave a reasonable answer to this question...

**Question 2:**  
question 7 of Q & A.  
[Click for question content](#)  
**learner's answer to this question:** Cloud's answer to the second question...  
**score:** 2      [Reference Answer](#)

The interface includes a navigation menu on the left, a header with "STOCHASTIKON" and "Magister", and a right sidebar with user ID "ball" and buttons for "Teacher", "Learner", and "Admin".

Figure 10.107: Magister 'Test' Management for teachers - Correct a Test (b).

The acknowledgement of a correction repeats the details of the correction. Actually, in 'Test' Management for teachers, the pages for displaying a corrected test are the same as given in Figure 10.107 except for the additional acknowledgement at the top of the page. The correction page (see Figure 10.107) offers also the possibility to send an E-mail not only to the

correcting teacher, but also to the learner who took the test. Moreover, the teacher can check the ‘Reference answer’ of each of the test question.

### 10.4.5 Learning Progress

‘Learner Progress’ Management is used to control and supervise the learning process of each learner.

Figure 10.108 is a UML swimlanes in activity diagram, which describes the Learner Progress Management process in Magister e-learning environment.

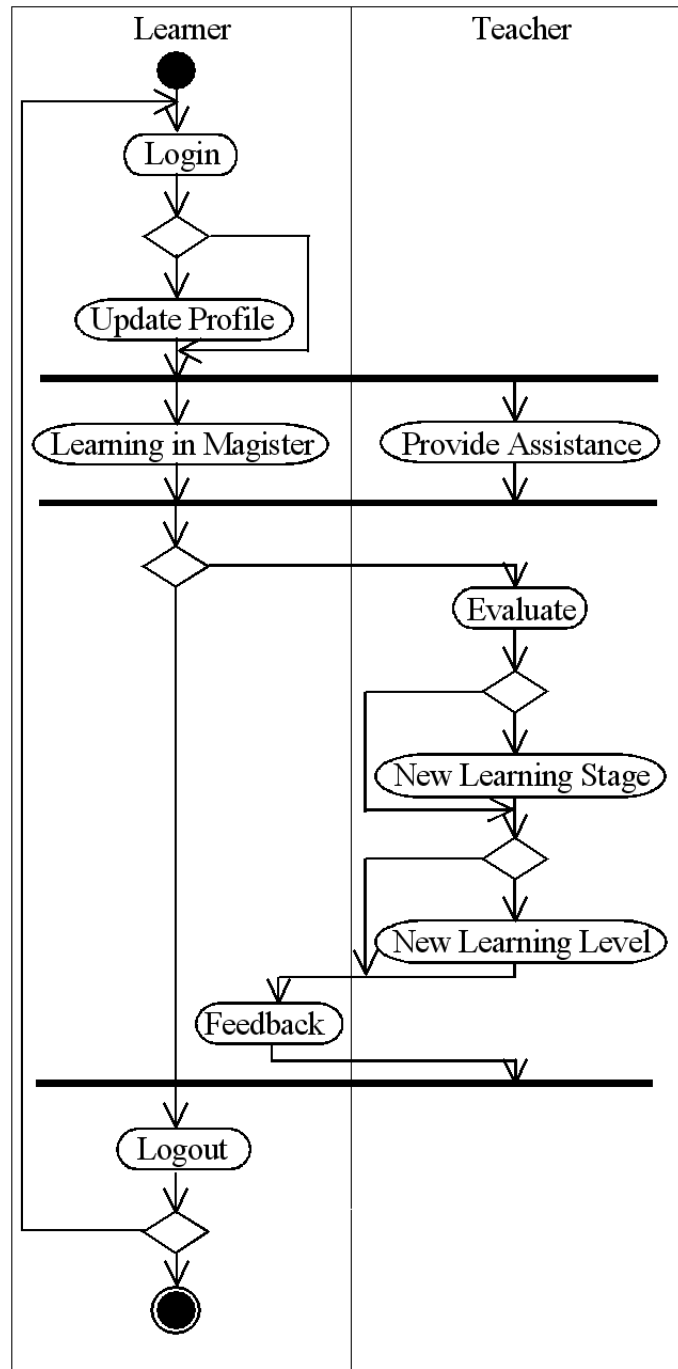


Figure 10.108: Learner Progress Management process for Learner and Teacher.

Figure 10.109 is a UML user case diagram that shows an overview of actors and the use cases present in the Magister Learner Progress Management system.

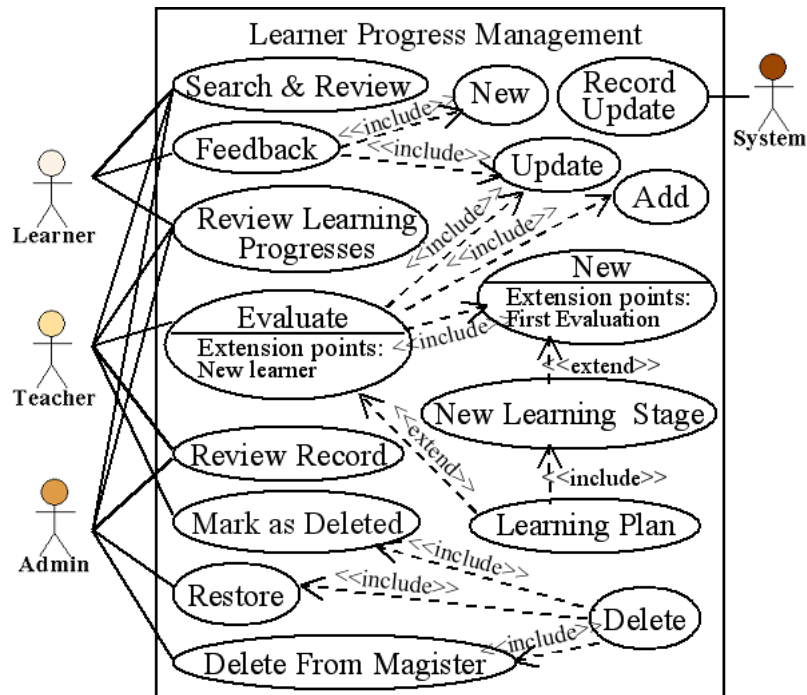


Figure 10.109: Learner progress management.

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User ID: cloud

Teacher  
Learner  
Admin

Learner

**'Learner Progress' Management**

Learner cloud's learning progress:

Review old evaluations:

Mark the evaluation you want to feedback:

	e-ID	teacherID	evaluation	feedback	creation time	feedback time
<input checked="" type="checkbox"/>	1	dana	Click for information		2005-04-22 17:14:19.0	
<input type="checkbox"/>		12345	add some comment on cloud's learning process.		2005-04-22 17:15:35.0	
<input type="checkbox"/>	2	sunshine	a new evaluation on cloud's learning progress.	cloud gives feedback to sunshine's evaluation...	2005-05-05 17:18:06.0	2005-05-08 20:17:15.0
<input type="checkbox"/>		12345	more suggestion on cloud's learning process.		2005-05-05 17:18:58.0	
<input type="checkbox"/>		dana	comment on cloud's learning progress.	cloud update feedback of dana's evaluation.	2005-05-07 17:23:20.0	2005-05-08 20:18:49.0
<input type="checkbox"/>	3	dana	a new evaluation of cloud's work.		2005-05-06 20:13:31.0	
<input type="checkbox"/>		12345	12345 gives some more comments on cloud's work of this period.	cloud gives feedback to 12345's evaluation.	2005-05-06 20:15:07.0	2005-05-10 20:22:41.0
<input type="checkbox"/>		ball	ball add some evaluation to cloud's work.		2005-05-10 20:21:53.0	

-- Finish listing

Reset Select

Check progresses learner cloud made after the latest evaluation Check

Figure 10.110: Magister Progress Management for learners - Review.

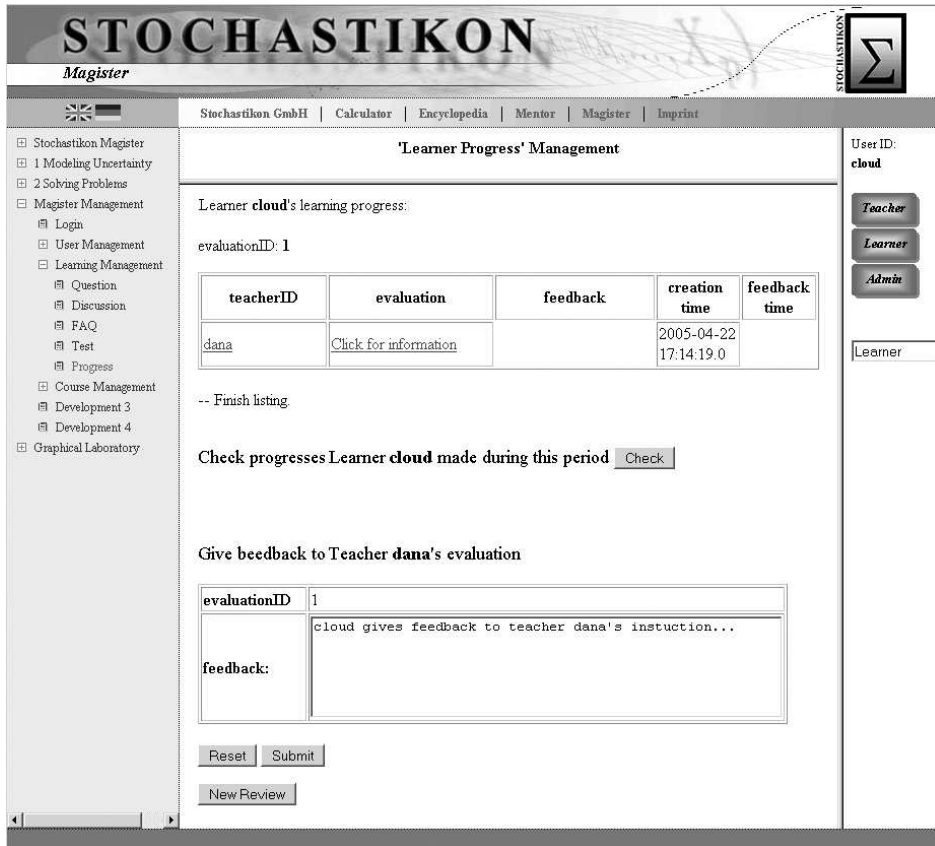


Figure 10.111: Magister Progress Management for learners - Feedback.

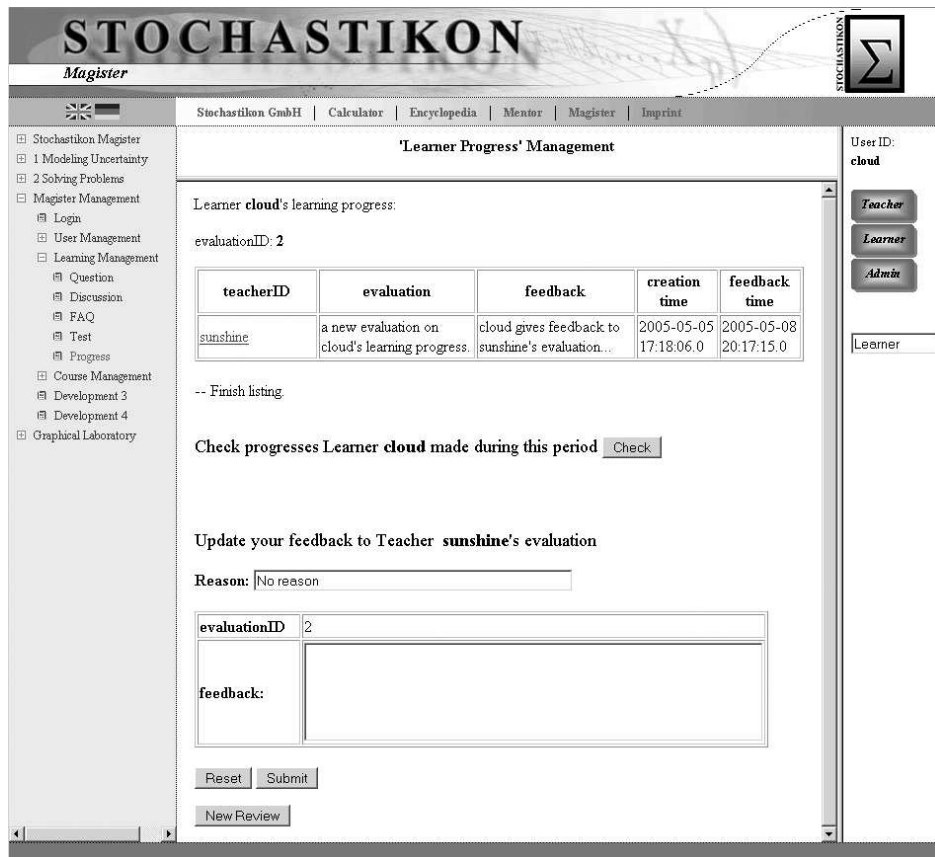


Figure 10.112: Magister Progress Management for learners - Update Feedback.

As can be seen from Figure 10.110, learners are required to give a feedback to each evaluation. By marking an evaluation and clicking the 'Select' button, a learner enters the feedback-page shown in Figure 10.111.

If Learner Cloud selects an evaluation, which he had responded before, then he can update his former feedback and record the reasons. Figure 10.112 shows the feedback-updating page.

The 'Check' buttons in Figure 10.112, Figure 10.117 and Figure 10.118 are used to review the progress of a learner during a selected period. If Learner Cloud clicks the 'Check' button in Figure 10.112, he will see all his learning activities, i. e., asked questions, contributions to discussions and taken tests in *Stochastikon Magister* during the first learning stage (evaluationID = 2) from 2005-04-22 to 2005-05-05 in Figure 10.113. He can check a detail of the activities by clicking the corresponding 'Detail' link.

# STOCHASTIKON

Magister

Stochastikon GmbH | Calculator | Encyclopedia | Mentor | Magister | Imprint

- ☐ Stochastikon Magister
- ☐ 1 Modeling Uncertainty
  - ☐ Introduction
  - ☐ 1 Basic Concepts
  - ☐ 2 Quantification
  - ☐ 3 Model of Uncertainty
- ☐ 2 Solving Problems
  - ☐ Introduction
  - ☐ 1 Stochastic Procedure
  - ☐ 2 Prediction
  - ☐ 3 Measurement
- ☐ Magister Management
  - ☐ Login
  - ☐ User Management
  - ☐ Learning Management
    - ☐ Question
    - ☐ Discussion
    - ☐ FAQ
    - ☐ Test
    - ☐ Progress
  - ☐ Course Management
    - ☐ Name
    - ☐ Literature
    - ☐ Development 3
    - ☐ Development 4
  - ☐ Graphical Laboratory

## 'Learner Progress' Management

Learning Period: 2005-04-22 ~ 2005-05-05

Questions asked by Learner cloud:

unitID	questionID	question	creation time	answe
2.2.13	1	cloud ask question about unit 2.2.13: minimum upper bound beta-uncertainty space for X	2005-04-29 20:34:58.0	0
2.1.3	1	cloud's question about unit 2.1.3 what is reliability?	2005-04-29 20:35:39.0	2
1.1.2	1	question 2 about unit 1.1.2 from cloud, about the connectivity. ask now.1@#%\$%&&^&^&(*)).+_!+==.90=-.13/.<>>?gh gfdhdfgdhgr54 9!hg'sfkl fgfgsdfg	2005-04-29 22:27:14.0	5

-- Finish listing

Discussions introduced by Learner cloud:

unitID	topicID	topic	creation time	comments	
1.1.11	1	topic 2, unit 1.1.11 by cloud.	2005-05-04 16:07:52.0	4	<a href="#">Detail</a>

-- Finish listing

Discussions joined by Learner cloud:

unitID	originatorID	topicID	comment	creation time	
1.1.2	rain	2	colorful colud responds to rain's topic.	2005-05-01 21:06:15.0	<a href="#">Detail</a>
1.1.2	ball	1	cloud gives comment to ball's topic.	2005-05-01 21:07:06.0	<a href="#">Detail</a>

-- Finish listing

Tests made by Learner cloud:

unitID	testID	level	group	score	teacherID	test time	correction time	
2.1.3	1		0,	5	ball	2005-05-03 12:47:43.0	2005-05-03 12:50:37.0	<a href="#">Detail</a>

-- Finish listing

New Review

User ID:  
**cloud**

[Teacher](#)

[Learner](#)

[Admin](#)

Learner

Figure 10.113: Magister Progress Management - Learning Progresses of a Learning Period.



The first step in the ‘Learner Progress’ Management for a teacher is to specify the learner by defining certain conditions as shown in Figure 10.114.

The screenshot shows the 'Learner Progress' Management interface. The header includes the 'STOCHASTIKON' logo and 'Magister' branding. A navigation menu on the left lists various sections like 'Stochastikon Magister', 'Modeling Uncertainty', and 'Learner Progress'. The main content area is titled 'Learner Progress' Management and contains a search form with the following fields:

- Conditions:
  - userID:
  - user level:  (0 ≤ 'user level' ≤ 9)
  - user status:  (0 ≤ 'user status' < yours)
  - user group:  (0 ≤ 'user group' < 28)
  - creation date:  (YYYY-MM-DD)
- Order by:  increase

Buttons for 'Reset' and 'Search' are located below the form. On the right side, there is a 'User ID: ball' field and three buttons labeled 'Teacher', 'Learner', and 'Admin', along with a 'Teacher' input field.

Figure 10.114: Magister Progress Management for teachers - Search a Learner.

The evaluation of a learner by a teacher can consist of a text, an image, a file attachment or a web link.

After a successful registration a learner is evaluated resulting in a learning plan constituting the first evaluation. Generally the learning plan is prepared in PDF format and saved in Magister as an attachment file. Figure 10.115 shows the request of a learning plan for a newly registered learner with name ‘Paper’.

The screenshot shows the 'Learner Progress' Management interface for creating a learning plan. The header and navigation menu are the same as in Figure 10.114. The main content area is titled 'Learner Progress' Management and displays the following information:

- Learner Paper's learning progress:
- \*\* Please first give a learning plan for Learner Paper!
- evaluationID:
- type:  text  image  file  link
- evaluation:

Buttons for 'Reset', 'Submit', and 'New Search' are located below the form. On the right side, there is a 'User ID: ball' field and three buttons labeled 'Teacher', 'Learner', and 'Admin', along with a 'Teacher' input field.

Figure 10.115: Magister Progress Management for teachers - Learning Plan.

If a learner has communication records with instructors, and a new evaluation needs to be generated by the teacher in charged which are not considered so far, the ‘Learner Progress’

Management panel supports two possibilities for a new evaluation. The teacher can either evaluate the learner's progress within the current learning stage (at the time period since the date of the first evaluation for the most recent fixed learning stage till now), or a supplement can be added to the assessment of the learners achievements in a former learning stage.

In Figure 10.116, Teacher Ball is to give the first evaluation with evaluationID 4 for the third Learning Stage, which was started in 2005-05-06. Teacher Ball can get information about Learner Cloud's learning achievements during the period, which need to be evaluated, by clicking the 'Check' button for assistance.

The screenshot displays the 'Learner Progress' Management interface. At the top, the header reads 'STOCHASTIKON Magister'. Below this is a navigation bar with links: Stochastikon GmbH, Calculator, Encyclopedia, Mentor, Magister, and Inprint. A sidebar on the left contains a menu with options like 'Stochastikon Magister', '1 Modeling Uncertainty', '2 Solving Problems', 'Magister Management', 'Login', 'User Management', 'Learning Management', 'Question', 'Discussion', 'FAQ', 'Test', 'Progress', 'Course Management', 'Development 3', 'Development 4', and 'Graphical Laboratory'. The main content area is titled 'Learner Progress' Management and features a table with columns for learner ID, name, description, and dates. The table contains three rows of data. Below the table are 'Reset' and 'Select' buttons. A 'Check' button is used to verify progress. A form titled 'Give a new evaluation to cloud's learning progress' includes an 'evaluationID' field set to 4, a 'type' dropdown set to 'text', and a text area with the text 'teacher ball give a new evaluation to cloud's study...'. 'Reset' and 'Submit' buttons are at the bottom of the form. On the right side, the user profile for 'ball' is shown with 'Teacher' buttons and a 'Teacher' label.

	learning progress.	dana's evaluation.	17.23.20.0	20.18.49.0
<input type="radio"/> 3	dana	a new evaluation of cloud's work.	2005-05-06 20:13:31.0	
<input type="radio"/>	12345	12345 gives some more comments on cloud's work of this period.	cloud gives feedback to 12345's evaluation.	2005-05-06 20:15:07.0 2005-05-10 20:22:41.0
<input type="radio"/> C	ball	ball add some evaluation to cloud's work.		2005-05-10 20:21:53.0

Figure 10.116: Magister Progress Management for teachers - The first evaluation for a new Learning Stage.

Note that the learning periods to be checked in Figure 10.112, Figure 10.117 and Figure 10.118 are past periods, which is indicated by the evaluationIDs smaller than 4 referring to given time periods. In contrast the learning period to be checked in Figure 10.110 and Figure 10.116 refers to the latest learning stage which is not completed unless the teacher has successfully submitted the evaluation for this learning stage, which also fix the time period of this stage.

Figure 10.117 shows the other possibility. Teacher Ball wants to add another assessment to Learner Cloud's study achievements during the first Learning Stage with evaluationID 2. To this end, Ball must enter the evaluation-adding page by marking any one of the evaluation belonging to this stage and clicking the 'Select' button.

If Learner Cloud's achievements during a selected Learning Stage have been evaluated by a teacher for example by Teacher Ball before, then it is not possible for Teacher Ball to input

the second evaluation, but it is possible for him to update the existing evaluation or to delete it by stating the reasons for this action.

The screenshot displays the 'Learner Progress' Management interface for teachers. At the top, the page title is 'STOCHASTIKON Magister'. The main content area is titled 'Learner Progress' Management and features a table of evaluations for learner 'cloud'. The table has the following data:

teacherID	evaluation	feedback	creation time	feedback time
sunshine	a new evaluation on cloud's learning progress.	cloud gives feedback to sunshine's evaluation..	2005-05-05 17:18:06.0	2005-05-08 20:17:15.0
12345	more suggestion on cloud's learning process.		2005-05-05 17:18:58.0	
dana	comment on cloud's learning progress.	cloud update feedback of dana's evaluation.	2005-05-07 17:23:20.0	2005-05-08 20:18:49.0

Below the table, there is a 'Finish listing' link and a 'Check progresses Learner cloud made during this period' button. A form titled 'Add a new evaluation to cloud's learning progress' is also present, with fields for 'evaluationID' (value: 2), 'type' (radio buttons for text, image, file, link), and a text area for the 'evaluation'. The form includes 'Reset', 'Submit', and 'Evaluate and Review' buttons, as well as a 'New Search' button.

Figure 10.117: Magister Progress Management for teachers - Add evaluations for a fixed Learning Stage.

Figure 10.118 illustrates the possibility to delete an evaluation. Teacher Ball wants to delete his assessment for Learner Cloud's achievements during the second Learning Stage (evaluationID = 3). The 'Learner Progress' Management for teachers makes it possible for a teacher to delete or restore his/her assessments. A deleted assessment is invisible for the learner, while teachers can check them, however, with an advice that it had been 'deleted'. In Figure 10.119 it is shown, how Teacher Ball wants to restore his deleted assessment for Learner Cloud's learning achievements during the second Learning Stage with evaluationID = 3.

In contrast to the other management panels, in the 'Learner Progress' Management panel, teachers are allowed to handle only their own evaluations independently of their user status by means of the functions 'Add', 'Update', 'Delete' or 'Restore'. Each teacher is entitled to give only one assessment of a specific learner during one learning stage. But, of course, the teacher can update his assessment at any time.

The 'Learner Progress' Management for administrators entitles administrators to completely delete a 'deleted' evaluation from *Stochastikon Magister* or mark it as 'deleted' so that learners cannot see it anymore no matter who authored the assessment.

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**'Learner Progress' Management**

User ID: ball

Teacher  
Learner  
Admin

Teacher

Learner cloud's learning progress:

evaluationID: 3

teacherID	evaluation	feedback	creation time	feedback time
dana	a new evaluation of cloud's work.		2005-05-06 20:13:31.0	
12345	12345 gives some more comments on cloud's work of this period.	cloud gives feedback to 12345's evaluation.	2005-05-06 20:15:07.0	2005-05-10 20:22:41.0
ball	ball add some evaluation to cloud's work.		2005-05-10 20:21:53.0	

-- Finish listing.

Check progresses Learner **cloud** made during this period

Delete your evaluation

Reason:

Update your evaluation to **cloud**'s learning progress

Reason:

evaluationID:

type:  text  image  file  link

evaluation:

Figure 10.118: Magister Progress Management for teachers - Update or Delete Evaluation.

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**'Learner Progress' Management**

User ID: ball

Teacher  
Learner  
Admin

Teacher

Learner cloud's learning progress:

evaluationID: 3

teacherID	evaluation	feedback	creation time	feedback time
dana	a new evaluation of cloud's work.		2005-05-06 20:13:31.0	
12345	12345 gives some more comments on cloud's work of this period.	cloud gives feedback to 12345's evaluation.	2005-05-06 20:15:07.0	2005-05-10 20:22:41.0
ball (deleted)	ball add some evaluation to cloud's work.		2005-05-10 20:21:53.0	

-- Finish listing.

Check progresses Learner **cloud** made during this period

Restore your evaluation

Reason:

Figure 10.119: Progress Management for teachers - Restore Evaluation.

The tasks of administrators in the ‘Learner Progress’ Management panel are similar as those in ‘Question’, ‘Discussion’, ‘FAQ’ and ‘Test’ Management panels. Administrators may review learners’ evaluations including the change records, check learners’ learning progresses within a specific learning stage or the entire learning processes, delete or restore deleted evaluations and so on. Figure 10.120 shows one of the corresponding interfaces.

The screenshot displays the 'Learner Progress' Management interface. The main content area features a table with the following data:

changerID	evaluation	feedback	reason	change type	date
ball	ball add some evaluation to cloud's work.	null			
ball	null	null			

Below the table, there are options to 'Restore your evaluation' and 'Delete this evaluation from Magister'. A 'Check' button is used to view the learner's progress. The detailed view shows the following data:

e-ID	teacherID	evaluation	feedback	creation time
1	dana	<a href="#">Click for information</a>		2005-04-21 17:14:19.0
	12345	add some comment on cloud's learning process.		2005-04-21 17:15:35.0
2	sunshine	a new evaluation on cloud's learning progress.	cloud gives feedback to sunshine's evaluation...	2005-05-01 17:18:06.0
	12345	more suggestion on cloud's learning process.		2005-05-01 17:18:58.0
	dana	comment on cloud's learning progress.	cloud update feedback of dana's evaluation.	2005-05-01 17:23:20.0
3	dana	a new evaluation of cloud's work.		2005-05-01 20:13:31.0
	12345	12345 gives some more comments on cloud's work of	cloud gives feedback to	2005-05-01

Figure 10.120: Magister Progress Management for administrators.

## 10.5 Graphical Laboratory

### 10.5.1 Objective of Graphical Laboratory

The Graphical Laboratory, as a special kind of learning material, shall serve as a means for dealing in a more playful way with stochastic concepts and procedures.

It is not easy to understand and practice Bernoulli Stochastics partly because Bernoulli Stochastics is based completely on mathematics in order to formulate and utilize unambiguously the stochastic relations between past and future and partly because it is based on a new way of thinking. It is well known that mathematics represents for a majority of persons an incomprehensible abstract formalism, which constitutes not only a barrier but act also as a deterrent.

At this point the computer-based stochastic ‘Graphical Laboratory’ comes in. It uses interactive programs, graphical representations, simulations and animations to visualize functions, sets, stochastic concepts and procedures. The interactive practicing and visualization shall help the learner to understand and bear in mind better the abstract concepts and the relationships between objects and variables. *Stochastikon Calculator* and Graphics (see Chapter 16) mediate the visualization quickly and comfortably and provide a dynamic real time illustration.

The Graphical Laboratory represents an additional dimension for learning Bernoulli Stochastics by combining numerical, symbolic and graphical capabilities. Learning of facts, skills,

algorithms and procedures is supported by illustrations. Learners can try and observe properties, explore and manipulate ideas, investigate patterns, conjecture and subsequently test propositions and thereby connect theory with visual objects. Learning by discovery and experiments is used for giving the learner the possibility to see, touch, feel and perform Bernoulli Stochastics.

Stochastic Graphical Laboratory shall support learning, practicing and doing Bernoulli Stochastics not as a special scientific enterprise but as an everyday activity. It shall facilitate gaining more insight into the functioning of evolution and the universal connectivity of the world.

## 10.5.2 Organization

The navigation structure of the Graphical Laboratory is the same as that of the Calculator. For each distribution it contains six sub-laboratories, which are called 'Properties', 'Probability of an event', 'Prediction', 'Measurement', 'Exclusion', and 'Classification'. In the following these sub-laboratories are introduced one by one for the Binomial Distribution.

## 10.5.3 The Sub-Laboratories

### The Sub-Laboratory 'Properties 2D'

In this Sub-Laboratory the properties of a probability distribution given by the mass (density-) function, the distribution function, the survival function and the two quantile functions can be called after inputting the values of the distribution parameters.

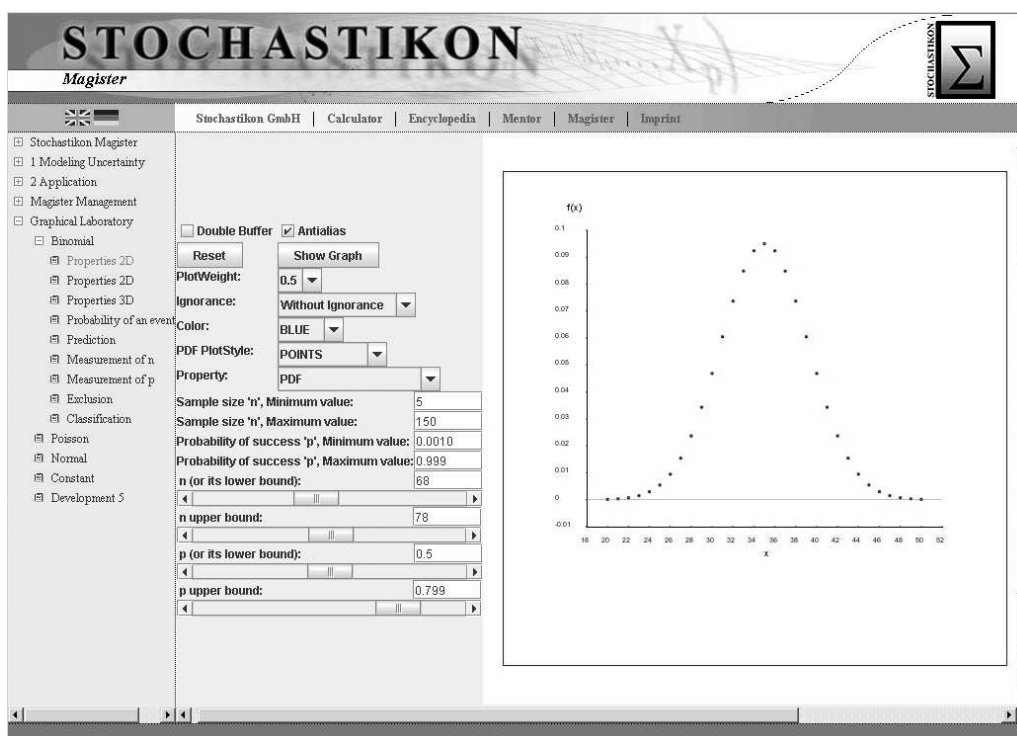


Figure 10.121: The interface of the Sub-Laboratory Properties 2D showing the mass function of a binomial distribution.

The functions are displayed by a simple 2D-graph for the case of no ignorance and the corresponding mean probability function in case of ignorance.

The interfaces of the input panels for the different sub-laboratories are very much alike. The differences refer to the different tasks to be resolved. The interface of the Sub-Laboratory Properties 2D offers the following functions:

- **Antialiasing:** Antialiasing graphics.
- **PlotWeight:** The size or width of plotting points, lines and curves can be selected.
- **Ignorance:** Indication whether there is ignorance about the values of the distribution parameters or not.
- **Color:** There are six alternative colors for the graphical objects (Blue, Red, Black, Green, Pink, Gray).
- **PDF PlotStyle:** Selection of different plotting styles for the curves (Points, Lines.Points, Lines, Dots).
- **Property:** The properties relate to a representation of the probability distribution, i. e., PDF = probability mass (density-) function, CDF = distribution function, Survival Function, Lower Quantile Function, and Upper Quantile Function.
- **$n$  (or its lower bound):** Value of the sample size  $n$ , or the lower bound of  $n$  in case of ignorance about  $n$ .
- **$n$  upper bound:** Upper bound of the sample size  $n$  in case of ignorance about  $n$ .
- **$p$  (or its lower bound):** Value of the success probability  $p$ , or the lower bound of  $p$  in case of ignorance about  $p$ .
- **$p$  upper bound:** Upper bound of the success probability  $p$  in case of ignorance about  $p$ .
- **Show Graph:** This shows the requested graph after having given the necessary information.

The minimum and the maximum values of the sample size and the success probability, respectively, supported by the Graphics Laboratory are given in the lines in the center of the input panel. Users can also change these parameters according to their conditions. As mentioned earlier the input panels are more or less the same for each of the sub-laboratories.

### The Sub-Laboratory ‘Properties 2D2’

The second Sub-Laboratory serves the same task as the first one. However, it shows the five curves, which represent the probability distribution simultaneously. Thus, the different effects of a change in the distribution parameter on the five curves can be simultaneously followed.

The control panel for this laboratory is the same as for the first one, except for the function ‘Property’, which now determines the curve to be displayed in the first place, i. e., which fixes the order of the displayed curves.

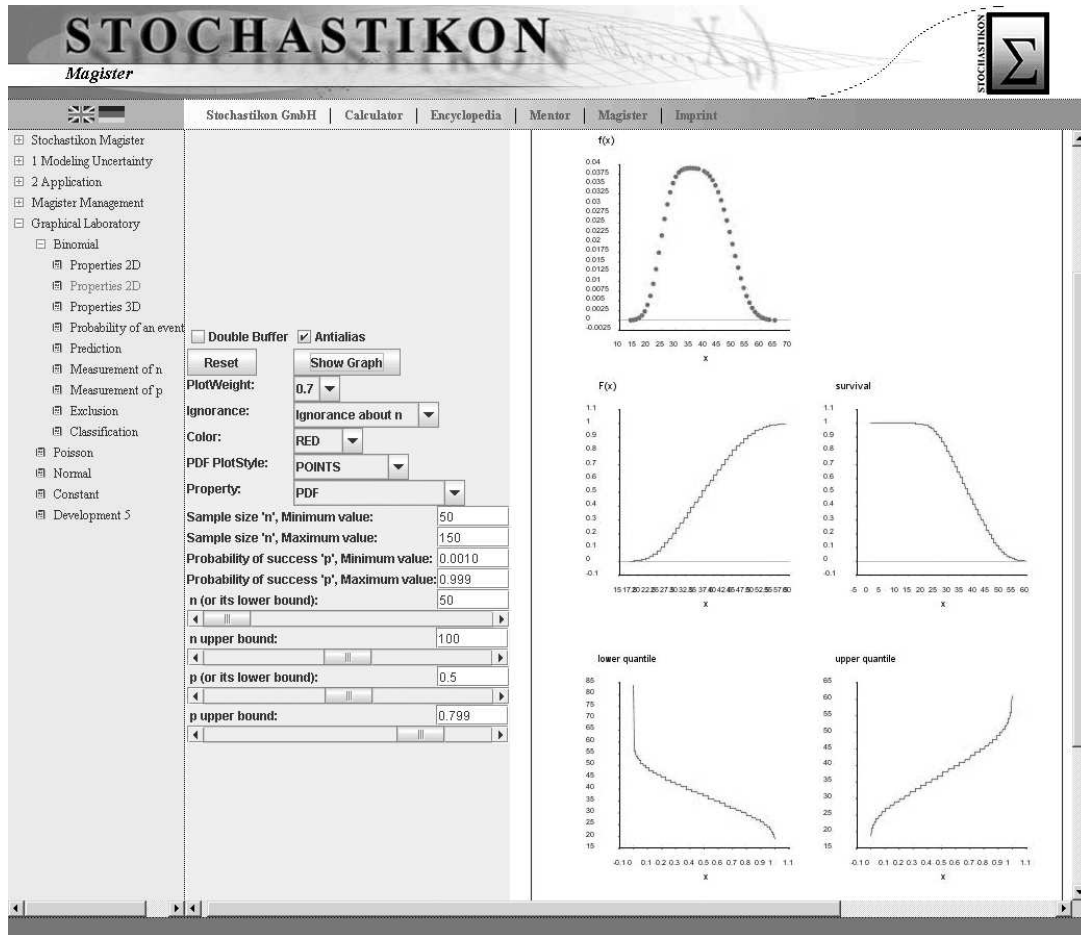


Figure 10.122: The interface of the Sub-Laboratory Properties 2D2 showing two-dimensional graphics of all the five curves representing the probability distribution with the probability mass function in the first place.

### The Sub-Laboratory ‘Properties 3D’

The control panel for this laboratory is similar as for the first two sub-laboratories. The difference refers to the type of illustration, which is here a 3D-graph.

Figures 10.123 to Figure 10.126 illustrate the interface of the Properties 3D-laboratory, they show the five different probability curves by means of 3-dimensional graphics. The size of these 3-dimensional backgrounds is determined by the minimum and the maximum values of the sample size  $n$  and/ or the success probability  $p$ . The case of no ignorance and the case of ignorance are supported.

Figure 10.123 shows a three dimensional graph representing the probability mass functions of a set of values of the input parameters in the case of no ignorance, where the value  $n$  of the sample size may vary in the graphics.

Figure 10.124 shows a three dimensional graph representing the probability mass functions of a set of values of the input parameters in the case of no ignorance, where the value  $p$  of the success probability may vary in the graphics.



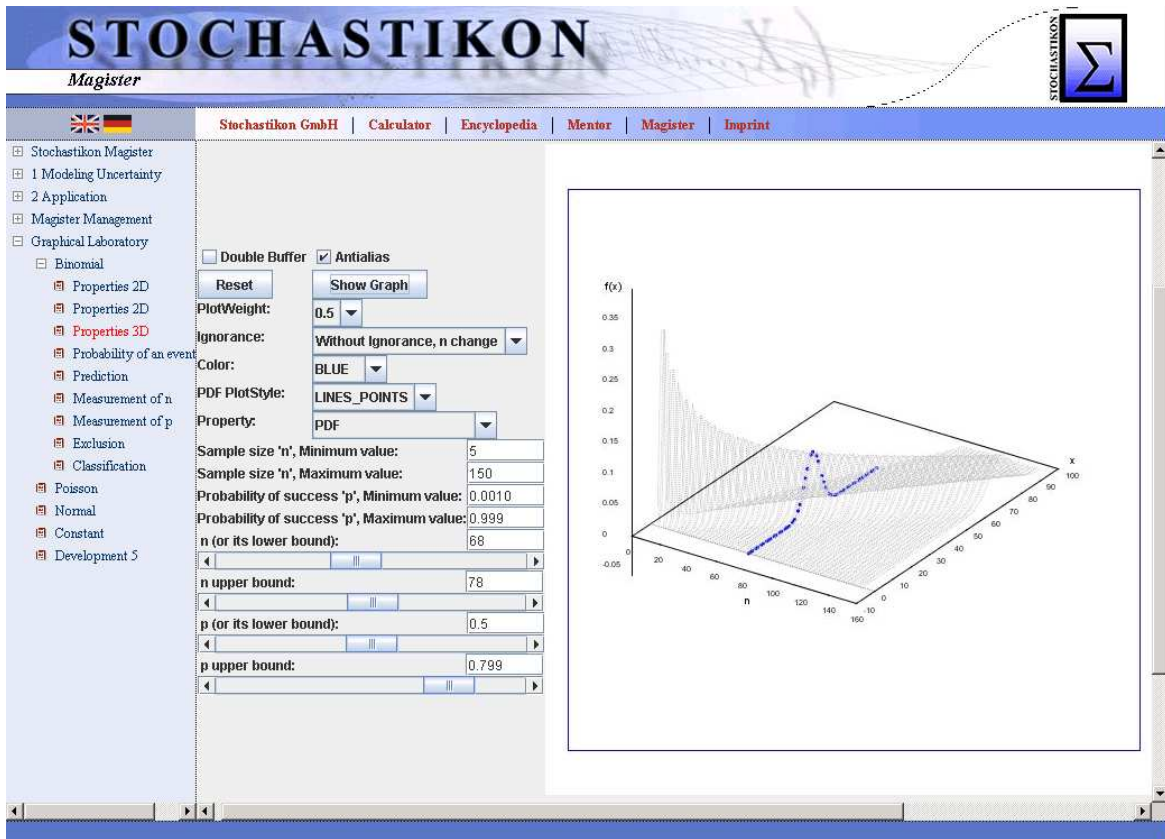


Figure 10.123: A graph shown in the ‘Properties 3D’ Sub-Laboratory (a).

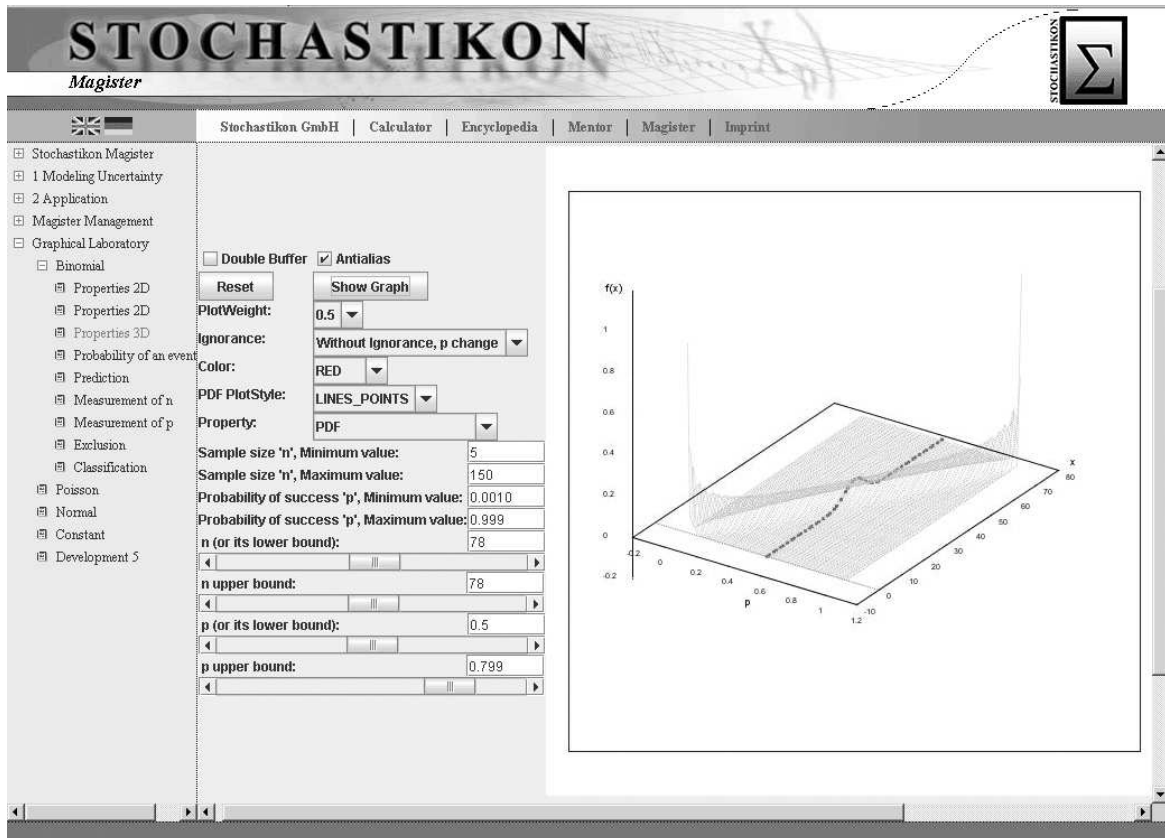


Figure 10.124: A graph shown in the ‘Properties 3D’ Sub-Laboratory (b).

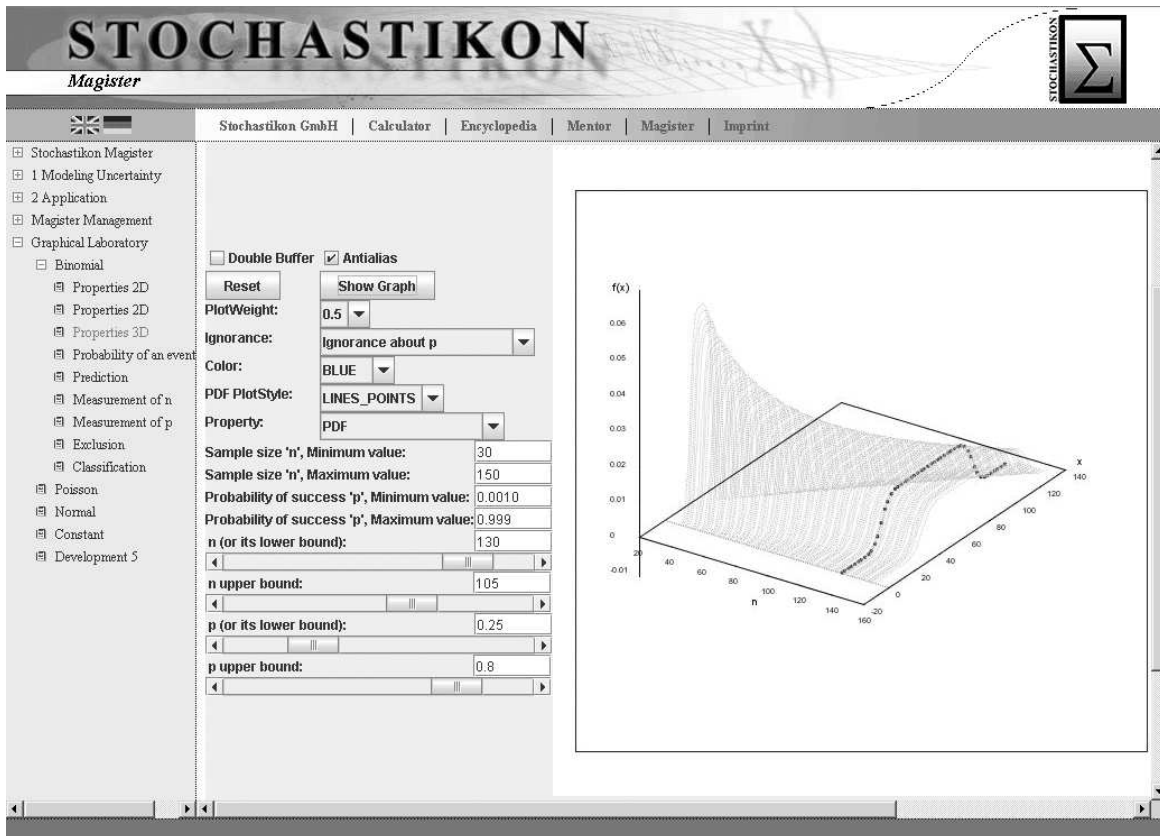


Figure 10.125: A graph shown in the 'Properties 3D' Sub-Laboratory (c).

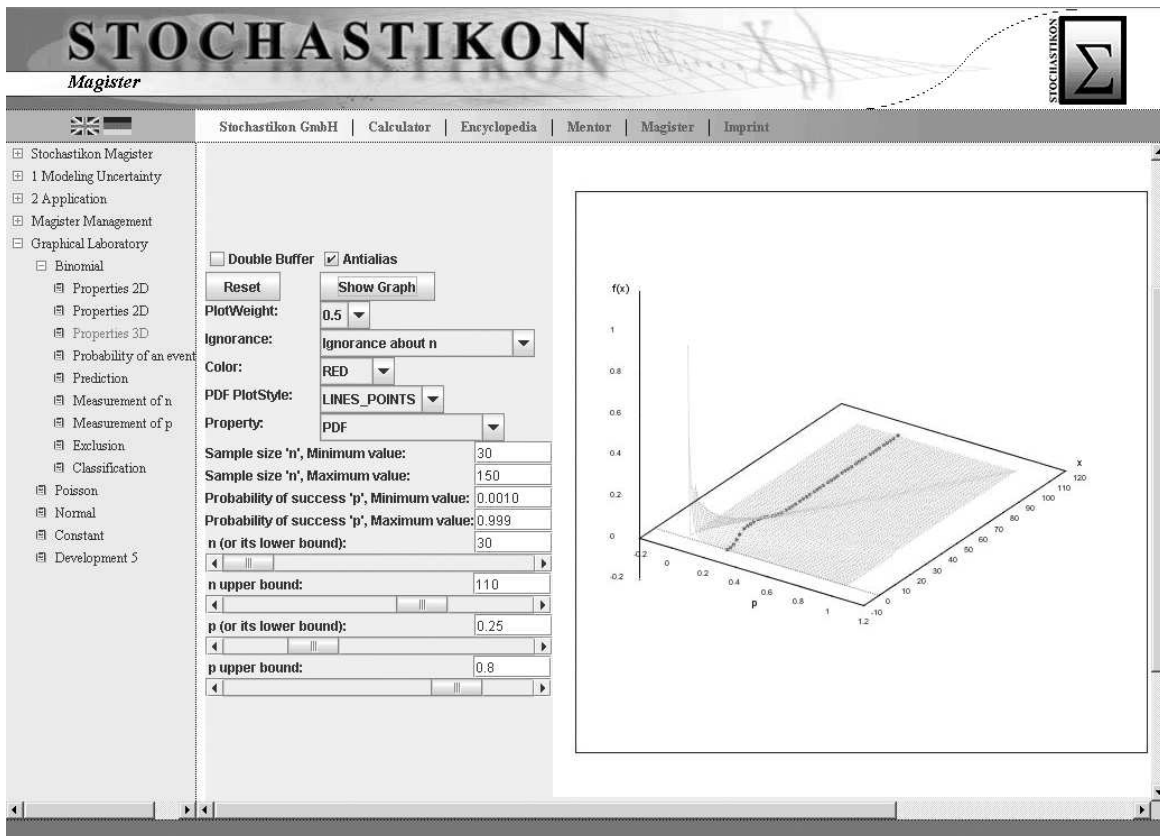


Figure 10.126: A graph shown in the 'Properties 3D' Sub-Laboratory (d).

Figure 10.125 shows a three dimensional graph representing the probability mass functions of a set of values of the input parameters in the case of ignorance with respect to the value  $p$  of the success probability, where the value  $n$  of the sample size may vary in the graphics.

Figure 10.126 shows a three dimensional graph representing the probability mass functions of a set of values of the input parameters in the case of ignorance with respect to the value  $n$  of the sample size, where the value  $p$  of the success probability may vary in the graphics.

### The Sub-Laboratory ‘Probability of an Event’

The Sub-Laboratory ‘Probability of an Event’ shall impart knowledge about the relations between the values of the success probability and the sample size, respectively, and the probability of given events. One should learn that only relatively few events have a noteworthy probability value while many other have almost no chance to occur.

The input panel of this Sub-Laboratory is similar to those of the previous panels with the exception of the last two queries, which refer to the lower and upper bounds of the event in question. (see Figure 10.127)

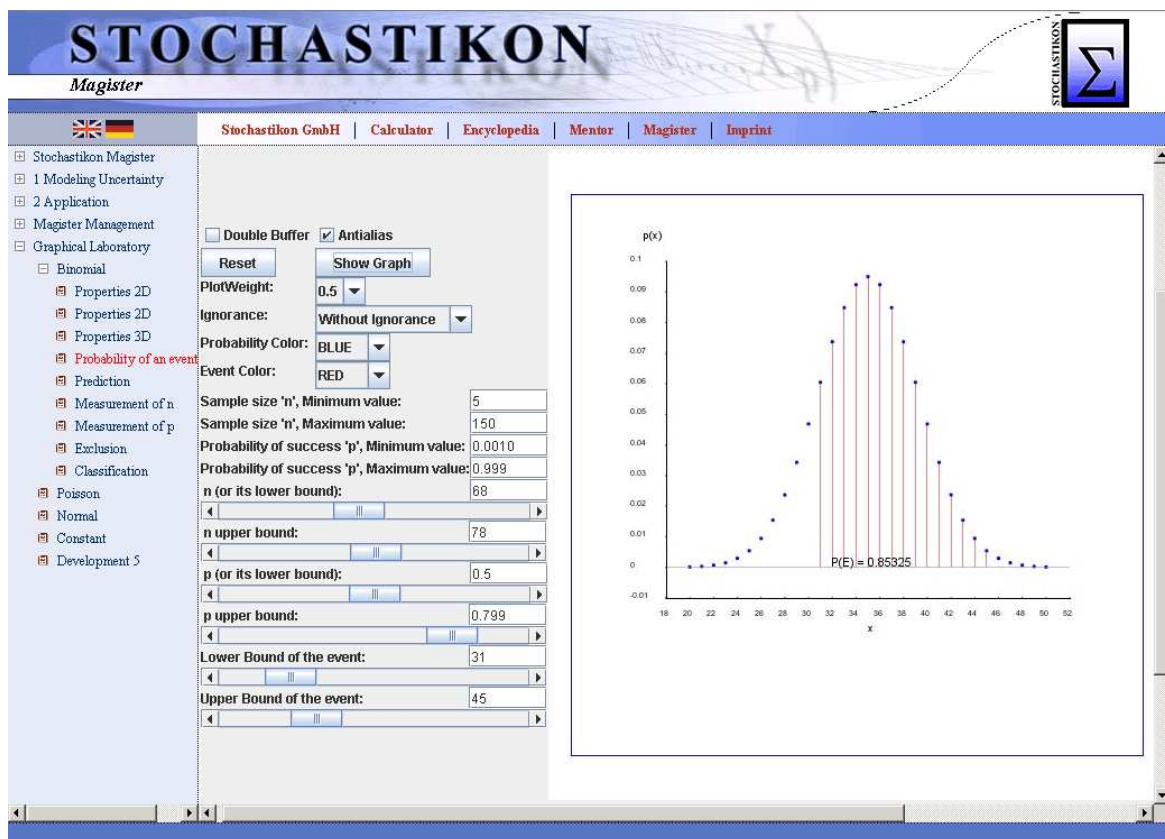


Figure 10.127: The interface of the Sub-Laboratory ‘Probability of an Event’ in the case of no ignorance showing the probability mass function with the event given in red color and the corresponding superimposed probability.

### The Sub-Laboratory ‘Prediction’

The input panel of this Sub-Laboratory is again similar to those of the previous panels with the exceptions that it is possible to specify the type of prediction (minimum, lower bound, upper bound) and, of course, the reliability level of the prediction procedure.

The graphics in this Sub-Laboratory shall illustrate how a prediction depends on the required reliability level on the one hand and on the state of ignorance on the other. If there is no ignorance, then the figures show the probability mass function together with the predicted event in red. If there is ignorance, then the graphics show the set determined by the ignorance space and the predicted event again in red.

This Sub-Laboratory offers besides the already known functions in the case of ignorance another one. It is possible to select different picture types. The one refers to the graphical representation of the predicted event, the other refers to the probability depending on the unknown value of the parameter with ignorance.

Figure 10.128 shows a minimum  $\beta$ -prediction in the case of no ignorance with the predicted event given by the red bars on the abscissa.

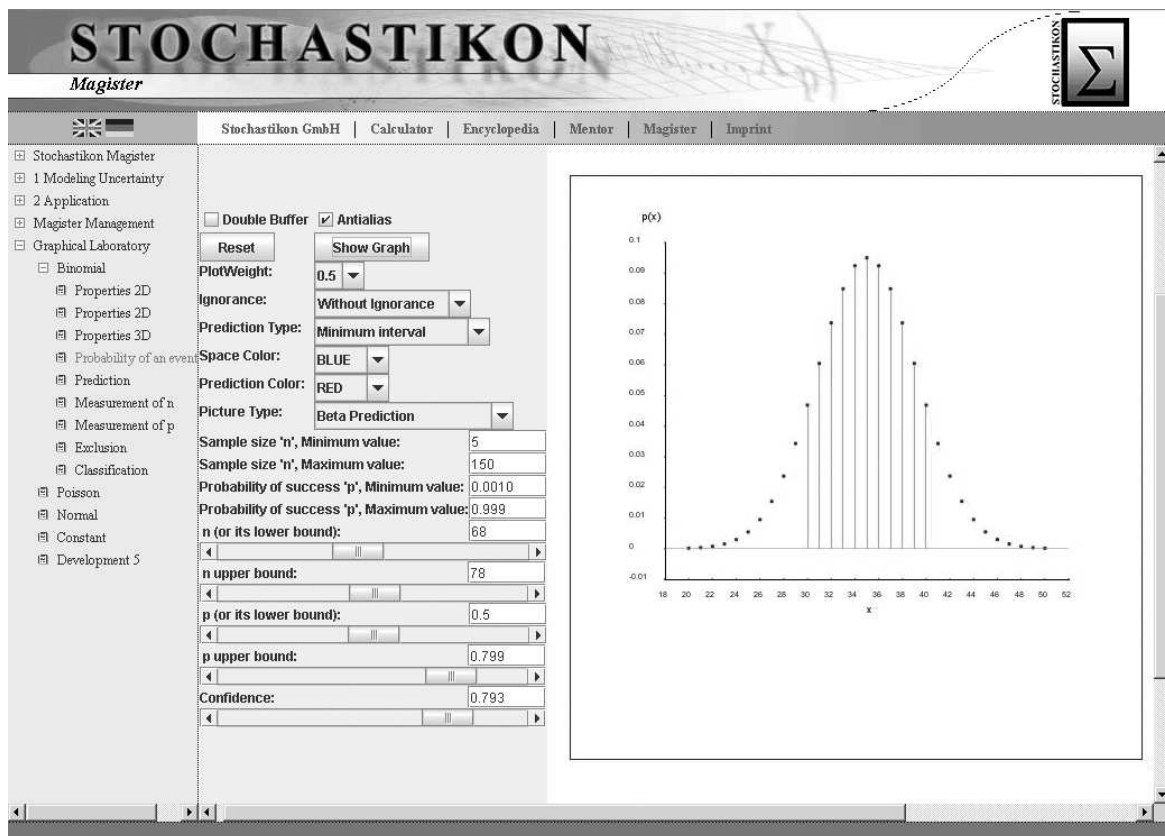


Figure 10.128: A  $\beta$ -prediction shown in the 'Prediction' Sub-Laboratory (a).

Figure 10.129 shows a maximum lower bound  $\beta$ -prediction (in red) with ignorance about  $n$ .

Figure 10.130 shows a minimum upper bound  $\beta$ -prediction (in red) with ignorance about  $p$ .

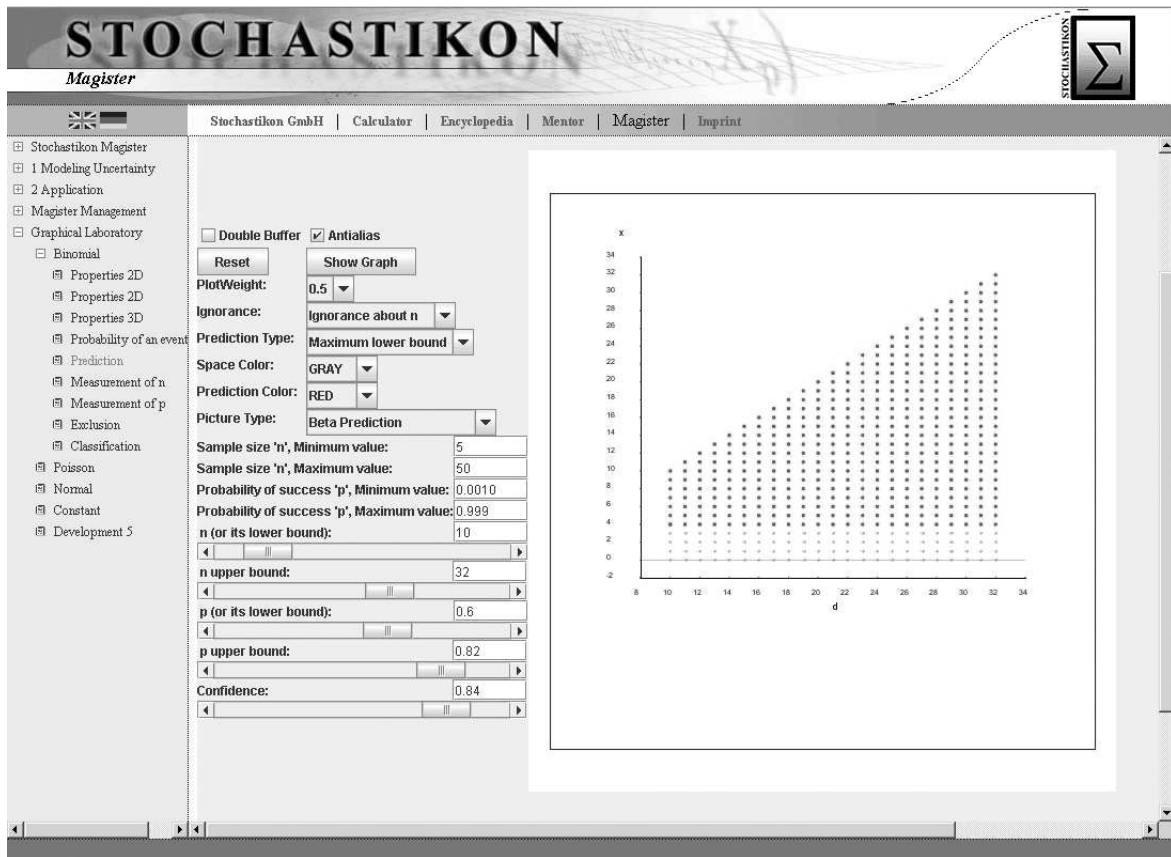


Figure 10.129: A  $\beta$ -prediction shown in the 'Prediction' Sub-Laboratory (b).

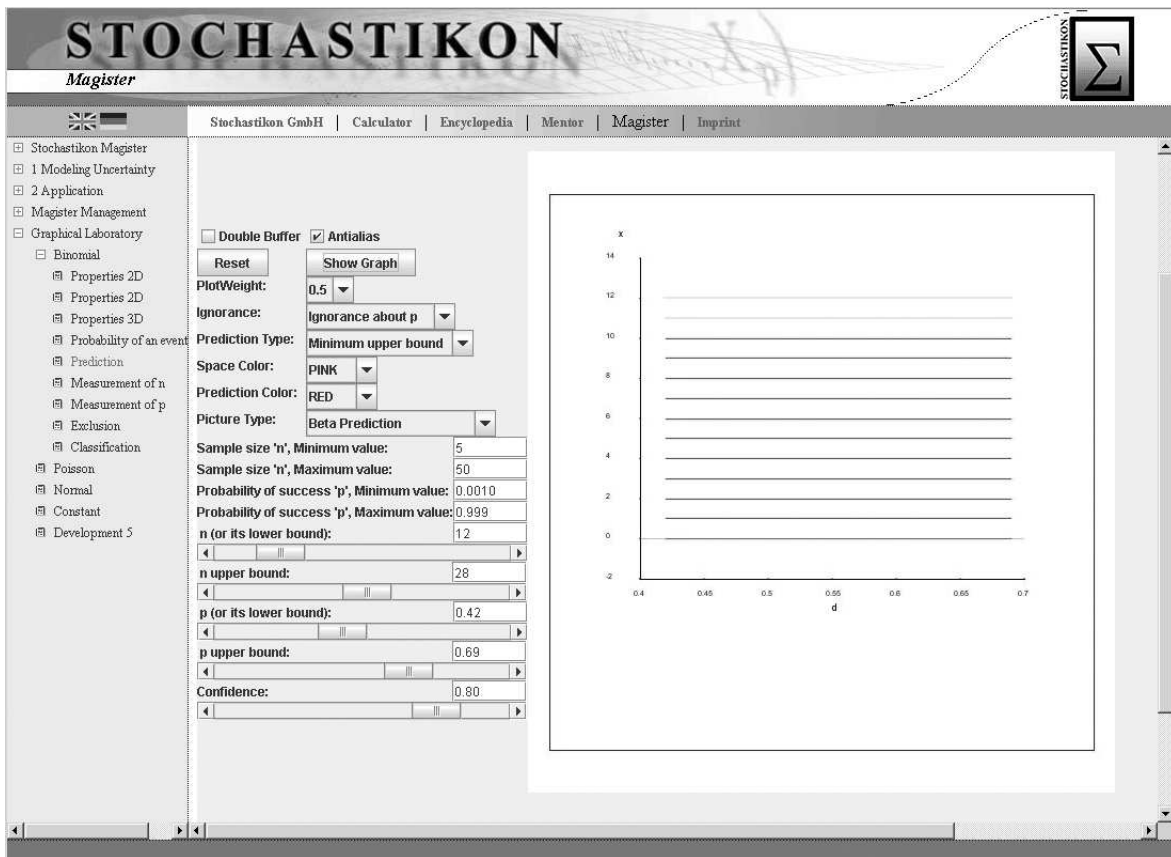


Figure 10.130: A  $\beta$ -prediction shown in the 'Prediction' Sub-Laboratory (c).

Figure 10.131 shows a minimum  $\beta$ -prediction (in red, upper figure) with ignorance about  $n$  and the corresponding probability curve (lower figure) giving the probability of the predicted event as function of the unknown value of  $p$ .

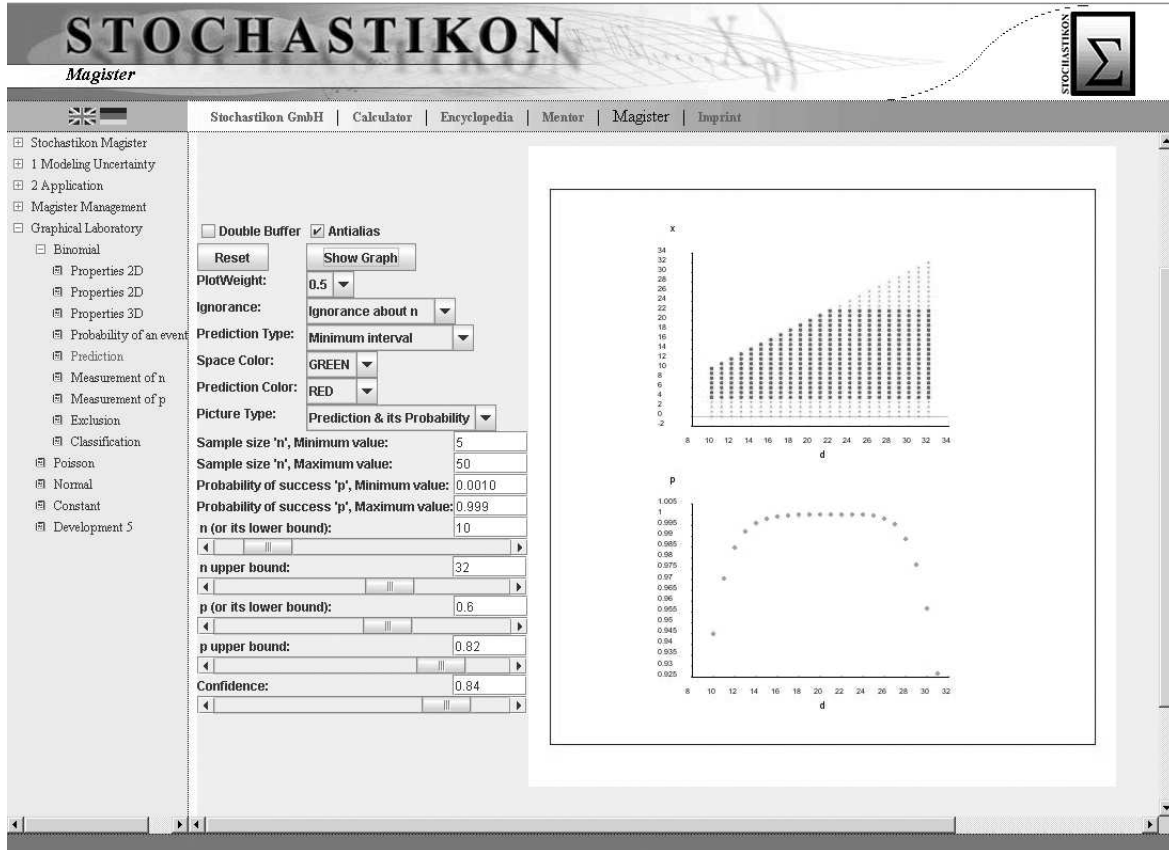


Figure 10.131: A  $\beta$ -prediction shown in the 'Prediction' Sub-Laboratory (d).

### The Sub-Laboratory 'Measurement of $n$ '

The input panel is similarly arranged as in the previous sub-laboratories with some exception, which refer to the different aim and, especially, to the fact that a measurement experiment is performed and the result, called realization, must be known.

Similar as in the case of the Sub-Laboratory Prediction, there is a difference between the graphics in case that there is no ignorance with respect to  $p$  or if there is ignorance.

Figure 10.132 shows a minimum  $\beta$ -measurement for the value  $n$  of the sample size without ignorance about the value  $p$  of the success probability.

Figure 10.133 shows a minimum  $\beta$ -measurement for the value  $n$  of the sample size with ignorance about the value  $p$  of the success probability.

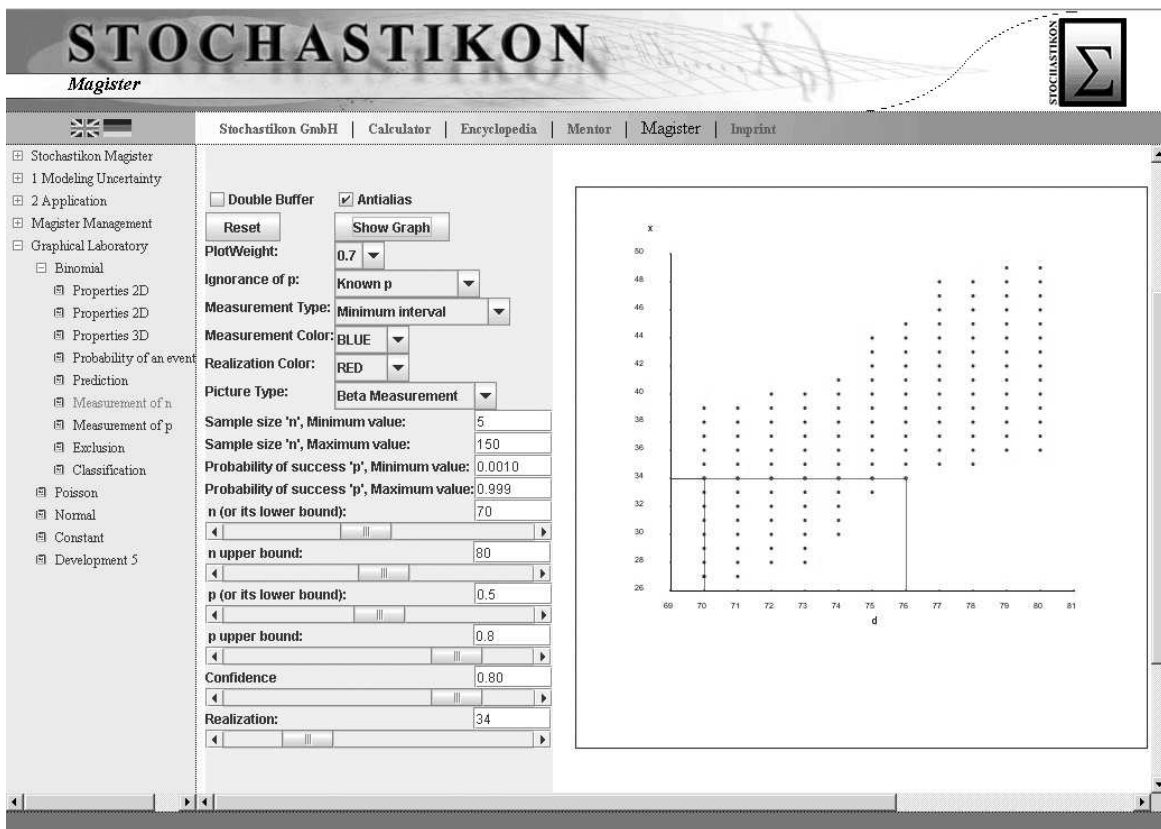


Figure 10.132: A minimum  $\beta$ -measurement for the value  $n$  shown in the ‘Measurement of  $n$ ’ Sub-Laboratory (a).

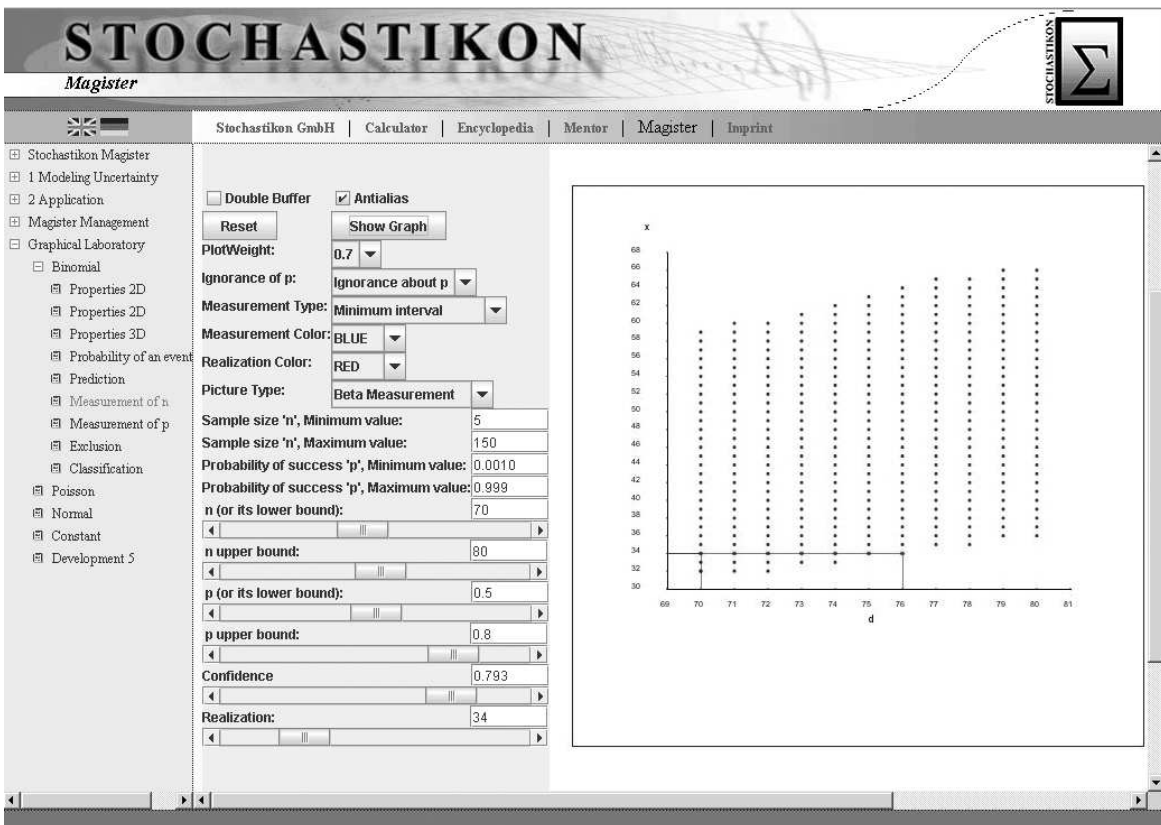


Figure 10.133: A minimum  $\beta$ -measurement for the value  $n$  shown in the ‘Measurement of  $n$ ’ Sub-Laboratory (b).

## The Sub-Laboratory ‘Measurement of $p$ ’

The input panel of this Sub-Laboratory is analogously arranged as the previous one. The only difference consists of the exchange of role of the two deterministic variables. In this Sub-Laboratory the value  $p$  of the success probability is measured, while the value  $n$  of the sample size may be known or not.

Figure 10.134 shows a minimum  $\beta$ -measurement for the value  $p$  of the success probability in the case of no ignorance with respect to the sample size  $n$ .

Figure 10.135 shows a minimum  $\beta$ -measurement for the value  $p$  of the success probability in case of ignorance with respect to the sample size  $n$ .

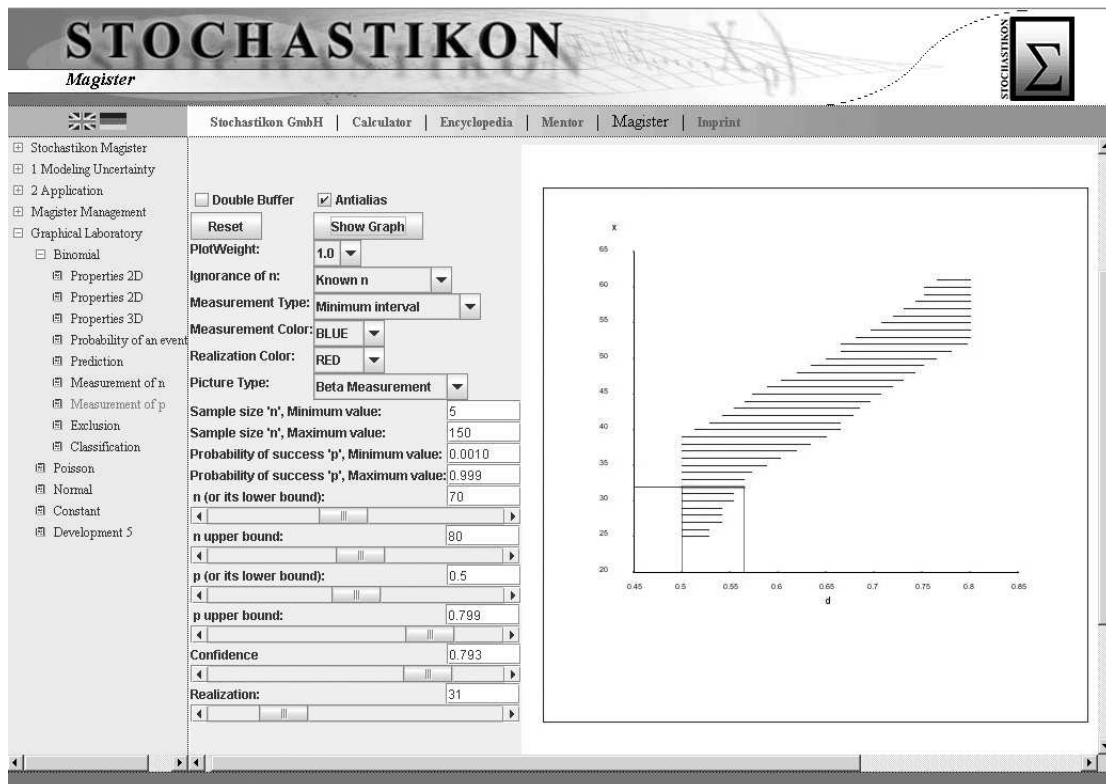


Figure 10.134: A minimum  $\beta$ -measurement for the value  $p$  of the success probability shown in the ‘Measurement of  $p$ ’ Sub-Laboratory (a).



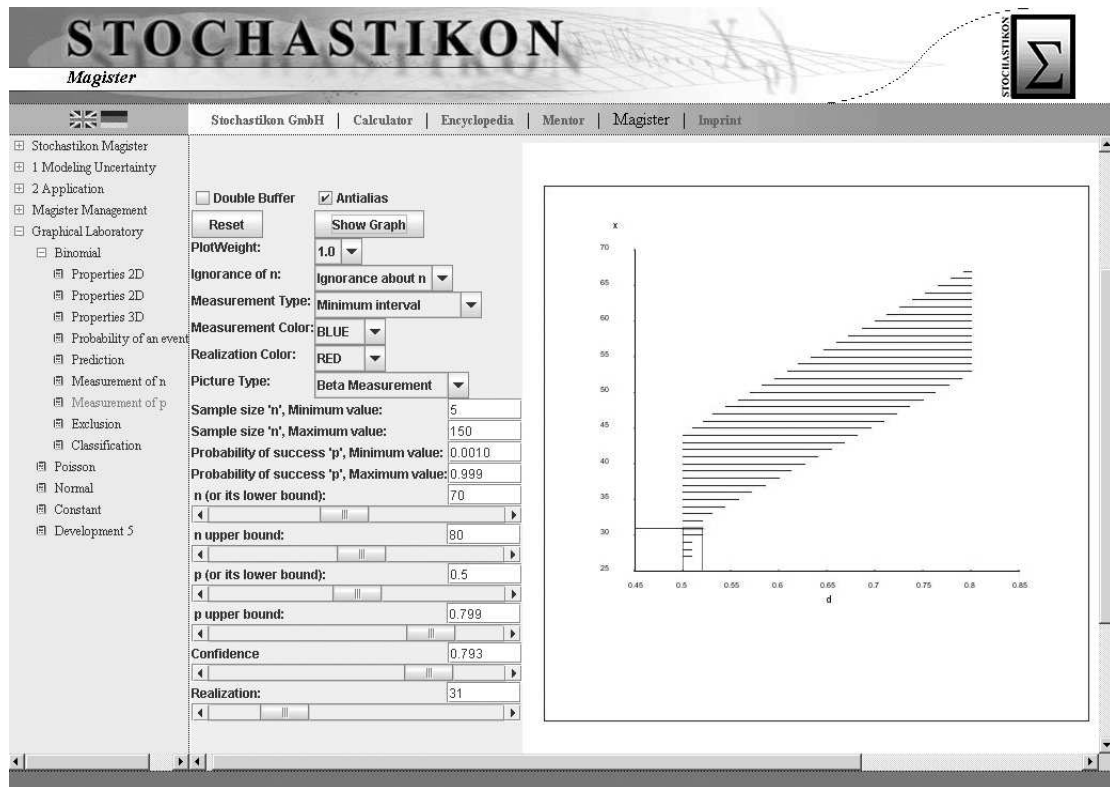


Figure 10.135: A minimum  $\beta$ -measurement for the value  $p$  of the success probability shown in the ‘Measurement of  $p$ ’ Sub-Laboratory (b).

### The Sub-Laboratory ‘Exclusion’

The input panel of the Sub-Laboratory Exclusion follows the same line as the previous ones. The differences refer to the possibility of inputting the sets of values with respect to the success probability or to the sample size to be excluded. Moreover, there are two ways of displaying the indifference set.

Figure 10.136 shows a  $\beta$ -exclusion of a set of values with respect to the sample size  $n$ , where the realization (red line) crosses the indifference set (blue points) and, thus, the exclusion is not possible.

Figure 10.137 shows a  $\beta$ -exclusion the same as in Figure 10.136 with the difference in displaying the indifference set, where the realization (red line) falls in the indifference set, which adopts the same color (with change) as the background uncertainty space, indicating that the hypothesis cannot be excluded.

Figure 10.138 shows a  $\beta$ -exclusion of a set of values with respect to the success probability  $p$ , where the hypothesis cannot be excluded.

Figure 10.139 shows a  $\beta$ -exclusion of a set of values with respect to the success probability  $p$  by using the ‘with change’ mode to display the result.

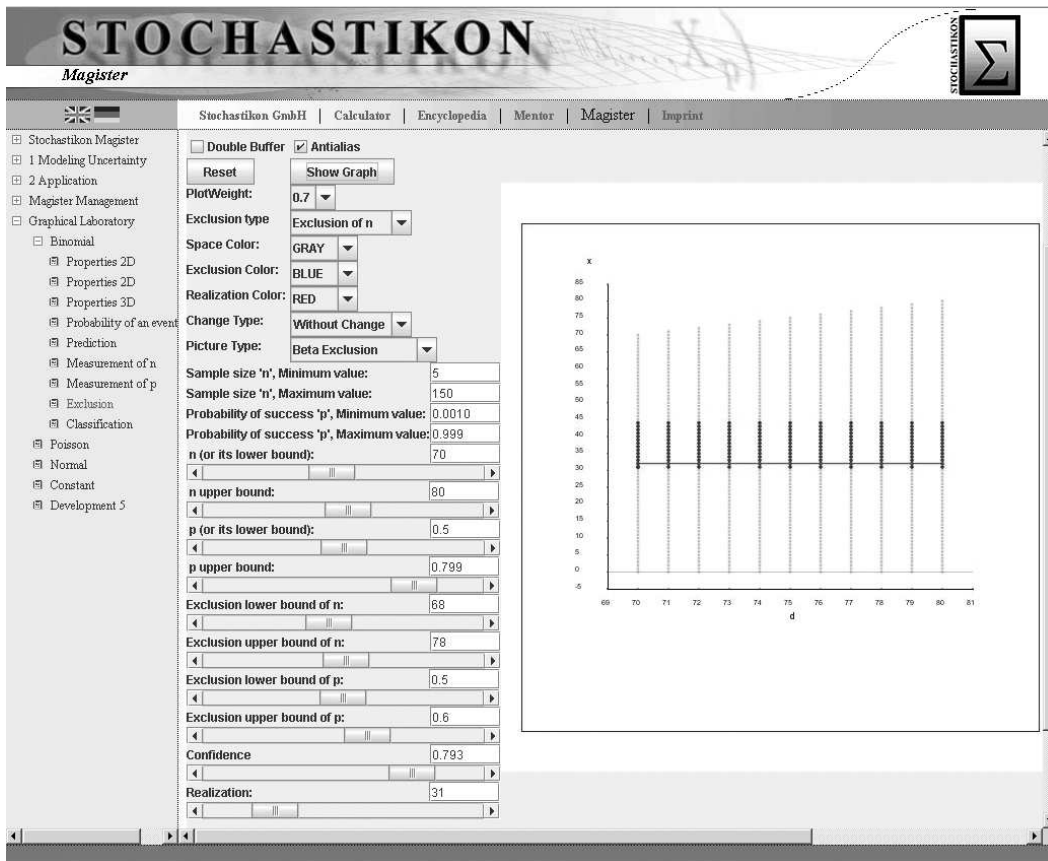


Figure 10.136: A  $\beta$ -exclusion shown in the 'Exclusion' Sub-Laboratory (a).

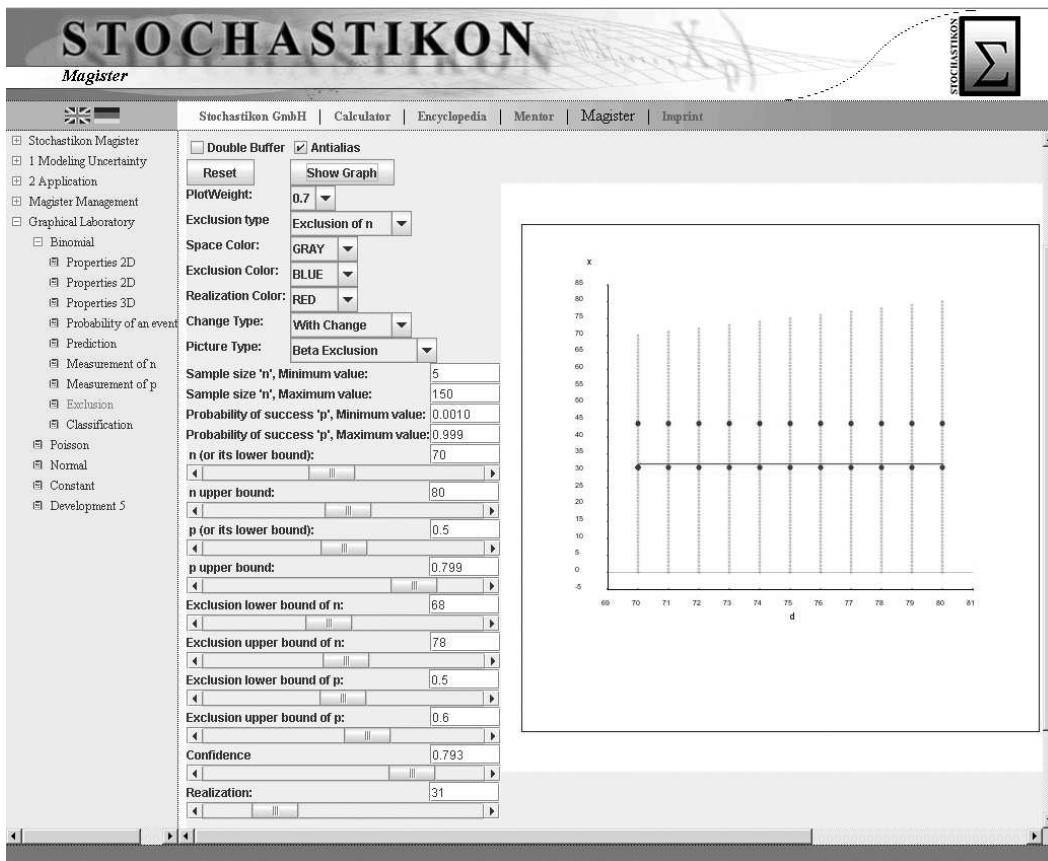


Figure 10.137: A  $\beta$ -exclusion shown in the 'Exclusion' Sub-Laboratory (b).

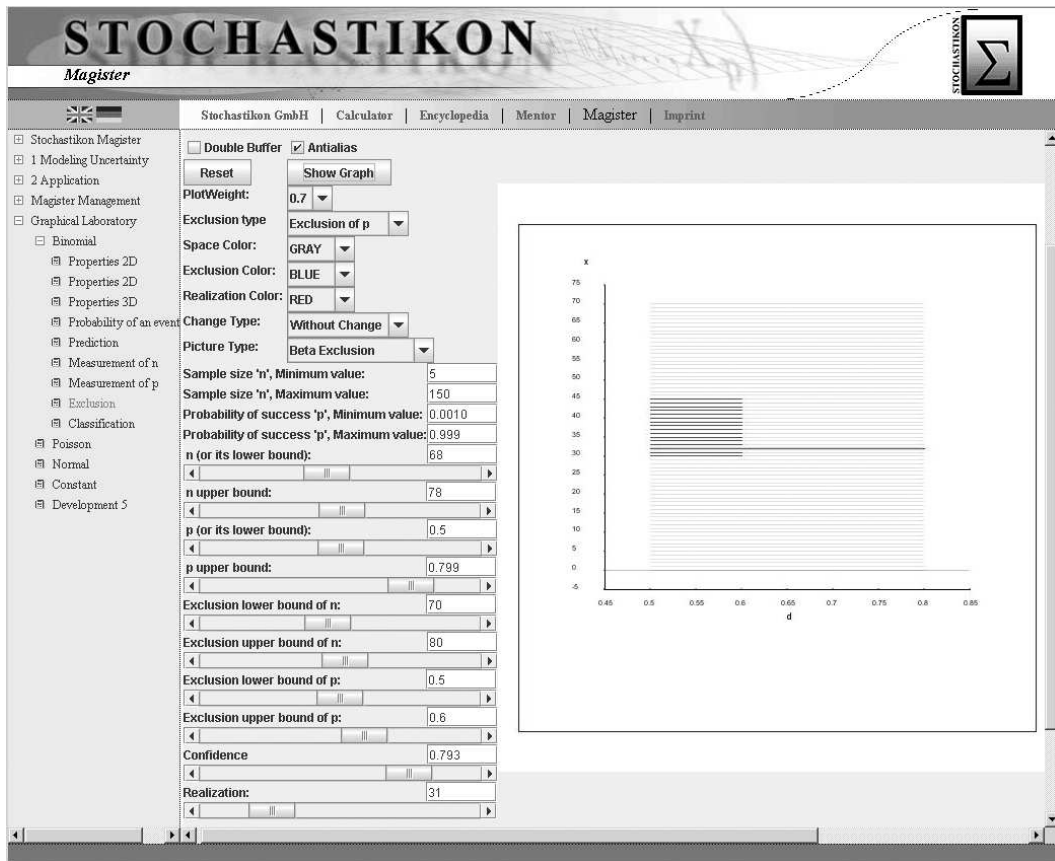


Figure 10.138: A  $\beta$ -exclusion shown in the 'Exclusion' Sub-Laboratory (c).

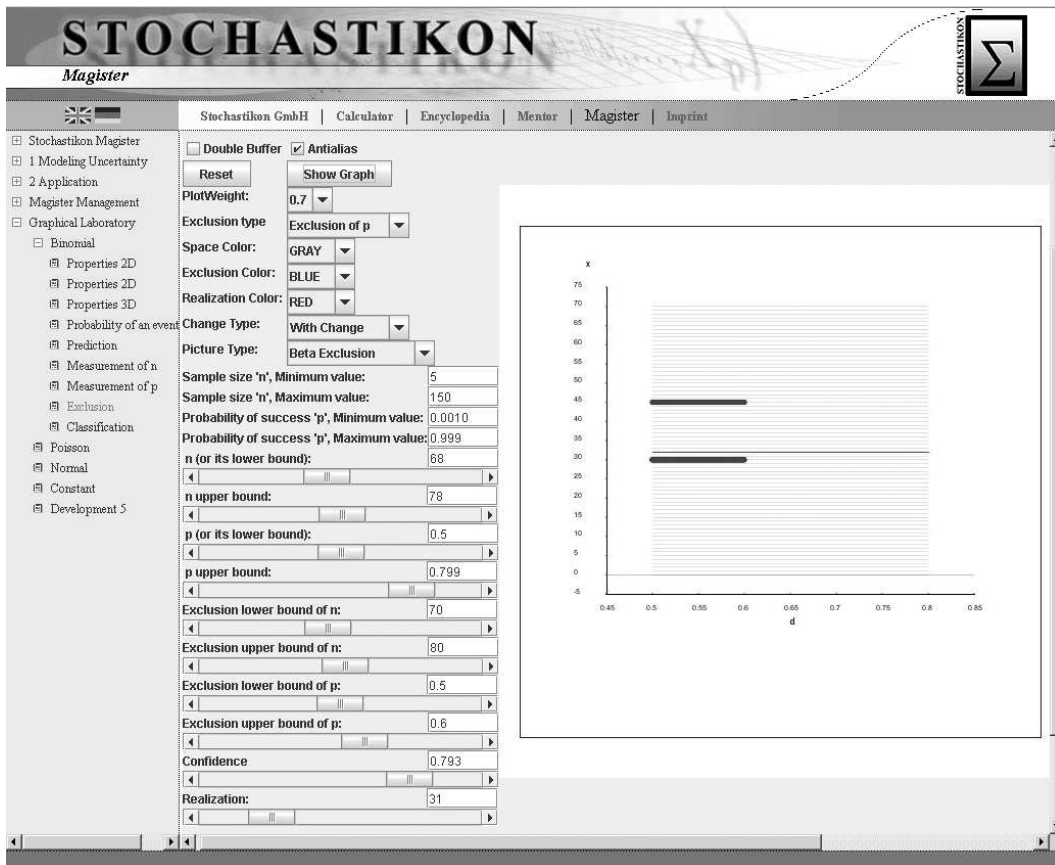


Figure 10.139: A  $\beta$ -exclusion shown in the 'Exclusion' Sub-Laboratory (d).

## The Sub-Laboratory ‘Classification’

The input panel provides the possibilities of classification for two or three alternatives with respect to the sample size  $n$  and the success probability  $p$ . Otherwise, there are no differences to the previous panels.

Figure 10.140 shows a  $(\beta_1, \beta_2)$ -classification of two alternatives with respect to the sample size  $n$ . The two blue sets show the two acceptance areas, while the small gap between them represents the indifference set. The red line indicates the realization.

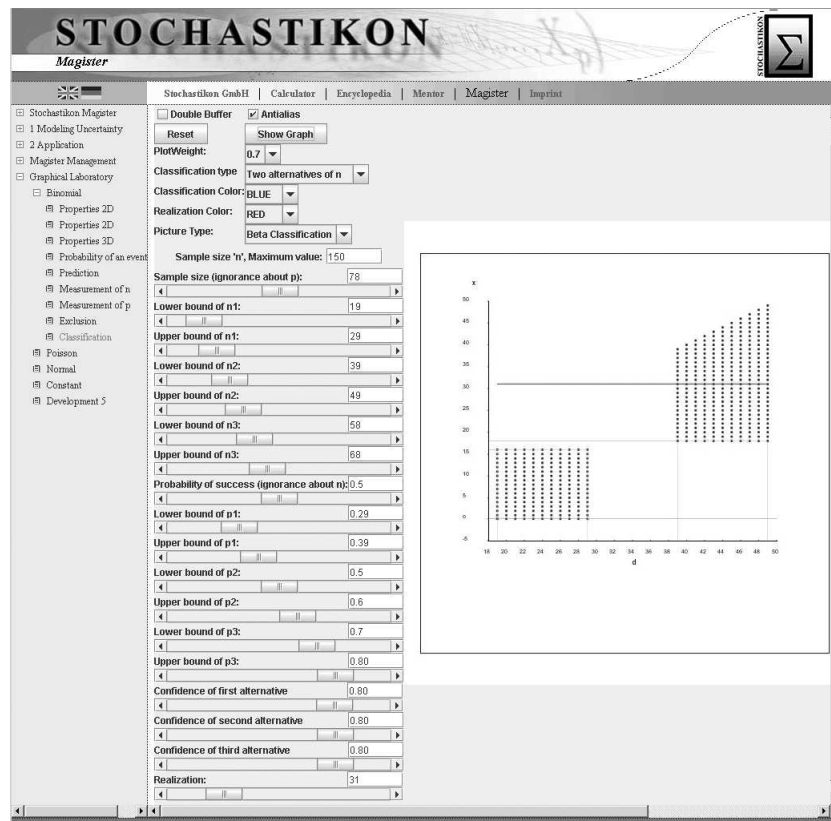


Figure 10.140: A  $(\beta_1, \beta_2)$ -classification of two alternatives with respect to the sample size  $n$ .

Figure 10.141 shows a  $(\beta_1, \beta_2, \beta_3)$ -classification of three alternatives with respect to the sample size  $n$ . The three blue sets show the acceptance areas, while the small gaps between them represent the indifference sets. The red line indicates the realization.

Figure 10.142 shows a  $(\beta_1, \beta_2)$ -classification of two alternatives with respect to the success probability  $p$ . The two blue sets show the two acceptance areas, while the small gap between them represents the indifference set. The red line stands for the realization.

Figure 10.143 shows a  $(\beta_1, \beta_2, \beta_3)$ -classification of three alternatives with respect to the success probability  $p$ . The three blue sets show the acceptance areas, while the small gaps between them represent the indifference sets. The red line stands for the realization.

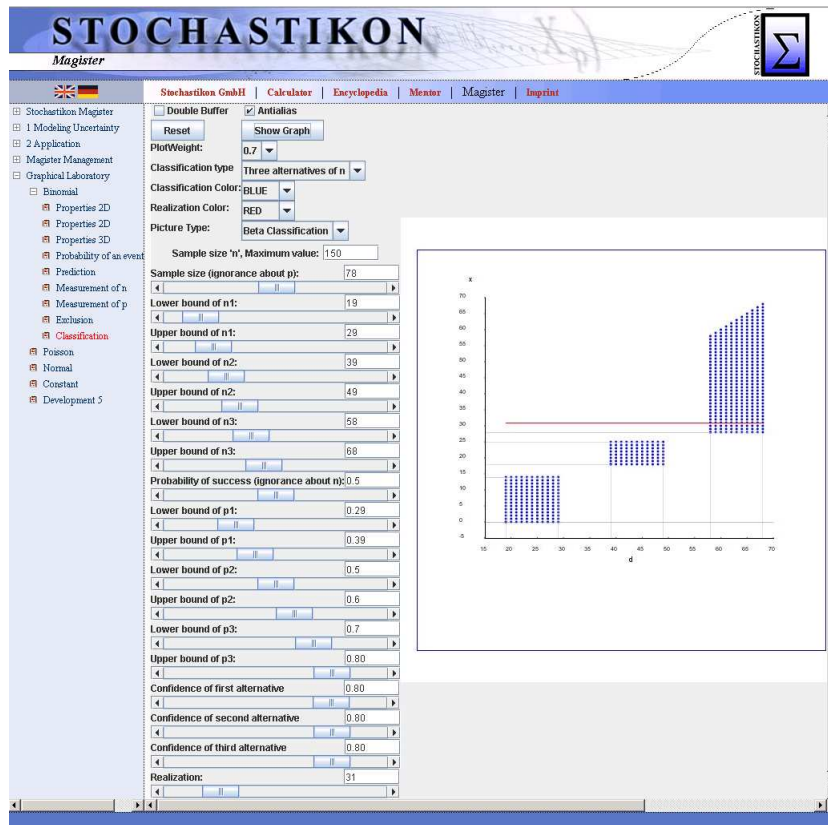


Figure 10.141: A  $(\beta_1, \beta_2, \beta_3)$ -classification of three alternatives with respect to the sample size  $n$ .

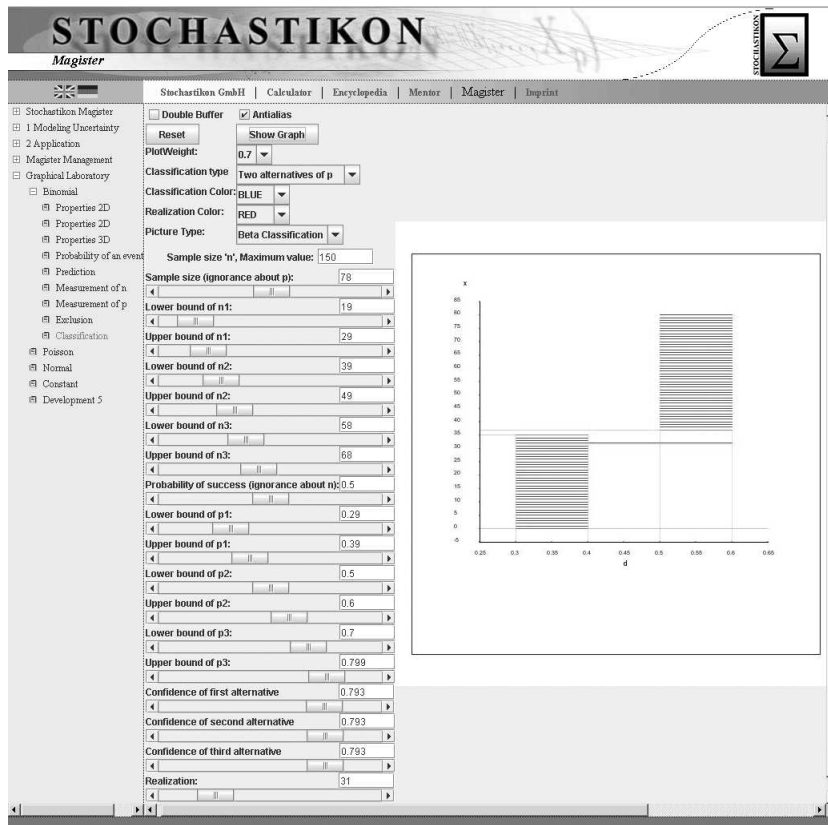


Figure 10.142: A  $(\beta_1, \beta_2)$ -classification of two alternatives with respect to the success probability  $p$ .

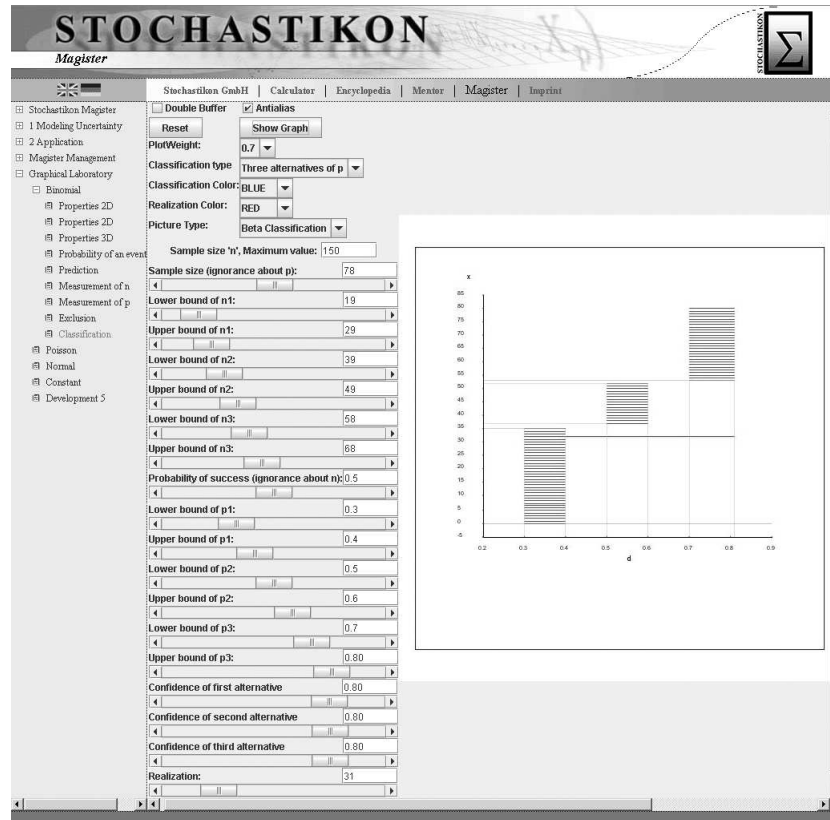


Figure 10.143: A  $(\beta_1, \beta_2, \beta_3)$ -classification of three alternatives with respect to the success probability  $p$ .

## 10.6 Summarized Results

Subsequently, the results of the implementation of *Stochastikon Magister* with respect to the listed requirements (see Section 8.4) are briefly outlined.

### 10.6.1 Bernoulli Stochastics

The first above listed set of requirements refer to the content of the curriculum in Bernoulli Stochastics, which should be based on the two seminal papers ([49],[50]), in which Bernoulli Stochastics is defined.

- Two courses have been developed as far as necessary for answering the relevant questions with respect to the implementation of *Stochastikon Magister*. One course covers the contents of [49] and the second course covers the contents [50]. Some teaching contents of these two Magister courses are presented in the second part of ‘Appendix’ (see Part V) .
- Uncertainty constitutes the central issue of Bernoulli Stochastics. Therefore, the first module introduces in detail the necessary basic concepts from *Evolution over Science to Knowledge*, which are related to uncertainty. The interrelations of these concepts are investigated. This module constitutes a first attempt to show the necessity of establishing Bernoulli Stochastics by discussing and introducing Bernoulli Stochastics in the context of general science. It acts on the suggestions given in [49] and [50], but represents them in a wider context.

- The second module is dedicated to quantification, i. e., to mathematizing, which besides the basic concepts is the second cornerstone of Bernoulli Stochastics and shows clearly the difference to traditional probability theory. As with the first module, a new stepwise approach is developed for quantifying or mathematizing a given situation with respect to uncertainty. Again, the ideas formulated in [49] and [50] are taken up and used as the basis for developing the content of the module.
- The topic *Bernoulli Space* of the last module of the first course refers to the stochastic model proposed in [49]. For the ways methods for representing ignorance and selecting probability distributions are presented and explained in detail on the level of a textbook.
- The second course contains the stochastic procedures necessary for exploiting a stochastic model given by a *Bernoulli Space* and for improving a stochastic model. Again the second course constitutes the first attempt of presenting stochastic procedures in a way that the required mathematical background does not exceed the one stated in the relevant requirement.
- The two courses are designed in a way, which is familiar to students used to classroom teaching. Aims, contents and benefits of each course and each module are introduced followed by learning units, consisting of separate issues for target, content, examples and exercises.
- The two courses, which are described in more detail in the following Part I of this thesis, constitute the first textbook in Bernoulli Stochastics. They are developed for graduates of secondary education and undergraduates of universities or colleges.
- Besides the didactic preparation and representation of the contents of the papers [49] and [50], the second course contains a new presentation and interpretation of known procedures and, moreover, two entirely new stochastic procedures:
  1. In Learning Unit 2.3.9 the well-known confidence interval for the value of a first moment based on the  $t$ -distribution is derived and presented in a new way as a stochastic measurement procedure.
  2. In Learning Unit 2.3.10 the well-known confidence interval for the value of the variance based on the  $\chi^2$ -distribution is derived and presented in a new way as a stochastic measurement procedure.
  3. In Learning Unit 2.3.11 a new measurement procedure is presented for the smallest value a random variable may adopt.
  4. In Learning Unit 2.3.12 a new measurement procedure is presented for the largest value a random variable may adopt.

### 10.6.2 Stochastikon Magister

The ninth requirement is one of the main reasons that a free or commercial shell could not be used for developing *Stochastikon Magister*. Instead we designed and developed *Stochastikon Magister* independently of any shell or development toolkit according to those issues, which seemed to be mandatory in order to have the Bernoulli Stochastics established:

- Personal supervision is one of the characteristic features of classroom teaching. Therefore, we not only implemented automatic machine responses to judge and guide the learner's performances, but also provide the possibility of personal service. Moreover, we developed

a complete learning support, guidance and management system, which directs, helps and documents a learner from the first login until the learner leaves the system. In all, a learner will find all those issues used in traditional classroom teaching.

- As required, we developed *Stochastikon Magister* as a multilingual system, which can incorporate courses in arbitrary many languages.
- Education in Bernoulli Stochastics is a new mathematically based field of teaching. For the pre-assigned content we developed an authoring system based on LaTeX and HTML templates, which allows an easy generation of new courses, modification or reuse of already existing learning units, and the incorporation of other formats. The LaTeX-files are automatically exported in PDF format used as input for *Stochastikon Magister*.
- The eighth requirement turned out to be the most challenging one. Stochastic thinking can be compared with thinking in sets, while deterministic thinking represents thinking in points. This comparison showed us how to meet the challenge. We designed and developed a virtual laboratory - called Magister Graphical Laboratory, which makes it possible that the stochastic models and procedures can be visualized by (online interactive dynamic) graphics, i. e., by sets, thus supporting the new way of thinking.
- Many e-learning products meant for mathematics contain dynamic interactive graphics. However, most online dynamic interactive graphics are ‘single graphics’, which bind the calculating part and the graphing part in one program such as a Java applet. This is possible, because these graphics are comparatively simple. However, these methods cannot be used to display the results of the complex calculations necessary for stochastic procedures, as performed by a huge calculating system - *Stochastikon Calculator* and a complex graphical system - *Stochastikon Graphics*. As discussed in Section 2.4.3 ‘Statistical Software and Virtual Labs’ of Chapter 2 ‘The State of the Art of E-Learning in Statistics Education’, we did not find any system which would fit to our needs. Therefore, Magister Graphical Laboratory had to be self-designed and self-developed. So far we cannot find any similar graphical system that is superior to the Magister Graphical Lab.
- Magister Graphical Laboratory also illustrates how the most important requirement has been met by *Stochastikon Magister*, which by its design is able to communicate with any existing or future component of the master system *Stochastikon*.

### 10.6.3 Other Specific Issues

*Stochastikon Magister* is the first and so far unique e-learning programme for Bernoulli Stochastics featuring the following issues of a required virtual classroom:

- *Stochastikon Magister* is an e-learning platform, which contains a closed pre-settled information flow and a dynamic open information flow, where the fixed information are seamlessly imbedded into to the dynamic information flow during the learning processes. Closed information flows are more popular than opened ones, because development cost and complexity of a closed information flow are lower than for an opened one. Moreover, there is almost no operating cost for a closed e-learning environment in contrast to an opened one, which might be rather cost-intensive. As most of the e-learning products are supplements for classroom teaching, closed e-learning environments are often sufficient because necessary information beyond the fixed information flow can be handled in the real classroom. In the case of Magister:



1. The learning contents *Target, Content, Example, Exercise* (including the Graphical Laboratory) and parts of the content *Literature* constitute the pre-settled or automatically generated information flow. These contents are either pre-assigned (as for example information stored in form of PDF-files) or pre-programmed (as in the case of the Graphical Laboratory).
  2. The information about *User, Question, Discussion, FAQ, Test*, parts of *Literature* and *Learning Progress* are open and dynamically changed along with the learners' learning processes.
- The management of the dynamic & fixed-combined information flow needs a high organizational level of the learning community. *Stochastikon Magister* uses a socialized mechanism and a humanized mode to run the information flow.
    1. Different authorities, responsibilities and rules of behaviors are defined in *Stochastikon Magister* by the “user status” called “organization by status”.
    2. In *Stochastikon Magister*, any registered user does not only adopt one specific role, but can adopt different roles in different conditions. This is called “role behavior”. The roles with respect to task, authority and responsibility are restricted by the user status and activity on hand, or other user specifics like group or study-level.
    3. Roles belonging to different categories for instance Learner, Teacher, Administrator and the Magister Master, have completely different tasks. The higher authority role-category automatically possesses the rights of the lowers. Roles in one category, but with different status, have different rights.
  - *Stochastikon Magister*, on the basis of the co-development and the collective intelligence of both the instructors and learners, constitutes a learner-centered environment for accomplishing a complete teaching-learning win-win process. For examples:
    1. The activities of all users are distinguished and recorded. In the Learner Progress panel, the data referring to the learners are recorded and regularly checked by teachers to evaluate and guide the learner's further learning process. Moreover, these data can also serve a learner to get information about his learning state and knowledge level. The Learner Progress panel acts as a main thread, which collects all the data about learning activities, which are used for the process-oriented service and supervision of *Stochastikon Magister*.
    2. In *Stochastikon Magister*, learners' questions and special discussion contributions can be marked for subsequent use for creating new frequently asked questions and new test questions. By this, it becomes possible that the teaching contents and the real learning conditions can be combined for providing supplements to the pre-settled assigned teaching content and accumulations to the existing teaching material. Thus, *Stochastikon Magister* not only provides an active learning enabling and encouraging environment with communication, participation and practice, but also a dynamic environment for the creation, improvement and update of the teaching contents. Actually, learning and teaching in *Stochastikon Magister* collaborate and stimulate each other simultaneously, which can be supported only by an open-information-flow environment as realized by *Stochastikon Magister*.

In one word, as has been designed in Section 9.8 ‘System Modeling’, the running of *Stochastikon Magister* e-learning programme will simultaneity accomplish two knowledge accumulation processes, one for Bernoulli Stochastics learners and the other for Magister knowledge system.

- *Stochastikon Magister* is a pure web application, therefore ever since it has been put to use, which has been for more than three years so far, the adjustment and improvement process has continued in accordance with the users' feedbacks and requirements, system operation analysis, and learning result evaluations.
- *Stochastikon Magister* has a lightweight user interfaces and is based on lightweight development models. Functional modules can be easily added or removed. The whole project is implemented in a cost-effective way.

Since the beginning of 2007 the first preliminary course of *Stochastikon Magister* was completed and offered online to students for preparing an examination in Stochastics without classroom teaching. So far about 60 students have taken the courses offered by *Stochastikon Magister*. They not only well-accepted *Stochastikon Magister*, but had also excellent examination results. In the coming part we present two evaluation studies for assessing the usability and learnability of *Stochastikon Magister*.

**Part IV**

**Empirical Evaluation of Stochastikon  
Magister**



# Chapter 11

## Significance of the Evaluation of Magister

### 11.1 Evaluation of E-Learning Programmes

The development of the course material in Bernoulli Stochastics and a virtual classroom environment for teaching it are only the first steps in developing an e-learning programme. The third and equally important step consists of an evaluation, which examines whether or not the e-learning system fulfills its teaching purpose and is accepted by the potential learners. Often this third step is omitted even in large-scale projects like EMILaA-stat and New Statistics (see Sections 2.3.2 and 2.3.4) with the consequence that in many cases the developments fall into oblivion. Therefore, the fourth part of this dissertation is devoted to an evaluation of *Stochastikon Magister*.

The concept of evaluation of software products, for example, e-learning programmes, corresponds to the concept of quality control in industry. Accordingly, evaluation means a systematic control of quality, i. e., functionality, effectivity and utility ([168], p.291). The basis for a quality evaluation is in general a random sample that is often taken after the object to be evaluated has been employed.

The evaluation of e-learning programmes has evolved into a large theoretical discipline that breaks the aim down to many different facets from usability, learnability, functionality, sustainability, impacts on people, practices, policies, utility, etc. The instruments for evaluation of e-learning products range from questionnaires, surveys, interviews, student personal diaries, tests and examinations, multiple-choice, to simulations and games. Which instrument shall be used depends on the purpose of the evaluation and the course components that shall be addressed, for example, course design, administration, technology, presentation, or learning outcome.

The evaluation process may consist of the following steps:

1. Definition of the activity to be evaluated: As to *Stochastikon Magister*, in this paper the evaluation refers to the *learning success*.
2. Purpose of evaluation: E-learning products are very popular, but when evaluated they often do not meet the expectations with respect to their effects on the learning process. Therefore, the purpose here is to investigate the effect of *Stochastikon Magister* on the *comprehension of uncertainty*.
3. Plan of the evaluation: *Stochastikon Magister* is still in the development phase. However, it has been offered and used for some years in the education of candidates for the teaching

profession (secondary schools) in Bavaria in parallel to classroom teaching in statistics. The results of the corresponding examinations shall represent the basis for the evaluation.

4. Sampling and analysis: The sample shall consist of the examinations in Bavaria performed during the years 2007 and 2008 in statistics against the examinations in Bernoulli Stochastics. The analysis shall be performed according to stochastic methods as derived and taught in *Stochastikon Magister*.
5. Interpretation of numerical results: The here performed evaluation is preliminary because of two reasons: As to the contents, the e-learning programme *Stochastikon Magister* is still in the development phase. Moreover, the number of users is still rather limited. Therefore, the interpretation must also have a preliminary character.

Many of the issues and instruments used in the evaluation of e-learning products are related to a comparison of different, competing learning systems that offer the same contents and have the same aims. In our case, there are no learning systems with identical contents, however, the two involved learning systems have aims, but handle the concepts and methods in a different way.

The aim of any learning system is to transfer knowledge and competence. In the case of *Stochastikon Magister*, the knowledge refers to Bernoulli Stochastics and the competence refers to solving problems taking into account uncertainty. Statistics has the more or less same purpose, but it is based on a different approach and at least partly on differently interpreted concepts.

The evaluation is therefore based on a comparison of the learning outcome between classroom teaching in statistics and e-learning teaching in Bernoulli Stochastics.

## 11.2 Teaching of Statistics and Bernoulli Stochastics

For many years the subject matter “stochastics”<sup>1</sup> has been taught at Bavarian secondary schools as part of the curriculum in mathematics. The candidates for the teaching profession therefore have to take a course in statistics at university, and have to pass a corresponding examination. Generally, the candidates attend a one-semester course in statistics and subsequently they take the required 30 minutes oral examination.

In many universities, there are special courses with lessons in statistics for the teacher candidates for secondary schools, each lesson lasts 45 minutes, two or four lessons per week for one semester. These courses are offered either with exercises or without exercises. Besides, there are universities without special courses for the candidates, who then have to attend a general introductory course in statistics offered to all students of mathematics.

In Würzburg, there is a one-semester special course for the candidates for the teaching profession with four lessons teaching and two lessons exercises per week which is offered in the summer semester. Additionally, there is a seminar offered in the same semester. The course is a typical introduction of statistics, starting with descriptive statistics and followed by the fundamental issues of probability theory. Finally, an introduction in elementary statistical estimation and test theory completes the course.

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<sup>1</sup>In Germany the words “Stochastics” and “statistics” are often used alternatively. For avoiding confusion between Bernoulli Stochastics and traditional stochastics and in line with the international practice, in the following, we only use the word “statistics” when it refers to the traditional class room teaching for statistics / Stochastics, and “Stochastics” for Bernoulli Stochastics.

Besides this more or less traditional program, the candidates in Würzburg can also select an alternative represented by Bernoulli Stochastics<sup>2</sup>[250, 251]. The differences between statistics and Bernoulli Stochastics are of principal nature and attending a course in statistics is of not much help for learning Bernoulli Stochastics. In order to illustrate the differences, the two subjects offered in Würzburg shall be briefly compared.

Statistics is most often described as a methodology, i. e., a set of methods for handling uncertainty, without, however, precisely defining uncertainty. As a consequence, uncertainty in statistics remains an ambiguous concept allowing in general several inconsistent interpretations. Statistics is divided into a descriptive part and an inductive one. The descriptive methods aim at representing a set of numbers (data) by graphics and by aggregated parameters. In contrast, inductive methods aim at drawing conclusions from the sample (data) to the “population” based on mathematical concepts derived in probability theory. As a matter of fact, there is no unified approach that comprises all the derived statistical methods. Contrary, the different methods stand more or less independently in parallel which makes understanding and learning difficult.

Accordingly, the statistics course in Würzburg consists of three parts the contents of which are listed on the homepage of the Chair of Mathematical Statistics<sup>3</sup>

1. Fundamentals of descriptive statistics: frequency distribution, graphical representation, statistical indices, robustness.
2. Random experiment, random variable, probability, combinatorics, conditional probability, stochastic independence, expectation and variance, covariance and correlation, law of large numbers, central limit theorem.
3. Estimation and test problems with binomial models,  $p$ -value method.

In contrast to statistics, Bernoulli Stochastics is not a set of methods for handling uncertainty, but represents a unified approach for defining, quantifying and subsequently modeling uncertainty of future developments. The uncertainty model is subsequently utilized for deriving methods for predicting indeterminate future developments as well as methods for revealing the determinate past.

Bernoulli Stochastics starts with a rigorous definition of uncertainty allowing its unique quantification and modeling. The introduced concepts are based on each other, and all the developed methods rest upon a unique approach and unique criteria based on predictions. Consequently, the course in Bernoulli Stochastics has a completely different structure than a traditional statistics course.

The current version of *Stochastikon Magister* is divided into two modularized courses. The first course “Modeling Uncertainty” is devoted to define unambiguously uncertainty. This definition enables the subsequent development of a unified model of uncertainty. The second course that is named “Application” contains the methods derived on the basis of the model, i. e., the model is exploited or applied for deriving methods for controlling uncertainty. The two courses and their modules taught in *Stochastikon Magister* are briefly described below.

1. The first course “Modeling Uncertainty” introduces uncertainty about future developments as the main problem of stochastic science and likewise of mankind. Uncertainty is quantified in the second module, and finally in the third module the fundamental model for

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<sup>2</sup><http://magister.stochastikon.com/>

<sup>3</sup>[http://statistik.mathematik.uni-wuerzburg.de/marohn/ss\\_09\\_stochastikLA\\_marohn.html](http://statistik.mathematik.uni-wuerzburg.de/marohn/ss_09_stochastikLA_marohn.html).

uncertainty called “Bernoulli Space” is introduced. Comparing this introductory course with a first course in statistics reveals that none of the above topics appears in the latter.

2. The second course “Application” answers the question how to exploit the model in order to solve problems in real life. The first module introduces the general stochastic procedures based on the Bernoulli Space. The following modules introduce and describe different procedures that are derived by a unified approach and unified criteria. The first class of procedures refers to predictions, which represents the basis of all stochastic procedures. They have no analog in statistics. The second course introduces so-called “Measurement Procedures” that can be looked upon as the basis for a general metrology. The corresponding statistical procedures are confidence intervals. However, generally the stochastic measurement procedures are more general and perform better than the corresponding confidence intervals, and, most important, the measurement procedures in contrast to the confidence intervals are derived along a unified approach. The next module under the heading “Exclusion Procedures” could be considered as a stochastic theory of significance tests. However, the exclusion procedures are again more general and less ambiguous than significance tests and, similar to the measurement procedures, they have a better performance than the analogous significance tests. The last module refers to classification procedures that could be compared with statistical alternative tests.

The main differences between statistics and Bernoulli Stochastics are as follows:

Statistical methods are generally based on pretended knowledge about “what is” with respect to distribution functions and parameter values. In contrast, the approach selected in Bernoulli Stochastics is based on knowledge about “what is not”. The result of statistical modelling is one probability distribution which with certainty is not the unknown true one. The result of stochastic modelling is a set of probability distributions, which cover the true one.

Generally statistics applies standard probability distributions as derived in mathematics, in particular the normal distribution. Bernoulli Stochastics applies probability distributions derived from the given conditions. A majority of statistical methods is based on point estimation referring to the past. The procedures derived in Bernoulli Stochastics are exclusively based on predictions referring to the future.

A comparison of statistics and Bernoulli Stochastics shows that the latter represents a stochastic basis for a holistic science while the former constitutes an attempt to consider uncertainty on a local level, but maintains otherwise the traditional deterministic approach. Thus, a statistics course does not support the learning process with respect to Bernoulli Stochastics. In contrast, Bernoulli Stochastics enables a better understanding of the statistical methods, their interrelations and specially their limitations. Exactly these points are important for teachers in secondary schools and Bernoulli Stochastics appears to be therefore well-suited for the education of the teacher candidates.

Actually, when comparing the contents of a textbook in statistics for secondary schools and the contents of the statistics course offered at universities, then it becomes evident that often the latter hardly covers the former. As a consequence, many teachers of secondary schools express their uneasiness about teaching statistics, because they are not able to classify statistics into the overall canon of science. These difficulties are addressed in Sträßer et al. ([214], p. 149). Although this article was written 25 years ago, the situation has not changed in principle. The authors state that there are statistics courses in almost all German universities, but, nevertheless, the teachers show educational shortcomings with respect to statistics. In this connection, they complain that many candidates for the teaching profession attend a statistics course only during the main study period and not during the basic one. Thus, the future



teachers cannot develop the so-called “stochastic (statistical) thinking” that is necessary to communicate statistics. A similar and in some sense even broader observation is made by Prediger [182]. She asks: “ Why do probability classes often fail to install the concept of probability as a strategic tool that is activated for decisions in random situations, even if the concepts are successfully learned in school?”

To overcome these shortcomings it would be desirable to impart a unified overview on science including uncertainty and randomness. Such an overview would enable the teachers to categorize, to better comprehend and, most importantly, to better motivate the set of more or less independent statistical methods and the employed mathematical concepts. Such an approach would leave the result-oriented scientific way and would come closer to the teacher’s way of looking at a subject matter namely to build up an organic connection between the scientific description and the students’ personal experiences. If this cannot be established the material remains purely formal and symbolic. The great educational reformer John Dewey puts it for children in the following words ([66] p. 159 and 160):

*Every study or subject thus has two aspects: one for the scientist as a scientist; the other for the teacher as a teacher. These two aspects are in no sense opposed or conflicting. But neither are they immediately identical. For the scientist, the subject-matter represents simply a given body of truth to be employed in locating new problems, instituting new researches, and carrying them through to a verified outcome. To him the subject-matter of the science is self-contained. He refers various portions of it to each other; he connects new facts with it. He is not, as a scientist, called upon to travel outside its particular bounds; if he does, it is only to get more facts of the same general sort. The problem of a teacher is a different one. As a teacher he is not concerned with adding new facts to the science he teaches; in propounding new hypotheses or in verifying them. He is concerned with the subject-matter of the science as representing a given stage and phase of the development of experience. His problem is that of inducing a vital and personal experiencing. Hence, what concerns him, as a teacher, is the ways in which the subject may become a part of experience; what there is in the child’s present that is usable with reference to it; how such elements are to be used; how his own knowledge of the subject-matter may assist in interpreting the child’s needs and doings, and determine the medium in which the child should be placed in order that his growth may be properly directed. He is concerned, not with the subject-matter as such, but with the subject matter as a related factor in a total and growing experience. Thus to see it is to psychologize it.*

Exactly this is one of the main aims of the e-learning programme *Stochastikon Magister*, namely to link the teaching contents in Bernoulli Stochastics with the students everyday experiences.

# Chapter 12

## Evaluation of Magister by Examination Scores

### 12.1 Oral Examinations in Bernoulli Stochastics

The evaluation of a teaching program shall measure its effectiveness in transferring new knowledge, in our case the learning outcome of the Magister programme. The effectiveness may be determined by oral or by written examinations that are rated by grades. After the first English version of the two courses of Magister were completed and made available online, it was offered as an extended subject matter for the examination in Stochastics to the candidates for the teaching profession majoring in mathematics.

Since 2005, in all about  $n = 60$  students (until 2009) in Würzburg have chosen the extended examination covering Bernoulli Stochastics as the subject matter instead of statistics. The examinations were either taken as final examinations or as an early examination according to the examination regulations. In the case of Bernoulli Stochastics, the examiner is in each case Professor von Collani. Besides him, there is a teacher from a secondary school (Gymnasium) as a co-examiner and observer, who has to put forward some few questions. The examinations last for 30 minutes.

The proceedings in Bernoulli Stochastics are as follows: After the students have enrolled for the oral examination in Bernoulli Stochastics they are advised by mail about the e-learning programme *Stochastikon Magister*. About one month before the examinations, an about three-hour lasting introduction to Bernoulli Stochastics is offered by Professor von Collani to make the students familiar with the contents, with himself and his way of posing the questions. One focus during this short introduction to Bernoulli Stochastics is to make clear that statistics and Bernoulli Stochastics are very different and that trying to use statistical knowledge for understanding Bernoulli Stochastics would fail. On the other hand, it is also stressed that Bernoulli Stochastics enables to improve the understanding of statistics.

After the initial introduction to Bernoulli Stochastics, the students have no further face-to-face instructions, but have to prepare for the examination only by using *Stochastikon Magister* and its functionalities. The students' preparations are additionally made difficult, because the German version of the courses in Magister is not completed so far and therefore they have to use the English version, which was prepared as part of this dissertation. The examinations in Bernoulli Stochastics are taken by Professor von Collani (about 25 minutes) and a teacher of a secondary school as an observer and a co-examiner (about 5 minutes). The students can get six marks: 1 = excellent, 2 = good, 3 = fair, 4 = sufficient, 5 = poor (failed) and 6 = unsatisfactory (failed).

The evaluation theory distinguishes between *norm-oriented* and *criteria-oriented* examinations ([81], p. 2). The norm-oriented tests are used for establishing an order between the candidates and are therefore particularly appropriate for qualifying examinations. In contrast, criteria-oriented tests are generally used for the evaluation of the students' knowledge. The examinations should exhibit a certain degree of objectivity and, in particular, they should not be affected heavily by the examiner. It is assumed that the second examiners from the secondary school, who may take part in both statistics and Bernoulli Stochastics examinations and who have a similar experience and background, guarantee that the last condition is at least approximately met, i. e., it is assumed here that the bias, which could be originated by the examiner, is marginal and can therefore be neglected.

The here performed evaluation is based on the examination grades obtained by candidates in the years 2007 and 2008 in Bavaria. The analysis includes the grades in statistics in Bavaria and in Würzburg, and compares them with the grades in Bernoulli Stochastics exclusively obtained in Würzburg. The evaluation aims at answering the question, whether the e-learning programme *Stochastikon Magister* can replace the traditional classroom teaching in statistics by determining the probability distributions of the grades obtained in the examination, and subsequently comparing them.

Modelling and comparing are based on the following quantities, which are used for describing the learning success by the two considered systems:

1. Let  $X_1$  denote the future grade of a candidate randomly selected from all Bavarian candidates to be tested in traditional statistics. The probability distribution  $P_{X_1}$  reflects the learning success of the student, and the analysis should therefore aim at determining  $P_{X_1}$ .
2. Let  $X_2$  denote the future grade of a candidate randomly selected from all Würzburg candidates to be tested in traditional statistics. The probability distribution  $P_{X_2}$  reflects the learning success of the student, and the analysis should therefore aim at determining  $P_{X_2}$ .
3. Let  $X_3$  denote the future grade of a candidate randomly selected from all Würzburg candidates to be tested in Bernoulli Stochastics. The probability distribution  $P_{X_3}$  reflects the learning success of the student, and the analysis should therefore aim at determining  $P_{X_3}$ .

What are needed are measurement procedures (in statistics: confidence intervals) for the relevant parameters that determine the desired probability distributions. The procedure used here to evaluate the quality of the e-learning programme Magister is introduced in [248].

## 12.2 The Bernoulli Space of the Evaluation Process

In a first step, the model of uncertainty, i. e., the Bernoulli Space (see Learning Module 1.3 of *Stochastikon Magister*) of the evaluation process must be developed. The aspect of interest is the future result of a student attending the examination. Because the result is indeterminate, it is represented by a random variable  $X$ , where  $X$  stands for the future result of a student, who is randomly selected from a given group of candidates. The study of mathematics for obtaining a teaching certificate is more or less fixed and, therefore, the students in each of the considered three groups take part in rather similar courses, they have a rather similar learning background, and the examinations are of similar character. It seems therefore justified to exclude a uniform distribution and distributions with more than one modal value, leaving

the sets of monotone and of uni-modal probability distributions for  $P_X$  (for details see Learning Unit 1.3.6 of Stochastikon Magister).

As explained in Magister Learning Unit 1.3.8, the deterministic variable  $D$  in this case can be selected following the principle of minimum information as the first moment  $E[X]$  and the variance  $V[X]$  which generally determine a sufficiently good approximation of  $P_X$ . Thus, we set:

$$D = (E[X], V[X]) \quad (12.1)$$

Assuming that there is no knowledge at all, except for the range of variability of the grade  $X$  given by  $\{1, 2, 3, 4, 5, 6\}$  and the information that the distribution is either monotonic or uni-modal, the following approximate Bernoulli Space (see Magister Learning Unit 1.2.9) is obtained:

$$\mathbb{B}_{X,D} = (\mathfrak{D}, \mathfrak{X}, \mathfrak{P}) \quad (12.2)$$

with

$$\mathfrak{D} = \left\{ (\mu, \sigma^2) \mid 1 < \mu < 6, 0 < \sigma^2 < 6.25 \right\} \quad (12.3)$$

$$\mathfrak{X}(\{(\mu, \sigma^2)\}) = \{1, 2, 3, 4, 5, 6\} \quad \text{for } (\mu, \sigma^2) \in \mathfrak{D} \quad (12.4)$$

$$\mathfrak{P}(\{(\mu, \sigma^2)\}) = P_{X|\{(\mu, \sigma^2)\}} \quad (12.5)$$

where the probability measure  $P_{X|\{(\mu, \sigma^2)\}}$  is given by its probability mass function  $f_{X|\{(\mu, \sigma^2)\}}$  (see Magister Learning Unit 1.3.5):

$$f_{X|\{(\mu, \sigma^2)\}}(k) = e^{\lambda_0(\mu, \sigma^2) + \lambda_1(\mu, \sigma^2)x + \lambda_2(\mu, \sigma^2)x^2} \quad \text{for } k \in \{1, 2, 3, 4, 5, 6\} \quad (12.6)$$

As introduced above, there are three random variables considered:  $X_1$  represents the future grade of a randomly selected student from all Bavarian candidates,  $X_2$  from all Würzburg candidates in statistics and  $X_3$  from all Würzburg candidates in Bernoulli Stochastics.

The probability distributions  $P_{X_i}$ ,  $i = 1, 2, 3$ , are determined by the parameter values  $(\lambda_{i,0}(\mu, \sigma^2), \lambda_{i,1}(\mu, \sigma^2), \lambda_{i,2}(\mu, \sigma^2))$  for each of the three random variables. This is not done directly, but in two steps. First, the values of the two moments  $E[X_i]$  and  $V[X_i]$  are determined and subsequently the values of the parameters (for details see Learning Unit 1.3.8 of Stochastikon Magister).

## 12.3 Stochastic Measurement Procedure for $E[X_i]$ and $V[X_i]$

A measurement is always based on a measurement process, which is given here by the oral examinations that define the samples for the three random variables:

$$(X_{1,1}, X_{1,2}, \dots, X_{1,n_1}) \quad (12.7)$$

$$(X_{2,1}, X_{2,2}, \dots, X_{2,n_2}) \quad (12.8)$$

$$(X_{3,1}, X_{3,2}, \dots, X_{3,n_3}) \quad (12.9)$$

where  $n_1$  is the number of students in the years 2007 and 2008 attending the statistics examination in Bavaria,  $n_2$  is the number of students attending the statistics examination in Würzburg, and finally  $n_3$  is the number of students attending the examination in Bernoulli Stochastics<sup>1</sup>.

<sup>1</sup>The examination results were provided by the Prüfungskanzlei of Würzburg University.

A measurement procedure for the unknown value of a deterministic variable is given by a function  $C_D^{(\beta)}$  that maps the observed result of the measurement process onto the corresponding measurement result, which is a subset of the ignorance space. The parameter  $\beta$  is called the reliability level (confidence level in statistics) of the measurement procedure that specifies a lower bound for the probability of obtaining a correct result. A measurement result is correct, if it contains the unknown true value of  $D$ .

The measurement procedures  $C_D^{(\beta)}$  for the values of the first moment and the variance by means of a sample size  $n$  are described in detail in the Magister Learning Unit 2.3.9 (first moment) and 2.3.10 (variance). Accordingly, each measurement procedure is defined by means of a prediction procedure  $A_X^{(\beta)}$  (see Magister Learning Unit 2.3.1). Actually, the general measurement procedure is defined by means of a prediction procedure as follows:

$$C_D^{(\beta)}(\{x\}) = \left\{ d \mid x \in A_X^{(\beta)}(\{d\}) \right\} \quad (12.10)$$

with  $d = (\mu, \sigma^2)$  and the prediction procedure  $A_X^{(\beta)}$  meets the following requirements:

Side conditions:

$$P_{X|\{d\}}(A_X^{(\beta)}(\{d\})) \geq \beta \quad \text{for } d \in \mathfrak{D} \text{ (reliability)} \quad (12.11)$$

$$\bigcup_{d \in \mathfrak{D}} A_X^{(\beta)}(\{d\}) = \mathfrak{X}(\mathfrak{D}) \text{ (completeness)} \quad (12.12)$$

Optimality with respect to accuracy:

$$\frac{1}{|\mathfrak{D}|} \sum_{d \in \mathfrak{D}} \sum_{x \in \mathfrak{X}(\{d\})} \left| \left\{ d \mid x \in A_X^{(\beta)}(\{d\}) \right\} \right| P_{X|\{d\}}(\{x\}) \stackrel{!}{=} \text{Minimum} \quad (12.13)$$

where the reliability and completeness of the measurement procedure are guaranteed by (12.11) and (12.12), while an optimal accuracy is given by (12.13).

Because this is a first and preliminary evaluation based on a very limited stock of data, the rather small reliability level of  $\beta = 0.80$  is selected.

## 12.4 The Samples for $X_1$ , $X_2$ and $X_3$ and their Analysis

The samples for 2007 and 2008 yield the following observations for the three considered groups:

1. The examination results in Bavaria based on traditional classroom teaching in statistics are as follows:

Total number of students	1	2	3	4	5	6
$n_1 = 541$	217	151	105	49	17	2

Thus the following observations with respect to the sample mean  $\bar{X}$  and the sample variance  $S^2$  are obtained:

$$\begin{aligned} \bar{x}_1 &= 2.0833 \\ s_1^2 &= 1.2813 \end{aligned}$$

2. The examination results in Würzburg based on traditional classroom teaching in statistics are as follows:

Total number of students	1	2	3	4	5	6
$n_2 = 58$	29	13	11	4	1	0

Thus the following observations with respect to the sample mean  $\bar{X}$  and the sample variance  $S^2$  are obtained:

$$\bar{x}_2 = 1.879$$

$$s_2^2 = 1.1255$$

3. The examination results in Würzburg based on the e-learning Programme *Stochastikon Magister* in Bernoulli Stochastics are as follows:

Total number of students	1	2	3	4	5	6
$n_3 = 25$	11	9	4	1	0	0

Thus the following observations with respect to the sample mean  $\bar{X}$  and the sample variance  $S^2$  are obtained:

$$\bar{x}_3 = 1.800$$

$$s_3^2 = 0.7565$$

Later on, the grade 1 and 2 are combined to be the grade class “with honor”, and the grade 5 and 6 to be the grade class “failed”.

Proceeding as described in Magister Learning Unit 2.3.9, the following measurement results are obtained for  $E[X_1]$ ,  $E[X_2]$ ,  $E[X_3]$  and  $V[X_1]$ ,  $V[X_2]$ ,  $V[X_3]$ :

- Examinations in Bavaria in statistics:

For the first moment  $E[X_1]$ , the following measurement result is obtained:

$$C_{E[X_1]}^{(0.80)}(\{(2.0832, 1.2813)\}) = \{\mu \mid 2.02073 \leq \mu \leq 2.14562\}$$

For the variance  $V[X_1]$  using the optimal measurement procedure (see Magister Learning Unit 2.3.10), the following measurement result is obtained:

$$C_{V[X_1]}^{(0.80)}(\{1.2813\}) = \{\sigma^2 \mid 1.18398 \leq \sigma^2 \leq 1.38454\}$$

- Examinations in Würzburg in statistics:

For the first moment  $E[X_2]$  the following measurement result is obtained:

$$C_{E[X_2]}^{(0.80)}(\{(1.879, 1.1255)\}) = \{\mu \mid 1.69869 \leq \mu \leq 2.05993\}$$

For the variance  $V[X_2]$  using the optimal measurement procedure, we obtain:

$$C_{V[X_2]}^{(0.80)}(\{1.1255\}) = \{\sigma^2 \mid 0.861494 \leq \sigma^2 \leq 1.4056\}$$

- Examinations in Würzburg in Bernoulli Stochastics:

For the first moment  $E[X_3]$  the following measurement result is obtained:

$$C_{E[X_3]}^{(0.80)}(\{(1.80, 0.7565)\}) = \{\mu \mid 1.57075 \leq \mu \leq 2.02925\}$$

For the variance  $V[X_3]$  using an optimal measurement procedure, we obtain:

$$C_{V[X_3]}^{(0.80)}(\{0.7565\}) = \{\sigma^2 \mid 0.48563 \leq \sigma^2 \leq 1.05564\}$$

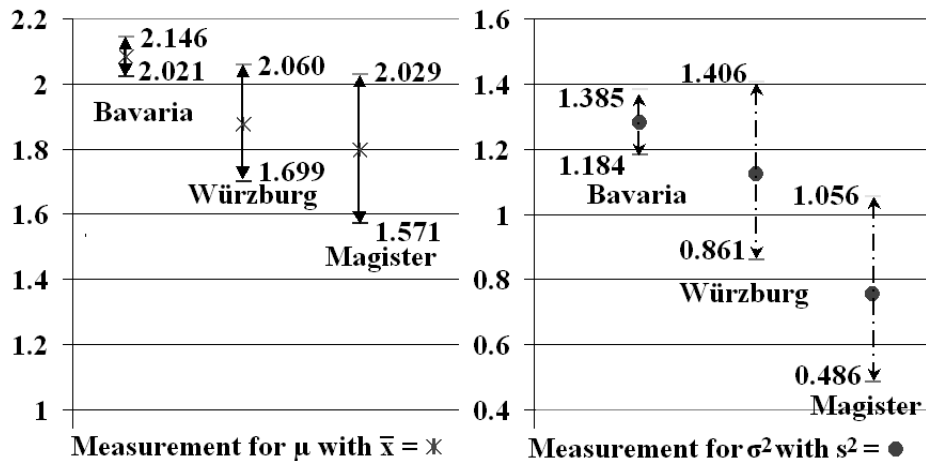


Figure 12.1: Measurement intervals for  $\mu_i$  and  $\sigma_i^2$ ,  $i = 1, 2, 3$  based on the reliability level  $\beta = 0.80$ .

Figure 12.1 shows very clearly how the sample size affects the measurement precision. In the case of the result concerning all Bavarian students with a large sample size, very short measurement intervals are obtained, while in the case of students who selected Bernoulli Stochastics, the small sample size leads to large measurement intervals. Each element of a measurement interval may be the true, but unknown one. Thus, the measurement intervals for  $\mu$  and  $\sigma^2$  represent the still existing ignorance which could be reduced but not completely eliminated by means of more costly measurement procedure, where “costly” refers to the sample size.

The measurement intervals based on the examination grades in statistics for the first moment (expectation) as well as for the variance obtained in Bavaria and in Würzburg overlap, which implies that the observed differences might well be explained by randomness, i. e., no difference with respect to the first moment and the variance in the statistic grades between all Bavarian candidates and the Würzburg candidates could be established and the same holds for a comparison between the results in statistics and in Bernoulli Stochastics in Würzburg.

However, comparing the measurement intervals for Bavaria in statistics and for Würzburg in Bernoulli Stochastics, then the measurement intervals for the variance are disjoint. In other words, based on a reliability level of  $\beta = 0.80$ , it is proved that there is a difference in the results obtained in Würzburg after attending the e-learning programme *Stochastikon Magister* in Bernoulli Stochastics and the results in Bavaria based on traditional classroom teaching in statistics. The difference refers to the variability of the grades given by the respective values of the variance. Moreover, the measurement intervals of the first moment hardly overlap, which indicates that possibly only the small sample in case of Bernoulli Stochastics prevented to prove an existing difference.

Starting from the results on the moments, the explicit probability distributions can be obtained for each of the admissible pairs  $(\mu, \sigma^2)$  in the three considered cases given by the corresponding measurement intervals.

## 12.5 Probability Distributions of $X_1$ , $X_2$ and $X_3$

For given values of the first moment and the second moment (or equivalently the variance), it is possible to obtain a probability distribution that is, in general, a good approximation for any probability distribution that is at most uni-modal. In the here considered cases each set of admissible values  $(\mu, \sigma^2)$  for the first moment and the variance yields a corresponding set of approximate probability distributions, which cover the true but unknown one.

In the following the two extreme cases with respect to the first moment and the variance are used in order to illustrate the range of probability distributions of  $X_i$ ,  $i = 1, 2, 3$ . The case with the largest values of  $E[X]$  and  $V[X]$  is called “worst case”, because the probability mass accumulates in the large (i. e., bad) grade values, where the case with the smallest values of  $E[X]$  and  $V[X]$  is called “best case” because the probability mass accumulates for the small (i. e., good) grade values.

Table 12.1: “Worst” and “best” probability distributions  $P_{X_1}$  in the case of all Bavarian examinations in statistics:

Grade	worst case, i. e., largest $E[X_1]$ and $V[X_1]$	best case, i. e., smallest $E[X_1]$ and $V[X_1]$
1	0.37259	0.40273
2	0.30133	0.31306
3	0.18908	0.17940
4	0.092046	0.075788
5	0.034766	0.023603
6	0.0101878	0.0054189
with honor	0.67393	0.71579
failed	0.044953	0.0290216

Table 12.2: ‘Worst’ and ‘best’ probability distribution  $P_{X_2}$  in the case of Würzburg examinations in statistics:

Grade	worst case, i. e., largest $E[X_2]$ and $V[X_2]$	best case, i. e., smallest $E[X_2]$ and $V[X_2]$
1	0.419323	0.57985
2	0.28686	0.26328
3	0.16626	0.10524
4	0.081632	0.037033
5	0.033957	0.0114723
6	0.011967	0.0031289
with honor	0.706189	0.843127
failed	0.045924	0.0146015



Table 12.3: “Worst” and “best” probability distribution  $P_{X_3}$  in the case of Würzburg examinations in Bernoulli Stochastics:

Grade	worst case, i. e., largest $E[X_3]$ and $V[X_3]$	best case, i. e., smallest $E[X_3]$ and $V[X_3]$
1	0.37428	0.53964
2	0.33793	0.35956
3	0.19549	0.091556
4	0.072465	0.0089091
5	0.017211	0.00033130
6	0.0026191	0.0000047080
with honor	0.71221	0.89920
failed	0.019830	0.00033601

## 12.6 Summary and Interpretation of Results

The measurement results show the good performance of the e-learning programme *Stochastikon Magister*. Although the students had only very limited time for preparation in Bernoulli Stochastics, and despite the fact that the courses are offered in English, the measurement intervals for the failure probability as well as for the probability of an honor examination contain much better values. The fact that the comparison does not yield significant results seems to be due to the small sample size as can be seen from Figure 12.2 and Figure 12.3.

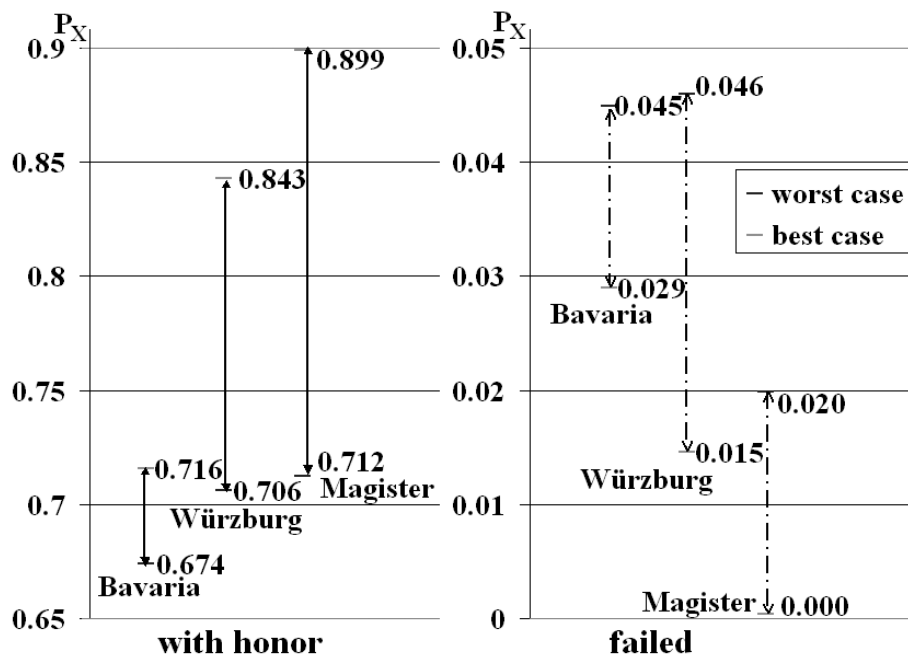


Figure 12.2: Measurement intervals for the probabilities of obtaining a grade “with honor” and “failed” in statistics and Bernoulli Stochastics based on the reliability level  $\beta = 0.80$ .

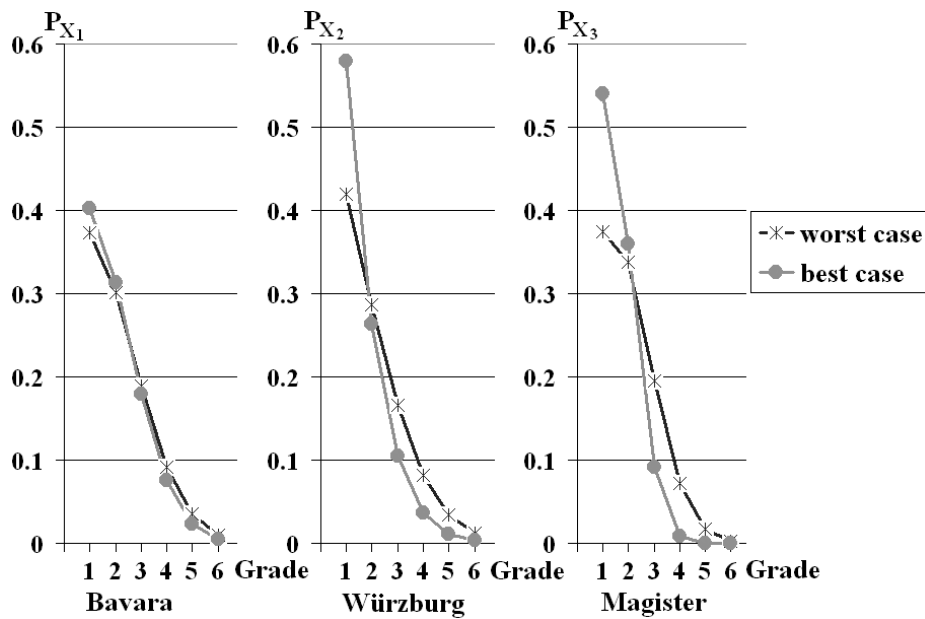


Figure 12.3: The probability distributions (worst and best case) for the grades in statistics and Bernoulli Stochastics.

Thus, the efficiency of *Stochastikon Magister* when compared with the classroom courses in statistics appears to be superior. One reason for this rather surprising outcome might be the fact that, in contrast to statistics, Bernoulli Stochastics follows a unified thread with respect to the introduced concepts and criteria implying that each new item is the logical consequence of the preceding ones. This is different in statistics where the applied criteria depend on the methods and are often not directly related to the pursued purpose. Furthermore, the extensive possibilities of *Stochastikon Magister* to illustrate the concepts and methods by means of graphics certainly contribute to the success of the Magister.

As stressed above, this evaluation is a preliminary one. But it shows clearly that the e-learning programme *Stochastikon Magister* meets its expectations and therefore represents a fully valid alternative to traditional classroom teaching in statistics. This holds true especially because Bernoulli Stochastics and its concepts can immediately be applied to other branches of science. Any natural law, for example, can easily be formulated as a Bernoulli Space. Such a representation immediately reveals the assumptions and postulates the law is based on.

If the good performance of *Stochastikon Magister* in the area of uncertainty can be approved by further and more in depth going evaluations, this alternative could fill a gap in the statistical education at German universities that *probably will occur after the bachelor/master study* programs have been implemented. It is to be suspected that the courses in statistics will be shortened or even be omitted, and this would affect also the teachers' education.

The here performed stochastic analysis also illustrates the difference between statistics and Bernoulli Stochastics. In statistics, the commonly performed evaluation analysis would have consisted of a significance test with respect to the values of the first moment of the three considered groups. Instead, in Bernoulli Stochastics, complete stochastic models are developed for each case and subsequently compared.

# Chapter 13

## Evaluation of Magister by Questionnaire

### 13.1 Introduction

The comparison of the examination scores of the teacher candidates, who took part in the course of Bernoulli Stochastics based on the e-learning programme *Stochastikon Magister* on the one hand and in a course of traditional stochastics based on classroom teaching on the other reveals that the scores are quite similar. The result therefore indicates the same degree of learnability for the two teaching alternatives.

However, this result may be argued because of the following reasons:

1. Oral examinations are rather subjective and may not reflect objectively the learnability of the two teaching alternatives.
2. Bernoulli Stochastics may be easier to learn than traditional stochastics and the scores reflect more the different contents than the different teaching methods.
3. The students did learn Bernoulli Stochastics not primarily with *Stochastikon Magister* but from examination protocols of other students, from the literature, in groups, etc.
4. The students can choose between Bernoulli Stochastics and traditional stochastics. Maybe the better students did choose Bernoulli Stochastics and the good scores are not due to Magister.

Any examination is to a certain degree subjective, but in this case subjectivity is reduced by the fact that there are two examiners, one professor (von Collani or Marohn) and one school teacher, and the school teachers are not only observers but also examiners, who must pose questions. Furthermore, the school teachers were partly the same persons for both the examinations in Bernoulli Stochastics and traditional stochastics. As already mentioned above it is therefore assumed here that a possible subjectivity of the examiners does not essentially affect the scores.

In order to check the validity of the other arguments, a questionnaire was prepared and the participants in the examination in Bernoulli Stochastics were asked to answer it. The aim of the questionnaire is to assess the usability and learnability of Magister compared with classroom teaching from the students' point of view. The questionnaire designed for the evaluation of *Stochastikon Magister* has taken into account the relevant ISO standard cited below.

According to the "Guidance on Usability" [114]

### 5.3 Usability measures

#### 5.3.1 Choice of measures

*A description of usability measures consists of target or actual values of effectiveness, efficiency, and satisfaction for the required contexts. It is normally necessary to provide at least one measure for each of effectiveness, efficiency and satisfaction. Because the relative importance of components of usability depends on the context of use and the purposes for which usability is being described, there is no general rule for how measures should be chosen or combined.....*

#### 5.3.2 Effectiveness

*Measures of effectiveness relate the goals or sub-goals of the user to the accuracy and completeness with which these goals can be achieved.....*

#### 5.3.3 Efficiency

*Measures of efficiency relate the level of effectiveness achieved to the expenditure of resources. ....*

#### 5.3.4 Satisfaction

*Measures of satisfaction describe the comfort and acceptability of the use.....*

## 13.2 Questionnaire

### 13.2.1 Content of Questionnaire

The invitation of the questionnaire was sent by e-mail to the students who had passed the oral examination and they were asked to fill the questionnaire and submit it. The Below the complete questionnaire is displayed.

#### *Magisterumfrage*

*Fragebogen zur Evaluierung des E-Learning-Programms Stochastikon Magister als Prüfungsfach für Lehramtskandidaten in den Lehrfächern Stochastik und Informationstheo.*

*Sie können bei jeder Frage nur eine Antwort ankreuzen, diese sollte die am meisten zutreffende Antwort sein.*

*Frage 1: Hauptgrund, warum Sie Stochastik und nicht Statistik für die mündliche Prüfung im Staatsexamen gewählt haben:*

- 1. Die Stochastik ist leichter zu verstehen und damit leichter zu lernen.*
- 2. Die Stochastik ist interessanter.*
- 3. Die Stochastik basiert auf einem einheitlichen Ansatz.*
- 4. Die Stochastik ist realitätsnäher.*

5. Die Stochastik stellt eine gute Vorbereitung für den Schulunterricht dar.
6. Für die Stochastik steht ein E-Learning-Programm zur Verfügung, was sehr nützlich für die Prüfungsvorbereitungen ist.
7. Mir wurde die Stochastik-Prüfung von Kommilitonen empfohlen.
8. Ich würde die Stochastik-Prüfung empfehlen.
9. Ich würde die Stochastik-Prüfung nicht empfehlen.

Frage 2: Wie haben Sie sich auf die Prüfung in Stochastik vorbereitet:

1. Vorbereitung on-line am Bildschirm entsprechend den Lektionen und Module des Magisters im CIP-Pool.
2. Vorbereitung on-line am Bildschirm entsprechend den Lektionen und Module des Magisters zu Hause.
3. Vorbereitung anhand der ausgedruckten Lektionen (Inhalt, Übung, Beispiele).
4. Vorbereitung anhand von Prüfungsprotokollen früherer Prüfungsteilnehmer.
5. Vorbereitung in Gruppen und mit anderen Prüfungsteilnehmern.

Frage 3: Stellt das E-Learning Programm Magister eine ausreichende Grundlage dar, um den Prüfungsstoff zu verstehen:

1. Ja, ohne Einschränkungen.
2. Ja, aber die Lehrinhalte sollten ausführlicher und anschaulicher sein.
3. Ja, aber es sollten mehr Beispiele und Übungen angeboten werden.
4. Ja, aber nur zusammen mit einer Einführung/Vorbesprechung.
5. Nein, auch nicht mit einer Einführung/Vorbesprechung, da die angebotenen Hilfen nicht ausreichend sind.

Frage 4: Fragen zur Prüfungsvorbereitung im Fach Stochastik:

1. Die Prüfungsvorbereitung wäre leichter gewesen wenn es eine reguläre Lehrveranstaltung in Stochastik, wie in Statistik, gegeben hätte?
2. Die Prüfungsvorbereitung war leichter als in anderen Fächern, weil es das E-Learning Programm Magister gibt.
3. Es wäre gut, wenn auch für andere Prüfungsfächer E-Learning Programme wie Magister zur Verfügung ständen.
4. Das E-Learning Program Magister war keine Hilfe bei der Prüfungsvorbereitung.

Frage 5: Fragen zum E-Learning Programm Magister:

5.1 Wie bewerten Sie die Bedienungsfreundlichkeit des Magisters?

1. Sehr gut
2. Gut

3. *Befriedigend*
4. *Ausreichend*
5. *Mangelhaft*

5.2 *Wie bewerten Sie die Verständlichkeit der Lektionen des Magisters?*

1. *Sehr gut*
2. *Gut*
3. *Befriedigend*
4. *Ausreichend*
5. *Mangelhaft*

5.3 *Wie bewerten Sie die Reaktion des Magisters?*

1. *Sehr gut*
2. *Gut*
3. *Befriedigend*
4. *Ausreichend*
5. *Mangelhaft*

5.4 *Wie bewerten Sie die Stabilität des Magisters als Software-Programms?*

1. *Sehr gut*
2. *Gut*
3. *Befriedigend*
4. *Ausreichend*
5. *Mangelhaft*

5.5 *Wie bewerten Sie die äusserliche Aufmachung des Magisters?*

1. *Sehr gut*
2. *Gut*
3. *Befriedigend*
4. *Ausreichend*
5. *Mangelhaft*

5.6 *Wie bewerten Sie das Arbeiten mit dem Magister?*

1. *Sehr zufriedenstellend*
2. *Zufriedenstellend*
3. *Weder zufriedenstellend noch frustrierend*
4. *Frustrierend*

5. *Sehr frustrierend*

5.7 *Häufigkeit der Arbeit mit dem Graphical Laboratory?*

1. *Sehr häufig*
2. *Weniger häufig*
3. *Manchmal*
4. *Selten*
5. *Nie*

5.8 *Wie bewerten Sie die Bedienungsfreundlichkeit des Graphical Laboratories?*

1. *Sehr gut*
2. *Gut*
3. *Befriedigend*
4. *Ausreichend*
5. *Mangelhaft*

*Frage 6: Fragen zum Einsatz des Magisters und von anderen E-Learning Programmen in der Lehre.*

6.1 *Bei welchem Unterricht (Statistik mit Präsenzunterricht) oder Stochastik (E-Learning-Programm) hatten Sie grössere Verständnisschwierigkeiten?*

1. *Statistik*
2. *Stochastik*

6.2 *In welchem der beiden Fälle (Statistik) oder Stochastik (E-Learning-Programm) konnten die Verständnisschwierigkeiten leichter behoben werden?*

1. *Statistik*
2. *Stochastik*

6.3 *Haben Sie neben Magister Erfahrungen mit anderen E-Learning Programmen?*

1. *ja*
2. *nein*

6.4 *Falls ja, wurden die anderen E-Learning Programme als Unterstützung des Präsenzunterrichts oder als Ersatz des Präsenzunterrichts angeboten?*

1. *Als Unterstützung*
2. *Als Ersatz*

6.5 Haben Sie bei der Prüfungsvorbereitung für die Stochastik das Fehlen des Präsenzunterrichts vermisst?

1. ja
2. nein

6.6 Könnten Sie sich, auf der Grundlage Ihrer Erfahrung mit dem Magister, vorstellen, dass auch anderen mathematischen und naturwissenschaftliche Teilgebieten die Lehre mit Hilfe eines E-Learning-Programms und ohne Präsenzunterricht erfolgreich sein kann (eventuell mit einigen wenigen Präsenztreffen, ähnlich zu der Einführung in die Stochastik und den Magister durch Prof. von Collani)?

1. ja
2. nein

6.7 Welche Teilgebiete kämen Ihrer Meinung dafür in Frage?

1. Analysis
2. Lineare Algebra
3. Numerische Mathematik
4. Geometrie
5. Algebra
6. Masstheorie
7. Wahrscheinlichkeitstheorie
8. Experimentelle Physik
9. Theoretische Physik

### 13.2.2 Response to Questionnaire

The call to participate the questionnaire was sent by email to 46 teacher candidates who had taken part in the oral examination in Bernoulli Stochastics. The questionnaire was available online, and the time to complete the questionnaire lasted from 16th February 2010 to 21st March 2010. During this time period 31 (67.4%) of the addressed students responded.

The evaluation of the examination scores was based on only 25 teacher candidates, namely those who took part in the examinations in 2007 and 2008. In contrast, the questionnaire was sent to all participants of the examinations in Bernoulli Stochastics, who were accessible by email connections.

The questionnaire aims at evaluating *Stochastikon Magister* in particular and e-learning in general with respect to the following points:

1. The reasons for selecting Bernoulli Stochastics instead of traditional stochastics.
2. The quality of teaching materials and learning environment of Magister.
3. The technical design and implementation of Magister system.
4. The comparison between Magister based e-learning and regular classroom teaching.



5. The usability and learnability of e-learning system like Magister for other subjects of mathematics and natural science.

Only very few students did not respond to all questions of the questionnaire. In all, only eleven questions were not answered, which include two follow-up questions, which could not be answered reasonably by the related two participants (this will be discussed later). Furthermore, one student was accountable for five of the eleven unanswered questions. The high response rate of 98.13% indicates that the questions of the questionnaire are understandable and reasonable.

In the remainder, traditional stochastics is named “statistics”, and Bernoulli Stochastics is named “Stochastics”. Moreover, “Question” is abbreviated by “Q”, for example, “Q3” means “Question 3”, and “Q5.1” means “Question 5.1”. The answers are specified by the corresponding number of the choice separated by a hyphen. For instance “Q2-2” stands for the second choice of Question 2, “Q6.5-1” means the first choice of Question 6.5, and “Q4-23” represents the second and the third choice of Question 4. Besides, in the following, students who made a specified selection are denoted by “group  $x$ ” where  $x$  represents the number of the choice or choices. For example, “group 1” stands for students who selected the first choice in the question, and “group 23” refers to students who selected the second or third choice of the question.

### 13.3 Analysis of the Responses to Questionnaire

The online questionnaire started with stating its aim and the instruction that only one answer should be selected to each question. This mode of fill-out was chosen to get a more differentiated result that would reflect the individual preferences better.

Below, each question of the questionnaire is shown together with the possible choices (in English). The response rate to this question and results in percent of the choices are given on the left hand side and are further illustrated by a diagram below. Subsequently the result is evaluated by comments and additional diagrams.

#### 13.3.1 Analysis of the Responses to Question 1

Question 1	The main reason for not selecting statistics but Stochastics for the oral examination is:	100.00%
1	Stochastics is easier to understand and thus easier to learn.	0.00%
2	Stochastics is more interesting.	6.45%
3	Stochastics is based on a unified approach.	3.23%
4	Stochastics is more realistic.	16.13%
5	Stochastics is a good preparation for classroom teaching.	9.68%
6	An e-learning program is available for Stochastics, which is very useful for the exam revision.	6.45%
7	The Stochastics examination was recommended by fellow students.	51.61%
8	I would recommend the Stochastic examination.	6.45%
9	I would not recommend the Stochastic examination.	0.00%

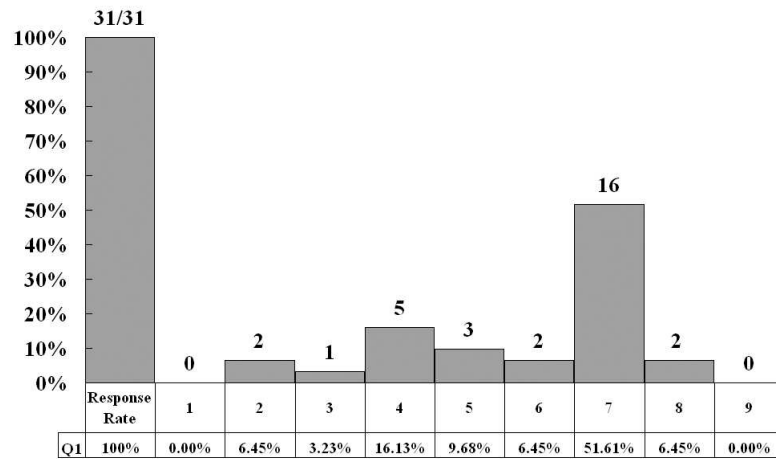


Figure 13.1: Diagram of responses to Question 1.

**Comments:**

This question, which was answered by all participants, aims at identifying the reasons for selecting Stochastics in order to check whether the scores in the examination and the selection are stochastically dependent or not. More than half of the students state that the most decisive reason for selecting Bernoulli Stochastics was a recommendation of fellow students. Others feel that Stochastics is more realistic (16.13%), interesting and useful (10%) and two students (6.45%) would recommend Stochastic. Only for two students (6.45%) the availability of the e-learning system Magister was the main reason to select Bernoulli Stochastics. Nobody states that the ease of understanding and learning of Bernoulli Stochastics was the most important reason for selecting it. Finally, it is noteworthy that all of the participants would recommend Stochastics to fellow students. However, only 12.5% of the students who selected Stochastics because of recommendation (Q1-7) also selected online learning at home (Q2-2), in contrast to 54.5% of those students who selected Stochastics because of more specific features (Q1-2345) also selected online learning at home (Q2-2). The answers to the first question indicate the following:

- The most frequently selected answer suggests that most of the participants did not have a clear understanding about the contents of Stochastics, about Magister or about the alternative statistics course when selecting Stochastics. Therefore, we may conclude that the participants in the Stochastics examination represent the outcome of a random sample.
- Those stated reasons, which indicate a relation between the selection and the content of Stochastics are probably a combination of the recommendation by fellow students and the students' own later learning experiences with Magister, as more than half of these students were learning online at home.
- Although nobody claimed that the reason for selecting Stochastics is because it is easier to understand and learn, it cannot be excluded that Stochastics is in fact easier to understand and learn. However, if there is a significant difference of difficulty between statistics and Stochastics, it should become apparent from the answers to the subsequent questions.

### 13.3.2 Analysis of the Responses to Question 2

Question 2	How did you prepare for the exam in Stochastics:	100.00%
1	Preparing on-line according to the lessons and modules of Magister in the CIP pool.	0.00%
2	Preparing on-line according to the lessons and modules of the Magister at home.	35.48%
3	Preparing on the basis of the printed lessons (content, exercises, and examples).	51.61%
4	Preparing on the basis of test protocols from former test participants.	3.23%
5	Preparing in groups and with other test participants.	9.68%

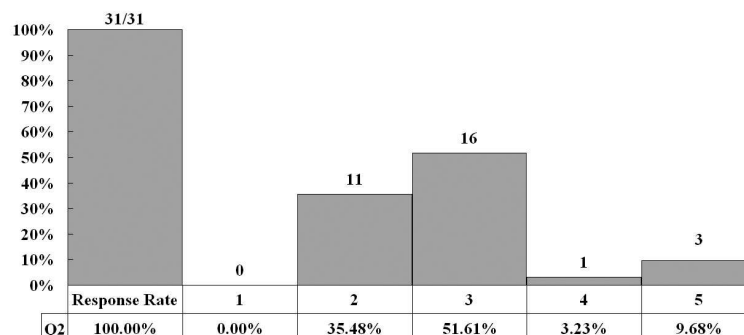


Figure 13.2: Diagram of responses to Question 2.

#### Comments:

All participants answered this question, which aims at checking the third argument that the students might have used not so much Magister for exam preparation but other sources.

More than half of the participants prepared their examinations on the basis of print-outs of Magister lessons (group 3). None of these students selected on-line learning in a CIP-pool, but 35.48% of them on-line learning at home (group 2). Less than 10% of the students worked in groups with other test participants (group 5), and only for one student protocols from former test participants were the main means for preparation.

- It may be concluded from the answers that the teacher candidates do not or at least do not often frequent a CIP pool.
- Magister was either directly (on-line) or indirectly (print-outs) the main source of learning. Examination protocols had only marginal significance for the learning process.

All the 20 students, who did not select online learning at home (Q2-2), answered all remaining questions (Q3 to Q6) about the usability and learnability of Magister. The overall answer rate of them for these questions is 96.76%. This result indicates that the questionnaire participants had in fact working experiences with Magister. .

It is not surprising that half of the students prepared examination mainly by print-outs instead of online studying, since 80.25% of these students had no prior experience with other e-learning systems (Q6.1-2). The preference for printed learning materials has been observed earlier:

- The main complaint about the famous CyberStats system is, according to [219], the fact of “not having a printed version of the textbook” and the authors are pleased that “recently, a printed version of CyberStats has been made available upon request.” In [218] it is stated that “Ideally an electronic textbook should be available in all three formats, i. e., CD, Web, and print...” Some educational statistics systems announce the provision of a printed format of the teaching content as a specific feature. A lot of persons do not like screen-reading, and for others a printed document, which is independent from computer and network, is more flexible and more convenient for study.
- The preference for learning from printed media was one of the reasons that Magister’s entire primary teaching material is provided in PDF format. Accordingly, Magister has a PDF creator for generating the documents making Magister a convenient environment for both instructors to develop the mathematical teaching material, and students to print and keep the teaching material for study.

Figure 13.3 shows the percentages of some responses related to Magister in particular and e-learning in general made by the three groups who selected different learning styles for the preparation. In Q4 it is stated that Magister was helpful in preparing Stochastics (Q4-2) and it would be good to have similar e-learning programs for other subjects (Q4-3). Q6.1-1 states that statistics caused more difficulties in understanding than Stochastics; accordingly Q6.3-1 stands for previous experiences with e-learning programs; and Q6.5-2 states that during the preparation classroom teaching was not missed.

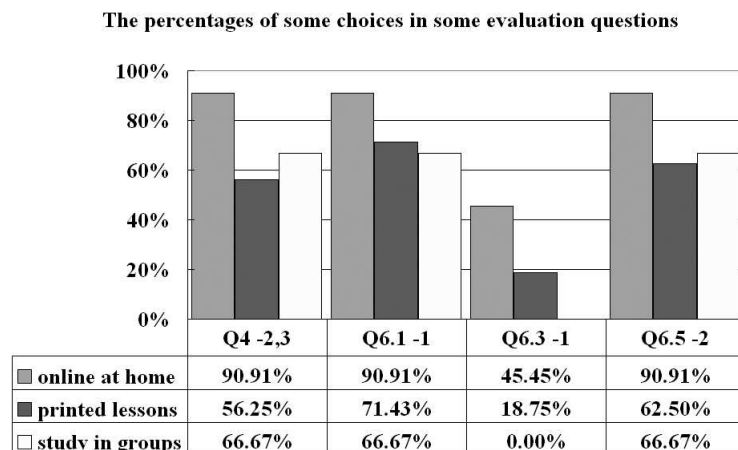


Figure 13.3: Analysis of responses to Question 2 (a).

90.91% of the students in group 2 (online learning at home) think that the exam preparation was easier because of Magister (Q4-2) and other examination subjects would be easier if there were e-learning systems such as Magister available (Q4-3), which is higher than that (56.25%) in group 3 (based on print-out lessons), and that (66.67%) in group 5 (studied in groups).

90.91% of the group 2 students feel that the instruction of statistics (with classroom teaching) caused more understanding difficulties to them in comparison with Stochastics (with e-learning system) (Q6.1-1), 71.43% of the group 3 students think so, and in group 5 only 2/3 of the students agree with this.

45.45% of the group 2 students had experiences of other e-learning systems (Q6.3-1). In group 3 only 18.75% of the students had experiences of other e-learning systems, and none of the group 5 students had other e-learning system experience.

As to Q6.5-2, during the preparation 90.91% of the group 2 students did not miss classroom teaching, 62.50% of group 3 students and 66.67% of the group 5 students did not get lost during the study in Magister.

- Figure 13.3 indicates that learning on-line coincides with a positive opinion about Magister and e-learning. Those who learned by means of print-outs or in study groups are less positive about e-learning.

Figure 13.4 shows the students' evaluation of technical performance characteristics (Q5) depending on the type of learning mode (on-line, print-out, groups). The technical criteria evaluated in Q5 are Q5.1 (ease of use), Q5.2 (clarity of the lessons), Q5.3 (system response), Q5.4 (stability), Q5.5 (layout), Q5.6 (satisfaction of working with Magister), Q5.7 (frequency of using the Graphical Lab), Q5.8 (ease of use of the Graphical Lab), and Q5 (overall rating). The evaluation grades range from 1 to 5 and stand for "very good", "good", "satisfactory", "sufficient" and "poor".

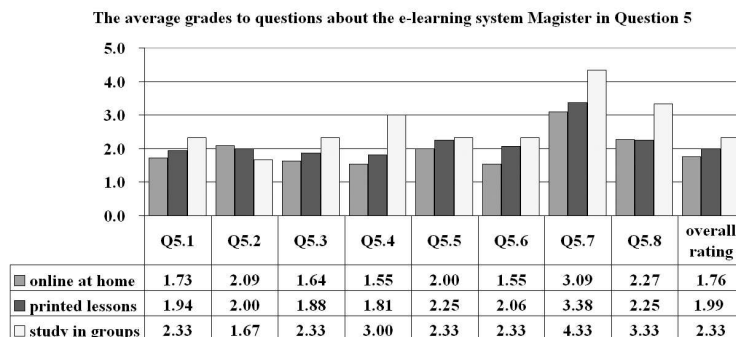


Figure 13.4: Analysis of responses to Question 2 (b).

The value of the overall rating in Figure 13.4 does not include the evaluation of Q5.7 and Q5.8. These two questions refer to the Graphical Lab ("frequency of use" and "ease of use"), which were not included since working with the Graphical Lab is not really necessary for passing the examination. The Graphical Lab has been developed for a further and deeper study and analysis of the stochastic procedures, particularly for those who plan to apply them.

- For questions listed in the diagram, the group of students who prepared online and therefore know Magister best, gave the best average scores, while the students who prepared in groups and probably know Magister less than the others, gave the worst scores. This coincides with the relation shown in Figure 13.4, the scores get better the more the students work with Magister.
- The above discussion about the results represented by Figure 13.4 did not include Q5.2 (comprehensibility), because for Q5.2 (clarity of the lessons) the response pattern is different. The students with group learning give the best average score, while the students with on-line learning give the worst average score. This behaviour reflects probably the fact that understanding achieved in a group is easier compared with learning alone, and students who learned in group or by print-outs note mainly the content of lessons and know less about the technical learning environment than those who learned from a monitor.

It is interesting that the only student who learned by means of examination protocols evaluated Magister's "layout" (Q5.5) with the worst score "sufficient". From his further responses it

appears that this student is an outlier. He has no e-learning experience (Q6.3-2), has never tried to use the Graphical Laboratory (Q5.7-5) and is indifferent with respect to working with Magister. In contrast, he expresses a positive attitude toward using e-learning without classroom teaching for other subjects in mathematics and natural science (Q6.6-1).

- Responses to Q2 show that participants of Stochastics examination prepared mainly by means of Magister and not by independent sources.
- The data represented in Figure 13.3 and Figure 13.4 indicates that previous e-learning experiences have an impact on the selection of the learning style. Besides, the more frequently the students work with Magister, the more benefits and better adaptation and the less difficulties and confusions they experience during the learning process. This can be concluded from the higher grades they gave to the usability and learnability of Magister. Moreover, these students also showed more interests in Stochastics as can be seen from their frequent use and evaluation of the Graphical Lab.

### 13.3.3 Analysis of the Responses to Question 3

Question 3	Is the e-learning system Magister a sufficient basis to understand the examination material?	96.77%
1	Yes, without restrictions.	33.33%
2	Yes, but the content should be more detailed and descriptive.	0.00%
3	Yes, but there should be more examples and exercises offered.	33.33%
4	Yes, but only together with an introduction / a preliminary meeting.	33.33%
5	No, the offered support is insufficient even with an introduction /preliminary meeting.	0.00%

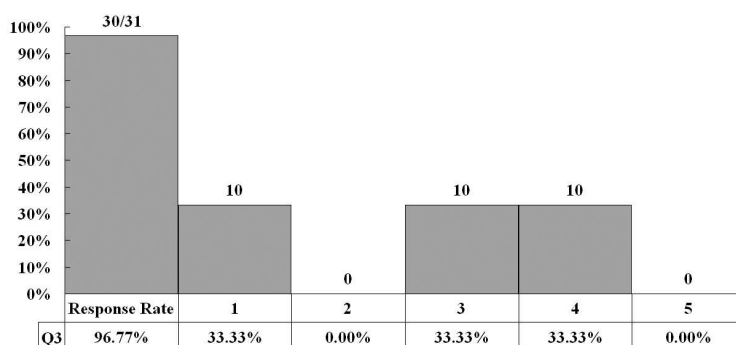


Figure 13.5: Diagram of responses to Question 3.

#### Comments:

This question aims at evaluating the overall learnability and usability of Magister by inquiring the sufficiency of teaching materials and learning services, and identifying possibilities for improvement. 30 of the 31 (96.77%) students responded and all of them positively. For one third of the students (group 1) Magister itself is a sufficient basis to understand the examination material without restrictions; another one third of the students (group 3) would prefer more examples and exercises; and the last one third (group 4) thinks that Magister should be supplemented by an introductory meeting. It is worthwhile noting that none of the students propose more detailed and elaborated lessons, and none of the students think that Magister is of no help for the preparation.

- The result indicates that Magister constitutes a satisfactory sufficient basis for learning Stochastics and for preparing the examination.
- Stochastics is a newly developed discipline for handling uncertainty, and Magister contains all resources for preparing the examination. Therefore the demand for more examples and exercises seem to be natural. The same holds for the demand for an introductory meeting that helps the students to get the right entrance to Stochastics and Magister. Figure 13.6 shows the relation between the above defined three groups and their responses to Q4-23 and Q6.5-2.

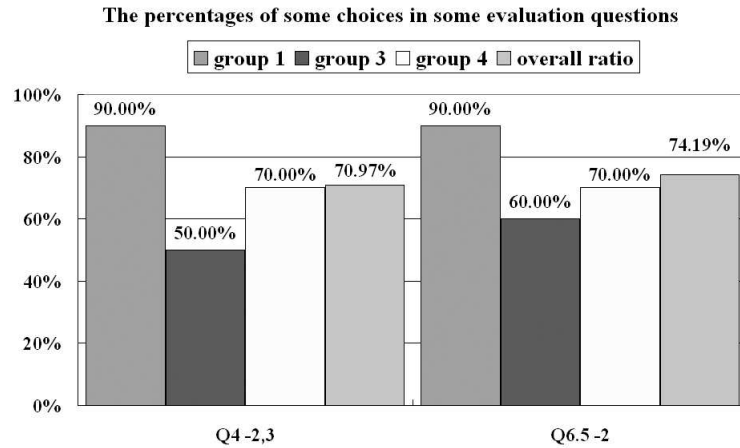


Figure 13.6: Analysis of responses to Question 3 (a).

As to Q4, 90.00% of the students in group 1 (Magister is a sufficient basis) think that the exam preparation was easier because of Magister (Q4-2) and other subjects would also benefit from e-learning programs such as Magister (Q4-3). The majority (70.00%) of the students in group 4 (an introductory meeting is necessary) have the same opinion. In group 3 (more examples and exercises should be better), however, the agreement decreases to 50%, yielding an overall agreement of 70.97%.

As to Q6.5, 90.00% of the group 1 students never missed classroom teaching (Q6.5-2), during the exam preparation. This is a higher percentage than that of students in group 3 (60.00%), and that of students in group 4 (70.00%). The total rate is 74.19%.

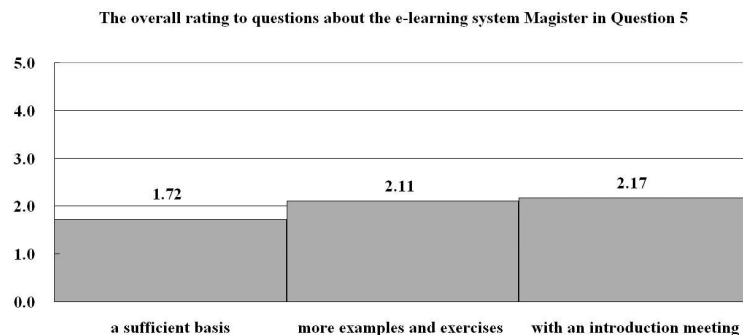


Figure 13.7: Analysis of responses to Question 3 (b).

Figure 13.7 shows the overall score to questions about Magister technical performance characteristics (Q5) given by students of the three above defined groups. The students, who feel Magister e-learning system is a sufficient basis for learning Stochastics, gave the highest overall score (1.72) to the technical environment of Magister. The scores of the other two groups are lower namely 2.11 and 2.17.

- These results show that most of the students, who think that Magister is a sufficient basis for learning Stochastics, highly appreciate e-learning for exam preparation and did not miss classroom teaching. The students, who think that additional examples and exercises are desirable and those who think that an introductory meeting is necessary, are less convinced about e-learning, but still a majority judges Magister as sufficient and beneficial.

### 13.3.4 Analysis of the Responses to Question 4

Question 4	Questions for the exam in the subject Stochastics:	100.00%
1	Would the exam revision have been easier if there would be a regular course in Stochastics like in Statistics?	22.58%
2	The exam revision was easier than in other subjects, because of the e-learning system Magister.	45.16%
3	It would be good if also for other examination subjects e-learning systems such as the Magister were available.	25.81%
4	The e-learning system Magister had no help for the exam revision.	6.45%

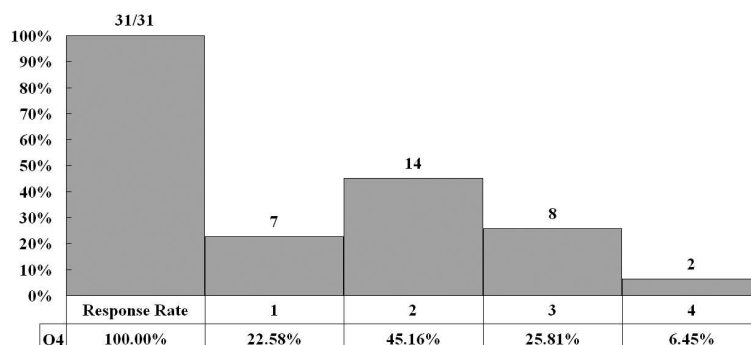


Figure 13.8: Diagram of responses to Question 4.

#### Comments:

Q4 aims at an overall evaluation of the usability and learnability of e-learning in general and Magister in particular. All participants responded to Q4. More than 20% of the students (group 1) feel that a regular course in classroom teaching would facilitate the preparation in Stochastics (Q4-1); in contrast more than 70% of the students (group 23) think that e-learning systems like Magister can make the exam preparation easier (Q4-23); two students (6.5%) state that Magister was of no help for the exam preparation.

- This result backs at least partly the previous results which reflected the positive attitude of the students towards Magister and e-learning.

The two students, who state that Magister was not helpful for exam preparation, answered all questions in the questionnaire, and their overall scores to Magister environment (Q5) are 2.17



and 2.5, respectively. Both of them use print-outs for the preparation (Q2-2), would like more examples and exercises (Q3-3), did not miss classroom teaching during Stochastics learning (Q6.5-2), and believe that e-learning would be useful also in other parts of mathematics and natural science (Q6.6-1). All these items are hardly consistent with their response Q4-4 and, therefore, we conclude that this choice was probably erroneously made.

Figure 13.9 shows the relation between the above defined group 1 and group 23 and various positive responses given to Q2, Q3 and some sub-questions of Q6.

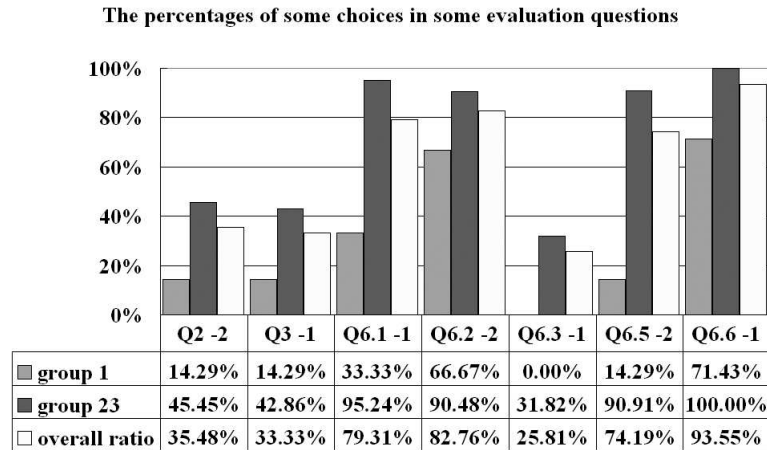


Figure 13.9: Analysis of responses to Question 4 (a).

As to Q2-2 (online learning at home), it was selected by 14.29% of group 1 in contrast to 45.45% of group 23. The situation is almost the same with Q3-1 (Magister as sufficient learning basis); it was selected by 14.29% of group 1 students, but by 42.86% of group 23 students.

Only one third of the group 1 students think that Stochastics instructed by Magister caused less comprehension difficulties to them compared with classroom teaching for statistics (Q6.1-1), but 95.24% of the group 23 students think so. The overall ratio is 79.31%.

As to Q6.2-2 (resolving of comprehension problems), two third of the group 1 students but 90.48% of group 23 students, think that problems may be overcome in Stochastics taught by Magister easier than in statistics taught by classroom teaching.

None of the seven students in group 1 (prefer a regular course in classroom teaching) has previous experiences with e-learning, but 31.82% of the group 23 students had e-learning experience besides Magister (Q6.3-1).

Six of the seven (85.71%) students of group 1 missed classroom teaching during the exam preparation (Q6.5-1) compared with only two of the 22 (9.09%) students in group 23. The overall ratio is 25.81%.

Also Q6.6 shows the difference between group 1 and group 23. Only 71.43% of the group 1 students think that other parts of mathematics and natural science can be taught by e-learning, while all students (100%) think so in group 23. The overall ratio is 93.55%.

Figure 13.10 shows the average score for the performance of Magister (Q5) given by the different groups of students (Q4-1, Q4-2, Q4-3 and Q4-4). The diagram shows that students who value Magister and e-learning highly gave higher average scores to the performance of Magister than students who prefer regular classroom teaching.

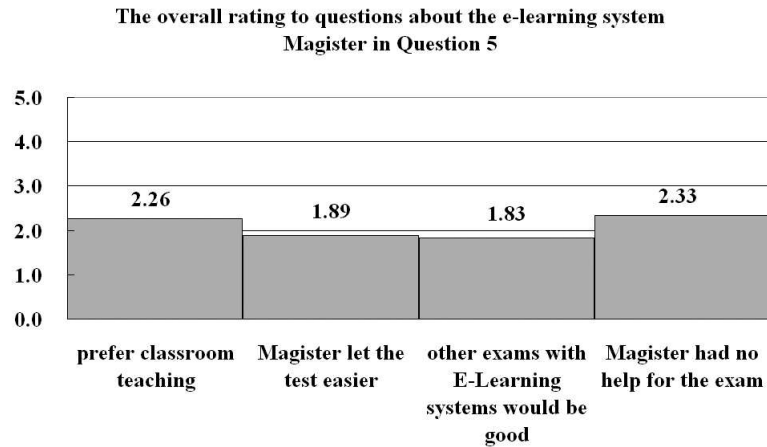


Figure 13.10: Analysis of responses to Question 4 (b).

- The response to Q4 divides the participants into two clear-cut parts with respect to attitudes, perceptions and experiences towards and with e-learning. More than 70% of the participants believe that Magister and e-learning are of advantage for the exam preparation. They have more experiences with e-learning, like to learn online at home, and judge the resources provided by e-learning system as being sufficient for the exam preparation. They experience less comprehension difficulties and are able to overcome the problems easier under the help of Magister. Consequently, these students value Magister highly and they are convinced of the benefits of e-learning. The smaller rest of students prefer classroom teaching and their attitude towards Magister, though positive, is less pronounced.

### 13.3.5 Analysis of the Responses to Question 5

Question 5: Questions about e-learning system Magister:

Figure 13.11 displays the average scores for the performance characteristics of Magister as given in Q5.

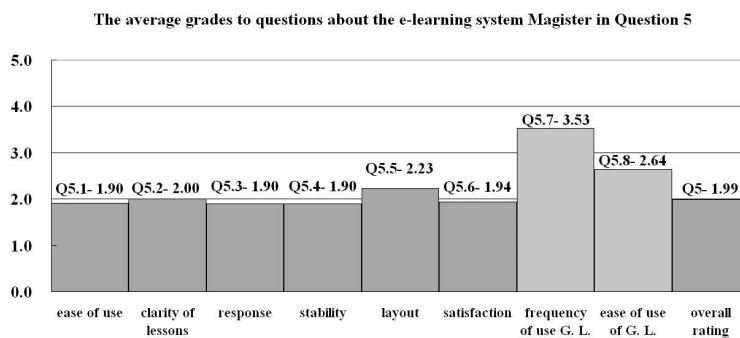


Figure 13.11: Analysis of responses to Question 5.

The grades in Q5 are the following:

1. Very good (very satisfactory, very often),

2. Good (satisfactory, less often),
  3. Satisfactory (neither satisfactory nor frustrating, sometimes),
  4. Sufficient (frustrating, seldom), and
  5. Poor (very frustrating, never).
- Most of the evaluation questions in Q5 are technical issues related to the usability and learnability of Magister virtual learning environment. The “ease of use”, system “response” and “stability” of Magister get the best score (1.90), followed by user satisfaction (1.94) and the clarity of the teaching contents (2.00), and finally the “layout” of the website (2.23).
  - As explained in the analysis of Q2, the overall rating of Q5 in the diagrams does not include the scores given to Q5.7 and Q5.8 which refer to the Graphical Lab that is not a part of the subject matter of the examination.
  - The average overall score of the technical performance criteria of Magister is 1.99. This result indicates that from the technical point of view, the usability and learnability, or in other words, the virtual classroom provided by Magister is judged by the participants to be “good”. Below the performance characteristics of Magister are analyzed individually.

Question 5.1	How do you rate the ease of use of Magister?	100.00%
1	Very good	25.81%
2	Good	61.29%
3	Satisfactory	9.68%
4	Sufficient	3.23%
5	Poor	0.00%

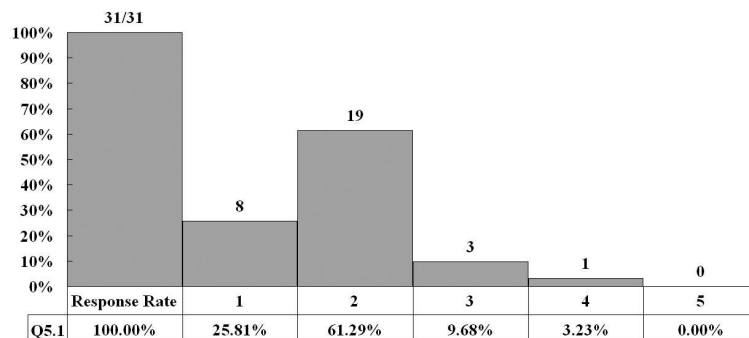


Figure 13.12: Diagram of responses to Question 5.1 (Average Score = 1.90).

### Comments:

Q5.1 evaluates the “ease of use” of Magister. All participants answered the question. More than 25% of the students (group 1) value the “ease of use” of Magister to be “very good”, more than 60% (group 2) “good”, less than 10% “satisfactory” (group 3), and one person selected only “sufficient”, but no person got the impression that Magister is difficult to use. The average score for the “ease of use” of Magister is 1.90, which is one of the best scores obtained in Q5.

- Based on this evaluation we conclude that Magister is ease of use and therefore meets one of the most essential requirements for an e-learning system.

Figure 13.13 shows the relation between the three Q5.1-groups with some selected choices namely Q2-2 (online learning at home), Q4-23 (e-learning supports comprehension) and Q6.5-2 (classroom teaching is not missed).

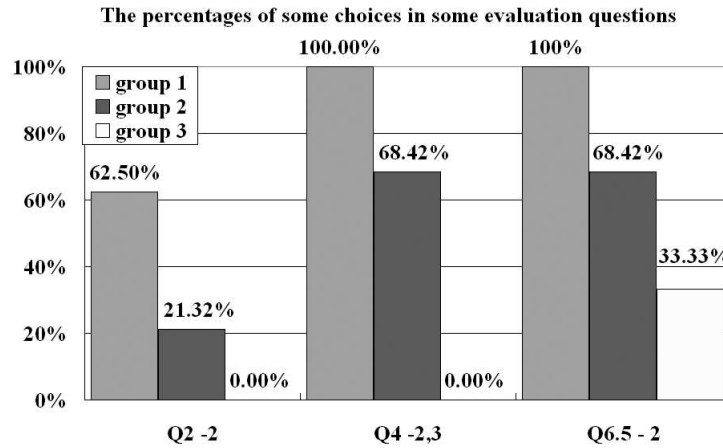


Figure 13.13: Analysis of responses to Question 5.1 (a).

62.5% students in group 1 selected online learning at home (Q2-2), and this is three times larger than the percentage in group 2. While in group 3 no student selected online learning.

As to Q4, all students in group 1 agree that e-learning supports comprehension of subjects like Stochastics (Q4-2 or Q4-3), two third of group 2 students think the same, while in group 3 all students prefer a regular course for Stochastics (Q4-1).

The reaction with respect to Q6.5 is similar, none of students in group 1 missed classroom teaching during Stochastics learning (Q6.5-1). In group 2 one third of the students missed classroom teaching, and this number increased to two third in the case of group 3.

Figure 13.14 shows the average scores given by the three groups of Q5.1 for other performance characteristics evaluated in Q5.

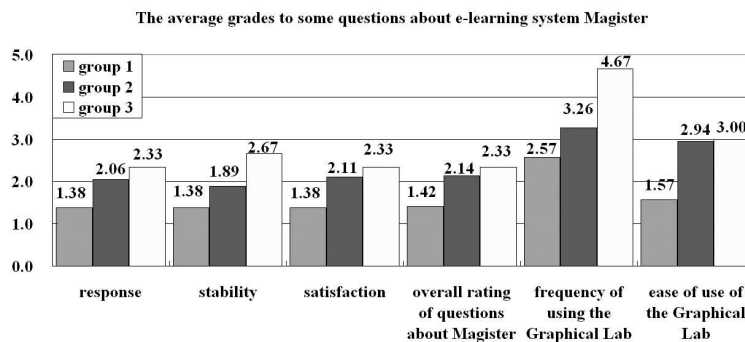


Figure 13.14: Analysis of responses to Question 5.1 (b).

As could be expected, the “ease of use” of Magister appears to be positively correlated to the system “response” (Q5.3), “stability” (Q5.4), “satisfaction of working with Magister” (Q5.6) and the overall rating. However, the relation between “clarity of the lessons” (Q5.2) and system “layout” (Q5.5) with the “ease of use” is not so clear from the data.

As to the single student who judged the “ease of use” of Magister to be only “sufficient”, he values the other characteristics either as “good” or “satisfactory” with an average score of 2.67, which is the second lowest score to Q5 in the questionnaire.

- As expected, the “ease of use” turns out to be a key characteristic for the acceptance of an e-learning system. It is closely related with most other performance characteristics, and it determines essentially the interests of the students in the subject as is indicated by the positive correlation between the “ease of use” and the “frequency of using the Graphical Lab”, which is taken as an indicator for the students’ interest to Magister and e-learning.
- Because of its significance, the data with respect to the “ease of use” shall also be used for a quantitative analysis of the acceptance of Magister. To this end the choices “very good” and “good” are merged and interpreted as acceptance of Magister. In order to check the students’ acceptance probability  $paM$ , the responses to the questionnaire are looked upon as a random sample of size  $n = 31$  with a number of  $x = 27$  acceptance. The acceptance probability shall be determined with a stochastic measurement procedure based on a reliability level  $\beta = 0.80$ . *Stochastikon Calculator* offers such a procedure online which yields the following measurement interval for  $paM$ :

$$\{p|0.78 \leq p \leq 0.94\}$$

Thus, with a probability of at least 78% the e-learning program Magister is accepted by the teacher candidates.

Question 5.2	How do you rate the clarity of the lessons of Magister?	100.00%
1	Very good	12.90%
2	Good	74.19%
3	Satisfactory	12.90%
4	Sufficient	0.00%
5	Poor	0.00%

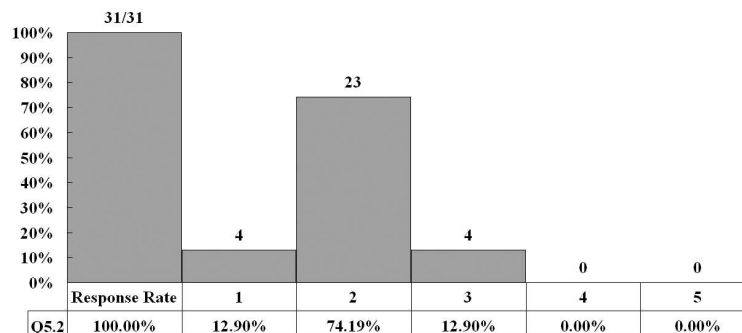


Figure 13.15: Diagram of responses to Question 5.2 (Average Score = 2.00).

### Comments:

Besides the “ease of use”, the quality of the teaching contents of an e-learning system is essential for its acceptance. All students responded to this question. 74% of the students evaluate the clarity of Stochastics lessons in Magister as being “good” (group 2), almost 13% as being “very good” (group 1), while the rest selected the grade “satisfactory” (group 3). No person evaluated the Stochastics lessons in Magister as being only “sufficient” or even “poor”. The “clarity of the lessons” got an average score of 2.00.

- By interpreting the grades, “very good” and “good” as acceptance of Stochastics lessons and the responses to the questionnaire as a random sample, it is possible to measure the acceptance probability  $paS$  for Stochastics lessons. Based on the reliability level  $\beta = 0.80$ , the same result as in case of  $paM$  is obtained, i. e., with a probability of at least 78% Stochastics as offered by Magister lessons is accepted by the teacher candidates.

All students in group 1 state that to overcome comprehension difficulties in Stochastics by Magister is easier than in statistics by classroom teaching (Q6.2-2). In group 2, more than 75% of the students agree about this, and even in group 3 two third of the student have the same opinion.

Figure 13.16 shows the relation between the above defined groups with respect to the evaluation of the clarity of Stochastics lessons and the choice made concerning Q3.

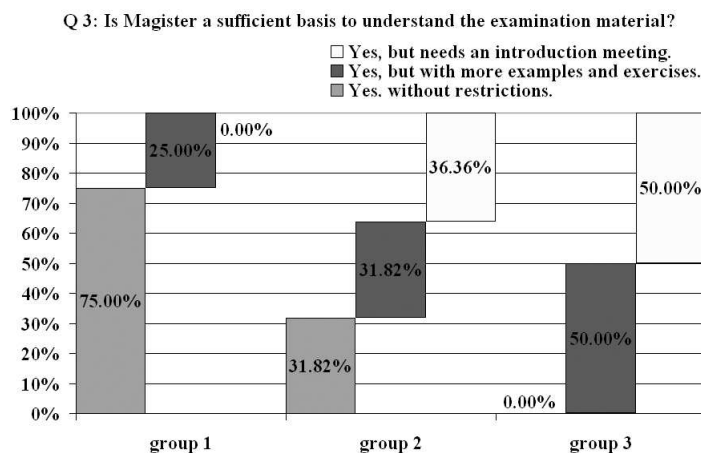


Figure 13.16: Analysis of responses to Question 5.2.

From the diagram we can see that 75% of the students, who valued the clarity of Magister lessons as being “very good” (group 1), consider Magister as a sufficient basis to understand the examination material (Q3-1). The remaining 25% think that it is sufficient but would appreciate more examples and exercises (Q3-3).

One third of the students of group 2 also consider the Magister lessons as being sufficient (Q3-1), another one third would appreciate more examples and exercises (Q3-3), while the remaining students think that an introductory meeting is necessary (Q3-4).

The situation in group 3 is different. None of the students considers the Magister lessons without further support as being sufficient (Q3-1). 50% of the students want additional examples and exercises (Q3-3) and 50% think that an introductory meeting is necessary (Q3-4).

- These results seem to be logical. A high degree of the clarity of Magister lessons causes less comprehension problems and hence requires less additional support by examples, exercises or face to face meetings. What is again surprising is the extremely good evaluation of the clarity of the Stochastics lessons when compared with similar courses in statistics. Stochastics deals with the same topic as statistics, however, its clarity seems to be better.

Question 5.3	How do you rate the response of Magister?	96.77%
1	Very good	30.00%
2	Good	50.00%
3	Satisfactory	20.00%
4	Sufficient	0.00%
5	Poor	0.00%

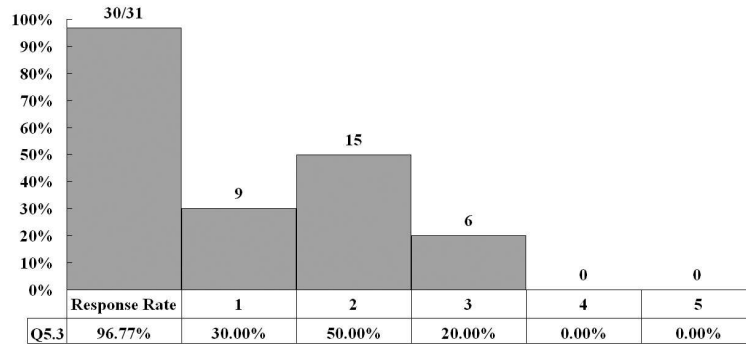


Figure 13.17: Diagram of responses to Question 5.3 (Average Score = 1.90).

### Comments:

This question is targeted to the purely technical response time and response accuracy of Magister. The system “response” is closely connected to the “ease of use”, as a long response time of a computer program is extremely tedious, and an irrelevant, erroneous or failed feedback is also frustrating.

30 of the 31 participants (96.77%) gave an answer. 30% of the students evaluate the system “response” as “very good” (group 1), 50% consider it as “good” (group 2) and the remaining 20% state that it is “satisfactory” (group 3). None of the students considers the system “response” as being only “sufficient” or “poor”. The average score for system “response” is 1.90, which represents another best score in Q5.

- Similar as the “ease of use”, the system “response” belongs to the most important characteristics for an e-learning system. The students’ evaluation shows that Magister is fast and accurate. This is probably due to the fact that PDF is the primary format for the teaching contents. Moreover, the structure of the learning environment is simple and clear, the programs developed for interactive teaching materials, interactive learning activities and teaching / learning / managing services are small, plain, sound and carefully tested. All these ensure a good response of Magister within any web-based running environment.
- Defining students’ satisfaction with the system “response” by the grades “very good” and “good”, and applying a stochastic measurement procedure with reliability level  $\beta = 0.80$  yields the following measurement interval for the satisfaction probability  $pr$ :

$$\{p|0.71 \leq p \leq 0.84\}$$

Figure 13.18 shows the relation between the above defined evaluation groups of Q5.3 and the positive responses in Q2, Q4, Q6.5 and Q6.6.

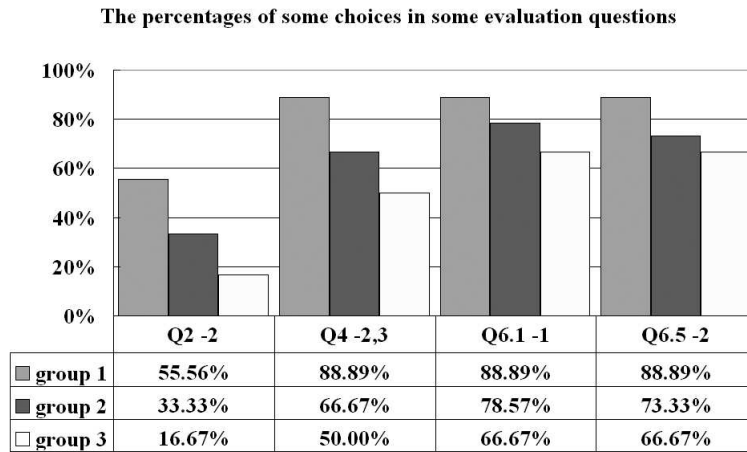


Figure 13.18: Analysis of responses to Question 5.3 (a).

55.56% of the group 1 students selected online learning at home (Q2-2), this percentage decreases to 33.33% in group 2 and to 16.67% in group 3.

88.89% of the group 1 students consider that Magister in particular and e-learning in general facilitate the exam preparation in Stochastics and in similar subjects (Q4-23), in group 2 the corresponding percentage decreases to 66.67% and in group 3 to 50%.

As to Q6.1, 88.89% of the group 1 students believe that statistics (with classroom teaching) caused more comprehension difficulties than Stochastics (with Magister) (Q6.1-1). 78.75% of the group 2 students think the same, and even in group 3 two third of the students have the same opinion.

Similar to Q6.5, 88.89% of the group 1 students never missed classroom teaching during Stochastics exam preparation, while 78.57% of the group 2 students and 66.67% of group 3 students have the same opinion.

One single student, who did not answer Q5.3, also did not answer Q5.4, Q5.8, Q6.1 and Q6.2. Actually, this student answered the least number of questions and gave the worst evaluation of Magister (Q5) with an average score of 2.75.

Figure 13.19 shows the relation between the three groups with respect to the evaluation of system “response” and some other performance characteristics investigated in Q5.

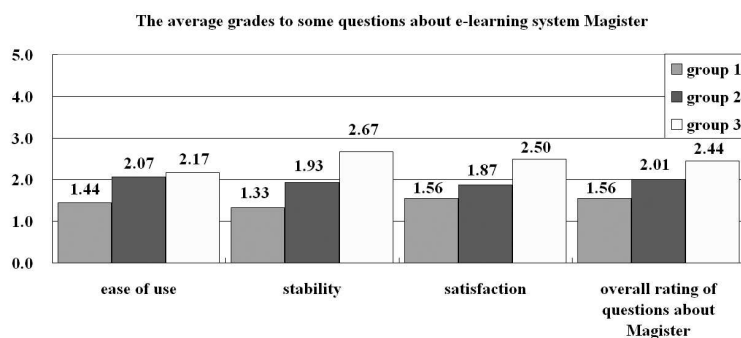


Figure 13.19: Analysis of responses to Question 5.3 (b).



The diagram indicate a positive correlation between the evaluation of the system “response” and the evaluation of the performance characteristics “ease of use” (Q5.1), “stability” (Q5.4) and “satisfaction of working with Magister” (Q5.6).

- Those students, who valued the system “response” highly, do the same with the other performance characteristics like “ease of use”, “stability” and “working satisfaction”. In all, the data indicate a positive correlation between the evaluations of the performance characteristics as considered in Q5.3.

Question 5.4	How do you rate the stability of the Magister as a software program?	96.77%
1	Very good	30.00%
2	Good	53.33%
3	Satisfactory	13.33%
4	Sufficient	3.33%
5	Poor	0.00%

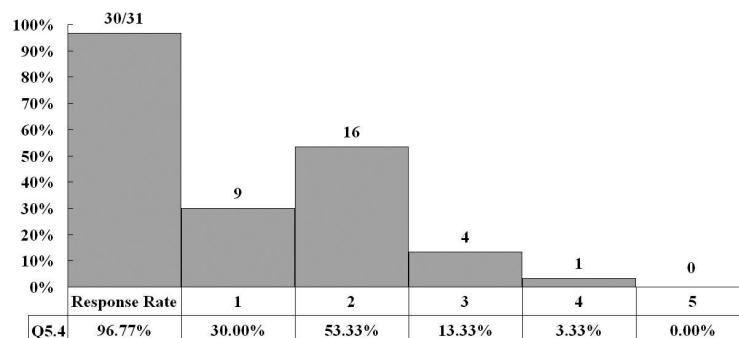


Figure 13.20: Diagram of responses to Question 5.4 (Average Score = 1.90).

### Comments:

The evaluation of the “stability” of Magister is similar to the evaluation of the other technical performance criteria. A large majority values the “stability” at least as “satisfactory” and only for a small minority the “stability” is “sufficient”. 30 of the 31 (96.77%) students made choices to this question. 30% of the students consider the “stability” of Magister as “very good” (group 1), more than half of the students value it as “good” (group 2), 13.33% as “satisfactory” (group 3), and one student as only “sufficient” (group 4). None of the students value the “stability” as being “poor”. The average score for the “stability” of Magister is 1.90.

- If the probability  $p_s$  of students’ satisfaction with the “stability” of Magister is defined by the responses “very good” and “good”, and a stochastic measurement procedure with reliability level  $\beta = 0.80$  is applied, the following measurement interval for  $p_s$  is obtained:

$$\{p|0.75 \leq p \leq 0.90\}$$

- Analogously to the other performance characteristics, the “stability” of Magister seems to be warranted. In other words, the students have experienced no problems with operating Magister. This is extremely important for an e-learning system as otherwise the students had to concentrate on technical problems and would neglect the actual contents.

In group 1, 88.89% of the students state that Magister in particular and e-learning in general facilitate the exam preparation (Q4-23). This proportion reduces to 66.67% in group 2 and to 50% in group 3.

The one student, who evaluates the “stability” of Magister as only “sufficient”, would prefer classroom teaching (Q4-1). The overall rating of Q5 by this student is 2.67, which is the second lowest grade in the questionnaire. As already mentioned, there is one student, who did not respond to Q5.3, Q5.4, Q5.8, Q6.1 and Q6.2, gave the worst overall score for Magister (2.75).

Figure 13.21 shows the relation between the four evaluation groups of Q5.4 and the evaluation to system “response” (Q5.3) and “satisfaction of working with Magister” (Q5.6).

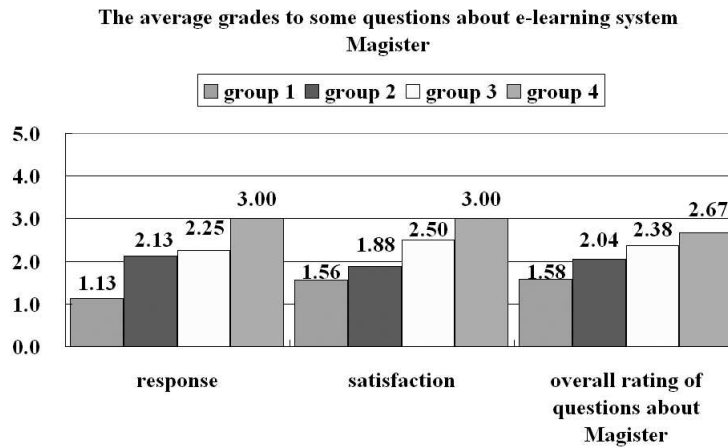


Figure 13.21: Analysis of responses to Question 5.4.

The diagram indicates again a positive correlation of the evaluations among different performance criteria “stability”, “response” and “satisfaction of working with Magister”. It also illustrates the relation between the evaluation of “stability” and the overall rating of the performance of Magister.

Question 5.5	How do you rate the layout of Magister?	100.00%
1	Very good	12.90%
2	Good	54.84%
3	Satisfactory	29.03%
4	Sufficient	3.23%
5	Poor	0.00%

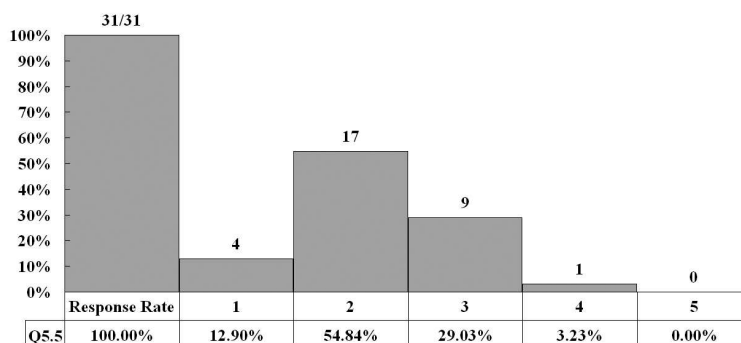


Figure 13.22: Diagram of responses to Question 5.5 (Average Score = 2.23).

**Comments:**

The evaluation of a system “layout” is much more subjective than the evaluation of other performance characteristics. Actually, all participants answered this question. Nearly 13% of the students like Magister “layout” very much (group 1), over half of the students consider it as being “good” (group 2), less than 30% of the students think it is “satisfactory” (group 3), and one student (3.23%) values it as being only “sufficient”. The average score is 2.23, the lowest value of all technical criteria evaluated in Q5.

- If again the grades “very good” and “good” are taken as an expression of students’ satisfaction with Magister “layout”, then the satisfaction probability  $pl$  can be measured by means of a stochastic measurement procedure. Based on a reliability level  $\beta= 0.80$ , the following measurement interval is obtained:

$$\{p|0.52 \leq p \leq 0.77\}$$

This result suggests a smaller degree of students’ satisfaction with the “layout” of Magister when compared with other technical performance characteristics.

In group 1, 75% of the students selected online learning at home (Q2-2), which is about double of the percentage in group 2 (35.29%), and about three times larger than that in group 3 (22.22%).

All students in group 1 consider Stochastics by Magister as being easier comprehensible than statistics by classroom teaching (Q6.2-2). In group 2, 75% of the students think the same, while in group 3 about two third of the students agree.

The scores given by the single student who considers the “layout” of Magister as being only “sufficient” (4) is not included in the further analysis since this student evaluates all technical items in Q5 as “good” (2) except the “layout”, therefore he/she is considered as an outlier and cannot constitute a “group”. Anyway his evaluation affected the average score of the “layout”.

Figure 13.23 shows the relation between the three evaluation groups of Q5.5 and the evaluation of some other issues in Q5.

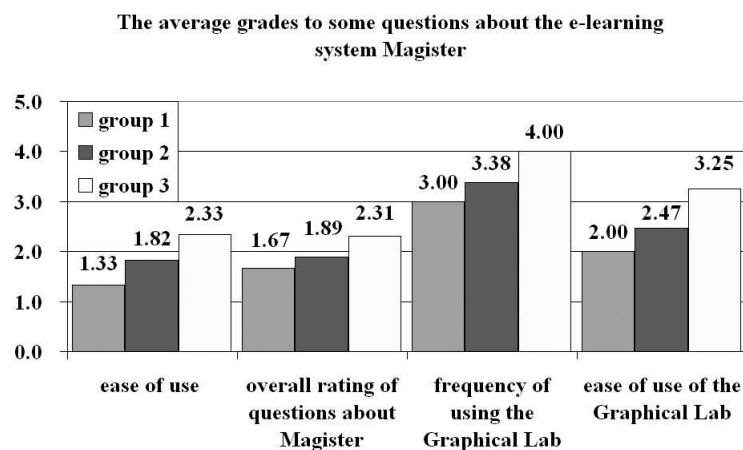


Figure 13.23: Analysis of responses to Question 5.5.

The data show the concurrence of the evaluation of the “layout” (Q5.5) and the “ease of use” (Q5.5) of Magister. The same holds for the overall evaluation (Q5) of Magister.

- The indicated relation between the evaluation of the “layout” and the “frequency of using the Graphical Lab” is rather interesting. It seems that a positive attitude towards the “layout” may increase the request to work with the system and thus try the Graphical Lab.

Question 5.6	How do you rate the working with the Magister?	100.00%
1	Very satisfactory	25.81%
2	Adequate	54.84%
3	Neither satisfactory nor frustrating	19.35%
4	Frustrating	0.00%
5	Very frustrating	0.00%

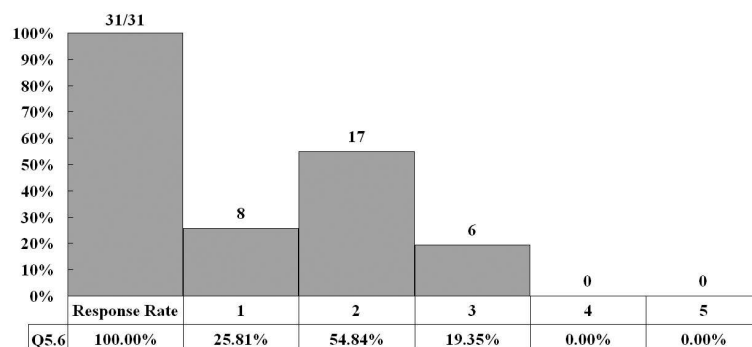


Figure 13.24: Diagram of responses to Question 5.6 (Average Score = 1.94).

### Comments:

This question aims at finding out how satisfactory it is to work with Magister. It is a comprehensive question and refers to the technical details as well as the contents and the representation of the lessons.

All participants responded to Q5.6. More than 25% of the students consider working with Magister to be “very satisfactory” (group 1). More than 50% state that it is “satisfactory” (group 2), and less than 20% say that it is “neither good nor bad” (group 3). For none of the students working with Magister was “frustrating” or even “very frustrating”. The average grade is 1.94.

- Considering the participants of the questionnaire as a random sample and the choices “very good” and “good” as representing satisfaction, the satisfaction probability  $pw$  can be determined by a stochastic measurement procedure. Based on the reliability level  $\beta = 0.80$ , the following measurement interval is obtained:

$$\{p|0.71 \leq p \leq 0.87\}$$

- The responses to Q5.6 show very clearly that Magister meets its purpose with respect to content and technical performance. None of the participating students felt frustrated or discouraged, but obviously most of the students like to work with Magister.

Figure 13.25 shows the relation between the three above defined groups with some evaluations concerning Q2, Q3, Q4, and Q6.

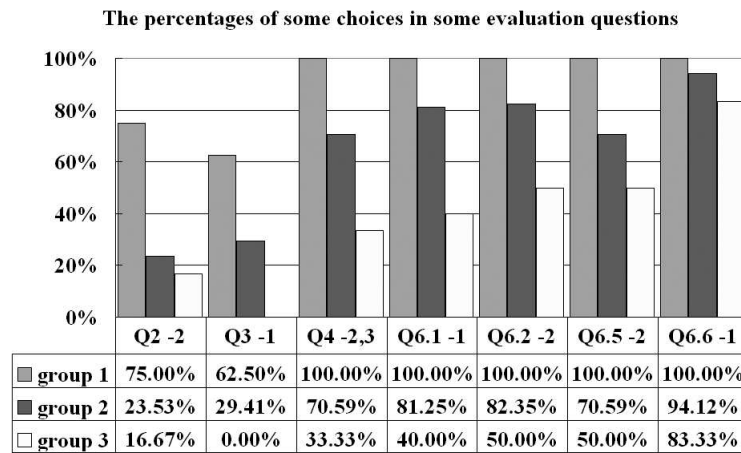


Figure 13.25: Analysis of responses to Question 5.6 (a).

75% of the group 1 students selected online learning at home (Q2-2), in contrast to 25% of group 2 students, and only 16% of the group 3 students.

62.5% of the group 1 students consider Magister as a sufficient basis for the exam preparation (Q3-1), in contrast 29.41% of the group 2 students agree, but none of the group 3 students.

All students in group 1 consider that Magister in particular and e-learning in general facilitate the exam preparation (Q4-23), the same opinion is expressed by 70.59% of the group 2 students, but only 33.33% of the group 3 students.

All group 1 students think that Stochastics by Magister causes less comprehension difficulties than statistics by classroom teaching (Q6.1-1), 81.25% of the group 2 students, and only 40% of the group 3 students agree.

The evaluation of Q6.2-2 is similar. All group 1 students state that it is easier to overcome the comprehension difficulties in Stochastics (Magister), 82.35% of the group 2 students and 50% of the group 3 students agree with this opinion.

All group 1 students did not miss classroom teaching during the exam preparation (Q6.5-2), the corresponding value is 70.59% for group 2 students and 50% for group 3 students.

Finally, based on their experiences with Magister all group 1 students agree that other branches of mathematics and natural science could be taught successfully by e-learning without classroom teaching (Q6.6-1); 94% of the group 2 students have the same opinion and still 83.33% of the group 3 students agree.

Figure 13.26 shows the relations between the above defined three evaluation groups of Q5.6 and the evaluation of other performance characteristics in Q5. Note that in Figure 13.26 G.L. stands for Graphical Lab.

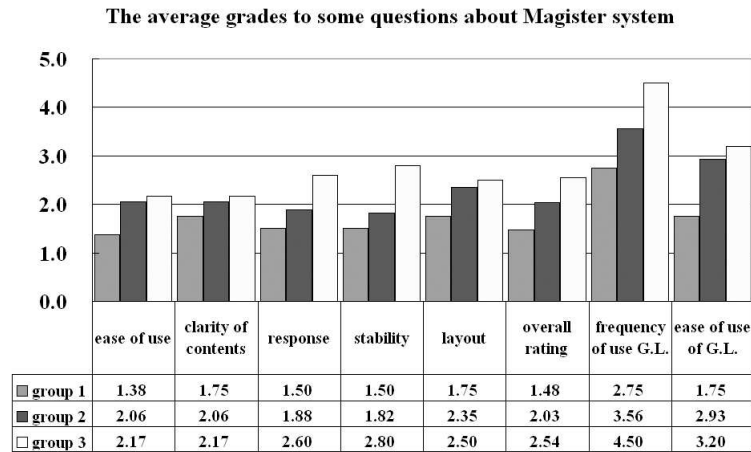


Figure 13.26: Analysis of responses to Question 5.6 (b).

- There is a strong indication for a positive correlation of satisfaction with the “ease of use” (Q5.1), system “response” (Q5.3), “stability” (Q5.4), and the overall rating of Q5, while there is less indication in the case of the “clarity of the lessons” (Q5.2) and “layout” (Q5.5) which seems to be plausible if one understands working as a more technical issue.
- Figure 13.25 and 13.26 show that the extent of “satisfaction of working with Magister” appears to be positively correlated with most of the other performance characteristics dealt with in Q5 and with the positive responses of most of the other questions in the questionnaire.

Question 5.7	Frequency of work with the Graphical Laboratory	96.77%
1	Very often	0.00%
2	Less often	16.67%
3	Sometimes	30.00%
4	Rarely	36.67%
5	Never	16.67%

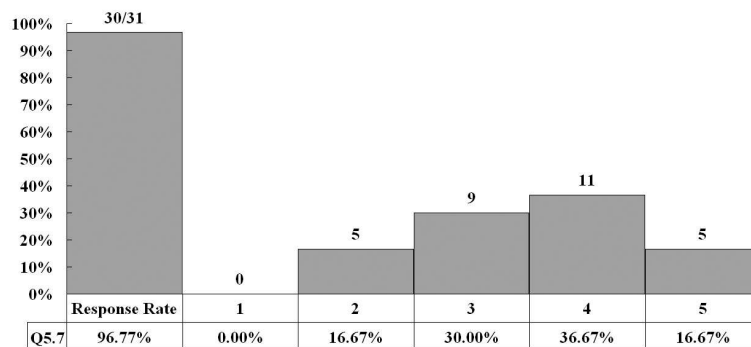


Figure 13.27: Diagram of responses to Question 5.7 (Average Score = 3.53).

**Comments:**

Familiarity with the Graphical Lab does not belong to the subject matter of the examination. Nevertheless, there are two questions related to the Graphical Lab in the questionnaire, which were used to assess the students’ interest for Stochastics.

- The Graphical Lab provides online dynamic interactive graphical representations for helping the students to learn, understand and practice stochastic procedures. The Graphical Lab can be used to visualize stochastic models and procedures and is therefore an important tool for supporting comprehension.

30 of the 31 students responded to the question about the "frequency of using the Graphical Lab" (Q5.7). None of the student used it "very frequently", 16.67% of them used it "less frequently" (group 2), 30% students "sometimes" (group 3), and 36.67% "rarely" (group 4), while 16.67% "never" worked with the Graphical Lab (group 5).

- This result is better than expected, as more than 80% of the participants at least tried to work with the Graphical Lab and nearly half of the students used it at least sometimes.

60.00% of the group 2 students consider Magister as being a sufficient basis for the exam preparation without restriction (Q3-1), in group 3 this percentage decreases to 44.44%, and in group 4 only 18.18% of the students, while in group 5 none of the students thinks that Magister is without restriction a sufficient basis.

All students in group 2 think that Stochastics by Magister is better qualified to overcome comprehension difficulties than statistics by classroom teaching (Q6.2-2), 88.89% of the group 3 students agree, 72.73% of the group 4 students, and still 66.67% of the group 5 students share the same opinion.

In group 2, none of the students ever missed classroom teaching because of learning in a virtual environment (Q6.5), 88.89% of the group 3 students and 63.64% of the group 4 students did not miss classroom teaching for Stochastics, and in group 5 the percentage decreases to 40.00%.

- These results show that working with Magister is satisfactory and that a majority of students get interested and try tools and contents of Magister which are not really needed for passing the examination.

Figure 13.28 shows the relation between the above defined four groups and some other performance characteristics evaluated in Q5.

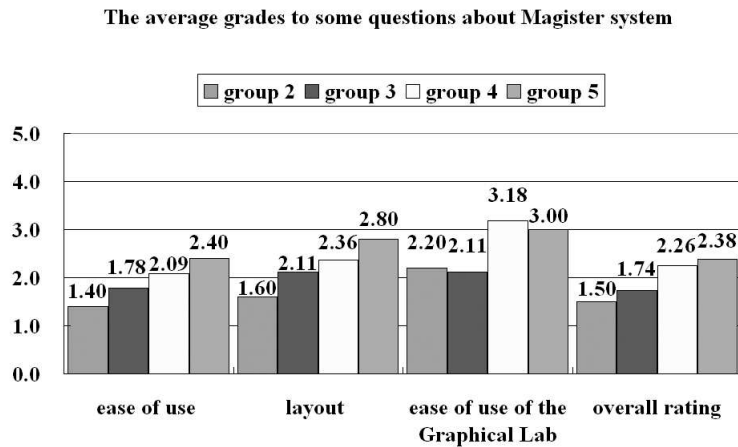


Figure 13.28: Analysis of responses to Question 5.7.

Figure 13.28 indicates a positive correlation between the “frequency of using the Graphical Lab” and the “ease of use” (Q5.1) of Magister, “layout” (Q5.5) of Magister and the overall rating to Magister (Q5). However, the relation between the “frequency of use” and the “ease of use” of the Graphical Lab seems not to be that clear.

- This result indicates that the interest for Stochastics together with Magister and the evaluation of “ease of use” as well as “layout” of the e-learning environment are closely dependent.

Question 5.8	How do you rate the ease of use of the Graphical Laboratory?	90.32%
1	Very good	10.71%
2	Good	32.14%
3	Satisfactory	39.29%
4	Sufficient	17.86%
5	Poor	0.00%

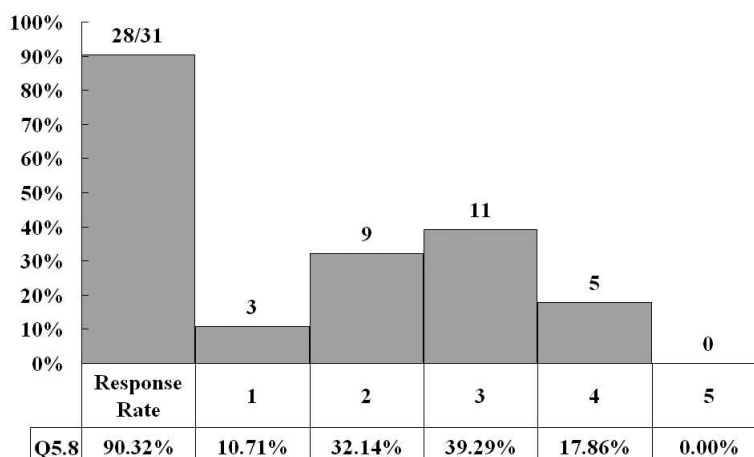


Figure 13.29: Diagram of responses to Question 5.8 (Average Score = 2.64).

### Comments:

The aim of this question is twofold: the response should be used for checking the consistency of the responses, and should yield some preliminary information about the usability of the Graphical Lab.

28 of the 31 participants responded to Q5.8, two of the three students who did not respond had never used the Graphical Lab (Q5.7), and the third person did not respond to Q5.7. There are three students who according to Q5.7 had no working experiences with the Graphical Lab, but nevertheless responded to Q5.8. Maybe these students visited the Graphical Lab, but did not actively work with it.

10.71% of the students consider the “ease of use of the Graphical Lab” as “very good” (group 1), 32.14% of the students think it is “good” (group 2), 39.29% judge it as being “satisfactory” (group 3) and 17.86% as being “sufficient” (group 4). None of the students selected the grade “poor”. The average score for the “ease of use of the Graphical Lab” is 2.64.

- The response to Q5.8 indicates that the Graphical Lab actually is able to meet its purpose namely to represent graphically stochastic models and stochastic procedure, and thereby



improve the comprehension of Stochastics. However, this result should be considered to be only preliminary because it is to be expected that most of the students had only little working experience with the Graphical Lab.

Figure 13.30 illustrates the relations between the four groups of Q5.8 and the evaluation of some other performance characteristics dealt within Q5.

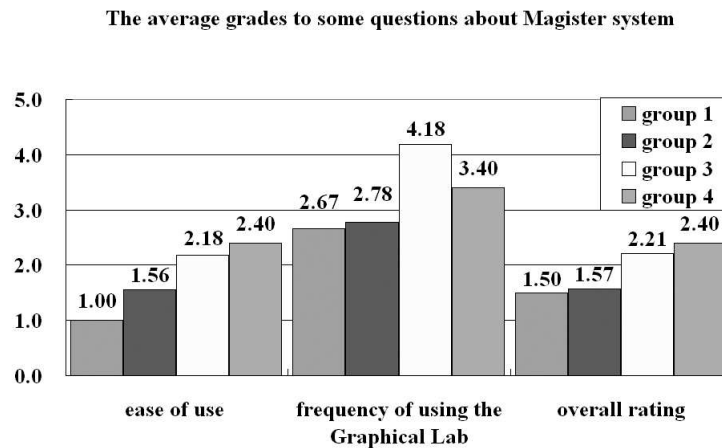


Figure 13.30: Analysis of responses to Question 5.8.

- Figure 13.30 indicates a positive correlation between the “ease of use” of the Graphical Lab” and of Magister. The same holds for the overall rating of Q5, while the relation to the “frequency of using the Graphical Lab” seems to be more complicated.

### 13.3.6 Analysis of the Responses to Question 6

Question 6: Questions about the use of Magister and other e-learning systems in teaching

This question aims to assess the general attitude of the students towards Magister in particular and e-learning in general. Q6 consists of seven questions which inquire the individual experiences and attitudes towards e-learning in order to obtain some information that may be used for deciding about the future offer of e-learning programs for the education of teacher candidates especially in the subject Stochastics and /or statistics.

Question 6.1	Which instruction Statistics (with classroom teaching) or Stochastics (with e-learning system) caused more understanding difficulties to you?	93.55%
1	Statistics	79.31%
2	Stochastics	20.69%

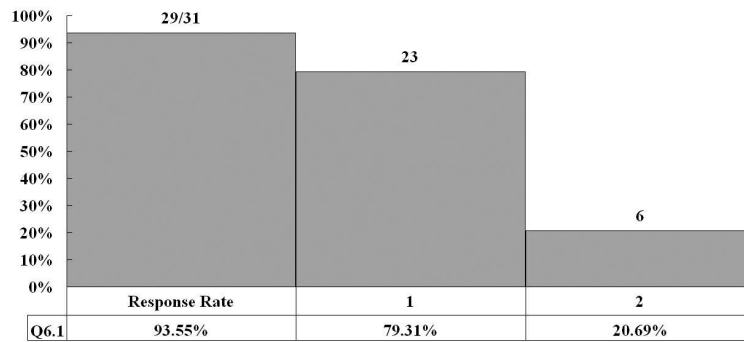


Figure 13.31: Diagram of responses to Question 6.1.

**Comments:**

Statistics is well-known for its comprehension difficulties and it is often said that these difficulties are caused by ambiguous concepts and missing principles. Stochastics on the other hand claims to be based on a unifying approach and on consistent concepts that make comprehension easier. Q6.1 aimed at scrutinizing this claim.

29 of the 31 participants responded to the question, nearly 80% (group 1) had less difficulties with Stochastics (by e-learning teaching) than with statistics (by classroom teaching), while about 20% had less difficulties with statistics (group 2).

- This result verifies the claim and shows that the students in fact appreciate Stochastics as a self-contained subject developed according to clearly formulated principles.
- In order to measure the probability that statistics causes more problems than Stochastics, the responses to the participants of the questionnaire are considered as a random sample of size  $n = 29$  with  $x = 23$  outcomes in favor of Stochastics with Magister e-learning. A stochastic measurement procedure based on a reliability level  $\beta = 0.80$  yields the following measurement interval for the desired probability:

$$\{p | 0.70 \leq p \leq 0.86\}$$

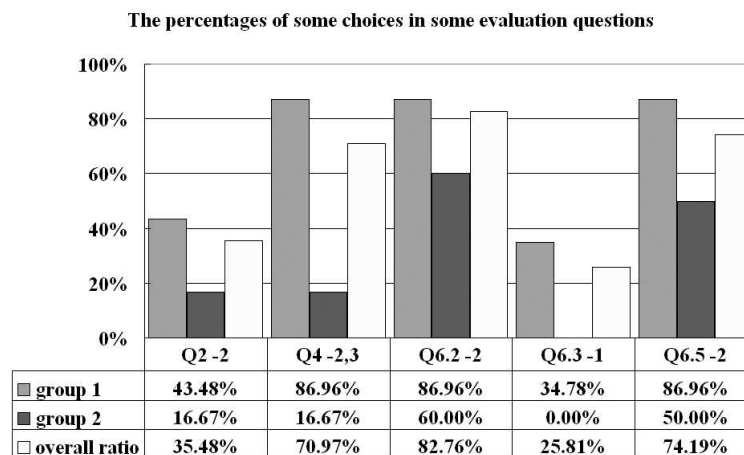


Figure 13.32: Analysis of responses to Question 6.1.

Figure 13.32 shows the relation between the students' attitude towards Stochastics vs. statistics and some other characteristics related to Magister in particular and e-learning in general.

As to Q2-2 (online preparation at home), 43.48% of the group 1 students selected online learning at home, in contrast to 16.67% of the group 2 students. As to Q4-23 (Magister facilitates exam preparation, e-learning also beneficial for other subjects) 86.96% of the students in group 1 had the impression that Magister facilitates the preparation and would appreciate e-learning for other subjects. In contrast, only 16.67% of the group 2 students share this opinion. As to the performance characteristics examined in Q5, the overall rating given by group 1 students is 1.88, and the overall rating given by group 2 students is 2.19. As to Q6.2-2 (comprehension difficulties can be overcome easier in Stochastics with Magister), 86.96% of the group 1 prefer Stochastics and Magister, and the same do 60% of the students of group 2, i. e., a majority of those students who had more comprehension problems with Stochastics nevertheless believe that these are easier to overcome by means of Magister than those in statistics by classroom teaching. As to Q6.3-1 (previous e-learning experience), 34.78% of the group 1 students had previous e-learning experiences, in contrast to none of the group 2 students. 86.96% of the students in group 1 confirm Q6.5-2 (classroom teaching was not missed during exam preparation) compared with 50% of the group 2 students.

A look at the students in group 2 reveals that none of them had previous experience with e-learning (Q6.3-1), 66.67% used print-outs for learning (Q2-3), and also 66.67% think that classroom teaching would have been beneficial for the exam preparation (Q4-1). However, even among these students, only one student opposed the claim that e-learning would also be beneficial for other mathematical or scientific subjects (Q6.6-1).

One of the two students who did not respond to Q6.1 did also not respond to Q5.3, Q5.4, Q5.8 and Q6.2. It is the student who responded to the least number of questions and gave the lowest overall rating 2.75 to the performance characteristics in Q5. Both students selected Stochastics because of recommendation (Q1-7), had no previous e-learning experiences, used print-outs for learning (Q2-3), and would like more examples and exercises in Magister (Q3-3). They missed classroom teaching during the exam preparation (Q6.5-1). Nevertheless, both of them think that other parts of mathematics and natural science can be taught successfully by e-learning (Q6.6-1).

- The responses to Q6.1 show that a vast majority of the participants consider Stochastics by Magister as the much better alternative. Even those students who had difficulties with Stochastics or with Magister consider e-learning as a serious substitute to classroom teaching.

As to Q2-2 (online preparation at home), 43.48% of the group 1 students selected online learning at home, in contrast to 16.67% of the group 2 students. As to Q4-23 (Magister facilitates exam preparation, e-learning also beneficial for other subjects) 86.96% of the students in group 1 had the impression that Magister facilitates the preparation and would appreciate e-learning for other subjects. In contrast, only 16.67% of the group 2 students share this opinion. As to the performance characteristics examined in Q5, the overall rating given by group 1 students is 1.88, and the overall rating given by group 2 students is 2.19. As to Q6.2-2 (comprehension difficulties can be overcome easier in Stochastics with Magister), 86.96% of the group 1 prefer Stochastics and Magister, and the same do 60% of the students of group 2, i. e., a majority of those students who had more comprehension problems with Stochastics nevertheless believe that these are easier to overcome by means of Magister than those in statistics by classroom teaching. As to Q6.3-1 (previous e-learning experience), 34.78% of the group 1 students had previous e-learning

experiences, in contrast to none of the group 2 students. 86.96% of the students in group 1 confirm Q6.5-2 (classroom teaching was not missed during exam preparation) compared with 50% of the group 2 students.

A look at the students in group 2 reveals that none of them had previous experience with e-learning (Q6.3-1), 66.67% used print-outs for learning (Q2-3), and also 66.67% think that classroom teaching would have been beneficial for the exam preparation (Q4-1). However, even among these students, only one student opposed the claim that e-learning would also be beneficial for other mathematical or scientific subjects (Q6.6-1).

One of the two students who did not respond to Q6.1 did also not respond to Q5.3, Q5.4, Q5.8 and Q6.2. It is the student who responded to the least number of questions and gave the lowest overall rating 2.75 to the performance characteristics in Q5. Both students selected Stochastics because of recommendation (Q1-7), had no previous e-learning experiences, used print-outs for learning (Q2-3), and would like more examples and exercises in Magister (Q3-3). They missed classroom teaching during the exam preparation (Q6.5-1). Nevertheless, both of them think that other parts of mathematics and natural science can be taught successfully by e-learning (Q6.6-1).

- The responses to Q6.1 show that a vast majority of the participants consider Stochastics by Magister as the much better alternative. Even those students who had difficulties with Stochastics or with Magister consider e-learning as a serious substitute to classroom teaching.

Question 6.2	In which of the two cases Statistics (classroom training) or Stochastics (e-learning system) the comprehension difficulties could be overcome easier?	93.55%
1	Statistics	17.24%
2	Stochastics	82.76%

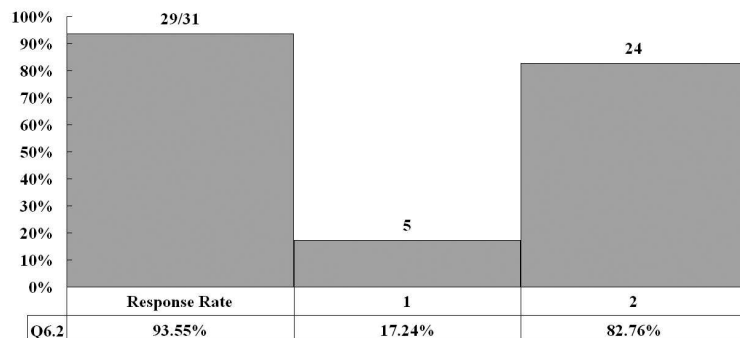


Figure 13.33: Diagram of responses to Question 6.2.

**Comments:**

Classroom teaching is generally looked upon as being more efficient than e-learning, because it is believed that problems and difficulties can be removed more easily in a face-to-face teaching environment. This may be true in general. However, Magister has been designed to represent a fully valid virtual classroom with almost all functions that are offered by classroom teaching. Q6.2 should therefore check the success of the virtual classroom simulated by Magister.

29 of the 31 participants responded to Q6.2 and 82.76% of the students are of the opinion that comprehension difficulties can be overcome with Stochastics by Magister easier in comparison

with statistics by classroom teaching (group 2) according to their former experiences. Only 17.24% of the students hold the opposite opinion (group 1).

- Applying a stochastic measurement procedure with reliability level  $\beta = 0.80$  to determine the probability of the event that overcoming comprehension problems in Stochastics by Magister is easier than in statistics in classroom, yields the following measurement interval:

$$\{p | 0.72 \leq p \leq 0.90\}$$

This result emphasizes the response to Q6.1, and it shows that the majority of the students consider Stochastics taught by Magister as not only being easier to understand, but also that occurring problems can be resolved easier in Stochastics than in statistics.

Figure 13.34 shows the relation between the opinion about Magister (Q6.2) and certain other characteristics.

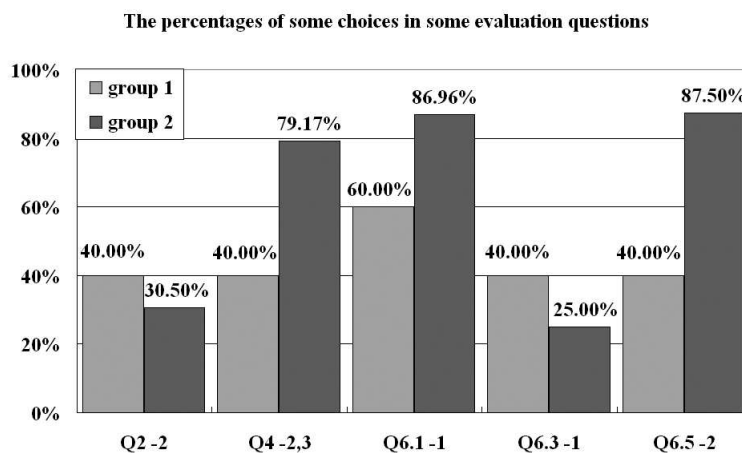


Figure 13.34: Analysis of responses to Question 6.2.

40.00% of the students in group 1 selected online learning at home (Q2-2), while in group 2 only 30.50% learned online. However, this rather surprising result may be explained by the small size of group 1 and the big size of group 2. Actually only two students in group 1 learned online compared with nine students in group 2. The response to Q6.3 (i.e., 40% of group 1 students against only 25% of group 2 students have previous e-learning experience) may be understood similarly as the response to Q2-2.

40% of the group 1 students think that the exam preparation was easier because of Magister and e-learning would be also good for other scientific subjects (Q4-23) although they judge the comprehension problems in Stochastics as more difficult to overcome (Q6.2-1). A similar unexpected result occurred with Q6.1-1, as still 60% of the students in group 1 state that Stochastics by Magister caused less comprehension problems. The overall rating to the performance characteristics examined in Q5 given by group 1 students is 2.17, and by group 2 students is 1.89. Finally, 40.00% of the group 1 students did not miss classroom teaching during exam preparation (Q6.5-2) compared with 87.50% of the group 2 students.

- These results show that students, who think that Stochastics taught by e-learning system Magister made it easier to overcome the comprehension difficulties, also think that e-learning systems such as Magister make the exam preparation easier, felt more familiar with the new virtual learning environment and appreciate Magister higher.

Question 6.3	Have you experience with other e-learning systems besides Magister?	100.00%
1	yes	25.81%
2	no	74.19%

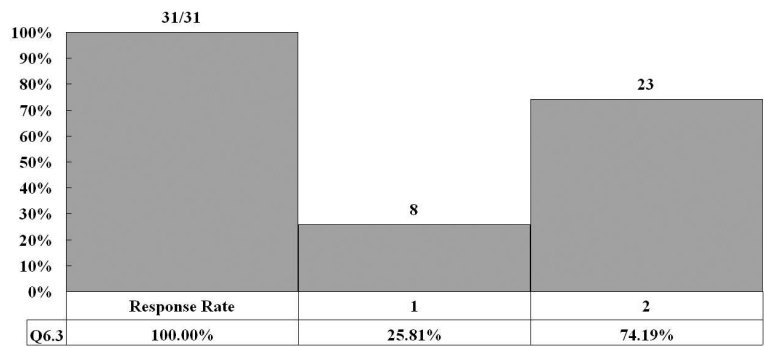


Figure 13.35: Diagram of responses to Question 6.3.

**Comments:**

During the last decade many e-learning programs in statistics were developed in Germany and a lot of them are offered for free in internet to be used as supplement for classroom teaching. Therefore, the fact is surprising that only about 25% of the participants (group 1) have experiences with e-learning systems besides Magister, while the rest (group 2) have not.

All participants responded to Q6.3 and in Figure 13.36 the relation between the experience with other e-learning system and some other characteristics is displayed.

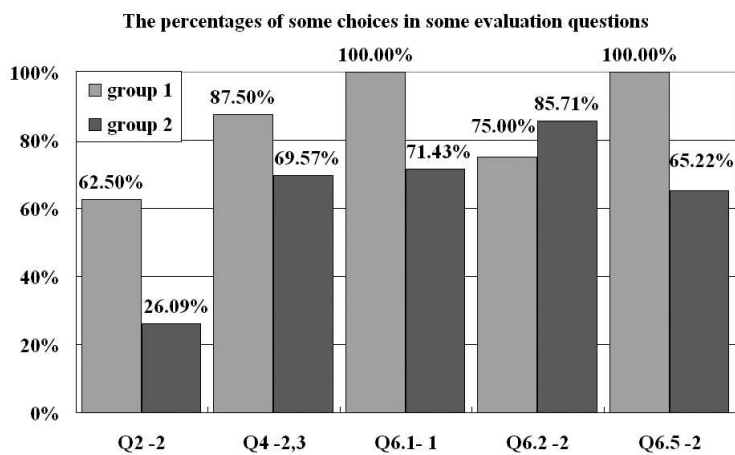


Figure 13.36: Analysis of responses to Question 6.3.

The most interesting question is whether previous experience with e-learning program would yield more negative or more positive responses with respect to Magister. 62.50% of the participants with e-learning experience (group 1) selected online learning, while only 26.09% of

the group 2 students learned online. The same to Q4-23 (Magister facilitates learning and e-learning would also be good for other subjects), Q6.1-1 (Stochastics causes less comprehension problems) and Q6.5-2 (classroom teaching is not missed when using Magister), percentages of group 1 are higher than those of group 2. The evaluation of the performance characteristics in Q5 is similar (average rating 1.96 in group 1 compared with 2.00 in group 2). To Q6.2-2 (comprehension problems can be overcome better by Magister) though the percentages exceed 75% in both groups, the pattern is opposite.

- Although the effect of e-learning experience is not really clear from these data, it is noteworthy that none of the eight students with e-learning experience (group 1) prefer a regular course (Q4-1) or miss classroom teaching for Stochastics (Q6.5-2), and all of them think Stochastics with Magister causes less comprehension problems (Q6.1-1).
- A positive correlation between Q6.3-1 (having other e-learning experiences) and Q6.1-1 (Stochastics causes less comprehension problems), and a negative correlation between Q6.3-1 (having other e-learning experiences) and Q6.2-1 (comprehension problems can be overcome better by Magister) are represented in Figure 13.32, Figure 13.34 and Figure 13.36, which may indicate that the amount of difficulties met by students, who have more e-learning experiences, are much less but these difficulties may not be easy to be handled.

Question 6.4	If yes, were the other e-learning systems offered as a support for classroom teaching or as a substitute for classroom teaching?	32.26%
1	As a support	70.00%
2	As a replacement	30.00%

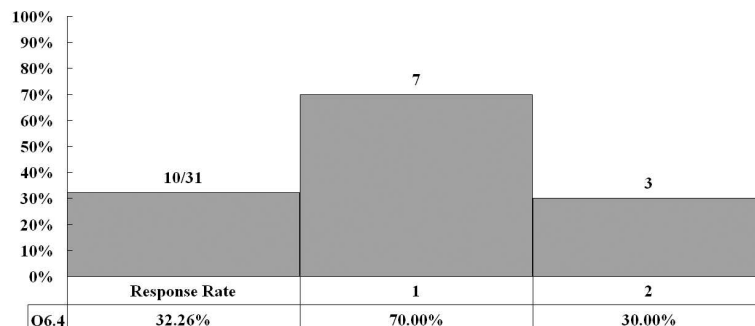


Figure 13.37: Diagram of responses to Question 6.4.

### Comments:

Q6.4 addressed those students with e-learning experience who should specify the type of e-learning program they had made their experience with.

There were eight students with e-learning experience (Q6.3-1), however ten students respond to Q6.4. These two students may have no personal experiences but may nevertheless hear about other e-learning systems. 70.00% of the students judge the other e-learning systems as supplements to classroom teaching, and 30.00% as replacement to classroom teaching.

- This result is consistent with the well-known fact that e-learning is often used to support classroom teaching but not to replace it.

Question 6.5	Have you missed classroom teaching during the exam preparation of Stochastics?	100.00%
1	yes	25.81%
2	no	74.19%

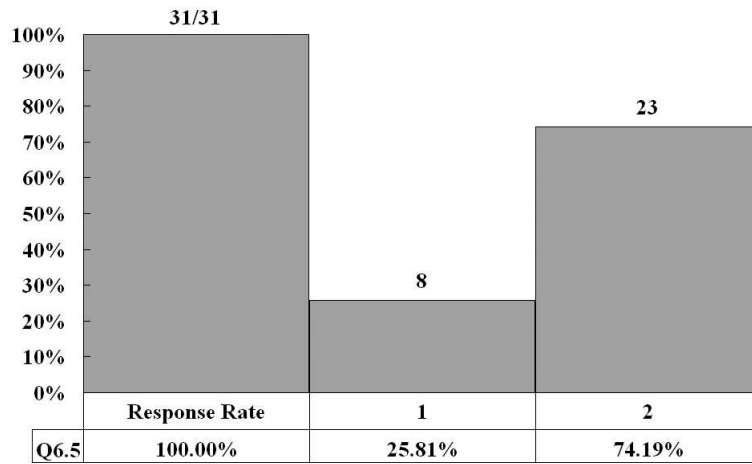


Figure 13.38: Diagram of responses to Question 6.5.

**Comments:**

The participants are accustomed to classroom teaching with making notes and using these notes during the exam preparation. This was impossible in the case of Stochastics and therefore it was of interest how many of the participants became successfully adapted to the new situation and how many missed their usual way of exam preparation. All students responded to Q6.5. About 25% missed classroom teaching (group 1) and probably felt occasionally lost during the preparation, while a majority (group 2) of about 75% accepted e-learning without feeling lost within the virtual teaching environment during exam preparation.

- Applying a stochastic measurement procedure with reliability level  $\beta= 0.80$  to determine the probability of the event that the student did not miss classroom teaching when preparing for the Stochastics examination, yields the following measurement interval:

$$\{p|0.63 \leq p \leq 0.81\}$$

This result indicates a hardly expected acceptance of Magister among the participating teacher candidates with three quarters of them had no any e-learning experiences before Magister, and it raises the question about the possible reasons for this success, which will be discussed later.

Next the differences between the responses of group 1 students and group 2 students are examined by means of those responses which indicate acceptance of e-learning in other questions and it is to be expected that group 1 students should agree less.

Figure 13.39 shows the relation between the selections about Magister (Q6.5) and certain other characteristics.



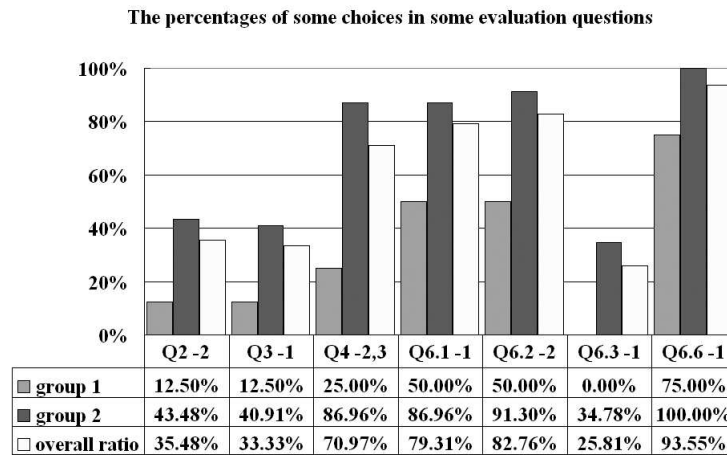


Figure 13.39: Analysis of responses to Question 6.5.

Only 12.50% of the group 1 students selected online learning at home (Q2-2), in contrast to 43.48% of the group 2 students. A similar result obtained with respect to Q3-1, 12.50% of the students in group 1 agree that Magister is a sufficient basis for the exam preparation in contrast to 40.91% of the group 2 students. 25.00% of the students in group 1 agree in the statements that Magister facilitates the exam preparation (Q4-2) and e-learning programs such as Magister would also be beneficial for other subjects (Q4-3) in contrast to 86.96% of the group 2 students. As to the performance characteristics examined in Q5, the overall rating given by group 1 students is 2.30, and by group 2 students 1.88.

As to comprehension difficulties (Q6.1), 50.00% of the group 1 students think that Stochastics by Magister causes less comprehension problems than statistics by classroom teaching, while in group 2, 86.96% of the students think so. The response to Q6.2-2 is similar, 50.00% of the group 1 students think that it is easier to overcome problems in Stochastics by Magister than in statistics by classroom teaching, and the corresponding percentage among group 2 students is 91.30%. As to Q6.3-1, none of the group 1 students has other e-learning experiences except Magister, but 34.78% of the group 2 students have. As to Q6.6-1, three quarters of the students in group 1 and all the students in group 2 think that other parts of mathematics and natural science can be taught successfully by e-learning programs such as Magister.

- Similar as with other questions, Q6.5 divides the participants in two groups, those who accept Stochastics and Magister without reservation consider the content, design and implementation of this e-learning project as superior to statistics by classroom teaching. However it is surprising that even among the students of group 1 who miss classroom teaching, the agreement with Stochastics and Magister is rather high.
- The size of the two groups is similar as in some of the previous questions, the group of Magister fans accounts for about 75% of the students, while the group of those who are more critical about Magister involves about 25%.

Question 6.6	Could you imagine based on your learning experience in Magister, that other parts of mathematics and natural science could also be taught successfully by using an e-learning system without classroom teaching (possibly with some few attendance meetings, similar to the introduction in Stochastics and Magister by Professor von Collani)?	100.00%
1	yes	93.55%
2	no	6.45%

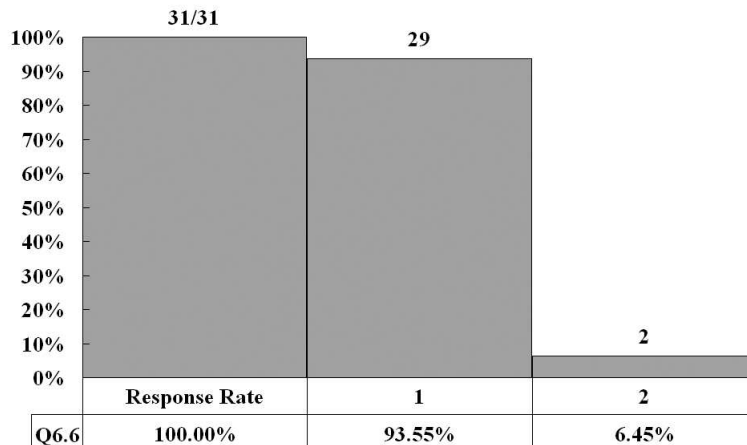


Figure 13.40: Diagram of responses to Question 6.6.

### Comments:

The participants had prepared themselves for the examination in Stochastics solely based on the e-learning program Magister. It follows that they have intensively worked with Magister and therefore they represent a most adequate population for checking the possibilities of e-learning for tertiary education. All students responded to Q6.6, and 93.55% of the students believe that other parts of mathematics and natural science can successfully be taught by means of an e-learning system. Only 6.45% students, i. e., two of the 31 students, disagree.

Both the two students, who doubt the ability of e-learning systems, have no e-learning system experience except Magister (Q6.3-2), feel like classroom teaching for Stochastics (Q4-1), and think Magister is sufficient learning basis only together with an introduction meeting (Q3-4). As the sample of this group is only 2, it is hard to identify special relations represented by the selections of this group.

- Applying a stochastic measurement procedure with reliability level  $\beta = 0.80$  to determine the probability of the event that the student feels confidence in a successful e-learning to other scientific subjects based on their Magister experiences, yields the following measurement interval:

$$\{p | 0.82 \leq p \leq 0.97\}$$

- This result should be taken as an overwhelming agreement to e-learning, but it also indicates the importance of a virtual classroom environment and the harmonization of content and technical design and implementation of an e-learning system like it is done by Magister.

Question 6.7	Which branches of mathematics may be considered?	96.77%
1	Analysis	0.00%
2	Linear Algebra	10.00%
3	Numerical Analysis	6.67%
4	Geometry	20.00%
5	Algebra	16.67%
6	Measure Theory	0.00%
7	Probability	33.33%
8	Experimental Physics	0.00%
9	Theoretical Physics	13.33%

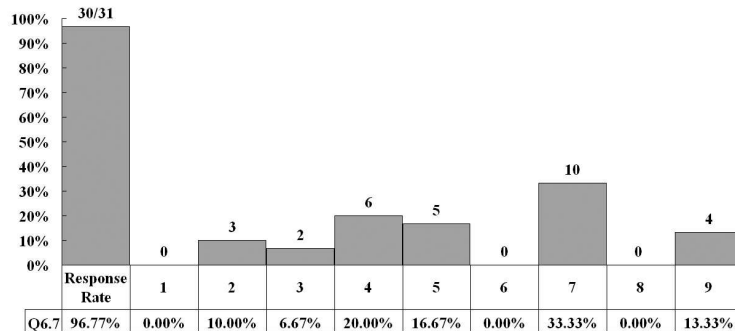


Figure 13.41: Diagram of responses to Question 6.7.

### Comments:

This question aims at checking the seriousness of responses to Q6.6. 30 of the 31 participants responded and marked six of the nine listed subjects.

- The three subjects which got no vote are analysis, measure theory and experimental physics. This seems to be adequate especially in the case of experimental physics.
- Probability theory got the most votes and again this appears to be natural since probability theory is the basis for Stochastics.

## 13.4 Further Discussion and Conclusion

One aim of the evaluation is to examine whether the better coordination of content-environment, design-implementation and target-outcome of Magister are reflected in the responses to the questionnaire.

Another aim of the questionnaire is to verify the conclusions gained by the comparison between the examination scores obtained in statistics on the one hand and Bernoulli Stochastics on the other.

Finally, the questionnaire should help to answer the question to which extent e-learning can be used as an efficient alternative to classroom teaching especially for newly emerging subjects that are based on mathematics and for which no adequately educated teachers and teaching material are available.

The questionnaire consists of six questions for assessing the usability and learnability and also the popularity of Magister in particular and e-learning in general from the viewpoint of the

learners. The different questions are not independent, but are closely related and can therefore also be used for checking the consistency of the responses. The sample size is rather small, therefore allows only rather imprecise statements. Nevertheless, the conclusions to be drawn from the empirical results shall be based, if possible, on stochastic procedures. However, the reliability level will be selected as  $\beta = 0.80$  which is rather low but reasonable when taking into account the small sample size.

The overall response rate to the six questions is 98% which shows that the questions were understandable and the questionnaire had a reasonable length. We conclude that the responses reflect without noteworthy bias the opinions of the participants. In the following, some points of interest with respect to the questionnaire are briefly treated.

### 13.4.1 Selection Criteria

The first issue that shall be examined refers to the reasons for selecting Stochastics instead of statistics. The responses show that the recommendations of fellow students represent the main reason for selecting Stochastics. We conclude that a majority of the students decided on learning Stochastics without having seriously engaged in Stochastics, while some few had a vague idea of some characteristic features of Stochastics. This means that the selection of Stochastics and the examination scores are more or less stochastically independent.

### 13.4.2 Mode of Learning

More than half of the students prepared their examination primarily on the basis of print-outs of the lessons, while one third of the students studied Stochastics online at home. One tenth of the students studied mainly in groups, and only one student stated that examination protocols of other students were his main learning materials. None of the students mentioned online learning in a CIP-pool. Note that the questionnaire allowed to specify only the most appropriate response, therefore someone who learned primarily from print-outs or in groups, may have frequented Magister online as well.

The responses confirm that all participants have Magister experiences, as even the single student who learned mainly from former protocols, answered all questions about Magister in the questionnaire. The differences in the main learning methods refer to the intensity of using a PC and the internet. The fact that none of the students mentioned learning in a CIP-pool and only one third learning on-line at home, while more than half of the students preferred to learn from print-outs might indicate that teacher candidates are less accustomed to use the internet for learning and preparing. However, whether or not such a conjecture is correct, should be investigated separately.

Almost three quarters of the student state that they had no e-learning experience except Magister. This might be an explanation to the above fact that nearly two third of the students did not select study online as their major learning method, but prefer to learn from printed media, as they were used during more than 15 years to classroom teaching and learning. This appears to be a reasonable behavior since a completely new learning mode such as online learning may lead to frustration, confusion and difficulties. Some students may feel lost during preparation based on e-learning because they miss the familiar classroom teaching. This effect could also be caused by the restrictions of the technical factors (equipment, environment, etc.) and the specific personal factors (learning habit, preference and (e-) learning experience). Therefore, it is not surprising that 64.52% of student did not select online learning and half of the students

accomplished their preparation mainly by means of the printed documents. Actually, the design of Magister had taken these situations into account by providing a convenient environment to handle these diversities.

### 13.4.3 Appreciation of Magister Learning Environment

The technical issue of the Magister virtual learning environment was assessed by the characteristics “ease of use”, “clarity of the lessons”, system “response”, “stability”, “layout” and “satisfaction of working with Magister”. The overall assessment of technical quality of Magister with reliability level 80% yields a value between 1.41 and 2.56, which seems to be rather good.

Figure 13.42 shows the ratio of positive responses to Magister. The diagram indicates that according to learners, Magister is well suited for learning Stochastics.

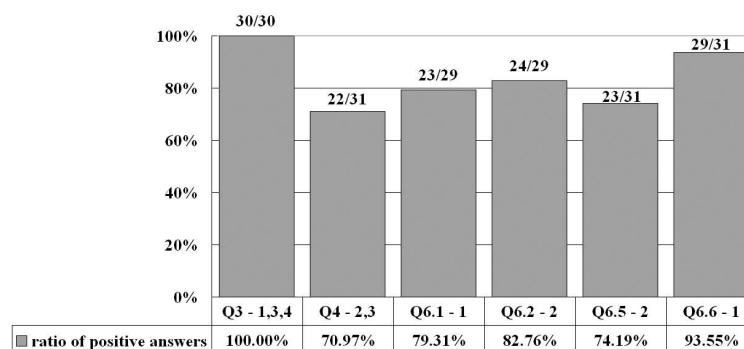


Figure 13.42: Analysis of responses to Question 3, 4 and 6.

The positive responses to the questionnaire and the positive scores obtained in the Stochastics examination indicate not only a high degree of acceptance of Magister, but also that e-learning can be a fully valid substitute for classroom teaching in Stochastics education. These results are in contrast to some experiences with e-learning programs in mathematics and some doubts about e-learning [67]. The reasons for the observed difference may be manifold, but some points in favor of Magister can be conjectured as follows:

1. Stochastics is a unified subject that is based on an unambiguously defined concept probability, on a clear model for uncertainty, and a methodology that is built up on prediction procedures. In contrast, statistics is a methodology that does not follow clear rules and, moreover, the concept probability has several inconsistent interpretations. The clarity of Stochastics on the one hand, and the blur of statistics on the other, facilitates teaching and learning of the former.
2. In contrast to many e-learning programs in statistics, Magister is designed to include a complete virtual classroom environment, which in case of understanding problems offers at least as much support as a real classroom. The possibility to get help whenever it is necessary does probably also contribute to the acceptance of Magister.
3. The fact that the development of content and technical issue of Magister lay in one hand has led to a more self-contained e-learning system, which facilitates the control of quality of the whole project.

4. Finally, in this evaluation, teacher candidates in mathematics are the target group of Magister. They have a rich mathematical background and enough self-study abilities. Therefore, if the learning environment is satisfactory, the learning support is strong and the learning material is sufficient, then achieving the learning goal in Stochastics should be not difficult for them, even in the case that the learning mode is completely new.

#### 13.4.4 Acceptance of E-Learning

Magister offers e-learning instead of classroom teaching for Stochastics, because there are no textbooks or trained teachers in Bernoulli Stochastics. Acceptance by the teacher candidates is one of the necessary conditions that Magister can meet its aim. The examination scores indicate that the aim is actually achieved. The questionnaire should support this conclusion, by providing information about how the teacher candidates used Magister, which impression they got about Magister and how the quality of Magister is assessed by them. The questionnaire shows that almost all of the students accepted e-learning by Magister and appreciate to work with Magister. Moreover, based on their Magister experiences almost all students agree that e-learning could replace classroom teaching, and, finally, all of them passed the examination successfully.

The questionnaire also reveals relations between the selected learning mode and previous e-learning experiences, the benefits obtained from Magister, the personal e-learning experiences with Magister, and the quality of the Magister learning environment. To sum up, the responses to the questionnaire by the participants show that Magister is widely accepted as a substitute for classroom teaching in statistics.

#### 13.4.5 Consistency of Examination Scores and Questionnaire Responses

The questionnaire about Magister reveals that there are two groups of students with rather different attitudes towards Magister and e-learning. There is a larger group of about 70%-85% of the students, and a smaller group of the remaining students. This division can be observed from the responses to Q4 (helpfulness of Magister for exam preparation), Q6.1 (understanding difficulties of Stochastics in Magister), Q6.2 (solving comprehension problems in Magister) and Q6.5 (missing classroom teaching during exam preparation) (see Figure 13.42).

Students in the larger group appreciate Magister by a much higher percentage of positive responses. They prefer e-learning for Stochastics to classroom teaching; they feel that Magister facilitates the exam preparation and consider that similar e-learning systems as Magister would also be beneficial to other subjects; they met less comprehension difficulties with Stochastics by Magister than with statistics by classroom teaching; they seldom or never miss classroom teaching when learning in the new virtual environment; they overcome comprehension difficulties easier in Stochastics with Magister than in statistics by classroom environment, and they are confident that e-learning could be used successfully instead of classroom teaching for other mathematical and natural scientific subjects.

Students in the smaller group would like to have a regular course for Stochastics; they learned Stochastics mainly on the basis of print-outs or in groups, they experienced more problems and found that it was more difficult to overcome the problems in the Magister environment than in a classroom environment. Moreover, they missed classroom teaching during the exam

preparation. Finally, their confidence in e-learning and in Magister is less pronounced than that of the larger group.

In order to check the consistency of the results of the two evaluations performed here, the group of students, who appreciate Magister, is compared with the group of students, who got an honor result in the examination. Table 12.3 is taken from Chapter 12 “Evaluation by Examination Scores”. It displays the probability distributions (best cases and worst cases) for the students’ scores in Würzburg examinations in Bernoulli Stochastics in the year 2007 and 2008 with reliability level of  $\beta = 0.80$ , and specifies the probability of getting an honor result in the oral examination.

Grade	worst case, i. e., largest $E[X_3]$ and $V[X_3]$	best case, i. e., smallest $E[X_3]$ and $V[X_3]$
1	0.37428	0.53964
2	0.33793	0.35956
3	0.19549	0.091556
4	0.072465	0.0089091
5	0.017211	0.00033130
6	0.0026191	0.0000047080
with honor	0.71221	0.89920
failed	0.019830	0.00033601

Table 12.3: Probability distributions of the scores in Bernoulli Stochastics.

Besides the probability distribution of the corresponding Bernoulli Space, we will also use the measurement result for the probability of obtaining an honor grade in Stochastics of a student with examination preparation based on *Stochastikon Magister*. The measurement result  $C_P^{80\%}(\{20\})$  is given below:

$$C_P^{80\%}(\{20\}) = \{p | 0.71 \leq p \leq 0.90\} \quad (13.1)$$

In order to compare the examination scores with the questionnaire responses we use the evaluation of the technical characteristics in Q5 to determine questionnaire scores about the Magister performance and compare them with the examination scores. Omitting the two questions referring to the Graphical Lab, there are six questions in Q5 with grades 1 to 5. The sum of the grades in each questionnaire may vary between 6 and 30, where 6 is achieved if each of the six characteristics is valued with the best grade 1, and 30 if each of the characteristics is valued with the worst grade 5. For defining an overall grade either the total sum of grades or the average grade is used as specified in Table 13.1.

Overall grade	Total sum of scores	Average score	Number of students
very good (1)	6 - 8	1.0 - 1.33	4
good (2)	9 - 14	1.5 - 2.33	21
satisfactory (3)	15 - 20	2.5 - 3.33	5
sufficient (4)	21 - 26	3.5 - 4.33	0
poor (5)	27 - 30	4.5 - 5.0	0

Table 13.1: Overall grades and grade distribution of the Magister technical performance.

30 of the 31 participants valued all six technical characteristics and are therefore considered here. The last column in Table 13.1 displays the observed grade distribution by the questionnaire.

We define the grade “very good” and “good” as “honor performance” of Magister and determine a measurement interval for the probability of an honor performance based on the questionnaire responses.

The sample size is  $n = 30$  and the number of the observed honor grades is  $x = 25$ . The probability is determined with a stochastic measurement procedure with reliability level  $\beta = 0.80$ . The measurement result for the probability of an honor result for the technical performance of Magister is given by the following interval:

$$C_P^{80\%}(\{25\}) = \{p|0.70 \leq p \leq 0.89\} \quad (13.2)$$

The conformance of the result with respect to the probability for an honor examination (13.1) and the probability for an honor performance (13.2) is striking. For both probabilities we obtain almost identical measurement intervals. This reflects the experience that students achieve good grades, if they are comfortable with the teaching method and the environment.

In the following, we analyze the responses of Q6 by first defining a positive performance of Magister and then determine the probability for a positive evaluation by means of stochastic measurement procedures with reliability level  $\beta = 0.80$  based on the questionnaire results.

Responses to Q6.1-1 (understanding difficulties with Stochastics in Magister), Q6.2-2 (solving comprehension problems in Magister), Q6.5-2 (missing classroom teaching during exam preparation) and Q6.6-1 (e-learning for other scientific subjects) are looked upon as a positive performance. A “very good” performance is defined, if all four responses are positive; a “good” performance is defined by three positive responses, a “satisfactory” performance by two positive responses, a “poor” performance by one positive response, and finally a “very poor” performance by no positive response. Next, we define the grades “very good” and “good” as an honor performance and wish to determine the probability of such an honor evaluation. 28 of the 31 participants gave responses to all the four questions.

In Table 13.2 the above grading system is given together with the questionnaire result.

Performance Grade	Number of positive response	Number of student
very good	4	18
good	3	6
satisfactory	2	2
poor	1	1
very poor	0	1

Table 13.2: A synthetic analysis of positive responses to questions in Question 6.

The sample size is  $n = 28$  and there is a number of  $x = 24$  of honour grades. The honour probability is again determined with a stochastic measurement procedure based on a reliability level of  $\beta = 0.80$ . The measurement result for the probability of an honour evaluation of Magister in particular and e-learning in general is given by the following interval:

$$C_P^{80\%}(\{24\}) = \{p|0.75 \leq p \leq 0.91\} \quad (13.3)$$

In Figure 13.43 the measurement intervals (reliability level  $\beta = 0.80$ ) for the probability of a positive evaluation of the various technical characteristics in Q5 and Q6 are displayed. Moreover, the honour evaluations for Q5, Q6 and students’ score with honour in Stochastics as given above are also included in 13.43.



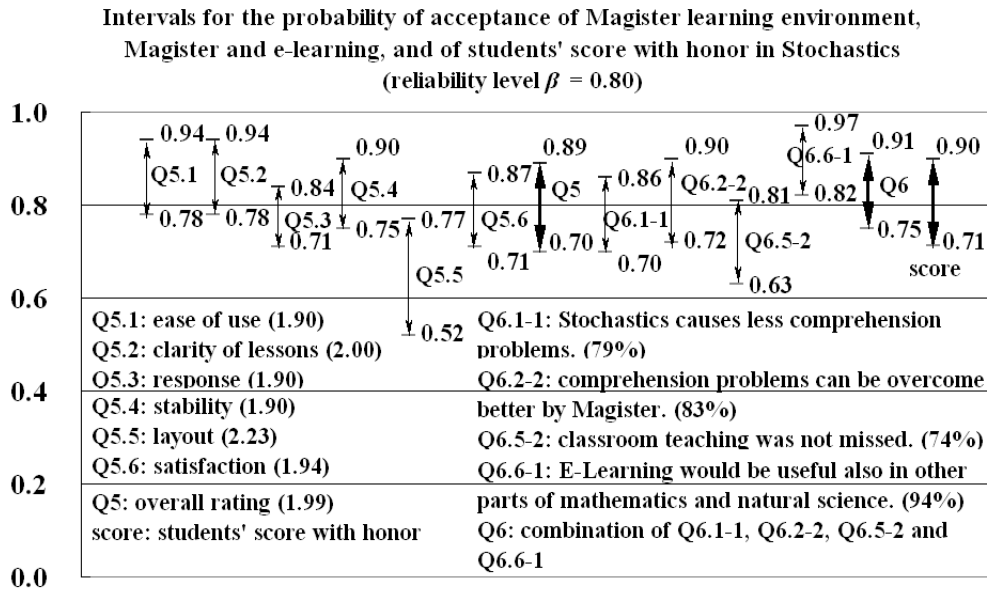


Figure 13.43: Analysis of responses to Question 5 and 6.

The measurement results of the probabilities of a positive evaluation are consistent with one another as the intervals overlap. One measurement result namely with respect to Q5.5 seems to be out of the ordinary. This question refers to the “layout” of Magister and we noted earlier that grading the “layout” is much more subjective than that of other technical characteristics.

The questionnaire was executed after the oral examinations and, therefore, the test scores may have had an influence on the students’ responses. On the other hand, the students had got their scores when they took part in the questionnaire, and therefore it is to be expected that the participants express their opinions in the questionnaire more frankly and truly. Anyway, the consistency of the score evaluation with the results of the questionnaire supports the reasonability and reliability of both investigations on Magister.

### 13.4.6 Influence of E-Learning Experiences

We divide the participants in those with only little e-learning experience and those with more e-learning experience. As an indicator for little experiences we take the responses Q4-1 (group 1) and Q6.5-1 (group 2), for none of the students in these two groups has other e-learning experience except Magister, while Q6.3-1 (group 3) and Q2-2 (group 4) serve as indicators for more experience with e-learning.

Four groups are defined and opposed with a number of responses to investigate the effects of the degree of e-learning experience. The groups and the considered responses are given below:

- Q4-1: preference for regular course for Stochastics. (group 1 contains 7 students)
- Q6.5-1: miss classroom teaching during exam preparation. (group 2 contains 8 students)
- Q6.3-1: experience with other e-learning systems besides Magister. (group 3 contains 8 students)
- Q2-2: preparation by online learning at home. (group 4 contains 11 students)

- Q2-3: preparation by print-out lessons.
- Q6.1-1: more comprehension difficulties with statistics and classroom teaching.
- Q6.6-1: e-learning system like Magister would also be successful for other subjects besides Stochastics.

Figure 13.44 shows the very pronounced difference between the groups 1 and 2 on the one hand and groups 3 and 4 on the other. Actually the groups 3 and 4 are more or less identical except for one single student in group 4, who responded negatively to Q4, Q6.1, Q6.5 and Q6.6. This student also gave the lowest ratings to Q5. He had no previous e-learning experience but selected nevertheless online learning at home. Maybe this was a wrong decision that may explain his negative responses that affect the values of group 4 by about 9%. His responses may be looked upon as not reflecting the overall opinion of group 4 students on e-learning in general and Magister in particular.

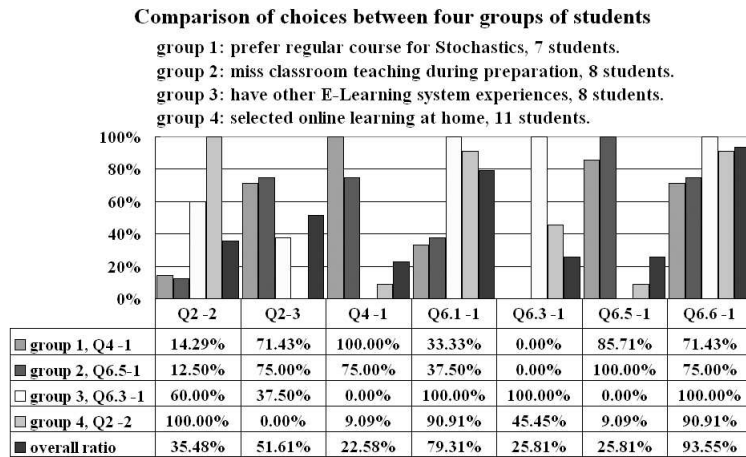


Figure 13.44: Analysis of responses to Question 2, 4 and 6.

Having this in mind we conclude that the result of the questionnaire shows that previous e-learning experiences as well as the amount of e-learning practices have a positive impact on the results obtained by working with Magister and on the attitudes toward e-learning.

Less than half of the students who selected online learning at home had previous e-learning experiences and only 60% of the students with previous e-learning experiences selected online learning at home. Thus, one may conclude that the relation between previous e-learning experiences and the selection of online learning is not really close. If this is correct then on-line learning may be much more related to technical factors (equipment, environment, etc.) and personal factors (habit, preference, etc.).

### 13.4.7 Overall Performance Rating

For evaluating Magister the overall performance rating is of particular significance. Figure 13.45 shows the overall performance rating with respect to Q5 for each response to Q2, Q3, Q4, and Q6, which was selected by more than one student independently whether the response was positive or negative. The bars representing average scores of negative responses are striped.

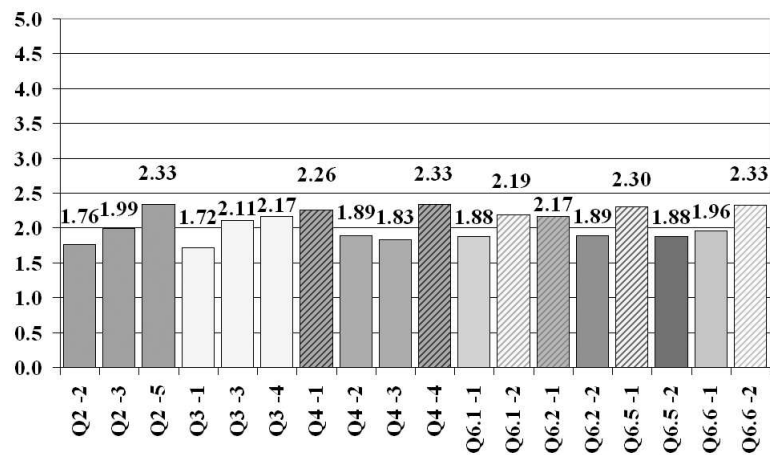


Figure 13.45: Analysis of the relationship between responses to Question 5 and responses to Question 2, 3, 4 and 6.

According to Figure 13.45 positive responses to Q2, Q3, Q4 and Q6 lead without exception to a better overall rating of the performance characteristics included in Q5 than negative responses. This fact shows the consistency of the response. The result also indicates that the subjective evaluation of Magister depends likewise on the technical environment and the scientific content. As to the overall rating of the performance criteria of Magister, the worst average score is 2.33 given by those students who prepared for the examination in groups, those who feel that Magister was no help for the exam preparation, and those who are not in favour of e-learning in other scientific subjects. For all positive responses the average scores for the performance characteristics are better than 2 (“good”).

### 13.4.8 Special Performance Characteristics

It is not surprising that the grades for the technical performance characteristics “ease of use”, system “response”, “stability” and “satisfaction of working with Magister” are positively correlated with one another and with positive responses in other questions.

In contrast, there seems to be no significant relations between the characteristic “clarity of the lessons” and other technical performance characteristics. However, “clarity of the lessons” seems to be closely related to the responses to the question whether Magister is a sufficient basis to understand the examination material. These two aspects may therefore be used to assess the quality of the teaching content in Magister.

The characteristic system “layout” is also more or less independent of the other performance characteristics, although Figure 13.23 indicates a positive correlation between system “layout” and “ease of use”. Actually, system “layout” got the worst overall evaluation score (2.23) among all evaluated characteristics. The evaluation of the “layout” depends more than all other characteristics on individual aesthetics and preferences. However, Figure 13.4 shows that especially those students who worked with Magister online and who are therefore familiar with the system “layout” evaluated it best. Maybe the first impression about the “layout” is less good, but it gets better by working with it. This change might be caused if a user working online with Magister switches to the other components like Calculator, Encyclopedia, Mentor, etc. of the *Stochastikon* system and feels immediately familiar with them because they have the same “layout” as Magister.

### 13.4.9 Graphical Laboratory

There are two sub-questions included in the questionnaire concerning the Graphical Lab, one refers to the “frequency of using the Graphical Lab”, and the other refers to the “ease of using the Graphical Lab”. These questions were included to provide information about the interest of the students in Stochastics that exceeds the needs for passing the examination, since familiarity with the Graphical Lab is not a subject of examination.

The responses to these questions reveal that more than 80% of the students at least tried the Graphical Lab, and more than half of these students have worked with it at least occasionally.

Having in mind the rather limited impact of the Graphical Lab for the exam preparation, the overall evaluation of its “ease of use” seems to be satisfactory.

The characteristic “ease of use” of Magister shows a positive correlation with both the “frequency of using the Graphical Lab” and the ease of use it. From this we conclude that “ease of use” of Magister may stimulate students’ enthusiasm to try new and different functions and technical tools provided by Magister.

### 13.4.10 Role of Magister and E-Learning

The questionnaire shows that three-quarters of the teacher candidates have no other e-learning experience except Magister. This fact seems to be surprising and it might indicate a shortcoming of the teachers’ education since e-learning is also gaining importance for school education and particularly for further education. It therefore seems to be desirable to have the teacher candidates work actively with e-learning programs during professional education at universities.

Actually, our time is characterized by the emergence of new technologies and new subjects that enter school curricula. As a matter of fact, school teachers especially of secondary schools must teach increasingly subjects they have not been trained for. Furthermore, basic education as well as higher education represent an ever shorter part of the lifetime of a person. This situation poses new challenges to the individuals as well as the societies with respect to further education. One way to meet this challenge is e-learning and this is another reason that teacher candidates should become familiar with e-learning during their university education. This task cannot be achieved theoretically, but must include practical exercises and learning. In other words, an e-learning course should be part of the professional training of teachers. This leads to the question which subject might be taught by means of e-learning.

One of the subjects which presently undergo big changes is statistics (traditional stochastics) although it is not at all a “new” school subject. In Germany, as early as in 1901 probability calculus and combinatory became part of the curriculum of the secondary schools in Prussia because of the eminent importance of probability theory and statistics for natural sciences [117]. However, during the Nazi-time probability calculus was abandoned and after World War II also statistics were taken off from German school curricula. Only in the 1970s statistics (traditional stochastics was) gradually included into German school curricula with the target to promote and train “stochastic thinking”. Unfortunately, the education of teacher candidates in statistics (traditional stochastics) is often restricted to one course, which gives a first brief overview on some selected statistical methods. These introductory courses generally omit a discussion of the fundamental issues like for instance the concept of probability, which is either introduced ambiguously or only as a mathematical entity. These educational shortcomings are frankly accepted, for example, Jäger and Schupp note “An explicit definition of the concept of

probability can be omitted, if adequate perceptions about and founded methods for the handling of probabilities are developed.” (Joachim Jäger & Hans Schupp: “Eine explizite Definition des Wahrscheinlichkeitsbegriffs kann unterbleiben, wenn angemessenen Vorstellungen vom und begründete Methoden für den Umgang mit Wahrscheinlichkeit entwickelt werden.”) [117] (p. 19)

To overcome these shortcomings in the statistics education of undergraduates, the German Federal Ministry of Education and Research (BMBF) has spent during the last decade a large amount of money and resources to support the development of e-learning programs to enhance the quality of statistical education at German universities. However, as the questionnaire illustrates a majority of the students had no e-learning experiences at all despite the fact that some large e-learning projects were completed during the recent years like EMILeA-stat and New Statistics.

From this perspective, the Stochastikon experiment to develop Magister and offer it as an e-learning alternative to the conventional course in statistics, seemed to be not very promising. However, in a relative short time it was accepted as a fully valid alternative to the conventional course. All participants in the questionnaire agreed that Magister is a learnable and usable educational tool. Even more surprising is the fact that more than 70% of the participants consider Magister with respect to the exam preparation as being superior to classroom teaching and 93.55% would appreciate such e-learning programs also for other subjects. The very positive opinion about Magister should therefore prompt to examine two questions:

- Which subjects of the curriculum of teacher candidates are adequate to be taught by e-learning systems?
- Should future e-learning systems be developed based on the virtual classroom as offered by Magister?

# Chapter 14

## Summary, Conclusions and Outlook

Stochastics is often divided into two parts. The one is represented by a special branch of mathematics, while the other part constitutes that branch of science which deals with the problem of how to model and how to deal with uncertainty about future developments. This thesis deals primarily with problems related to the second part, which is called Bernoulli Stochastics here.

In view of the fact that uncertainty constitutes a central problem of science and of society, quick dissemination of any new knowledge about how to handle it, is of great significance. This is in particular true, because in many cases when decisions in all fields of human societies have to be made, uncertainty is not or not appropriately taken into account. In practice mathematical modelling of uncertainty generally follows the ISO *Guide to the Expression of Uncertainty in Measurement* (GUM) [115], which according to its scope “establishes general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields - from the shop floor to fundamental research.” The GUM is restricted to uncertainty in measurement and, moreover, it is based on unrealistic assumptions and outdated methods. In this thesis different rules for evaluating and expressing uncertainty are developed which avoid the shortcomings of the GUM and which aim at replacing the GUM.

Actually, the entire project *Stochastikon* serves to develop and spread the stochastic approach for handling and controlling uncertainty. One main difference between the new approach and the traditional ones is the fact that the always existing ignorance is explicitly incorporated into models and procedures making reliable and accurate predictions possible. This advantage is paid by an essentially increased analytical complexity, which requires the introduction of new mathematical concepts and the reinterpretation of the old ones. Another characteristic feature of the new stochastic approach is the necessity to change the way of thinking. For understanding stochastic models and procedures, the traditional way of causal thinking has to be given up in favor of stochastic thinking. Clearly, the change of thinking patterns constitutes an extraordinary didactic and methodological challenge for teaching Bernoulli Stochastics.

The activities in mathematical sciences aim at generating knowledge, representing knowledge, and disseminating knowledge. This thesis serves for all three purposes, however, the focus is on the two latter ones. It represents knowledge by means of the courses developed and implemented into *Stochastikon Magister*, and it supports dissemination of the new stochastic approach by developing an IT-based virtual classroom for supporting the realization of an e-learning environment within the *Stochastikon* system. Finally, new knowledge is generated for example by developing new stochastic procedures for measuring the minimum and the maximum values of the range of variability of a random variable.

Computer science deals with the development of hardware and software for various purposes including e-learning platform, which are often developed based on existing shell programs.

However, the e-learning programme *Stochastikon Magister* is designed in order to be able to work closely with the other components of the *Stochastikon* system. The required cooperation between the system components constituted one of the most important preconditions for the design, and led already at an early state of the development (in 2004) to the decision not to use a commercially available e-learning shell, but to rely on a self-development. As an immediate consequence the entire programming work for *Stochastikon Magister* was part of the dissertation project.

Another requirement for the e-learning environment meant for Bernoulli Stochastics refers to the addressees: it is not only for persons having a university degree in mathematics, but also for mathematically less skilled persons. The new stochastic approach is almost exclusively based on sets and set-functions, which constitute a special difficulty for mathematically less skilled persons. In order to support understanding the new approach, a Graphical Laboratory was developed, which is build on *Stochastikon Graphics* and *Stochastikon Calculator*. By means of which stochastic models and stochastic procedures can be visualized in two and three dimensions.

The first part of this thesis defines the state of the art of e-learning development for statistics and Bernoulli Stochastics by means of the educational statistics systems with focus on Germany.

The second part of the thesis illustrates the novelty of the approach by outlining the content of the first two courses of the e-learning project, which introduce a newly developed stochastic terminology and the stochastic approach to uncertainty. This part shall show the necessity and the initial range of the e-learning programme *Stochastikon Magister*.

The third part contains general guidelines for the development of e-learning programmes. Subsequently, the design and functionality of *Stochastikon Magister* is described in some detail. The selected structure of the e-learning programme, as well as the support activities for learners and teachers in the learning community are listed.

The fourth part is devoted to an evaluation of the e-learning programme Magister. The evaluation is performed by means of a comparison with a statistics course and traditional classroom teaching. The evaluation is performed in two different ways. The first one is done from the (more or less objective) point of view of learning success, based on the scores obtained in the oral state examination. The second one is done from more subjective point of view as expressed in a questionnaire for the students.

The Appendix contains the original response data of the questionnaire, and an introduction to *Stochastikon Graphics*, which constitutes one of the tasks within this dissertation project, since it is the basis for the Graphical Laboratory. Finally the course materials for the two courses already implemented in Magister are added as a supplement to the thesis.

The first version of two online courses in Bernoulli Stochastics are offered to candidates for the teacher profession as subject “stochastics” for the final state examination. The students may register free to take the Magister e-learning courses. The experiences with these students are evaluated with respect to the presentation of the teaching contents and the technical operation by analyzing their examination scores and questionnaire responses. The results of the evaluations when compared with a statistics course based on traditional classroom teaching indicate that *Stochastikon Magister* e-learning programme for Bernoulli Stochastics constitute a fully valid substitute for the statistics course.

For the future, it is planned to extend the contents of *Stochastikon Magister* by developing new courses addressing different branches of science and different fields of application. Particularly,

it is planned to include an introductory course in mathematics for those learners, who need some coaching in mathematics.

Moreover, the cooperation between the different components of the *Stochastikon* system shall be intensified in course of their advancements. For instance, as soon as *Stochastikon Calculator* contains more distribution families, the examples and exercises in *Stochastikon Magister* can deal with more realistic examples and problems and solve them numerically by using the computing power of *Stochastikon Calculator*. The same holds for the other two components, namely *Stochastikon Encyclopedia* and *Stochastikon Mentor*. Now all important concepts introduced in *Stochastikon Magister* teaching content are linked to their explanations in *Stochastikon Encyclopedia*. In future these two components shall be further used for supporting the e-learning platform and offer additional information and assistance in teaching how to solve real problems by taking the uncertainty into account.

The *Stochastikon* system is developed with the purpose of supporting a change of the worldview from an essentially deterministic one to a stochastic world view. The consequences of a successful change can hardly be foreseen, as mankind in thinking and behaving follows the belief in cause-effect relations, which is almost the opposite of stochastic relations.

There are at least two necessary conditions for a success of the *Stochastikon* system, namely:

- The stochastic world view must prove to be superior in allowing reliable and accurate predictions.
- The *Stochastikon* system must be able to present and explain the stochastic world view and get it across to everybody, who is interested.

While meeting the first condition seems to be trivial, the second condition is the by far more challenging one. Besides *Stochastikon Magister* it is the component *Stochastikon Graphics*, which shall meet the challenge by providing illustrative graphs making the mathematical concepts and procedures more easy to grasp.

Thus, *Graphics* takes a central position within the *Stochastikon* system. It supports *Stochastikon Calculator* by developing suitable scientific reports, it enables the designing of the Graphical Laboratory within *Stochastikon Magister* and provides *Magister* with graphics used in the learning units and, although, so far not being implemented, will also provide *Stochastikon Encyclopedia* with illustrative material.

*Stochastikon Graphics*, similar as the other components of the *Stochastikon* system, is to be further developed to comprise even more powerful functions and abilities for supporting the entire *Stochastikon* system.



**Part V**

**Appendix**



# Chapter 15

## Original Response Data of the Questionnaire

Q1	Q2	Q3	Q4	Q5.1	Q5.2	Q5.3	Q5.4	Q5.5	Q5.6	Q5.7	Q5.8	Q6.1	Q6.2	Q6.3	Q6.4	Q6.5	Q6.6	Q6.7	Date
7	3	4	3	2	2	3	2	2	2	4	3	1	2	2	0	2	1	2	16.02.2010 11:16
2	3	1	2	2	2	2	2	2	2	4	2	1	1	2	0	1	1	4	16.02.2010 12:28
7	3	4	2	2	2	3	3	3	2	4	4	1	2	1	2	2	1	9	17.02.2010 09:20
8	5	4	3	2	2	3	2	2	2	4	4	1	2	2	0	2	1	7	19.02.2010 15:26
7	3	1	2	2	2	2	2	2	2	2	4	1	2	2	0	2	1	4	19.02.2010 17:38
4	2	1	3	1	2	1	1	2	1	3	2	1	2	1	1	2	1	5	19.02.2010 19:33
7	3	1	1	3	1	2	2	3	2	4	3	2	2	2	0	2	1	4	22.02.2010 13:54
2	2	1	2	2	2	1	1	2	1	2	2	1	2	2	0	2	1	5	22.02.2010 19:54
4	3	1	2	2	1	2	2	2	1	3	2	1	2	2	1	2	1	7	24.02.2010 12:40
7	3	4	1	3	2	2	2	2	2	5	0	1	2	2	0	1	1	7	03.03.2010 11:10
7	3	3	4	2	2	2	3	1	3	4	3	2	2	2	0	2	1	4	03.03.2010 12:31
7	3	3	1	2	3	2	2	3	2	4	3	0	2	2	0	1	1	9	03.03.2010 15:00
7	4	4	2	2	2	2	2	4	3	5	3	1	2	2	0	2	1	5	03.03.2010 19:19
7	3	0	3	1	2	2	1	2	2	0	0	2	2	2	0	2	1	5	03.03.2010 21:30
5	2	4	1	2	3	3	3	2	3	4	4	2	1	2	0	1	2	0	04.03.2010 12:32
4	2	1	2	2	2	2	2	2	2	3	2	1	2	1	1	2	1	3	04.03.2010 20:37
7	3	1	2	1	2	1	2	1	1	2	2	1	2	1	1	2	1	4	05.03.2010 06:33
7	3	4	3	1	2	1	1	2	2	2	1	1	2	2	0	2	1	5	05.03.2010 12:37
7	2	4	2	1	2	1	1	1	2	2	2	1	2	2	0	2	1	7	05.03.2010 14:02
8	2	3	3	1	2	2	2	1	1	3	1	1	2	2	0	2	1	3	07.03.2010 15:58
4	2	3	2	2	2	1	1	3	2	5	3	1	1	1	2	2	1	2	10.03.2010 08:50
7	5	3	2	2	2	1	3	3	2	4	3	1	2	2	0	2	1	7	11.03.2010 11:10
4	2	3	2	1	2	1	1	2	1	3	2	1	2	2	0	2	1	9	11.03.2010 11:47
7	3	3	2	2	3	0	0	3	3	5	0	0	0	2	1	1	1	7	14.03.2010 12:13
6	2	4	3	4	3	2	2	3	2	3	4	1	2	1	2	2	1	7	14.03.2010 12:22
3	5	3	1	3	1	3	4	2	3	5	3	2	0	2	0	1	1	4	14.03.2010 14:22
7	2	1	2	1	1	2	2	2	1	3	1	1	2	1	1	2	1	9	14.03.2010 16:29
5	3	3	4	2	2	3	2	3	3	4	3	1	1	1	1	2	1	7	14.03.2010 19:13
6	2	1	3	2	2	2	1	2	1	3	2	1	2	2	0	2	1	7	15.03.2010 12:01
7	3	4	1	2	2	2	2	2	2	4	3	1	2	2	0	1	2	2	15.03.2010 20:27
5	3	3	1	2	2	1	1	3	2	3	3	2	1	2	0	1	1	7	21.03.2010 12:03

Table 15.1: Original Response Data of the Questionnaire

# Chapter 16

## Stochastikon Graphics

As mentioned earlier sets and set-functions play a decisive role in the development and application of Bernoulli Stochastics. Their graphical representations are used for illustrating the meaning and the use of sets and set-functions in particular for teaching purposes. Therefore, another part of this thesis refers to the introduction of the development of the component *Stochastikon Graphics* as the fifth subsystem of the *Stochastikon* system. As a matter of fact, *Stochastikon Graphics* has been mentioned in the preceding sections when introducing the realization of the Graphical Laboratory in *Stochastikon Magister*.

So far *Stochastikon Graphics* is only employed in the framework of the two components *Stochastikon Calculator* and *Stochastikon Magister*. However, in the course of further development of the entire *Stochastikon* system it is planned to make use of it in each of the other subsystems.

### 16.1 Basic Plotting Objects of Graphs

Any graphical representation is composed of some basic plotting objects, thus supporting the production of graphical representations. *Stochastikon Graphics* provides the following basic objects: Point, Line, Curve, Shape, Polygon, Label and Axis.

The representation of these basic objects depends on the requested dimensions of the graphics. *Stochastikon Graphics* provides representation for two and three dimensions.

#### 16.1.1 Plotting Objects for 2-dimensional Graphs

Table 16.1 lists all Plotting Objects for 2-dimensional graphs, in which:

- ‘StyledPoint’ is a point in a plotting style.
- ‘MarkedPoint’ is a point from which with vertical and horizontal lines in a specific plotting style are drawn. MarkedPoint marks a specific position.
- ‘Label’ stands for characters or numbers written at a specific position.
- ‘Line’ draws a line between two points in a specific line style.
- ‘Curve’, ‘Shape’ and ‘Polygon’ are plotted according to one or two functions. ‘Polygon’ can also be plotted according to a series of points. In Table 16.1, ‘y(x)’ means value  $y = f(x)$ , ‘x’ is the independent variable. The result can be a single value or an interval with lower bound and upper bound. Functions can also be plotted in rotation of  $90^\circ$ , i. e.,

plotting x value on y-axis and y value on x-axis. In Table 16.1 this is expressed by  $x(y)$ , or  $x(yLowerbound, yUpperbound)$  when the value of y is an interval.

- ‘Axes’ fix the dimension characters of a graph. Each axis has a name, a type (continuous or discrete) and a bound.

Plotting Object	Plotting Element		Plotting Style	
	Point2d*(s)	IFunction(s) <y(x) stands for $y = f(x)$ >	LineStyle **	FillStyle***
StyledPoint2d	1 point		√	
MarkedPoint2d	1 point		√	
Label2d	1 point			
Line2d	2 points		√	
Curve2d		1 function, y(x) is a value or an interval	√	√ (only PlotStyle)
Shape2d		1 function, y(x) is a value or an interval 2 functions, y1(x),y2(x) are two values	√	√
		plot above function(s) y(x) as x(y) or yInterval(x) as x(yLowerbound, yUpperbound)		
Polygon2d	An array of points	1 function, y(x) is a value or an interval 2 functions, y1(x),y2(x) are two values	√	√
		plot above function(s) y(x) as x(y) or yInterval(x) as x(yLowerbound, yUpperbound)		
Axe	<b>Axe</b> (String axeName, int axeType, IInterval axeBounds), plot x-axis and y-axis.			

*Point2d\**: A 2-dimensional Point determined by a pair of values: x-value and y -value. ‘Point’ is the most basic element for all Plotting Objects, ‘Function’ is points with special functional relations between dependent and independent variables.

*LineStyle\*\**: Determines the color, line width, point shape (star, round, cross, etc.), point size of each Plotting Object.

*FillStyle\*\*\**: Determines the filling direction, filling or plotting type (with points, lines, dots, lines and points etc.).

Table 16.1: Plotting Objects for 2-dimensional graphs.

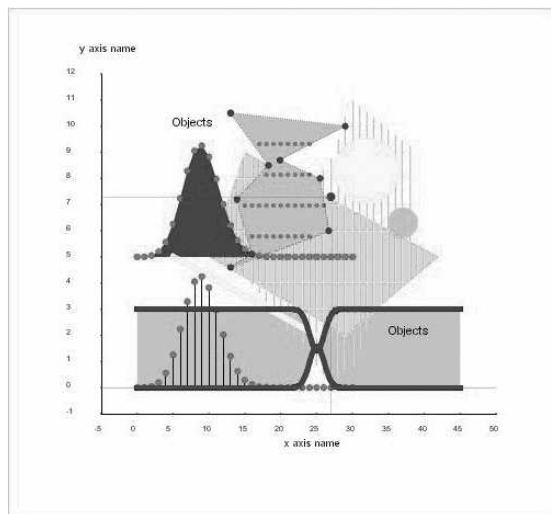


Figure 16.1: Different kinds of Plotting Objects in a 2-dimensional graph.

Figure 16.1 illustrates different kinds of 2-dimensional Plotting Objects.

### 16.1.2 Plotting Objects for 3-dimensional Graphs

The 3-dimensional Plotting Objects are generated by adding one dimension (a value or a series of values) to the original 2-dimensional Plotting Objects. Table 16.2 lists the Plotting Objects for the 3-dimensional graphs provided by *Stochastikon Graphics*.

Plotting Object	Plotting Element	
	Point3d(s) <x value, y value, z value>	add one more dimension to the original 2d Objects <x value / y value / z value>
Label3d	1 point	
StyledPoint3d	1 point	
MarkedPoint3d	1 point	
Line3d	2 points	
Curve3d		Curve3dXY: plot original 2d curve at x, y-axes, add a z-axis value to the curve.
		Curve3dXZ: plot original 2d curve at x, z-axes, add a y-axis value to the curve.
		Curve3dYZ: plot original 2d curve at y, z-axes, add an x-axis value to the curve.
Shape3d		Shape3dXY: plot original 2d shape at x, y-axes; add a z-axis value to the shape.
		Shape3dXZ: plot original 2d shape at x, z-axes; add a y-axis value to the shape.
		Shape3dYZ: plot original 2d shape at y, z-axes, add an x-axis value to the shape.
Ploygon3d	add a more dimension value (x value / y value / z value) to an array of 2d points	Polygon3dXY: plot original 2d polygon at x, y-axes, add a z-axis value to the polygon.
		Polygon3dXZ: plot original 2d polygon at x, z-axes, add a y-axis value to the polygon.
		Polygon3dYZ: plot original 2d polygon at y, z-axes, add an x-axis value to the polygon.
Axe	Axe(String axeName, int axeType, IInterval axeBounds) especially for plotting x-axis, y-axis and z-axis.	

Table 16.2: Plotting Objects for the 3-dimensional graphs.

Figure 16.2 illustrates the examples of a 3-dimensional Plotting Object, the Polygon, displayed in a 3-dimensional coordinate system (x,y,z). Figure 16.2 also demonstrates different filling styles of the example Polygon: full fill blue, no fill red point outline, line fill black, no fill green line outline. The Polygon is plotted in direction  $x = f(z)$  and the different polygons are distinguished by means of their different y values.

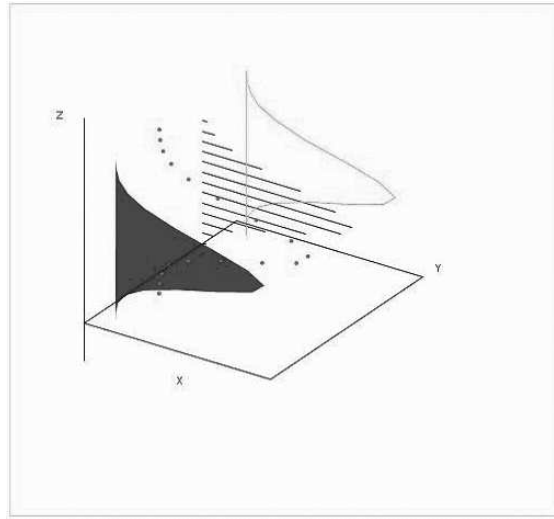


Figure 16.2: Change 2-dimensional Plotting Objects to 3 dimensions.

### 16.1.3 Difference between Shape and Polygon

The Plotting Objects ‘Shape’ and ‘Polygon’ are used for plotting 2-dimensional areas or 3-dimensional objects. There are the following differences between them:

- 1.) The filling of a ‘Shape’ is completely determined by its function(s) values. So if its function is a discrete PDF (Probability Distribution Function), in a Gnuplot graph program, with the ‘full filled’ method, it only fills the shape with few lines. (In a Java image program it is the same as fill a ‘Polygon’)

The filling of a ‘Polygon’ is not only determined by its plotting points, because the connections between two neighbouring-points are treated as lines. So, if its function is a discrete PDF, it can also ‘full fill’ the Polygon with a specific color. The default distance between each two filling lines is ‘1’ when the whole plotting range is large than 1 and the plotting style is not ‘full filled’. Otherwise it will automatically full fill the plotting area. The algorithm of how to fill a Polygon is rather complex and will be discussed later in Section 16.2 ‘Plotting Data’.

- 2.) The filling of a ‘Shape’ means to fill the area within two curves or within a curve and one axis. It needs symmetric points at its plotting direction. So if a Shape is determined by two functions, they should be both continuous or both discrete. If a Shape is determined by a function curve that is not ‘closed’, the plotting area is between the curve and a specific axis. This specific axis is determined by the Shape’s plotting direction.

A ‘Polygon’ automatically connects its first and its last point and, thus, builds a ‘closed’ curve, which is filled. As mentioned above Polygon does not require symmetric points for plotting.

- 3.) A ‘Polygon’ can be determined by a series of points, so one can use this Plotting Object to plot any irregular ‘shape’ according to the given series of points.

### 16.1.4 Methods of the Plotting Objects

Each Plotting Object implements the interface `IPlotObject2d` or `IPlotObject3d`. In Table 16.3, the basic methods of these Plotting Objects are listed.

With these methods any Plotting Object can be stored into a graph file using the Gnuplot graphical software or into a Java image (file) using Java programs.

void	<u><a href="#">addToPlotData(PlotData2d data)</a></u>	add the plotting object into a PlotData2d to plot a 2-dimension graph.
void	<u><a href="#">addToPlotData(PlotData3d data)</a></u>	add the plotting object into a PlotData3d to plot a 3-dimension graph.
IInterval	<u><a href="#">getPreferredXRange()</a></u>	return an interval, indicate the preferred X plotting range of the plotting object.
IInterval	<u><a href="#">getPreferredYRange()</a></u>	return an interval, indicate the preferred Y plotting range of the plotting object.
IInterval	<u><a href="#">getPreferredZRange()</a></u>	return an interval, indicate the preferred Z plotting range of the plotting object for 3d graph.
IInterval	<u><a href="#">getMaximumXRange()</a></u>	return an interval, indicate the maximum X plotting range of the plotting object.
IInterval	<u><a href="#">getMaximumYRange()</a></u>	return an interval, indicate the maximum Y plotting range of the plotting object.
IInterval	<u><a href="#">getMaximumZRange()</a></u>	return an interval, indicate the maximum Z plotting range of the plotting object for 3d graph.
int	<u><a href="#">getPreferredXType()</a></u>	return an integer value, indicate the X axis type of the plotting object.
int	<u><a href="#">getPreferredYType()</a></u>	return an integer value, indicate the Y axis type of the plotting object.
int	<u><a href="#">getPreferredZType()</a></u>	return an integer value, indicate the Z axis type of the plotting object for 3-dimension graph.

Table 16.3: Basic Methods of the Plotting Objects.

PlotObjectSet2d and PlotObjectSet3d define two special Plotting Objects. They have a special method:

[add\(IPlotObject2d Object\)](#) / [add\(IPlotObject3d Object\)](#)

which compose several basic Plotting Objects to a big Plotting Object - a Plotting Object Set. Figure 16.3 illustrates this process. As a graphical illustration see Figure 16.5, where the composed object is a fish.

Moreover, a PlotObjectSet2d or PlotObjectSet3d can be entered as an element Plotting Object within a larger Plotting Object Set as shown in Figure 16.4. Figure 16.6 shows as a graphical example several fishes.



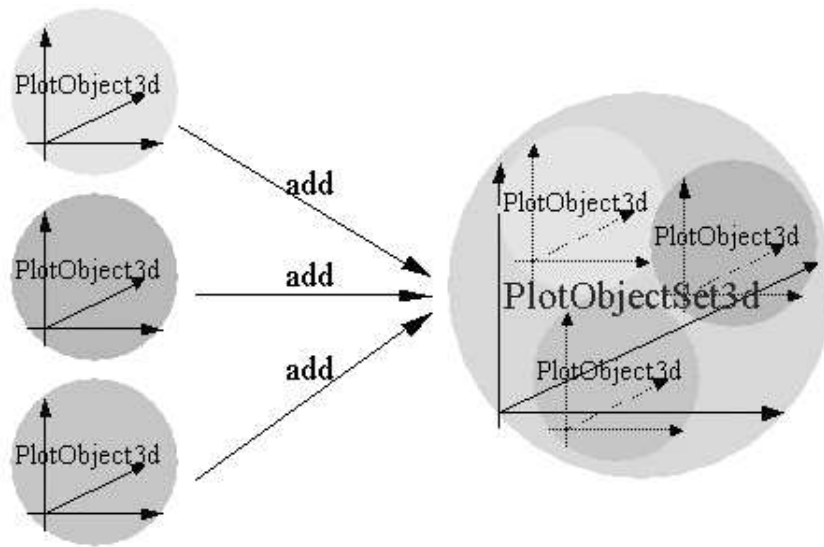


Figure 16.3: Add several Plotting Objects to a Plotting Object Set.

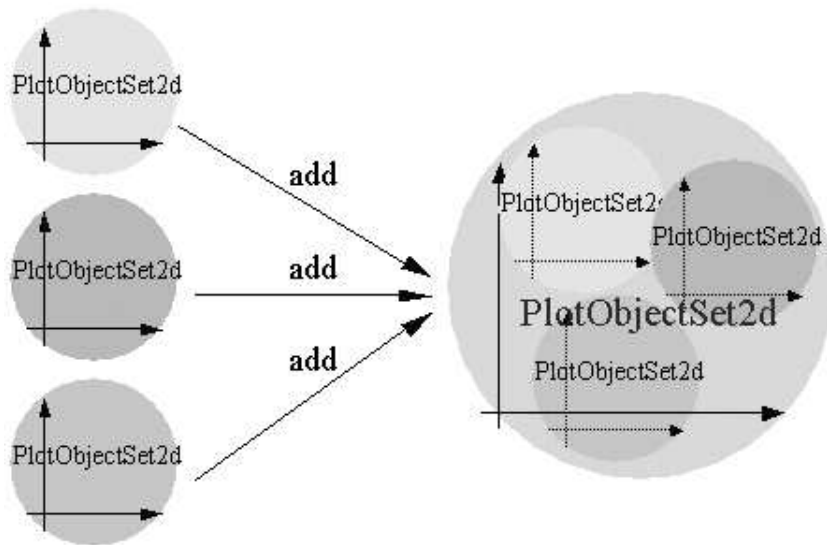


Figure 16.4: Add small Plotting Object Sets to a big Plotting Object Set.

### 16.1.5 Illustrations of Plotting Objects

In Figure 16.5, the ‘fish’ is a `PlotObjectSet`, which contains `Curves`, `StylePoints`, `Polygons` and `Lines`.

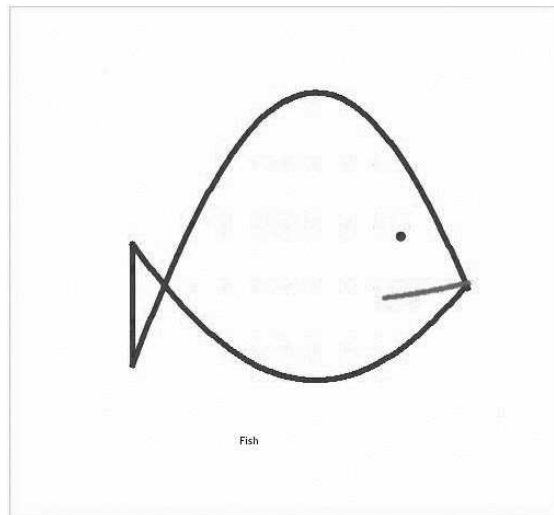


Figure 16.5: Some Plotting Objects combine a Plotting Object Set: a fish.

The following Java Program yields the ‘fish’ shown in Figure 16.5:

```
public class Fish extends PlotObjectSet2d {
    public static final int BODYFILL_FULL = FillStyle.STYLE_FULL_FILLED;
    public static final int BODYFILL_LINES = FillStyle.STYLE_Y_DIRECTION_LINES_FILLED;
    public static final int BODYFILL_POINTS = FillStyle.STYLE_Y_DIRECTION_POINTS_FILLED ;
    public static final int BODYFILL_LINES_POINTS = FillStyle.STYLE_Y_DIRECTION_LINES_POINTS_FILLED;
    private static final IPDF F1 = NormalPDF.create(0, 9);
    private static final IPDF F2 = NormalPDF.create(0, 11.5);
    private static final IPDF F3 = NormalPDF.create(0, 20);
    private static final int BASIC_POINT_NUMBER = 12;
    private Point2d position = new Point2d(0, 0);
    private double size = 1;
    private int bodyColor = LineStyle.YELLOW_COLOR;
    private int bodyBoundColor = LineStyle.BLUE_COLOR;
    private int eyeColor = LineStyle.BLUE_COLOR;
    private int bodyFilling = BODYFILL_POINTS;
    private int bodyMarking = LineStyle.STAR_POINT;
    public Fish() {}
    public void drawFish() {
        IInterval fishBound = new DoubleInterval(-6 * size + position.getX(), 5 * size + position.getX());
        IRealToRealFunction f1 = new AbstractRealToRealFunction(fishBound) {
            public final double realValue(double x) {
                return (F1.realValue((x - position.getX()) / size) * 500 * size + position.getY());
            }
        };
        IRealToRealFunction f2 = new AbstractRealToRealFunction(fishBound) {
            public final double realValue(double x) {
                return ((0.0697 - F2.realValue((x - position.getX()) / size)) * 500 * size + position.getY());
            }
        };
        LineStyle bodyBoundLs = new LineStyle(bodyBoundColor, 5, 3, 1);
        Curve2d bodyLine1 = new Curve2d(new Function2dParameter1(f1), bodyBoundLs);
        Curve2d bodyLine2 = new Curve2d(new Function2dParameter1(f2), bodyBoundLs);
    }
}
```

```

double xlb = fishBound.doubleLowerBound();
double ylb = bodyLine2.getPreferredYRange().doubleLowerBound(),
yub = bodyLine1.getPreferredYRange().doubleUpperBound();
Interval fishHeight = new DoubleInterval(ylb, yub);
Line2d2 tailLine = new Line2d2(new Point2d(xlb, f1.realValue(xlb)), new Point2d(xlb, f2.realValue(xlb)), bodyBoundLs);
LineStyle eyeLs = new LineStyle(eyeColor, 1, LineStyle.ROUND_POINT, 2.25);
StyledPoint2d2 eye = new StyledPoint2d2(
new Point2d(xlb + fishBound.getVolume() * 0.8, ylb + fishHeight.getVolume() * 0.5), eyeLs);
IRealToRealFunction f3 = new AbstractRealToRealFunction(
new DoubleInterval(xlb + fishBound.getVolume() * 0.75, fishBound.doubleUpperBound())) {
public final double realValue(double x) {
return ((0.0575 - F3.realValue((x - position.getX()) / size)) * 500 * size + position.getY());
};
};
LineStyle mouthLs = new LineStyle(LineStyle.RED_COLOR, 4, 3, 1);
Curve2d mouth = new Curve2d(new Function2dParameter1(f3), mouthLs);
LineStyle bodyLs = new LineStyle(bodyColor, 1.5, bodyMarking, 1.5);
Polygon2d2 body = new Polygon2d2(new Shape2dParameter(f1,f2), bodyLs, bodyFilling);
add(bodyLine1);
add(bodyLine2);
add(body);
add(tailLine);
add(eye);
add(mouth);
}
public void setPosition(Point2d myPosition) {
position = myPosition;
}
public void setSize(double mySize) {
size = mySize;
}
public void setBodyColor(int myBodyColor) {
bodyColor = myBodyColor;
}
public void setBodyBoundColor(int myBodyBoundColor) {
bodyBoundColor = myBodyBoundColor;
}
public void setEyeColor(int myEyeColor) {
eyeColor = myEyeColor;
}
public void setBodyFilling(int myBodyFilling) {
bodyFilling = myBodyFilling;
}
public void setBodyMarking(int myBodyMarking) {
bodyMarking = myBodyMarking;
}
}

```

Some small Plotting Object Sets and Plotting Objects can be combined to a big Plotting Object Set. For example several fishes may compose a flock of fishes as shown in Figure 16.6.

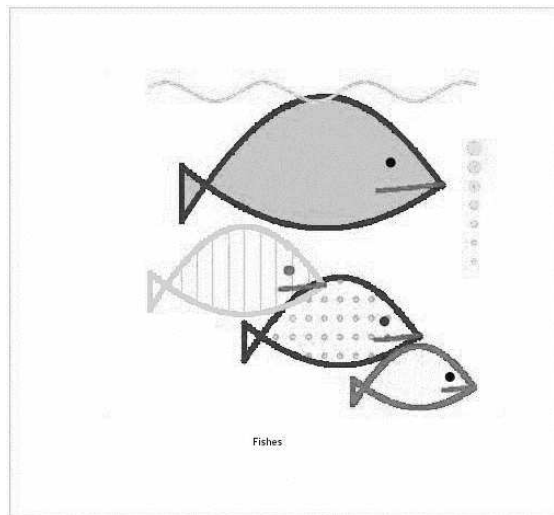


Figure 16.6: A big Plotting Object Set: fishes.

The following Java Program yields the above shown flock of fishes:

```
public class Fishes extends PlotObjectSet2d {
    public Fishes(Point2d[] positions, double[] sizes, int[] bodyColors, int[] bodyBoundColors, int[] eyeColors,
        int[] bodyFillings, int[] bodyMarkings) {
        for (int i = 0; i < sizes.length; i++) {
            Fish fish = new Fish();
            if (!(positions[i] == null)) {
                fish.setPosition(positions[i]);
            }
            if (!(sizes[i] == -100)) {
                fish.setSize(sizes[i]);
            }
            if (!(bodyColors[i] == -100)) {
                fish.setBodyColor(bodyColors[i]);
            }
            if (!(bodyBoundColors[i] == -100)) {
                fish.setBodyBoundColor(bodyBoundColors[i]);
            }
            if (!(eyeColors[i] == -100)) {
                fish.setEyeColor(eyeColors[i]);
            }
            if (!(bodyFillings[i] == -100)) {
                fish.setBodyFilling(bodyFillings[i]);
            }
            if (!(bodyMarkings[i] == -100)) {
                fish.setBodyMarking(bodyMarkings[i]);
            }
            fish.drawFish();
            add(fish);
        }
    }
}
```

The following program contains two parts marked by `/*...*/`, one for plotting an original ‘Fish’ and the other for ‘Fishes’ (note: they should run separately). The result of each running is a Java image (a fish or fishes) on the screen and a corresponding ‘myfile2d.jpg’ file stored in the assigned folder.

```
public class TestJavaGPlotData2d {
    private final static int[] IMAGE_SIZE = JavaG.IMAGE_SIZE;
    private final static JButton button = JavaG.button;
    private static int setTics = 1;
    public static void main(String[] s) throws Exception {
        PlotData2d data = new PlotData2d();

        /*
        // an original Fish
        Fish fish = new Fish();
        fish.drawFish();
        data.setXAxis(new Axis("Fish", fish.getPreferredXType(), fish.getPreferredXRange()));
        data.setYAxis(new Axis("", fish.getPreferredYType(), fish.getPreferredYRange()));
        fish.addToPlotData(data);
        */

        /*
        // Fishes
        Point2d[] positions = {null, new Point2d(5,3), new Point2d(-1,-1.5), new Point2d(-6,2.7)};
        double[] sizes = {-100,0.7, 1.5, -100};
        int[] bodyColors = {LineStyle.GREEN_COLOR,-100,LineStyle.LIGHT_GRAY_COLOR,LineStyle.PINK_COLOR };
        int[] bodyBoundColors = {-100, LineStyle.RED_COLOR,-100, LineStyle.ORANGE_COLOR};
        int[] eyeColors = {-100, LineStyle.BLACK_COLOR, LineStyle.BLACK_COLOR, LineStyle.RED_COLOR};
        int[] bodyFillings = {-100, Fish.BODYFILL_LINES_POINTS, Fish.BODYFILL_FULL, Fish.BODYFILL_LINES};
        int[] bodyMarkings = {-100, LineStyle.ROUND_POINT, -100, -100 };
        Fishes fishes = new Fishes(positions, sizes, bodyColors, bodyBoundColors, eyeColors, bodyFillings, bodyMarkings);
        //pops
        fishes.add(
            new StyledPoint2d2(new Point2d(8.5, 23), new LineStyle(LineStyle.LIGHT_BLUE_COLOR, 0.1, LineStyle.ROUND_POINT, 1.5)));
        fishes.add(
            new StyledPoint2d2(new Point2d(8.5, 24), new LineStyle(LineStyle.LIGHT_BLUE_COLOR, 0.1, LineStyle.ROUND_POINT, 1.5)));
        fishes.add(
            new StyledPoint2d2(new Point2d(8.5, 25), new LineStyle(LineStyle.LIGHT_BLUE_COLOR, 0.1, LineStyle.ROUND_POINT, 1.8)));
        fishes.add(
            new StyledPoint2d2(new Point2d(8.5, 26), new LineStyle(LineStyle.LIGHT_BLUE_COLOR, 0.1, LineStyle.ROUND_POINT, 2.3)));
        fishes.add(
            new StyledPoint2d2(new Point2d(8.5, 27), new LineStyle(LineStyle.LIGHT_BLUE_COLOR, 0.1, LineStyle.ROUND_POINT, 2.7)));
        fishes.add(
            new StyledPoint2d2(new Point2d(8.5, 28), new LineStyle(LineStyle.LIGHT_BLUE_COLOR, 0.1, LineStyle.ROUND_POINT, 3.1)));
        fishes.add(
            new StyledPoint2d2(new Point2d(8.5, 29), new LineStyle(LineStyle.LIGHT_BLUE_COLOR, 0.1, LineStyle.ROUND_POINT, 3.5)));
        //water
        IRealToRealFunction water = new AbstractRealToRealFunction(fishes.getPreferredXRange()) {
            public final double realValue(double x) {
                return Math.sin(x) / 2 + 32;
            }
        };
    }
}
```

```

fishes.add(new Curve2d(new Function2dParameter1(water), new LineStyle(LineStyle.LIGHT.BLUE.COLOR, 3, 3, 1)));

data.setXAxis(new Axis("Fishes", fishes.getPreferredXType(), fishes.getPreferredXRange()));

data.setYAxis(new Axis("", fishes.getPreferredYType(), fishes.getPreferredYRange()));

fishes.addToPlotData(data);

*/

WindowListener l = new WindowAdapter() {

public void windowClosing(WindowEvent e) { System.exit(0); }

public void windowClosed(WindowEvent e) { System.exit(0); }

};

JFrame f = new JFrame();

f.addWindowListener(l);

JPanel panel = new JPanel();

panel.add(button);

JavaG j = new G_PlotData2d(data);

j.setNoTics();

f.getContentPane().add(BorderLayout.SOUTH, panel);

f.getContentPane().add(BorderLayout.CENTER, j);

f.pack();

f.setSize(new Dimension(IMAGE.SIZE[0], IMAGE.SIZE[1]));

f.show();

j.getJPEG("c://stoch//temp//report//myfile2d.jpg");

System.out.println("finished.");

}

}

```

### 16.1.6 Stochastic Objects and Plotting Objects

In this section the use of Plotting Objects (Set) for representing Stochastic Objects is briefly outlined. Stochastic Objects are stochastic procedures and their results based on the calculations of *Stochastikon Calculator*. Table 16.4 lists the Stochastic Objects and the Plotting Objects used for their construction.

Beta Measurement Space	Lower Bound	<u>Shape2d12</u> (IFunction2dParameter param,LineStyle lineStyle, int fillStyle) <u>Marked Point2d2</u> (Point2d p,LineStyle style)
	Upper Bound	<u>Line2d2</u> (double x1, double y1, double x2, double y2,LineStyle style) <u>StyledPoint2d2</u> (Point2d p,LineStyle style)
	Minimum	<u>Shape2d22</u> (IShape2dParameter param,LineStyle lineStyle, int fillStyle) <u>Marked Point2d2</u> (Point2d p,LineStyle style) <u>Line2d2</u> (double x1, double y1, double x2, double y2,LineStyle style) <u>StyledPoint2d2</u> (Point2d p,LineStyle style) <u>Shape2d12</u> (IFunction2dParameter param,LineStyle lineStyle, int fillStyle) <u>Marked Point2d2</u> (Point2d p,LineStyle style) <u>Line2d2</u> (double x1, double y1, double x2, double y2,LineStyle style) <u>StyledPoint2d2</u> (Point2d p,LineStyle style)
Beta Uncertainty & Prediction Space	Lower Bound	<u>Shape2d11</u> (IFunction2dParameter param, int lineStyleIndex, int fillStyle)
	Upper Bound	<u>StyledPoint2d</u> (Point2d p, int style) <maybe>
	Minimum	
Classification	2 Alternatives	<u>Shape2d11</u> (IFunction2dParameter param, int lineStyleIndex, int fillStyle) <u>Line2d2</u> (double x1, double y1, double x2, double y2,LineStyle style) <u>StyledPoint2d</u> (Point2d p, int style) <maybe>
	3 Alternatives	
Beta Exclusion		<u>Shape2d11</u> (IFunction2dParameter param, int lineStyleIndex, int fillStyle) <u>Line2d2</u> (double x1, double y1, double x2, double y2,LineStyle style) <u>StyledPoint2d</u> (Point2d p, int style) <maybe>
Ignorance Space		<u>Polygon2d1</u> (IFunction2dParameter param,LineStyle lineStyle, int fillStyle)
Uncertainty Space		<u>Polygon2d1</u> (IFunction2dParameter param,LineStyle lineStyle, int fillStyle)
Property Function		<u>Curve2d</u> (IFunction2dParameter param,LineStyle lineStyle) <u>Curve2d</u> (IFunction2dParameter param, int lineStyleIndex)
Probability of an Event		<u>Curve2d</u> (IFunction2dParameter param,LineStyle lineStyle) <u>Shape2d12</u> (IFunction2dParameter param,LineStyle lineStyle, int fillStyle)

Table 16.4: Stochastic Objects and Plotting Objects.

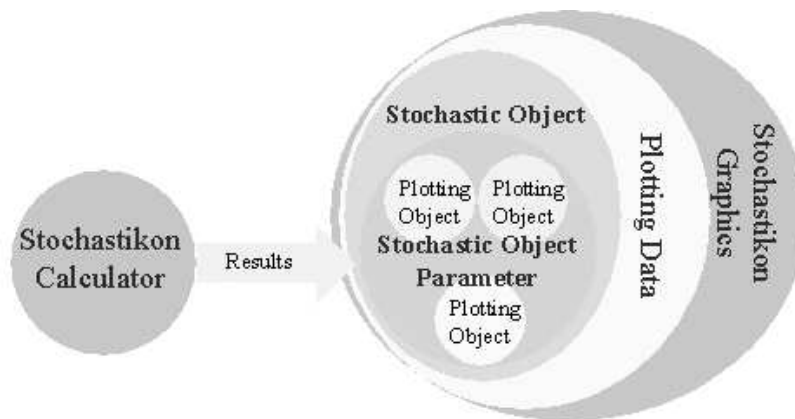


Figure 16.7: Construction of a Stochastic Object according to the results provided by *Stochastikon Calculator*.

Figure 16.7 depicts how a Stochastic Object is constructed using Plotting Objects which are created according the calculations of *Stochastikon Calculator*.

The following Java Program shows exemplarily how to construct a Stochastic Object Parameter for the ‘Minimum  $\beta$ -Measurement’ Object according to the results from *Stochastikon Calculator*:

```

public class MinimumBetaMeasurementSpace2dParameter1 implements IParameter, IPlotProperties {
    final IShape2dParameter param;
    final int xType;
    final int yType;
    private final LineStyle measurementStyle;
    private final LineStyle shapeStyle;
    private final LineStyle markedPointStyle;
    private final int fillStyle;
    private final double realization;
    final IInterval preferredXRange;
    private double x1;
    private double x2;
    public MinimumBetaMeasurementSpace2dParameter1(IFunction f1, IFunction f2, IInterval dRange, double realization,
    IInterval measurement, LineStyle shapeStyle, LineStyle measurementStyle, LineStyle markedPointStyle) {
    this.param = new Shape2dParameter(f1, f2);
    this.preferredXRange = dRange;
    this.measurementStyle = measurementStyle;
    this.shapeStyle = shapeStyle;
    this.markedPointStyle = markedPointStyle;
    this.realization = realization;
    this.xType = param.getYType();
    this.yType = param.getXType();
    x1 = measurement.doubleLowerBound();
    x2 = measurement.doubleUpperBound();
    if (this.xType == Axe.INTEGRAL_AXE) {
    this.fillStyle = FillStyle.X_DIRECTION — FillStyle.POINTS_PLOTTYPE;
    } else {
    this.fillStyle = FillStyle.X_DIRECTION — FillStyle.LINES_PLOTTYPE;
    }
    }
    public MinimumBetaMeasurementSpace2dParameter1(IFunction f1, IFunction f2, IInterval dRange, double realization,
    IInterval measurement, int shapeStyleIndex, int measurementStyleIndex, int markedPointStyleIndex) {
    this(f1, f2, dRange, realization, measurement, LineStyle.defaultStyles[shapeStyleIndex],
    LineStyle.defaultStyles[measurementStyleIndex], LineStyle.defaultStyles[markedPointStyleIndex]);
    }
    public IShape2dParameter getShapeParameter() {
    return new IShape2dParameter() {
    public int getXType() {
    return param.getXType();
    }
    public int getYType() {
    return param.getYType();
    }
    }
    }
}

```



```

public IInterval getPreferredXRange() {
return param.getPreferredXRange();
}
public IInterval getPreferredYRange() {
return preferredXRange;
}
public IInterval getMaximumXRange() {
return param.getMaximumXRange();
}
public IInterval getMaximumYRange() {
return preferredXRange;
}
public IFunction2dParameter getFunctionParameter1() {
return param.getFunctionParameter1();
}
public IFunction2dParameter getFunctionParameter2() {
return param.getFunctionParameter2();
}
};
}
public Line2d2 getMeasurement() {
return new Line2d2(x1, this.realization, x2, this.realization, measurementStyle);
}
public MarkedPoint2d2 getMarkedPoint1() {
return new MarkedPoint2d2(new Point2d(x1, this.realization), markedPointStyle);
}
public MarkedPoint2d2 getMarkedPoint2() {
return new MarkedPoint2d2(new Point2d(x2, this.realization), markedPointStyle);
}
public IInterval getPreferredXRange() {
return this.preferredXRange;
}
public IInterval getPreferredYRange() {
return param.getPreferredXRange();
}
final public IInterval getMaximumXRange() {
return this.preferredXRange;
}
public IInterval getMaximumYRange() {
return param.getMaximumXRange();
}
public int getFillStyle() {
return this.fillStyle;
}
publicLineStyle getShapeStyle() {
return this.shapeStyle;
}
public int getXType() {
return this.xType;
}
}

```

```

public int getYType() {

return this.yType;

}

}

```

The next Java Program creates a Stochastic Object (Minimum  $\beta$ -Measurement Space) based on the parameters constructed by the previous Java Program:

```

public class MinimumBetaMeasurementSpace2d1 extends PlotObjectSet2d {

public MinimumBetaMeasurementSpace2d1(MinimumBetaMeasurementSpace2dParameter1 param) {

this.add(new Shape2d22(param.getShapeParameter(), param.getShapeStyle(), param.getFillStyle()));

this.add(param.getMarkedPoint1());

this.add(param.getMarkedPoint2());

Line2d2 line = param.getMeasurement();

if (param.getXType() == Axe.INTEGRAL_AXE) {

double y = line.getP1().getY();

double x = Math.min(line.getP1().getX(), line.getP2().getX());

for (int i = 0; i <= Math.abs(line.getP1().getX() - line.getP2().getX()); i++) {

this.add(new StyledPoint2d2(new Point2d(x + i, y), line.getLineStyle()));

}

} else {

this.add(line);

}

}

}

```

Two classes are extremely important for transforming the results obtained from *Stochastikon Calculator* to Plotting Objects. They implement IFunction2dParameter, which contains only one method:

**getFunction()**

In Table 16.5 methods of these two classes are listed.

Function2dParameter1(IFunction f)

<u>IFunction</u>	<b>getFunction()</b>
<u>IInterval</u>	<b>getMaximumXRange()</b>
<u>IInterval</u>	<b>getMaximumYRange()</b>
<u>IInterval</u>	<b>getPreferredXRange()</b>
<u>IInterval</u>	<b>getPreferredYRange()</b>
int	<b>getXType()</b>
int	<b>getYType()</b>
<u>IInterval</u>	<b>preferredYRange(IInterval yRange, double[] y)</b>
<u>IInterval</u>	<b>yRange(IInterval xBound)</b>

Function2dParameter2(IFunction f)

<u>IFunction</u>	<b>getFunction()</b>
<u>IFunction</u>	<b>getLBFunction()</b>
<u>IInterval</u>	<b>getMaximumXRange()</b>
<u>IInterval</u>	<b>getMaximumYRange()</b>
<u>IInterval</u>	<b>getPreferredXRange()</b>
<u>IInterval</u>	<b>getPreferredYRange()</b>
<u>IFunction</u>	<b>getUBFunction()</b>
int	<b>getXType()</b>
int	<b>getYType()</b>
double[]	<b>getYValues(IInterval xBound, IFunction f)</b>
<u>IInterval</u>	<b>preferredYRange(IInterval yRange, double[] y)</b>
<u>IInterval</u>	<b>yRange(IInterval xBound, IFunction f1, IFunction f2)</b>

Table 16.5: The two most important Java classes for Stochastic Objects.

These two classes calculate and decide on the plotting range for the parameter according to the functional results from *Stochastikon Calculator* and according to properties of the function to be plotted, i. e., continuous or discrete, converging, unimodal, probability density function, and so on.

In Table 16.5, the ‘preferred’ range means that range, which contains only the relevant parts of the function, while the irrelevant parts are omitted. Certainly the function graph is meaningful when plotted in any range within the ‘maximum’ range, however, often using the maximum range would yield hardly understandable graphics. The maximum range becomes important, when different functions with different ranges are combined into one graph.

The differences between these two classes are the following:

- `Function2dParameter1(IFunction f)` handles real-valued functions, i. e., when  $y = f(x)$  is real value. This class can be used to handle the probability functions.
- `Function2dParameter2(IFunction f)` handles set functions, i. e., when  $y = f(x)$  is given by an interval. This class is useful for a graphical representation of stochastic procedures and all related functions.

### 16.1.7 Illustrations of Stochastic Objects

The following figures illustrate Stochastic Objects generated by *Stochastikon Graphics* in cooperation with *Stochastikon Calculator*. Because so far only the Binomial distribution can be more or less completely handled by *Stochastikon Calculator*, all the graphics displayed below belong to the Binomial distribution.

The examples include single graphics, sets of graphics, 2D-graphics and 3D-graphics. Moreover, there are real-valued functions displayed and set functions. Hence, all the possibilities to represent graphically Stochastic Objects by means of *Stochastikon Graphics* are represented.

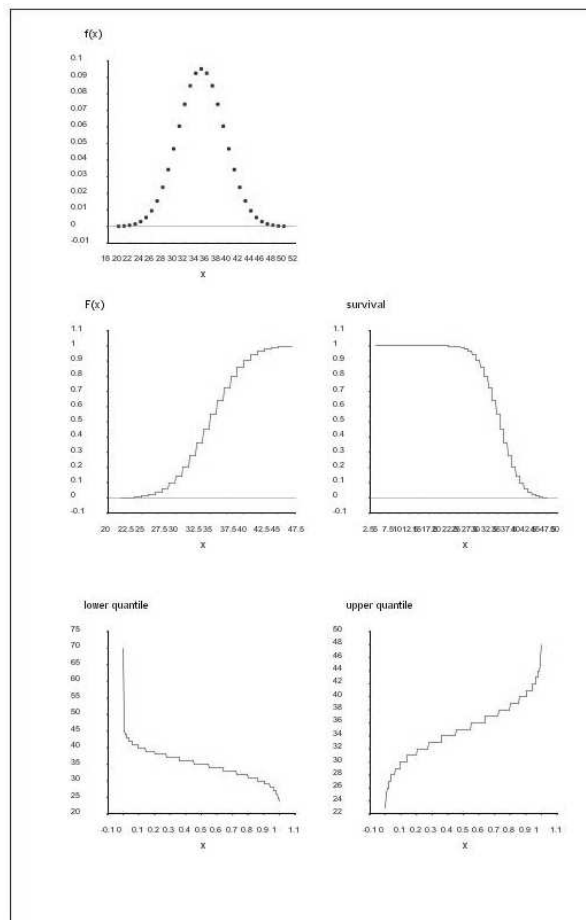


Figure 16.8: Properties of the Binomial distribution.

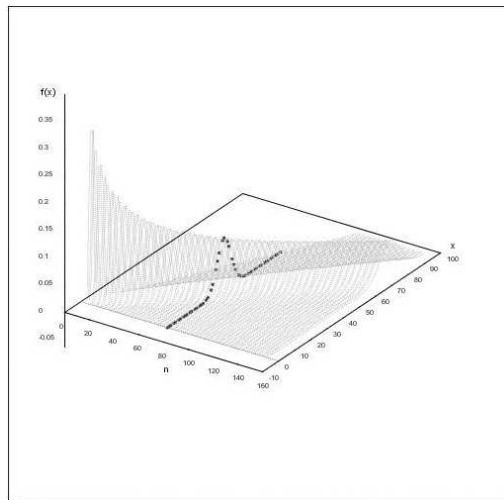


Figure 16.9: Properties of the Binomial distribution in a 3-dimensional graph.

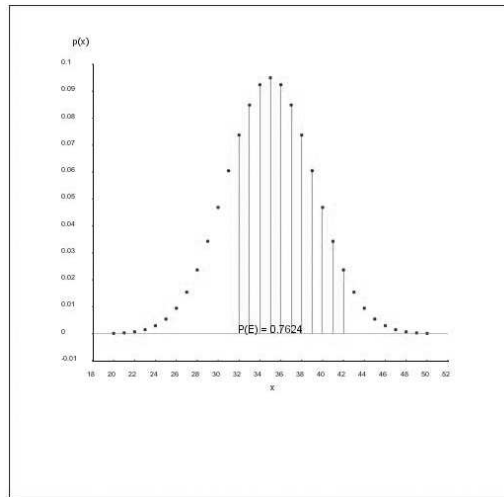


Figure 16.10: Probability of an Event assuming the Binomial distribution.

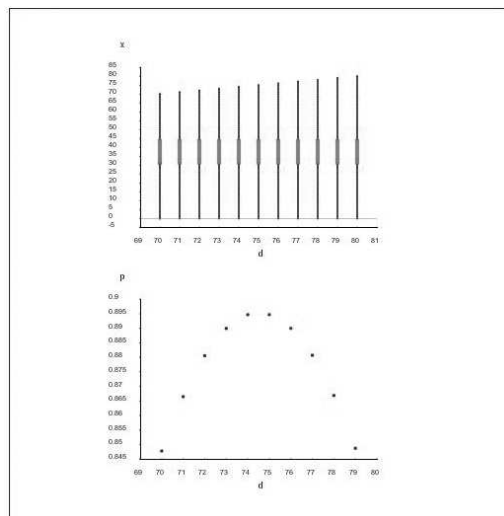


Figure 16.11: Binomial  $\beta$ -Uncertainty Space (blue) and Predictions (red) with ignorance about  $n$  (upper part), and (lower part) the probability of a prediction.

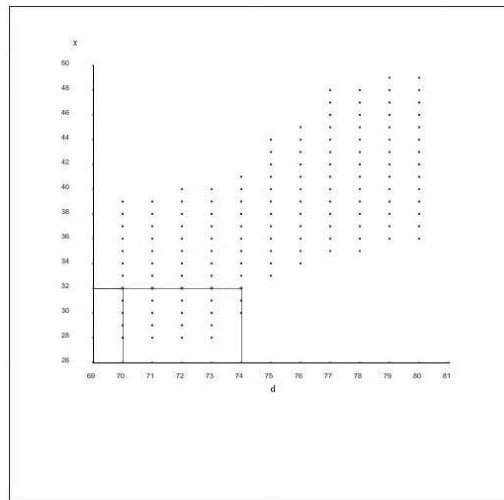


Figure 16.12: Binomial  $\beta$ -Measurement for  $n$  with  $x$  value corresponding to the red line being the realization.

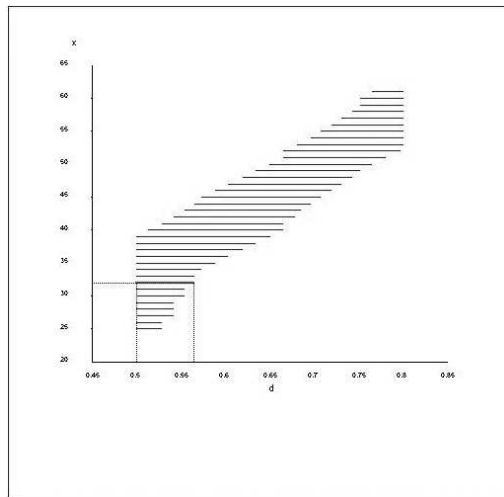


Figure 16.13: Binomial  $\beta$ -Measurement for  $p$  and the  $x$  value corresponding to the red line being the realization.

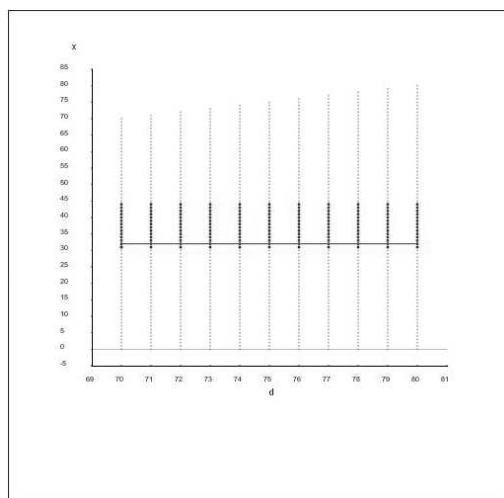


Figure 16.14: Binomial  $\beta$ -Exclusion (blue) and the  $x$  value corresponding to the red line being the realization.

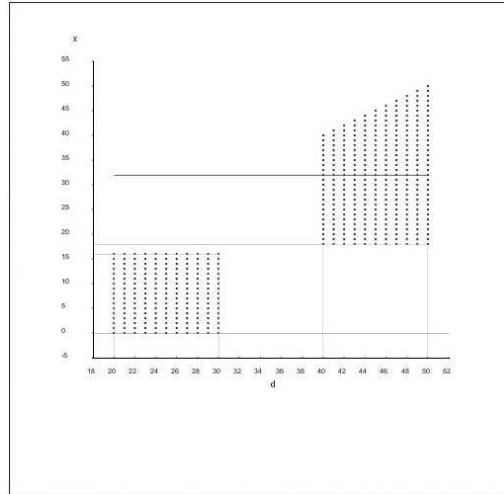


Figure 16.15: Binomial  $\beta$ -Classification for two alternatives about  $n$  and the  $x$  value corresponding to the red line being the realization.

## 16.2 Plotting Data

The Plotting Data classes are used to store, transfer and offer Plotting Objects for drawing graphs. They implement interfaces that contain methods primarily for ‘getting’ Plotting Objects. In the following section three interfaces and two classes are introduced.

### 16.2.1 IPlotData, IPlotData2d and IPlotData3d

- IPlotData is the basic common interface and is extended by other two specialize interfaces: IPlotData2d and IPlotData3d. Table 16.6 lists the main methods contained in the IPlotData interface.

int	<u>getCurveLineStyle</u> (int index)
int	<u>getCurvePlotStyle</u> (int index)
LineStyle[]	<u>getLineStyles</u> ()
int	<u>getPolygonFillStyle</u> (int index)
int	<u>getPolygonLineStyle</u> (int index)
int	<u>getShapeFillStyle</u> (int index)
int	<u>getShapeLineStyle</u> (int index)
Axe	<u>getXAxe</u> ()
Axe	<u>getYAxe</u> ()
int	<u>numberOfCurves</u> ()
int	<u>numberOfPolygons</u> ()
int	<u>numberOfShapes</u> ()

Table 16.6: Methods in the IPlotData interface.

- IPlotData2d is the interface for 2-dimensional Plotting Data, which extends IPlotData. Table 16.7 lists the main methods contained in the IPlotData2d interface.

Point2d[]	<u>getCurvePoints</u> (int index)
Label2d[]	<u>getLabels</u> ()
Line2d[]	<u>getLines</u> ()
StyledPoint2d[]	<u>getMarkedPoints</u> ()
StyledPoint2d[]	<u>getPoints</u> ()
Point2d[]	<u>getPolygonPoints</u> (int index)
Point2d[]	<u>getShapePoints</u> (int index)

Table 16.7: Methods in the IPlotData2d interface.

- IPlotData3d is the interface for 3-dimensional Plotting Data, which also extends IPlotData. Table 16.8 lists the main methods contained in the IPlotData3d interface.



<u>Point3d[]</u>	<u>getCurvePoints</u> (int index)
<u>Label3d[]</u>	<u>getLabels</u> ()
<u>Line3d[]</u>	<u>getLines</u> ()
<u>StyledPoint3d[]</u>	<u>getMarkedPoints</u> ()
<u>StyledPoint3d[]</u>	<u>getPoints</u> ()
<u>Point3d[]</u>	<u>getPolygonPoints</u> (int index)
<u>Point3d[]</u>	<u>getShapePoints</u> (int index)
<u>Axe</u>	<u>getZAxe</u> ()

Table 16.8: Methods in the IPlotData3d interface.

## 16.2.2 PlotData2d and PlotData3d

PlotData2d and PlotData3d are two classes which implement the above introduced two interfaces: IPlotData2d and IPlotData3d respectively. As indicated by their names the first one is used for 2-dimensional objects, while the second is used for 3-dimensional objects.

- The PlotData2d is used to store, transfer and offer Plotting Objects for 2-dimensional graphs. Besides methods implemented from IPlotData2d, it has methods listed in Table 16.9.

Methods for adding 2-dimensional Plotting Objects into a 2-dimensional plotting data.	<b><code>addCurve(Point2d[] points, int lineStyle, int plotStyle)</code></b> <b><code>addLabel(Label2d label), addLabels(Label2d[] newLabels)</code></b> <b><code>addLine(Line2d arrow), addLines(Line2d[] newArrows)</code></b> <b><code>addLineStyle(LineStyle style)</code></b> <b><code>addMarkedPoint(StyledPoint2d point),</code></b> <b><code>addMarkedPoints(StyledPoint2d[] newPoints)</code></b> <b><code>addPoint(StyledPoint2d point), addPoints(StyledPoint2d[] newPoints)</code></b> <b><code>addPolygon(Point2d[] points, int lineStyle, int fillStyle)</code></b> <b><code>addShape(Point2d[] points, int lineStyle, int fillStyle)</code></b>
Methods for cleaning a specific 2d Plotting Object from a 2d plotting data.	<b><code>resetCurves(), resetLabels(), resetLines(), resetMarkedPoints(),</code></b> <b><code>resetPoints(), resetPolygons(), resetShapes()</code></b>
Methods for setting specific 2-dimensional Plotting Object into a 2-dimensional plotting data which has no such Object before.	<b><code>setLabels(Label2d[] values), setLines(Line2d[] values),</code></b> <b><code>setLineStyles(LineStyle[] styles),</code></b> <b><code>setMarkedPoints(StyledPoint2d[] values),</code></b> <b><code>setPoints(StyledPoint2d[] values),</code></b> <b><code>setXAxe(Axe xAxe), setYAxe(Axe yAxe)</code></b>
Methods for getting the index (position) of a specific LineStyle in this plotting data's existing LineStyle list, if the list does not contain this LineStyle list, add it to the list then return its position.	<b><code>getLineStyleIndex(LineStyle lineStyle)</code></b>
Methods for writing, reading, creating a 2-dimensional plotting data from a stream or a string.	<b><code>create(DataInputStream in), writeToStream(DataOutputStream out), readFromStream(DataInputStream in),</code></b> <b><code>createFromString(String s), toString(), valueOf(String s)</code></b>

Table 16.9: More methods in PlotData2d class.

- The PlotData3d is used to store, transfer and offer Plotting Objects for 3-dimensional graphs. Besides methods implemented from IPlotData3d, it has methods listed in Table 16.10.

The methods in Table 16.10 are mainly used to ‘add’ a 3-dimensional Plotting Objects into a 3-dimensional Plotting Data. There are two ways of constructing a 3-dimensional Plotting Data:

<p>Methods for adding 3-dimensional Plotting Objects into a 3-dimensional plotting data.</p>	<pre> <b>add Curve</b>(Point3d[] points, int lineStyle, int plotStyle), <b>add CurveXY</b>(Point2d[] points, int lineStyle, int plotStyle, double z), <b>add CurveXZ</b>(Point2d[] points, int lineStyle, int plotStyle, double y), <b>add CurveYZ</b>(Point2d[] points, int lineStyle, int plotStyle, double x) <b>add Label</b>(Label3d label), <b>add Labels</b>(Label3d[] newLabels), <b>add LabelsXY</b>(Label2d[] newLabels, double z), <b>add LabelsXZ</b>(Label2d[] newLabels, double y), <b>add LabelsYZ</b>(Label2d[] newLabels, double x), <b>add LabelXY</b>(Label2d label, double z), <b>add LabelXZ</b>(Label2d label, double y), <b>add LabelYZ</b>(Label2d label, double x), <b>add Line</b>(Line3d arrow), <b>add Lines</b>(Line3d[] newArrows), <b>add LineStyle</b>(LineStyle style), <b>add LinesXY</b>(Line2d[] newArrows, double z), <b>add LinesXZ</b>(Line2d[] newArrows, double y), <b>add LinesYZ</b>(Line2d[] newArrows, double x), <b>add LineXY</b>(Line2d arrow, double z), <b>add LineXZ</b>(Line2d arrow, double y), <b>add LineYZ</b>(Line2d arrow, double x), <b>add MarkedPoint</b>(StyledPoint3d point), <b>add MarkedPoints</b>(StyledPoint3d[] newPoints), <b>add MarkedPointsXY</b>(StyledPoint2d[] newPoints, double z), <b>add MarkedPointsXZ</b>(StyledPoint2d[] newPoints, double y), <b>add MarkedPointsYZ</b>(StyledPoint2d[] newPoints, double x), <b>add MarkedPointXY</b>(StyledPoint2d point, double z), <b>add MarkedPointXZ</b>(StyledPoint2d point, double y), <b>add MarkedPointYZ</b>(StyledPoint2d point, double x), <b>add Point</b>(StyledPoint3d point), <b>add Points</b>(StyledPoint3d[] newPoints), <b>add PointsXY</b>(StyledPoint2d[] newPoints, double z), <b>add PointsXZ</b>(StyledPoint2d[] newPoints, double y), <b>add PointsYZ</b>(StyledPoint2d[] newPoints, double x), <b>add PointXY</b>(StyledPoint2d point, double z), <b>add PointXZ</b>(StyledPoint2d point, double y), <b>add PointYZ</b>(StyledPoint2d point, double x), <b>add Polygon</b>(Point3d[] points, int lineStyle, int fillStyle), <b>add PolygonXY</b>(Point2d[] points, int lineStyle, int fillStyle, double z), <b>add PolygonXZ</b>(Point2d[] points, int lineStyle, int fillStyle, double y), <b>add PolygonYZ</b>(Point2d[] points, int lineStyle, int fillStyle, double x), <b>add Shape</b>(Point3d[] points, int lineStyle, int fillStyle), <b>add ShapeXY</b>(Point2d[] points, int lineStyle, int fillStyle, double z), <b>add ShapeXZ</b>(Point2d[] points, int lineStyle, int fillStyle, double y), <b>add ShapeYZ</b>(Point2d[] points, int lineStyle, int fillStyle, double x) </pre>
--	---

Table 16.10: More methods in PlotData3d class.

- A Plotting Object is originally a 3-dimensional Object, and then the 3-dimensional Plotting Data is obtained by applying a method like:

```
addMarkedPoint(StyledPoint3d point)
```

- The second way consists of adding one more dimension to a 2-dimensional Plotting Object, by means of methods like:

`addMarkedPointXY(StyledPoint2d point, double z)`

`addMarkedPointYZ(StyledPoint2d point, double x)`

`addMarkedPointXZ(StyledPoint2d point, double y)`

Other methods of the `PlotData3d` class are listed in Table 16.11.

Methods for cleaning a specific 3-dimensional Plotting Object from a 3-dimensional plotting data.	<code>resetCurves()</code> , <code>resetLabels()</code> , <code>resetLines()</code> , <code>resetMarkedPoints()</code> , <code>resetPoints()</code> , <code>resetPolygons()</code> , <code>resetShapes()</code>
Methods for setting specific 3-dimensional Plotting Object into a 3-dimensional plotting data which has no such Object before.	<code>setLabels(Label3d[] values)</code> , <code>setLines(Line3d[] values)</code> , <code>setLineStyles(LineStyle[] styles)</code> , <code>setMarkedPoints(StyledPoint3d[] values)</code> , <code>setPoints(StyledPoint3d[] values)</code> , <code>setXAxis(Axis xAxis)</code> , <code>setYAxis(Axis yAxis)</code> , <code>setZAxis(Axis zAxis)</code>
The same as <code>PlotData2d</code>	<code>getLineStyleIndex(LineStyle lineStyle)</code>
Methods for writing, reading, creating a 3-dimensional plotting data from a stream or a string.	<code>create(DataInputStream in)</code> , <code>writeToStream(DataOutputStream out)</code> , <code>readFromStream(DataInputStream in)</code> , <code>createFromString(String s)</code> , <code>toString()</code> , <code>valueOf(String s)</code>

Table 16.11: Other methods in `PlotData3d` class.

Some algorithms are used for expressing Plotting Objects stored in Plotting Data in form of graphs, especially for drawing ‘Shape’ and ‘Polygon’. For example, for ‘filling’ a Polygon, every symmetric point pair in the plotting direction has to be identified. To this end not only a counting algorithm is needed, but also a number of side conditions have to be met, for instance, for concave and the convex point series.

The following Java Program constitutes the main part of the ‘filling’ algorithm:

```
public static List getPolygonFillList(double[] px, double[] py, double[] xValues) {
    java.util.List yList = new ArrayList();
    Set inSet = inSet(py, px); //change (px,py)
    IInterval[] edges = new DoubleInterval[px.length];
    for (int i = 0; i < px.length - 1; i++) {
        if (px[i] < px[i + 1]) {
            edges[i] = new DoubleInterval(px[i], px[i + 1]);
        } else {
            edges[i] = new DoubleInterval(px[i + 1], px[i]);
        }
    }
    if (px[px.length - 1] < px[0]) {
        edges[px.length - 1] = new DoubleInterval(px[px.length - 1], px[0]);
    } else {
        edges[px.length - 1] = new DoubleInterval(px[0], px[px.length - 1]);
    }
}
```

```

for (int i = 0; i < xValues.length; i++) {
java.util.List yRange = new ArrayList();
yRange.add(new Double(xValues[i]));
for (int j = 0; j < px.length; j++) {
boolean inEdge = false;
if (edges[j].contains(xValues[i]) && !(edges[j].doubleLowerBound() == edges[j].doubleUpperBound())) {
if (!inEdge && ((j == 0) —— (j == px.length - 1) &&&
(edges[0].doubleLowerBound() == edges[0].doubleUpperBound()) —— (j > 0) &&&
!(xValues[i] == edges[j - 1].doubleUpperBound()) && !(xValues[i] == edges[j - 1].doubleLowerBound()) &&&
!(edges[j - 1].doubleUpperBound() == edges[j - 1].doubleLowerBound())) —— inSet.contains(new Double(xValues[i])) —— (j > 0) &&&
(edges[j - 1].doubleUpperBound() == edges[j - 1].doubleLowerBound())) { //!inEdge
inEdge = true;
if (j == px.length - 1) {
double t = (xValues[i] - px[j]) / (px[0] - px[j]);
double yValue = py[j] + t * (py[0] - py[j]);
yRange.add(new Double(yValue));
} else {
double t = (xValues[i] - px[j]) / (px[j + 1] - px[j]);
double yValue = py[j] + t * (py[j + 1] - py[j]);
yRange.add(new Double(yValue));
}
} else {
inEdge = false;
}
} else {
inEdge = false;
}
}
yList.add(yRange);
}
return yList;
}

```

The next class supports the above algorithm by checking and storing points, which need special care.

```

public static Set inSet(double[] px, double[] py) {
Set inSet = new HashSet();
for (int i = 1; i < py.length - 1; i++) {
if (((py[i - 1] > py[i] && py[i + 1] > py[i]) —— (py[i - 1] < py[i] && py[i + 1] < py[i])) {
inSet.add(new Double(px[i]));
}
}
if (((py[py.length - 1] > py[0] && py[1] > py[0]) —— (py[py.length - 1] < py[0] && py[1] < py[0])) {
inSet.add(new Double(px[0]));
}
if (((py[py.length - 2] > py[py.length - 1] && py[0] > py[py.length - 1]) —— (py[py.length - 2] < py[py.length - 1] && py[0] < py[py.length - 1])) {
inSet.add(new Double(px[py.length - 1]));
}
return inSet;
}

```

## 16.3 Stochastic Graph Systems

The Stochastic Plotting Objects can be stored and transferred by Plotting Data for building up a complete Stochastic Graph System. The ways of how to generate graphs by Plotting Data and further how to construct different independent complete Stochastic Graph Systems are briefly described in the following sub-sections.

### 16.3.1 Two ways of generating graphs by Plotting Data

Plotting Data can be used to generate graphs in two different ways:

1. The GNUPLOT graphical software is used for generating graphs, which are saved in form of \*.eps, \*.gif or \*.tex files.

In the ‘Gnuplot Homepage’ <http://www.gnuplot.info/> the Gnuplot is described as follows:

*Gnuplot is a portable command-line driven interactive data file (text or binary) and function plotting utility for UNIX, IBM OS/2, MS Windows, DOS, Apple Macintosh, VMS, Atari and many other platforms. The software is copyrighted but freely distributed (i. e., you don't have to pay for it). It was originally intended as graphical program, which would allow scientists and students to visualize mathematical functions and data. It does this job pretty well, and in addition it serves as non-interactive plotting engine for miscellaneous portable third-party applications, like Octave. Gnuplot is developed and supported since 1986, and having its scripts and commands easy to understand text files, it is time-portable as well.*

The first step consists of transforming the Plotting Data, which contain Plotting Objects, to a \*.gnu file. This is done for 2-dimensional graphs and for 3-dimensional graphs, respectively, by the following classes

`GnuplotFile2d(IPlotData2d plotData, int outputFileType, String path, String size, int tics)`

`GnuplotFile3d(IPlotData3d plotData, int outputFileType, String path, String size, int tics)`

The main function of the above two transforming classes is to change the Plotting Objects contained in the Plotting Data into corresponding Gnuplot plotting commands and to store these commands into a drawable \*.gnu file, which can be used by the Gnuplot software for generating the \*.eps, \*.gif or \*.tex graph file.

2. Java programs are used for generating Java images, which subsequently are transformed into \*.jpeg, \*.png files or used as Java image streams.

All classes in Table 16.12 are used to transform Plotting Objects from the Plotting Data form into standard Java.awt.Graphics2D objects. With these classes Plotting Objects are stored into Java images (BufferedImage) and can be shown on screen, saved as \*.jpeg, \*.png files or as Java image streams.

<b><u>G 3D</u></b>	This class contains basic methods used to calculate and change a 2-dimension position to a 3-dimension position.
<b><u>G Axe</u></b>	This class contains methods used to calculate and draw axes (axis lines, axis labels and axis tics) into a Java image graph.
<b><u>G Tic</u></b>	This class contains methods used to calculate and set tic positions and tic labels containing in a Java image graph.
<b><u>G Label</u></b>	This class contains basic methods used to set and draw labels from different kinds of parameters into a Java image graph.
<b><u>G Line</u></b>	This class contains methods used to draw lines from different kinds of parameters into a Java image graph.
<b><u>G Point</u></b>	This class contains methods used to draw points from different kinds of parameters into a Java image graph.
<b><u>G Shape</u></b>	This class contains methods used to draw shapes from different kinds of parameters into a Java graph.
<b><u>G LineStyle</u></b>	This class contains methods used to change the LineStyle from IPlotData style to proper Java image graph style.
<b><u>G PlotStyle</u></b>	This class contains methods used to change the PlotStyle from IPlotData style to proper Java image graph style.
<b><u>G Graph</u></b>	This class "plots" a Java image graph according to a plot data.
<b><u>G PlotData2d</u></b>	This class used to draw a Java image graph according an IPlotData2d object.
<b><u>G PlotData3d</u></b>	This class used to draw a Java image graph according an IPlotData3d object.

Table 16.12: Classes transforming Plotting Objects into Java.awt.Graphics2D objects.

Clearly, more functions are needed to generate Java images from a Plotting Data than to generate Gnuplot files. This is due to the fact that Gnuplot is a graphical software package which includes many functions that specially serve for plotting graphs.

For example Gnuplot software can automatically plot axes with tics and tic labels when fixing the corresponding parameters, i. e., range of each axis, length of each tic and name of axes. In contrast, for a Java image the positions of axes, axis names, tics and tic labels have to be counted and then lines for the axes have to be drawn, where points stand for the tics and labels for names of each axis and tic.

Moreover, Gnuplot software can easily generate a 3-dimensional graph, but for a Java image one needs to count the position of every 3-dimensional point in a 2-dimensional graph.

Although Gnuplot software can automatically count tic length and plot tic labels, there are a number of requirements for *Stochastikon Graphics*, which necessitates the development of special algorithms, for instance for meeting the requirements with respect to tic length (no overlap in case of long tic labels) and tic label (last character should be 2 or 5 and 0 if larger than 10) within the graphs.

The following Java program is the main part of this algorithm for determining the standard tics in a Gnuplot file. The idea is nearly the same but more complex when used for Java images, because in a Java image one needs to distinguish between 2-dimensional axes and 3-dimensional axes.

```

public final String getTic(String ticName, boolean D3, int setTics) throws PlotGeneratorException {
    if (this.bounds.getVolume() <= Double.MIN_VALUE ) {
        return "1";
    } else {
        int p = (D3 && !(setTics == NO_TIC)) ? 6 : 9;
        String tic = null;
        if (getType() == INTEGRAL_AXE) { //integer range
            int tint = 1;
            int range = (int)this.bounds.getVolume();
            if (range == 0 && !ticName.equals("xAxe") && !ticName.equals("yAxe")) {
                tint = 0;
            } else if (range < 0 && (ticName.equals("xAxe") —— ticName.equals("yAxe"))) {
                throw new PlotGeneratorException("Error - " + ticName + " range is 0 or negative. ");
            } else {
                if (ticName.equals("xAxe") —— (D3 && ticName.equals("yAxe"))) {
                    tint = Math.max(range / p, 1);
                    int ml = Math.max(String.valueOf(getBounds().intLowerBound()).length(),
                        String.valueOf(getBounds().intUpperBound()).length());
                    if (ml > 6) {
                        p = p / (((ml - String.valueOf(tint).length() + 5) / 5) + 1);
                    } else {
                        p = p / ((ml / 5) + 1);
                    }
                    tint = Math.max(range / p, 1);
                } else {
                    if (range > 10) {
                        tint = range / p + 1;
                    }
                }
                tic = String.valueOf(tint);
            } else { //double range
                tic = "0.1";
                double range = this.bounds.getVolume();
                if (range <= 0.0 && !ticName.equals("xAxe") && !ticName.equals("yAxe")) {
                    D3 = false;
                    tic = "0.0";
                } else if (range < 0.0 && (ticName.equals("xAxe") —— ticName.equals("yAxe"))) {
                    throw new PlotGeneratorException("Error - " + ticName + " range is 0 or negative. ");
                } else {
                    double tdouble = range / p;
                    if (tdouble > 1) {
                        tic = NumberTool.format(tdouble, 1);
                    } else {
                        tic = NumberTool.format(tdouble, (int)(-Math.log(tdouble) / Math.log(10) + 1));
                    }
                }
                if (ticName.equals("xAxe") —— (D3 && ticName.equals("yAxe"))) {
                    int pt = tic.length() - tic.indexOf(".");
                    int ml = Math.max(String.valueOf((int)getBounds().doubleLowerBound()).length(),
                        String.valueOf((int)getBounds().doubleUpperBound()).length());

```



```

if (ml > 6) {
p = p / (((ml - String.valueOf((int)tdouble).length() + 5 + pt) / 5) + 1);
} else if (getBounds().doubleUpperBound() < 1) {
ml = (int)(-Math.log(getBounds().doubleUpperBound()) / Math.log(10)) + 1;
if (ml > 4) {
p = p / (((5 + pt - ml) / 5) + 1);
} else {
p = p / ((pt + 2) / 5 + 1);
}
} else {
p = p / (((ml + pt) / 5) + 1);
}
tdouble = range / p;
if (tdouble > 1) {
tic = NumberTool.format(tdoube, 1);
} else {
tic = NumberTool.format(tdoube, (int)(-Math.log(tdoube) / Math.log(10) + 1));
}
}
}
}
return polishTic(tic);
}
}

```

### 16.3.2 Three Stochastic Graph Systems

Till now three different kinds of Stochastic Graph Systems have been implemented by *Stochastikon Graphics* based on Plotting Objects, Stochastic Objects and Plotting Data. Each of these three systems serves a different purpose.

#### Report System

The Report System refers to the generation of scientific reports by *Stochastikon Calculator* which incorporate graphs for illustrating stochastic procedures and results.

According to certain requirement and parameters, *Stochastikon Calculator* provides a user with a complete scientific report of the result. Each report contains a number of graphs showing the corresponding stochastic concepts, procedures and results.

For compiling a report, *Stochastikon Calculator* sends the calculated results to *Stochastikon Graphics* asking for graphs. *Stochastikon Graphics* uses the results together with Gnuplot software to generate graphs and sends them back to *Stochastikon Calculator*, which inserts these graphs into specific places within the report. The Stochastic Graph System, which generates graphs for a stochastic report is called the ‘Report System’.

The Report System can be used in both stand-alone computer and on-line via internet. Users can download or print these final \*.pdf reports from *Stochastikon Calculator*.

### Stand-Alone Computer Graph System

When installing the *Stochastikon* system in a local computer, users can use the ‘Stand-Alone Computer Graph System’, which is an interactive dynamic graphical system for Bernoulli Stochastics.

To use the system one has to select a ‘Distribution’ and a ‘Stochastic Object’, subsequently the needed parameters have to be input and some other choices made. According to the input parameters, the graph system will plot corresponding pictures on the graph panel using the calculating results obtained from *Stochastikon Calculator*. Thus, users can watch changes in the graphics due to the changes of the parameter values. Moreover, each generated picture can also be printed if desired.

### Web Graph System

The system for using Stochastic Graphics via internet without installing *Stochastikon* in the local computer is called ‘Web Graph System’. Analogously, as the stand-alone system it is an interactive dynamic graphical system for Bernoulli Stochastics.

After having selected the ‘Distribution’ and the ‘Stochastic Object’ by clicking different links on the web page, an applet will run and wait that the parameters are input and some other choices are made. By clicking the ‘Show Graph’ button to identify the request, the requirements are sent to the web server. The resulting graphs are \*.jpeg files, and will be shown on the applet panel. Users can also get the graph files from some other web pages. The ability of Web Graph System is almost the same as Stand-Alone Computer Graph System except for the ‘Show Graph’ button.

Web Graph System has been used for the Graphical (Stochastic Procedure) Laboratory in *Stochastikon Magister*, and it works perfectly well (see Section 10.3.2 ‘Exercises in Course 2’ Figure 10.28 and Section 10.5 ‘Graphical Laboratory’).

### 16.3.3 Realization of the Stochastic Graph Systems

All these implemented Stochastic Graph Systems are developed by Java, only in the Report System Gnuplot software is added to help the generating of \*.eps graph files.

#### Realization of the Report System

Each scientific report produced by *Stochastikon Calculator* includes graphical representations of certain aspects of the given situation, of the involved probability distributions and the stochastic results. By obtaining the request of *Stochastikon Calculator*, some Plotting Data are generated by *Stochastikon Graphics* according to the users’ requirements and the *Stochastikon Calculator*’s calculating results. Each Plotting Data contains a Stochastic Object which will be used to form a \*.gnu file for the Gnuplot software to produce a \*.eps graph file.

After being generated by *Stochastikon Graphics* and sent to *Stochastikon Calculator*, the required graphs will be inserted into their proper places in the final \*.pdf report. In Figure 16.16 the flowchart of Report System is displayed:

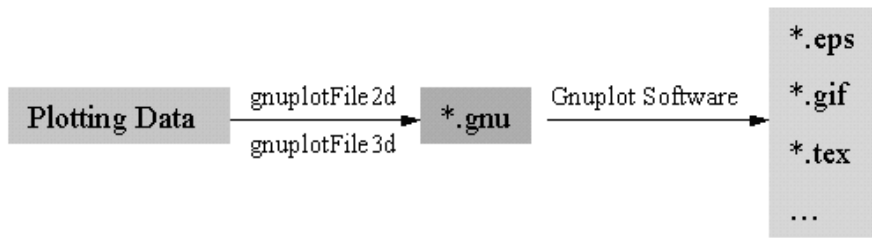


Figure 16.16: Flowchart of the Report System.

### Realization of the Stand-Alone Computer Graph System

The Stand-Alone Computer Graph System is a multi-lever system. All Plotting Data are generated according to the users' requirements and the calculated results from *Stochastikon Calculator*.

Figure 16.17 outlines the structure of the Stand-Alone Computer Graph System, which also shows the most important class pair for the Stand-Alone Computer Graph System - JavaP and P\_Component. Functions of this class pair are listed in Table 16.13.

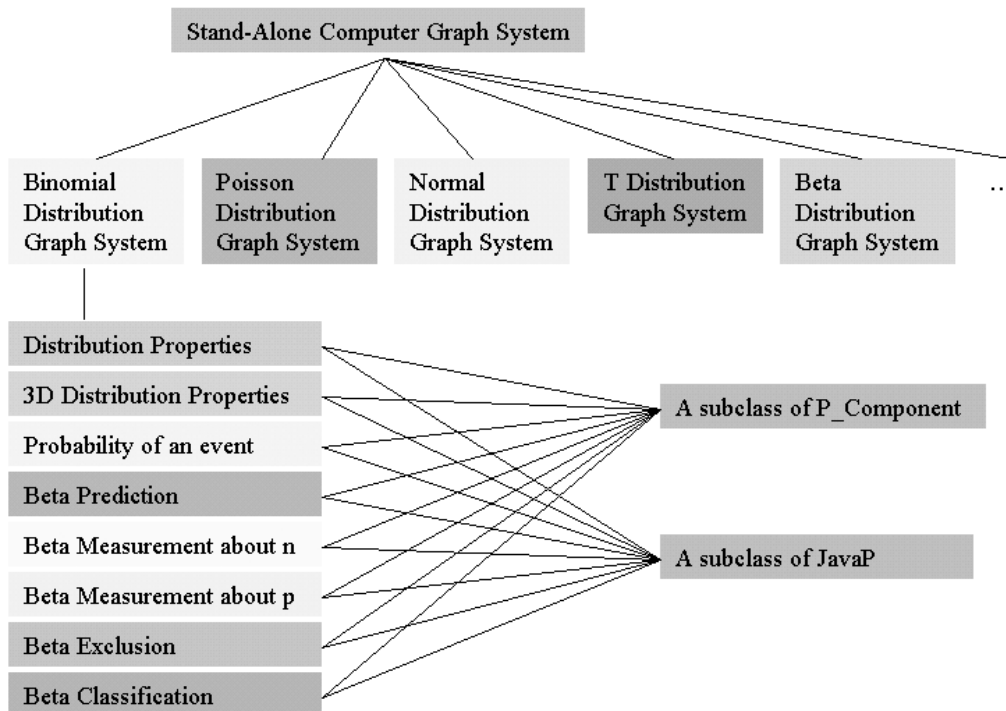


Figure 16.17: Structure of the Stand-Alone Computer Graph System.

<u>JavaP</u>	<p>extends <a href="#">JavaImage</a></p> <p>implements <a href="#">AdjustmentListener</a>, <a href="#">ItemListener</a></p> <p>This class is an abstract class used to generate an image graph in a printable container according to different parameters. It is one of the basic classes of the stand-alone-computer graph system which generate graphs according to the user's input Parameters.</p> <p>All classes extends this class are used <b>to settle java swing objects for user to input different parameters.</b></p>
<u>P Component</u>	<p>extends <a href="#">IComponent</a></p> <p>implements <a href="#">IReportKeys</a></p> <p>This is an abstract class contains the common methods and parameter variables used to generate an image graph in a printable container according to different parameters. It is one of the basic classes of the stand-alone-computer graph system which generate graphs according to the user's input Parameters.</p> <p>All classes extends this class are used <b>to draw graphs according to parameters transferred from corresponding <a href="#">JavaP</a> subclasses.</b></p>

Table 16.13: The basic class pair for the Stand-Alone Computer Graph System.

'GraphGroups' is a concrete example system of the Stand-Alone Computer Graph System. It can easily be extended for handling more distributions. Table 16.14 below lists the structural components and functions of the 'GraphGroups' class.

<u>GraphGroups</u>	<p>extends <a href="#">JFrame</a></p> <p>implements <a href="#">ActionListener</a></p> <p>This class is the final product of stand-alone-computer graph system which generate graphs according to use's input parameters.</p> <p>It contains several concrete <a href="#">JIntervalFrame</a> classes which are the "extends" of <a href="#">IntervalGraphs</a> such as: <a href="#">IGBinomial</a>, <a href="#">IGPoisson</a>,...;</p> <p>Each concrete <a href="#">JIntervalFrame</a> contains a <a href="#">JTabbedPane</a> such as: <a href="#">BinomialTabbedPane</a>, <a href="#">PoissonTabbedPane</a>,...;</p> <p>Each <a href="#">JTabbedPane</a> corresponding to a special distribution: Binomial, Poisson...;</p> <p>Each distribution's <a href="#">JTabbedPane</a> contains several different panels (created by couples of <a href="#">JavaP</a> - <a href="#">P Component</a> "extends" classes, such as a couple for Binomial distribution properties 2d graph plotting : <a href="#">Binomial2dPDF1 - C_Binomial2dPDF1</a>...).</p> <p>These panels show graphs of different characters (<b>Stochastic Objects</b>): distribution properties, beta measurement, beta prediction, classification, and exclusion... of this distribution.</p> <p><b>See Also:</b> <a href="#">JavaP</a>, <a href="#">P Component</a>, <a href="#">IntervalGraphs</a></p>
--------------------	---

Table 16.14: An example system of the Stand-Alone Computer Graph System.

## Realization of the Web Graph System

The Web Graph System has almost the same structure as the Stand-Alone Computer System. From the internet, users can chose the desired distribution and the Stochastic Object by different paths and links, such as:

<http://.../Binomial/> (for Binomial distribution graph system)

<http://.../Poisson/> (for Poisson distribution graph system)

Each of these web pages, lists a series of links to \*.html files which arouse graphical applets for specific Stochastic Objects.

Below, the Web Graph System for the Binomial Distribution is given:

[Binomial2dPDFClient1.htm](#)  
[Binomial2dPDFClient2.htm](#)  
[Binomial2dProbWithEventClient.htm](#)  
[Binomial3dPDFClient.htm](#)  
[BinomialPredictionClient.htm](#)  
[BinomialMeasurementPClient.htm](#)  
[BinomialMeasurementNClient.htm](#)  
[BinomialExclusionClient.htm](#)  
[BinomialClassificationClient.htm](#)

Figure 16.18 displays the flowchart of Web Graph System.

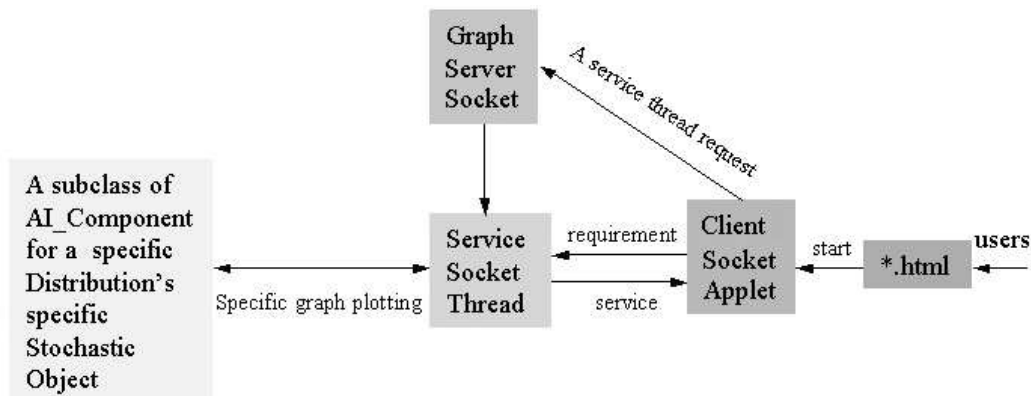


Figure 16.18: Flowchart of the Web Graph System.

The most important pair of classes for the Web Graph System is called AI\_Component and GraphServer. Their functions are listed in Table 16.15.

Table 16.16 lists the program groups for Binomial Distribution in the Web Graph System. Each Stochastic Object corresponds to a group of four program files: a html, a socket applet, a socket thread and a subclass of AI\_Component.

<u>AI Component</u>	<p>extends BufferedImage implements IReportKeys</p> <p>This is an abstract class contains the common methods and parameter variables used to generate an image graph in a BufferedImage according to different parameters. It is the basic class of web graph system which generate graphs according to the internet user's input Parameters.</p> <p>All classes extends this class are <b>used to draw graphs according to parameters transferred from corresponding parameter-acceptor, the Client Socket Applets.</b></p> <p>It can also save the result BufferedImage as a jpeg file or file stream.</p> <p>The correspondent class in stand-alone-computer graph system is <u>P_Component</u>.</p> <p><b>See Also:</b> <u>GraphServer</u></p>
GraphServer	<p>This class likes a graph "server" gives services according to different "clients" requirements.</p> <p>It is a <u>Server Socket</u>. When accept a requirement from a specific client Socket, it will <b>start a corresponding service Socket thread.</b></p> <p>Each <u>Client Socket</u> is a Swing <u>Applet</u> used <b>to accept user's input parameters and send them to the graph server.</b></p> <p>The corresponding <u>service Socket Thread</u> will <b>call a specific subclass of <u>AI_Component</u> to generate a graph according to the parameters transferred from the client, save the graph, tell graph name to its client.</b></p> <p>Then the Client Socket Applet will <b>show the result graph on its panel to user.</b></p>

Table 16.15: The most important class pair for the Web Graph System.

The Stand-Alone Computer Graph System as well as the Web Graph System are interactive dynamic graphical systems using Java images to store, convey and plot graphs. There is only one essential difference:

- The Stand-Alone Computer Graph System directly shows the generated graph images. Thus the change of the graph follows immediately any change of the input parameters. This real time pattern enables the user to watch the relation between past given by the input values and future represented by the images more closely and in more detail.
- The Web Graph System uses applets to show the resulting graphs on the web browser with the graphs being transformed from Java images into \*.jpeg files. The Web Graph System also has a real time version, i. e., the changes of the graph follow immediately those of the inputs, however, the effect is unstable. As shown in the Web Graph System flowchart in Figure 16.18, there are a number of stages in the process of changing the input data to the final graph, which contains a lot of communications between Client Socket Applet and Socket Thread through web and also the time consuming for the saving of \*.jpeg files. If a \*.jpeg file is generated whenever the input is changed, the server usually can hardly stand such burden and satisfy the speed. Therefore, when the Client Socket gets the required graph file name and wants to get the graph, the server may still be busy

in generating the graph. Therefore, in the Web Graph System the ‘ShowGraph’ button is provided, which has to be clicked for getting a graph whenever the user finishes the parameter settings.

Stochastic Objects	HTML file*	Client Socket Applet**	service Socket Thread***	sub-class of AI_Component****
Distribution Properties (2d plotting)	Binomial2dPDFClient1 Binomial2dPDFClient2	Binomial2dPDFClient1 Binomial2dPDFClient2	Binomial2dPDFThread1 Binomial2dPDFThread2	AI_Binomial2dPDF1 AI_Binomial2dPDF2
Distribution Properties (3d plotting)	Binomial3dPDFClient	Binomial3dPDFClient	Binomial3dPDFThread	AI_Binomial3dPDF
Probability of an event	Binomial2dProbOfEventClient	Binomial2dProbOfEventClient	Binomial2dProbOfEventThread	AI_Binomial2dProbOfEvent
Beta Measurement about n	BinomialMeasurementNClient	BinomialMeasurementNClient	BinomialMeasurementNThread	AI_BinomialMeasurementN
Beta Measurement about p	BinomialMeasurementPClient	BinomialMeasurementPClient	BinomialMeasurementPThread	AI_BinomialMeasurementP
Beta Prediction	BinomialPredictionClient	BinomialPredictionClient	BinomialPredictionThread	AI_BinomialPrediction
Beta Exclusion	BinomialExclusionClient	BinomialExclusionClient	BinomialExclusionThread	AI_BinomialExclusion
Classification	BinomialClassificationClient	BinomialClassificationClient	BinomialClassificationThread	AI_BinomialClassification

html file\*: (to start Client Socket Applet)  
 Client Socket Applet \*\*: (to accept requirements and show result graphs)  
 service Socket Thread \*\*\*: (to transfer requirements, call graph generating program and send out name of result graphs)  
 subclass of AI\_Component \*\*\*\*: (to generate specific graph according to requirements)

Table 16.16: Program groups for Binomial Distribution in the Web Graph System.

The Web Graph System uses Socket to realize web graphs, as it is the only way to implement interactive dynamic graphs on the web for *Stochastikon*. Usually a graph applet running on the web contains all the calculating part or stores all calculating programs in a jar file. But in *Stochastikon*, the size of *Stochastikon Calculator* and *Stochastikon Graphics* are already too large and will be extended by a multiple in its final version. Therefore, it is impossible to zip them into a jar file and running the whole program from an applet. The only way is to run some small applets on the web part, while *Stochastikon Calculator* and the main *Stochastikon Graphics* still run on the server. These methods proved to be highly efficient and need only very short download time on the users' side.

Moreover, the Client Socket Applet only gets the generated graph file name from the Socket Thread, takes the graph file from a specified place follows the gotten name and shows it on the web. This procedure operates very stable because transmitting only a name is much less than transmitting graph file. At the same time, showing a formed graph is much more stable than showing it after the entire graph image has been transmitted from the Socket Thread through internet to the user part. As a by-product users can download the generated \*.jpeg graph file from a specific address.

The Web Graph System allows users running several Client Socket Applets (graph windows) simultaneously. So users can learn by comparisons.

#### 16.3.4 Illustrations

In this sub-section some illustrations of these implemented Stochastic Graph Systems are displayed.

Figure 16.19 illustrates a layout of the Report System, which shows the second page of a scientific report created by *Stochastikon Calculator* that contains a graph showing the uncertainty space.

Figure 16.20 illustrates a layout of the Report System, which shows the third page of a scientific report created by Calculator that contains two graphs showing the uncertainty space (light blue), prediction (blue), and probability of the prediction (the blue curve below).

Figure 16.21 and Figure 16.22 are interfaces of Stand-Alone Computer Graph System, one for selecting a 'Distribution' and the other for selecting a 'Stochastic Object'.

Figure 16.23 illustrates a layout of the Stand-Alone Computer Graph System, which shows a Classification with three alternatives for  $n$ . When users 'move' the parameter values, the graph will change accordingly.

Figure 16.24 illustrates a layout of the Stand-Alone Computer Graph System, which shows that a user can open several inner windows (for different Distributions and different Stochastic Objects) simultaneously.

Illustrations of the Web Graph System implemented by *Stochastikon Graphics* have been given in Section 10.5, in which the Web Graph System is used to build a 'Graphical Laboratory' for *Stochastikon Magister*.



**STOCHASTIKON**  
Calculator

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Binomial Distribution

- Properties
- Probability of an event
- Prediction
  - Without ignorance
  - Ignorance about p
    - Minimum interval
    - Maximum lower bound
    - Minimum upper bound
  - Ignorance about n
  - Ignorance about n, p
- Measurement of p
- Measurement of n
- Exclusion
- Classification

Poisson Distribution

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Degree of ignorance about the variability of the random variable:

- Complete knowledge about random variability.

6.) Uncertainty space of  $(X, D)$ :

$$U_{X,D} = \{((80, p), x) \mid (n, p) \in D, x \in \mathbb{N}_0, 0 \leq x \leq 80\}$$

The graph shows a horizontal line at  $p(x) = 80$  for  $x$  values between 0.5 and 0.75. The x-axis is labeled 'x' and ranges from 0.5 to 0.75. The y-axis is labeled 'p(x)' and ranges from 0 to 85.

- The uncertainty space  $U_{X,D}$  of the pair of variables  $(X, D)$  represents the entire variability which has to be taken into account:  
The true variability due to randomness and the pretended variability in case of ignorance about the initial conditions.

2. Problem and Result

- Required reliability:  
 $\beta = 0.8$
- Prediction of outcome of  $X$ :  
 $A_X^{(0.8)}(\{D\}) = \{x \mid x \in \mathbb{N}_0, 36 \leq x \leq 59\}$

2 of 3

Print reports with Ctrl-P

Figure 16.19: Report System: a page of a scientific report by *Stochastikon Calculator* that contains a graph (a).

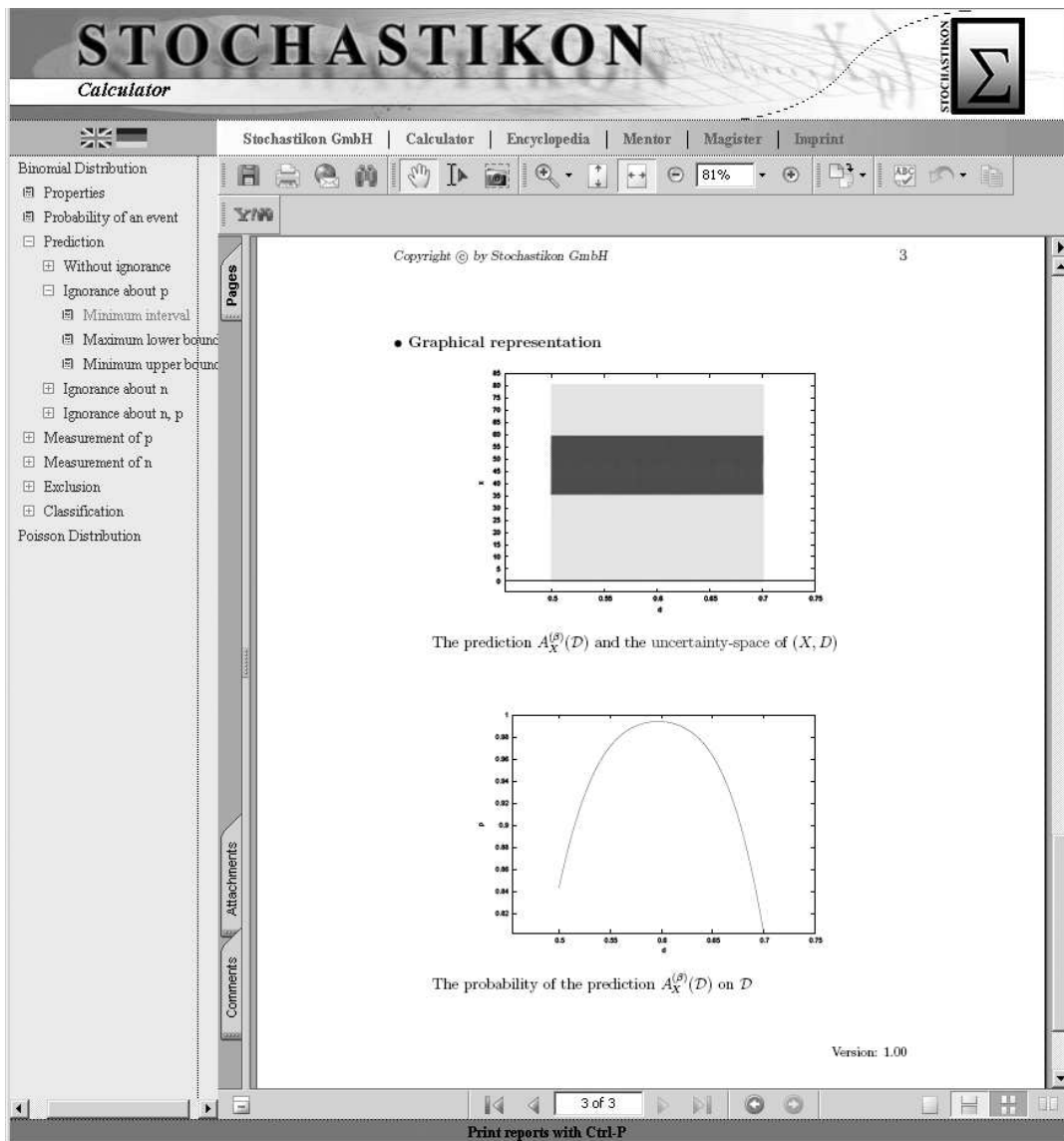


Figure 16.20: Report System: a page of a scientific report by *Stochastikon Calculator* that contains a graph (b).



Figure 16.21: Stand-Alone Computer Graph System: Select a 'Distribution'.



Figure 16.22: Stand-Alone Computer Graph System: Select a ‘Stochastic Object’.

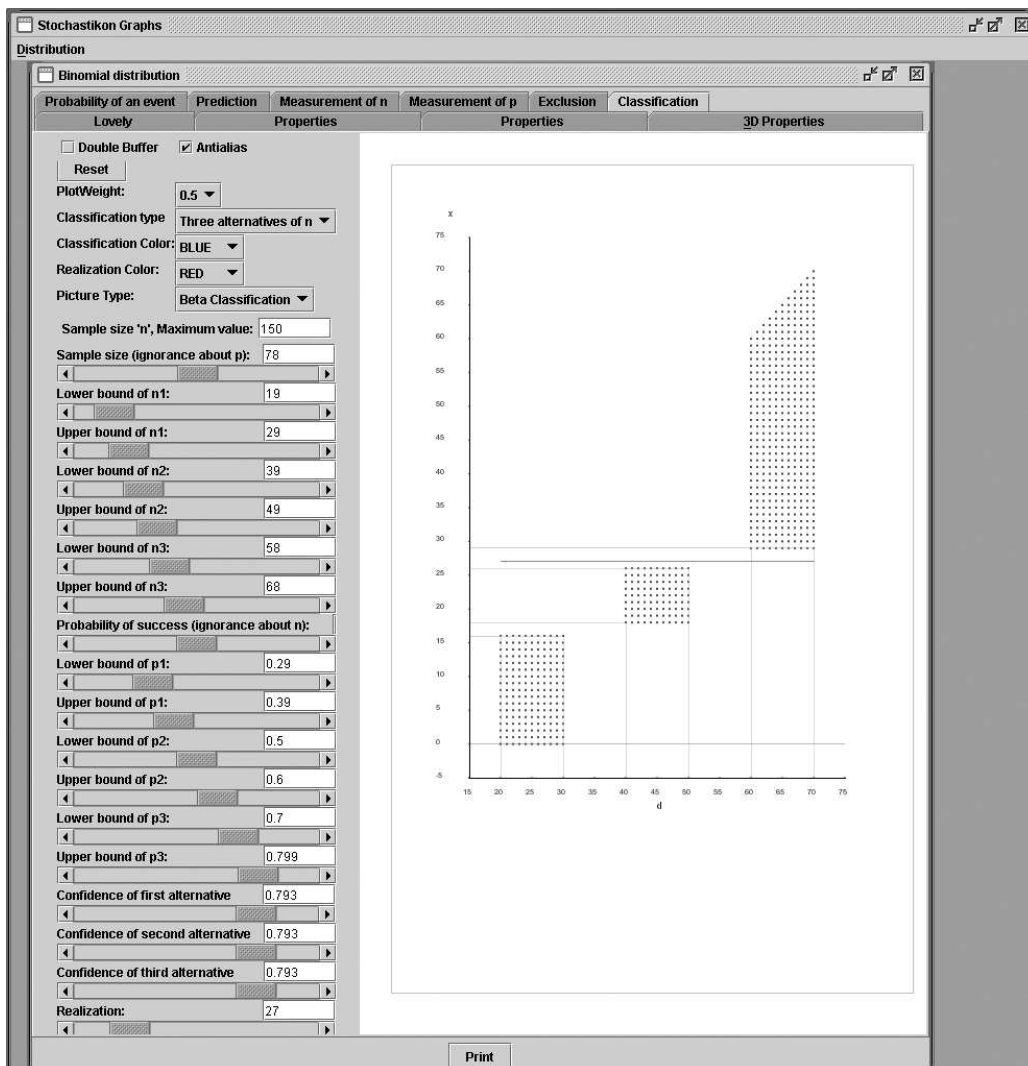


Figure 16.23: Stand-Alone Computer Graph System: Classification with three alternatives for  $n$ .

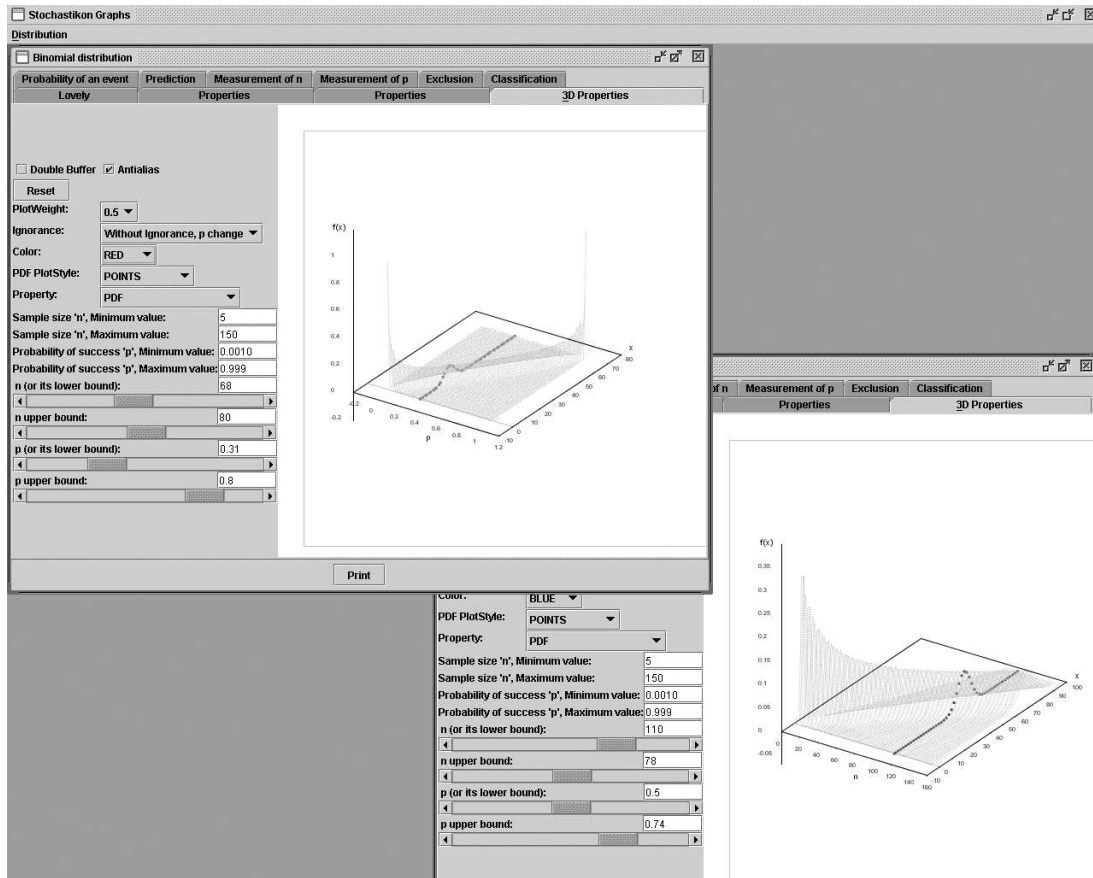


Figure 16.24: Stand-Alone Computer Graph System: Open several inner windows simultaneously.

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