ABDUCTIVE ARGUMENTATION FOR PROVING IN DYNAMIC GEOMETRY ENVIRONMENT

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Abstract. The continuity and gap between argumentation and proof may explain the reason why students usually make some mistakes during proving process and unable to write their formal proof. In order to construct the proofs the students use deduction based on produced arguments and previous theorems. However, they tend to use abduction during the process of formulating the conjectures and bringing up the idea of proof. In this paper we would like to investigate how a dynamic geometry environment encourage the students in producing abductive argumentation for proving and making sense of proof at the tertiary level. We have also singled out a sequence of phases in which the students pass from recognizing invariants to increasing levels of deductive argumentation.

Keywords: abduction; argumentation; proof and proving; dynamic geometry environment.

I. INTRODUCTION

Proving is a crucial activity in the learning and teaching of mathematics. It makes a contribution to develop students' mathematical thinking and argumentation. The concept of proof has been approached on different directions at the educational levels. The bulk of our research regards abduction as the seed of creativity and seizing the essence of geometric proving situations at the tertiary level. Proofs at this level tend to be longer, more complex and more rigorous than those at earlier levels (Annie Selden, 2010). They also involve creativity, deep understanding and formal-writing style. In order to scrutinize the students' arguments producing and analyze how the second-year mathematics students come to their formal proof, we have used a frame analysis method. The students worked in a group of three and they were encouraged to use dynamic geometry software in realizing geometric transformation for solving geometric problems. In this analysis, more attention is paid to the structural gap between argumentation and proof. This gap may explain the students' difficulties in constructing a formal proof.

In our research, we have proposed four main consecutive proving phases by using geometric transformations method:

- *Invariance:* realize geometric invariants including static and dynamic invariants;
- *Conjecture:* formulate conjectures based on realized invariants;
- *Argumentation:* produce arguments to validate the conjectures;
- *Proof:* organize produced arguments to write a formal proof.

These phases provide a rich opportunity to interpret the birth of new proof ideas by using abduction and the students' proving styles. The shift from conjecture phase to proof was also embodied throughout this paper aimed at clarifying the process of producing plausible arguments for proof and proving.

II. ABDUCTIVE ARGUMENTASTION IN TOULMIN'S MODEL

The term "abduction"[1] was coined by Peirce (1960) to differentiate this type of reasoning from deduction and induction. *Abductive argumentation* is a concept stemming from abduction. It is a kind of guessing by a process of forming a plausible hypothesis that explains a given set of facts or data. It has often been considered as a kind of 'backwards' reasoning and as an 'inference to the best explanation' because it starts from the observed facts and probes backwards into the reasons or explanations for these facts (Douglas Walton, 2001). In general, abductive argumentation is crucial in introducing new ideas and supports the transition to the proving modality (Peirce, 1960; Arzarello et al., 1998b). Using this type of argumentation, we can analyze students' proving styles while they formulating conjectures and generating the ideas of proof. Therefore, it supports explanatory conjectures and the subsequent related proof. In a dynamic geometry environment (such as GeoGebra), the strength of abductive argumentation depends on all evidence and data which are collected by dragging, observing, measuring, and checking the relationship between objects. The produced data in this environment sow the seed of generating abductive argumentation during proving process.

In mathematics, proof is deductive, but the discovering and conjecturing processes is often characterized by abductive argumentation. When students are engaged in the mathematical practice of proving, they often "come up" with an idea. To analyze what students are doing when this happens, one can refer to abduction (B. Pedemonte & D. Reid, 2010).

To scrutinize the relationship between abductive argumentation and deductive proof, Toulmin's model of argumentation was utilized. Through this model, argumentation and proof can be analyzed and compared from a structural point of view (Pedemonte, 2007). In Toulmin's model, a step appears as a deductive step (data and warrants lead to a claim) but this model is also a powerful tool to represent an abductive step (including abductive argumentation) by using three basic elements of arguments (Toulmin, 1958; Pedemonte & Reid, 2010) as follows:

C (*claim*): the statement of the speaker

D (*data*): data justifying the claim **C**

W (*warrant*): the inference rule that allows data to be connected to the claim

Fig. 1. *Toulmin's basic model of argumentation*

These models are suitable to describe an argument in dynamic geometry environment because such argument includes three steps. The first step is expressed by *a claim* (or a hypothesis). The second step consists of the production of *data* supporting the claim. A *warrant* provides the justification for using the data. The warrant, which can be expressed by a principle or a rule, acts as a bridge between the data and the claim. In tandem with the approach to teaching proof through abductive argumentation, the students' conjectures production were analyzed according to Toulmin's basic model in order to highlight and to understand the cognitive relation between abductive argumentation and deductive proof.

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[1] *Abduction* is an inference which allows the construction of a claim starting from an observed fact (Magnani, 2001; Peirce, 1960; Polya, 1962).

According to the principle of *cognitive unity*[2] (Boero, Garuti, Mariotti, 1996; Pedemonte, 2005) prior argumentation can be used by students in the construction of proofs if they can organize some of the previously produced arguments into a logical chain. Hence, the students have to seize an opportunity to exploit the power of the dynamic geometry environment aimed at taking inspiration for producing arguments and focusing on the arguments to support proofs. This phenomenon refers to the concept of cognitive unity. During a problem solving process, an argumentation activity is usually developed in order to produce a conjecture. The hypothesis of cognitive unity is that in some cases this argumentation can be used in the construction of proof by organizing some of the previously produced arguments in a logical chain (Pedemonte, 2005).

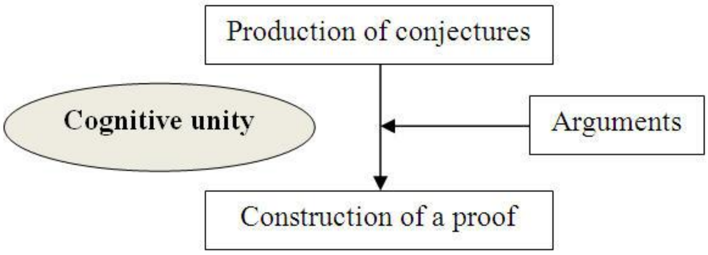


Fig. 2. *Cognitive unity in the proving process*

Cognitive unity may bring abductive argumentation to the light and simultaneously realize the importance of inchoate arguments which are generated during the process of modifying the understanding and constructing of a proof. This concept also explains and makes intuitive the proof as process. In our research, we have shown that three kinds of inferences play an essential role in different consecutive phases of proving processes: *realizing invariants*, *formulating conjectures, producing arguments*, *validating conjectures* and *writing deductive proofs*.

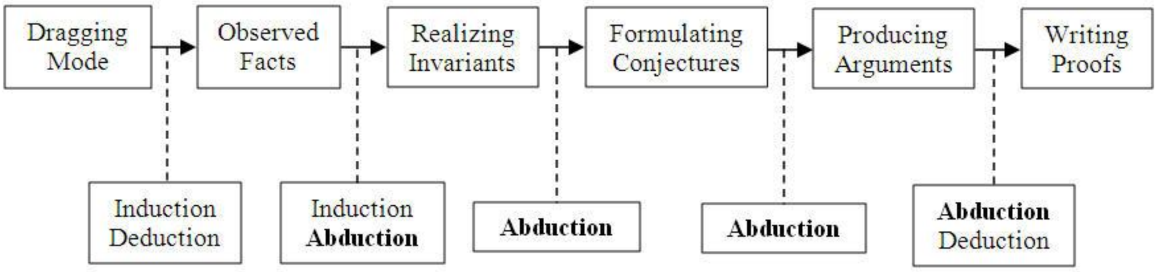


Fig. 3. *Three kinds of inferences during proving processes*

III. TRANSITION FROM ABDUCTIVE ARGUMENTATION TO DEDUCTIVE PROOF

The proof of the pudding is in the eating, therefore, we always spur the students to formulate conjectures during the resolution process. This activity was set on a par with the proving because the production of conjectures motivates the students producing arguments and constructing proofs on their own. Argumentation structure is often abductive but proof is deductive. Hence, the structural distance from an abductive argumentation to a deductive proof is not always

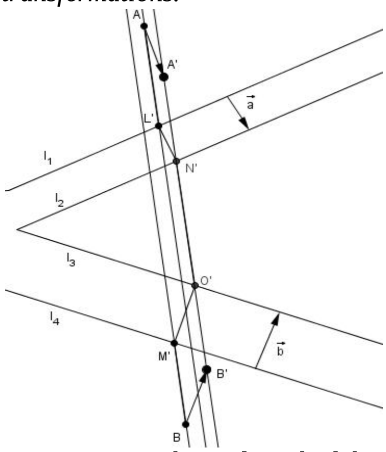
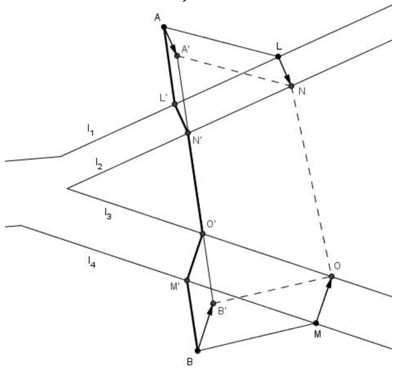
[2] *Cognitive unity* is the following phenomenon: “During the production of the conjecture, the student progressively works out his/her statement through an intensive argumentative activity functionally intermingled with the justification of the plausibility of his/her choices. During the subsequent statement-proving stage, the student links up with this process in a coherent way, organizing some of previously produced arguments according to a logical chain” (Boero, Garuti, Mariotti, 1996).

covered by the students, who sometime produce incorrect proofs because they are not able to transform the structure of argumentation in deductive structure for proof (Pedemonte, 2007).

In order to monitor and track students' proving processes, we have recorded the students' working frame[3] in a dynamic geometry environment by using the screen-casting *Wink®* software[4] (Kumar, 2007; Reis & Karadag, 2008). The students were required to form conjectures, taken notes and written a formal proof. The following example with the students' protocol gives us a deep understanding about the scaffolding to bridge the gap between abductive argumentation and deductive proof.

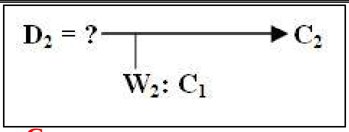
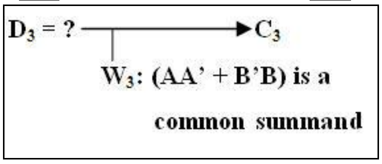
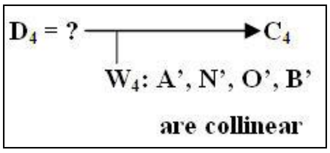
Two-Bridge Problem

Where would you build two bridges over the two sleeves of a river with parallel straight sides to minimize the length of the path between the cities A and B? (Bridges have to perpendicular to the sides of the river).

Students' Protocols	Abductive Argumention	Deductive Proof
9. V: hey… look! I think at this position, the length of the broken line ALNOMB is minimal. **13. S:** yes,… but what are special characteristics in this figure? **15. N:** we try to draw the lines NO, MB, and AL? **20. S:** hey... may be these lines are parallel! **25. S:** we would suppose that they are parallel, which transformation does exist here? **28. V:** translation! because we have two fixed vectors $\vec{a}$ and $\vec{b}$ and the translation preserves parallelism. **29. N:** so we have $T_{\vec{a}}(AL') = N'O'$ and $T_{\vec{b}}(BM') = O'N'$. **34. S:** from the results of measurement, we obtain the following equalities? *AA' = L'N' = LN* *BB' = M'O' = MO* *AL = A'N; AL' = A'N'* *BM = B'O; BM' = B'O'* **37. V:** according to the properties of the translations $T_{\vec{a}}$ and $T_{\vec{b}}$! **41. N:** the length of the broken line AL'N'O'M'B is always smaller then that of broken line ALNOMB! **44. V:** it means that we need to prove the inequality? *AL'+ L'N' + N'O'+ O'M'+ M'B ≤ AL+ LN+ NO+ OM+ MB* ***(1)*** **48. S:** we have the following data: The left side of inequality ***(1)*** *= AA'+ A'N'+ N'O'+ O'B'+ B'B* *= AA' + A'B' + B'B* ***(2)***	*After realizing the invariants, the students have used abductive argumentation in order to determine geometric transformations.*  *By measuring, the students had the following data:* **C_1:** *AA' = L'N' = LN* *BB' = M'O' = MO* *AL = A'N, AL' = A'N* *BM = B'O, BM' = B'O'* $D_1 = ? \xrightarrow{W_1:\ \text{Translation}} C_1$ **D_1:** $T_{\vec{a}}(A) = A'$; $T_{\vec{b}}(B) = B'$ *By measuring and using abductive argumentation, the students produced the following claims:* **C_2:** *AL'+ L'N' + N'O'+ O'M'+ M'B ≤ AL+ LN+ NO+ OM+ MB*	*Firstly, the students constructed two points N' and O' where they could situate the two bridges:* Let A' be image of A under the translation of vector $\vec{a}$ and B' be image of B under the translation of vector $\vec{b}$ and connect A' and B' by a line. Let N', O' be the intersections of A'B' and l_2, l_3, respectively. So L', N' and O', M' are the end points to build two bridges. *Based on the properties of the translation, the students gathered initial data. They wrote:* From the properties of a translation, we derive that: AA' = L'N' = LN BB' = M'O' = MO AL = A'N; AL' = A'N' BM = B'O; BM' = B'O'  *The students reversed abductive structure in order to write the formal proof , starting from the inequality (4) à (3) à (2) à (1) as follows:* We always have the following

[3] A frame is defined as the snapshots of the computer screen at a specified moment.

[4] This software also allowed us to zoom into any frame recorded and to annotate it. This feature delivered our messages and jotted our notes down on the desired frames. It also made the communication easier because we can easily navigate the frames, describe the moment of action, and deliver the message in order to provide opportunity of just-in-time commenting.

The right side of inequality ***(1)*** *= A'N + AA' + NO + B'B + OB'* *= AA'+ A'N + NO + OB'+ B'B* ***(3)*** **51. V:** so from (2) and (3) we have to prove the following inequality: *A'B' ≤ A'N + NO + OB'?* ***(4)*** **57. N:** but it is always true because the length of segment A'B' is always smaller than the length of the broken line A'NOB'! **61. S:** exactly! so we will start from the final inequality ***(4)*** to prove the required inequality ***(1)***.	 **$D_2=C_3$:** *<u>AA'</u>+ A'N' + N'O' + O'B' + <u>B'B</u>* *≤ <u>AA'</u> + A'N + NO + OB' + <u>B'B</u>*  **$D_3 = C_4$:** *A'N'+ N'O'+O'B'≤ A'N+NO+OB'*  **$D_4 = C_5$:** *A'B' ≤ A'N + NO + OB'* The final claim **C_5** is always a true statement (or a theorem).	inequality: A'B' ≤ A'N + NO + OB' Add the sum (AA' + B'B) to both sides of that inequality, we have: AA' + A'B' + B'B ≤ AA' + A'N + NO + OB' + B'B ⇒ AA'+ A'N'+ N'O' + O'B'+ B'B ≤ AL + LN + NO + OM + MB (since A'N = AL, AA' = LN B'B = OM, O'B' = MB) ⇒ AL'+ L'N'+ N'O'+ O'M'+ M'B ≤ AL + LN + NO + OM + MB (since AA' = L'N', A'N' = AL' O'B' = M'B, B'B = O'M').

Through two-bridge problem, we have revealed that a dynamic geometry environment has often amplified the potential of providing the students claims (observed facta) and the students only have to look for data and some rules to justify the claims. Therefore, if the data are collected and a certain rule is selected, there is a smooth transition from abductive argumentation to deductive proof. Additionally, we used *abductive argumentation scheme* (AAS) to describe this valuable transition. This scheme is argument form representing inferential structures of abductive arguments. It also offers a means of characterizing stereotypical patterns of reasoning in Toulmin's model.

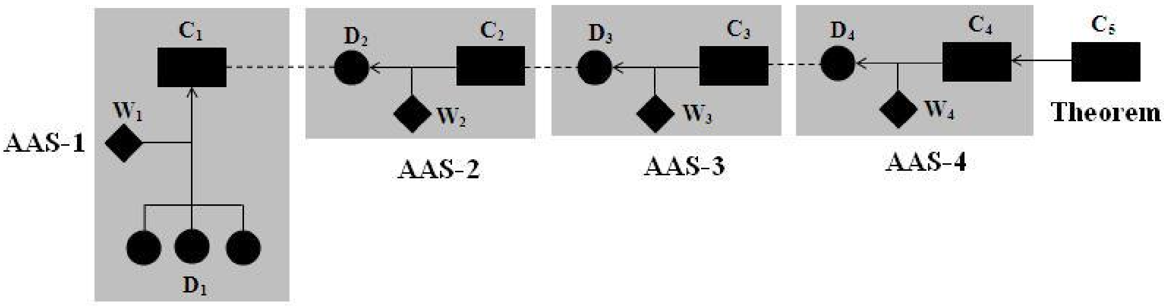


Fig. 4. *Two-bridge problem's abductive argumentation scheme*

The strength of this abductive chain (including four main arguments) seems to be so strong that there were a lot of students who could not construct continuity in the *referential system*[5] with the argumentation. They have lost the connection with the referential system. Therefore, they are also not able to transform their abductive argumentations into deductive proofs (Pedemonte, 2007). However, if the students can *reserve* the abductive structures, the structural continuity between argumentation and proof will be appear and they come easier to a formal proof.

[5] The *referential system* included both the representations system (language, the heuristic, the drawing) and the knowledge system (conceptions, theorems) of argumentation and proof. These factors have been used in the argumentation supporting the conjecture.

IV. CONCLUSION

Proof is more "accessible" to students if an argumentation activity leads to the construction of a conjecture (Garuti, Boero, Lemut & Mariotti, 1996). That is the reason why the teachers should encourage their students using dynamic geometry software to formulate conjectures and generate abductive argumentation during the construction of a proof. This strategy also *decreases* the distance between realizing invariants and writing proofs. As a consequence, the students are motivated to explore and 'flash' the idea of proving by using abductive argumentation.

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