Three Essays on Market Concentration and Welfare

Inauguraldissertation

zur Erlangung des akademischen Grades
eines Doktors der Wirtschaftswissenschaften an der
wirtschaftswissenschaftlichen Fakultät
der Julius-Maximilians-Universität Würzburg

Vorgelegt von:
Jens Jurgan
Würzburg, im August 2009

Betreuer der Arbeit:

Professor Norbert Schulz, Ph.D.

Meiner Familie

Danksagung

Die vorliegende Arbeit ist im Rahmen meiner Tätigkeit als wissenschaftlicher Mitarbeiter am Lehrstuhl für Volkswirtschaftslehre, insbesondere Industrieökonomik von Professor Norbert Schulz, Ph.D. an der Universität Würzburg entstanden.

An dieser Stelle möchte ich mich bei allen herzlich bedanken, die mich währenddessen unterstützt haben, allen voran meinem Betreuer Norbert Schulz. Er hat mir immer wieder den Blick für das Wesentliche geschärft und mich stets ermuntert, den Stift nicht fallen zu lassen. Die Arbeit an seinem Lehrstuhl hat mir großen Spass gemacht. Weiterhin bedanken möchte ich mich bei meinen Kollegen Sebastian Wismer, Alex Steinmetz und Robin Kleer, die sich als standfeste Diskussionspartner erwiesen haben und nicht zuletzt Ute Reich für das unermüdliche Korrekturlesen.

Zu guter Letzt möchte ich mich noch bei meiner allerliebsten Schnuppeliese bedanken, die mich während meiner Promotion mit allen Launen ertragen hat. Ich hab dich ganz arg lieb!

Zusammenfassung

Die vorliegende Dissertation untersucht den Zusammenhang zwischen der Konzentration eines Marktes und der Effizienz des Marktergebnisses in einem Modell mit differenzierten Gütern aus unterschiedlichen Sichtweisen. Die Marktkonzentration ist für Wettbewerbsbehörden, wie zum Beispiel für die US-amerikanische Federal Trade Commission (FTC) oder die Wettbewerbsbehörde der Europäischen Union, das Directorate-General for Competition (DG-Comp), ein bedeutender Indikator für die Wettbewerbsfähigkeit eines Marktes und soll Rückschlüsse auf dessen Effizienz ermöglichen.

Zunächst werden in dem einführenden Kapitel die Ziele der Wettbewerbspolitik erörtert. Weiterhin wird auf die Rolle der Marktkonzentration als indirekter Indikator für die Marktmacht einzelner Unternehmen eingegangen. Anschließend wird die Bedeutung der Marktkonzentration im Rahmen der Bewertung horizontaler Unternehmenszusammenschlüsse gemäß den Europäischen sowie US-amerikanischen Fusionsrichtlinien deutlich gemacht. Im Folgenden werden einige ausgewählte theoretische Ergebnisse dargelegt, die einen grundlegenden Überblick über das Zusammenspiel von Konzentration und Effizienz verschaffen sollen. Abschließend zeigt das erste Kapitel mehrere Lücken in der Modelltheorie auf, die im Laufe der Arbeit durch eigene Untersuchungen geschlossen werden. Die Kapitel 2 bis 4 stellen eigenständige Abschnitte dar, doch bauen sie inhaltlich aufeinander auf und beleuchten das Thema aus unterschiedlichen Sichtweisen.

Das zweite Kapitel analysiert den Zusammenhang zwischen sozialem Überschuss und der Heterogenität der Marktstruktur im Rahmen eines differenzierten Cournot Oligopols. Einzige Ursache für die Heterogenität sind hier die in der Produktion unterschiedlich effiziente Unternehmen, die jeweils eine Ausprägung eines differen-

zierten Gutes herstellen. Die Kostenstruktur aller Unternehmen weist konstante, aber unterschiedliche Grenzkosten ohne fixe Kosten auf. Die Präferenzen der Haushalte werden durch eine quadratische Nutzenfunktion abgebildet, die auf Dixit (1979) zurückgeht. Da die Präferenzen quasi-linear sind, wird der soziale Überschuss als Maßstab für die Paretianische Effizienz herangezogen. Der Einfluss der Marktheterogenität auf die Effizienz des Marktergebnisses wird untersucht, indem der Zusammenhang zwischen der Verteilung der Grenzkosten und der Konsumenten- sowie Produzentenrente analysiert wird.

Die Untersuchungen dieser Arbeit knüpfen inhaltlich an eine Vielzahl von Studien aus dem Bereich der homogenen Güter an, deren Ergebnisse als Spezialfall repliziert werden können. Da in meinem Modellrahmen eine maximale Bandbreite unterschiedlicher Differenzierungsgrade abgebildet werden, beginnend von perfekten Substituten über teilweise substituierbare Güter bis hin zu völlig unabhängigen Gütern, stellen meine Untersuchungen eine Verallgemeinerung der bisherigen Modelltheorie dar, die überdies wertvolle neue Einblicke liefern.

Die zentrale Erkenntnis ist, dass im Fall differenzierter Güter nicht nur die Produzentenrente, sondern auch die Konsumentenrente mit der Varianz der Grenzkosten steigt, sofern die durchschnittlichen Grenzkosten konstant sind. Bei Vorliegen homogener Güter sind die Konsumenten indifferent bezüglich unterschiedlicher Marktstrukturen, sofern die durchschnittlichen Grenzkosten der Unternehmen identisch sind. Erst mit zunehmender Differenzierung der einzelnen Produkte beginnen die Konsumenten von der Heterogenität des Marktes zu profitieren.

Dieses Ergebnis ist aus zweierlei Gründen völlig unerwartet: Zum einen entsteht den Unternehmen gerade durch die Differenzierung ihrer Produkte Marktmacht in Form eines Preisspielraumes. Dieser nimmt sogar mit dem Grad der Differenzierung zu. Je unterschiedlicher die Güter, desto weniger sind Produkte der Konkurrenz im Falle einer Preiserhöhung als Substitute geeignet. Darüberhinaus bevorzugen Konsumenten unterschiedliche Produkte in gleicher Menge, da jedes einzelne Gut einen abnehmenden Grenznutzen aufweist. Es lässt sich jedoch zeigen, dass die Ausübung der Marktmacht großer Unternehmen zu einer effizienteren Verteilung der Gesamtproduktion auf die einzelnen Unternehmen führt. Die Größe eines Unternehmens steht mit der Effizienz seiner Produktionstechnologie in positivem

Zusammenhang. Ein effizientes Unternehmen wählt im Vergleich zu seinen weniger effizienten Konkurrenten eine höhere Ausbringungsmenge aber bringt diese zu einem geringeren Preis auf den Markt. Durch dieses aggressive Wettbewerbsverhalten erzielt das effizientere Unternehmen einen höheren Gewinn im Vergleich zu seinen weniger effizienten Konkurrenten. Darüberhinaus besitzt dieses Wettbewerbsverhalten effizienzsteigernde Wirkung, da weniger effiziente Unternehmen mit einer Reduktion ihrer Ausbringungsmenge reagieren. Analog zu homogenen Gütern sinken die Gesamtkosten der Produktion mit der Varianz der Grenzkosten, falls die durchschnittlichen Grenzkosten konstant sind. Im Gegensatz zu homogenen Gütern sinken jedoch nicht nur die gesamten Produktionskosten, sondern auch der Gesamtumsatz, welcher den aggregierten Ausgaben der Haushalte entspricht. Obgleich der Bruttonutzen der Haushalte mit der Varianz der Grenzkosten abnimmt, steigt die Konsumentenrente, da der positive Effekt auf die reduzierten Ausgaben überwiegt. Im Fall differenzierter Güter profitieren sowohl Konsumenten als auch Produzenten von einer heterogenen Marktstruktur.

Dieses Resultat ist für die wohlfahrtstheoretische Beurteilung von Märkten von eminenter Bedeutung, da sowohl die europäischen und US-amerikanischen Fusionsrichtlinien als auch die deutsche Wettbewerbspolitik zwar nicht explizit die Konsumentenrente als Bewertungsmaßstab heranziehen, dieser jedoch eine erhebliche Bedeutung beimessen. Darüberhinaus wird von einem negativen Einfluss der Konzentration auf die Wettbewerbsfähigkeit und somit auf die Effizienz des Marktergebnisses ausgegangen. Große Marktanteile sowie eine hohe Konzentration werden als Indikator für Marktmacht einzelner Unternehmen angesehen. Die befürchteten Folgen sind verzerrter Wettbewerb und damit verbundene Effizienzverluste, vor allem auf Seiten der Konsumenten. Jedoch ist es gerade die Marktmacht der großen Unternehmen die zu einer effizienteren Verteilung der Gesamtproduktion auf die einzelnen Unternehmen führt und sowohl Konsumenten als auch Produzenten besserstellt. Die Erkenntnisse des ersten Kapitels bilden das Fundament für die weiteren Untersuchungen dieser Arbeit.

Das dritte Kapitel untersucht den Zusammenhang zwischen dem Herfindahl-Hirschman Index und der Verteilung der Grenzkosten im gleichen Modellrahmen. Bei der Berechnung des Index wird die Summe der quadrierten Marktanteile aller Unternehmen gebildet. Obwohl grundsätzlich der Marktanteil aller Unternehmen Berücksichtigung findet, werden große Unternehmen durch das Quadrieren der Anteile stärker gewichtet. Der Herfindahl-Hirschman Index ist daher für Wettbewerbsbehörden von herausragender Bedeutung.

Bei Vorliegen homogener Güter besteht ein monotoner Zusammenhang zwischen der Varianz der Grenzkosten und dem Herfindahl-Hirschman Index bei konstanten durchschnittlichen Grenzkosten. Die Marktanteile entsprechen den Mengenanteilen, da der Marktpreis aller Produkte gleich ist. Der Gesamtoutput der Industrie hängt wiederum lediglich von den durchschnittlichen Grenzkosten aller Unternehmen ab. Der Output eines Unternehmens wird positiv von den durchschnittlichen Grenzkosten seiner Konkurrenten beeinflusst und sinkt mit den eigenen Grenzkosten. Die Marktkonzentration sowie die Varianz der Grenzkosten steht somit in einem positiven Verhältnis. Die Verteilung der Marktanteile und die entsprechende Konzentration ist ein Spiegelbild der zugrundeliegenden Kostenstruktur.

Bei differenzierten Gütern ist dieser Zusammenhang jedoch von vornherein nicht klar ersichtlich und deutlich schwieriger nachzuweisen. Dies liegt zum einem daran, dass die Ausbringungsmenge eines Unternehmens mit seinen Grenzkosten sinkt, während der zugehörige Marktpreis steigt. Eine Veränderung der Kosten beeinflusst den Umsatz folglich auf gegensätzliche Art und Weise. Darüberhinaus hängt der Gesamtumsatz aller Unternehmen von der Varianz der Grenzkosten ab, wie sich im Laufe von Kapitel 2 zeigt. Eine hypothetische Veränderung der Kostenstruktur, welche die durchschnittlichen Grenzkosten unverändert lässt, beeinflusst somit auch die Marktanteile der Unternehmen, die von der Kostenveränderung nicht betroffen sind, da sich der Gesamtumsatz als Bemessungsgrundlage aller Marktanteile ändert.

Dennoch besteht zwischen dem Herfindahl-Hirschman Index und der Verteilung der Grenzkosten weiterhin ein monotoner Zusammenhang. Weiterhin kann gezeigt werden, dass eine (hypothetische) gleichmäßige Erhöhung aller Grenzkosten die Marktkonzentration erhöht, obwohl das Marktumfeld für alle Marktteilnehmer schwieriger wird. Die von vornherein ineffizienteren Unternehmen sind überdurchschnittlich von der Kostenerhöhung benachteiligt. Folglich kann eine hohe Marktkonzentration entweder darauf hindeuten, dass das Marktumfeld für alle Unternehmen sehr schwierig ist (da die Grenzkosten aller Unternehmen sehr nah an der maximalen Zahlungsbereitschaft liegen) oder der Markt durch wenige, sehr effiziente Unternehmen versorgt wird. In keinem der beiden Fälle sind große Unternehmen verantwortlich für eine ineffiziente Versorgung des Marktes.

Der positive Zusammenhang zwischen dem Herfindahl-Hirschman Index und dem sozialen Überschuss im Kontext differenzierter Güter ist als Ergebnis der Kapitel 2 und 3 zum grundlegenden Verständnis des folgenden Abschnitts von besonderer Bedeutung.

Das vierte Kapitel untersucht wohlfahrtstheoretische Implikationen der aktuellen Fusionsrichtlinien auf Grundlage des theoretischen Modells der vorangegangenen Kapitel. Sowohl die europäischen als auch die US-amerikanischen Richtlinien zur Bewertung horizontaler Unternehmenszusammenschlüsse gehen bei ihrer Beurteilung grundsätzlich von einem negativen Einfluss der Marktkonzentration auf die Wettbewerbsfähigkeit des Marktes und somit auf die Effizienz des Marktergebnisses aus. Die vorangegangenen beiden Kapitel haben jedoch gezeigt, dass die Richtlinien in diesem Punkt auf falschen Annahmen basieren. Gemäß den aktuellen Fusionsrichtlinien steigt die Wahrscheinlichkeit einer Ablehnung ceteris paribus mit dem aggregierten Marktanteil der beteiligten Unternehmen. Im Rahmen des vierten Kapitels werden die Auswirkungen dieser Richtlinien auf das Verhalten der Unternehmen sowie die Effizienz des damit verbundenen Marktergebnisses untersucht. Der Schwerpunkt der Untersuchungen liegt hierbei nicht auf möglichen koordinierten Effekten eines Zusammenschlusses, sondern allein auf dessen unilateralen Effekten.

Die aktuelle Bewertungspraxis der entsprechenden Wettbewerbsbehörden sieht einen Vergleich der Situation nach Zusammenschluss mit der Situation ohne Zusammenschluss vor. Dieses Vorgehen lässt jedoch außer Acht, dass ein Zusammenschluss zwischen kleineren Unternehmen die Folge oder Alternative eines abgelehnten Zusammenschlusses größerer Unternehmen sein kann. Hierbei muss einem kleinen Zusammenschluss nicht zwingend ein tatsächlich abgelehnter großer Zusammenschluss vorausgegangen sein, da es aus Sicht der Unternehmen rational ist, die Kriterien des Zusammenschlusses zu antizipieren um Zeit und Kosten zu sparen. Um diesem Umstand Rechnung zu tragen, werden im Rahmen meiner Untersuchungen zwei Situationen zum Vergleich angeführt, die auf Grund unterschiedlich großer Zusammenschlüsse entstanden sind. Die Größe eines Zusammenschlusses ist hierbei durch den aggregierten Marktanteil der beteiligten Unternehmen gegeben.

Im Rahmen der Analyse werden in der Hauptsache zwei Annahmen der aktuellen Richtlinien kritisch hinterfragt. Erstens: Besteht tatsächlich ein positiver Zusammenhang zwischen dem gemeinsamen Gewinn nach dem Zusammenschluss und dem aggregierten Marktanteil der betroffenen Unternehmen vor dem Zusammenschluss? Zweitens: Liegt in der Tat ein negativer Zusammenhang zwischen dem aggregierten Marktanteil vor dem Zusammenschluss und der Effizienz des Marktergebnisses nach dem Zusammenschluss vor?

Die Untersuchungen belegen, dass die erste Annahme vorbehaltlos unterstützt werden kann. Dieses Resultat ist intuitiv, da vor dem Zusammenschluss Gewinn und Absatzmenge in einem positiven Verhältnis stehen. Beispielsweise stellt ein Zusammenschluss der beiden größten Unternehmen eine Kombination der beiden effizientesten und somit profitabelsten Unternehmen dar.

Weiterhin kann gezeigt werden, dass die Effizienz des Marktergebnisses nach dem Zusammenschluss sowie der gemeinsame Marktanteil vor dem Zusammenschluss tatsächlich in einem negativen Zusammenhang stehen. Abgesehen von einer Ausnahme in der die Marktkonzentration bereits vor dem Zusammenschluss derart hoch ist, dass die Genehmigung eines Zusammenschlusses jedweder Art fraglich ist. In dieser Situation ist mit Sicherheit von einer Schließung des kleineren Fusionspartners auszugehen, ungeachtet der Größe des Zusammenschlusses.

Da die Richtlinien in der Bedeutung der Marktkonzentration für die Effizienz des Marktergebnisses von dieser falschen Annahme ausgehen, ist deren effizienzsteigernde Wirkung erst auf den zweiten Blick einleuchtend. Hinsichtlich der Marktstruktur nach dem Zusammenschluss lassen sich drei unterschiedliche Fälle unterscheiden, wobei der erste Fall bereits oben erwähnt wurde. Im zweiten Fall kommt es ungeachtet der Größe des Zusammenschlusses nicht zu einer Schließung des kleineren Teils des fusionierten Unternehmens. In dieser Situation führt ein Zusammenschluss kleiner Unternehmen zu einer Erhöhung der Disparität der unterschiedlichen Ausbringungsmengen, da die Insider[1] die externen Effekte auf den gemeinsamen Gewinn internalisieren und ihre Ausbringungsmenge verkleinern. Infolgedessen reagieren die Outsider mit einer Ausweitung ihres Outputs. Die Erhöhung der Disparität wirkt sich positiv auf den sozialen Überschuss aus, da die effizienteren Outsider ihre Produktion ausweiten, wohingegen es zu einer Kontraktion der Mengen der ineffizienteren Insider kommt. Bei einem Zusammenschluss großer Unternehmen würde das Gegenteil eintreten. Die ineffizienten Outsider vergrößern ihren Output während die effizienten Insider ihre Mengen verkleinern. Somit steht dieses Resultat in Einklang mit denen des zweiten Kapitels.

Im letzten Fall kommt es ausschließlich bei einem Zusammenschluss kleiner Unternehmen zur Schließung des ineffizienteren Teils des fusionierten Unternehmes. Nach einem großen Zusammenschluss werden weiterhin alle Güter in positiver Menge produziert. Die aktuelle Zusammenschlusspolitik führt somit zur Schließung eines Teils des fusionierten Unternehmens und damit zur einer Reduktion der Produktvielfalt, da ein Gut nicht länger angeboten wird. Jedoch ist die effizienzsteigernde Wirkung gerade auf diese Schließung zurückzuführen, da sie eine Markt bereinigende Funktion besitzt. Ein vergleichsweise ineffizient herzustellendes Gut wird nicht länger produziert, während die Produktion der günstiger herzustellenden Güter ausgeweitet wird. Die Schließung ist für die Insider profitabel sowie effizienzsteigernd.

Zusammenfassend kann man festhalten, dass die aktuelle Fusionskontrollpolitik die Disparität der Güterversorgung und somit eigentlich die Konzentration grundsätzlich erhöht, obwohl das genaue Gegenteil beabsichtigt ist. Dies kann unter Umständen auch zur Entfernung eines ineffizient herzustellenden Gutes aus dem Markt führen. Gemäß den US-amerikanischen wie auch den europäischen Fusionsrichtlinien wird ein Zusammenschluss anhand der zu erwartenden Konzentration kategorisiert. Der Anstieg des Herfindahl-Hirschman Indexes wird hierbei durch das doppelte Produkt der betroffenen Marktanteile geschätzt.[2] Zum Beispiel ein Zusammenschluss der beiden kleinsten Unternehmen wird demnach die kleinste

[1] Die am Zusammenschluss beteiligten Unternehmen werden im Folgenden als Insider bezeichnet, wohingegen die unbeteiligten Unternehmen als Outsider bezeichnet werden.

[2] Werden (1991) hat in seinem Kommentar klargestellt, dass es hier nicht um eine echte Schätzung der Konzentration sondern vielmehr um eine Kategorisierung des Zusammenschlusses geht und somit den entsprechenden Kritikpunkt von Farrell und Shapiro (1990) zurückgewiesen.

Konzentration erwarten lassen. Tatsächlich steigt bei einem kleinen Zusammenschluss die Unterschiedlichkeit der Ausbringungsmengen, was die Konzentration eigentlich erhöhen sollte. Dieser Effekt wird jedoch durch die Reduktion der Anzahl an Unternehmen verschleiert. Ein Zusammenschluss der größten Unternehmen verkleinert eigentlich die Disparität der Mengen, lässt aber gemäß den Richtlinien die höchste Konzentration erwarten.

Abschließend bleibt festzuhalten, dass weder eine maximale Anzahl an Wettbewerbern noch eine größtmögliche Gleichverteilung der Marktanteile wettbewerbsfördernd und somit effizienzsteigernd sein muss.

Abstract

This thesis analyzes the relationship between market concentration and efficiency of the market outcome in a differentiated good context from different points of view. For antitrust and competition authorities, such as the American Federal Trade Commission (FTC) or European Directorate-General for Competition (DG-Comp), market concentration is a crucial indicator for competitiveness and, therefore, efficiency of the market outcome.

In the first chapter I introduce the objectives of competition policy and antitrust authorities. Afterwards, I outline the importance of market concentration as indirect measure for market power in general. Then I turn to the role of market concentration with regard to US and European merger guidelines, in particular. Subsequently, I present some theoretical results concerning the relationship between market concentration and efficiency. Finally, I reveal a lack of theoretical analysis and motivates my studies. In principle all chapters are independent. Nevertheless, my studies are closely connected since each chapter bases on the previous chapter(s) and each of them examines the object of investigation from a different point of view.

In chapter 2 I analyze the relationship between social surplus and market heterogeneity in a differentiated Cournot oligopoly. Market heterogeneity is due to differently efficient firms, each of them producing one variety of a differentiated good. All firms exhibit constant but different marginal costs without fixed costs. Consumers preferences are given by standard quadratic utility originated by Dixit (1979). Since preferences are quasi-linear social surplus is the measure for Pareto-optimality. I investigate the impact of market heterogeneity on efficiency of the market outcome by analyzing the relationship between the distribution of marginal costs and consumer surplus as well as producer surplus.

My analysis is in tie with many studies investigating the same subject in the homogeneous good case whose central results are just a special case of my studies. Since I allow for a wide range of degrees of product differentiation, starting from perfect substitutes up to independent goods, my analysis is a valuable generalization of existing theory with striking new insights.

The main finding is that consumer surplus as well as producer surplus increases with the variance of marginal costs if average marginal costs are constant. In case of homogeneous goods consumers are indifferent between different market structures provided that average marginal costs are unchanged. Consumers do not benefit from market heterogeneity until products are differentiated.

This result is surprising due to two reasons: On the one hand market power arises from product differentiation since the degree of substitutability decreases with the degree of product differentiation. Concurrence products are less good substitutes in case of a rise in price. Moreover, consumers prefer each good in the same quantity since there is a diminishing marginal utility of each differentiated good. However, it is just the market power of big firms which promotes a more efficient distribution of aggregated output on each firm. Equilibrium output and marginal costs are inversely proportional. A more efficient firm chooses a higher quantity and sells the good at a lower price compared to a less efficient competitor. This aggressive behavior is not only profitable for the firm itself but it is also welfare enhancing since output of less efficient firms is suppressed. Analogously to the homogeneous good case total costs of production decrease with the dispersion of marginal costs provided that average marginal costs are constant. In contrast to the homogeneous good case gross revenue, equal to households expenditures, decreases with the dispersion of marginal costs, too. Even though gross utility decreases with the disparity of marginal costs, consumer surplus increases with market heterogeneity since the positive effect on aggregated expenditures overcompensates.

This finding is crucial for the assessment of market structures since European and US Merger Guidelines as well as German competition law are at least partially consumer orientated. Moreover, the guidelines presume a negative impact of market

concentration on competitiveness and, therefore, social surplus in general. Big market shares and a highly concentrated market are indicators for market power of dominant firms. Distorted competition is presumed to be a consequence of market power. At least consumers are supposed to be worse off in case of distorted competition. However, the exact opposite is true. It is just the market power of big firms which promotes a more efficient distribution of total production on the differently efficient firms making producers as well as consumers better off. Hence, my first study is the basis for my further research.

The third chapter deals with the relationship between market concentration measured by the Herfindahl-Hirschman Index and the distribution of marginal costs in the same differentiated good context. The Herfindahl-Hirschman Index is the sum of squared market shares of all firms active in the market. Since market shares are squared, big firms have more weight compared to small firms. The Herfindahl-Hirschman Index plays a prominent role for competition policy and antitrust authorities.

In the homogeneous good case there is a monotonic relationship between the variance of marginal costs and market concentration measured by the Herfindahl-Hirschman Index if average marginal costs are unchanged. Market shares equal respective output shares since equilibrium price is equal. Aggregated output in turn solely depends on average marginal costs. Equilibrium output of a single firm increases with average marginal costs of its competitors and decreases with own marginal costs. Therefore, market concentration increases with the variance of marginal costs if average marginal costs are constant. Therefore, the distribution of market shares is just a reflection of underlying cost structure.

In case of differentiated goods, however, this relationship is not unambiguous at first sight and difficult to prove as you can see in chapter 3. Comparable to the homogeneous good case equilibrium output decreases with own marginal costs. Corresponding equilibrium price, however, increases with respective marginal costs. The impact of a conjectural cost variation on respective revenue is ambiguous since the effect on output and price are counteracting. Moreover, as you can see in chapter 2 gross revenue decreases with the variance of marginal costs. Thus, all market shares vary in case of a mean preserving cost variation even those of unaffected firms. This is due to aggregated revenue which is the basis for the calculation of market shares.

Nonetheless, there is no evidence for a non-monotonic relationship between market concentration and the distribution of marginal costs in the differentiated good context. Furthermore, it can be shown that an (conjectural) increase of all marginal costs raises the Herfindahl-Hirschman Index since already less efficient firms are disadvantaged above average. Thus, a highly concentrated market can indicate that the market is unfavorable since all marginal costs are close to the maximum willingness to pay or, alternatively, the market is supplied by some fairly efficient firms. In no case big firms entail an inefficient supply with the differentiated goods.

The results of the third chapter in conjunction of the results of the second chapter are a valuable contribution for a better understanding of the positive relationship between market structure and efficiency in case of differentiated goods and are basis for my final research.

In the fourth chapter I analyze welfare implications of present antitrust enforcement policy on basis of the same theoretical model. European as well as the US Merger Guidelines presume a negative impact of market concentration on the competitiveness of the market and, therefore, on the efficiency of the market outcome. The results of the previous chapters indicate that this assumption is false. According to present merger guidelines the likelihood of an objection increases ceteris paribus with aggregated market share of the merger candidates. The fourth chapter investigates the impact of present antitrust enforcement policy on firms' merger activities as well as efficiency of the market outcome. My analysis does not encompass possible coordinated effects but focuses solely on unilateral effects of horizontal mergers.

According to present Merger Guidelines the market outcome after the requested horizontal merger and the market outcome without any merger is compared. This proceeding abstracts away from the fact that a horizontal merger between smaller firms may be consequence of a rejected horizontal merger comprising bigger firms. Furthermore, firms anticipate the criteria of antitrust authorities and request a small horizontal merger from the outset. Firms know that the likelihood of an approval decreases with aggregated market shares of the merger candidates. It is rational for firms to request a horizontal merger which is likely to be approved to save time and money. Thus, a merger comprising smaller firms instead of bigger can be treated as a consequence of present antitrust enforcement policy. Therefore, I compare post-merger market outcome in case of a big merger and the post-merger market outcome in case of a small merger. The size of the merger is determined by aggregated market share of the merger candidates.

In the course of my analysis I question two assumptions of present merger guidelines: Firstly, is there actually a positive relationship between post-merger joint profit and aggregated market shares of concerned firms and, secondly, is there there a negative relationship between aggregated market share and efficiency of the market outcome.

My studies affirm that the first assumption is true. This result is intuitive since a merger between the biggest firms, for instance, constitutes a coalition of the most profitable firms. Remember that equilibrium profit increases with equilibrium output.

Moreover, it can be shown that efficiency of the market outcome actually decreases with aggregated market share apart from one exception. However, this case is pathologic since market concentration is already very high in the starting position. Hence, an approval of any horizontal merger is unlikely. The shutdown of the less efficient part of the merged entity is consequence of any merger irrespective its size.

At first glance it is amazing that present antitrust enforcement policy is welfare enhancing even though the merger regulation base on a false assumption. Only after a second glance the results become more intuitive. With respect to post-merger market structure a case differentiation is necessary whereas the first case was already mentioned above. Under some circumstances there is no shutdown at any kind of merger. A merger between small firms increases the disparity of output levels compared to the pre-merger situation since the insiders[3] internalize the external effects on joint profit and reduce their output levels. The outsiders enlarge their output levels as a consequence. This is welfare enhancing since output is reshuffled from

[3]The firms involved in the horizontal merger are referred to as insider whereas the remaining firms are referred to as outsider.

less efficient to more efficient firms. A horizontal merger between big firms decreases the disparity of output levels, in contrast, since the big insiders reduce their output levels whereas the small outsiders increase their output. Therefore, these results are in line with those of chapter 2.

In the last case there is a shutdown of the less efficient part of the merged entity if there is a merger between small firms whereas there is no reduction of product diversity in case of a big merger. Present antitrust enforcement policy entails the shutdown of the less efficient part of the merged entity and reduces product diversity. Paradoxically, the positive welfare effect can be traced back on this provoked shutdown since it is merely kind of a shake-out. The production of a good which can be produced expensively is stopped whereas the production of cheaper products is enlarged. The shutdown is not only profitable for the Insiders but also welfare enhancing.

In summary, present enforcement policy increases the disparity of the different products even though the reverse is intended. Under some circumstances the removal of a good which has to be produced costly can be the consequence. According to US as well as european merger guidelines a merger is categorized with respect to expected increase in market concentration. The rise of market concentration is estimated by twice the product of concerned market shares.[4] A merger between the two biggest firms, for instance, is expected to increase market concentration most. Such a big merger should decrease market concentration since the disparity of output levels is decreased. However, this effect is counteracted by the reduction of number of firms.

Ultimately, it has to be noted that neither a maximum of competitors nor an equal distribution of market shares is welfare enhancing.

[4]Farrell and Shapiro (1990) criticize this proceeding since it disregards the insiders' incentive to deviate from pre-merger output levels. In his comment Werden (1991) clarifies that this approach rather intends to categorizes the horizontal merger than to estimate real post-merger market concentration.

Contents

List of Figures

Chapter 1

Introduction

The thesis at hand analyzes the relationship between market concentration and efficiency of the market outcome in a differentiated good context and, therefore, fills a gap in theoretical analysis. The relationship between market concentration and efficiency is a matter of particular interest for competition policy as well as antitrust authorities. The aim is to prevent the market from distorted competition which is a consequence of market power arising from dominant positions. A direct measurement of market power requires information about market parameters such as the residual price elasticity of demand or firms´ cost structure. But, these information are mostly unknown due to lack of information since respective data are hardly to observe. Market concentration as an indirect measure of market power is fairly easy to determine since it only requires information about market shares.

Conventional wisdom suggests that there is a negative impact of market concentration on efficiency of the market outcome. This notion is branded by extremal cases such as the monopoly on the one hand and the perfectly competitive market on the other hand. The former is characterized by welfare losses due to monopolistic price setting and a maximum of market concentration while the latter yields an efficient market outcome with a negligible degree of concentration. However, there are many counterexamples such as the theory of contestable markets originated by Baumol et al. (1988) arguing that even a monopoly can yield an efficient market outcome if there is enough competitive pressure from outside the market. Further, Schulz (2003) argues that a free-entry homogeneous Cournot oligopoly with fixed costs is characterized by excessive entry while Koh (2008) has comparable findings in a differentiated good context. Nevertheless, the determination of the relevant market and the subsequent calculation of market shares as well as market concentration is an inherent part of the typical procedure followed by antitrust authorities and courts assessing the market power of firms. Even though many factors such as potential efficiency gains or potential entry are taken into account, firms with big market shares in a highly concentrated market are presumed to have market power and, therefore, a negative impact on competitiveness and the efficiency of the market outcome.

The relationship between market structure and efficiency in the homogeneous good case is analyzed extensively. Even though most of the goods are rather imperfect substitutes than perfect substitutes the analysis of differentiated good oligopolies is sparse. However, the assumption of homogeneous goods is inappropriate for the analysis of a differentiated good context mainly due to two reasons: Firstly, market power arises from product differentiation and, secondly, in reality most of the goods are rather imperfect substitutes than perfect substitutes. Ultimately, it can be shown that results in context of differentiated goods differ from those of the homogeneous good case. In the latter consumers are indifferent between different market structures if average marginal costs is constant whereas producer surplus increases with the dispersion of marginal costs. At first sight one would expect that consumer surplus decreases with the dispersion of marginal costs in the differentiated good context since consumers exhibit a diminishing marginal utility of each good assuming standard quadratic utility according to Dixit and, therefore, prefer each good in same quantity. Moreover, firms' market power arises from product differentiation since the degree of substitutability is inversely proportional to the degree of product differentiation. The second chapter shows that the exact opposite is true. Consumer surplus as well as producer surplus and, therefore, social surplus, increase with the dispersion of marginal costs.

The insights of chapter 2 have striking implications on the role of market concentration in course of the standard procedure followed by antitrust authorities assessing firms' market power since consumer surplus is the main criteria.[1] Yet, before I ana-

[1] The objectives of antitrust authorities and competition policy are discussed below more detailed.

lyze the welfare implications of present antitrust enforcement policy the relationship between market concentration measured by the Herfindahl-Hirschman Index and the distribution of marginal costs is investigated in chapter 3. Market concentration is commonly measured by the Herfindahl-Hirschman Index which is just the sum of all squared market shares. In case of homogeneous goods market shares are simple output shares since the equilibrium price cancels. Since aggregated output (equal to the denominator of each market share) solely depends on average marginal costs there is a monotonic relationship between the dispersion of marginal costs and market concentration measured by the Herfindahl-Hirschman Index. In the differentiated good case market shares are not just output shares since equilibrium prices differ in general. Furthermore, aggregated revenue no longer solely depends on average marginal costs but decreases with its dispersion. Thus, all market shares vary in case of a mean preserving cost variation even those of non-affected firms. The third chapter analyzes the relationship between market structure determined by the distribution of marginal costs and market concentration in a differentiated good context.

Even though there are many countervailing effects (i.e. aggregated revenue decreases with the dispersion of marginal costs while equilibrium output is inversely proportional to respective marginal costs whereas equilibrium price increases with respective marginal costs) there is no evidence that market concentration measured by the Herfindahl-Hirschman Index is no longer a valid measure for market heterogeneity in the differentiated good context comparable to the homogeneous good case.

Chapters 2 and 3 show that antitrust law and competition policy are based, at least partially, on false assumptions with regard to the importance of market concentration on welfare. Firms with big market shares in highly concentrated markets are presumed to have market power and, therefore, raise anticompetitive concern. The first two studies of this thesis show that consumers as well as producers and, therefore, society is better off in more concentrated market structures compared to less concentrated markets. Therefore, chapter 4 finally analyzes two important relationships: Firstly, does the profitability of a horizontal merger actually increase with the size of aggregated market shares? Secondly, is there a negative relationship between post-merger social surplus and aggregated market share of the concerned firms? The first question matters since firms' market power is presumed to increase with respective market share and, therefore, a higher aggregated market share is suspected to yield a higher post-merger joint profit. The second question is interesting since antitrust authorities and competition policy compare in course of the appraisal of a requested horizontal merger social surplus in case of the requested merger with social surplus without any horizontal merger. The standard procedure neglect the fact that another horizontal merger with a smaller aggregated market share can be the consequence of the objection of a requested merger. Moreover, it is possible that merger candidates anticipate this negative standard appraisal and, therefore, request a merger comprising smaller from the very first.[2]

Firstly, it can be shown that the profitability of the horizontal merger indeed increases with the size of aggregated market shares. Under some circumstances there is a shutdown of the less efficient part of the merged entity. Secondly, in the majority of cases present antitrust enforcement policy is welfare enhancing even though it bases partially on false assumptions. Under some special parameter constellations antitrust enforcement policy reinforces the shutdown of the less efficient part of the merged entity. Paradoxically, the welfare gains of antitrust enforcement policy can be traced back to this provoked shutdown. In this respect my results are in line with those Wang and Zhao (2007) showing that the elimination of a fairly inefficient firm can be welfare enhancing.

Therefore, this present thesis is a valuable contribution for a better understanding of the interdependencies between market concentration and social surplus and a helpful guidance for antitrust and competition policy.

1.1 Objectives of competition policy and antitrust law

The main objective of competition policy and antitrust law is to prevent the market from distorted competition. The regulatory framework of European competition law bases mainly on §§81, 82 of the Treaty establishing the European Union (in the following "the Treaty"). According to §82 of the Treaty any abuse of

[2] Lyons (2008) argues that firms have a considerable incentive to propose merger that are likely to be accepted and, therefore, anticipate merger control.

a dominant position is incompatible with the common market of the European Union. §82 prohibits among other things unfair pricing or limiting production to the disadvantage of consumers. The EC Merger Regulation[3] contains rules applying to the concentration of firms. The unifying theme of the EC Merger Regulation is that mergers which create or strengthen a dominant position are incompatible with the common market. Section 2 of the preamble as well as §2 section 1 (a) of the EC Merger Regulation states that the primary aim of the EC Merger Regulation is to prevent the internal market from distorted competition. According to §2 section 2 a concentration of firms is compatible with the common market if it does not significantly impede competition as a result of creation or strengthening a dominant position otherwise the concentration of firms is declared to be incompatible with the common market.[4] The recent amendment of the Council Regulation No 4064/89 of 21 December 1989 on the control of concentrations between undertakings which results to the Council Regulation No 139/2004 (the EC Merger Regulation) is evident for the actuality of this topic.

German competition law is targeted on the abuse of dominant positions, too. According to §19 section 1 GWB[5] the abuse of a dominant position by exercising market power is explicitly forbidden.

US Merger Guidelines are also concerned about the creation of market power by horizontal merger. According to section 0.1 of the US Merger Guidelines market power is presumed to be the source of financial gains of firms. Further, market power is expected to have a negative impact on the competitiveness of the market and, therefore, on the efficiency of the market outcome. "The unifying theme of the Guidelines is that mergers should not be permitted to create or enhance market power or to facilitate its exercise."[6]

But, the legislation leaves open which benchmark has to be used assessing the

[3]Cf. Council Regulation (EC) No 139/2004 of 20 January 2004 on the control of concentrations between undertakings (the EC Merger Regulation).

[4]Cf. §2 section 3 of the EC Merger Regulation.

[5]Abbr. for "Gesetz gegen Wettbewerbsbeschränkungen"

[6]Cf. US Merger Guidelines, section 0.1.

market outcome. According to §81 section 3 of the Treaty, certain actions, which are declared to be incompatible with the common market according to section 1, are exceptionally allowed if respective action "contributes to improving the production or distribution of goods or to promoting technical or economic progress, while allowing consumers a fair share of the resulting benefit [...]". According to §2 section 1(b) of the EC Merger Regulation "the development of technical and economic progress" has to be taken into account appraising the anticompetitive impact of a horizontal merger "provided that it is to consumers‘ advantage". Both articles suggests that there is an attempt to realize at least a minimum of equity between consumers and producers if there is not even a consumer surplus based assessment.

US Merger Guidelines are much more distinct with reference to the assessment criteria. According to section 0.1 of the US Merger Guidelines market power is defined as the ability to maintain prices above the competitive level which results in "[...] a transfer of wealth from buyers to sellers or a misallocation of resources." Section 4 of the US Merger Guidelines deals with potential efficiency gains generated by horizontal mergers and its seriousness for the appraisal of the anticompetitive effect of horizontal mergers. The importance of consumer surplus in course of the appraisal comes to the fore especially in the following expression: "To make the requisite determination, the Agency considers whether cognizable efficiencies likely would be sufficient to revers the merger´s potential to harm consumers in the relevant market, e.g., by preventing price increases in that market." Obviously, consumer surplus is the benchmark to assess the anticompetitive effect of requested horizontal mergers.

In scientific analysis social surplus is the commonly used assessment criteria. Motta (2004), for instance, prefers social surplus as measure for the performance of a market outcome even though its use totally abstracts from the aspect of income distribution. He argues that social surplus is adequate since it is a measure how good does an industry perform as whole. Schulz (2003) agrees and points out that in case of quasi-linear preferences social surplus is a measure for Pareto-optimality which is a commonly used standard of evaluation in industrial economics. Moreover, the distinction between consumers and producers is not possible without further ado since consumers can also be shareholders and benefit from producer surplus, too. Audretsch et al. (2001) criticize antitrust law since the maximization of social surplus is not part of the primary objectives. Rather, antitrust law prohibits several actions reducing competition. They argue that there is not necessarily a positive relationship between the strength of competition and efficiency of the market outcome neither in a static sense nor in a dynamic context. I am in line with these arguments. Therefore, in my analysis social surplus is the measure for Pareto-optimality.

1.2 Importance of market concentration for present antitrust enforcement policy

The previous section has outlined the objectives of competition policy and antitrust law in Europe and the United States. One is concerned about market power accruing from dominant positions. Therefore, antitrust authorities eye firms´ activities such as horizontal mergers, for instance, which are likely to create or strength a dominant position. The following section deals with the pivotal question how to detect market power arising from dominant positions. Furthermore, what is the importance of market concentration in the course of this analysis. Firstly, I focus on the theoretical analysis of market power, I reveal the problems of the direct measurement of market power and, afterwards, I turn towards the standard procedure practiced by antitrust authorities. As you can see in the following not only the objectives but also the standard procedure of antitrust authorities to assess the anticompetitive harm of horizontal mergers are similar on both sides of the Atlantic. Whinston (2006)[7] states: "[...] in recent years there has been a striking convergence in merger laws and enforcement around the globe toward a model in which mergers are evaluated prospectively for their potential competitive harms according to fairly similar standards."

In theory the detection of market power is fairly easy. Schulz (2003) argues that market power arises if there is a failing of competitive pressure irrespective whether the pressure is exercised by firms already active in the market or by potential entrants. Neven et al. (1993) point out that the own-price elasticity of demand is crucial for the determination of market power. But Neven et al. (1993) append: "However, it is usually very difficult to infer the magnitude of this elasticity of

[7]Cf. Whinston (2006), p. 2390.

demand from observable evidence".[8] Mainly, this is due to a lack of empirical data. The direct measure of market power is difficult and laborious. Therefore, market concentration is commonly used as an indirect measure of market power since its calculation solely requires information about the distribution of market shares. These information are fairly easy to obtain.

The following section reveals the importance of market concentration in context of the US Merger Guidelines. I ignore central elements of the standard procedure such as the definition of the relevant market, for instance, but I focus on the role of market concentration.

Indeed, US Merger Guidelines are concerned about market concentration per se. "Other things being equal, market concentration affects the likelihood that one firm, or a small group of firms, could successfully exercise market power."[9] But, the Guidelines amend beneath that the calculation of market shares and the subsequent determination of market concentration is just the starting point of the analysis.

Section 1.51 contains a case differentiation with respect to estimated post-merger market concentration and its expected increase. According to section 1.51 a merger is considered to be harmless if its post-merger market concentration measured by the Herfindahl-Hirschman Index is below 1.000 points. Normally, in this case no further analysis is necessary. In case of a post-merger market concentration between 1.000 and 1.800 points antitrust authorities are not concerned about the merger if the expected increase is less than 100. Otherwise, the horizontal merger raises significant concern and further analysis is the consequence. If the post-merger market concentration exceeds 1.800 the merger is still harmless if expected increase is less than 50 points. If the expected increase exceeds 50 points there is further analysis. In case of an increase of more than 50 points the merger is assumed to create or enhance a dominant position.

With regard to this classification of potential horizontal mergers with respect to market concentration the likelihood of a rejection increases, ceteris paribus, with post-merger market concentration as well as expected increase of concentration.

[8] Cf. Neven et al. (1993), p. 16.

[9] Cf. US Merger Guidelines, section 2.0.

This will be the motivation for my analysis in chapter 4. In contrast to Farrell and Shapiro (1990) I consider an asymmetric differentiated good oligopoly and analyze the relationship between aggregated market share of concerned firms and social surplus in the post-merger equilibrium. For this purpose I compare the post-merger social surplus in case of a horizontal merger between the two biggest firms in the market[10] with post-merger social surplus in case of a horizontal merger between two smaller firms. Intuitively, a big merger is expected to increase market concentration more than a small merger and results in a higher concentrated market structure. In this context the welfare implications of antitrust enforcement policy are given by the difference of both social surpluses. The assumption of differentiated goods is reasonable. In reality most of the goods are rather imperfect substitutes than perfect substitutes. Moreover, market power arises from product differentiation since products are less substitutable. Farrell and Shapiro (1991) already point out in their reply that the differentiated good context is worth to be analyzed.

In the following I give a brief overview over the importance of market concentration in antitrust policy in the European Union. §§81, 82 of the Treaty establishing the European Union constitute the basis for European competition law. Among other things §81 prohibits actions, concerted practices and agreements which may affect trade or result in distorted competition. §82 prohibits the abuse of dominant positions, in general. Both articles contain several concrete actions which are not compatible with the common market, such as the control of selling prices. But, there are no threshold values for market share or market concentration indicating a dominant position.

According to section 7 of the preamble of the EC Merger Regulation the articles 81 and 82 of the Treaty are applicable to horizontal mergers in general, but are insufficient to capture all types of horizontal mergers conflicting with the aim of undistorted competition. According to §2 section 3 of the EC Merger Regulation a horizontal merger which creates or strengthens a dominant position is declared to be incompatible with the common market since a dominant position is likely to impede effective competition.

[10]For notational ease the first type of horizontal merger is henceforth referred to as "big merger" while the latter is referred to as "small merger"

The guidelines[11] of the EC Merger Regulation (henceforth the "EC Merger Guidelines") sketch the analytical procedure of Commission´s appraisal of horizontal mergers. As you can see in the following the EC Merger Guidelines are similar to US Merger Guidelines with respect to the importance of market concentration. According to section 19 a horizontal merger is unlikely to impede competition if expected post-merger market concentration measured by the Herfindahl-Hirschman Index is below 1.000 points. According to section 20 a merger with a post-merger market concentration between 1.000 and 2.000 points raises no anticompetitive concern if the expected increase is less than 250 points. If expected post-merger concentration exceeds 2.000 points the merger is still harmless if expected increase does not exceed 150 points. However, the EC Merger Guidelines stress in section 21 that market concentration is just a first indicator of the existence of market power. Solely, a market share which exceeds 50% can be evidence for market power on its own[12] whereas an aggregated market share of less than 25% indicates that an impediment of effective competition is unlikely.[13] Obviously, US Merger Guidelines as well as EC Merger Regulation agree with the impact of market concentration and market share, respectively, on competitiveness. "The larger the market share, the more likely a firm is to possess market power. And the larger the addition of market share, the more likely it is that a merger will lead to a significant increase in market power."[14] Finally, it must be stated that there is a convergence of European and US competition law.

Finally, I turn towards German competition law. In general, the abuse of a dominant position is forbidden according to §19 section 1 GWB. Section 2 mentions some general facts, such as financial power or interdependencies to other firms which have to be taken into account in course of the appraisal. Section 4 contains a blacklist of concrete activities such as payments or hindered access to markets suggesting the abuse of a dominant position. Section 3 focuses on the role of the market shares. A single firm is supposed to have a dominant position if its market

[11]Cf. 2004/C31/03 - "Guidelines on the assessment of horizontal merges under the Council Regulation on the control of concentrations between undertakings"

[12]Cf. section 17 of the Guidelines of EC Merger Regulation.

[13]Cf. section 32 of the preamble of the EC Merger Regulation.

[14]Cf. section 27 of the Guidelines on the assessment of horizontal mergers 2004/C 31/03.

share exceeds one third. A conglomerate of three firms or less is supposed to be dominant if its aggregated market share exceeds 50% or if five firms or less have at least a market share of two third unless concerned firms satisfactorily show that there is still effective competition. Therefore, a big market share is presumed to facilitate the abuse of a dominant position.

In the following I consider some EU cases and I highlight the importance of market concentration and market share, respectively, within the appraisal and corresponding justification. As you can see below European antitrust authorities use their area of discretion provided by section 14[15] of respective Guidelines and consider not only the market concentration but also the impact of competitiveness of remaining firms.

The European Commission imposed a fine on Intel for the illegal abuse of a dominant position in the market for computer chips according to §82 of the Treaty on May 13th 2009.[16] The actions of Intel were illegal since the aim was to exclude Intel´s competitors. In course of the justification of the dominant position and its abuse the European Commission referred to Intel´s market share of more than 70% but focus on illegal rebates and payments. At first sight the importance of Intel´s market share seems to be insignificant since the Commission not go into details. However, the effectiveness of those illegal actions increases intuitively with respective market share since the proportion of profits made with competitors´ chips is getting less important.

At the beginning of 2009 the European Commission has approved the requested acquisition of Broström by A.P. Møller-Mærsk[17] even though the business activities of both firms are overlapping in several fields such as the liquid bulk tanker sector. But, the European Commission pointed out that the expected post-merger market share is fairly small and consumers still have the possibility to chose an alternative supplier. Furthermore, there are almost no capacity constraints in the liquid bulk tanker sector and no barriers to entry. Therefore, the requested merger can be approved since it is not expected to impede effective competition.

[15]"Market shares and concentration level provide useful first indications of he market structure and of the competitive importance of both the merging parties and their competitors."

[16]Cf. IP 09/745 of European press release.

[17]Cf. IP 09/51 of European press release.

The European Commission cleared the requested acquisition of the two Spanish low-cost airlines Vueling and Clickair by Iberia under some conditions. Initially, the requested merger raised serious anticompetitive concern since a dominant position or even a monopoly with associated damages to competition would have been consequence of the merger on some air routes. This was the result of a route-by-route analysis of all concerned air routes served by the merger participants. The anticompetitive concern mainly based on the high aggregated share of transfer slots[18] at some airports such as Barcelona-El Prat or Madrid-Barajas. The anticompetitive concern could be abolished if the requestors offer transfer slots free at charge to their competitors. Hereby, entry of new competitors would be facilitated establishing effective competitive pressure. Since all requirements were met the requested acquisition could be approved by the European Commission.

It is noticeable that the European Commission lays little stress on the mere number of remaining competitors but on the effectiveness of competition. In the Broström - A.P. Møller-Mærsk case the Commission has emphasized that there are no capacity constraints and, therefore, a competitive market outcome can be the consequence in spite of the merger. This approach is in line with adequate theory. A homogeneous good duopoly with Bertrand competition is an extreme example for the case in which a single competitor can be sufficient for a perfectly competitive market outcome especially if there are no capacity constraints.[19] Nevertheless, the big aggregated market share of slots in the Iberia case raised anticompetitive concern which could only be suppressed by offering transfer slots to competitors which equals a reduction of market shares measured by slots per time. The justification of EU cases is in accordance with regulatory framework due to two reasons: Firstly, anticompetitive concern increases with aggregated market share. Secondly, aggregated market share as well as market concentration is just the starting point since other factors such as potential competition is taken into account.

I will close this section with a citation of Lyons (2008) who analyzes European

[18]A transfer slot contains the right to take-off or land at a certain time.

[19]Levitan and Shubik (1972) illustrate in context of a price-setting homogeneous good duopoly the relationship between capacities and equilibrium price in a simple 2-period context. The larger the capacities the more equals the equilibrium price to the perfectly competitive price.

merger control and concludes on page 50 as follows:

> "Actual decisions are only the tip of the iceberg in terms of the impact of merger control, because these decisions along with guidelines and policy pronouncements influence the type of mergers that firms propose. Given the costs of delay and compliance, firms have considerable incentive to propose mergers that will be acceptable. This makes it crucial to publish the right argument behind a decision. If this guidance is sufficiently clear and if firms rationally anticipate merger control, they will only propose acceptable or marginally harmful mergers - this is why Phase II merger control is, or should be, very difficult to call. In this sense, the analysis is more important than the decision itself."

1.3 Theoretical background

The target-setting of antitrust law and competition policy is to promote an efficient outcome by preventing the market from distorted competition. Even though the objective is well-defined, the detection of dominant positions is far from being simple since it requires in effect information about market parameters such as price-elasticity of demand and the underlying cost structure. The previous section presented the importance of market concentration in context of the detection of a dominant position. A highly concentrated market is an indicator for a dominant position in both US Merger Guidelines and EC Merger Regulation. In the following I summarize some theoretical results concerning the relationship between market concentration and social surplus. Moreover, I reveal a lack in theoretical analysis which is a margin for my own research. In this connection I focus solely on unilateral effects of horizontal merger, not on coordinated effects.[20]

A horizontal merger has a multifaceted impact on both market concentration and social surplus since, firstly, the number of firms is reduced by one at least and, secondly, the insiders internalize their external effects on joint-profit and, therefore, deviate from pre-merger equilibrium. In quantity setting models firms' output levels are strategic substitutes and, therefore, the insiders reduce their output whereas

[20] See Whinston (2006), for instance, for further discussion.

the outsiders rise their output levels as a consequence, but by less. Hence, there is not only a reduction of aggregated output but also a redistribution of total production between insiders and outsiders. It is decisive for social surplus whether small firms or big firms merge since equilibrium output is inversely proportional to respective marginal costs. There is an output shift from less (more) efficient to more (less) efficient firms in case of a merger between small (big) firms. This reshuffling from less efficient to more efficient firms, for instance, decreases total costs of production and can overcompensate welfare losses caused by reduced industry output. These effects apply analogously to market concentration wherefore the impact of horizontal mergers on market concentration is sophisticated, too.

The impact of a horizontal merger on market concentration and social surplus can be decomposed into two components. Firstly, the number of firms is reduced by one and, secondly, output is redistributed among insiders and outsiders. The former effect increases market concentration whereas the latter effect decreases (increases) market concentration in case of a merger between big (small) firms since the distribution of output gets more even (uneven). The reduction of the number of firms usually decreases industry output and, therefore, decreases consumer surplus for sure. But, social surplus may increase subsequently in the presence of fixed costs due to its saving.[21]

In the following I highlight some theoretical results analyzing the impact of redistribution of production among the firms on social surplus and market concentration. In case of an asymmetric homogeneous Cournot oligopoly market power measured by the firm specific Lerner-Index is proportional to respective market share but inversely proportional to price elasticity of demand in equilibrium.[22] Thus, a big market share indicates market power since price elasticity of demand is identical for all firms in equilibrium provided that the price-elasticity of demand is not infinite. At first glance one would expect that an even distribution of market shares characterized by a small market concentration corresponds with an efficient market outcome since any firm has significant market power measured

[21] Cf. Schulz (2003), p. 45 in case of homogeneous goods, for instance, or Koh (2008) in case of differentiated goods.

[22] Cf. Schulz (2003), p. 50, for instance.

by the firm specific Lerner-Index. However, Février and Linnemer (2004), for instance, show that the exact opposite is true. Social surplus increases with market concentration provided that average marginal costs are constant. Consumer surplus solely depends on industry output which again only depends on average marginal costs. Therefore, consumers are indifferent between different market structures if average marginal costs as well as the number of firms are constant. Producer surplus increases with market concentration since total costs of production decrease with the diversity of marginal costs whereas gross revenue (equal to total expenditures) is constant. But, it is noteworthy that welfare gains of highly concentrated markets base solely on producer surplus in the homogeneous good case.

Farrell and Shapiro (1990) consider an asymmetric homogeneous Cournot oligopoly, too, but analyze the relationship between welfare and market concentration from a different point of view. They analyze the external effects (that is social surplus without joint-profit of the insiders) of a conjectural deviation of a single firm (or a subgroup of firms) from the Cournot-Nash equilibrium whereas the infinitesimal merger is assumed to be profitable for the insiders. Such a deviation from the Cournot-Nash equilibrium, which is referred to as infinitesimal merger, is supposed to be consequence of the internalization of external effects on joint profit. They show that the external effect of an infinitesimal merger is positive and, therefore, increases social surplus if market concentration measured by the Herfindahl-Hirschman Index increases sufficiently. Thus, there is not necessarily a negative relationship between market concentration and social surplus. Their criticism of US Merger Guidelines presuming a negative relationship between welfare and market concentration in general, bases on this insight.[23]

The intuition behind the results of Farrell and Shapiro (1990) is the following: Suppose that a single firm (or a sub-group of firms) deviates from the Cournot-Nash equilibrium and reduces its output. In case of strategic substitutes their competitors increase output as a consequence, but by less. If respective firm (or the sub-group of firms) is fairly inefficient there is a reduction of total costs of production since there is an output shift from less efficient to more efficient firms. If the firm which reduces its output is sufficiently inefficient (i.e. its marginal costs exceeds a threshold

[23] Cf. US Merger Guidelines, section 2.0: "Other things being equal, market concentration affects the likelihood that one firm, or a small group of firms, could successfully exercise market power."

value) the positive welfare effect of cost saving outweighs the negative effect of less industry output. In this case an increase of market concentration comes along with an increase of social surplus.

Moreover, Farrell and Shapiro (1990) derive critical values for aggregated market shares ensuring that an infinitesimal merger increases social surplus.[24] Since equilibrium output is inversely proportional to respective marginal costs critical values for market shares can be derived. These insights have striking implications on Farrell and Shapiro's (1990) criticism. The categorization of horizontal mergers with respect to expected post-merger market concentration coincides with their finding that small aggregated market shares are less harmful. Further, Farrell and Shapiro (1990) criticize US Merger Guidelines since expected increase of market concentration is estimate by twice the product concerned pre-merger market shares. This approach neglects the fact that the insiders internalize the external effects on joint profit. In his comment on Farrell and Shapiro's (1990) horizontal merger analysis Werden (1991)[25] makes clear that this approach of US Merger Guidelines is rather a simple rule to classify requested mergers than an attempt to predict exact post-merger market concentration.

The analysis of Farrell and Shapiro (1990) sheds an interesting light on present antitrust enforcement policy since US Merger Guidelines as well as EC Merger Regulation presume a negative relationship between market concentration and efficiency of the market outcome. Farrell and Shapiro (1990) show that a (infinitesimal) privately profitable merger increases social surplus even though aggregated output decreases if market concentration raises sufficiently. Though, the application of present Merger Guidelines tends to increase market concentration. This is due to the fact that a merger between small (big) firms increases (decreases) the disparity of output levels since the small (big) insiders decrease their output whereas the big (small) outsider expand their quantities.

[24]According to Farrell and Shapiro (1990), p.111 the threshold level for aggregated market share is 50% in case of linear demand and constant marginal costs, for instance.

[25]In 1991 Gregory J. Werden was member of the Antitrust Division of the US Department of Justice.

1.4 Outline

This thesis is organized as follows: The second chapter analyzes the relationship between the distribution of marginal costs and consumer surplus as well as producer surplus and, therefore, social surplus in a differentiated good oligopoly. Since I allow for a wide range of degrees of substitutability, starting from perfect substitutes up to completely independent goods, this analysis is a generalization of the standard analysis in context of homogeneous Cournot oligopolies. The main result is that consumer surplus as well as producer surplus increases with the dispersion of marginal costs. Therefore, there is a positive relationship between market heterogeneity and efficiency of the market outcome.

The third chapter analyzes the relationship between the distribution of marginal costs and market concentration measured by the Herfindahl-Hirschman Index.

The fourth chapter analyzes the relationship between joint-profit and aggregated market shares of the merger candidates. Furthermore, the relationship between social surplus and aggregated market share is analyzed. For this purpose I consider an asymmetric oligopoly consisting of three firms differing in marginal costs. I compare post-merger joint profit if there is a big merger (i.e. a merger between the market leader and more efficient competitor) and in case of a small merger (between the market leader and the less efficient competitor). Afterwards, I compare post-merger social surplus in both types of mergers. In contrast to Farrell and Shapiro (1990) I do not analyze infinitesimal horizontal merger but Nash equilibria. This proceeding is reasonable since it is more realistic. The first part of the analysis shows that post-merger joint profit increases with aggregated pre-merger market share (as presumed by antitrust authorities). Moreover, it can be shown that application of present US as well as EC Merger Regulation increases social surplus even though Merger Regulation bases on false assumption with respect to the relationship of market concentration and efficiency of the market outcome. The reasons for enhanced social surplus are, at least partially, unexpected. Under some circumstances present antitrust enforcement policy fosters a more efficient distribution of output (i.e. more efficient firms are stimulated to produce more output whereas less efficient firms produce less). In some cases, antitrust policy provokes the shutdown of the less efficient part of the merged entity. Paradoxically, the welfare gains can be reduced to this shutdown.

Chapter 2

Conjectural Cost Variations in a Differentiated Good Oligopoly

2.1 Introduction

This paper analyzes the relationship between efficiency and market heterogeneity in a differentiated good oligopoly. Market heterogeneity is caused by differently efficient firms. The pivotal question is whether society is better off in case of a more heterogeneous market structure or not. Assuming standard quadratic utility according to Dixit, social surplus is the measure for Pareto-optimality since preferences are quasi-linear. The impact of a conjectural marginal cost variation on consumer surplus as well as producer surplus and therefore social surplus is analyzed. An arbitrarily marginal cost variation is decomposed into an average component and a heterogeneity component. The former increases or decreases all marginal costs to the same degree. The latter increases or decreases the dispersion of marginal costs and lets average marginal costs unchanged.

In the homogenous good case there is a positive relationship between market heterogeneity and efficiency. Consumer surplus solely depends on aggregated output which in turn only depends on average marginal costs. Total cost of production decreases with the dispersion of marginal costs. Since total revenue (equal to aggregated expenditure) is constant, producer surplus increases with the dispersion of marginal costs. There is a positive relationship between market heterogeneity (given by the distribution of marginal costs) and efficiency in the homogenous good case.

In case of differentiated goods consumer surplus not only depends on aggregated output but also on its distribution. The goods are not perfectly substitutable and marginal utility of each good diminishes. Therefore, consumers prefer the differentiated goods in equal quantity. Gross utility decreases with the diversity of the goods if aggregated output is constant. Since the willingness to pay for each good does not only depend on aggregated quantity but also on its distribution, aggregated expenditures (equal to total revenue) varies in case of a mean preserving cost variation. In contrast to the homogenous good case total revenue (equal to total expenditures) is not constant in case of a mean preserving cost variation. Gross utility, aggregated expenditures, total revenue and total cost of production changes. Hence, the relationship between market heterogeneity and consumer surplus as well as producer surplus and therefore social surplus is ambiguous. Furthermore, there may be additional inefficiencies due to firms exercising their market power since goods are no longer perfect substitutes. One would expect that at least consumers should be worse off in more heterogeneous market structures.

But, it can be shown that the exact opposite is true. Diminishing total expenditures outweigh declining gross utility. Consequently, consumer surplus increases with the dispersion of marginal costs and vice versa. Declining total costs of production overcompensate sales collapse. Thus, producer surplus increases with the dispersion of marginal costs, too. Since consumers and producers are better off in case of a mean preserving conjectural cost variation there remains a positive relationship between market heterogeneity and efficiency as in the homogenous good case.

In the context of homogenous goods there is a huge amount of literature analyzing the relationship between market structure and producer surplus as well as consumer surplus (thus welfare). Dixit and Stern (1982) analyze a homogenous good oligopoly with iso-elastic demand. They show that equilibrium prices depend on average marginal costs and decrease with the number of firms and elasticity of demand. Industry profits are increasing with the Herfindahl-Hirschman Index. Market concentration (hence industry profits) increases in case of a cost reduction of a single firm if the respective firm is more efficient than the average firm.

Consumers benefit from this cost reduction. Dixit and Stern allow for different reaction functions including the Cournot case. Farrell and Shapiro (1990) consider a homogenous Cournot oligopoly and analyze the relationship between market concentration and welfare. They show that even a (conjectural) reduction of the output of a single firm increases welfare if the market concentration measured by the Herfindahl-Hirschman Index increases sufficiently. This is due to a shift in production from less efficient to more efficient firms. Kimmel (1992) analyzes the impact of an increase of all marginal costs on equilibrium profits and the market price in context of homogenous goods. While consumers are always worse off, the equilibrium profit of a firm increases if inverse demand is sufficiently concave (convex) and respective market share is sufficiently small (big). Salant and Shaffer (1999) use the results from Bergstrom and Varian (1985) and show that aggregate cost of production strictly decreases with the variance of marginal costs. Since gross revenue is invariant, industry profits increase while consumer surplus remains unchanged. Van Long and Soubeyran (2001) show that aggregated profits are an increasing function of the dispersion of marginal costs if average marginal costs are constant. Since aggregate output and consumer surplus remains unchanged, social welfare increases with the dispersion of marginal costs too. Furthermore there is a stringent (inverse) relationship between the market concentration measured by the Herfindahl-Hirschman index and the distribution of marginal costs. Février and Linnemer (2004) analyze the impact of an arbitrary marginal cost variation on consumer surplus, producer surplus and welfare as well as on market concentration in a homogenous Cournot oligopoly in an extensive manner. They replicate the results of the aforementioned papers and allow for a simultaneous change of all marginal costs. The effect of an arbitrary cost variation on the variables of interest is decomposed into an average impact and a heterogeneity impact.

Lahiri and Ono (1988) show that a reduction of the marginal costs of a single firm may reduce welfare if respective firm is relatively inefficient. They also show that closing down a sufficiently inefficient firm increases social surplus. Zhao (2001) continues the analysis of Lahiri and Ono (1988) and derives threshold values for marginal cost and respective market shares such that a cost reduction reduces welfare. Smythe and Zhao (2006) refine the analysis of Zhao (2001) and allow for nonlinear demand and nonlinear costs as well as technological spill-over. Wang and

Zhao (2007) extend the analysis of Lahiri and Ono (1988) and Zhao (2001) in a differentiated good context. Assuming a utility originated by Shubik (1980) they derive conditions under which marginal cost reductions reduce welfare in Cournot and Bertrand competition.

Even though most of the goods are not perfectly substitutable, there are only a few studies analyzing the relationship between efficiency and market heterogeneity in a differentiated good context. Assuming Dixit-utility, Singh and Vives (1984) compare equilibrium prices under Bertrand and Cournot competition in a differentiated good duopoly. They show that consumer surplus and social surplus are higher under Bertrand competition whereas producer surplus is higher under Cournot (Bertrand) competition if the goods are substitutes (complements). Häckner (2000) continues the analysis of Singh and Vives (1984) and shows that duopoly results do not hold generally in the oligopoly case. Koh (2008) assumes a Dixit-utility and analyzes a symmetric oligopoly with fixed cost under Bertrand and Cournot competition. He shows that profits are always lower under Bertrand competition and derives conditions depending on the fixed cost under which there is excessive entry. Zanchettin (2006) investigates an asymmetric differentiated good duopoly allowing for quality and cost asymmetries. Depending on the degree of substitutability he derives conditions under which (industry) profits are higher under Cournot compared to Bertrand competition. Symeonidis (2003) analyzes the impact of quality heterogeneity on consumer surplus and producer surplus thus on social welfare in a vertically differentiated good context. Assuming a Dixit-utility he finds that consumer surplus as well as producer surplus and therefore social welfare increase with the quality heterogeneity if the average quality is unvaried. The market heterogeneity is caused only by quality differences since firms are assumed to have identical cost functions.

The aim of the paper is to analyze the relationship between efficiency and market structure in a differentiated good oligopoly in an extensive manner. Firms are assumed to compete in quantities and have constant return to scale without fixed cost. The impact of an arbitrary marginal cost variation is decomposed into an average and a heterogeneity impact. While the former influences all firms in equal manner, the latter is a mean preserving cost variation. Furthermore the effect of a cost variation on social surplus is decomposed into its components consumer surplus and producer surplus. The results are contrasted to the homogeneous good case.

This paper is organized as follows: the following section describes the framework of the model. Section 3 presents the central results. Section 4 finally concludes.

2.2 The model

Consider an oligopoly consisting of $n \geq 2$ firms competing in quantities. Each firm produces one differentiated good Q_i with $i = 1, \ldots, n$. Abstracting from fixed cost, each firm incurs constant marginal cost c_i. Let q_i denote the quantity produced by firm $i = 1, \ldots, n$. The quasi-linear preferences of the representative household are described by a quadratic utility according to Dixit (1979). Firm $i = 1, \ldots, n$ faces the following inverse demand:

$$p_i = 1 - q_i - \nu Q_{-i} \tag{2.1}$$

$Q_{-i} := \sum_{j \neq i} q_j$ denotes aggregated output of the competitors of firm $i = 1, \ldots, n$ and ν denotes the parameter of substitution. In case of $\nu > 0$ goods are substitutes and in case of $\nu < 0$ goods are complements. For $\nu = 0$ the goods are independent. To secure that utility is concave the parameter of substitution is assumed to be $\nu \in \left(-\frac{1}{n-1}, 1\right)$. For further insight see appendix 2.5.1. Each firm maximizes its profit choosing an optimal quantity. Let Q^* denote aggregated output in equilibrium. Summing up all first order conditions given by $1 - 2q_i^* - \nu Q_{-i}^* - c_i = 0$ and solving for Q^* yields:

$$Q^* = \frac{n(1 - \overline{c})}{2 + \nu(n-1)} \tag{2.2}$$

Let $\overline{c} := \frac{1}{n} \sum_{i=1}^{n} c_i$ denote average efficiency which is assumed not to exceed 1. Comparable to the homogenous good case, aggregated output depends just on the average of marginal costs and not on its distribution. Industry output Q^* is unchanged in case of a mean preserving cost variation. Since goods are differentiated the (heterogeneity) impact of a mean preserving cost variation on consumer surplus is different to the homogenous good case. I will come back to this point later. In contrast to aggregated output the derivation of equilibrium output q_i^* is little more tricky. The derivation is delegated to the appendix.

Lemma 1 (Equilibrium output). *Equilibrium output of firm $i = 1, \dots, n$ is given as follows:*

$$q_i^* = \frac{(2-\nu) - [2+\nu(n-2)]c_i + \nu \sum_{j \neq i} c_j}{(2-\nu)[2+\nu(n-1)]}$$

Intuitively equilibrium output is in reverse proportion to its marginal costs and increases with the sum of competitors marginal costs irrespective its distribution. As shown in the appendix, corresponding equilibrium price p_i^* is given by $p_i^* = q_i^* + c_i$. Comparable to the homogenous good case, equilibrium profit $\Pi_i^* := (p_i^* - c_i)q_i^*$ equals its squared quantity.

$$\Pi_i^* = (q_i^*)^2 \tag{2.3}$$

Since entry or exit is not subject of investigation I assume $p_i^* - c_i = q_i^* > 0$ for $i = 1, \dots, n$. Solving $q_i^* > 0$ for c_i yields the expression is the following assumption:

Assumption 2.2.1 (Oligopoly of n firms). *To ensure an oligopoly consisting of n firms, I assume $q_i^* > 0$ for $i = 1, \dots, n$ which is equivalent to the following inequality:*

$$c_i < \frac{2-\nu}{2+\nu(n-2)} + \frac{\nu}{2+\nu(n-2)} \sum_{j \neq i} c_j$$

Note that in case of substitutes assumption 2.2.1 requires marginal costs not to exceed 1 (equal to the maximum willingness to pay). In case of complements marginal cost may exceed 1 if rivals are sufficiently efficient. In case of complements the willingness to pay for a good increases with the consumption of rivals' output which in turn is in reverse proportion to respective marginal costs.

2.3 Results

In the following the central results concerning producer surplus, consumer surplus and social surplus are presented. In the terminology of Février and Linnemer (2004) the impact of an arbitrary conjectural marginal cost variation on the aforementioned variables is decomposed into an average and a heterogeneity impact. Analytically speaking the average impact and the heterogeneity impact are given by directional derivatives. The average effect reduces (increases) marginal cost of all firms to the same degree while the variance is constant. The heterogeneity effect comprises the reduction of the marginal cost of a single firm. In return the marginal cost of another firm increases to the same degree. The heterogeneity component increases or decreases the variance of marginal costs while average efficiency is unchanged.

Definition 2 (Average and heterogeneity impact)**.** *Let* AIF *denote the average impact and* HIF *the heterogeneity impact on F. In this study F is given by producer surplus* PS*, consumer surplus* CS *and social surplus* W*. The total derivative of F is given by $dF = \sum_{k=1}^{n} \frac{\partial F}{\partial c_k} dc_k$. The average impact is characterized by $dc_1 = \ldots = dc_n = dc$. Without loss of generality the heterogeneity impact is given by a conjectural variation of c_k and c_l with $k < l$ and $dc_k = -dc_l > 0$.* AIF *and* HIF *are given as follows:*

$$\text{AIF} := \sum_{i=1}^{n} \frac{\partial F}{\partial c_i} \qquad \text{HIF} := \frac{\partial F}{\partial c_k} - \frac{\partial F}{\partial c_l}$$

Note that the 'directions' $dc_1 = \ldots = dc_n$ and $dc_k = -dc_l$ just equal the Eigenvectors of the matrix of coefficients characterizing the Cournot-Nash equilibrium given by (2.18).

2.3.1 Producer surplus

In the following, the relationship between producer surplus and market structure is analyzed. Producer surplus $\text{PS}^* := \sum_i \Pi_i^*(q_i^*, Q_{-i}^*)$ is just the sum of all equilibrium profits.

Proposition 2.1 (Average Impact)**.** *The average impact on equilibrium profit of firm $i = 1, \ldots, n$ and producer surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation.*

Proof: Due to linearity the average impact on producer surplus is just the sum of the average impact on Π_i^*. $\text{AIPS}^* = \sum_j \text{AIPS}_j^*$ with $\text{AIPS}_j^* = \sum_i \partial_i \left(q_j^*\right)^2$ since $\Pi_i^* = (q_i^*)^2$.

It holds:

$$\begin{aligned}
\text{AIPS}_j^* &= \sum_i \partial_i \left(q_j^*\right)^2 = 2q_j^* \sum_i \partial_i q_j^* \\
&= 2q_j^* \left(\frac{-[2+\nu(n-2)] + \nu(n-1)}{(2+\nu)[2+\nu(n-1)]} \right) \qquad (2.4) \\
&= \frac{-2q_j^*}{2+\nu(n-1)} < 0 \qquad (2.5)
\end{aligned}$$

The average impact on producer surplus is just the sum of all AIPS_j^*.

$$\text{AIPS}^* = \sum_j \frac{-2q_j^*}{2+\nu(n-1)} = \frac{-2Q^*}{2+\nu(n-1)} < 0 \qquad (2.6)$$

All firms are worse off in case of a cost variation making all firms less efficient and vice versa. □

The average impact on equilibrium profit has two opposite components. On the one hand making all competitors more efficient has a negative effect on the equilibrium profit since all substitutes of the product are getting cheaper and, therefore, more attractive. This effect is given by $\nu(n-1)$ in (2.4). On the other, hand each firm benefits by a reduction of its marginal cost. This effect is given by $-[2+\nu(n-2)]$ in (2.4). The latter effect outweighs the former effect. The profit of each firm increases in case of a cost variation decreasing all marginal costs and vice versa.

This result coincides with the homogenous good case since producer surplus decreases if all firms are negatively affected unless market concentration is sufficiently high and inverse demand is sufficiently concave. Since inverse demand is linear in this model, firms are always worse off increasing all marginal costs. In context of homogenous goods a firm benefits by an increase of all marginal costs if its market share is sufficiently big and inverse demand sufficiently concave. This is due to a shift in production from the inefficient to the efficient firms. Compare Seade (1985), Kimmel (1992) or Février and Linnemer (2004).

In the following, the dispersion of marginal costs is varied while keeping average efficiency constant. The results concerning the heterogeneity impact on equilibrium profit and producer surplus are summarized in the following proposition.

Proposition 2.2 (Heterogeneity Impact)**.** *Producer surplus increases with the dispersion of marginal costs and vice versa.*

Proof: According to (2.3) equilibrium profit is given by $\Pi_i^* = (q_i^*)^2$. The heterogeneity impact $\text{HIQ}^* := \partial_k q_i^* - \partial_l q_i^*$ on equilibrium output q_i^* is given as follows:

$$\text{HIQ}_i^* = \begin{cases} \frac{-1}{2-\nu}, \text{ for } i = k, \\ \frac{1}{2-\nu}, \text{ for } i = l, \\ 0, \text{ else.} \end{cases} \tag{2.7}$$

Intuitively equilibrium output of the firm which is positively (negatively) affected by the cost variation increases (decreases). The heterogeneity impact on the equilibrium profit of the unaffected firms $i \neq k, l$ is zero. The heterogeneity impact on producer surplus is composed of the heterogeneity impacts on Π_k^* and Π_l^*.

$$\begin{aligned} \text{HIPS}^* &= \text{HIPS}_k^* + \text{HIPS}_l^* \\ &= \left[\partial_k (q_k^*)^2 - \partial_l (q_k^*)^2\right] + \left[\partial_k (q_l^*)^2 - \partial_l (q_l^*)^2\right] \qquad (2.8) \\ &= 2q_k^* \,\text{HIQ}_k^* + 2q_l^* \,\text{HIQ}_l^* \\ &\overset{(2.7)}{=} 2\,\text{HIQ}_k^* (q_k^* - q_l^*) \qquad (2.9) \end{aligned}$$

Equilibrium quantity is in reverse proportion to efficiency.

$$\begin{aligned} q_k^* - q_l^* &= \frac{-[2 + \nu(n-2)](c_k - c_l) + \nu(c_l - c_k)}{(2+\nu)[2 + \nu(n-1)]} \\ &= \frac{-1}{2-\nu}(c_k - c_l) \qquad (2.10) \end{aligned}$$

Inserting (2.10) in (2.9) yields:

$$\text{HIPS}^* = \frac{2}{(2-\nu)^2}(c_k - c_l) \tag{2.11}$$

Producer surplus increases in case of a cost variation increasing the dispersion of marginal costs and vice versa. □

Intuitively, the firm which is advantaged by the cost variation profits and the disadvantaged firm looses. Reducing the marginal cost of a firm increases its equilibrium output as well as its price-cost margin since $p_i^* - c_i = q_i^*$. The heterogeneity effect on the more efficient firm outweighs the effect on the less efficient one. Producer surplus increases with the dispersion of marginal costs.

The heterogeneity impact on equilibrium profit and producer surplus coincides with the homogenous good case. Compare Bergstrom and Varian (1985) or Février and Linnemer (2004). This result, however, is not self-evident. In contrast to the homogenous good case, the heterogeneity impact on total revenue is not constant but falls with the diversity of marginal costs. But, the effect on total costs overcompensates the effect on total revenue. I will get back to this later.

These results coincide with those of Symeonidis (2003), too. Assuming Dixit-utility he analyzes a vertically differentiated good oligopoly. He finds that industry profits under Cournot competition increase with the dispersion of quality levels if average quality is constant.

In the following the heterogeneity impact on producer surplus is explained by an alternative point of view. Let us investigate the heterogeneity effect on producer surplus by analyzing its components: total revenue and total cost. In contrast to the homogenous good case gross revenue decreases with the dispersion of marginal costs.

Lemma 3 (Total revenue versus total cost)**.** *Both total revenue and total cost decreases with the disparity of marginal costs. The heterogeneity impact on total cost outweighs the heterogeneity impact on total revenue. Producer surplus increases with the disparity of marginal costs.*

In the homogenous good case producer surplus increases with the dispersion of marginal cost, since gross revenue (equal to total expenditure) is unchanged and total cost decrease with the disparity of marginal cost. Compare Salant and Shaffer (1999) for instance. In the homogeneous good case as well as in the differentiated good context producer surplus increases with market heterogeneity.

2.3.2 Consumer surplus

Are consumers better off in a more heterogeneous market structure characterized by some big and several small firms? Does a more homogeneous market structure solely consisting of equipollent firms involve more favorable conditions? Consumer surplus caused by the consumption of the goods q_i^* with $i = 1, \ldots, n$ is defined as follows: $\text{CS}^* := U(m - \sum_{i=1}^n p_i^* q_i^*, q_1^*, \ldots, q_n^*) - U(m, 0, \ldots, 0)$. The consumption of the numeraire good q_0 is given by $q_0^* = m - \sum_{i=1}^n p_i^* q_i^*$. Let m denote the income of the representative household which is assumed to be exogenous. In the following the average effect on consumer surplus is analyzed.

Proposition 2.3 (Average Impact)**.** *Consumer surplus decreases with average marginal costs and vice versa.*

A reduction of all marginal costs increases all equilibrium quantities and, therefore, consumers are unambiguously better off. This result again coincides with the homogenous good case. Compare Février and Linnemer (2004) for instance. In case of homogenous goods consumer surplus increases with industry output which again is negatively correlated with average efficiency.

In the following, the relationship between the dispersion of marginal costs and consumer surplus is analyzed. Are there inefficiencies due to firms exercising their market power in highly concentrated markets? Since goods are not perfectly substitutable, firms have more market power to enforce higher price-cost margins. As shown above, the price-cost margin increases with efficiency. Compare (2.7) and (2.3). Since marginal utility decreases, consumers prefer the goods in equal quantity if aggregated output is constant. Indeed, gross utility decreases with the dispersion of marginal costs. Therefore, the results concerning the heterogeneity impact on consumer surplus are surprising.

Proposition 2.4 (Heterogeneity Impact)**.** *Consumer surplus increases with the dispersion of marginal costs.*

In case of differentiated goods a more heterogeneous market structure is favorable not only for producers but also for consumers. Although price-cost margin increases with efficiency and variance of equilibrium output increases, consumers are better off in case of heterogeneous market structures. In the limit case of perfect substitutes (i.e. $\nu \to 1$) the result coincides with classical homogenous good models. Consumer surplus solely depends on industry output which again depends on average efficiency. Compare Février and Linnemer (2004), for instance.

This result also corresponds with the insight of Symeonidis (2003). Assuming a Dixit-utility he finds that in a vertically differentiated good oligopoly producer surplus as well as consumer surplus increase with the variance of the quality levels if average quality is constant.

The heterogeneity impact on consumer surplus can be explained by decomposing the effect on its components: gross utility and total expenditure. Since households‘ expenditures just equal gross revenue, the results concerning firms revenue given by (2.27) can be employed for this analysis. It remains to analyze the heterogeneity impact on gross utility.

Lemma 4 (Total expenditure versus gross utility)**.** *Total expenditures as well as gross utility decrease with the disparity of marginal costs. The heterogeneity impact on total expenditure outweighs the effect on gross utility. Consumer surplus increases with the dispersion of marginal cost.*

This result is essentially different to the homogenous good case since gross utility as well as total expenditures decrease with market heterogeneity. Ultimately, consumers are better off in more heterogeneous market structures. In the following the heterogeneity effect on consumer surplus is analyzed by another point of view. Consumer surplus is just the sum of the net benefits of each single commodity. Let CS_i denote the net utility caused by the consumption of good $i = 1, \ldots, n$:

$$\text{CS}_i := q_i - \frac{1}{2}q_i^2 - \frac{\nu}{2}q_i Q_{-i} - p_i q_i$$

The term $q_i - \frac{1}{2}q_i^2$ reflects the direct utility caused by the consumption of commodity q_i^*. The term $\frac{\nu}{2}q_i Q_{-i}$ describes the additional utility (or disutility) caused by simultaneous consumption of the other commodities. Associated expenditures are given by $p_i q_i$. It is easy to prove that consumer surplus CS is just aggregated net utility of all n goods.

Obviously, the net utility of the non-affected goods is unchanged in case of mean preserving cost variation since aggregated concurrence output is unchanged and according to (2.7) the heterogeneity impact on non-affected quantities and equilibrium prices is zero. Due to linearity, the heterogeneity impact on consumer surplus is the sum of heterogeneity impacts on the affected goods.

Lemma 5 (Net utility of a single commodity)**.** *The net utility of a single commodity is in reverse proportion to its marginal costs. The absolute value of the heterogeneity effect is proportional to efficiency. The effect on the more efficient firm outweighs the effect on the less efficient one. Consumer surplus increases with the dispersion of marginal costs.*

Proof: Consumer surplus can be expressed as follows:

$$\begin{aligned} \text{CS}^* =& q_k - \frac{1}{2}q_k^2 - \frac{\nu}{2}q_k Q_{-k} - p_k q_k && (2.12)\\ &+ q_l - \frac{1}{2}q_l^2 - \frac{\nu}{2}q_l Q_{-l} - p_l q_l && (2.13)\\ &+ \sum_{j\neq k,l}\left(q_j - \frac{1}{2}q_j^2 - \frac{\nu}{2}q_j Q_{-j} - p_j q_j\right) \end{aligned}$$

According to (2.7) the impact on equilibrium quantity and price of the unaffected goods is zero. Since aggregated output solely depends on average efficiency (cf. (3.2)) the heterogeneity impact on aggregated concurrence output is zero. Hence, the effect on the net utility of the unaffected goods $j \neq k, l$ is zero. The heterogeneity impact on consumer surplus is just the sum of HICS_k and HICS_l.

$$\begin{aligned} \text{HICS}_k^* =& \partial_k q_k^* \left(1 - q_k^* - \frac{\nu}{2}Q_{-k}^*\right) - \frac{\nu}{2}q_k^*\partial_k Q_{-k}^* - \partial_k p_k^* q_k^* - p_k^*\partial_k q_k^*\\ &- \partial_l q_k^* \left(1 - q_k^* - \frac{\nu}{2}Q_{-k}^*\right) + \frac{\nu}{2}q_k^*\partial_l Q_{-k}^* + \partial_l p_k^* q_k^* + p_k^*\partial_l q_k^* \end{aligned}$$

Note that the equilibrium price is just given by $p_i^* = 1 - q_i^* - \nu Q_{-i}^*$ for $i = 1, \ldots, n$. Furthermore $p_i^* = q_i^* + c_i$ for $i = 1, \ldots, n$. It holds:

$$\begin{aligned} \text{HICS}_k =& -\frac{\nu}{2}q_k^*\partial_k Q_{-k}^* + \partial_k q_k^*\frac{\nu}{2}Q_{-k}^* - \partial_k p_k^* q_k^*\\ &+ \frac{\nu}{2}q_k^*\partial_l Q_{-k}^* - \partial_l q_k^*\frac{\nu}{2}Q_{-k}^* + \partial_l p_k^* q_k^* \end{aligned}$$

The heterogeneity impact on equilibrium output q_i^* is denoted by HIQ_i^*. Furthermore the heterogeneity impact $\mathrm{HIQ}_{-k} := \partial_k Q_{-k}^* - \partial_l Q_{-k}^*$ on aggregated concurrence output is given by $\mathrm{HIQ}_{-k} = \mathrm{HIQ}_l^* = -\,\mathrm{HIQ}_k^*$. The equilibrium price is given by $p_i^* = q_i^* + c_i$ and the heterogeneity impact on p_i^* is given by $\mathrm{HIP}_i^* = \mathrm{HIQ}^* + \mathrm{HIC}_i$. Let HIC_i denote the 'heterogeneity impact' on the marginal cost of firm $i = 1, \ldots, n$ with $\mathrm{HIC}_k = 1$, $\mathrm{HIC}_l = -1$ and $\mathrm{HIC}_i = 0$ for $i \neq k, l$.

$$\begin{aligned}\mathrm{HICS}_k &= \frac{\nu}{2} Q_{-k}^* \,\mathrm{HIQ}_k^* + \frac{\nu}{2} q_k^* \,\mathrm{HIQ}_k^* - q_k^* \,\mathrm{HIQ}_k^* - q_k^* \\ &= \frac{\nu}{2} Q^* \,\mathrm{HIQ}_k^* - \left(\frac{1-\nu}{2-\nu}\right) q_k^* < 0\end{aligned}$$

Since $\mathrm{HIQ}_k^* < 0$ the heterogeneity impact on CS_k is negative irrespective of the distribution of marginal costs or the degree of substitutability ν. Similarly CS_l can be derived which is given by $\mathrm{CS}_l = -\frac{\nu}{2} Q^* \,\mathrm{HIQ}_k^* + \left(\frac{1-\nu}{2-\nu}\right) q_l^* > 0$. Summing up CS_k and CS_l yield the heterogeneity impact on consumer surplus given by (2.31). □

Consumers haven´t worry about heterogeneous market structures. Net utility of a commodity is in reverse proportion to its marginal costs. A mean preserving cost variation increasing the disparity of marginal costs makes consumers better off. In the homogeneous good case consumers have no preferences about the distribution of marginal cost as long as average efficiency is constant.

2.3.3 Social surplus

In the following the relationship between market structure and efficiency is analyzed. It can be shown that a heterogeneous market structure is not a hostile environment for society. It provides a more efficient market outcome compared to more homogenous market structures. Social surplus is an increasing function of the dispersion of marginal costs, if average marginal costs are constant. Since preferences are quasi-linear, social surplus is the measure for Pareto-optimality.

$$W := U\left(m - \sum_{i=1}^{n} c_i q_i, q_1, \ldots, q_n\right) - U(m, 0, \ldots, 0)$$

The consumption of the numeraire-good q_0 is given by $q_0 = m - \sum_i c_i q_i$. Naturally, social surplus abstracts from the distribution of total surplus on consumers and producers. Social surplus is just the sum of producer surplus and consumer surplus. Therefore, the average impact on social surplus is the sum of the average impacts on both components.

Corollary 6 (Average Impact)**.** *The average impact on social surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation.*

Decreasing all marginal cost makes society unambiguously better off and vice versa. In the homogeneous good case social surplus increases due to a cost variation making all firms less efficient if inverse demand is sufficiently concave and market concentration is sufficiently high. In this case there is a shift in production from inefficient firms to efficient firms. This phenomena cannot occur since demand is linear in this model.

Since consumer surplus as well as producer surplus increases with the dispersion of marginal costs, the following result is no longer surprising.

Corollary 7 (Heterogeneity Impact)**.** *Social surplus increases with the disparity of marginal costs if average marginal costs is constant.*

Society benefits from a mean preserving cost variation increasing the market heterogeneity irrespective the distribution of marginal costs or parameter of substitution. A more heterogeneous market structure is beneficial for both consumers as well as producers and therefore society. This result is well known in the homogeneous good case and can be brought forward into the differentiated good context. In the homogeneous good case consumers are indifferent between market structures with same average efficiency. In case of differentiated goods society is better off since producer surplus as well as consumer surplus increases with market heterogeneity.

This result also coincides with related research in vertically differentiated good models (cf. Symeonidis (2003)). Consumer surplus as well as producer surplus increases with the dispersion of quality levels if average quality is constant. Therefore, market heterogeneity either in terms of quality differences or in terms of differently efficient firms provides favorable conditions for efficient market outcomes.

2.3.4 Incentive to innovate

In the following there are some remarks concerning the profit incentive of the firms to innovate. Even though innovation activities are not the main object of investigations there are some insights which can directly be deduced from results made so far. They pertain to the relationship of market structure and innovation incentives and give an intuition of how the structure of an asymmetric oligopoly may change in consequence of innovation activities.

Belleflamme and Vergari (2006) have a similar framework since they consider a differentiated good oligopoly consisting of n firms assuming Dixit utility. But, they analyze the profit incentive of a cost reducing process innovation if solely one firm uses the new technology. Moreover, they consider an ex ante symmetric oligopoly since all firms are assumed to produce initially with identical marginal costs. They focus on the impact of the intensity of competition (Bertrand versus Cournot competition, number of firms and the degree of substitution) on the profit incentive. The study of Belleflamme and Vergari gives no insight if the market structure is ex ante asymmetric or if there is no kind of patent race if production technologies are heterogeneous and, therefore, there are no spill-overs or externalities. Bester and Petrakis (1993) analyze a differentiated good duopoly assuming standard Dixit utility, too. They compare the optimal innovation level of a single firm under Bertrand and Cournot competition with the social optimal innovation level. Boone (2001) analyzes the effect of the intensity of competition on the incentive to innovate in a asymmetric oligopoly whereas the intensity of competition is founded axiomatically. He finds that there is a non-monotonic relationship between competitive pressure and the innovation incentive. Vives (2004) gives an overview about literature dealing with the relationship between innovation and competitive pressure.

In the following section I analyze the incentive of each firm to reduce its marginal costs in the course of a process innovation. Suppose that each firm has its own production technology and there are no spill-overs. Intuitively, the absolute value of marginal profit with respect to own marginal costs is the incentive to innovate. Let $\triangle_i^m$ denote the incentive of firm $i = 1, \ldots, n$ to innovate. The index m indicates that $\triangle_i^m$ can be treated as a marginal or infinitesimal innovation.

$$\triangle_i^m := -\partial_i \Pi_i^*$$

The following proposition contains the main result concerning the incentive to make a cost reducing process innovation.

Proposition 2.5 (Incentive to innovate)**.** *The incentive to innovate is inversely proportional to respective marginal costs.*

Proof: According to (2.3) the equilibrium profit of firm i is just its squared equilibrium output. The incentive to innovate is given by $\triangle_i^m = -2q_i^* \partial_i q_i^*$. It holds:

$$\begin{aligned}\triangle_i^m &= -2q_i \partial_i q_i \\ &= \left(\frac{2[2+\nu(n-2)]}{(2-\nu)[2+\nu(n-1)]}\right)\left(\frac{(2-\nu)-[2+\nu(n-2)]c_i + \nu\sum_{i\neq j} c_j}{(2-\nu)[2+\nu(n-1)]}\right)\end{aligned}$$

The partial derivative of $\triangle_i^m$ with respect to own marginal costs is given as follows:

$$\frac{\partial}{\partial c_i}(\triangle_i^m) = \frac{-2[2-\nu(n-2)]^2}{(2+\nu)^2[2-\nu(n-1)]^2} < 0$$

Hence, the incentive to innovate is in reverse proportion to marginal costs. □

At first glance this result is counterintuitive since fairly efficient and, therefore, big firms impose competitive pressure on less efficient firms. Less efficient firms should have a bigger incentive to elude this pressure. But, more efficient firms have a bigger incentive to reduce their marginal costs since their marginal profit exceeds those of less efficient firms. A cost reduction is more profitable since it applies to a bigger quantity.

Let us resolve the incentive to innovate given by $\triangle_i^m = -2q_i^* \partial_i q_i^*$ into its components. The impact of a cost reduction on equilibrium output given by $-2\partial_i q_i^* = \frac{2[2+\nu(n-2)]}{(2-\nu)[2+\nu(n-1)]}$ is independent of respective marginal costs. But, equilibrium output is the multiplier of marginal output. Thus, a cost reducing process innovation is more profitable to the more efficient firm since equilibrium output is inversely proportional to respective marginal costs.

Suppose now that each firm has the possibility to increase its efficiency by a cost reducing process innovation. Furthermore, suppose that there are no patents or licences, spill-overs or externalities in connection with the innovation activities and all firms have the possibility to innovate simultaneously. It is an direct implication of the previous proposition that market heterogeneity increases due to innovation activities.

Corollary 8 (Market structure and innovation)**.** *The incentive to innovate is inversely proportional to respective marginal costs. Thus, simultaneous innovation activities of all firms increase market heterogeneity. The dispersion of marginal costs increases.*

In the following I assess this result from a welfare point of view. Already big firms have a bigger incentive to invest in cost reducing process innovations compared to less efficient firms. The disparity of output levels increases. Is it wise to subsidize smaller firms, for instance, to gain on the big firms? Without such an intervention the market has the tendency to get more heterogeneous. With regard to the results of the previous sections the following corollary is not surprising.

Corollary 9 (Welfare implication)**.** *Simultaneous innovation activities of all firms move the market towards a market structure which is characterized by a higher social surplus.*

Analytically speaking an arbitrary cost combination has the tendency to move away from the 45°-line characterized by $c_1 = \ldots = c_n$. Even though marginal costs of each firm decreases the distance between the present marginal costs combination and the 45°-line increases due to innovation activities. Without loss of generality let assume $c_1 \leq \ldots \leq c_n$ and consider the vector given by $(\triangle_1^m, \ldots, \triangle_n^m)$. This vector characterizes the innovation incentives of the whole market. According to proposition 2.5 it holds: $\triangle_1^m \geq \ldots \geq \triangle_n^m$. Thus, an arbitrary cost combination "moves" in direction of the vector given by $\triangle^{\mathbf{m}} := (-\triangle_1^m, \ldots, -\triangle_n^m)$ due to innovation activities. Let us now decompose this innovation vector into its average component and heterogeneity component. According to definition 2 this is just a principle axis transformation. For this purpose just multiply the innovation vector $\triangle^{\mathbf{m}}$ with the matrix P^T consisting of the Eigenvectors given by (2.18).

It holds:

$$P^T \triangle^{\mathbf{m}} = \begin{pmatrix} \triangle_2^m - \triangle_1^m \\ \dots \\ \triangle_n^m - \triangle_{n-1}^m \\ -(\triangle_1^m + \dots + \triangle_n^m) \end{pmatrix}$$

Since $\triangle_1^m \geq \ldots \geq \triangle_n^m \geq 0$ each component of $P^T \triangle^{\mathbf{m}}$ is negative. The first $n-1$ components equal the heterogeneity impact and the $n-th$ component equals the average effect of $\triangle^{\mathbf{m}}$. According to proposition 6 and proposition 7 both the average impact and the heterogeneity impact of $\triangle^{\mathbf{m}}$ on social surplus are positive.

In the following I investigate the importance of the degree of substitutability ν on the innovation vector $\triangle^{\mathbf{m}}$. How does the degree of substitution affects the disparity of innovation incentives of differently efficient firms? Does the disparity of marginal profits increases with the degree of substitution since products are more substitutable and bigger firms can more easily gain demand from less efficient firms? It can be easily shown that the parameter of substitution ν tightens the competitive pressure since it increases the disparity of innvation incentives.

Proposition 2.6 (Market pressure and the incentive to innovate)**.** *The higher the degree of substitutability ν, the higher is the incentive of an efficient firm to innovate compared to a less efficient one.*

Proof: Let us subtract the incentives to innovate of firm k and l. It holds:

$$\begin{aligned} \triangle_k^m - \triangle_l^m &= \frac{2[2+\nu(n-2)]}{(2-\nu)[2+\nu(n-1)]} \left(\frac{-[2+\nu(n-2)](c_k - c_l) + \nu(c_l - c_k)}{(2-\nu)[2+\nu(n-1)]} \right) \\ &= \frac{2[2+\nu(n-2)]}{(2-\nu)[2+\nu(n-1)]} \left(\frac{-[2+\nu(n-1)](c_k - c_l)}{(2-\nu)[2+\nu(n-1)]} \right) \\ &= \frac{-2[2+\nu(n-2)]}{(2-\nu)^2[2+\nu(n-1)]}(c_k - c_l) \qquad (2.14) \end{aligned}$$

It is easy to verify that the fraction in (2.14) is negative for $n > 2$. This corresponds to $\triangle_k^m - \triangle_l^m > 0$ for $c_k < c_l$ and vice versa.

The partial derivative of the fraction with respect to the degree of substitution ν is given as follows:

$$\frac{\partial}{\partial \nu}(\cdot) = \frac{4\Big[2+\nu\Big(-5+4n+[2+(-3+n)n]\nu\Big)\Big]}{(-2+\nu)^3[2+(-1+n)\nu]^2} \tag{2.15}$$

It is easy to proof that the partial derivative given by (2.15) is negative for $n \geq 2$ and $\nu \in [0,1]$. Thus, the difference between $\triangle_k^m$ and $\triangle_l^m$ increases with ν. $\square$

This result generalizes those of Sacco (2008) who analyzes the relationship between competitive pressure and the incentive to reduce marginal costs in a differentiated good duopoly. For $n = 2$ the results of Sacco (2008) can be replicated by my analysis. But, Sacco lays more stress on the relationship between the competitive pressure and the level of innovation. Assuming convex cost of innovation he analyzes the optimal level of a cost reducing process innovation in a 2-stage game. Athey and Schmutzler (2001) come to similar results in a much more general framework. They analyze, inter alia, ongoing decision to invest in cost reducing process innovations.

2.4 Conclusions

This chapter analyzes the relationship between the dispersion of marginal costs and consumer surplus, producer surplus as well as social surplus in a differentiated good context. The effect of an arbitrary cost variation on the aforementioned variables is decomposed into an average and a heterogeneity component. It can be shown that there is a positive relationship between the dispersion of marginal costs and efficiency of the market outcome. In contrast to the homogenous good case consumer surplus as well as producer surplus increases with the dispersion of marginal costs.

On the one hand these results coincide with the homogenous good case since there is a positive relationship between the dispersion of marginal costs and efficiency of the market outcome. In contrast to homogeneous goods not only producer surplus but also consumer surplus increases with the variance of marginal costs.

On the other hand these results are similar to related research analyzing vertically differentiated good oligopolies. Consumer surplus as well as producer surplus increase with the dispersion of quality levels if average quality is constant. Heterogeneous market structures provide favorable conditions not only for producers but also for consumers.

2.5 Appendix

2.5.1 Utility

The quadratic utility according to Dixit (1979) is given as follows:

$$U(q_0, q_1, \ldots, q_n) = q_0 + \sum_i q_i - \frac{1}{2}\boldsymbol{q}^T H \boldsymbol{q}$$

Let q_0 denote the numeraire good and the matrix of substitution H is given as follows:

$$H = \begin{pmatrix} 1 & \nu & \cdots & \nu \\ \nu & 1 & \cdots & \nu \\ \vdots & \vdots & \ddots & \vdots \\ \nu & \nu & \cdots & 1 \end{pmatrix}$$

The corresponding Hessian $\nabla^2 U = -H$ is real, symmetric and can be decomposed by $P^{-1}DP = -H$. Let D denote the matrix containing the Eigenvalues and let P denote the matrix consisting of the Eigenvalues of the Hessian H. The correctness can be proved by calculating $-HP = PD$. Compare Jänich (2002), p. 219.

$$D = \begin{pmatrix} -1+\nu & 0 & \cdots & 0 & 0 \\ 0 & -1+\nu & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & -1+\nu & 0 \\ 0 & 0 & \cdots & 0 & [-1-\nu(n-1)] \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 1 \\ -1 & 1 & \cdots & 0 & 0 & 1 \\ 0 & -1 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & 1 & 1 \\ 0 & 0 & \ldots & 0 & -1 & 1 \end{pmatrix}$$

Utility is concave if the corresponding Hessian is negative definit. This requires negative Eigenvalues. Compare Königsberger (1993), p.74. The expression $-1+\nu < 0$ is equivalent to $\nu < 1$ and $-1-\nu(n-1) < 0$ is equivalent to $\nu > -\frac{1}{n-1}$. Thus, I assume: $\nu \in \left(-\frac{1}{n-1}, 1\right)$. Utility can also be expressed as follows:

$$U(q_0, q_1, q_2, \ldots, q_n) = \sum_i q_i - \frac{1}{2}\sum_i (q_i)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i q_j \tag{2.16}$$

2.5.2 Proof of lemma 1

Competing in quantities, firm $i = 1, \ldots, n$ maximizes its profit $\Pi_i = p(q_i + Q_{-i})q_i - c_i q_i$ choosing an optimal q_i. Inverse demand $p(q_i + Q_{-i}) = 1 - q_i - \nu Q_{-i}$ is given by (3.1). The first order condition of firm $i = 1, \ldots, n$ is given by $1-2q_i-\nu Q_{-i}-c_i = 0$. In matrix form all first order conditions can be expressed as follows:

$$\begin{pmatrix} 2 & \nu & \cdots & \nu \\ \nu & 2 & \cdots & \nu \\ \vdots & \vdots & \ddots & \vdots \\ \nu & \nu & \cdots & 2 \end{pmatrix} \boldsymbol{q} = \begin{pmatrix} 1-c_1 \\ 1-c_2 \\ 1-c_3 \\ \vdots \\ 1-c_n \end{pmatrix}$$

Let A denote the matrix of coefficients. $\boldsymbol{c}^T = (1-c_1, \ldots, 1-c_n)$ is the vector of constants. A is real, symmetric and can be decomposed by $A = PDP^{-1}$. $A\mathbf{q} = \mathbf{c}$ can be expressed by $PDP^{-1}\boldsymbol{q} = \boldsymbol{c}$. Let P denote the matrix of Eigenvectors. The diagonal matrix D contains the corresponding Eigenvalues. It is easy to proof that $\lambda_1 = 2-\nu$ is an $n-1$ fold Eigenvalue of A and $\lambda_2 = 2+\nu(n-1)$ is the n-th Eigenvalue. The diagonal matrix D is given as follows:

$$D = \begin{pmatrix} 2-\nu & 0 & \cdots & 0 & 0 \\ 0 & 2-\nu & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 2-\nu & 0 \\ 0 & 0 & \cdots & 0 & [2+\nu(n-1)] \end{pmatrix} \tag{2.17}$$

The matrix P contains the corresponding Eigenvectors $\boldsymbol{v}_i$ with $i = 1, \ldots, n$. It holds:

$$P = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & 1 \\ -1 & 1 & \cdots & 0 & 0 & 1 \\ 0 & -1 & \cdots & 0 & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & -1 & 1 & 1 \\ 0 & 0 & \ldots & 0 & -1 & 1 \end{pmatrix} \tag{2.18}$$

Prove the accuracy of (2.17) and (2.18) by calculating $AP = PD$. The Cournot-Nash equilibrium q_i^* for $i = 1, \ldots, n$ is determined by solving $PDP^{-1}\boldsymbol{q}^* = \boldsymbol{c}$ in two steps. Firstly, $PD\boldsymbol{z}^* = \boldsymbol{c}$ is solved for $\boldsymbol{z}^* := P^{-1}\boldsymbol{q}$. Then the solution of $\boldsymbol{q}^*$ can be derived by calculating $\boldsymbol{q}^* = P\boldsymbol{z}^*$. The optimal $\boldsymbol{z}^*$ must solve the following system of linear equations $PD\boldsymbol{z}^* = \boldsymbol{c}$:

$$\begin{pmatrix} 2-\nu & 0 & \cdots & 0 & 0 & [2+\nu(n-1)] \\ -(2-\nu) & 2-\nu & \cdots & 0 & 0 & [2+\nu(n-1)] \\ 0 & -(2-\nu) & \cdots & 0 & 0 & [2+\nu(n-1)] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -(2-\nu) & 2-\nu & [2+\nu(n-1)] \\ 0 & 0 & \cdots & 0 & -(2-\nu) & [2+\nu(n-1)] \end{pmatrix} \boldsymbol{z}^* = \begin{pmatrix} 1-c_1 \\ 1-c_2 \\ \vdots \\ 1-c_n \end{pmatrix}$$

Summing up the first and the second row yields the new second row. The new second row is added to the third row which again yields the new third row et cetera. The resulting row echelon form is given as follows:

$$\begin{pmatrix} 2-\nu & 0 & \cdots & 0 & 0 & [2+\nu(n-1)] \\ 0 & 2-\nu & \cdots & 0 & 0 & 2[2+\nu(n-1)] \\ 0 & 0 & \cdots & 0 & 0 & 3[2+\nu(n-1)] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 2-\nu & (n-1)[2+\nu(n-1)] \\ 0 & 0 & \cdots & 0 & 0 & n[2+\nu(n-1)] \end{pmatrix} \boldsymbol{z}^* = \begin{pmatrix} 1-c_1 \\ 2-c_1-c_2 \\ \vdots \\ (n-1)-\sum_{i=1}^{n-1} c_i \\ n-\sum_{i=1}^{n} c_i \end{pmatrix}$$

Solving the last row for z_n^* yields:

$$z_n^* = \frac{n - \sum_{i=1}^n c_i}{n[2 + \nu(n-1)]} \tag{2.19}$$

Inserting z_n^* given by (2.19) in the row before last which is given as follows

$$(2-\nu)z_{n-1}^* + (n-1)[2+\nu(n-1)]z_n^* = (n-1) - \sum_{i=1}^{n-1} c_i$$

yields the solution for z_{n-1}^* which is given as follows:

$$\begin{aligned}
z_{n-1}^* &= \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1)[2+\nu(n-1)]z_n^*\right) \\
&\overset{(2.19)}{=} \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1)[2+\nu(n-1)]\left(\frac{n - \sum_{i=1}^n c_i}{n[2+\nu(n-1)]}\right)\right) \\
&= \frac{1}{2-\nu}\left((n-1) - \sum_{i=1}^{n-1} c_i - (n-1) + \frac{n-1}{n}\sum_{i=1}^{n} c_i\right) \\
&= \frac{1}{2-\nu}\left(\frac{n-1}{n}\sum_{i=1}^{n} c_i - \sum_{i=1}^{n-1} c_i\right)
\end{aligned} \tag{2.20}$$

Equilibrium quantities q_i^* are given by $\boldsymbol{q}^* = P\boldsymbol{z}^*$. The solution for q_n^* is given by $q_n^* = -z_{n-1}^* + z_n^*$ with z_{n-1}^* and z_n^* given by (2.19) and (2.20) respective. It holds:

$$\begin{aligned}
q_n^* &= \frac{-1}{2-\nu}\left(\frac{n-1}{n}\sum_{i=1}^{n} c_i - \sum_{i=1}^{n-1} c_i\right) + \frac{n - \sum_{i=1}^n c_i}{n[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - \frac{2-\nu}{n}\sum_{i=1}^n c_i + [2+(n-1)\nu]\left(\sum_{i=1}^{n-1} c_i - \frac{n-1}{n}\sum_{i=1}^n c_i\right)}{(2-\nu)[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - \frac{2-\nu}{n}\sum_{i=1}^n c_i + 2\sum_{i=1}^{n-1} c_i + (n-1)\nu\sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
&\quad + \frac{-2\frac{n-1}{n}\sum_{i=1}^n c_i - \frac{(n-1)^2}{n}\nu\sum_{j=1}^n c_j}{(2-\nu)[2+(n-1)\nu]}
\end{aligned}$$

Rearranging the terms by collecting the coefficients of c_n and c_i for $i \neq n$ yields:

$$\begin{aligned}
q_n^* &= \frac{(2-\nu) + \left[-\frac{2-\nu}{n} - 2\frac{n-1}{n} - \frac{n-1}{n}(n-1)\nu\right] c_n}{(2-\nu)[2+(n-1)\nu]} \\
&\quad + \frac{\left[-\frac{2-\nu}{n} + 2 + (n-1)\nu - 2\frac{n-1}{n} - \frac{(n-1)^2}{n}\nu\right] \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
&= \frac{(2-\nu) + \left[-2 + \nu\left(\frac{1}{n} - \frac{(n-1)^2}{n}\right)\right] c_n + \left[\nu\left(\frac{1}{n} + (n-1) - \frac{(n-1)^2}{2}\right)\right] \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]} \\
&= \frac{(2-\nu) - [2+\nu(n-2)]c_n + \nu \sum_{i=1}^{n-1} c_i}{(2-\nu)[2+(n-1)\nu]}
\end{aligned}$$

Equilibrium output q_i^* for $i = 1, \ldots, n-1$ can be derived analogously. q_i^* for $n = 1, \ldots, n$ is given as follows:

$$q_i^* = \frac{(2-\nu) - [2+\nu(n-2)]c_i + \nu \sum_{j \neq i} c_i}{(2-\nu)[2+\nu(n-1)]} \quad \square \tag{2.21}$$

2.5.3 Proof of equation 2.3

In the following I show that equilibrium profit Π_i^* is just its squared quantity. Equilibrium price can be obtained by inserting equilibrium quantities given by (2.21) in the inverse demand. It holds: $p_i^* - c_i = 1 - q_i^* - \nu Q_{-i}^* - c_i$. Aggregated concurrence output $Q_{-i}^* = \sum_{j \neq i} q_j^*$ is given as follows:

$$Q_{-i}^* = \frac{(n-1)(2-\nu) - [2+\nu(n-2)] \sum_{j \neq i} c_j + \nu(n-1)c_i + \nu(n-2) \sum_{j \neq i} c_j}{(2-\nu)[2+\nu(n-1)]}$$

It remains to show that $p_i^* - c_i = q_i^*$:

$$\begin{aligned}
p_i^* - c_i =& - q_i^* + \frac{(2-\nu)[2+\nu(n-1)] - \nu(n-1)(2-\nu)}{(2-\nu)[2+\nu(n-1)]} \\
&+ \frac{-\nu^2(n-1) - (2-\nu)[2+\nu(n-1)]}{(2-\nu)[2+\nu(n-1)]} c_i \\
&+ \frac{\nu[2+\nu(n-2)] - \nu^2(n-2)]}{(2-\nu)[2+\nu(n-1)]} \sum_{j \neq i} c_j \\
=& - q_i^* + \frac{2(2-\nu) + [-4 + (2-n)2\nu]c_i + 2\nu \sum_{j \neq i} c_j}{(2-\nu)[2+\nu(n-1)]} \\
=& - q_i^* + 2q_i^* \\
=& q_i^* \quad \square
\end{aligned}$$

2.5.4 Proof of lemma 3

The heterogeneity impact $\mathrm{HIR}^* := \partial_k R^* - \partial_l R^*$ on total revenue $R^* := \sum_i p_i^* q_i^*$ is just the sum of the heterogeneity impacts on each firms revenue.

$$\begin{aligned}
\mathrm{HIR}^* :=& \partial_k R^* - \partial_l R^* \\
=& \partial_k \sum_i R_i^* - \partial_l^* \sum_i R_i^* \\
=& \sum_i \Big(\partial_k R_i^* - \partial_l R_i^* \Big) \\
=& \sum_i \mathrm{HIR}_i^*
\end{aligned}$$

According to (2.7) the heterogeneity impact on the output of the unaffected firms is zero. Since $p_i^* = q_i^* + c_i$ the heterogeneity impact on the equilibrium price of the unaffected firms is zero. The heterogeneity impact on total revenue is given as follows:

$$\mathrm{HIR}^* = \mathrm{HIR}_k^* + \mathrm{HIR}_l^* \tag{2.22}$$

The heterogeneity impact HIR_i^* on equilibrium revenue of firm $i = 1, \ldots, n$ is given as follows:

$$\begin{aligned}
\mathrm{HIR}_i^* &:= \partial_k R_i^* - \partial_l R_i^* \\
&= \partial_k (p_i^* q_i^*) - \partial_l (p_i^* q_i^*) \\
&= \partial_k p_i^* q_i^* + p_i^* \partial_k q_i^* - (\partial_l p_i^* q_i^* + p_i^* \partial_l q_i^*) \\
&= (\partial_k p_i^* - \partial_l p_i^*) q_i^* + (\partial_k q_i^* - \partial_l q_i^*) p_i^* \\
&= \mathrm{HIP}_i^* \, q_i^* + \mathrm{HIQ}_i^* \, p_i^* \qquad (2.23)
\end{aligned}$$

Let HIP_i^* denote the heterogeneity impact on the equilibrium price of firm i. The equilibrium price p_i^* is given by $p_i^* \overset{(2.3)}{=} q_i^* + c_i$. It holds:

$$\begin{aligned}
\mathrm{HIP}_i^* &= \partial_k (q_i^* + c_i) - \partial_l (q_i^* + c_i) \\
&= \mathrm{HIQ}_i^* + \mathrm{HIC}_i \qquad (2.24)
\end{aligned}$$

Let $\mathrm{HIC}_i := \partial_k c_i - \partial_l c_i$ denote the 'heterogeneity impact' on the marginal cost of firm $i = 1, \ldots, n$ with

$$\mathrm{HIC}_i = \begin{cases} 1, & \text{for } i = k, \\ -1, & \text{for } i = l, \\ 0, & \text{else.} \end{cases} \qquad (2.25)$$

The heterogeneity impact on revenue is given as follows:

$$\begin{aligned}
\mathrm{HIR}_i^* &\overset{(2.23)}{=} \mathrm{HIP}_i^* \, q_i^* + \mathrm{HIQ}_i^* \, p_i^* \\
&\overset{(2.24)}{=} (\mathrm{HIQ}_i^* + \mathrm{HIC}_i) q_i^* + \mathrm{HIQ}_i^* \, p_i^* \\
&= \mathrm{HIQ}_i^* (q_i^* + p_i^*) + \mathrm{HIC}_i \, q_i^* \\
&\overset{(2.3)}{=} \mathrm{HIQ}_i^* (2 q_i^* + c_i) + \mathrm{HIC}_i \, q_i^*
\end{aligned}$$

The heterogeneity impact on equilibrium quantity and marginal costs of the unaffected firms $j \neq k, l$ is zero. Moreover, it holds: $\mathrm{HIQ}_k = -\,\mathrm{HIQ}_l = \frac{-1}{2-\nu}$.

The heterogeneity impact on revenue is given as follows:

$$\mathrm{HIR}_i^* = \begin{cases} \frac{-1}{2-\nu}(2q_k^* + c_k) + q_k^*, & \text{for } i = k, \\ \frac{1}{2-\nu}(2q_l^* + c_l) - q_l^*, & \text{for } i = l, \\ 0, & \text{else.} \end{cases}$$

It holds: $2\,\mathrm{HIQ}_k + 1 = \frac{-\nu}{2-\nu}$. The heterogeneity impact on the revenue of firm i is given as follows:

$$\mathrm{HIR}_i^* = \begin{cases} \left(\frac{-\nu}{2-\nu}\right) q_k^* + \frac{-1}{2-\nu} c_k, & \text{for } i = k, \\ \left(\frac{\nu}{2-\nu}\right) q_l^* - \frac{-1}{2-\nu} c_l, & \text{for } i = l, \\ = 0, & \text{else.} \end{cases} \tag{2.26}$$

In case of substitutes (i.e. $\nu \geq 0$) the heterogeneity impact on revenue k is negative and the heterogeneity impact on revenue l is positive. If there are complements this is not true in general. The heterogeneity impact on total revenue is given as follows:

$$\begin{aligned} \mathrm{HIR}^* &\overset{(2.22)}{=} \mathrm{HIR}_k^* + \mathrm{HIR}_l^* \\ &\overset{(2.26)}{=} \left[\left(\frac{-\nu}{2-\nu}\right) q_k^* + \frac{-1}{2-\nu} c_k\right] + \left[\left(\frac{\nu}{2-\nu}\right) q_l^* - \frac{-1}{2-\nu} c_l\right] \\ &= \left(\frac{-\nu}{2-\nu}\right)(q_k^* - q_l^*) + \frac{-1}{2-\nu}(c_k - c_l) \\ &\overset{(2.10)}{=} \frac{-\nu}{2-\nu}\left[\frac{-(c_k - c_l)}{(2-\nu)}\right] + \left(\frac{-1}{2-\nu}\right)(c_k - c_l) \\ &= -2\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l) \begin{cases} < 0, & \text{for } c_k > c_l, \\ = 0, & \text{for } c_k = c_l, \\ > 0, & \text{for } c_k < c_l. \end{cases} \end{aligned} \tag{2.27}$$

Total revenue diminishes (increases) if the more (less) efficient firm is getting more efficient. This result is true in case of substitutes and complements even though the heterogeneity impact on revenue R_k must not be negative in case of complements (cf. (2.26)).

In the following the heterogeneity impact on total costs $C^* := \sum_i c_i q_i^*$is investigated.

$$\begin{aligned}
\partial_k C^* - \partial_l C^* &= \partial_k \sum_i c_i q_i^* - \partial_l \sum_i c_i q_i^* \\
&= \sum_i \Big(\partial_k (c_i q_i^*) - \partial_l (c_i q_i^*) \Big) \\
&= \sum_i \Big(\partial_k c_i q_i^* + c_i \partial_k q_i^* - \partial_l c_i q_i^* - c_i \partial_l q_i^* \Big) \\
&= \sum_i \Big(\mathrm{HIC}_i \, q_i^* + \mathrm{HIQ}_i^* \, c_i \Big) \\
&\overset{(2.25)}{=} q_k^* - q_l^* + \mathrm{HIQ}_k^* \, c_k + \mathrm{HIQ}_l^* \, c_l \\
&\overset{(2.10)}{=} \frac{-2(c_k - c_l)}{(2-\nu)} \begin{cases} > 0, & \text{for } c_k < c_l, \\ = 0, & \text{for } c_k = c_l, \\ < 0, & \text{for } c_k > c_l. \end{cases}
\end{aligned} \tag{2.28}$$

The heterogeneity impact on total costs is negative (positive) if the more (less) efficient firm is getting more efficient. Obviously the heterogeneity impact on total revenue outweighs the heterogeneity impact on total costs for $c_k > c_l$:

$$\begin{aligned}
\partial_k C^* - \partial_l C^* \overset{(2.28)}{=} \frac{-2}{2-\nu}(c_k - c_l) < \frac{-2(1-\nu)}{(2-\nu)^2}(c_k - c_l) \overset{(2.27)}{=} \partial_k R^* - \partial_l R^* \\
\Leftrightarrow 1 > \frac{1-\nu}{2-\nu}
\end{aligned}$$

In case of $c_k > c_l$ the diminishing total costs outweigh the diminishing revenue and vice versa. The heterogeneity impact on producer surplus is positive (negative) if the more (less) efficient firm is getting more efficient. Note that the heterogeneity impact on producer surplus is just the difference between the heterogeneity impact on revenue and total costs. Subtracting (2.28) from (2.27) yields (2.11). □

2.5.5 Proof of proposition 2.3

According to (2.16) the Dixit-utility is given as follows:

$$U(q_1^*, q_2^*, \ldots, q_n^*) = \sum_i q_i^* - \frac{1}{2} \sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^*$$

The average impact $\mathrm{AICS}^* := \sum_{i=1}^{n} \partial_i \, \mathrm{CS}^*$ on consumer surplus in equilibrium is given as follows:

$$\begin{aligned}\mathrm{AICS}^* &= \sum_k \partial_k \left(\sum_i q_i^* - \frac{1}{2} \sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^* - \sum_i p_i^* q_i^* \right) \\ &= \sum_k \Bigg\{ \sum_i \partial_k q_i^* - \sum_i q_i^* \partial_k q_i^* - \nu \left(\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (\partial_k q_i^* q_j^* + q_i^* \partial_k q_j^*) \right) \\ &\qquad - \sum_i (\partial_k p_i^* q_i^* + p_i^* \partial_k q_i^*) \Bigg\}\end{aligned}$$

Market price p_i^* of firm $i = 1, \ldots, n$, is given by $p_i^* = q_i^* + c_i$. The average impact on consumer surplus is given as follows:

$$\begin{aligned}\mathrm{AICS}^* = \sum_k \Bigg\{ &\sum_i \partial_k q_i^* - \sum_i \partial_k q_i^* q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_j^* \partial_k q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* \partial_k q_j^* \\ &- \sum_i q_i^* \partial_k (q_i^* + c_i) - \sum_i (q_i^* + c_i) \partial_k q_i^* \Bigg\}\end{aligned}$$

Rearranging the terms deftly allows to factor out $p_i^* = 1 - q_i^* - \nu Q_{-i}^*$.

$$\begin{aligned}\mathrm{AICS}^* &= \sum_k \left\{ \sum_i \partial_k q_i^* \left(1 - q_i^* - \nu Q_{-i}^* - c_i\right) - 2 \sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \\ &= \sum_k \left\{ \sum_i \partial_k q_i^* \left(p_i^* - c_i\right) - 2 \sum_i q_i^* \partial_k q_i^* - q_k^* \right\}\end{aligned}$$

Remember that $p_i^* - c_i = q_i^*$. It holds:

$$\begin{aligned}\mathrm{AICS}^* &= \sum_k \left\{ \sum_i q_i^* \partial_k q_i^* - 2 \sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \\ &= \sum_k \left\{ - \sum_i q_i^* \partial_k q_i^* - q_k^* \right\} \qquad (2.29) \\ &= - \sum_i q_i^* \sum_k \partial_k q_i^* - Q^*\end{aligned}$$

The average impact $\mathrm{AIQ}_i^* := \sum_k \partial_k q_i^*$ on the equilibrium output of firm $i = 1, \ldots, n$ is given by $\mathrm{AIQ}_i^* = \frac{-1}{2+\nu(n-1)}$.

$$\begin{aligned}
\text{AICS}^* &= -\sum_i q_i^* \, \text{AIQ}_i - Q^* \\
&= -\text{AIQ}_i \, Q^* - Q^* \\
&= -(\text{AIQ}_i + 1) Q^* \\
&= -\frac{1+\nu(n-1)}{2+\nu(n-1)} Q^* \qquad (2.30)
\end{aligned}$$

The term $-\frac{1+\nu(n-1)}{2+\nu(n-1)}$ is non-positive for $\nu \in \left(-\frac{1}{n-1}, 1\right)$. The average impact on consumer surplus is positive (negative) if all firms are positively (negatively) affected by the cost variation. □

2.5.6 Proof of proposition 2.4

In the following the heterogeneity impact on consumer surplus $\text{HICS}^* := \partial_k \, \text{CS}^* - \partial_l \, \text{CS}^*$ is derived. The partial derivatives $\partial_k \, \text{CS}^*$ and $\partial_l \, \text{CS}^*$ are given as follows:

$$\begin{aligned}
\partial_k \, \text{CS}^* &= \partial_k \left(\sum_i q_i^* - \frac{1}{2} \sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^* - \sum_i p_i^* q_i^* \right) \\
&\overset{(2.29)}{=} -\sum_i q_i^* \partial_k q_i^* - q_k^* \\
\partial_l \, \text{CS}^* &\overset{(2.29)}{=} -\sum_i q_i^* \partial_l q_i^* - q_l^*
\end{aligned}$$

The heterogeneity impact on consumer surplus $\text{HICS}^* := \partial_k \, \text{CS} - \partial_l \, \text{CS}$ is given as follows:

$$\begin{aligned}
\text{HICS}^* &= -\sum_i q_i^* \partial_k q_i^* - q_k^* - \left(-\sum_i q_i^* \partial_l q_i^* - q_l^* \right) \\
&= -\sum_i q_i^* \left(\partial_k q_i^* - \partial_l q_i^* \right) - (q_k^* - q_l^*) \\
&= -\sum_i q_i^* \, \text{HIQ}_i - (q_k^* - q_l^*)
\end{aligned}$$

According to (2.7) the heterogeneity impact on the equilibrium output of the unaffected firms is zero.

$$\begin{aligned}
\text{HICS}^* &= -q_k^* \,\text{HIQ}_k - q_l^* \,\text{HIQ}_l - (q_k^* - q_l^*) \\
&= -(q_k^* - q_l^*)\,\text{HIQ}_k - (q_k^* - q_l^*) \\
&= -(\text{HIQ}_k + 1)(q_k^* - q_l^*) \\
&= -\left(\frac{1-\nu}{2-\nu}\right)(q_k^* - q_l^*) \\
&\overset{(2.10)}{=} \frac{1-\nu}{(2-\nu)^2}(c_k - c_l) \qquad (2.31)
\end{aligned}$$

Thus the heterogeneity impact on consumer surplus is positive (negative) if the more inefficient (efficient) firm is getting more efficient. □

2.5.7 Proof of lemma 4

Households' expenditures just equal to firms' total revenue which was analyzed already in appendix 2.5.4. The heterogeneity impact on households expenditures is given by (2.27). It remains to analyze the heterogeneity impact on consumers utility $U(q_0^*, q_1^*, \ldots, q_n^*)$ given by $\partial_k U^* - \partial_l U^*$.

$$\begin{aligned}
\partial_k U^* &= \partial_k \left(\sum_i q_i^* - \frac{1}{2}\sum_i (q_i^*)^2 - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* q_j^* \right) \\
&= \sum_i \partial_k q_i^* - \sum_i \partial_k q_i^* q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_j^* \partial_k q_i^* - \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} q_i^* \partial_k q_j^* \\
&= \sum_i \partial_k q_i^* \left(1 - q_i^* - \nu Q_{-i}^*\right) \\
&\overset{(3.1)}{=} \sum_i \partial_k q_i^* p_i^*
\end{aligned}$$

The heterogeneity impact on consumers utility is given as follows:

$$\begin{aligned}
\partial_k U^* - \partial_l U^* &= \sum_i (\partial_k q_i^* - \partial_l q_i^*)\, p_i^* \\
&= \sum_i \text{HIQ}_i^* \, p_i^*
\end{aligned}$$

According to (2.7) the heterogeneity impact on the output of the unaffected firms is zero.

For $\mathrm{HIQ}_k = -\,\mathrm{HIQ}_l$ it holds:

$$\begin{aligned}
\partial_k U^* - \partial_l U^* &= \mathrm{HIQ}_k\, p_k^* + \mathrm{HIQ}_l\, p_l^* \\
&= \mathrm{HIQ}_k (p_k^* - p_l^*) \\
&\overset{(2.3)}{=} \mathrm{HIQ}_k [q_k^* + c_k - (q_l^* + c_l)] \\
&= \mathrm{HIQ}_k (q_k^* - q_l^*) + \mathrm{HIQ}_k (c_k - c_l) \\
&\overset{(2.10)}{=} \frac{-1}{2-\nu}\left[\frac{-1}{2-\nu}(c_k - c_l)\right] + \frac{-1}{2-\nu}(c_k - c_l) \\
&= -\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l)
\end{aligned} \tag{2.32}$$

The heterogeneity impact on consumer surplus is negative (positive) if the more (less) efficient firm is positively affected by the cost variation. It is easy to check that the heterogeneity impact on consumer expenditures outweighs the heterogeneity impact on consumer utility.

$$\partial_k U^* - \partial_l U^* \overset{(2.32)}{=} -\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l) \overset{c_k > c_l}{>} -2\frac{(1-\nu)}{(2-\nu)^2}(c_k - c_l) \overset{(2.27)}{=} \partial_k R^* - \partial_l R^*$$

The heterogeneity impact on consumers expenditures outweighs the heterogeneity impact on consumers utility. The heterogeneity impact on consumer surplus is positive (negative) if the more (less) efficient firm is getting more efficient. □

Chapter 3

The Herfindahl-Hirschman Index in a Differentiated Good Oligoply

3.1 Introduction

Market concentration measured by the Herfindahl-Hirschman Index is vitally important for antitrust authorities and competition policy to assess the anticompetitive harm of horizontal mergers. According to US Merger Guidelines anticompetitive harm seems to be an increasing function of market concentration. "A merger is unlikely to create or enhance market power or to facilitate its exercise unless it significantly increases concentration and results in a concentrated market, properly defined and measured. Merger that either do not significantly increase concentration or do not result in a an concentrated market ordinarily require no further analysis".[1] Furthermore, antitrust authorities presume a negative relationship between market concentration and the likelihood of distorted competition. "Other things being equal, market concentration affects the likelihood that one firm, or a small group of firms, could successfully exercise market power".[2]

Scientific analysis, however, show that there is not necessarily a negative relationship between market concentration and social surplus. In the homogeneous good case there is a positive relationship between market concentration and social surplus even though consumer surplus solely depends on average marginal costs since producer

[1]Cf. US Merger Guidelines, section 1.0.

[2]Cf. US Merger Guidelines, section 2.0.

surplus increases with the dispersion of marginal costs.[3] In the context of differentiated goods chapter 2 shows that consumer surplus as well as producer surplus and, therefore, social surplus increases with the dispersion of marginal costs.

In the homogeneous good case there is a monotonic relationship between market concentration and the dispersion of marginal costs. Market shares are output shares since equilibrium prices cancels. Aggregated output solely depends on average marginal costs. Marginal costs and respective equilibrium output are in reverse proportion. Therefore, there is a monotonic relationship between marginal costs and corresponding market share.[4]

In the differentiated good context, however, market shares are not just output shares since equilibrium prices differ in general. Equilibrium output and marginal costs are in reverse proportion whereas equilibrium prices increases with respective marginal costs. In contrast to the homogeneous good case aggregated revenue decreases with the dispersion of marginal costs.[5] Therefore, all market shares change in case of a mean preserving conjectural cost variation of a subgroup of firms. Therefore, a monotonic relationship between market concentration measured by the Herfindahl-Hirschman Index and the distribution of marginal costs is not self-evident.

Assuming standard quadratic utility according to Dixit (1979) I consider a differentiated good oligopoly consisting of n firms competing in quantities. According to Kreps and Scheinkman (1983) the assumption of quantity competition is reasonable in capacity constrained industries. Furthermore, in reality most of the goods are rather imperfect substitutes than perfect substitutes. All firms are assumed to exhibit constant marginal costs. Firms differ in their production technology and, therefore, have different marginal costs. Since entry and/or exit is not object of investigations I abstract from fixed cost.

This chapter analyzes the relationship between the distribution of marginal costs and the market concentration measured by the Herfindahl-Hirschman Index. The impact of an arbitrary marginal cost variation on market concentration is decomposed into an average impact and a heterogeneity impact. The average component affects

[3]Cf. Février and Linnemer (2004) for further insight.

[4]Cf. Février and Linnemer (2004), for instance.

[5]Cf. chapter 2.

all marginal cost in equal manner whereas the heterogeneity component lets the average of marginal costs constant.

This study shows that market concentration increases (decreases) if all firms are negatively (positively) affected by the cost variation since the less efficient firms are disadvantaged (favored) above average. Even though there are many effects acting in opposite direction there is no evidence that there is a non-monotonic relationship between market concentration measured by the Herfindahl-Hirschman Index and the distribution of marginal costs .

This chapter is organized as follows: section 2 describes the framework of the model. Section 3 presents the central results and section 4 finally concludes.

3.2 The model

Let us consider an asymmetric differentiated good oligopoly consisting of $n \geq 2$ firms competing in quantities. Each firm produces one differentiated good Q_i with $i = 1, \ldots, n$. Each firm incurs constant marginal cost c_i without fixed cost. Let q_i denote the quantity produced by firm $i = 1, \ldots, n$. The quasi-linear preferences of the representative household are described by a quadratic utility according to Dixit (1979) given by $U(q_0, q_1, \ldots, q_n) = q_0 + \sum_{i=1}^{n} q_i - \frac{1}{2}\sum_i q_i^2 - \frac{\nu}{2}\sum_i q_i Q_{-i}$. Let q_i denote the quantity of firm $i = 1, \ldots, n$ whereas $Q_{-i} := \sum_{j \neq i} q_j$ denotes the aggregated output of i´s competitors. Finally, let q_0 denote the consumption of the numeraire-good. The degree of substitution is given by $\nu \in [0; 1]$. Firm $i = 1, \ldots, n$ faces the following inverse demand:

$$p_i = 1 - q_i - \nu Q_{-i} \tag{3.1}$$

Each firm is assumed to maximize its profit choosing an optimal output. Let Q^* denote aggregated output in equilibrium. Summing up the n first order conditions given by $1 - 2q_i^* - \nu Q_{-i}^* - c_i = 0$ and solving for Q^* yields:

$$Q^* = \frac{n(1 - \bar{c})}{2 + \nu(n-1)} \tag{3.2}$$

Let $\overline{c} := \frac{1}{n}\sum_{i=1}^{n} c_i$ denote average marginal costs which is assumed not to exceed 1. Analogously to the homogenous good case industry output Q^* solely depends on average marginal costs and not on its distribution. Thus, Q^* is unaffected by a mean preserving cost variation. The Cournot-Nash equilibrium output of firm $i = 1, \ldots, n$ is given as follows:[6]

$$q_i^* = \frac{(2-\nu) - (2+\nu)c_i + \nu \sum_{j \neq i} c_j}{2(2-\nu)(2+\nu)} \tag{3.3}$$

Equilibrium output and respective marginal costs are in reverse proportion and q_i^* increases with the sum of i's competitors marginal costs. It is easy to verify that the equilibrium price p_i^* is just given by $p_i^* = q_i^* + c_i$. Therefore, comparably to the homogenous good case equilibrium profit $\Pi_i^* := (p_i^* - c_i)q_i^*$ equals its squared quantity. Since entry or exit is not object of my investigations I assume $p_i^* - c_i = q_i^* > 0$ for $i = 1, \ldots, n$.

Assumption 3.2.1 (Oligopoly of n firms)**.** *To ensure non-negative equilibrium output of firm $i = 1, \ldots, n$ I assume $q_i^* \geq 0$ which corresponds to the following inequality:*

$$c_i \leq \frac{2-\nu}{2+\nu(n-2)} + \frac{\nu}{2+\nu(n-2)} \sum_{j \neq i} c_j$$

Let $s_i := \frac{R_i}{\sum_j R_j}$ denote the market share of firm $i = 1, \ldots, n$ while $R_i = p_i q_i$ denotes i's revenue. Let HHI denote market concentration measured by the Herfindahl-Hirschman Index which is defined by $\text{HHI} := \sum_i s_i^2$. The object of my investigations is HHI evaluated in the Cournot-Nash equilibrium:

$$\text{HHI} = \sum_i \left(\frac{R_i^*}{\sum_j R_j^*} \right)^2 \tag{3.4}$$

Since equilibrium prices p_i^* differ in general the market shares are not simple output shares as in the homogeneous good case. Therefore, the impact of a mean preserving cost variation on market concentration is not unambiguous. Suppose an increase in c_k while c_l decreases to the same extent. According to (3.3) the equilibrium output of the unaffected firms $i \neq k, l$ is unchanged as well as corresponding

[6] Cf. appendix 2.5.2 for a detailed derivation.

equilibrium price. Furthermore, chapter 2 shows that even though equilibrium revenue of the unaffected firms is unchanged respective market share changes since aggregated revenue not only depends on average marginal costs but also on its distribution. Therefrom, an analytical investigation of the relationship between market concentration and the dispersion of marginal costs is necessary.

3.3 Results

In the following the impact of an arbitrary cost variation on market concentration measured by the Herfindahl-Hirschman Index HHI is analyzed. Since market concentration is calculated in the equilibrium HHI^* can be expressed as a function of marginal costs. The procedural method equals Février and Linnemer (2004) or chapter 2. An arbitrary conjectural cost variation is decomposed into an average component and a heterogeneity component. Analytically, the average impact and the heterogeneity impact is given by directional derivatives of HHI^* in the equilibrium. The average component positively or negatively affects the marginal costs of all firms to the same degree whereas the variance of marginal costs is constant. The heterogeneity component comprises the mutual alteration of the marginal costs of two firms so that average marginal costs are constant.

Definition 10 (Decomposition of an arbitrary cost variation)**.** *Let* AIH *denote the average impact and* HIH *the heterogeneity impact on market concentration* HHI*. The total derivative of* HHI^* *is given by* $d\,\text{HHI}^* = \sum_{k=1}^{n} \frac{\partial\,\text{HHI}^*}{\partial c_k} dc_k$*. The average impact is characterized by* $dc_1 = \ldots = dc_n = dc$*. Without loss of generality the heterogeneity impact is given by a conjectural variation of* c_k *and* c_l *with* $k < l$ *and* $dc_k = -dc_l > 0$*. Thus,* AIH *and* HIH *are given as follows:*

$$\text{AIH} := \sum_{i=1}^{n} \frac{\partial\,\text{HHI}^*}{\partial c_i} \qquad \text{HIH} := \frac{\partial\,\text{HHI}^*}{\partial c_k} - \frac{\partial\,\text{HHI}^*}{\partial c_l}$$

Note that the 'directions' $dc_1 = \ldots = dc_n$ and $dc_k = -dc_l$ just equal the Eigenvectors of the matrix of coefficients characterizing the Cournot-Nash equilibrium. Cf. the appendix 2.5.2 for further insight.

In the following, the average impact on market concentration is analyzed. According to chapter 2 all equilibrium quantities decrease if all firms are negatively affected by the cost variation. On the one hand the distribution of the different brands gets more uneven since the market share of the already least efficient firm is getting even smaller and converges to zero. Hence, market concentration increases. On the other hand market concentration decreases since the distribution of all firms except for the least efficient gets more even. The average impact on market concentration is unambiguous at first sight.

Proposition 3.1 (Average Impact)**.** *Market concentration increases if all firms are negatively affected by the cost variation and vice versa. The average impact on market concentration is zero if all firms are equally efficient.*

Proof: See appendix 3.5.1.

If all firms are negatively affected by the cost variation the market becomes less favorable for all firms. The spread between the maximum willingness to pay and marginal costs gets smaller for all firms. Intuitively, the more efficient firm has bigger sales compared to the less efficient one. Furthermore, the average impact on the revenue is commensurate with respective marginal costs. Hence, the sales collapse caused by an increase of all marginal costs disadvantages the more inefficient firms above average. In the consequence the disparity of revenue increases (decreases) if all firms are negatively (positively) affected by the cost variation. Thus, market concentration measured by the Herfindahl-Hirschman Index increases (decreases) in consequence of an increase (reduction) of all marginal costs.

In contrast to the homogeneous good case the heterogeneity impact on market concentration is not unambiguous. Even though the equilibrium output and, therefore, the revenue of unaffected firms is unchanged, respective market shares vary since aggregated revenue changes.[7] A mean preserving cost variation of a subset of firms which increases the dispersion of marginal costs, for instance, increases the market shares of all unaffected firms since aggregated revenue decreases. A mean preserving cost variation of only two firms causes a multiplicity of effects on market concentration acting in opposite direction.

Proposition 3.2 (Heterogeneity Impact)**.** *(i) In case of a duopoly market concentration increases with the disparity of marginal costs and vice versa. (ii) In case of*

[7] Cf. appendix 2.5.4.

$n \geq 3$ firms it can be easily shown that there is a monotonous relationship between the dispersion of marginal costs and market concentration if the smallest market share of the affected firms is not bigger compared to aggregated market shares of all unaffected competitors. However, there is no evidence that this relationship does not hold true in this case. No numerical counterexample can be found documenting a non-monotonous relationship between the dispersion of marginal costs and market concentration.

Proof: See appendix 3.5.2.

Let us first consider the case of a simple duopoly. Without loss of generality let us assume $c_1 \leq c_2$ and $dc_1 = -dc_2 > 0$. Prior the cost variation the market share of firm 1 exceeds the market share of firm 2 since $R_2^* \geq R_1^*$. The market share of firm 2 decreases since respective equilibrium revenue decreases whereas aggregated revenue increases. The denominator as well as the numerator of the market share of firm 1 increases. The effect on the numerator, however, dominates since equilibrium revenue of firm 1 increases more than aggregated revenue does. Therefore, the market share of firm 1 increases in consequence of this mean preserving cost variation. Note that in case of $c_1 \geq c_2$ all effects are the other way around and the interpretation is analogously. Therefore, market concentration measured by the Herfindahl-Hirschman Index decreases with the dispersion of marginal costs in case of 2 firms. For better understanding the effects on each component is illustrated by an arrow either upward or downward directed in the following expression:

$$\text{HHI}^* \downarrow = \left(\frac{R_1^* \downarrow\downarrow}{(R_1^* + R_2^*) \uparrow} \right)^2 + \left(\frac{R_2^* \uparrow\uparrow\uparrow}{(R_1^* + R_2^*) \uparrow} \right)^2$$

Let us now expand our duopoly above by introducing a third firm. Three cases can occur: firstly, the third firm is most efficient, secondly, the third firm is less efficient than 1 but more efficient than 2 and, finally, the third firm is least efficient. In the first case firm 3 has the biggest market share since $c_3 < c_1 < c_2$. Even though equilibrium revenue of firm 3 is unaffected, respective market share decreases since aggregated revenue increases. The heterogeneity impact on the market shares of firm 1 and 2 is analogously to the duopoly above. As you can see in the appendix respective proof is not so unambiguous as expected since there are two effects acting in opposite direction. On the one hand the market shares of 1 and 2 converge and, therefore, market concentration decreases. On the other hand market concentration increases since the market share of the biggest affected firm (i.e. firm 2) decreases. Note that the Herfindahl-Hirschman Index is the sum over the squared market shares and, naturally, the square increases (decreases) disproportionately high (small) compared to respective market share.

Let us now turn to case 2 in which the unaffect firm is more efficient than firm 2 but less efficient than 1. The biggest market share 1 decreases most, the second largest market share 3 decreases little and the smallest market share 2 increases. The mean preserving cost variation shifts all market shares towards uniform distribution and, therefore, market concentration decreases. As you can see in the appendix, respective proof is very easy since all components act in the same direction.

Finally, let us consider the third case characterized by $c_1 < c_2 < c_3$. The impact on the affected firms 1 and 2 is similiar to all aforementioned cases. The market share of firm 1 decreases and those of firm 2 increases. Firm 3 has the smallest market share which decreases in consequence of the mean preserving cost variation since aggregated revenue equals to the denominator of respective market share increases.

Analyzing the heterogeneity impact on market concentration it can be shown that for $c_3 = c_2$ as well as for $c_3 = \frac{2-\nu}{2+\nu(n-2)} + \frac{\nu}{2+\nu(n-2)} \sum_{j \neq i} c_j$ the heterogeneity impact on market concentration is negative for $c_1 < c_2$ and $dc_1 = -dc_2 > 0$. Remember that $q_3^* = 0$ for $c_3 = \frac{2-\nu}{2+\nu(n-2)} + \frac{\nu}{2+\nu(n-2)} \sum_{j \neq i} c_j$ according to assumption 3.2.1. For $c_3 \in \left(c_2, \frac{2-\nu}{2+\nu(n-2)} + \frac{\nu}{2+\nu(n-2)} \sum_{j \neq i} c_j\right)$ there is no evidence in terms of a numerical counterexample that there is no monotonous relationship between the dispersion of marginal costs and market concentration measured by HHI.

The generalization on n firms is fairly easy since it is irrelevant whether the revenue of the unaffected firm accrues from 1, 2 or n firms. The impact on the market shares of the unaffected firms is comparable to those of firm 3 in the case of $n = 3$.

Even though there are many effects acting in opposite direction there is no evidence that there is no monotonous relationship between market concentration measured by the Herfindahl-Hirschman Index and the dispersion of marginal costs. Respective proof is straightforward apart from one exception. If the (sum of) market share(s) of the unaffected firm(s) is less compared to the smallest market shares of the affected firms there is a lack in the proof. But, in this case no numerical counterexample can be found.

3.4 Conclusions

This chapter analyzes the relationship between market heterogeneity and market concentration measured by the Herfindahl-Hirschman Index in a differentiated good context. Market concentration increases with the dispersion of marginal costs if average marginal costs are constant. The Herfindahl-Hirschman Index increases if the marginal costs of all firms are increased simultaneously and vice versa.

The results are comparable to the homogeneous good case. Making all firms less efficient makes the market less favorable for all firms. In this case less efficient firms are disadvantaged above average and, therefore, market concentration increases. In case of a conjectural cost variation which makes all firms more efficient the reverse holds true. The Herfindahl-Hirschman Index is a valid measure for market heterogeneity in the differentiated good context, too. Thus, it is possible to draw conclusions from market concentration measured on the underlying cost structure.

In conjunction with the second chapter this study is a valuable contribution for a better understanding of the relationship between market concentration and social surplus in the differentiated good context. Social surplus as well as market concentration increases with the dispersion of marginal costs. In contrast to the homogeneous good case not only producer surplus but also consumer surplus increases with market concentration provided that average marginal costs are constant.

For antitrust authorities such as the Federal Trade Commission or the Directorate-General for Competition, DG-Comp, market concentration measured by the Herfindahl-Hirschman Index plays a prominent role in the course of the appraisal of horizontal mergers, for instance. The direct measurement of market power is difficult due to a lack of data. Typically, information about the price-elasticity of demand are required. Big firms are presumed to have more market power compared to small firms. Thus, big market shares and a highly concentrated market, respectively, are indicators for market power.

The insights of chapter 2 and chapter 3 shed new light on the importance of market concentration for antitrust authorities. In the course of the appraisal of horizontal mergers a highly concentrated post-merger equilibrium raises significant anticompetitive concern. According to present merger guidelines distorted competition is presumed to be the consequence of the abuse of market power. Thus, antitrust authorities intend to prevent the market from horizontal mergers resulting in highly concentrated markets. But, present merger regulation are based on false assumptions with regard to the presumed negative relationship between market concentration and efficiency of the market outcome. The analysis of welfare implications of present antitrust enforcement policy is dedicated to the next chapter.

3.5 Appendix

3.5.1 Proof of proposition 3.1

The Herfindahl-Hirschman Index can be expressed by $\mathrm{HHI}^* = \frac{\sum_i (R_i^*)^2}{\left(\sum_j R_j^*\right)^2}$. Hence the average impact $\mathrm{AIH} := \sum_k \partial_k \,\mathrm{HHI}$ on market concentration is given as follows:

$$
\begin{aligned}
\mathrm{AIH}^* &= \sum_k \partial_k \frac{\sum_i (R_i^*)^2}{\left(\sum_j R_j^*\right)^2} \\
&= \sum_k \frac{\partial_k\left(\sum_i (R_i^*)^2\right)\left(\sum_j R_j^*\right)^2 - \left(\sum_i (R_i^*)^2\right)\partial_k\left(\sum_j R_j^*\right)^2}{\left(\sum_j R_j^*\right)^4}
\end{aligned}
\tag{3.5}
$$

The denominator of (3.5) is positive. The sign of AIH^* is determined by respective numerator which is given as follows:

$$
\begin{aligned}
&\sum_k \left(\sum_i 2R_i^* \partial_k R_i^*\right)\left(\sum_j R_j^*\right)^2 - \sum_k \left(\sum_i (R_i^*)^2\right) 2\left(\sum_j R_j^*\right)\sum_j \partial_k R_j^* \\
=&2\sum_j R_j^* \left[\sum_k \left(\sum_i R_i^* \partial_k R_i^*\right)\left(\sum_j R_j^*\right) - \sum_k \left(\sum_i (R_i^*)^2\right)\sum_j \partial_k R_j^*\right]
\end{aligned}
$$

Since $2\sum_j R_j^*$ is positive, the sign of AIH^* is determined by the sign of the term in squared brackets which can be rearranged as follows:

$$
\left(\sum_i R_i^* \sum_k \partial_k R_i^*\right)\left(\sum_j R_j^*\right) - \left(\sum_j \sum_k \partial_k R_j^*\right)\left(\sum_i (R_i^*)^2\right) \tag{3.6}
$$

Let $\mathrm{AIR}_i^* := \sum_k \partial_k R_i^*$ denote the average impact on the revenue of firm $i = 1, \ldots, n$. Therefore, (3.6) can be expressed as follows:

$$
\left(\sum_i R_i^* \,\mathrm{AIR}_i^*\right)\left(\sum_j R_j^*\right) - \left(\sum_j \mathrm{AIR}_j^*\right)\left(\sum_i (R_i^*)^2\right) \tag{3.7}
$$

The first summand of (3.7) is given as follows:

$$\begin{aligned}\left(\sum_i R_i^* \operatorname{AIR}_i^*\right)\left(\sum_j R_j^*\right) &= (R_1^*)^2 \operatorname{AIR}_1^* + R_1^* R_2^* \operatorname{AIR}_2^* + \ldots + R_1^* R_n^* \operatorname{AIR}_n^* \\ &+ R_1^* R_2^* \operatorname{AIR}_1^* + (R_2^*)^2 \operatorname{AIR}_2^* + \ldots + R_2^* R_n^* \operatorname{AIR}_n^* \\ &\vdots \\ &+ R_1^* R_n^* \operatorname{AIR}_1^* + R_2^* R_n^* \operatorname{AIR}_2^* + \ldots + (R_n^*)^2 \operatorname{AIR}_n^*\end{aligned}$$

The second summand of (3.7) can be expressed as follows:

$$\begin{aligned}\left(\sum_j \operatorname{AIR}_j^*\right)\left(\sum_i (R_i^*)^2\right) &= (R_1^*)^2 \operatorname{AIR}_1^* + \ldots + (R_1^*)^2 \operatorname{AIR}_n^* \\ &+ (R_2^*)^2 \operatorname{AIR}_1^* + \ldots + (R_2^*)^2 \operatorname{AIR}_n^* \\ &\vdots \\ &+ (R_n^*)^2 \operatorname{AIR}_1^* + \ldots + (R_n^*)^2 \operatorname{AIR}_n^*\end{aligned}$$

Calculating (3.7) dissolves the squared terms $(R_i^*)^2 \operatorname{AIR}_i^*$ for $i = 1, \ldots, n$. Rearranging the remaining terms deftly (3.7) is given by:

$$\begin{aligned}(3.7) =& R_1^* R_2^* \operatorname{AIR}_2^* + R_1^* R_2^* \operatorname{AIR}_1^* - (R_1^*)^2 \operatorname{AIR}_2^* - (R_2^*)^2 \operatorname{AIR}_1^* \\ &+ R_1^* R_3^* \operatorname{AIR}_3^* + R_1^* R_3^* \operatorname{AIR}_1^* - (R_1^*)^2 \operatorname{AIR}_3^* - (R_3^*)^2 \operatorname{AIR}_1^* \\ &\vdots \\ &+ R_1^* R_n^* \operatorname{AIR}_n^* + R_1^* R_n^* \operatorname{AIR}_1^* - (R_1^*)^2 \operatorname{AIR}_n^* - (R_n^*)^2 \operatorname{AIR}_1^* \\ &+ R_2^* R_3^* \operatorname{AIR}_3^* + R_2^* R_3^* \operatorname{AIR}_2^* - (R_3^*)^2 \operatorname{AIR}_1^* - (R_2^*)^2 \operatorname{AIR}_3^* \\ &+ R_2^* R_4^* \operatorname{AIR}_4^* + R_2^* R_4^* \operatorname{AIR}_2^* - (R_2^*)^2 \operatorname{AIR}_4^* - (R_4^*)^2 \operatorname{AIR}_2^* \\ &\vdots \\ &+ R_2^* R_n^* \operatorname{AIR}_n^* + R_2^* R_n^* \operatorname{AIR}_2^* - (R_2^*)^2 \operatorname{AIR}_n^* - (R_n^*)^2 \operatorname{AIR}_2^* \\ &\vdots \\ &+ R_{n-1}^* R_n^* \operatorname{AIR}_n^* + R_{n-1}^* R_1^* \operatorname{AIR}_{n-1}^* - (R_n^*)^2 \operatorname{AIR}_{n-1}^* - (R_{n-1}^*)^2 \operatorname{AIR}_n^*\end{aligned}$$

These terms can be rearranged as follows:

$$
\begin{aligned}
(3.7) = & R_1^* \Big(R_2^* \,\mathrm{AIR}_1^* - R_1^* \,\mathrm{AIR}_2^* \Big) + R_2^* \Big(R_1^* \,\mathrm{AIR}_2^* - R_2^* \,\mathrm{AIR}_1^* \Big) \\
& + R_1^* \Big(R_3^* \,\mathrm{AIR}_1^* - R_1^* \,\mathrm{AIR}_3^* \Big) + R_3^* \Big(R_1^* \,\mathrm{AIR}_3^* - R_3^* \,\mathrm{AIR}_1^* \Big) \\
& \vdots \\
& + R_1^* \Big(R_n^* \,\mathrm{AIR}_1^* - R_1^* \,\mathrm{AIR}_n^* \Big) + R_n^* \Big(R_1^* \,\mathrm{AIR}_n^* - R_n^* \,\mathrm{AIR}_1^* \Big) \\
& + R_2^* \Big(R_3^* \,\mathrm{AIR}_2^* - R_2^* \,\mathrm{AIR}_3^* \Big) + R_3^* \Big(R_2^* \,\mathrm{AIR}_3^* - R_3^* \,\mathrm{AIR}_2^* \Big) \\
& + R_2^* \Big(R_4^* \,\mathrm{AIR}_2^* - R_2^* \,\mathrm{AIR}_4^* \Big) + R_4^* \Big(R_2^* \,\mathrm{AIR}_4^* - R_4^* \,\mathrm{AIR}_2^* \Big) \\
& \vdots \\
& + R_2^* \Big(R_n^* \,\mathrm{AIR}_2^* - R_2^* \,\mathrm{AIR}_n^* \Big) + R_n^* \Big(R_2^* \,\mathrm{AIR}_n^* - R_n^* \,\mathrm{AIR}_2^* \Big) \\
& \vdots \\
& + R_{n-1}^* \Big(R_n^* \,\mathrm{AIR}_{n-1}^* - R_{n-1}^* \,\mathrm{AIR}_n^* \Big) + R_n^* \Big(R_{n-1}^* \,\mathrm{AIR}_n^* - R_n^* \,\mathrm{AIR}_{n-1}^* \Big)
\end{aligned}
$$

Rearranging the above given terms deftly yields:

$$
\begin{aligned}
(3.7) = & \Big(R_1^* - R_2^* \Big) \Big(R_2^* \,\mathrm{AIR}_1^* - R_1^* \,\mathrm{AIR}_2^* \Big) \\
& + \Big(R_1^* - R_3^* \Big) \Big(R_3^* \,\mathrm{AIR}_1^* - R_1^* \,\mathrm{AIR}_3^* \Big) \\
& \vdots \\
& + \Big(R_1^* - R_n^* \Big) \Big(R_n^* \,\mathrm{AIR}_1^* - R_1^* \,\mathrm{AIR}_n^* \Big) \\
& + \Big(R_2^* - R_3^* \Big) \Big(R_3^* \,\mathrm{AIR}_2^* - R_2^* \,\mathrm{AIR}_3^* \Big) \\
& + \Big(R_2^* - R_4^* \Big) \Big(R_4^* \,\mathrm{AIR}_2^* - R_2^* \,\mathrm{AIR}_4^* \Big) \\
& \vdots \\
& + \Big(R_2^* - R_n^* \Big) \Big(R_n^* \,\mathrm{AIR}_2^* - R_2^* \,\mathrm{AIR}_n^* \Big) \\
& \vdots \\
& + \Big(R_{n-1}^* - R_n^* \Big) \Big(R_n^* \,\mathrm{AIR}_{n-1}^* - R_{n-1}^* \,\mathrm{AIR}_n^* \Big)
\end{aligned}
$$

The term (3.7) is given by:

$$
(3.7) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \Big(R_i^* - R_j^* \Big) \Big(R_j^* \,\mathrm{AIR}_i^* - R_i^* \,\mathrm{AIR}_j^* \Big) \tag{3.8}
$$

Without loss of generality I assume $c_1 \geq c_2 \geq \ldots \geq c_n$. According to appendix 2.5.4 the heterogeneity impact on revenue is negative (positive) if the firm is negatively (positively) affected by the cost variation. The more efficient firm has bigger sales. Thus, $R_1^* \leq R_2^* \leq \ldots \leq R_n^*$ for $c_1 \geq c_2 \geq \ldots \geq c_n$. In the following I show that the more inefficient firm is harmed by the cost variation above average (i.e. $|\,\mathrm{AIR}_k^*\,| > |\,\mathrm{AIR}_l^*\,|$ for $c_k > c_l$).

Lemma 11 (Average Impact on revenue)**.** *The absolute value of the average impact on revenue is proportional to the marginal costs.*

Proof: The average impact $\mathrm{AIR}_k^* := \sum_i \partial_i R_k^*$ on the revenue of firm $k = 1, \ldots, n$ is given as follows:

$$\begin{aligned}
\mathrm{AIR}_k^* &= \sum_i \partial_i (p_k^* q_k^*) \\
&= \sum_i \Big(\partial_i p_k^* q_k^* + p_k^* \partial_i q_k^* \Big) \\
&= \left(\sum_i \partial_i p_k^* \right) q_k^* + p_k^* \left(\sum_i \partial_i q_k^* \right) \\
&= \mathrm{AIP}_k^*\, q_k^* + p_k^*\, \mathrm{AIQ}_k^*
\end{aligned}$$

Since $p_i^* = q_i^* + c_i$ in equilibrium the average impact on the market price AIP_k^* for $k = 1, \ldots, n$ is given as follows:

$$\begin{aligned}
\mathrm{AIP}_k^* &= \sum_i \partial_i (q_k^* + c_k) \\
&= \mathrm{AIQ}_k^* + \mathrm{AIC}_k
\end{aligned}$$

with $\mathrm{AIC}_k = 1$. The average impact on quantity of firm k is given as follows:

$$\begin{aligned}
\mathrm{AIQ}_k^* &= \sum_i \partial_i q_k^* \\
&= \frac{-[2 + \nu(n-2)] + \nu(n-1)}{(2-\nu)[2 + \nu(n-1)]} \\
&= \frac{-1}{2 + \nu(n-1)} < 0
\end{aligned}$$

Note that the average impact on the equilibrium price is positive (negative) if all firms are positively (negatively) affected by the cost variation.

$$\begin{aligned}\mathrm{AIP}_k^* &= \frac{-1+[2+\nu(n-1)]}{2+\nu(n-1)} \\ &= \frac{1+\nu(n-1)}{2+\nu(n-1)} > 0\end{aligned}$$

The average impact on the revenue of firm $k = 1, \ldots, n$ is given as follows:

$$\begin{aligned}\mathrm{AIR}_k^* &= \frac{1+\nu(n-1)}{2+\nu(n-1)} q_k^* + (q_k^* + c_k)\,\mathrm{AIQ}_k^* \\ &= \left[\frac{1+\nu(n-1)}{2+\nu(n-1)} + \frac{-1}{2+\nu(n-1)}\right] q_k^* - \frac{-1}{2+\nu(n-1)} c_k \\ &= \frac{\nu(n-1)}{2+\nu(n-1)} q_k^* - \frac{1}{2+\nu(n-1)} c_k\end{aligned}$$

As you can see below the more efficient firm has bigger sales:

$$\begin{aligned}\mathrm{AIR}_k^* - \mathrm{AIR}_l^* &= \frac{\nu(n-1)}{2+\nu(n-1)}(q_k^* - q_l^*) - \frac{1}{2+\nu(n-1)}(c_k - c_l) \\ &= \frac{\nu(n-1)}{2+\nu(n-1)}\left(\frac{-1}{2-\nu}\right)(c_k - c_l) - \frac{1}{2+\nu(n-1)}(c_k - c_l) \\ &= \frac{-[2+\nu(n-2)]}{(2-\nu)[2+\nu(n-1)]}(c_k - c_l) \begin{cases} < 0, & \text{for } c_k > c_l, \\ = 0, & \text{for } c_k = c_l, \\ > 0, & \text{for } c_k < c_l. \end{cases}\end{aligned}$$

Revenue of firm k diminishes more than revenue of firm l (i.e. $\mathrm{AIR}_k^* \leq \mathrm{AIR}_l^* < 0$ for $c_k \geq c_l$. It holds: $|\,\mathrm{AIR}_1\,| \geq \ldots \geq |\,\mathrm{AIR}_n\,|$ for $c_1 \geq \ldots \geq c_n$. □

The sign of the average impact on market concentration given by (3.8) is positive (negative) if all firms are negatively (positively) affected by the cost variation. Since $R_1^* = \ldots = R_n^*$ and $|\,\mathrm{AIR}_1^*\,| = \ldots = |\,\mathrm{AIR}_n^*\,|$ for $c_1 = \ldots = c_n$ the average impact on market concentration is zero in case of equally efficient firms. □

3.5.2 Proof of proposition 3.2

Omitting the sum over k we can employ (3.6) to compute the heterogeneity impact on market concentration $\mathrm{HIHHI}^* := \partial_k \mathrm{HHI}^* - \partial_l \mathrm{HHI}^*$. The sign of HIHHI^* is determined by the following expression:

$$\sum_i R_i^* \partial_k R_i^* \sum_j R_j^* - \sum_i (R_i^*)^2 \sum_j \partial_k R_j^* \qquad (3.9)$$

$$- \left(\sum_i R_i^* \partial_l R_i^* \sum_j R_j^* - \sum_i (R_i^*)^2 \sum_j \partial_l R_j^* \right)$$

$$= \sum_j R_j^* \sum_i R_i^* \left(\partial_k R_i^* - \partial_l R_i^*\right) - \sum_i (R_i^*)^2 \left(\sum_j \left(\partial_k R_j^* - \partial_l R_j^*\right) \right)$$

$$= \sum_j R_j^* \sum_i R_i^* \operatorname{HIR}_i^* - \sum_i (R_i^*)^2 \sum_j \operatorname{HIR}_j^* \qquad (3.10)$$

According to (2.26) the heterogeneity impact on the revenue of the unaffected firms is zero. The term in (3.10) is given as follows:

$$(3.10) = \sum_j R_j^* \Big(R_k^* \operatorname{HIR}_k^* + R_l^* \operatorname{HIR}_l^* \Big) - \sum_i (R_i^*)^2 \Big(\operatorname{HIR}_k^* + \operatorname{HIR}_l^* \Big) \qquad (3.11)$$

The second summand of (3.11) represents the impact of the variation of aggregated revenue on each market share. In case of a mean preserving cost variation which decreases the dispersion of marginal costs aggregated revenue increases according to (2.27). Since aggregated revenue affects the market shares by varying the denominator this impact decreases (increases) all market shares in case of a mean preserving cost variation which decreases (increases) the dispersion of marginal costs.

The first summand of (3.11) represents the impact on the market shares of the affected firms. Since the heterogeneity impact on the revenue of the unaffected firms is zero, this impact only affects the market shares of the affected firms k and l. But, the sign of $R_k^* \operatorname{HIR}_k^* + R_l^* \operatorname{HIR}_l^*$ is ambiguous since $c_k < c_l \Leftrightarrow R_k > R_l$ but $|\operatorname{HIR}_k^*| < |\operatorname{HIR}_l^*|$ and vice versa.

Firstly, let us consider the case of 2 firms. As you can see in the following the sign of the heterogeneity impact on market concentration given by (3.10) is unambiguous. For $n = 2$, $k = 1$ and $l = 2$ the sign of the heterogeneity impact on market concentration is determined by the sign of the following expression:

$$\begin{aligned}
(3.11) \overset{(n=2)}{=} & \Big(R_1^* + R_2^*\Big)\Big(R_1^* \,\mathrm{HIR}_1^* + R_2^* \,\mathrm{HIR}_2^*\Big) \\
& - \Big((R_1^*)^2 + (R_2^*)^2\Big)\Big(\mathrm{HIR}_1^* + \mathrm{HIR}_2^*\Big) \\
& = R_1^*\Big(R_2^* \,\mathrm{HIR}_1^* - R_1^* \,\mathrm{HIR}_2^*\Big) + R_2^*\Big(R_1^* \,\mathrm{HIR}_2^* - R_2^* \,\mathrm{HIR}_1^*\Big) \\
& = \Big(R_1^* - R_2^*\Big)\Big(R_2^* \,\mathrm{HIR}_1^* - R_1^* \,\mathrm{HIR}_2^*\Big) \qquad (3.12)
\end{aligned}$$

Note that $R_1^* > R_2^*$ and $|\,\mathrm{HIR}_1^*\,| < |\,\mathrm{HIR}_2^*\,|$ for $c_1 < c_2$ and vice versa. Since $\mathrm{HIR}_1^* < 0$ and $\mathrm{HIR}_2^* > 0$ the heterogeneity impact on market concentration given by (3.12) is negative for $c_1 < c_2$. In case of $c_1 > c_2$ the reverse holds true. If there is a duopoly there is a monotonic relationship between market concentration and the dispersion of marginal costs.

Let us now turn to the case of 3 firms. In this case the heterogeneity impact on market concentration is given as follows:

$$(3.11) \overset{(n=3)}{=} (3.12) + R_3^*\Big[\Big(R_1^* - R_3^*\Big)\,\mathrm{HIR}_1^* + \Big(R_2^* - R_3^*\Big)\,\mathrm{HIR}_2^*\Big] \qquad (3.13)$$

Three cases can occur: Firstly, $c_3 < c_1 < c_2$, secondly, $c_1 < c_3 < c_2$ and, finally, $c_1 < c_2 < c_3$.

In the first case R_3 exceeds R_1. Since HIR_1^* is negative the sign of $(R_1^* - R_3^*)\,\mathrm{HIR}_1^*$ is positive. Since R_3^* exceeds R_2^* the sign of $(R_2^* - R_3^*)\,\mathrm{HIR}_2^*$ is negative since HIR_2^* is positive. Since $|\,\mathrm{HIR}_1^*\,| < |\,\mathrm{HIR}_2^*\,|$ and $|R_1^* - R_3^*| < |R_2^* - R_3^*|$, however, the term $(R_2^* - R_3^*)\,\mathrm{HIR}_2^*$ outweighs $(R_1^* - R_3^*)\,\mathrm{HIR}_1^*$. Remember that $R_3^* > R_1^* > R_2^* \Leftrightarrow c_3 < c_1 < c_2$. The sum in brackets is negative and, therefore, the heterogeneity impact on revenue in case of 3 firms is negative for $c_1 < c_2$ and vice versa.

In the second case R_1^* exceeds R_3^*. The first summand in the brackets of (3.13) is negative. Since R_3^* exceeds R_2^* and $\mathrm{HIR}_2^* > 0$ the second summand in the brackets of (3.13) is negative, too. The heterogeneity impact on market concentration is also negative for $c_1 < c_3 < c_2$.

Finally, let us analyze the last case in which the third (unaffected) firm is least efficient. The lower bound for c_3 is given by c_2 per assumption 4.2.1. The upper bound for c_3 is given by $\frac{2-\nu}{2+\nu} + \frac{\nu}{2+\nu} \sum_{j \neq 3} c_j$ per assumption 4.2.2. Unfortunately, the sign of the heterogeneity impact on market concentration cannot by derived immediately from (3.13) for $c_1 < c_2 < c_3$ since the sign of $(R_1^* - R_3^*)\,\mathrm{HIR}_1$ is negative but the term $(R_2^* - R_3^*)\,\mathrm{HIR}_2$, however, is positive. Furthermore, the difference $(R_1^* - R_3^*)$ exceeds $(R_2^* - R_3^*)$ since $R_1^* > R_2^* > R_3^*$ whereas $|\,\mathrm{HIR}_1\,| < |\,\mathrm{HIR}_2\,|$.

Firstly, suppose that c_3 equals its lower bound equal to c_2. In this case (i.e. for $c_3 = c_2 \geq c_1$) it holds: $R_1 \geq R_2 = R_3$.

$$(3.11) \stackrel{(c_3=c_2)}{=} (3.12) + R_3^* \left[\left(R_1^* - R_3^* \right) \mathrm{HIR}_1^* \right] \tag{3.14}$$

Since the term in brackets is unambiguously negative the sign of heterogeneity impact on market concentration unambiguous, too. Now let increase c_3 starting from c_2 until the upper bound for c_3 equal to $\frac{2-\nu}{2+\nu} + \frac{\nu}{2+\nu} \sum_{j \neq 3} c_j$ is reached. Equilibrium revenue R_3^* decreases with c_3 and reaches zero if c_3 equals its upper bound. Remember that the upper bound for c_3 is deduced from $q_3^* = 0$. For $c_3 = \frac{2-\nu}{2+\nu} + \frac{\nu}{2+\nu} \sum_{j \neq i} c_j$ it holds: (3.11) = (3.12) since $R_3^* = 0$. It remains to analyze the relationship between $c_3 \in \left[c_2; \frac{2-\nu}{2+\nu} + \frac{\nu}{2+\nu} \sum_{j \neq 3} c_j \right]$ and the term in squared brackets in (3.13) equal to $\left(R_1^* - R_3^* \right) \mathrm{HIR}_1^* + \left(R_2^* - R_3^* \right) \mathrm{HIR}_2^*$. Since $R_3^* \geq 0$ for all considered values for c_3 the proof would end here if the term in squared brackets has no roots for $c_3 \in \left[c_2; \frac{2-\nu}{2+\nu} + \frac{\nu}{2+\nu} \sum_{j \neq 3} c_j \right]$. Unfortunately, it can be easily verified that there exists one root in respective interval. Just check this by evaluating $\left(R_1^* - R_3^* \right) \mathrm{HIR}_1^* + \left(R_2^* - R_3^* \right) \mathrm{HIR}_2^*$ for $\nu = \frac{1}{2}$, $c_1 = 0$, $c_2 = \frac{1}{1}$ and $c_3 = \frac{1}{830} \left(-146 + 3\sqrt{17041} \right)$.

It can easily be verified, however, that the heterogeneity impact on market concentration is not positive. Evaluating (3.13) at the aforementioned numerical example yields negative values. The term (3.12) equals -0.000664665 approximately and the second summand $R_3^* \left[\left(R_1^* - R_3^* \right) \mathrm{HIR}_1^* + \left(R_2^* - R_3^* \right) \mathrm{HIR}_2^* \right]$ equals 0.0000769787 approximately. The sum (3.13) is given by -0.000587687 approximately.

Chapter 4

Welfare Implications of present Antitrust Enforcement Policy - A Theoretical Analysis in Context of a Differentiated Good Oligopoly.

4.1 Introduction

Antitrust authorities, such as the Federal Trade Commission (FTC) or the Directorate-General for Competition (DG-Comp), are concerned about horizontal mergers creating or strengthening dominant positions. Distorted competition is presumed to be consequence of the abuse of market power arising from dominant positions. According to US merger guidelines "mergers should not be permitted to create or enhance market power or to facilitate its exercise".[1] The possible negative consequences of distorted competition can be divided into unilateral effects and coordinated effects. The latter effects comprise facilitated tacit collusion, for instance. Some negative (unilateral) effects of market power are referred in section 2.2 of US Merger Guidelines: "A merger may diminish competition even if it does not lead to increased likelihood of successful coordinated interaction, because

[1] Cf. US Merger Guidelines, section 0.1.

merging firms may find it profitable to alter their behavior unilaterally following the acquisition by elevating price and suppressing output". In context of differentiated goods the Merger Guidelines appends:[2] "A merger between firms in a market for differentiated products may diminish competition by enabling the merged firm to profit by unilaterally raising the price of one or both products above the premerger level". My analysis does not include possible coordinated effects but focus solely on unilateral effects of horizontal mergers. The insiders[3] internalize the external effects on joint profit and, therefore, deviate from the pre-merger equilibrium, in general. In case of substitutes and quantity competition, for instance, an output reduction of the insiders is consequence of the horizontal merger. The outsiders increase their output as a consequence. The effect of this output reshuffling on social surplus is multifaceted and depends on marginal costs of each firm active in the market.

Market concentration measured by the Herfindahl-Hirschman Index plays a prominent role for antitrust authorities in the course of the assessment of the anticompetitive harm of a requested horizontal merger. Present US as well as European merger guidelines presume a negative relationship between market concentration and efficiency of the market outcome, in general. A merger which increases market concentration significantly or results in a highly concentrated market raises anticompetitive concern since the merger is suspected to increase the likelihood of distorted competition. For this purpose, antitrust authorities estimate the expected increase of market concentration as twice the product of the corresponding market shares. According to present merger guidelines the likelihood of an objection increases ceteris paribus with aggregated pre-merger market share.

Scientific analysis shows that there is not necessarily a negative relationship between market concentration and efficiency of the market outcome. In case of homogeneous goods Février and Linnemer (2004) show that producer surplus increases with the dispersion of marginal costs if average marginal costs are constant. Consumer surplus solely depends on average marginal costs. Thus, social surplus increases with market concentration provided that average marginal costs

[2] Cf. US Merger Guidelines, section 2.21.

[3] The firms involved in the merger are referred to as insider whereas the remaining firms are referred to as outsider.

are constant. In context of differentiated goods the previous chapters show that not only producer surplus but also consumer surplus increases with the dispersion of marginal costs.

This study mainly answers two questions: Firstly, is there actually a positive relationship between post-merger joint profit and aggregated market share of the merger candidates? Secondly, is there a negative relationship between aggregated market share in the pre-merger situation and social surplus in the post-merger situation?

The first question is interesting since a highly concentrated market is presumed to facilitate the creation of a dominant position which is harmful for consumers but profitable and, therefore, desirable for the insiders. Section 0.1 of the US Merger Guidelines state: "Mergers are motivated by the prospect of financial gains" whereas "the Guidelines focus on the one potential source of gain that is of concern under the antitrust laws: market power". If this presumption is true, big mergers should yield a higher post-merger joint profit than small mergers. Intuitively, the size of a merger is determined by aggregated market share of the merger candidates in the pre-merger situation. Moreover, this part of my analysis gives information about the preferred merging partner since post-merger joint profit is crucial for this choice.

The second question matters since a horizontal merger comprising smaller firms can be the consequence or the alternative of an objected big merger. According to present Merger Guidelines the likelihood of an approval decreases with aggregated market share of merger candidates. Apart from that an objected big merger is not essential since it is rational for merger candidates to anticipate the importance of aggregated market share for the appraisal. Merger candidates have an incentive to request merger which is likely to be approved to save time and money. Hence, merger candidates may request a smaller merger from the outset. According to Lyons (2008) there is empirical evidence supporting this conjecture.

According to present merger guidelines, antitrust authorities compare social surplus in the situation after the requested merger and the social surplus in the situation without any horizontal merger. However, this approach ignores the above mentioned signalling effect completely. In my analysis I compare post-merger market outcomes resulting from two horizontal mergers differing in their size. The profitability of horizontal mergers in the classical sense[4] is not subject of my investigations.[5]

Assuming standard quadratic utility according to Dixit (1979) I consider a differentiated good oligopoly consisting of 3 firms competing in quantities. The assumption of differentiated goods is reasonable since in reality most of the goods are rather imperfect substitutes than perfect substitutes. According to Kreps and Scheinkman (1983) the assumption of quantity competition is suggestive in capacity constrained industries. All firms are assumed to exhibit constant but different marginal costs without fixed cost. Without loss of generality firm 1 is assumed to have lowest marginal costs. It is referred to as market leader since firm 1 has the biggest pre-merger equilibrium output. Firm 2 is the more efficient competitor whereas firm 3 is least efficient. For simplicity firm 1 is assumed to merge either with firm 2 or with firm 3. The former is referred to as big merger whereas the latter is referred to as small merger.

The model of Kao and Menezes (2007) is closely related to my setup. They consider an asymmetric duopoly with differentiated goods according to Singh and Vives (1984) or Zanchettin (2006) and derive conditions for cost asymmetry providing that horizontal mergers are welfare enhancing. In contrast to my analysis they compare social surplus in the pre-merger equilibrium with social surplus in the post-merger situation. But, their model setup is very special since it is a merger to monopoly. Furthermore, they do not allow for the importance of present antitrust enforcement policy. In my study I compare the post-merger equilibrium of two different types of horizontal mergers with respect to social surplus and joint profit of the insiders depending on the size of merger.

[4]Usually, a horizontal merger is said to be profitable if post-merger joint profit exceeds the sum of the pre-merger profits of the merger candidates.

[5]Many authors (e.g. Deneckere and Davidson (1985), Perry and Porter (1985), Gaudet and Salant (1991), Cheung (1992), Fauli-Oller (1997), Kleer (2006), Heywood and McGinty (2007) and Mialon (2008)) study how to resolve the "merger paradox" which reveals in the findings of Salant et al. (1983), for instance.

Since I assume no synergies, all marginal costs are unaffected by the merger. The insiders maximize joint profit since the former independent firms are operating as plants under corporate governance.[6] In the post-merger equilibrium the insiders and the outsider compete as Cournot players. The insiders reduce their output levels compared to the pre-merger situation in case of substitutes since they internalize the external effects on joint profit. The shutdown of the less efficient part of the merged entity can be the consequence even though all firms produce non-negative output in the pre-merger situation. Intuitively, a shutdown occurs predominantly if there is a merger with a fairly small merging partner with little output already in the pre-merger situation.

As you can see in the following section present antitrust enforcement policy is welfare enhancing in the majority of cases since it fosters a more uneven distribution of aggregated output on the different firms. In case of a small merger the disparity of output levels increases since the already small insiders even decrease their output levels. The increased disparity of output levels is welfare enhancing. This result corresponds with those of chapter 2.

Under some circumstances antitrust policy provokes the shutdown of the less efficient part of the merged entity. There is a shutdown in case of a small merger whereas there is no shutdown in case of a big merger. Paradoxically, the positive welfare effects of this antitrust intervention can be traced back to the provoked shutdown.[7] Least efficient firm 3 as insider is closed down. A brand with an inferior production technology is removed from the market whereas output of the remaining brands increases. These results coincide with those of Kao and Menezes (2007) since efficiency gains are caused by the potential shutdown the less efficient part of the merged entity.

Although my assumptions (no fixed cost and constant marginal costs) differ sig-

[6] The role of divisionalization on profitability (in the classical sense) is studied by Kamien and Zang (1990, 1991), Prechel et al. (1999), Huck et al. (2001) and Creane and Davidson (2004), for instance. The main result is that divisionalization enhances the profitability of horizontal mergers and supports to resolve the merger paradox.

[7] Remember that I do not compare the pre-merger equilibrium and the post-merger equilibrium but two post-merger equilibria with two different merging partners.

nificantly from those of Koh (2008),[8] the central results are the same: The removal of a brand can be welfare enhancing. But, in my study the welfare gains are rather due to a more efficient distribution of production on the different brands than the saving of fixed costs. In quantity setting models the best response is typically downward sloping. Internalizing the (negative) external effect of each brand on joint profit, the insiders reduce their post-merger output. In case of a merger between the market leader and the more efficient competitor, the two most efficient firms reduce their output which is intuitively detrimental for social surplus. The shutdown of the least efficient firm as merging partner is welfare enhancing since production of the remaining brands is enlarged.

This result is in line with the findings of several authors analyzing the negative welfare effects of marginal costs reductions. Lahiri and Ono (1988), Zhao (2001) as well as Smythe and Zhao (2006) analyze welfare effects of a marginal costs reduction in homogeneous good oligopolies. They come to the result that a cost reduction reduces welfare if respective marginal costs are sufficiently high. Wang and Zhao (2007) extend the analysis to the differentiated good case and derive comparable results. The converse argument is that the removal of a fairly inefficient firm is welfare enhancing.

This chapter is organized as follows: the next section describes the framework of the model. In section 3 I analyze the relationship between the size of the merger and corresponding joint profit of the insiders. I investigate the impact of aggregated market share of the merger candidates and social surplus in the post-merger equilibrium in section 4. Finally, section 5 concludes.

4.2 The model

The following section provides the model and the post-merger equilibria. Moreover, some secondary results concerning the profitability of the outsider are presented.

Let us consider an asymmetric differentiated good oligopoly consisting of 3 firms

[8] Assuming fixed cost and zero marginal costs, Koh (2008) shows that there are too many brands offered in a differentiated good oligopoly with free entry. Therefore, the removal of a brand is welfare enhancing.

competing in quantities. The cost structure of firm $i = 1, 2, 3$ is given by $C_i(q_i) = c_i q_i$. In the pre-merger situation firm i produces one variety of a differentiated good with quantity q_i for $i = 1, 2, 3$. Firms are assumed to differ in marginal costs.

Assumption 4.2.1 (Efficiency of the firms)**.** *Without loss of generality I assume:* $c_1 \leq c_2 \leq c_3$.

Since I assume no synergies the post-merger cost structure is equal to the pre-merger cost structure. Consumers preferences are characterized by a standard quadratic utility originated by Dixit (1979) which is given by $U(q_0, q_1, \ldots, q_n) = q_0 + \sum_{i=1}^{n} q_i - \frac{1}{2}\sum_i q_i^2 - \frac{\nu}{2}\sum_i q_i Q_{-i}$. Let q_i denote the quantity of firm $i = 1, \ldots, n$ whereas $Q_{-i} := \sum_{j \neq i} q_j$ denotes the aggregated output of i´s competitors. Finally, let q_0 denote the consumption of the numeraire-good. The degree of substitutability is given by ν. To ensure concavity of utility I assume $\nu \in \left(-\frac{1}{n-1}, 1\right)$.[9] Corresponding inverse demand of good i is given by $p_i = 1 - q_i - \nu Q_{-i}$. In case of $\nu > 0$ the goods are substitutes, for $\nu = 0$ there are n independent goods and for $\nu < 0$ the goods are complements. In the pre-merger situation each firm is assumed to maximize its profit given by $\Pi_i := (p_i - c_i)q_i$ choosing an optimal output q_i.[10]

Lemma 12 (Pre-merger equilibrium)**.** *The Cournot-Nash equilibrium in the pre-merger situation is given as follows for $i = 1, 2, 3$:*

$$q_i^* = \frac{(2-\nu) - [2 + \nu(n-2)]c_i + \nu \sum_{j \neq i} c_j}{(2-\nu)[2 + \nu(n-1)]}$$

To ensure a pre-merger oligopoly consisting of $n = 3$ firms, the following assumption is made:

Assumption 4.2.2 (Pre-merger oligopoly)**.** *In the pre-merger equilibrium each of the three firms is assumed to produce a non-negative quantity. Therefore, I assume $q_i^* \geq 0$ for $i = 1, 2, 3$ which corresponds to the following inequality:*

$$c_i \leq \frac{2-\nu}{2+\nu} + \frac{\nu}{2+\nu}\sum_{j \neq i} c_j$$

According to (12) equilibrium output q_i^* is inversely proportional to respective marginal cost. Under assumption 4.2.1 firm 1 has the biggest output and, therefore,

[9] Compare appendix 2.5.1 for further insight.

[10] Compare appendix 2.5.2 for a detailed derivation.

is referred to as market leader. According to assumption 4.2.2 a firm has a non-negative equilibrium output if its marginal costs does not exceed the sum of competitors' marginal costs weighted by the degree of substitution ν. In case of independent goods (i.e. $\nu = 0$), for instance, marginal costs of each firm must not exceed 1 equal to the maximum willingness to pay. In this case competitors' marginal costs have no impact. The slope of the threshold value on the right hand side of the condition increases with the degree of substitution whereas the intersection of the y-axis decreases with ν. Therefore, the upper limit for c_i (given by the right hand side of assumption 4.2.2) decreases with the degree of substitution ceteris paribus.

After the horizontal merger both plants of the merged entity are governed by a joint management. In case of a merger between firm 1 and $j = 2, 3$ the insiders maximize joint profit given by $\Pi_1 + \Pi_j$ with respect to q_1 and q_j. Let q_i^{1j} denote the post-merger equilibrium quantity of firm $i = 1, 2, 3$ in case of a merger between firm 1 and $j = 2, 3$. The post-merger equilibrium is given as follows:

Lemma 13 (Post-merger equilibrium)**.** *If all post-merger equilibrium quantities are positive, the post-merger equilibrium in case of a merger between firm 1 and 2 is given as follows:*[11]

$$
\begin{aligned}
q_1^{12} &= \frac{1}{4}\left(\frac{c_2 - c_1}{1-\nu} + \frac{2[2 - c_1 - c_2 - \nu(1 - c_3)]}{2 + \nu(2-\nu)}\right)\\
q_2^{12} &= \frac{1}{4}\left(\frac{c_1 - c_2}{1-\nu} + \frac{2[2 - c_1 - c_2 - \nu(1 - c_3)]}{2 + \nu(2-\nu)}\right)\\
q_3^{12} &= \frac{2[1 - (1+\nu)c_3] + \nu(c_1 + c_2)}{2[2 + \nu(2-\nu)]}
\end{aligned}
$$

In case of a shutdown of the less efficient part of the merged entity the post-merger equilibrium is characterized by a simple duopoly. In this case the post-merger equilibrium is characterized by lemma 12 for $n = 2$.

Proof: See appendix 4.6.1.

[11]The post-merger equilibrium in case of a merger between 1 and 3 can be easily derived by interchanging the indices of firm 2 and 3.

Intuitively, the post-merger equilibrium output of brand 1 exceeds the post-merger output of brand 2 since $c_1 \leq c_2$ per assumption. Note that not every marginal cost combination satisfying assumption 4.2.2 yields non-negative post-merger equilibrium quantities given by lemma 13. There is a shutdown of the less efficient part of the merged entity if c_2 exceeds c_1 by far such that the second summand of q_2^{12} does not outweigh the negative first summand. In this case the post-merger equilibrium is characterized by a simple duopoly. The shutdown of the less efficient part of the merged entity is caused by an output reduction of the insiders internalizing the external effect of each brand on joint profit. Intuitively, the shutdown of firm 2 as insider implies the shutdown of firm 3 as insider. The reverse implication, however, is not true. I will come to this in detail later. The outsider, however, always profits by the merger.

Proposition 4.1 (Profitability - outsider). *A horizontal merger is always beneficial to the outsider irrespective the degree of substitutability or the distribution of marginal costs.*

Proof: Firstly, let us consider a merger between firm 1 and 2. It is easy to verify that the pre-merger equilibrium profit Π_3^* and the post-merger equilibrium profit Π_3^{12} just equals its squared quantity given by lemma 12 and lemma 13, respectively. The merger is beneficial to firm 3 if $q_3^{12} \geq q_3^*$. It holds:

$$q_3^{12} - q_3^* = \frac{\nu^2(2 - \nu - c_1 - c_2 + \nu c_3)}{2(2-\nu)(1+\nu)[2+(2-\nu)\nu]}$$

The denominator is unambiguously positive for $\nu \in \left(-\frac{1}{2}, 1\right)$. Hence, the merger is beneficial to the outsider if the term in brackets in the numerator is positive. The correctness of $q_3^{12} \geq q_3^*$ is proved by contradicting a necessary condition for $q_3^{12} < q_3^* \Leftrightarrow (2 - \nu - c_1 - c_2 + \nu c_3) < 0$ which is equivalent to $2 - \nu - c_1 - c_2 < -\nu c_3 \overset{\nu > 0}{\Leftrightarrow} -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_2) > c_3 \Rightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_2) > c_2 \Leftrightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}c_1 > \left(\frac{\nu-1}{\nu}\right) c_2$. This, in turn, is equivalent to $-(2-\nu) + c_1 > -(1-\nu)c_2 \Leftrightarrow \frac{2-\nu}{1-\nu} - \frac{1}{1-\nu}c_1 < c_2 \Rightarrow \frac{2-\nu}{1-\nu} - \frac{1}{1-\nu}c_1 < 1 \Leftrightarrow c_1 > 1$ which contradicts with the basic assumption $c_1 \leq 1$. In case of complements (i.e. for $\nu < 0$) it holds: $2 - \nu - c_1 - c_2 < -\nu c_3 \Leftrightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_2) < c_3 \Rightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_2) < 1$. This term is equivalent to $-\frac{2}{\nu} + \frac{1}{\nu}c_2 < -\frac{c_1}{\nu} \Leftrightarrow 2 - c_2 < c_1$ for $\nu < 0$. The last inequality requires $c_2 > 1$ which cannot be fulfilled since $c_2 < 1$ per assumption.

In the following I show that firm 2 as outsider benefits, too. In case of a merger between firm 1 and 3 the merger is profitable to firm 2 as outsider if $2 - \nu - c_1 - c_3 + \nu c_2 > 0$.[12] The accuracy of $2 - \nu - c_1 - c_3 + \nu c_2 > 0$ is shown by contradicting the contrary inequality. In case of substitutes it holds: $2 - \nu - c_1 - c_2 < -\nu c_2 \Leftrightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_3) > c_2 \Rightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_3) > c_1 \Leftrightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}c_3 > \left(\frac{\nu-1}{\nu}\right) c_1 \Leftrightarrow -2(2-\nu) + c_3 > -(1-\nu)c_1 \Leftrightarrow \frac{2-\nu}{1-\nu} - \frac{c_3}{1-\nu} < c_1 \Rightarrow \frac{2-\nu}{1-\nu} - \frac{c_3}{1-\nu} < c_3 \Leftrightarrow \frac{2-\nu}{1-\nu} < \frac{2-\nu}{1-\nu}c_3 \Leftrightarrow 1 < c_3$ which contradicts with the basic assumption $c_3 \leq 1$. In case of complements the following holds true: $2 - \nu - c_1 - c_3 < -\nu c_2 \Leftrightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_3) < c_2 \Rightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}(c_1 + c_3) < c_3 \Leftrightarrow -\frac{2-\nu}{\nu} + \frac{1}{\nu}c_1 < \frac{\nu-1}{\nu}c_3 \Leftrightarrow -(2-\nu) + c_1 > -(1-\nu)c_3 \Leftrightarrow \frac{2-\nu}{1-\nu} - \frac{c_1}{1-\nu} < c_3 \Rightarrow \frac{2-\nu}{1-\nu} - \frac{c_1}{1-\nu} < 1 \Leftrightarrow \frac{1}{1-\nu} < \frac{c_1}{1-\nu} \Leftrightarrow 1 < c_1$ which again is a contradiction since $c_1 \leq 1$ per assumption. Therefore, $q_i^{1j} \geq q_i^* \Leftrightarrow \Pi_i^{1j} \geq \Pi_i^*$ for $i, j = 2, 3$ and $j \neq i$ in case of substitutes and complements except for independent goods and any distribution of marginal costs. Intuitively, in case of independent goods (i.e. $\nu = 0$) the profit of the outsider is unchanged. □

The insiders internalize the external effect $\frac{d\Pi_i}{dq_1} = -\nu q_i$ and $\frac{d\Pi_1}{dq_i} = -\nu q_1$ of each brand q_1 and q_i for $i = 2, 3$, respectively, on joint profit $\Pi_1 + \Pi_i$. In case of substitutes they reduce the post-merger output of their brands compared to the pre-merger levels since the external effect is negative. The willingness to pay for the outsider's good decreases with aggregated output of his competitors. Thus, the outsider benefits by the output reduction of the insider. In case of complements, the insiders increase their output compared to their pre-merger level since the external effect is positive for $\nu > 0$. In case of complements the willingness to pay increases with aggregated output of the competitors. Thus, the outsider benefits by the merger, too. Solely in case of independent goods outsider's profit is unchanged by merger activities.

[12] This inequality can be easily derived by interchanging the indices of firm 3 and 2.

4.3 Size of merger and post-merger joint profit

In the following section I analyze the relationship between the size of a horizontal merger and corresponding post-merger joint profit. The size of the merger is determined by aggregated market share of the merger candidates. The central question is whether post-merger joint-profit actually increases with the size of the merger as presumed by antitrust authorities. A big market share and a highly concentrated market, respectively, are indirect indicators for market power whose exercise is presumed to be profitable for firms but harmful for consumers. The impact of the size of a merger on efficiency of the market outcome is object of investigations of the next section.

Merger guidelines presume highly concentrated markets and big market shares of the merger candidates, respectively, to be (indirect) indicators for dominant positions. The abuse of dominant positions is presumed to be profitable for respective firms but harmful at least for consumers. However, the previous chapters have shown that there is a positive relationship between market concentration and social surplus. Moreover, not only producers but also consumers are better off in more heterogenous market structures. Therefore, the following section analyzes the relationship between the size of a horizontal merger and corresponding post-merger joint profit of the insiders. The size of the merger is determined by aggregated market share of concerned merger candidates. For this purpose, I compare post-merger joint profit in case of a merger between firm 1 and 2 with post-merger joint profit in case of a merger between firm 1 and 3. According to assumption 4.2.1 firm 1 has the biggest output and, therefore, is referred to as market leader. A merger between firm 1 and firm 2 is referred to as big merger while a merger between firm 1 and firm 3 is referred to as small merger.

A comparison of both post-merger joint profits yields clear-cut results: There is a positive relationship between the size of a merger and post-merger joint profit. A coalition of the biggest firms yields a higher post-merger joint profit compared to a smaller merger.

Proposition 4.2 (Post-merger joint profit)**.** *(i) In case of substitutes (i.e. for $\nu \geq 0$) post-merger joint profit in case of a horizontal merger between the market leader and* *the more efficient competitor exceeds post-merger joint profit in case of a merger with the less efficient competitor irrespective the distribution of marginal costs. (ii) In some cases there is a shutdown of the less efficient part of the merged entity. In this case there are only two brands left after the merger.*

Proof: See appendix 4.6.2.

The intuition behind this result is the following: If there is no shutdown a horizontal merger between the market leader and the more efficient competitor yields a higher joint profit since the merger constitutes a coalition of the most efficient firms in the market. Already in the pre-merger situation the most efficient firms realize biggest pre-merger profits. Remember that there is a monotonous relationship between output and profit in equilibrium. The pre-merger equilibrium profit just equals squared output which again is in reverse proportion to respective marginal costs. The price-cost margin equals output in equilibrium and increases with efficiency in production. If there is a shutdown of the less efficient part of the merged entity, the market leader realizes a higher profit in case of a merger with firm 2 since the remaining firm is least efficient and, therefore, an inferior competitor.

4.4 Size of merger and post-merger social surplus

As expected by antitrust authorities there is actually a positive relationship between post-merger joint profit and the size of the horizontal merger. The output of a firm is positively affected by respective marginal costs. The price-cost margin in turn increases with equilibrium output. Therefore, big firms realize high profits. In the following section I analyze the relationship between the size of the merger and efficiency of the market outcome. Merger Guidelines presume that this relationship is negative. Chapter 2, however, has shown that there is a positive relationship between market concentration and social surplus in general. Moreover, consumer surplus as well as producer surplus increases with market concentration in case of differentiated goods. Thus, why should be no longer a positive relationship between market concentration and efficiency in context of horizontal merger?

This question matters since a small horizontal merger can be consequence of a rejected big merger. According to present European and US Merger Guidelines the likelihood of an objection increases with aggregated market share of the merger candidates. Moreover, present merger guidelines have a signalling effect: Firms anticipate the importance of market concentration for antitrust authorities. It is rational for firms to request a merger which is likely to be approved to save time and money. In the course of the appraisal antitrust authorities compare the situation without any merger with the situation after the requested horizontal merger. This approach disregards that a small horizontal merger can be consequence of an impeded big merger and the fact that firms anticipate the importance of market shares for merger regulation. Therefore, I analyze the relationship between the size of a horizontal merger and corresponding efficiency of the market outcome. To confirm the object of investigations of this section the following definition is made:

Definition 14 (Welfare implications of antitrust intervention)**.** *Within this welfare analysis, social surplus in case of a big merger (i.e. a merger between the market leader and the more efficient competitor) is compared with social surplus in case of a small merger (i.e. a merger between the market leader and the less efficient firm). A small merger instead of a big merger is assumed to be consequence of present antitrust enforcement policy.*

In my analysis social surplus serves as measure for Pareto-optimality since preferences are quasi-linear. As usual, let m denote household´s income which is assumed to be exogenous. The consumption of the numeraire-good is given by $q_0 = m - \sum_{i=1}^{3} c_i q_i$. Social surplus is defined as follows:

$$W := U\left(m - \sum_{i=1}^{3} c_i q_i, q_1, q_2, q_3\right) - U(m, 0, 0, 0)$$

With regard to post-merger market structure it can be shown that 3 different cases can occur: Firstly, all post-merger quantities are positive in any kind of merger. Secondly, there is a shutdown of the less efficient part of the merged entity in both kinds of merger. Thirdly, there is a shutdown of the less efficient part of the merged entity in case of a merger between 1 and 3 while all brands are offered after the merger between 1 and 2. Thus, present antitrust enforcement policy has no impact on the diversity of brands in case 1 and 2 while in case 3 the variety of products is reduced. In the following each case is analyzed separately.

Let W^{1j} for $j = 2, 3$ denote social surplus in case of a merger between 1 and j. It holds: $W^{1j} := W(q_1^{1j}, q_2^{1j}, q_3^{1j})$ whereas the post-merger equilibrium quantities are given by lemma 13. The object of investigation is the welfare impact of present antitrust enforcement policy. For this purpose I investigate the sign of $W^{12} - W^{13}$. Remember that a merger between the market leader and the more efficient competitor is referred to as big merger whereas the merger between the market leader and the less efficient competitor is referred to as small merger. Therefore, the analysis of the sign of $W^{12} - W^{13}$ gives information about the relationship between the size of the merger and efficiency of corresponding market outcome.

Proposition 4.3 (Case 1: without shutdown)**.** *In case of substitutes a horizontal merger between the market leader and the less efficient competitor yields a higher social surplus compared to a merger with the more efficient competitor if there is no shutdown at any kind of merger.*

Proof: Since social surplus is analyzed in the post-merger equilibrium it can be expressed as a function of marginal costs. The valid set of marginal cost combinations is characterized by assumption 4.2.1 and 4.2.2. In the following I prove $W^{12} \leq W^{13}$ by contradicting a necessary condition for $W^{12} > W^{13}$. It is easy to compute that the difference between W^{12} and W^{13} is given as follows:

$$W^{12} - W^{13} = \frac{\nu(c_2 - c_3)\Big\{-w_0 + w_1 c_1 - w_2(c_2 + c_3)\Big\}}{-16(1-\nu)[2 + (2-\nu)\nu]^2} \tag{4.1}$$

with $w_0 := 16 - 16\nu^2$, $w_1 := 16 + 28\nu + 12\nu^2 - 2\nu^3$ and $w_2 := 14\nu + 14\nu^2 - \nu^3$. Note that w_0, w_1 and w_2 are positive for $\nu \in (0, 1)$. The denominator of (4.1) is negative for $\nu \in (0, 1)$. Since $c_2 \leq c_3$ per assumption W^{12} exceeds W^{13} if the term in curly brackets is positive. In the following I solve $\{\cdot\} > 0 \Leftrightarrow W^{12} > W^{13}$ for c_1: $-w_0 + w_1 c_1 - w_2(c_2 + c_3) > 0 \Leftrightarrow -\frac{w_0}{w_2} + \frac{w_1}{w_2}c_1 - c_2 > c_3 \Rightarrow -\frac{w_0}{w_2} + \frac{w_1}{w_2}c_1 - c_2 > c_2 \Leftrightarrow -\frac{w_0}{2w_2} + \frac{w_1}{2w_2}c_1 > c_2 \Rightarrow -\frac{w_0}{2w_2} + \frac{w_1}{2w_2}c_1 > c_1 \Leftrightarrow -\frac{w_0}{2w_2} > \left(\frac{2w_2 - w_1}{2w_2}\right)c_1 \Leftrightarrow c_1 > -\frac{w_0}{2w_2 - w_1} = 1$ since $\frac{2w_2 - w_1}{2w_2} < 0$ for $\nu \in (0, 1)$. Therefore, $c_1 > 1$ is a necessary condition for

$W^{12} > W^{13}$. In case of substitutes (i.e. for $\nu > 0$) assumption 4.2.1 and 4.2.2 implies $0 \leq c_1 \leq c_2 \leq c_3 \leq 1$ which contradicts with $c_1 > 1$. Therefore, social surplus in case of a merger between firm 1 and 3 exceeds social surplus in case of a merger between firm 1 and 2. □

The intuition behind this result is the following: Generally, the insiders internalize the external effect of each insider's brand on joint profit and, therefore, reduce each quantity compared to the pre-merger level in case of substitutes. Consequently, the outsider increases his output level since quantities are strategic substitutes. In case of a merger between the market leader and the more efficient competitor the two most efficient firms reduce their output levels whereas the least efficient firm expands its output. Intuitively, this output reshuffling is detrimental with regard to Pareto optimality. It is easy to prove that in case of a merger between the market leader and the less efficient competitor equilibrium output of the market leader exceeds respective quantity in case of merger with the more efficient competitor. Furthermore, the output of the two most efficient firms 1 and 2 as non-cooperative players exceed respective output of 1 and 2 in case of insiders. Moreover, the least efficient firm 3 produces less as insider compared to respective output as outsider. Therefore, present antitrust enforcement policy is welfare enhancing since it fosters a more efficient supply with all brands if there is no shutdown at any kind of merger. In this case there is a negative relationship between the size of the merger measured by aggregated pre-merger market shares and efficiency of the market outcome.

In the following I consider case 2 characterized by the shutdown of the less efficient part of the merged entity irrespective the kind of merger. The post-merger equilibrium is given by a simple duopoly consisting of firm 1 and 3 (2) in case of a merger between 1 and 2 (3). Intuitively, a marginal cost combination corresponding with a shutdown of the more efficient competitor as merging partner involves the shutdown of the less efficient firm 3 as merging partner. However, a marginal cost combination corresponding with a shutdown of firm 3 does not imply the shutdown of firm 2 as merging partner.

At first sight one would expect that a duopoly consisting of the most efficient firms 1 and 2 yields a higher social surplus compared to a duopoly consisting of firm 1 and least efficient firm 3. However, as you can see in the following under some circumstances the exact opposite is true.

Proposition 4.4 (Case 2: shutdown in any kind of merger). *In case of a marginal cost combination satisfying* $\{(c_1, c_2, c_3)|q_j^{1j} = 0 \text{ for } j = 2, 3\}$ *a horizontal merger between the market leader and the less efficient competitor yields a less efficient market outcome compared to a merger between the market leader and the more efficient competitor if the following condition is satisfied:*[13]

$$c_2 > \frac{(2-\nu)^2(3+\nu)}{12-\nu^2} + \frac{(8\nu-\nu^3)}{12-\nu^2}c_1$$

Proof: It is easy to verify that $q_2^{12} = 0$ implies $q_3^{13} = 0$ whereas the reverse implication is not true. Therefore, in the following I consider marginal cost combinations satisfying $q_2^{12} = 0$. Let k_i for $i = 0, \dots, 3$ be defined as follows: $k_0 := 2[2 - \nu(3 - \nu)]$, $k_1 := \nu(4 - \nu)$, $k_2 := 4 - \nu^2$ and $k_3 := 2\nu(1 - \nu)$. I consider the set of marginal cost combinations satisfying $q_2^{12} \le 0$ which is equivalent to $k_0 + k_1c_1 - k_2c_2 + k_3c_3 \le 0$.

In case of a merger between 1 and 2 (3) the market outcome is a simple duopoly consisting of firm 1 and 3 (3) competing as Cournot players. In the following I derive conditions with respect to marginal costs corresponding with $W^{12}(q_1^{12}, 0, q_3^{12}) > W^{13}(q_1^{13}, q_2^{13}, 0)$. Let q_i^* for $i = 1, 2$ denote the equilibrium quantity of firm $i = 1, 2$ in case of a simple Cournot-Nash duopoly. Furthermore, let W^* denote social surplus in case of a duopoly with $W^* := q_1^* + q_2^* - \frac{1}{2}\left[(q_1^*)^2 + (q_2^*)^2\right] - \nu q_1^* q_2^* - c_1 q_1^* - c_2 q_2^*$. It holds:

$$\begin{aligned}
\frac{dW^*}{dc_2} &= \partial_2 q_1^* + \partial_2 q_2^* - q_1^* \partial q_1^* - q_2^* \partial_2 q_2^* - \nu\left(\partial_2 q_1^* q_2^* + q_1^* \partial_2 q_2^*\right) - c_1 \partial_2 q_1^* - q_2^* - c_2 \partial_2 q_2^* \\
&= \partial_2 q_1^*(1 - c_1) + \partial_2 q_2^*(1 - c_2) - q_1^*(\partial_2 q_1^* + \nu \partial_2 q_2^*) - q_2^*(\partial_2 q_2^* + \nu \partial_2 q_1^* + 1)
\end{aligned}$$

According to lemma 12 the equilibrium quantities are given by $q_i^* = \frac{(2-\nu)-2c_i+\nu c_j}{(2-\nu)(2+\nu)}$ for $i, j = 1, 2$ and $i \ne j$. Respective partial derivatives of equilibrium output are given by: $\partial_1 q_1^* = \partial_2 q_2^* = \frac{-2}{(2-\nu)(2+\nu)}$ and $\partial_2 q_1^* = \frac{\nu}{(2-\nu)(2+\nu)}$. It holds:

[13] As you can see in the following this is a sufficient condition.

$$\begin{aligned}\frac{dW^*}{dc_2} =& \frac{\nu(1-c_1)-2(1-c_2)}{(2-\nu)(2+\nu)} \\ &- \frac{(2-\nu)-2c_1+\nu c_2}{(2-\nu)(2+\nu)}\left[\frac{-\nu}{(2-\nu)(2+\nu)}\right] \\ &- \frac{(2-\nu)+\nu c_1-2c_2}{(2-\nu)(2+\nu)}\left[\frac{2}{(2-\nu)(2+\nu)}\right] \\ =& \frac{-12+8\nu+\nu^2-\nu^3-(8\nu-\nu^3)c_1+(12-\nu^2)c_2}{(4-\nu^2)^2}\end{aligned}$$

Solving $\frac{dW^*}{dc_2} > 0$ for c_2 yields:

$$c_2 > \frac{(2-\nu)^2(3+\nu)}{12-\nu^2} + \frac{(8\nu-\nu^3)}{12-\nu^2}c_1 \tag{4.2}$$

It is an immediate implication that two marginal cost combinations (c_1, c_2') and (c_1, c_2'') satisfying (4.2) are characterized by $W(c_1, c_2') < W(c_1, c_2'')$ for $c_2' < c_2''$. The condition (4.2) is not pathologic as it can be fulfilled for reasonable values of marginal costs. A non-negative equilibrium output $q_i^* \geq 0$ for $i = 1, 2$ is equivalent to $c_i \leq \frac{2-\nu}{2} + \frac{\nu}{2}c_j$ for $j = 1, 2$ and $j \neq i$. It is easy to prove that $\frac{(2-\nu)^2(3+\nu)}{12-\nu^2} < \frac{2-\nu}{2} \Leftrightarrow 2+\nu > 0$ and $\frac{8\nu-\nu^3}{12-\nu^2} > \frac{\nu}{2} \Leftrightarrow 4-\nu^2 > 0$. Furthermore, the right hand side of (4.2) equals 1 for $c_1 = 1$. Thus, for any $c_1 \in (0, 1)$ there exists a $c_2 \in (c_1, 1)$ with $q_i^* \geq 0$ for $i = 1, 2$ such that the condition (4.2) is fulfilled.

In the following I derive the smallest value for c_2 involving a shutdown of firm 2 and, therefore, implying the shutdown of firm 3 as insider, too. As you have seen above, $q_2^{12} \leq 0$ is equivalent to $k_0 + k_1c_1 - k_2c_2 + k_3c_3 \leq 0$. It is easy to verify that $q_3^{13} = 0$ is equivalent to $k_0 + k_1c_1 + k_3c_2 - k_2c_3 = 0$. Solving $q_2^{12} = 0$ and $q_3^{13} = 0$ for c_3 and equating to each other yields: $c_2 = \left(\frac{k_0}{k_2-k_3}\right) + \left(\frac{k_1}{k_2-k_3}\right)c_1 = \left[2 - \frac{2(2+\nu)}{4-\nu(2-\nu)}\right] + \frac{(4-\nu)\nu}{4-\nu(2-\nu)}c_1$. It is easy to prove that $\frac{(2-\nu)^2(3+\nu)}{12-\nu^2} > 2 - \frac{2(2+\nu)}{4-\nu(2-\nu)}$ and $\frac{(8\nu-\nu^3)}{12-\nu^2} < \frac{(4-\nu)\nu}{4-\nu(2-\nu)}$ for $\nu \in (0, 1)$. Therefore, a marginal cost combinations corresponding with a shutdown of firm 2 (3) in case of a merger with firm 2 (3) can satisfy (4.2) or not. Therefore, a merger between 1 and 2 with a subsequent shutdown of firm 2 implies a higher social surplus if the marginal cost combinations satisfies (4.2) even though $c_2 < c_3$. □

The intuition behind this result is the following: On the one hand the reduction[14] of the marginal cost of a single firm increases aggregated output and, therefore, is welfare enhancing. This can be verified by summing up all first order conditions $1 - 2q_i^* - \nu Q_{-i}^* = 0$ characterizing the Cournot-Nash equilibrium and solving for aggregated output Q^*. It holds: $Q^* = \frac{n(1-\overline{c})}{2+\nu(n-1)}$ whereas $\overline{c} := \frac{1}{n}\sum_i c_i$ denotes average marginal costs. On the other hand a reduction of marginal cost generates an output shift from the unaffected firms to the firm which reduces its marginal costs. If the firm which is about to reduce its marginal cost is less efficient compared to its competitor this output shift increases total costs of production and, therefore, decreases social surplus. Thus, there are two effects influencing social surplus in opposite direction. If the firm which reduces its marginal costs is sufficiently inefficient (compared to its competitors), the negative effect of this output shift outweighs the positive effect of increased aggregated output. This finding is in line with studies from several authors. In the homogenous good case the negative impact of a cost reduction on social surplus is analyzed by Lahiri et al. (1988), Zhao (2001) and Smythe et al. (2006), for instance. In the differentiated good context, Wang et al. (2007) have comparable findings.

Let us now turn towards the analysis of the third case. For better understanding and illustration the restrictions $q_i^* \geq 0 \Leftrightarrow 2-\nu-(2+\nu)c_i+\nu\sum_{j\neq i} c_j \geq 0$ for $i = 1, 2, 3$ ensuring a real pre-merger oligopoly are depicted in figure 4.1. The marginal cost of firm 1 are on the x-axis, those of firm 2 on the y-axis and, finally, the marginal cost of firm 3 on the z-axis. A marginal cost combination satisfying $q_i^* \geq 0$ for $i = 1, 2, 3$ must lie in the polyhedron constituted by the planes which are implicitly defined by $q_i^* = 0$ for $i = 1, 2, 3$.

Figure 4.2 contains the graphics already depicted in figure 4.1 complemented by the restriction $q_j^{1j} = 0$ for the potential insider $j = 2, 3$. Intuitively, the $(q_3^{13} = 0)$-plane lies below the $(q_3^* = 0)$-plane since the insiders internalize the external effect of their brands on joint profit and, therefore, reduce their output. Marginal cost combinations limited by the $(q_3^* = 0)$-plane, the $(q_3^{13} = 0)$-plane and the $(q_2^{13} = 0)$-plane are characterized by a shutdown of firm 3 as insider while firm

[14] Analytically, the impact of an alternative merging partner (which is differently efficient) on market outcome can be treated analogously to a reduction (or increase) of respective marginal cost.

2 as insider produces non-negative quantity. Marginal cost combinations restricted by the $(q_3^* = 0)$-plane, $(q_2^* = 0)$-plane, $(q_2^{12} = 0)$-plane and the $(q_3^{13} = 0)$-plane are characterized by shutdown of the less efficient part of the merged entity at any kind of merger. Marginal cost combinations restricted by the $(q_2^* = 0)$-plane, $(q_2^{12} = 0)$-plane and lying below the $(q_3^{13} = 0)$-plane are excluded per assumption $0 \leq c_1 \leq c_2 \leq c_3 \leq 1$. It is easy to verify that the intersection of the $(q_2^{12} = 0)$-plane and the $(q_3^{13} = 0)$-plane are characterized by $c_2 = c_3$.

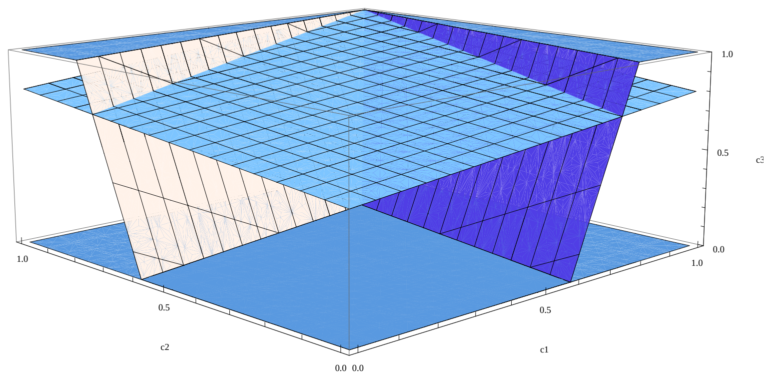

Figure 4.1: Restrictions $q_i^* = 0$ for $i = 1, 2, 3$ as implicit function of marginal cost with $\nu = \frac{1}{2}$.

The third case is characterized by the shutdown of firm 3 as insider whereas firm 2 as insider produces positive output. This case is characterized by different effects acting in opposite direction. This makes an analytical investigation of the welfare effects of present antitrust enforcement policy indispensable. On the one hand in case of a merger between 1 and 2 there are 3 brands in the market which is, apparently, welfare enhancing. On the other hand in case of a merger between 1 and 3 there are larger quantities of brand 1 and 2 since firm 1 and firm 2 behave as non-cooperative Cournot competitors. Moreover, the shutdown of a very inefficient firm can be welfare enhancing. Compare Lahiri and Ono (1988), Zhao (2001), Smythe

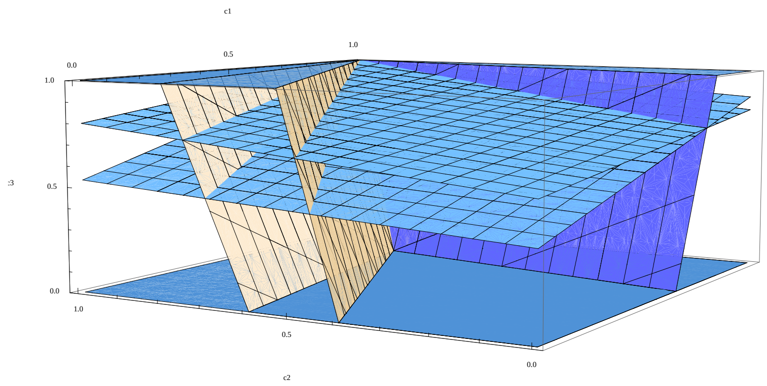

Figure 4.2: Restrictions $q_i^* = 0$ for $i = 1, 2, 3$ and $q_j^{1j} = 0$ for $j = 2, 3$ as implicit functions of marginal costs with $\nu = \frac{1}{2}$.

and Zhao (2006) or Wang and Zhao (2007). Nevertheless, the following analytical investigation yields clear-cut results.

Proposition 4.5 (Case 3 - provoked shutdown)**.** *In case of a marginal cost combination corresponding with $q_3^{13} = 0$ and $q_2^{12} \geq 0$ there is a shutdown of firm 3 as insider whereas firm 2 as insider produces non-negative output. In this case present antitrust enforcement policy provokes the shutdown of the less efficient part of the merged entity. However, the positive welfare effect of antitrust intervention can be traced back to this enforced shutdown.*

Proof: For simplicity I assume $c_1 = 0$ and $0 \leq c_2 \leq c_3 \leq 1$. The proof takes place in two steps: starting at the point characterized by the intersection of the $(q_3^{13} = 0)$-line and the $(q_2^{12} = 0)$-line, say point A, I 'walk along' the $(q_3^{13} = 0)$-line (i.e. I reduce c_2 and vary c_3 such that $q_3^{13} = 0$ is ensured) and show that W^{13} exceeds W^{12}. Since the $(q_3^{13} = 0)$-line is the lower boundary for c_3 of the considered set of marginal cost combinations I afterwards analyze the welfare impact of an increase of c_3 for an arbitrary marginal cost combination lying on the $(q_3^{13} = 0)$-line. Naturally, the market outcome and, therefore, social surplus in case of a merger between 1 and 3 is unchanged by an increase of c_3 starting from the $(q_3^{13} = 0)$-line. In case of a merger between 1 and 2 it can be shown that social surplus firstly decreases with c_3. As soon as c_3 reaches the threshold value characterizing $\frac{dW^{12}}{dc_3} = 0$, social surplus increases with c_3 which is referred to as 'welfare reducing cost reduction' phenomena. Naturally, c_3 is limited by the $(q_3^* \geq 0)$ condition ensuring a pre-merger oligopoly. If the threshold value characterizing $\frac{dW^{12}}{dc_3} = 0$ is more restrictive compared to the $(q_3^* = 0)$ condition W^{12} has a local maximum at the $(q_3^* = 0)$-line. In some cases, however, the $(q_3^* = 0)$-condition is more restrictive compared to the $\left(\frac{dW^{13}}{dc_3} = 0\right)$-condition. Finally, I show that social surplus on the $(q_3^* = 0)$-line (equal to the upper boundary of c_3) in case of a merger between 1 and 2 is always smaller compared to social surplus in case of a merger between 1 and 3 on the $(q_3^{13} = 0)$-line.

Step 1: According to the proof of proposition 4.4, the point A is given by $(c_1, c_2, c_3) = \left(0, \frac{k_0}{k_2-k_3}, \frac{k_0}{k_2-k_3}\right)$ and characterized by $W^{12} = W^{13}$ since $c_2 = c_3$. Starting from A, I consider $W\big(0, c_2, c_3(c_2)\big)$ and decrease c_2 with $c_3(c_2)$ such that $q_3^{13} = 0$. According to lemma 13 the post-merger equilibrium quantity of firm 3 as insider is given as follows: $q_3^{13} = \frac{1}{4}\left(\frac{-c_3}{1-\nu} + \frac{2[2-c_3-\nu(1-c_2)]}{2+\nu(2-\nu)}\right)$. According to the proof of lemma 4.4, let us define: $k_0 := 2[2-\nu(3-\nu)]$, $k_1 := \nu(4-\nu)$, $k_2 := 4-\nu^2$ and $k_3 := 2\nu(1-\nu)$. Solving $q_3^{13} = 0$ for c_3 yields: $c_3 = \frac{k_0}{k_2} + \frac{k_3}{k_2}c_2$. It is easy to compute that $W^{13}(0, c_2, c_3(c_2))$ exceeds $W^{12}(0, c_2, c_3(c_2))$ with $c_3(c_2) = \frac{k_0}{k_2} + \frac{k_3}{k_2}c_2$ for $c_2 \in \left(\frac{k_0}{k_2-k_3}, 0\right]$ and $\nu \in (0, 1)$.

Social surplus W^{12} (dashed lines) and W^{13} (solid lines) as a function of c_2 with $c_3(c_2)|_{q_3^{13}=0}$ are depicted in figure 4.3 for different values of substitution. Note that W^{12} and W^{13} is evaluated for $c_2 \in \left(0, \frac{k_0}{k_2-k_3}\right]$. Intuitively, the upper limit $\frac{k_0}{k_2-k_3}$ for c_2 decreases with ν. In case of independent goods (i.e. for $\nu = 0$) there are no externalities the insiders have to internalize. Therefore, the upper limit for c_2 equals the maximum willingness to pay for brand 3 (i.e. $\frac{k_0}{k_2-k_3} = 1$ for $\nu = 0$).

Step 2: In the next step I consider an increase of c_3 starting from an arbitrary point, let say B, on the $(q_3^{13} = 0)$-line. According to the first part of this proof, an arbitrary point B on the $(q_3^{13} = 0)$-plane is characterized by $W^{13}(q_1^{13}, q_2^{13}, 0) \geq W^{12}(q_1^{12}, q_2^{12}, q_3^{12})$. It is self-evident that $W^{13}(q_1^{13}, q_2^{13}, 0)$ is unchanged by an increase of c_3 since $q_3^{13} = 0$ for all considered values of c_3.

The impact on W^{12}, however, is ambiguous. Intuitively, an increase of c_3 reduces the outsider´s output q_3^{12} but increases the insiders´ output q_1^{12} and q_2^{12}. The outsider always profits and, therefore, the pre-merger oligopoly assumption $q_3^* \geq 0$ is the

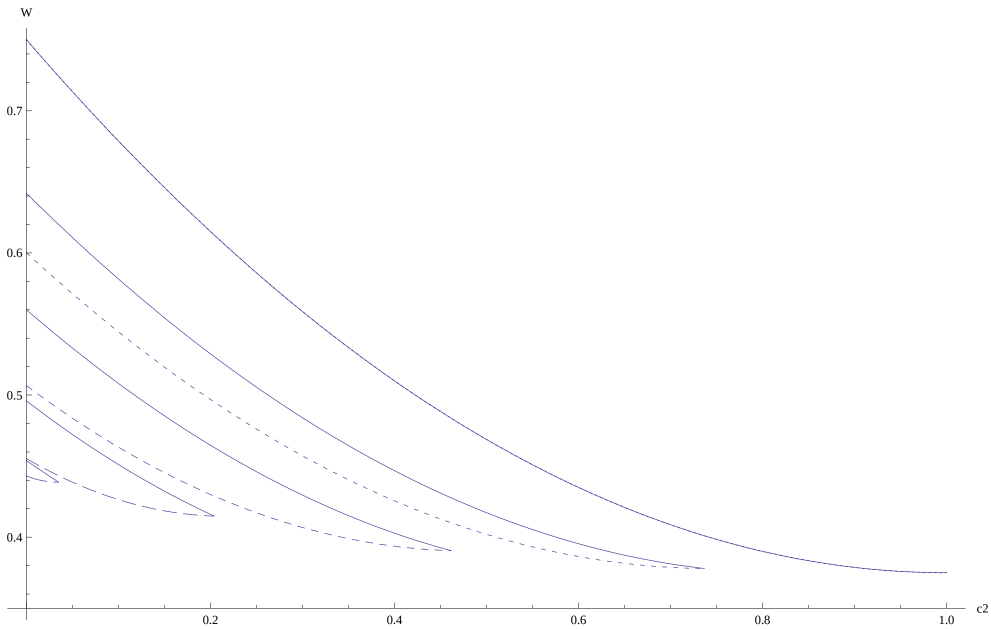

Figure 4.3: W^{12} (dashed lines) and W^{13} (solid lines) for $c_1 = 0$ and $c_3(c_2)$ such that $q_3^{13} = 0$ for $\nu \in \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{95}{100}\right\}$ top down.

relevant restriction (upper bound) for c_3. Compare proposition 4.1. According to assumption 4.2.2 the condition $q_3^* = 0$ for $n = 3$ and $c_1 = 0$ is given as follows:

$$c_3 \leq \frac{2-\nu}{2+\nu} + \frac{\nu}{2+\nu} c_2 \tag{4.3}$$

It can be shown that in some cases the threshold value c_3' characterizing $\frac{dW^{12}}{dc_3} = 0$ is smaller than the threshold value given by the right hand side of (4.3). Otherwise the proof would be finished at this point since W^{13} exceeds W^{12} on the $(q_3^{13} = 0)$-line, W^{13} is unchanged by an increase of c_3 and W^{12} decreases with c_3 until c_3' is reached. The threshold value c_3' can be obtained by solving $\frac{dW\left(q_1^{12}, q_2^{12}, q_3^{12}\right)}{dc_3} \leq 0$ for c_3 which is given as follows:

$$c_3' \leq c_2 + \frac{2-c_2}{1+\nu} + \frac{2(2-c_2)\nu}{6+(6-\nu)\nu} - 1 \tag{4.4}$$

Both conditions given by (4.3) and (4.4) are identical for $\nu = \sqrt{\frac{2}{3}}$. The condition $q_3^* \geq 0$ given by (4.3) is more restrictive in case of $\nu > \sqrt{\frac{2}{3}}$. In this case a merger between 1 and 3 with $q_3^{13} = 0$ is more efficient compared to a merger between 1 and 2. In case of $\nu < \sqrt{\frac{2}{3}}$, social surplus in case of a merger between 1 and 2 decreases with c_3 until c_3' is reached and afterwards increases with c_3. Therefore, the pivotal question is whether social surplus W^{12} at the $(q_3^* = 0)$-line achieves or even exceeds the level of W^{12} at the $(q_3^{13} = 0)$-line. Note that for $\nu < \sqrt{\frac{2}{3}}$, social surplus W^{12} exhibits a relative maximum at the $(q_3^* = 0)$-line. It is easy to compute that the following holds true for $c_2 \in \left[0, \frac{k_0}{k_2 - k_3}\right]$ with $A = \frac{k_0}{k_2 - k_3}$:

$$W^{13}\left(0, c_2, c_3(c_2)\Big|_{q_3^{13}=0}\right) \geq W^{12}\left(0, c_2, c_3(c_2)\Big|_{q_3^{*}=0}\right)$$

Therefore, social surplus in case of a merger between 1 and 3 exceeds social surplus in case of a merger between 1 and 3 for $\{(0, c_2, c_3) | q_3^{13} = 0 \wedge q_2^{12} > 0\}$ even though there are only 2 brands produced. $\Box$

The intuition behind these results is the following: Generally, the insiders internalize the external effect of each insider's brand on joint profit and, therefore, reduce the post-merger quantity of their brands compared to the pre-merger levels in case of substitutes. During the course of this output reduction firm 3 as insider is closed down whereas it would produce positive output as non-cooperative Cournot competitor. Thus, in case of a merger between 1 and 3 there is a shutdown of the least efficient firm 3. The more efficient firms 1 and 2, however, remain as Cournot competitors. The post-merger output of firm 1 and 2 as non-cooperative Cournot players exceeds respective post-merger output of 1 and 2 as insiders. Since $c_1 \leq c_2 \leq c_3$ large quantities of brand 1 and 2 can be produced fairly efficient. In case of a merger between 1 and 2 the most efficient firms reduce their output and the least efficient firm expands its output. Both effects are welfare reducing. Therefore, social surplus in case of a merger between the market leader and the less efficient competitor with a subsequent shutdown of firm 3 exceeds social surplus in case of a merger between the market leader and the less efficient competitor.

In summary present antitrust enforcement policy is welfare enhancing in most cases. However, the sole exception is characterized by a marginal cost combination which exhibits fairly big marginal cost of firm 2 and 3 compared to marginal costs of the market leader. In this case the approval of any horizontal merger is unlikely since the pre-merger equilibrium is already fairly concentrated.

At first glance this finding is amazing since merger guidelines base on false assumptions concerning the relationship between market concentration and efficiency of the market outcome according to chapter 2 of present thesis. After a closer look, however, the results become intuitive especially in the light of chapter 2. According to present merger guidelines the likelihood of an approval decreases with aggregated market share of merger candidates. Therefore, a small merger instead of a big merger can be considered as the consequence of present antitrust policy. A small merger, however, increases the disparity of the different brands compared to the pre-merger levels since the insiders reduce their output levels whereas the outsider increases the output level. In case of a big merger the market leader and the more efficient competitor reduces their output levels whereas the least efficient firm increases its output level as outsider. The decreased disparity of the different brands normally decreases market concentration. This effect, however, is covered by the reduction of number of firms which increases market concentration. In case of a small merger the reverse holds true. Taking into account the fact that a small merger is consequence of a rejected big merger or firms request a small merger from outset present antitrust enforcement policy increases the disparity of the different brands at the end.

4.5 Conclusions

The objective of antitrust authorities such as the Federal Trade Commission (FTC) or the Directorate-General for Competition (DG-Comp) is the prohibition of horizontal mergers which are likely to create or strengthen a dominant position. Distorted competition is expected to be the consequence of the abuse of market arising from a dominant position. A high market concentration and a big aggregated market share of the merger candidates, respectively, are indirect indicators for market power. However, the second chapter of present thesis shows that this assumption is false since there is a positive relationship between market concentration and consumer surplus as well as producer surplus. Nevertheless, it can be shown that present antitrust enforcement policy is welfare enhancing in the majority of cases even though underlying merger guidelines base on false assumptions. Present antitrust enforcement policy fosters a more uneven distribution of aggregated output on the different brands. Thus, the findings are in line with those of chapter 2 since output of brands with low marginal costs increases whereas output of brands with high marginal costs is reduced. Paradoxically, present antitrust enforcement policy increases the disparity of output levels even though the opposite is intended. Remember that a horizontal merger is unlikely to be approved if it results in a highly concentrated market. In a special case antitrust policy provokes the shutdown of the less efficient part of the merged entity and reduces the variety of goods. The positive welfare effect, however, can be traced back to this shutdown since a fairly inefficient firm is removed from the market. Therefore, my findings are in line with those of Kao and Menezes (2007), for instance.

An even distribution of market shares or a maximum of product diversity are indicators of the strength of competition. However, neither of them assures an efficient market outcome. A certain degree of market concentration is the natural result of the market process in case of differently efficient firms. A deviation from the Cournot-Nash equilibrium caused by horizontal mergers, for instance, increases social surplus if the disparity of output levels is increased. This may imply the removal of a product with an inferior production technology. Thus, the existence of fixed costs is not a precondition for firm's exit to be welfare enhancing. At the end present merger guidelines are welfare enhancing although the reasons differ from what is expected.

4.6 Appendix

4.6.1 Proof of lemma 13

In case of a horizontal merger between 1 and 2 the insider maximize $\Pi_1+\Pi_2 = (p_1 - c_1)q_1 + (p_2 - c_2)q_2$ with respect to q_1 and q_2. The outsider maximizes $\Pi_3 = (p_3 - c_3)q_3$ with respect to q_3. In matrix form the first order conditions can be expressed as follows:

$$\begin{pmatrix} 2 & 2\nu & \nu \\ 2\nu & 2 & \nu \\ \nu & \nu & 2 \end{pmatrix} \boldsymbol{q} = \begin{pmatrix} 1-c_1 \\ 1-c_2 \\ 1-c_3 \end{pmatrix}$$

From Cramer follows:

$$q_1^{12} = \frac{\det \begin{pmatrix} 1-c_1 & 2\nu & \nu \\ 1-c_2 & 2 & \nu \\ 1-c_3 & \nu & 2 \end{pmatrix}}{\det \begin{pmatrix} 2 & 2\nu & \nu \\ 2\nu & 2 & \nu \\ \nu & \nu & 2 \end{pmatrix}}$$

Expanding the above given determinant along the last row yields:

$$\begin{aligned} q_1^{12} &= \frac{1}{4}\left[\frac{(1-c_1)(4-\nu^2)-(1-c_2)(4\nu-\nu^2)+(1-c_3)(2\nu^2-2\nu)}{2-3\nu^2+\nu^3}\right] \\ &= \frac{1}{4}\left[\frac{(4-6\nu+2\nu^2)-(4-\nu^2)c_1+(4\nu-\nu^2)c_2-(2\nu^2-2\nu)c_3}{2-3\nu^2+\nu^3}\right] \\ &= \frac{1}{4}\left[\frac{[2+\nu(2-\nu)](c_2-c_1)+2(1-\nu)[2-c_1-c_2-\nu(1-c_3)]}{(1-\nu)(2+2\nu-\nu^2)}\right] \\ &= \frac{1}{4}\left(\frac{c_2-c_1}{1-\nu}+\frac{2[2-c_1-c_2-\nu(1-c_3)]}{2+\nu(2-\nu)}\right) \end{aligned}$$

Analogously the post-merger equilibrium quantity of brand 2 can be obtained:

$$q_2^{12} = \frac{\det\begin{pmatrix} 2 & 1-c_1 & \nu \\ 2\nu & 1-c_2 & \nu \\ \nu & 1-c_3 & 2 \end{pmatrix}}{\det\begin{pmatrix} 2 & 2\nu & \nu \\ 2\nu & 2 & \nu \\ \nu & \nu & 2 \end{pmatrix}}$$

$$= \frac{1}{4}\left[\frac{(4-6\nu+2\nu^2)+(4\nu-\nu^2)c_1-(4-\nu^2)c_2-(2\nu^2-2\nu)c_3}{2-3\nu^2+\nu^3}\right]$$

$$= \frac{1}{4}\left[\frac{[2+\nu(2-\nu)](c_1-c_2)+2(1-\nu)[2-c_1-c_2-\nu(1-c_3)]}{(1-\nu)(2+2\nu-\nu^2)}\right]$$

$$= \frac{1}{4}\left(\frac{c_1-c_2}{1-\nu}+\frac{2[2-c_1-c_2-\nu(1-c_3)]}{2+\nu(2-\nu)}\right)$$

Finally, the post-merger equilibrium output of the outsider is given as follows:

$$q_3^{12} = \frac{\det\begin{pmatrix} 2 & 2\nu & 1-c_1 \\ 2\nu & 2 & 1-c_2 \\ \nu & \nu & 1-c_3 \end{pmatrix}}{\det\begin{pmatrix} 2 & 2\nu & \nu \\ 2\nu & 2 & \nu \\ \nu & \nu & 2 \end{pmatrix}}$$

$$= \frac{(1-c_1)(2\nu^2-2\nu)-(1-c_2)(2\nu-2\nu^2)+(1-c_3)(4-4\nu^2)}{4(2-3\nu^2+\nu^3)}$$

$$= \frac{2(1-\nu^2)-\nu(1-\nu)(2-c_1-c_2)-2(1-\nu^2)c_3}{2(1-\nu)[2+\nu(2-\nu)]}$$

$$= \frac{2[1-(1+\nu)c_3]+\nu(c_1+c_2)}{2[2+\nu(2-\nu)]} \qquad \square$$

4.6.2 Proof of lemma 4.2

Let denote $\Pi_{1/j}$ for $j = 2, 3$ the post-merger joint profit in case of a merger with firm j. It is easy to compute that the difference between $\Pi_{1/2}$ and $\Pi_{1/3}$ is given as follows:[15]

[15] Provided that all post-merger quantities given by lemma 13 are positive. Otherwise, a simple duopoly is the market outcome. I will come to this later.

$$\Pi_{1/2} - \Pi_{1/3} = \frac{(c_2 - c_3)\Big\{k_0(\nu) + k_1(\nu)c_1 - k_2(\nu)(c_2 + c_3)\Big\}}{-8(1-\nu)[2+\nu(2-\nu)]^2} \tag{4.5}$$

The coefficients in the numerator of (4.5) are given as follows: $k_0 = 16 + 8\nu - 24\nu^2 - 8\nu^3 + 8\nu^4$, $k_1 = 8\nu + 8\nu^2 + 2\nu^4$ and $k_2 = 8 + 8\nu - 8\nu^2 - 4\nu^3 + 5\nu^4$ with $k_i > 0$ for $i = 1, 2, 3$ and $\nu \in (0, 1)$. The denominator in (4.5) is negative for $\nu \in (0, 1)$. Since $c_2 < c_3$ per assumption, $\Pi_{1/2}$ exceeds $\Pi_{1/3}$ if the term in curly brackets is non-negative. Solving $\{\cdot\} \geq 0$ for $c_2 + c_3$ yields:

$$c_2 + c_3 \leq \frac{k_0}{k_2} + \frac{k_1}{k_2}c_1 \tag{4.6}$$

Let $\overline{c}_q(c_1) := \frac{k_0}{k_2} + \frac{k_1}{k_2}c_1$ denote the threshold value for the sum of c_2 and c_3 given by the right hand side of (4.6). It is easy to check that $\frac{k_0}{k_2}(\nu)$ has the following properties: $\frac{k_0}{k_2}(0) = 2$, $\frac{k_0}{k_2}(1) = 0$ and $\frac{\partial}{\partial \nu}\left(\frac{k_0}{k_2}\right) < 0$ for $\nu \in (0, 1)$. It holds: $\frac{k_1}{k_2}(0) = 0$, $\frac{k_1}{k_2}(1) = 2$ and $\frac{\partial}{\partial \nu}\left(\frac{k_1}{k_2}\right) > 0$ for $\nu \in (0, 1)$. Finally, $\overline{c}_q(1) = \frac{k_0}{k_2} + \frac{k_1}{k_2} = 2$. The threshold values for different degrees of substitution intersects at $(1, 2)$ in the $(c_1, c_2 + c_3)$-coordinate system.

According to assumption 4.2.1 a marginal cost combination must satisfy $0 < c_1 < c_2 < c_3 < 1$. Summing up both inequalities $c_1 < c_2$ and $c_1 < c_3$ yields:

$$c_2 + c_3 > 2c_1 \tag{4.7}$$

The condition (4.7) and the threshold value $\overline{c}_q$ given by (4.6) are depicted in figure 4.4 for $\nu \in \{0, 0.3, 0.5, 0.7, 0.9, 1\}$. The slope of the threshold value $\overline{c}_q$ (dashed lines) increases with the degree of substitutability ν. Note that for $\nu = 1$ both the dashed line and the thick line are identical. The thick line illustrates the condition (4.7). Hence a marginal costs combination (c_1, c_2, c_3) satisfying $\Pi_{1/2} > \Pi_{1/3}$ must lie above the thick line and below the corresponding dashed line.

Figure 1 suggests that a marginal cost combination lying above the threshold value $\overline{c}_q$ (dashed line) leads to a merger with firm 3. But, it can be shown that such a marginal cost combination (i.e. not satisfying (4.6)) involves the shutdown of brand 3 in case of a merger between 1 and 3. In case of a shutdown of the

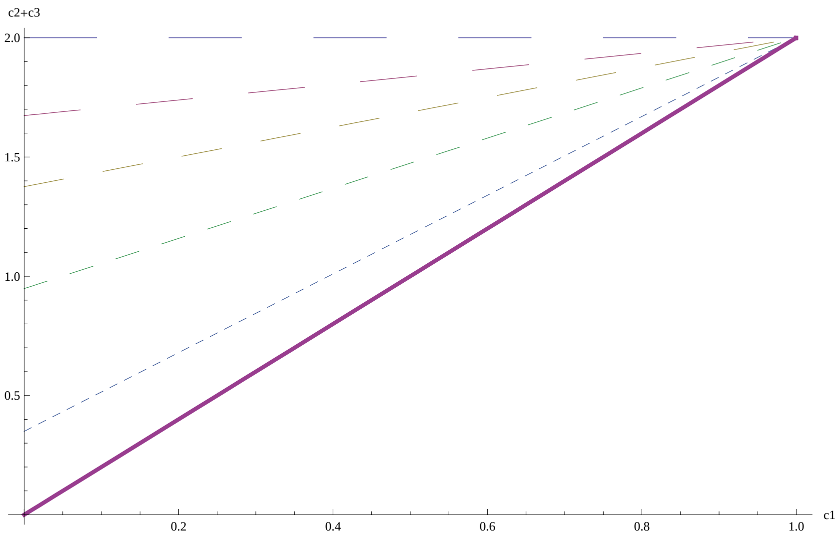

Figure 4.4: The threshold value $\overline{c}^q$ (dashed lines) and (4.7) (solid line).

acquired brand firm 1 always prefers a merger with 2 instead of a merger with firm 3 since the remaining competitor (i.e. firm 3) is least efficient and, therefore, an inferior rival. Equation (4.6) is depicted as the dashed line in figure 4.4. This line characterizes the shutdown of the acquired brand (former firm 2) in case of a merger between firm 1 and firm 2.

In the following I show that a marginal cost combination not satisfying (4.6) involves the shutdown of brand 3 in case of a merger between firm 1 and 3. The post-merger equilibrium quantity of firm 3 as insider is given as follows:

$$q_3^{13} = \frac{1}{4}\left(\frac{c_1 - c_3}{1-\nu} + \frac{2[2 - c_1 - c_3 - \nu(1-c_2)]}{2+\nu(2-\nu)}\right)$$

Let us define $z_0 := 4 - 6\nu + 2\nu^2$, $z_1 := 4\nu - \nu^2$, $z_2 := 2\nu - 2\nu^2$ and $z_3 := 4 - \nu^2$. It is easy to verify that $q_3^{13} \geq 0$ requires $z_0 + z_1 c_1 + z_2 c_2 - z_3 c_3 \geq 0$ which can be expressed as follows:

$$c_3 \leq \frac{z_0}{z_3} + \frac{z_1}{z_3}c_1 + \frac{z_2}{z_3}c_2 \tag{4.8}$$

The condition (4.6) is equivalent to the following inequality:

$$c_3 > \frac{k_0}{k_2} + \frac{k_1}{k_2}c_1 - c_2 \tag{4.9}$$

In the following I show that the set of (c_1, c_2, c_3) satisfying the inequalities given by (4.8), (4.9) and $0 \leq c_1 \leq c_2 \leq c_3 < 1$ is an empty set. The threshold values for c_3 given by the right hand side of (4.8) and (4.9), respectively, are just (two-dimensional) hyperplanes in the three-dimensional (c_1, c_2, c_3)-coordinate system whereas the marginal costs of firm 1 is the x-axis, c_2 is the y-axis and finally c_3 is the z-axis. Since $\frac{k_0}{k_2} \geq \frac{z_0}{z_3} > 0$ for $\nu \in (0,1)$ the intersection with the c_3-axis of (4.9) exceeds the intersection of (4.8). Furthermore since $\frac{k_1}{k_2} \geq \frac{z_1}{z_3} > 0$ ($\frac{z_2}{z_3} \geq 0$) the partial derivative of the (4.9)-hyperplane in direction of the c_1-axis (c_2-axis) exceeds the respective partial derivative of the (4.8)-hyperplane. In the following I show that the intersection of both hyperplanes is outside the polyhedron characterized by the basic assumption $0 \leq c_1 \leq c_2 \leq c_3 < 1$. Note that the intersection of both hyperplanes is just a one-dimensional hyperplane (i.e. a straight line). Equating (4.8) with (4.9) and solving for c_2 yields:

$$\begin{aligned}
\frac{z_0}{z_3} + \frac{z_1}{z_3}c_1 + \frac{z_2}{z_3}c_2 &= \frac{k_0}{k_2} + \frac{k_1}{k_2}c_1 - c_2 \\
\left(\frac{z_2}{z_3} + 1\right)c_2 &= \left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right) + \left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)c_1 \\
c_2(c_1) &= \left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right)\left(\frac{z_3}{z_2 + z_3}\right) + \left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2 + z_3}\right)c_1 \qquad (4.10)
\end{aligned}$$

Note that the sign of the constant term in (4.10) as well as the sign of the factor of c_1 are positive for $\nu \in (0,1)$. It cannot be shown that (4.10) implies $c_2(c_1) \not> c_1$ which would contradict with the central assumption $0 \leq c_1 \leq c_2 \leq c_3 < 1$. The solution for c_3 as a function of c_1 can be derived by inserting $c_2(c_1)$ given by (4.10) in (4.9):

$$\begin{aligned}
c_3(c_1) &= \frac{k_0}{k_2} + \frac{k_1}{k_2}c_1 - c_2(c_1) \\
&= \frac{k_0}{k_2} + \frac{k_1}{k_2}c_1 - \left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right)\left(\frac{z_3}{z_2 + z_3}\right) - \left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2 + z_3}\right)c_1 \\
&= \frac{k_0}{k_2} - \left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right)\left(\frac{z_3}{z_2 + z_3}\right) + \left[\frac{k_1}{k_2} - \left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2 + z_3}\right)\right]c_1 \qquad (4.11)
\end{aligned}$$

In the following I show that $c_3(c_1)$ given by (4.11) does not exceed $c_2(c_1)$ given by (4.10) for $\nu \in (0,1)$. The inequality $c_3(c_1) > c_2(c_1)$ is given by:

$$\frac{k_0}{k_2} - \left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right) + \left[\frac{k_1}{k_2} - \left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right)\right] c_1$$
$$> \left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right) + \left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right) c_1$$

which is equivalent to

$$\left[\frac{k_1}{k_2} - 2\left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right)\right] c_1 > 2\left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right) - \frac{k_0}{k_2}$$
$$\Leftrightarrow c_1 > \frac{2\left(\frac{k_0}{k_2} - \frac{z_0}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right) - \frac{k_0}{k_2}}{\frac{k_1}{k_2} - 2\left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right)} = 1$$

since $\frac{k_1}{k_2} - 2\left(\frac{k_1}{k_2} - \frac{z_1}{z_3}\right)\left(\frac{z_3}{z_2+z_3}\right) > 0$ for $\nu \in (0,1)$. Obviously $c_1 > 1$ contradicts with the assumption $0 \le c_1 \le c_2 \le c_3 < 1$. Hence the one-dimensional hyperplane given by the intersection of both two-dimensional hyperplanes does not intersect the polyhedron characterized by $0 \le c_1 \le c_2 \le c_3 < 1$. Therefore, the set of marginal costs combinations satisfying (4.8), (4.9) and $0 \le c_1 \le c_2 \le c_3 < 1$ is an empty set. A shutdown of brand 3 is be the consequence if there is a merger between firm 1 and firm 3 if the marginal costs combination does not satisfy (4.6).

If there is a shutdown of the less efficient part of the merged entity joint profit in case of a merger with firm 2 exceeds the joint profit if there is a merger with firm 3. If there is a shutdown of the less efficient part of the merged entity a simple duopoly is the consequence. Profits of the remaining firms are given as follows:

$$\Pi_{12}^{cd2} = (1 - q_1 - \nu q_3 - c_1)q_1$$
$$\Pi_3^{cd2} = (1 - \nu q_1 - q_3 - c_3)q_3$$

According to lemma (12) the corresponding Cournot-Nash equilibrium is given as follows:

$$q_1^{cd2} = \frac{2 - \nu - 2c_1 + \nu c_3}{4 - \nu^2}$$
$$q_3^{cd2} = \frac{2 - \nu + \nu c_1 - 2c_3}{4 - \nu^2}$$

The equilibrium quantity of firm 1 in case of a merger with firm 3 (and subsequent shutdown of the acquired brand) is given by $q_1^{cd3} = \frac{2-\nu-2c_1+\nu c_2}{4-\nu^2}$. Since the equilibrium profit is given by squared equilibrium output it is easy to verify that firm 1 realizes a bigger profit with firm 3 as rival instead of firm 2 as rival for $c_2 \leq c_3$. $\square$

Chapter 5

Concluding Remarks

The thesis at hand sheds new light on the relationship between market concentration and social surplus. Analyzing an asymmetric Cournot oligopoly in a differentiated good context it fills a gap in existing theoretical analysis. It can be shown that the positive relationship between market concentration and social surplus applies to differentiated goods, too. In contrast to the homogeneous good case not only producer surplus but also consumer surplus increases with market heterogeneity.

Antitrust authorities presume that firm's market power increases with its size. This presumption is true since big firms actually realize bigger profits compared to smaller firms. But, the exercise of market power does neither harm consumers nor causes welfare losses. Market power of big firms reinforces a more uneven and, therefore, efficient distribution of total production on the different firms. Firms with a more efficient and, therefore, superior production technology have bigger quantities compared to less efficient firms. However, big firms not only produce higher quantities but sell their good at a lower price than their competitors. The consequence of this aggressive competitive conduct is that social surplus as well as consumer surplus increases with market heterogeneity. Thus, there is no conflict of objectives between firm's (non-coordinated) profit maximizing behavior and the aim of antitrust authorities: the maximization of social surplus and consumer surplus, respectively.

These insights contradict the structure-conduct-performance paradigm. The Harvard School argues that a highly concentrated market is a bad starting point for the market process since the market structure determines the competitive conduct. Distorted competition with associated welfare losses are presumed to be consequence. My analysis rather supports the Chicago point of view that market structure is a result of market process. The distribution of market shares and concentration is just the reflection of the underlying cost structure.

In principle, present merger guidelines allow for the fact that horizontal mergers are a part of the natural market process. In the sense of Stigler's (1950) seminal study merger and acquisitions are an appropriate remedy for a firm to expand and diversify its products range. The aim of antitrust authorities to promote an efficient market outcome is correct. Even though the interdependencies between market concentration and efficiency are misunderstood, the remedies of antitrust policy recorded in present merger guidelines are proper. Present merger regulation tends to increase the disparity of output levels. Under some circumstances the exit of an already small firm is the consequence. At first sight the removal of a brand which comes along with a growth of already big firms seems to be detrimental at least for consumers. However, neither a maximum of product diversity nor an even distribution of quantity levels is necessarily welfare enhancing as shown in chapter 4. The existence of fixed costs is not necessary for this result. The positive relationship between competitive pressure and efficiency of the market outcome in context of the (free-entry) Marshall equilibrium in case of a symmetric homogeneous Cournot oligopoly with constant marginal cost without fixed cost is rather an exception than the normal case. The removal of a good with an inferior production technology is also part of the natural market process as the market launch of new products. The crowding out of no longer contemporary products is rather a kind of natural shake-out than a manifestation of distorted competition.

Bibliography

[1] Athey, S. and A. Schmutzler (2001): Investment and Market Dominance, *RAND Journal of Economics*, 32, 1-16

[2] Audretsch, D.B., W.J. Baumol and A.E. Burke (2001): Competition policy in dynamic markets, *International Journal of Industrial Organization*, 19, 613-634

[3] Baumol, W.J., J. Panzar and D. Willig (1988): Contestable Markets and the Theory of Industry Structure, Harcourt Brace Jovanovich, New York

[4] Belleflamme, P. and C. Vergari (2006): Incentives to innovate in Oligopolies, *Core Discussion Paper*, 2006/-8

[5] Bergstrom, T.C. and H.R. Varian (1985): Two Remarks on Cournot Equilibria, *Economics Letters*, 19, 5-8

[6] Bester, H. and E. Petrakis (1993): The incentives for cost reduction in a differentiated industry, *International Journal of Industrial Organization*, 11, 519-534

[7] Boone, J. (2001): Intensity of Competition and the Incentive to Innovate, International Journal of Industrial Organization, 19, 705-726

[8] Carlton, D.M. and J.M. Perloff (2005): Modern Industrial Organization, Pearson, Addison Wesley, Boston

[9] Cheung, F.K. (1992): Two Remarks on Equilibrium Analysis of Horizontal Mergers, *Economics Letters*, 40, 119-123

[10] Creane A. and C. Davidson (2004): Multidivisional Firms, Internal competition and the Merger Paradox, *Canadian Journal of Economics*, 37, 951-977

[11] Deneckere, R. and C. Davidson (1985): Incentives to form coalitions with Bertrand Competition, *Rand Journal of Economics*, 16, 473-86

[12] Dixit, A (1979): A Model of Duopoly Sugggesting a Theory of Entry Barriers, *Journal of Economics*, 10, 20-32

[13] Dixit, A. and N. Stern (1982): Oligopoly and Welfare: A unified Presentation and Application to Trade and Development, *European Economic Review*, 19, 123-143

[14] Farrell, J. and C. Shapiro (1990): Horizontal Mergers: an Equilibrium Analysis, *American Economic Review*, 80, 107-126

[15] Farrell, J. and C. Shapiro (1991): Horizontal Mergers: Reply, *American Economic Review*, 81, 1007-1011

[16] Fauli-Oller, R. (1997): On Merger Profitability in a Cournot Setting, *Economics Letters*, 54, 75-79

[17] Février, P. and L. Linnemer (2004): Idiosyncratic Shocks in an Asymmetric Cournot Oligopoly, *International Journal of Industrial Organization*, 22, 835-848

[18] Gaudet, G. and S.W. Salant (1991): Increasing the profits of a subset of firms in oligopoly models with strategic substitutes, *American Economic Review*, 81, 658-665

[19] Häckner, J. (2000): A Note on Price and Quantity Competition in Differentiated Oligopolies, *Journal of Economic Theory*, 93, 233-239

[20] Heywood, J.S. and M. McGinty (2007): Convex costs and the merger paradox revisited, *Economic Inquiry*, 45, 342-349

[21] Huck, S., K.A. Konrad and W. Müller (2001): Profitable Horiontal Mergers without cost advantages: the role of internal organization, information, and market structure, *CESifo Working Papers*, No. 435

[22] Jänich, K. (2002): Lineare Algebra, 9. Auflage, Springer Verlag, Berlin

[23] Kamien, M.I. and I. Zang (1990): The Limits of Monopolization through Acquisition, *Quarterly Journal of Economics*, 105, 465-499

[24] Kamien, M.I. and I. Zang (1991): Competitively Cost Advantageous Mergers and Monopolization, *Games and Economic Behavior*, 3, 323-338

[25] Kao T. and F. Menezes (2007): Welfare Enancing Mergers under Product Differentiation, *Discussion Paper No. 350*, School of Economics, The university of Queensland

[26] Kimmel, S. (1992): Effects of Cost Changes on Oligopolists´ Profits, *Journal of Industrial Economics*, 40, 441-449

[27] Kleer, R. (2006): The Effect of Mergers on the Incentive to Invest in Cost Reducing Innovations, *BGPE Discussion Paper*, No. 11

[28] Koh, W. (2008): Market competition, social welfare in an entry-constrained differentiated-good oligopoly, *Economics Letters*, 100, 229-233

[29] Königsberger, K. (1993): Analysis 2, Springer Verlag, Berlin

[30] Kreps, D. and J. Scheinkman (1983), Quantity Precommitment and Bertrand Competition yield Cournot Outcomes, *Bell Journal of Economics*, 14, 326-337

[31] Lahiri, S. and Y. Ono (1988): Helping Minor Firms Reduces Welfare, *The Economic Journal*, 98, 1199-1202

[32] Levin, D. (1990): Horizontal Mergers: The 50-Percent Benchmark, *American Economic Review*, 80, 1238-1245

[33] Levitan, R. and M. Shubik (1972): Price Duopoly and Capacity Constraints, *International Economic Review*, 13, 111-122

[34] Lyons, B. (2008): An Economic Assessment of EC Merger Control: 1958-2007, *CCP Working Paper*, 08-17

[35] Mialon, S.H. (2008): Efficient horizontal mergers: The effects of internal capital reallocation and organizational form, *International Journal of Industrial Organization*, 26, 861-877

[36] Motta, M. (2004): Competition Policy: Theory and Practice, Cambridge University Press, New York

[37] Neven, D., Nutall, R. and P. Seabright (1993): Merger in Daylight, CEPR

[38] Perry, M.K. and R.H. Porter (1985): Oligopoly and the incentive for horizontal merger, *American Economic Review*, 75, 219-227

[39] Prechel, H., J. Boies and T. Woods (1999): Debt, mergers and acquisitions, institutional arrangemnts and change to the multilayered subsidiary form, *Social Sciences Quarterly*, 80, 115-135

[40] Sacco, D. (2008): Is there a U-shaped Relation between Competition and Investment?, *Working Paper* No. 0808, University of Zurich

[41] Salant, S.W. and G. Shaffer (1999): Unequal Treatment of Identical Agents in Cournot Equilibrium, *Amercican Economic Review*, 89, 585-604

[42] Salant, S.W., S. Switzer and R.J. Reynolds (1983): Losses from Horizontal Merger: the Effects of an Exogenous Change in Industry Strucutre on Cournot-Nash Equilibrium, *Quarterly Journal of Economics*, 98, 185-199

[43] Schulz, N. (2003), Wettbewerbspolitik, Mohr Siebeck, Tübingen

[44] Seade, J. (1985): Profitable Cost Increases and the Shifting of Taxation: Equilibrium Response of Markets in Oligopoly, *The Warwick Economic Research Paper Series*, 260

[45] Shubik, M. (1980): Market Structure and Behavior, Havard University Press, Cambridge

[46] Singh, N. and X. Vives (1984): Price and quantity competition in a differentiated duopoly, *Rand Journal of Economics*, 15, 546-554

[47] Smythe, D.J. and J. Zhao (2006): The Complete Welfare Effects of Cost Reductions in a Cournot Oligopoly, *Journal of Economics*, 87, 181-193

[48] Stigler, G.J. (1950): Monopoly and oligopoly by merger, *American Economic Review*, 40, 23-34

[49] Symeonidis, G. (2003): Quality heterogeneity and welfare, *Economics Letters*, 78 (1), 1-7

[50] US Merger Guidelines (1997)
URL: http://www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html, 18.08.2009

[51] van Long, N. and A. Soubeyran (2001): Cost manipulation games in oligopoly, with cost of manipulating, *International Economic Review*, 42 (2), 505-533

[52] Vives, X. (2004): Innovation and competitive pressure, *CEPR Discussion paper*, No. 4369

[53] Wang, H. and J. Zhao (2007): Welfare reductions from small cost reductions in differentiated oligopoly, *International Journal of Industrial Organization*, 25, 173-185

[54] Werden, G.J. (1991): Horizontal mergers: Comment, *American Economic Review*, 81, 1002-1006

[55] Whinston, M.D. (2006): Antitrust Policy Toward Horizontal Mergers, Handbook of Industrial Organization, Vol. 3

[56] Zanchettin, P. (2006): Differentiated Duopoly with Asymmetric Costs, *Journal of Economics and Management Strategy*, 15, 999-1015

[57] Zhao, J (2001): A Characterization for the Negative Welfare Effects of Cost Reduction in Cournot Oligopoly, *International Journal of Industrial Organization*, 19, 455-469

Lebenslauf

Persönliche Angaben

Name	Jens Jurgan
Geburtsdatum	12. April 1977
Geburtsort	Bad Mergentheim
Staatsangehörigkeit	Deutsch
Familienstand	ledig

Ausbildung / Tätigkeiten

2005-2007	**Incentives – Bavarian Graduate Program in Economics** Internationales Doktorandenkolleg des Elitenetzwerks Bayern
Seit 2004	**Julius-Maximilians-Universität, Würzburg** Wissenschaftlicher Mitarbeiter am Lehrstuhl für Volkswirtschaftslehre, insb. Industrieökonomik. (Prof. Norbert Schulz, Ph.D.)
2003-2004	**Julius-Maximilians-Universität, Würzburg** Wissenschaftlicher Mitarbeiter am Lehrstuhl für Volkswirtschaftslehre, insb. Wirtschaftspolitik (Prof. Dr. Hans G. Monissen)
2003	**Julius-Maximilians-Universität, Würzburg** Abschluss als Diplom Volkswirt
2000-2003	**Julius-Maximilians-Universität, Würzburg** Studium der Volkswirtschaftslehre
1997-2000	**Julius-Maximilians-Universität, Würzburg** Studium der Betriebswirtschaftslehre
1996-1997	**Bundeswehr** Grundwehrdienst
1996	**Deutschorden-Gymnasium Bad Mergentheim** Allgemeine Hochschulreife